Nuclear Science Computing Center at CCNU

Search for the QCD Critical Endpoint at a Temperature of 108 MeV

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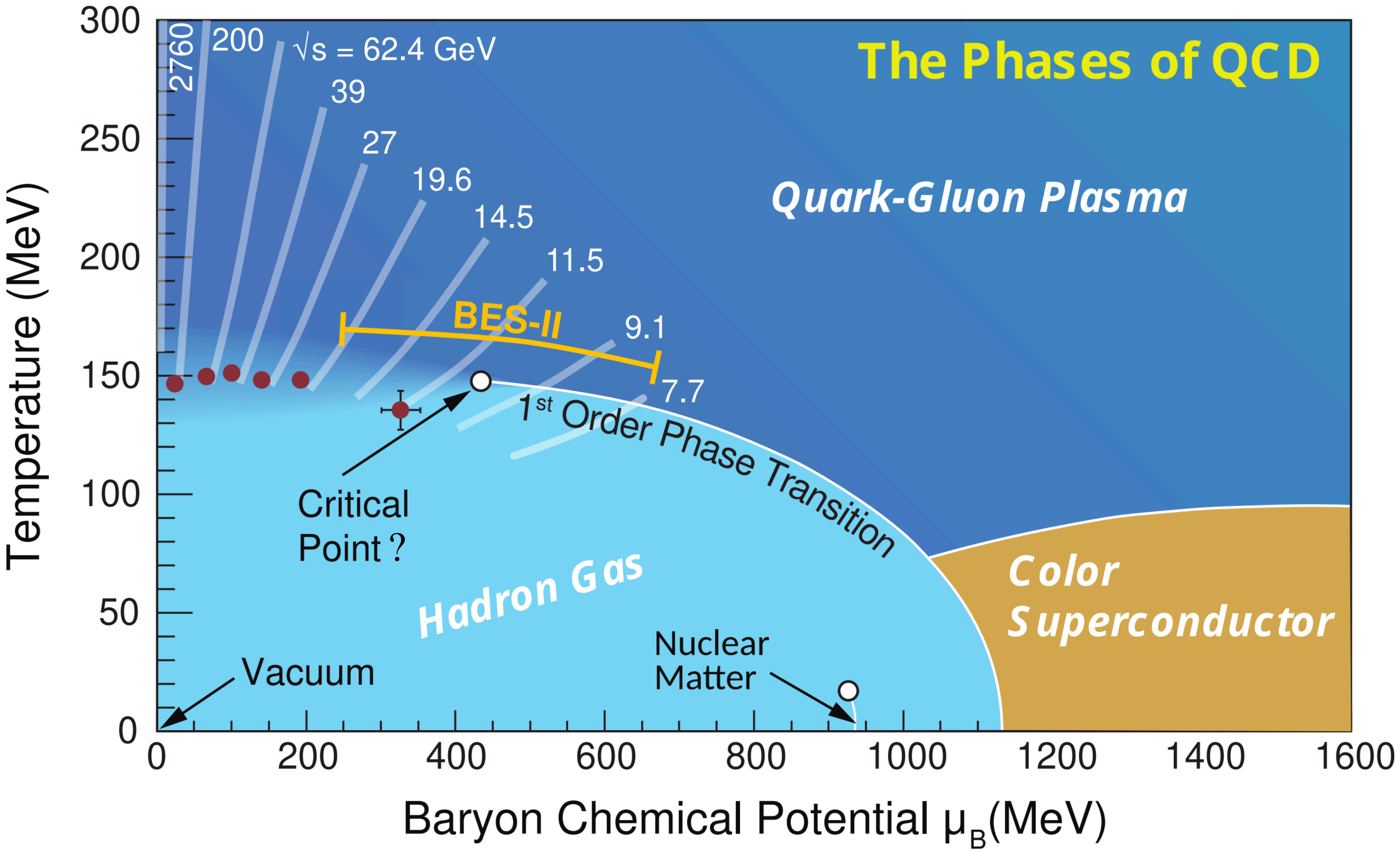
QCD phase diagram



A. Bzdak et al., arxiv:1906.00936

$\mu_B = 0$: (Lattice QCD)

- A rapid but smooth crossover

Y. Aoki et al., Nature 443 (2006) 675
A. Bazavov et al., [HotQCD], PLB 795 (2019) 15
S. Borsanyi et al., PRL 125 (2020) 052001

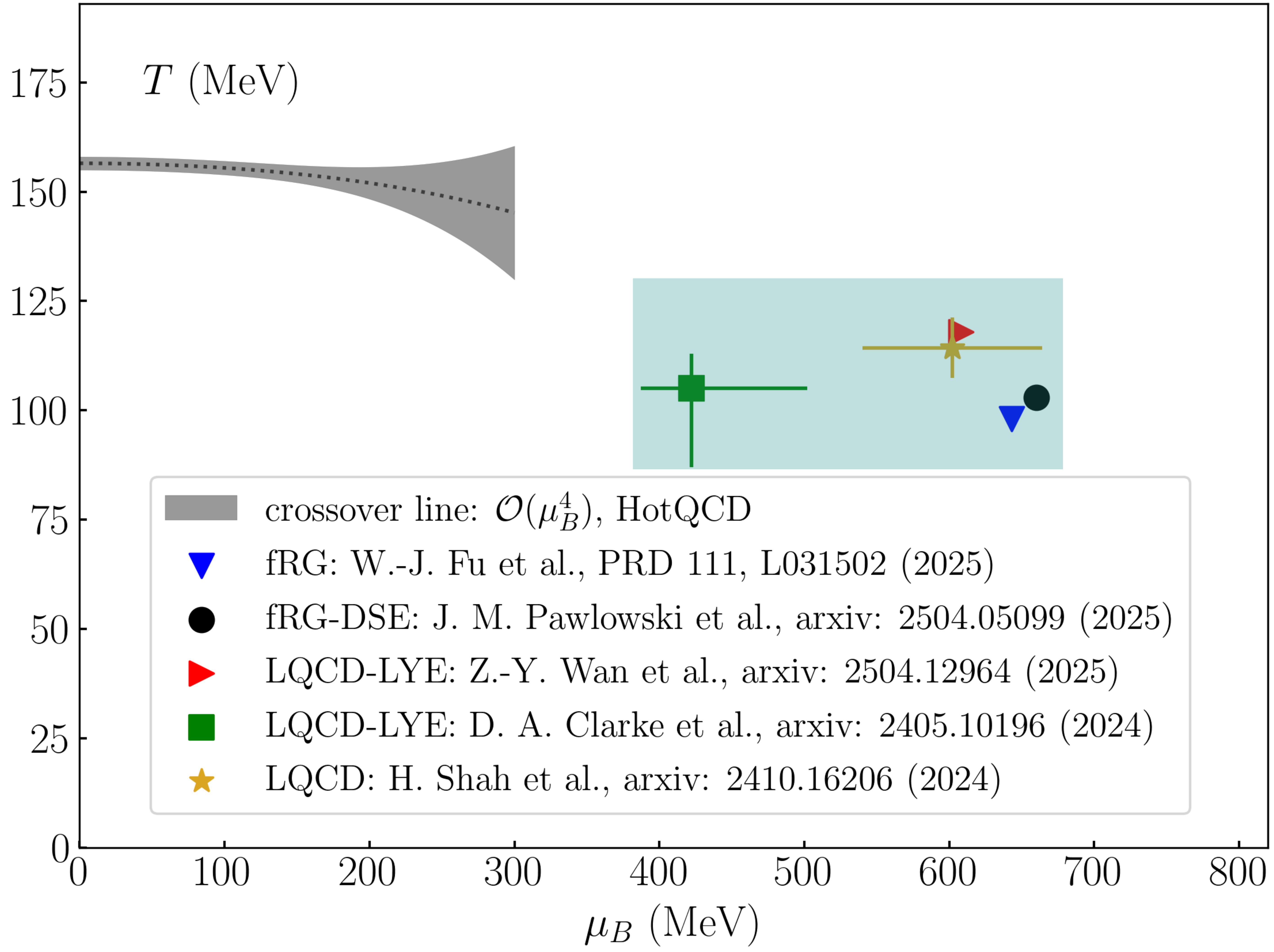
Large μ_B , low T (model approach, not robust)

- A first-order phase transition line

M.A. Stephanov, Prog. Theor. Phys. Suppl. 153 (2004) 139

Where is the location of CEP?

Recent results on the CEP search



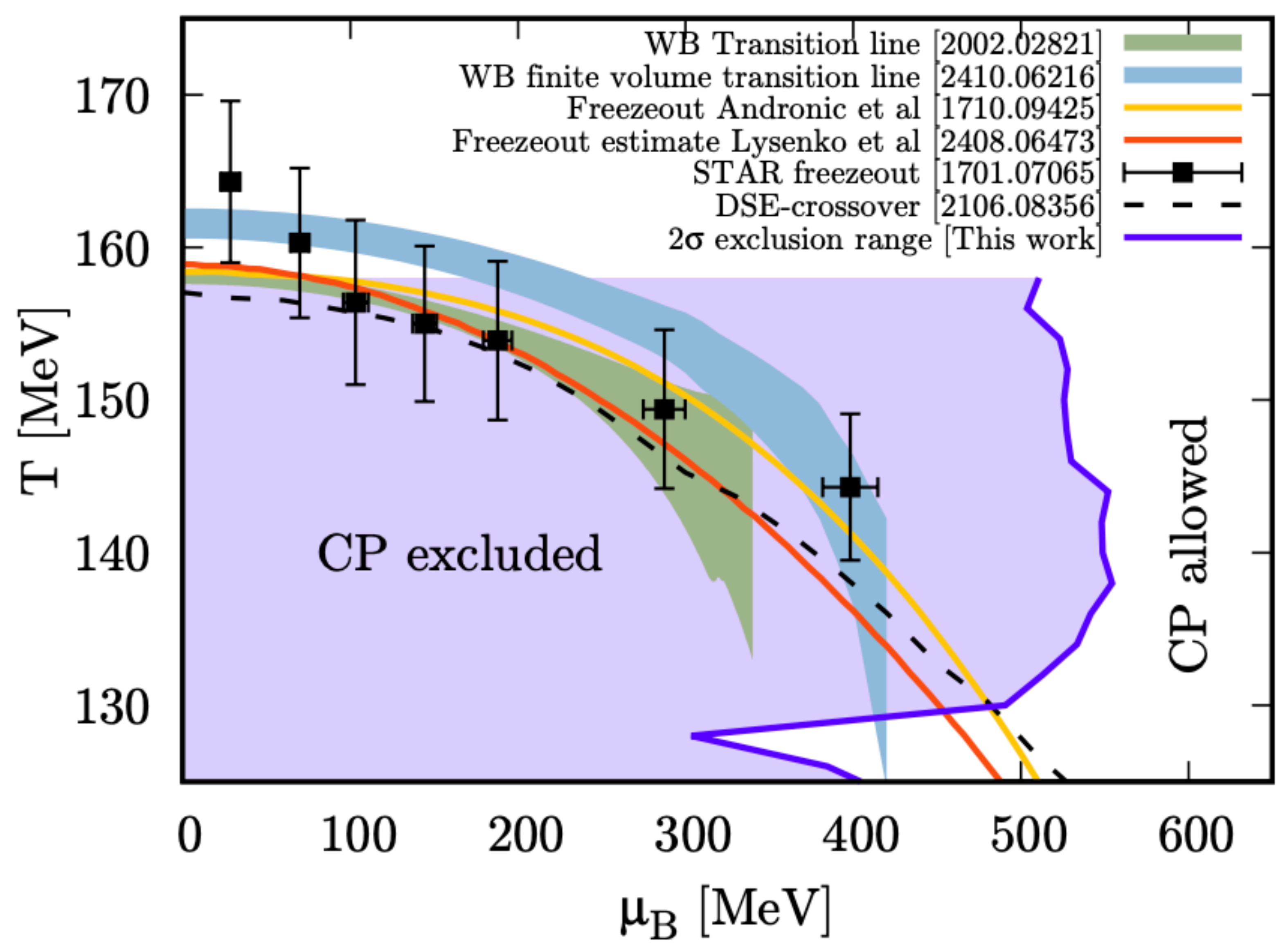
Locations of CEP as predictions:

- $T_c^{\rm CEP} \approx 83 - 120\,\mathrm{MeV}$
- $\mu_{B,c}^{\rm CEP} \approx 387 - 664\,\mathrm{MeV}$

Or CEP does not exist

Recent results on the CEP search in lattice QCD

Contour of constant entropy



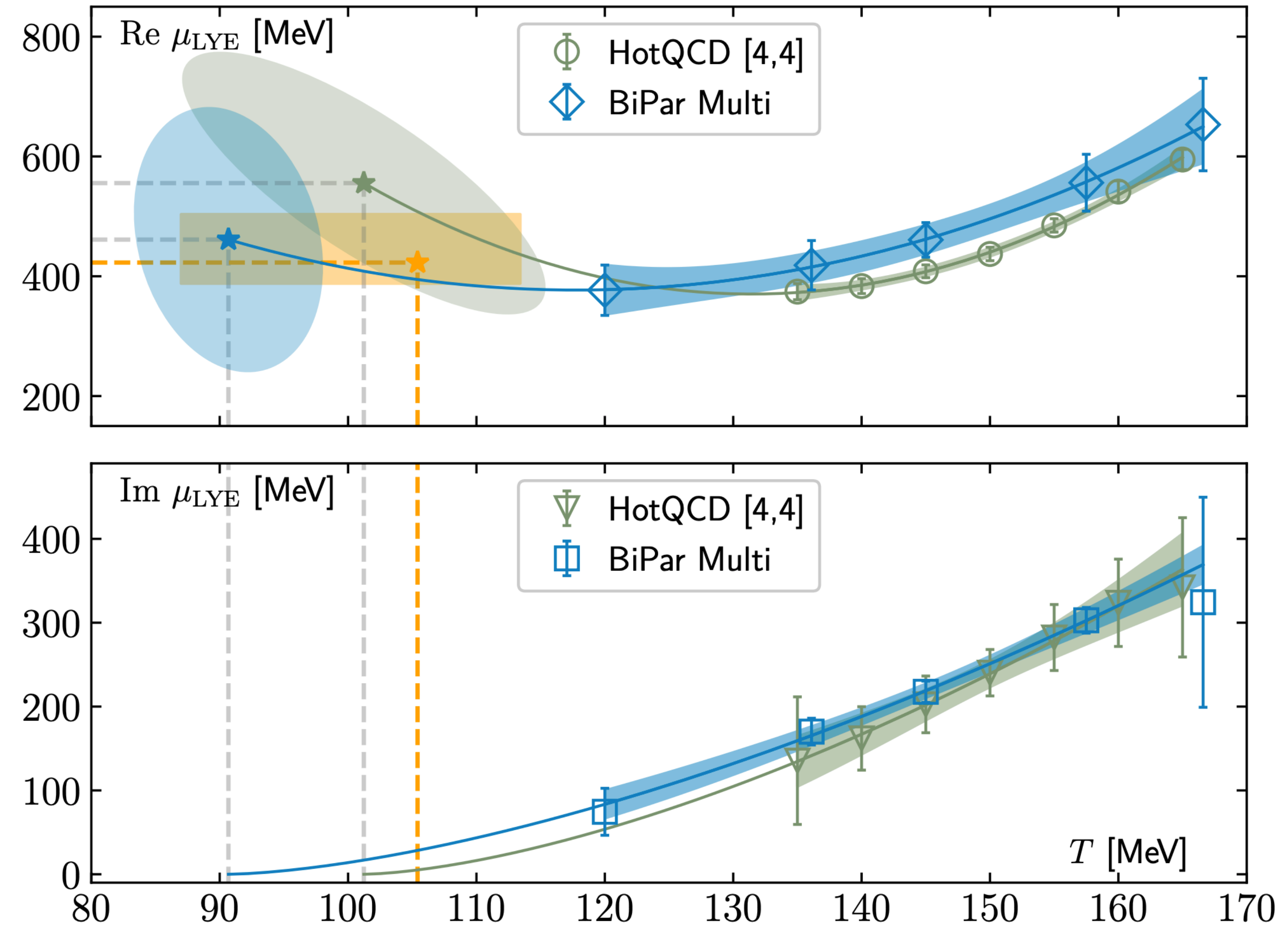
No CEP at $\mu_B < 450$ MeV at the 2σ level

Borsányi et al., arXiv:2502.10267

$$T^{\mathrm{CEP}} = 114.3 \pm 6.9\ \mathrm{MeV}$$
$$\mu_B^{\mathrm{CEP}} = 602.1 \pm 62.1\ \mathrm{MeV}$$

Shah et al., arXiv:2410.16206

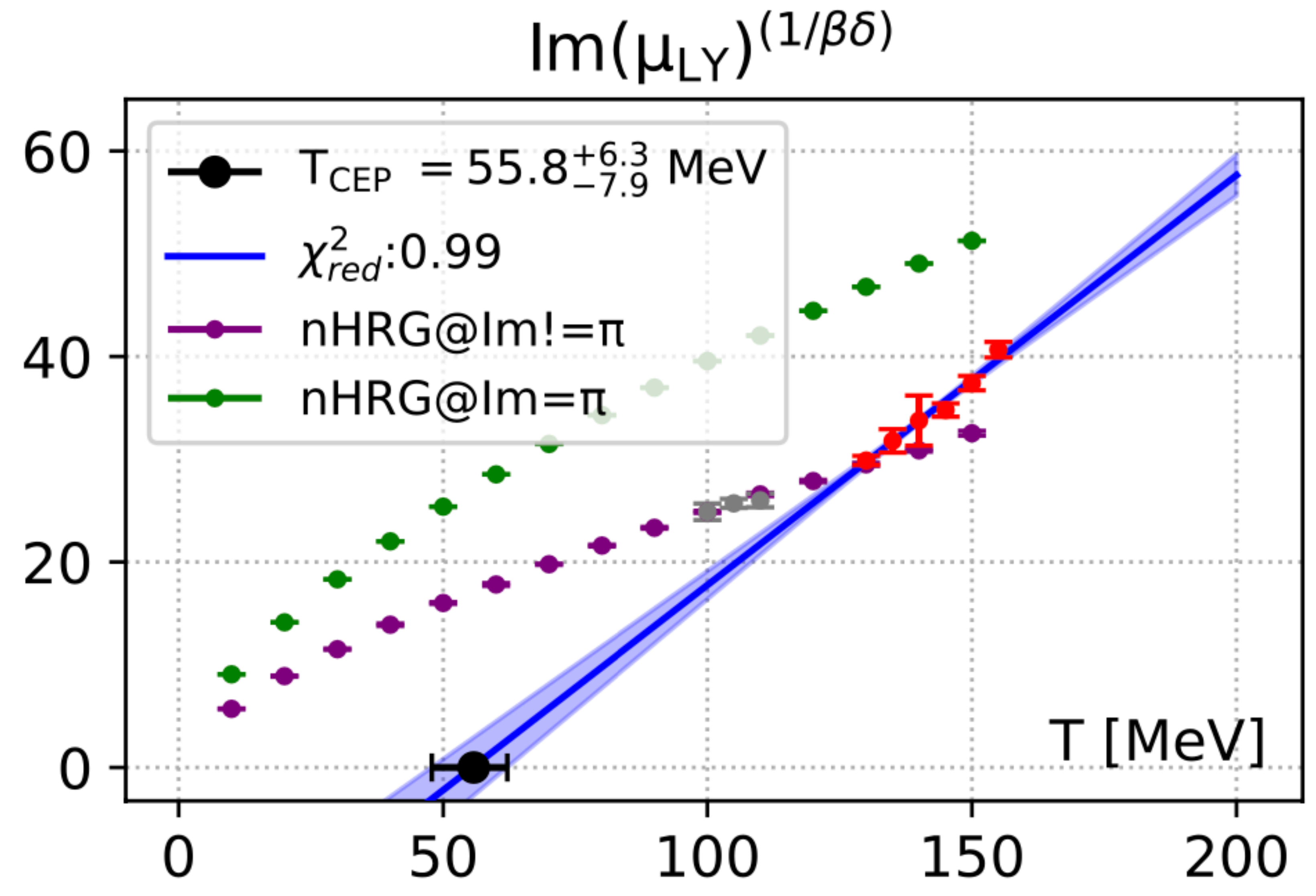
Lee-Yang edge singularity



$$T^{\mathrm{CEP}} = 105^{+8}_{-18}\ \mathrm{MeV}$$
$$\mu_B^{\mathrm{CEP}} = 422^{+80}_{-35}\ \mathrm{MeV}$$

Bielefeld-Parma, arXiv:2405.10196

Lee-Yang edge singularity



$T^{\mathrm{CEP}} < 103$ MeV
with 84% probability or
CEP does not exist

Adam et al., arXiv:2507.13254

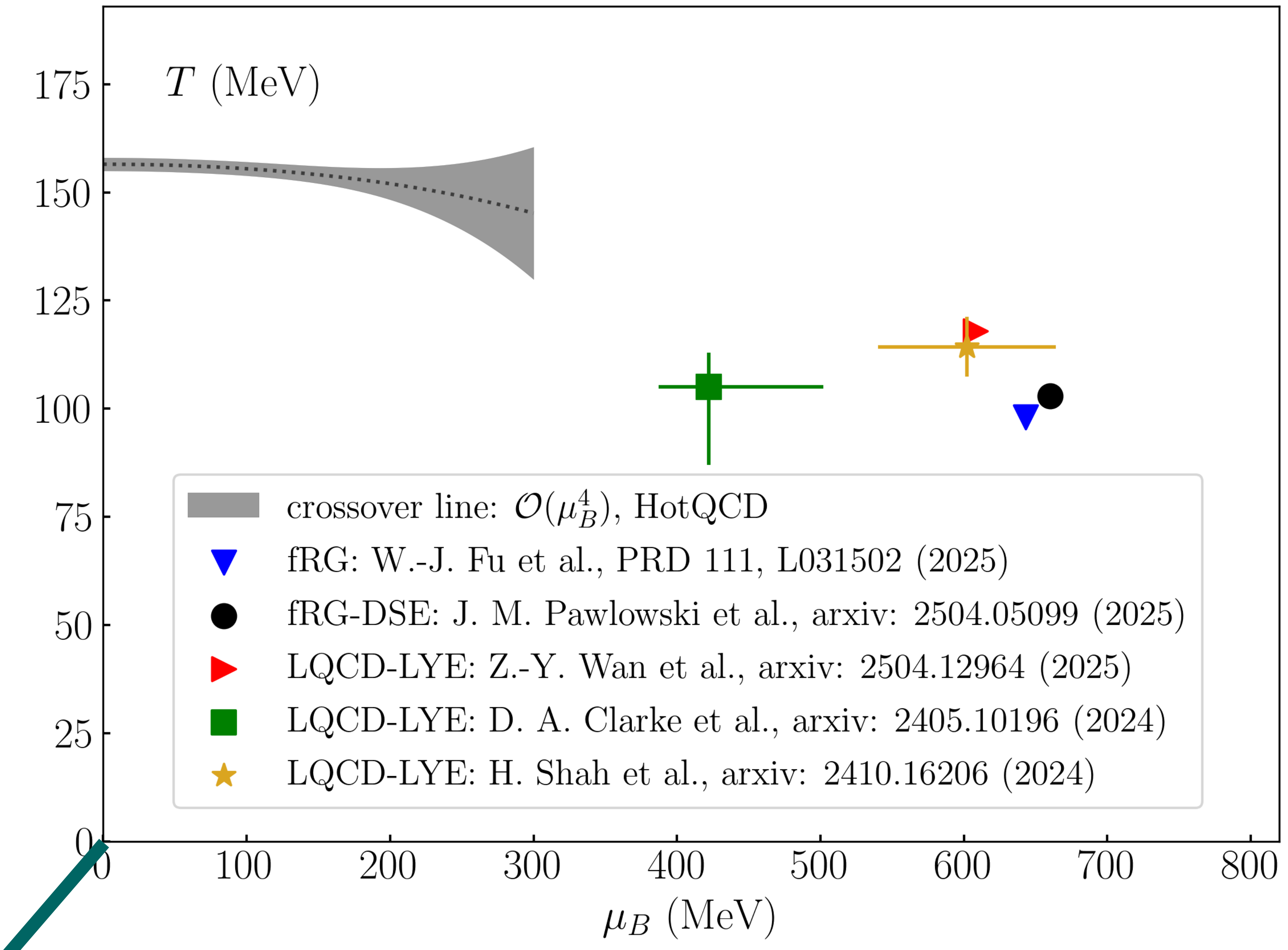
Recent results on the CEP search in lattice QCD

- $T^{\mathrm{CEP}} = 114.3 \pm 6.9$ MeV
- $T^{\mathrm{CEP}} = 105^{+8}_{-18}$ MeV
- $T^{\mathrm{CEP}} < 103$ MeV w. 84% prob. or no CEP

Current LQCD simulations at T ≥ 120 MeV

$T_c^{\mathrm{CEP}} \approx 83 - 120$ MeV

How about perform a direct LQCD simulations in the currently predicted T^{CEP} region? E.g. 108 MeV



Im μ_B

Simulation along the imaginary μ_B

HotQCD, PRD 108 (2023) 014510
P. Dimopoulos, et al., PRD 105 (2022) 034513
S. Mukherjee, et al., PRD 105 (2022) 014026

Lattice Setup at $T \approx 108$ MeV

- **Temperature:** 107.7 MeV
- **Lattice sizes:** $20^3 \times 10$
- **Action:** 2+1 flavor HISQ fermions and a tree-level improved Symanzik gauge action
- **Quark mass:** $m_s^{\rm phy}/m_l = 27 \to M_\pi \approx 135\,\mathrm{MeV},\ l = \{u, d\}$
- **Partition function:** $Z(T, V, \mu_l, \mu_s) = \int \mathscr{D}U\, \mathrm{e}^{-S_g} \left[\det M\left(m_l, i\mu_l^I\right)\right]^{\frac{1}{2}} \left[\det M\left(m_s, i\mu_s^I\right)\right]^{\frac{1}{4}},\ \mu_l = \mu_s \to \mu_B = 3\mu_q$
- **Imaginary μ_B^I window**: 24 μ_B^I points with $\mu_B^I/T \in [0, \pi]$
- **Statistics:** ~400k configurations for zero chemical potential, ~230k configurations for the other μ_B^I point.
- Additional $32^3 \times 10$ lattices generated to check the volume dependence

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Observables: baryon number cumulants

Pressure: $P/T^4 \propto \dfrac{\log Z(T, V, \mu_f)}{VT^3}$

Cumulants: $\chi_n^B(T, V, i\hat{\mu}_B^I) = \dfrac{\partial^n}{\partial(i\hat{\mu}_B^I)^n}\dfrac{P}{T^4}$

Real/Imaginary Observables

Z and P are real functions.

$\chi_1^B, \chi_3^B, \cdots$(odd) are pure imaginary.

$\chi_2^B, \chi_4^B, \cdots$(even) are real.

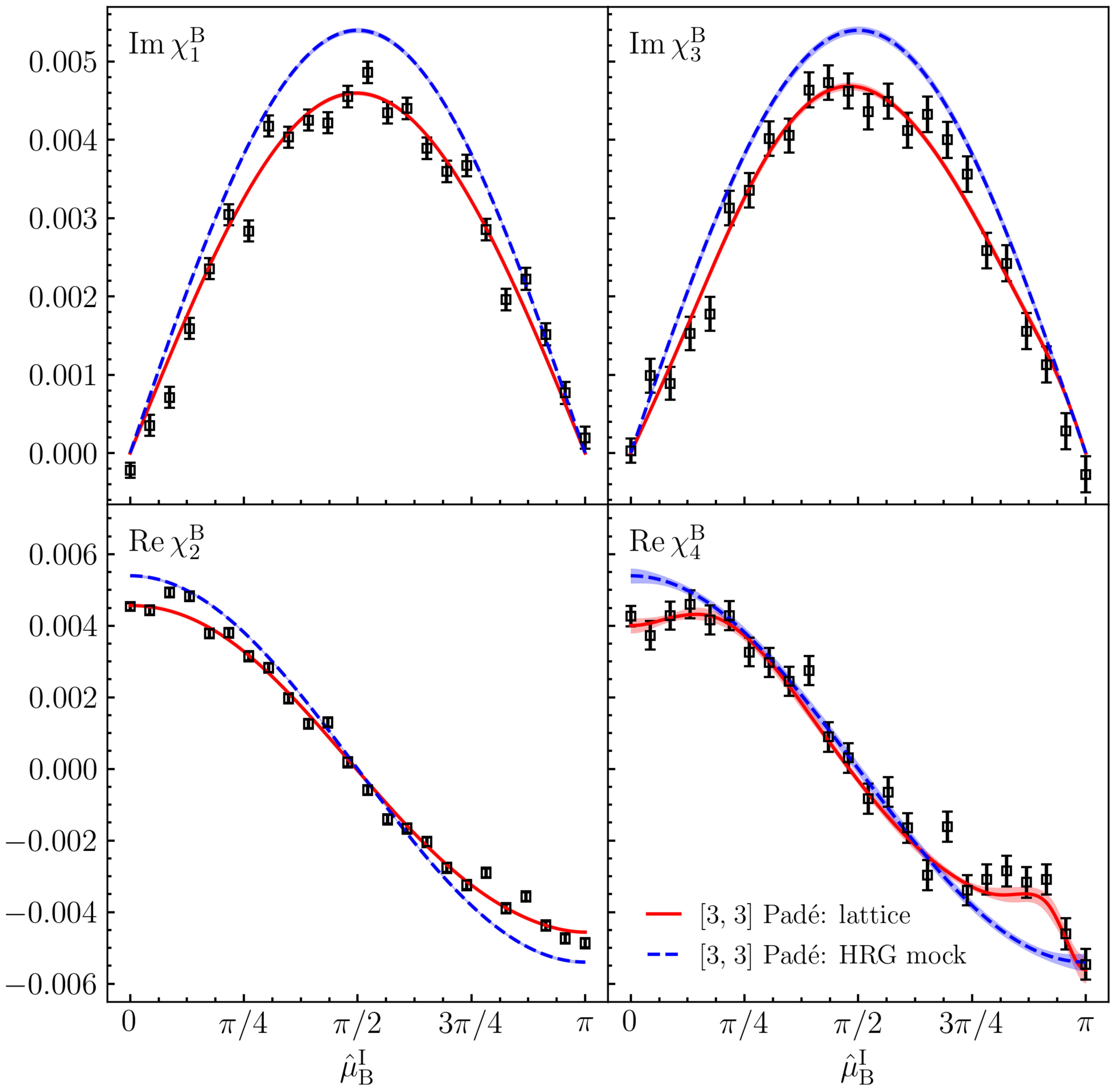
Physical Correspondence

$\chi_1^B \rightarrow$ imaginary n_B

$\chi_2^B, \chi_3^B, \chi_4^B, \cdots \rightarrow$ fluctuations of imaginary n_B

We measured $\chi_1^B, \chi_2^B, \chi_3^B, \chi_4^B$.

Padé fits to $\chi_1^B, \chi_2^B, \chi_3^B, \chi_4^B$ along Im $\hat{\mu}_B$ axis

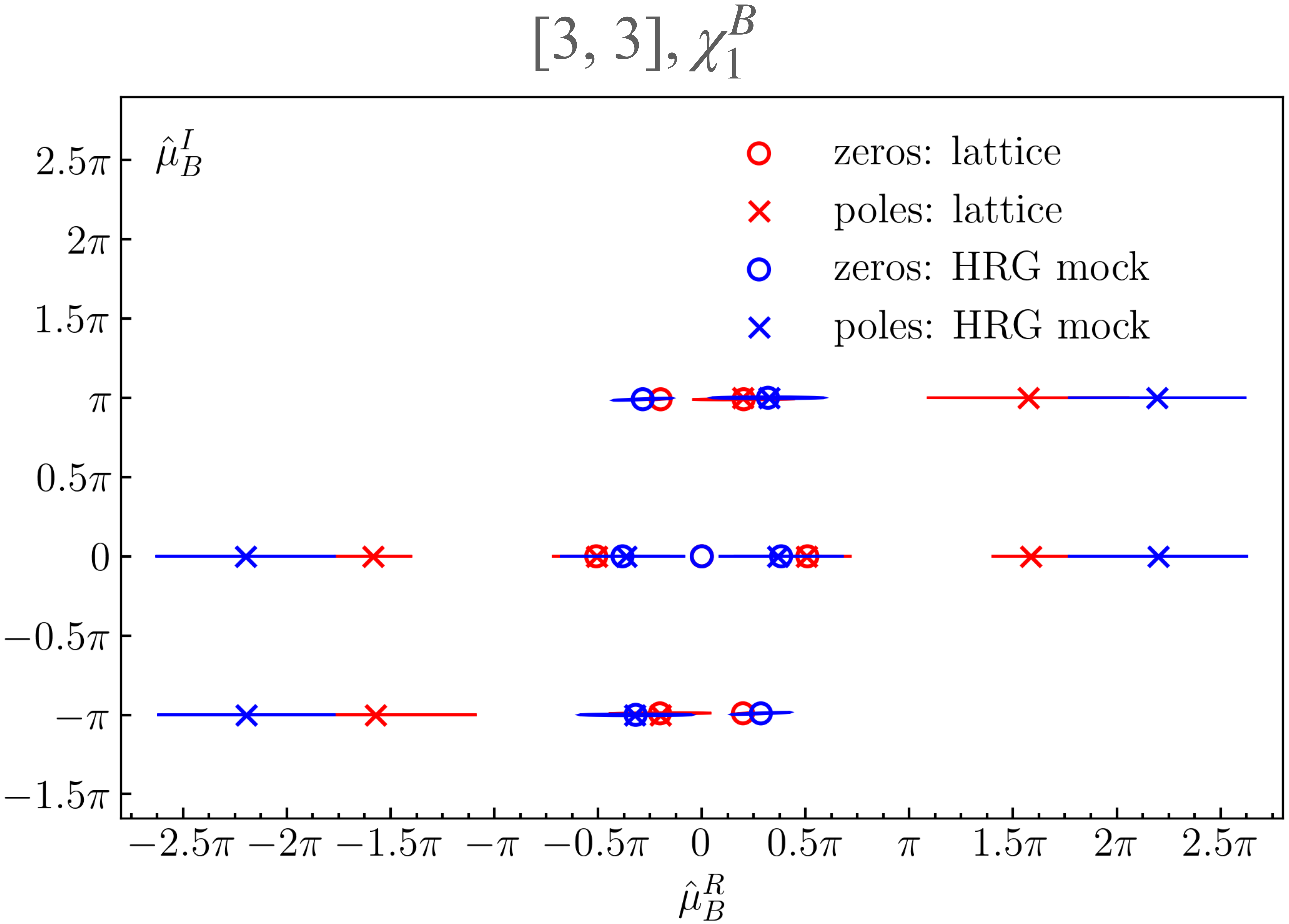


$$\text{Padé [3, 3]: } R_3^3(x) = \frac{\sum_{i=0}^{3} p_i x^i}{1 + \sum_{j=1}^{3} q_j x^j} \xrightarrow{\text{fit}} P(i\hat{\mu}_B^I)/T^4$$

$$x \to \cosh(i\hat{\mu}_B^I) - 1$$

- Generate N_{bs} bootstrap samples by resampling $N_{\text{conf},i}$ configurations allowing replacement for each $\mu_{B,i}$
- Joint fit to χ_n^B with $\partial^n P(x)/\partial\mu_B^n$ in each bootstrap sample with covariance matrix
- Final uncertainties from 68% confidence interval over bootstrap fits

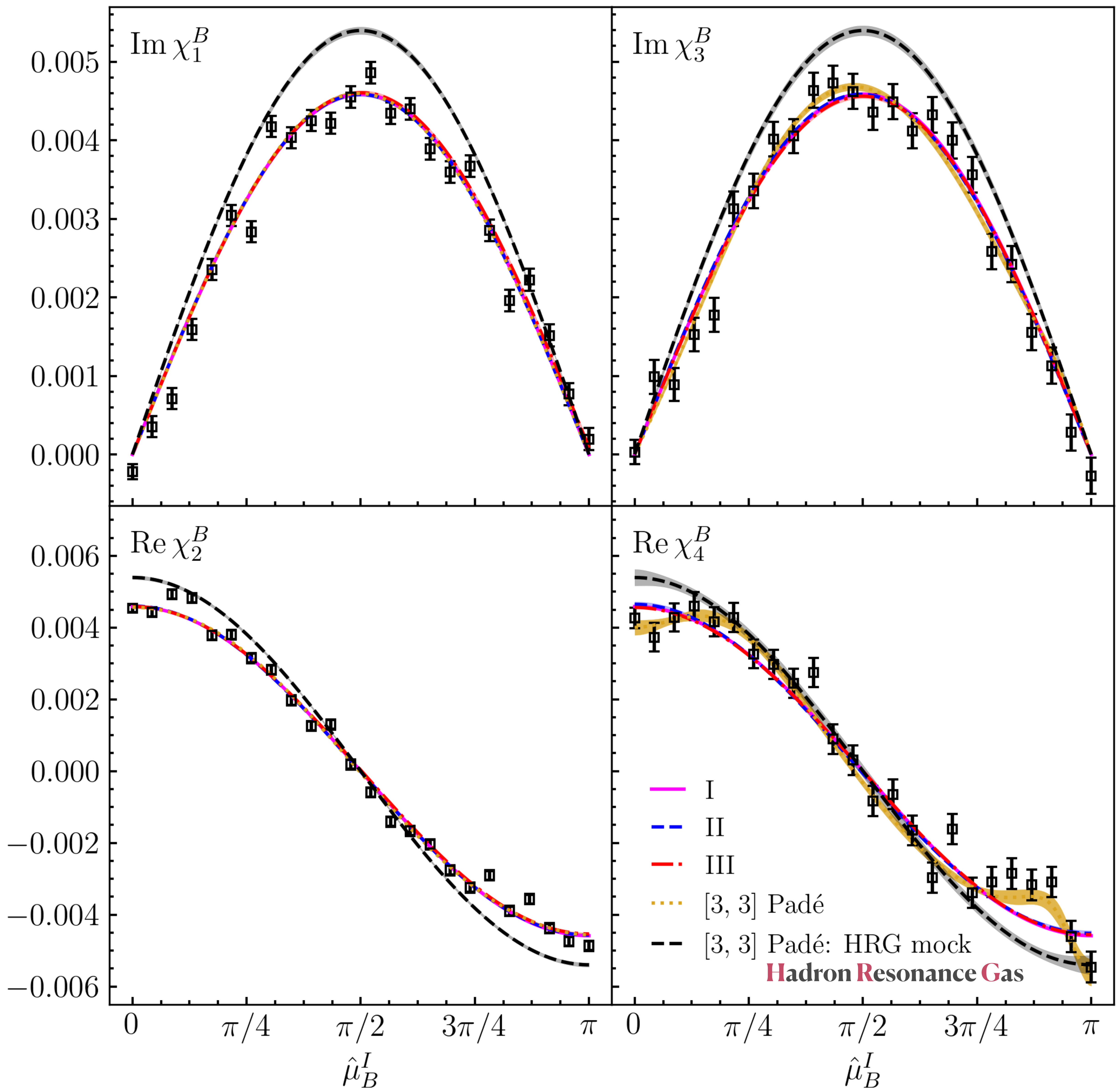
Zero/poles extracted from Padé [3, 3] analyses



$$\Delta P = \frac{\sum_{i=0}^{3} p_i x^i = 0 \rightarrow \text{zeros}}{1 + \sum_{j=1}^{3} q_j x^j = 0 \rightarrow \text{poles}}$$

- No uncanceled poles
- Similar pattern between Lattice and HRG mocks

Fits with physical-motivated non-critical ansatz



Non-critical Ansatz for pressure:

Ansatz I: $f_{\rm I}(\hat{\mu}_B) = A\cosh(\hat{\mu}_B)$

Ansatz II: $f_{\rm II}(\hat{\mu}_B) = A\cosh(\hat{\mu}_B) + B\cosh(2\hat{\mu}_B)$

	Lattice QCD Data			
	All 24 points		Random 12 points	
	Ansatz I	**Ansatz II**	**Ansatz I**	**Ansatz II**
Parameters				
A $(\times 10^{-3})$	4.584(35)	4.582(34)	4.528(48)	4.662(50)
B $(\times 10^{-5})$	–	0.38(64)	–	0.001(898)
Statistics				
χ^2/d.o.f.	4.20(55)	4.13(55)	4.45(82)	4.23(81)
AICc	401(52)	401(52)	211(38)	199(37)

Singularities in the real plane?

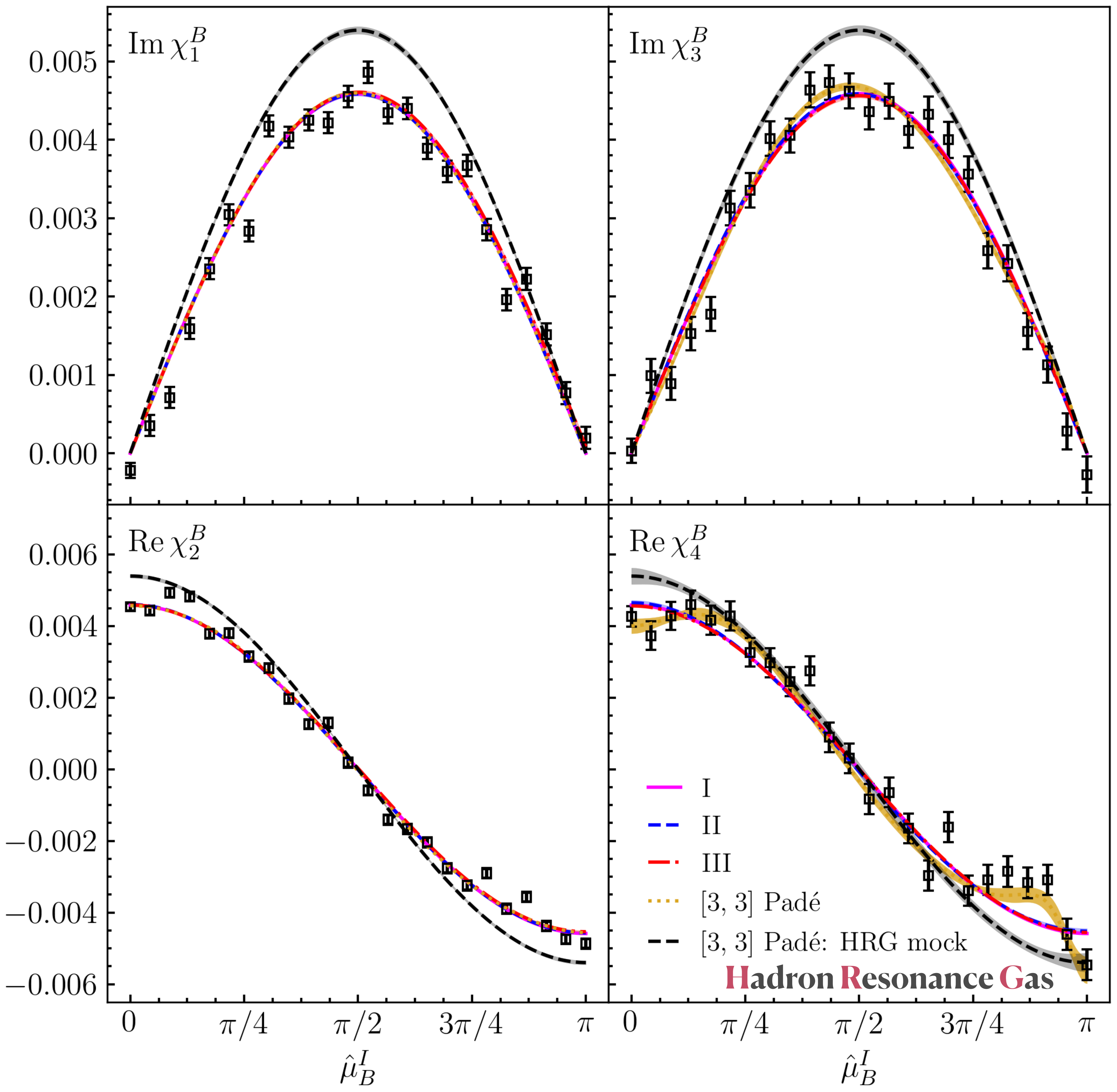
Critical ansatz III: $f(\hat{\mu}_B) = A\cosh(\hat{\mu}_B) + C(|\hat{\mu}_B - \hat{\mu}_{B,c}|^{\alpha} + |\hat{\mu}_B + \hat{\mu}_{B,c}|^{\alpha})$

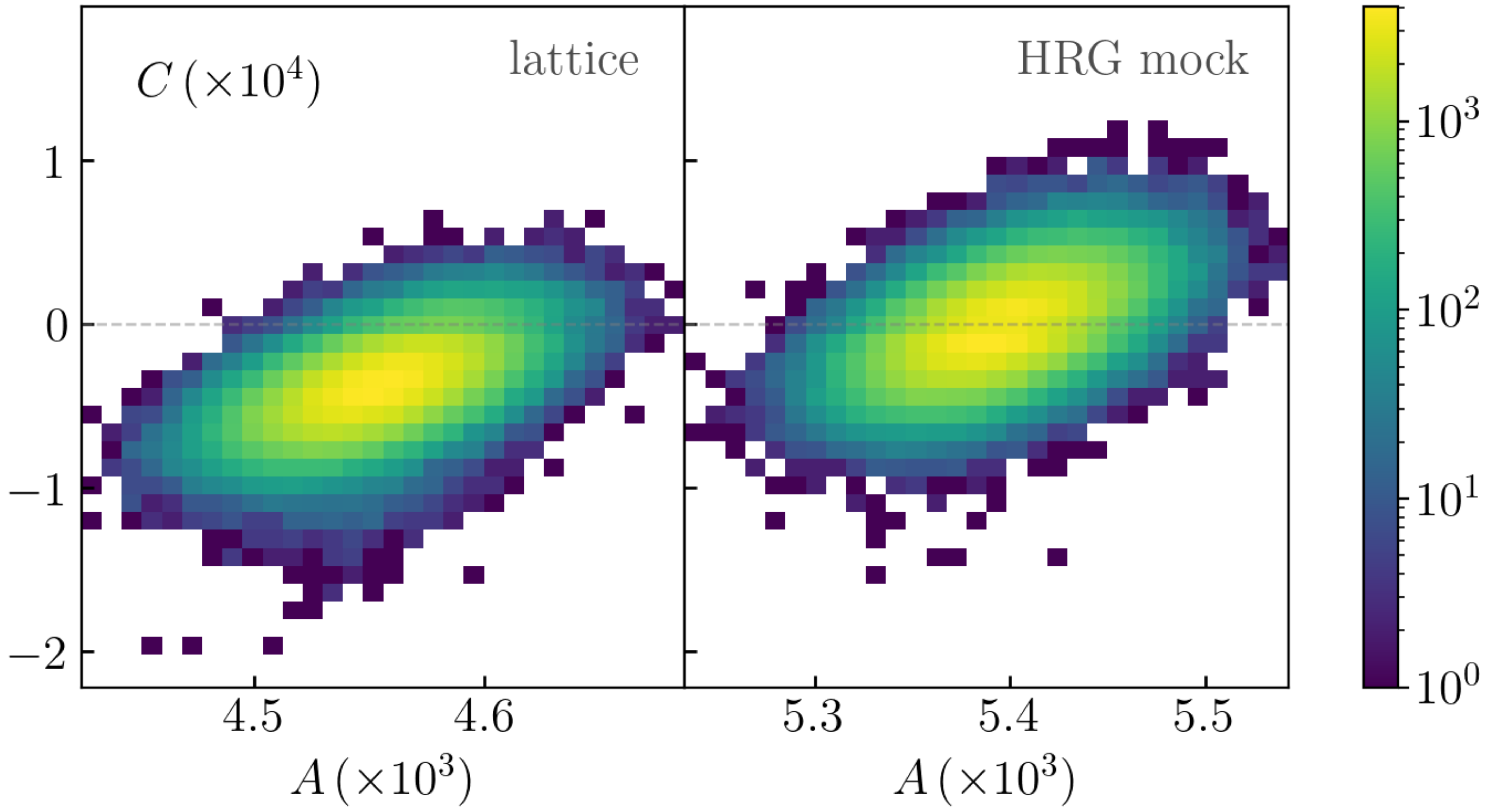
Impose lower bound of $\mu_{B,c}$: 350, 360, 380, 400 MeV

$\alpha = 1 + \delta_{\text{Ising}} = 1.208$

Hasenbusch , PRB 82(2010)174433

Karch, Schmidt & Singh, PRD109(2024)014508





Diagnostic measure of $\mu_{B,c}$ in a certain region $\mathscr{R}$

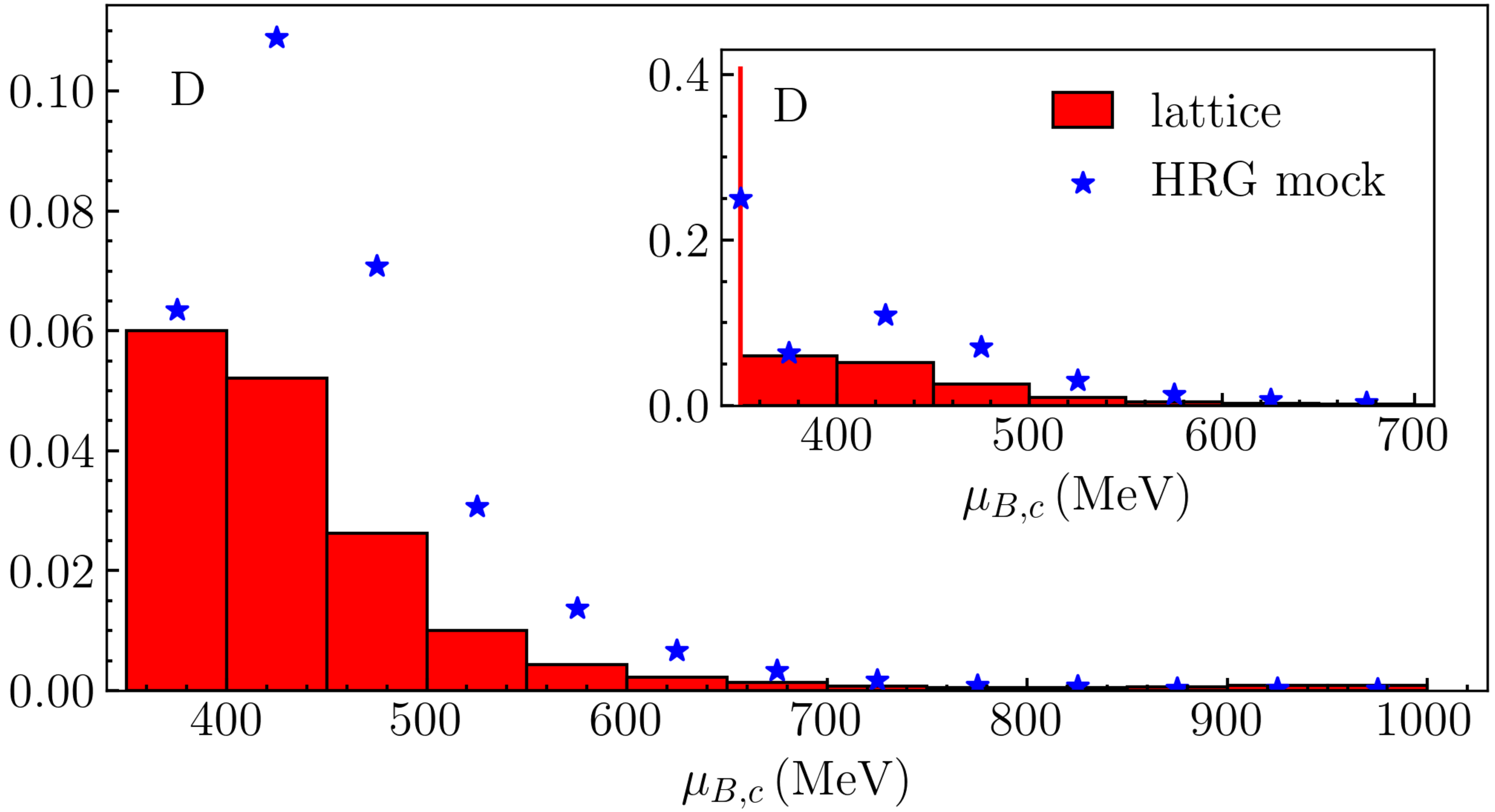
define a Diagnostic measure D for $\mu_{B,c}$ in a region $\mathscr{R}$:

$$D \equiv \frac{N_{\mathscr{R}}}{N_{\text{total}}}$$

Bootstrap samples in $\mathscr{R}$

Total bootstrap samples

Lower bound of $\mu_{B,c}$=350 MeV

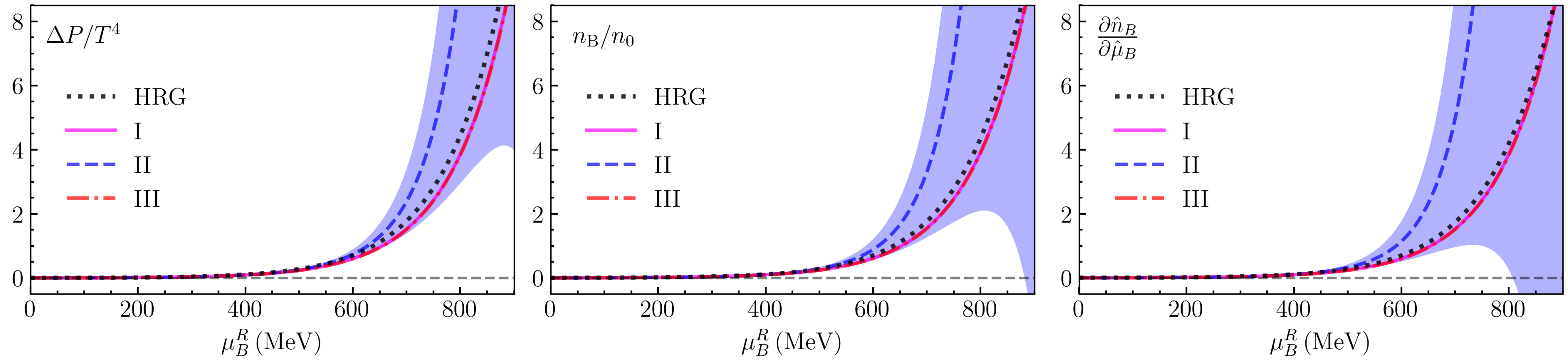


- Nonzero false-positive $D_{\mathrm{HRG}} > D_{\mathrm{LAT}}$!

Region $\mathcal{R}$ (MeV)	$D(\mathcal{R})$ lattice	HRG mock
387–502 (Predicted Region I)	9.1%	20.4%
420–750 (Experimental Search Region I)	7.3%	19.4%
420–632 (Experimental Search Region II)	7.1%	18.7%
540.0–664.2 (Predicted Region II)	0.86%	2.6%

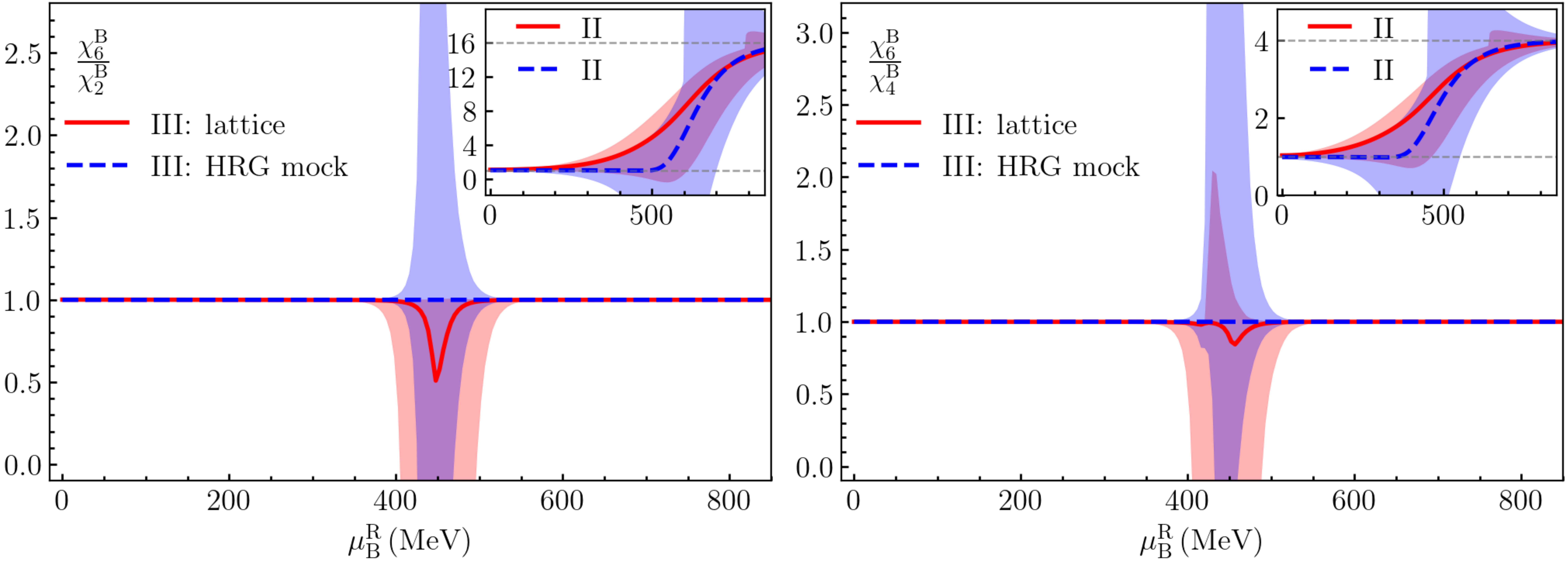
- Accumulation of large samples at lower bound

Analytic continuation



- Baryon number density $\hat{n}_B$ and $\partial\hat{n}_B/\partial\hat{\mu}_B$ shall be positive, which restricts $\mu_B^R \lesssim 800$MeV
- Results from ansatz II (interacting) differ from others at $\mu_B^R \gtrsim 500$MeV
- Results from ansatz I (non-interacting) and III (critical) are very much similar

Manifestation in the high order susceptibilities



Although about 7% bootstrap samples allowed to have $\mu_{B,c} \in (420,750)$ MeV

No significant sign change, deviation from HRG etc are observed

No signal in Exp

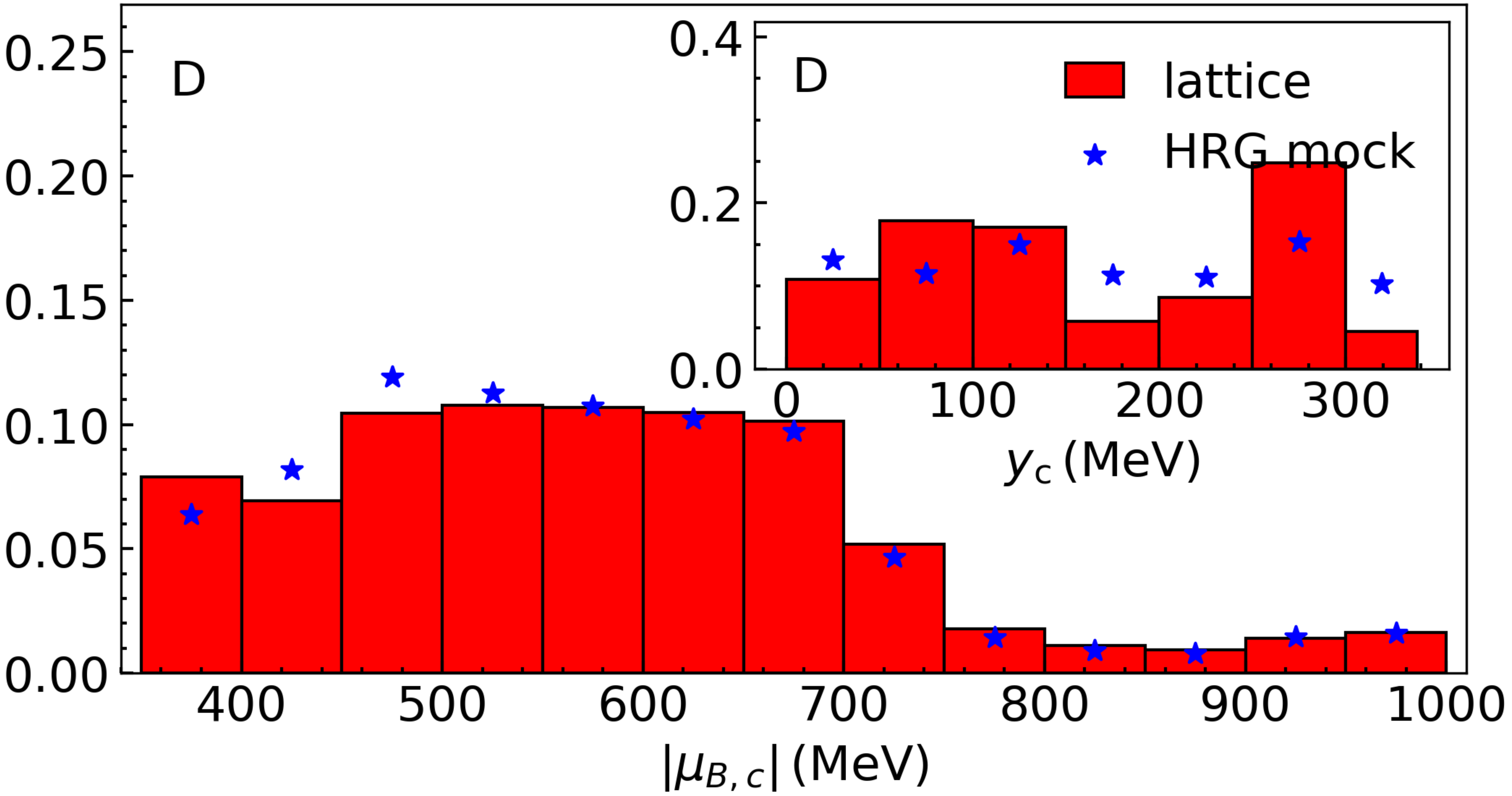
Criticality in the complex μ_B plane

$$f(\hat{\mu}_B) = A(T)\cosh(\hat{\mu}_B) + C\,(\hat{\mu}_B - \hat{\mu}_{B,c})^{\alpha_{\rm LY}} + C\,(-\hat{\mu}_B - \hat{\mu}_{B,c})^{\alpha_{\rm LY}} + C^*\,(\hat{\mu}_B - \hat{\mu}^*_{B,c})^{\alpha_{\rm LY}} + C^*\,(-\hat{\mu}_B - \hat{\mu}^*_{B,c})^{\alpha_{\rm LY}}$$

$C, \hat{\mu}_{B,c} \in \mathbb{C}$

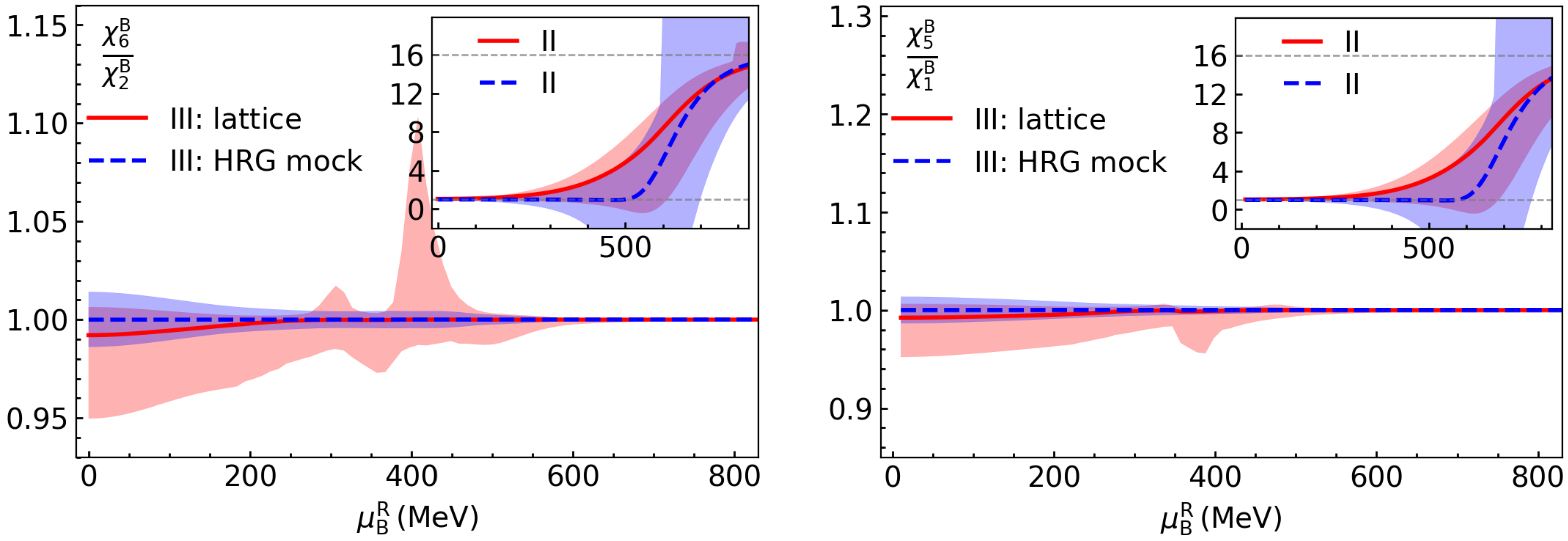
$\alpha_{\rm LY} = 1.085$

Gliozzi and Rago, JHEP 10 (2014) 042



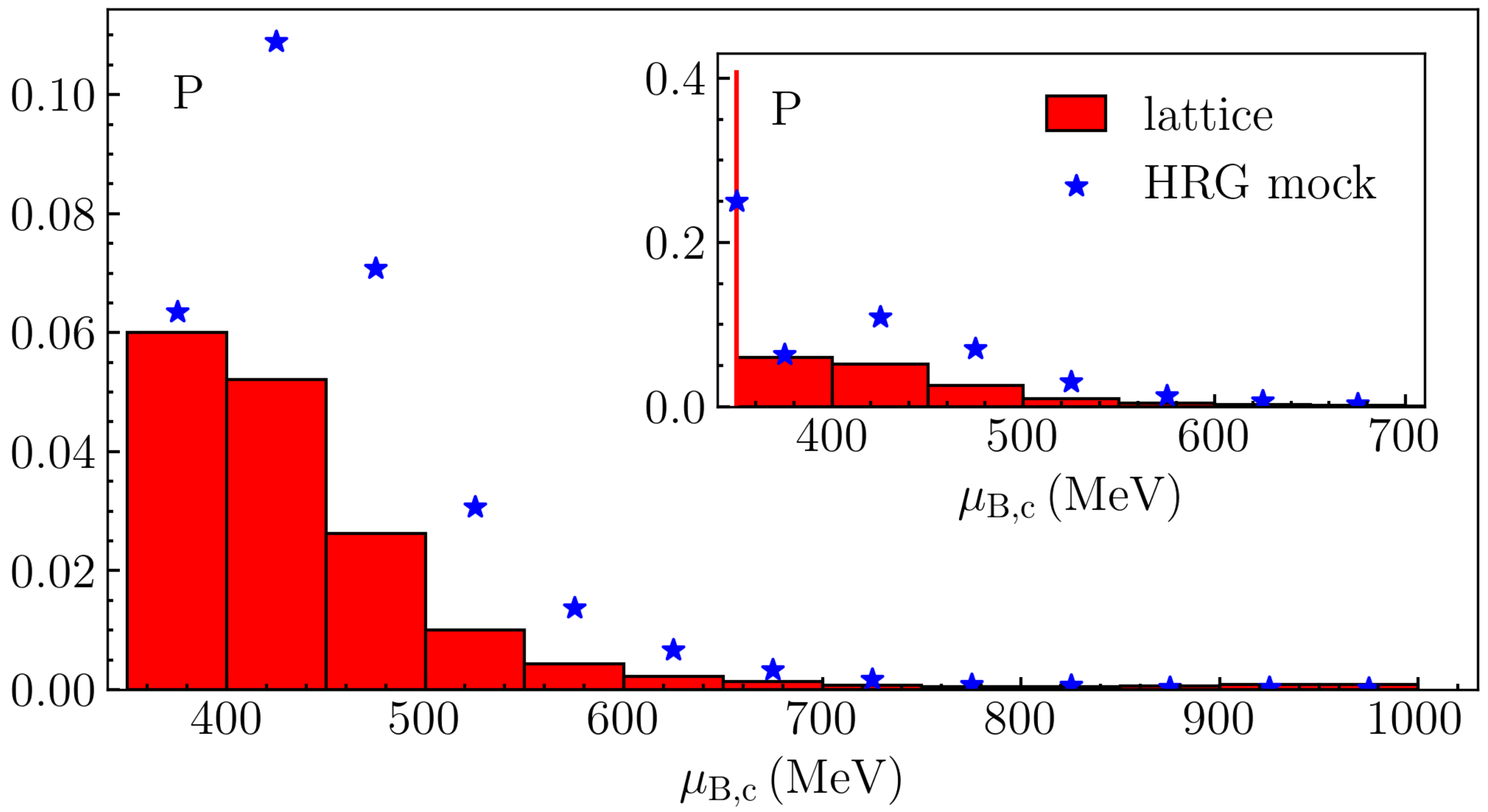
Ratios of susceptibilities at real μ_B

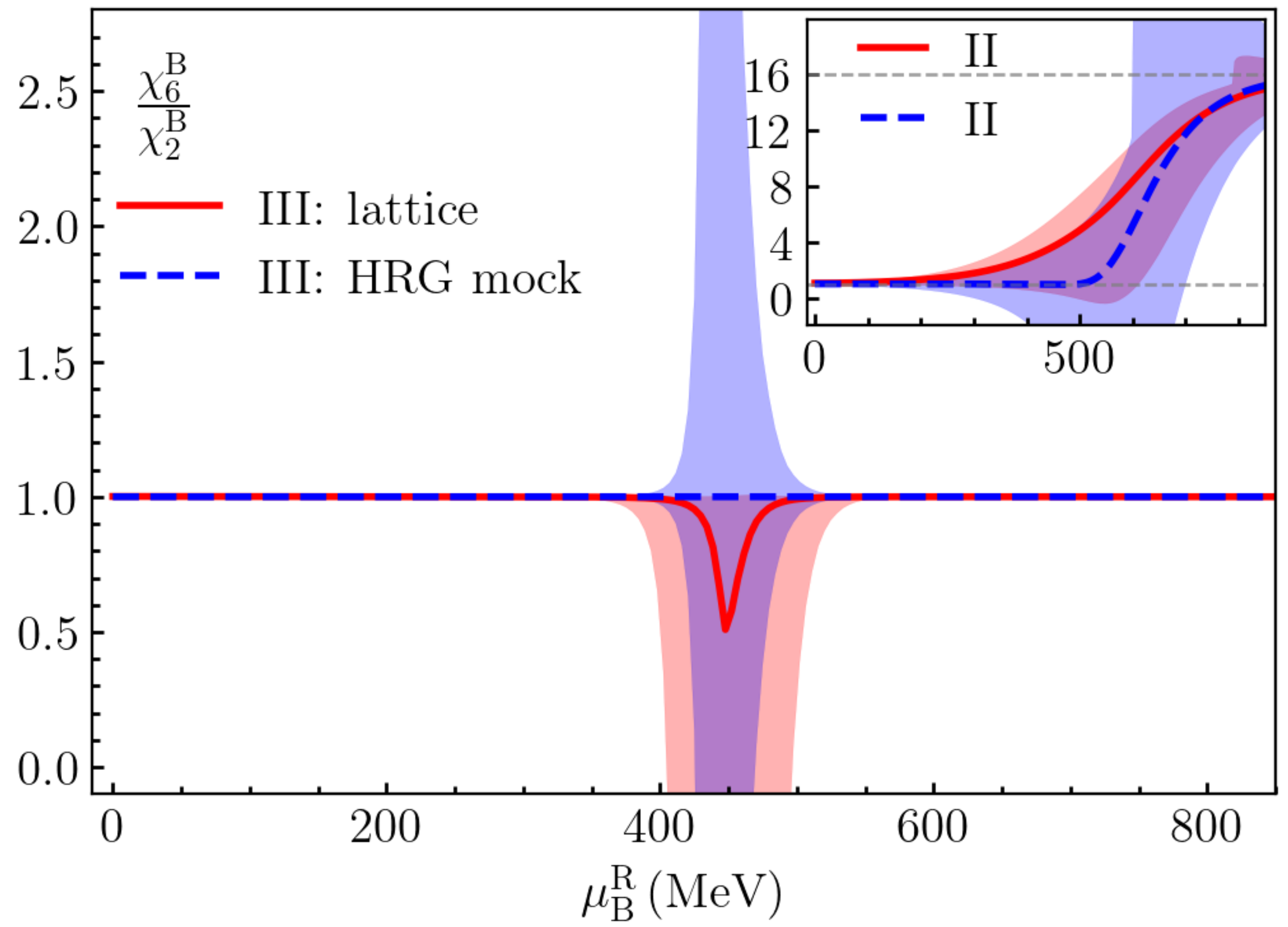
$$f(\hat{\mu}_B) = A(T)\cosh(\hat{\mu}_B) + C\,(\hat{\mu}_B - \hat{\mu}_{B,c})^{\alpha_{\mathrm{LY}}} + C\,(-\hat{\mu}_B - \hat{\mu}_{B,c})^{\alpha_{\mathrm{LY}}} + C^*\,(\hat{\mu}_B - \hat{\mu}^*_{B,c})^{\alpha_{\mathrm{LY}}} + C^*\,(-\hat{\mu}_B - \hat{\mu}^*_{B,c})^{\alpha_{\mathrm{LY}}}$$



Summary

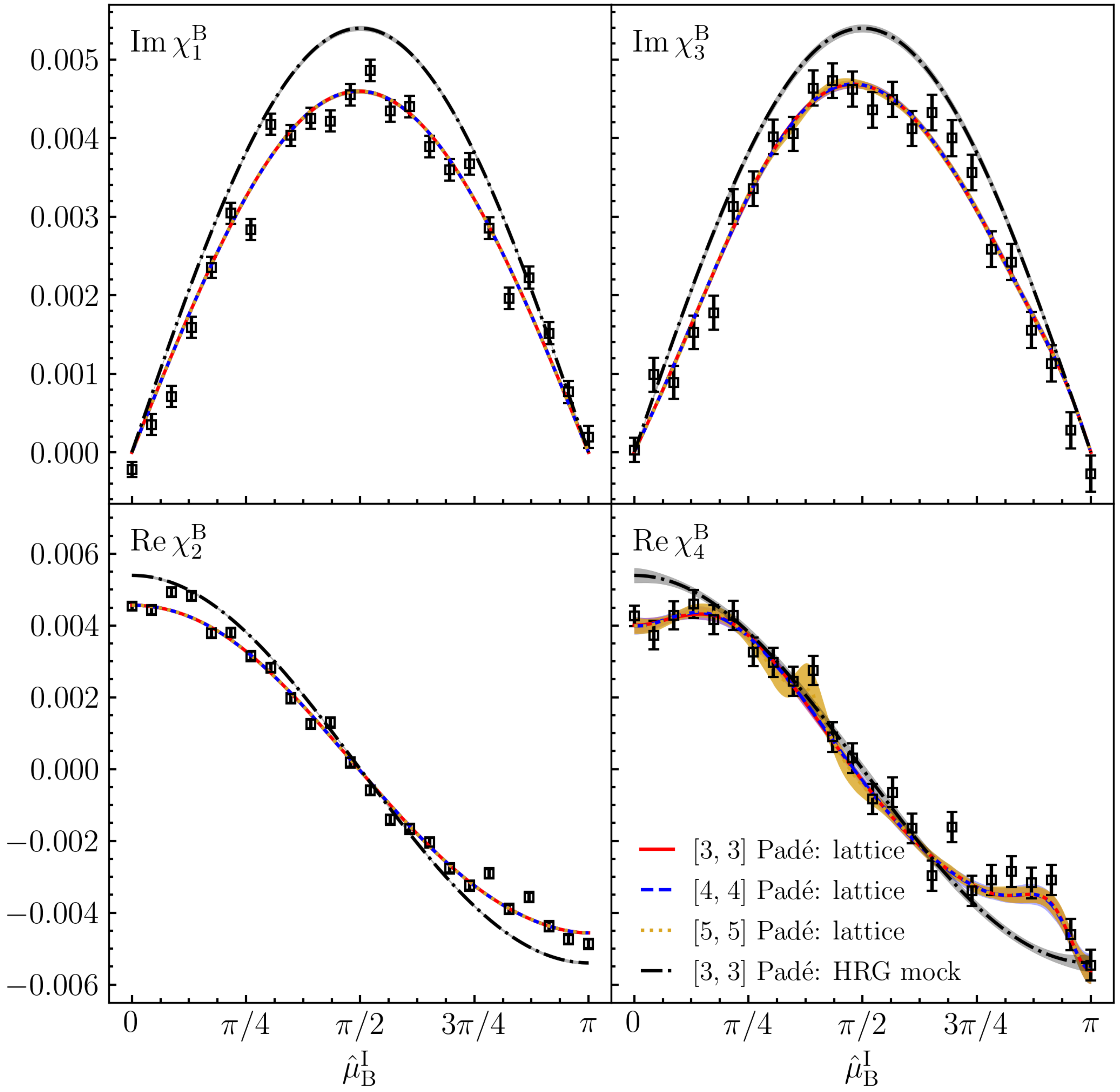
- Recent predictions on $T_c^{CEP} \approx 83 - 120$ MeV, extrapolated based on the lattice simulations at $T \gtrsim 120$ MeV
- Our LQCD simulations at Im(μ_B) @T=108 MeV: data compatible with smooth, analytic thermodynamics, obtained using fits with direct Padé approximants, physical motivated non-interacting, interacting and critical Ansatz
- Only ~7% bootstrap samples gives for a CEP in $\mu_{B,c}^{CEP} \in$ 420-750MeV, however, it does not manifest in the highest baryon susceptibilities that EXP can measure with reasonable precisions





Backup

Padé fits to $\chi_1^B, \chi_2^B, \chi_3^B, \chi_4^B$ along Im $\hat{\mu}_B$ axis



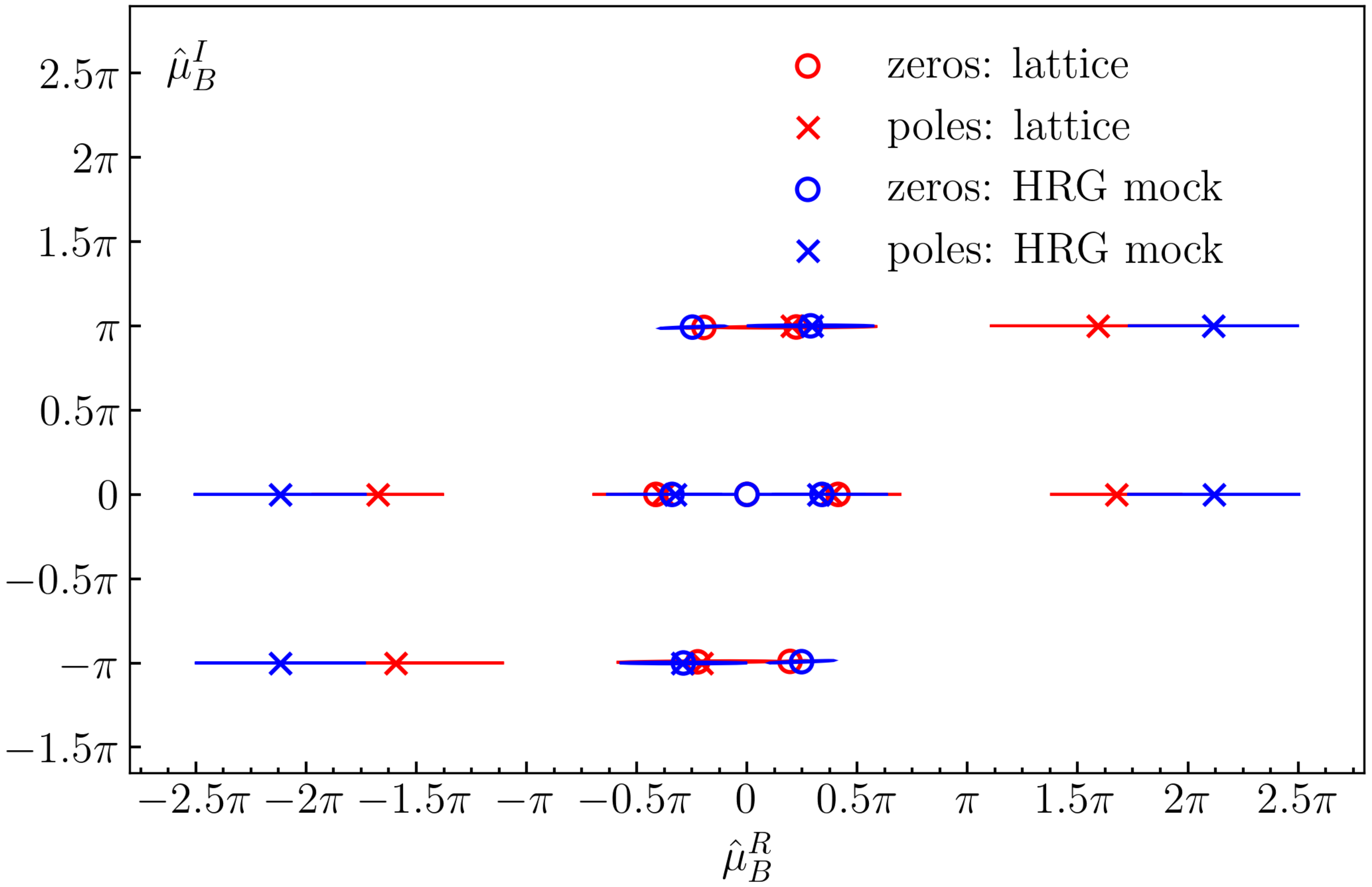
$$\text{Padé } [m,\ n]: R_n^m(x) = \frac{\sum_{i=0}^m p_i x^i}{1+\sum_{j=1}^n q_j x^j} \xrightarrow{\text{fit}} P(i\hat{\mu}_B^I)/T^4$$

$$x \to \cosh(i\hat{\mu}_B^I) - 1$$

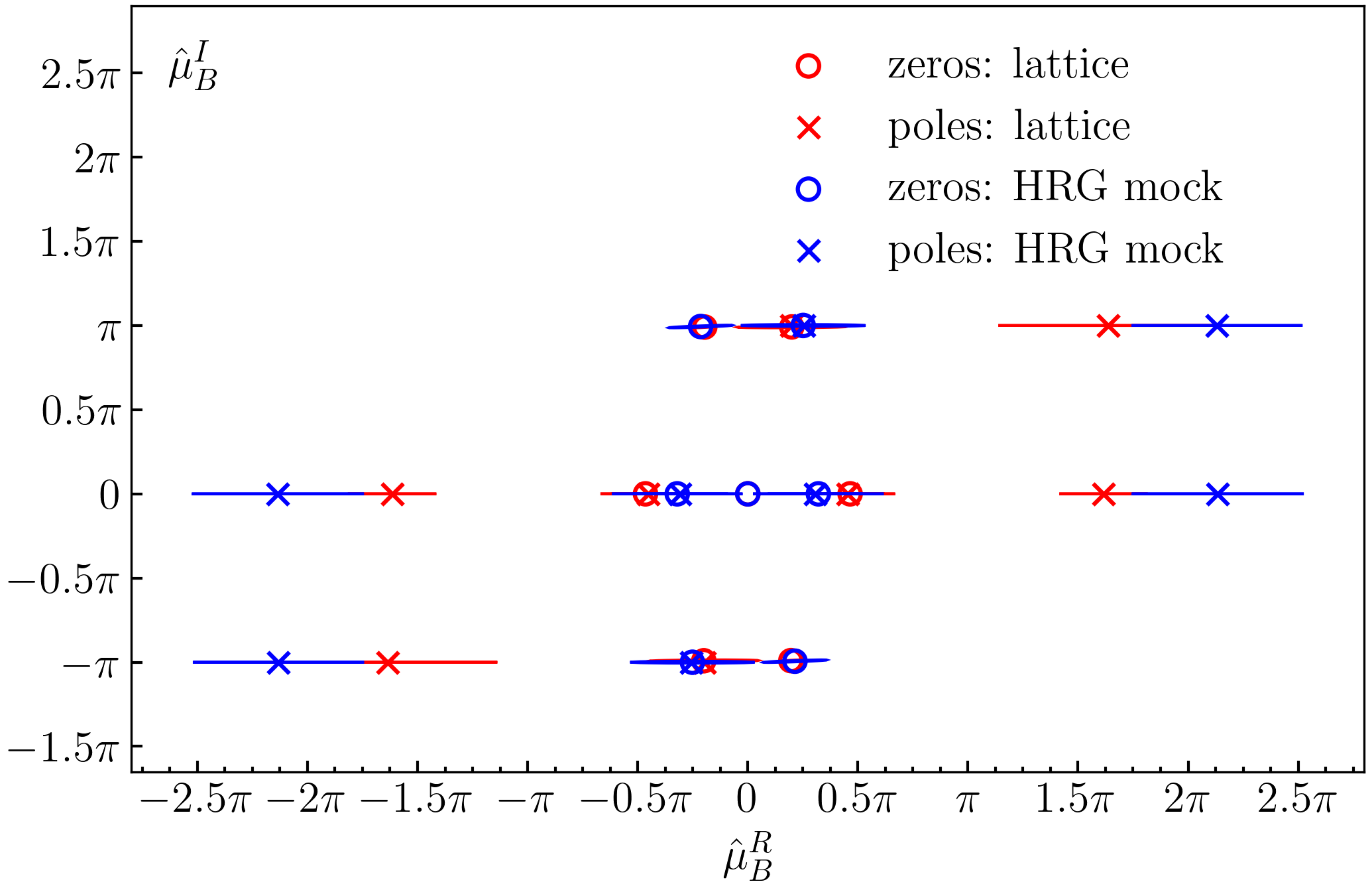
		Padé [3,3]		Padé [4,4]		Padé [5,5]	
		All 24 points	Random 12 points	All 24 points	Random 12 points	All 24 points	Random 12 points
χ^2/d.o.f.	lattice	4.00(55)	3.39(75)	4.06(57)	4.45(90)	4.03(57)	3.99(85)
	HRG mock	0.97(21)	0.95(30)	0.97(21)	0.94(31)	0.96(21)	0.93(31)
AICc	lattice	373(50)	156(31)	375(50)	198(36)	370(49)	178(32)
	HRG mock	101(19)	54(13)	103(18)	57(12)	105(18)	61(12)

Zero/poles extracted from higher order Padé analyses

$[4, 4], \chi_1^B$

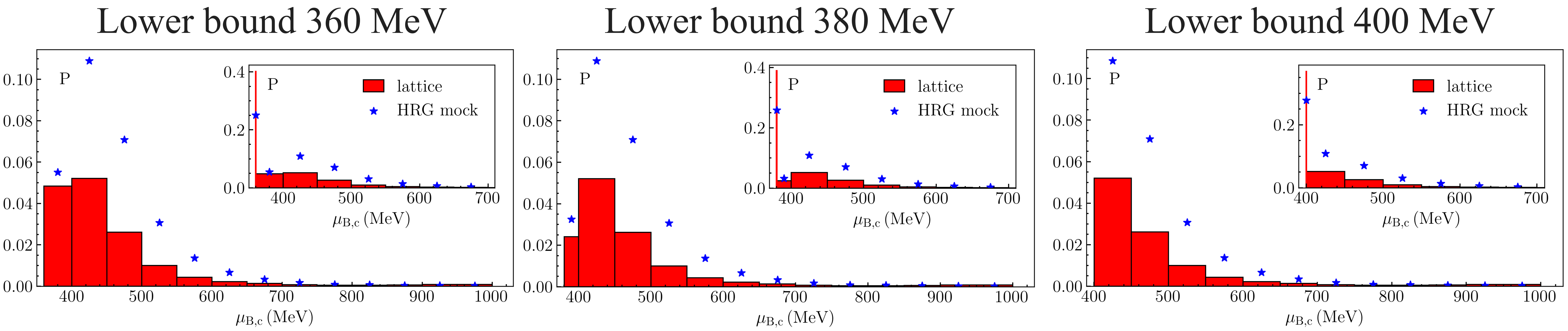


$[5, 5], \chi_1^B$



Similar observation as in the case of Padé [3, 3]

Dependence on the lower bound & # of bootstrap samples



	$D(\mathcal{R})$											
	100K Samples				200K Samples				300K Samples			
Region $\mathcal{R}$ (MeV)	350	360	380	400	350	360	380	400	350	360	380	400
387–502	0.091	0.091	0.091	-	0.092	0.091	0.091	-	0.092	0.092	0.092	-
420–750	0.074	0.074	0.074	0.074	0.073	0.073	0.073	0.073	0.073	0.073	0.073	0.073
420–632	0.071	0.071	0.071	0.071	0.071	0.071	0.071	0.071	0.071	0.071	0.071	0.071
540.0–664.2	0.0086	0.0086	0.0087	0.0087	0.0087	0.0087	0.0086	0.0086	0.0085	0.0085	0.0084	0.0085

Dependence on the reduced/full data sets

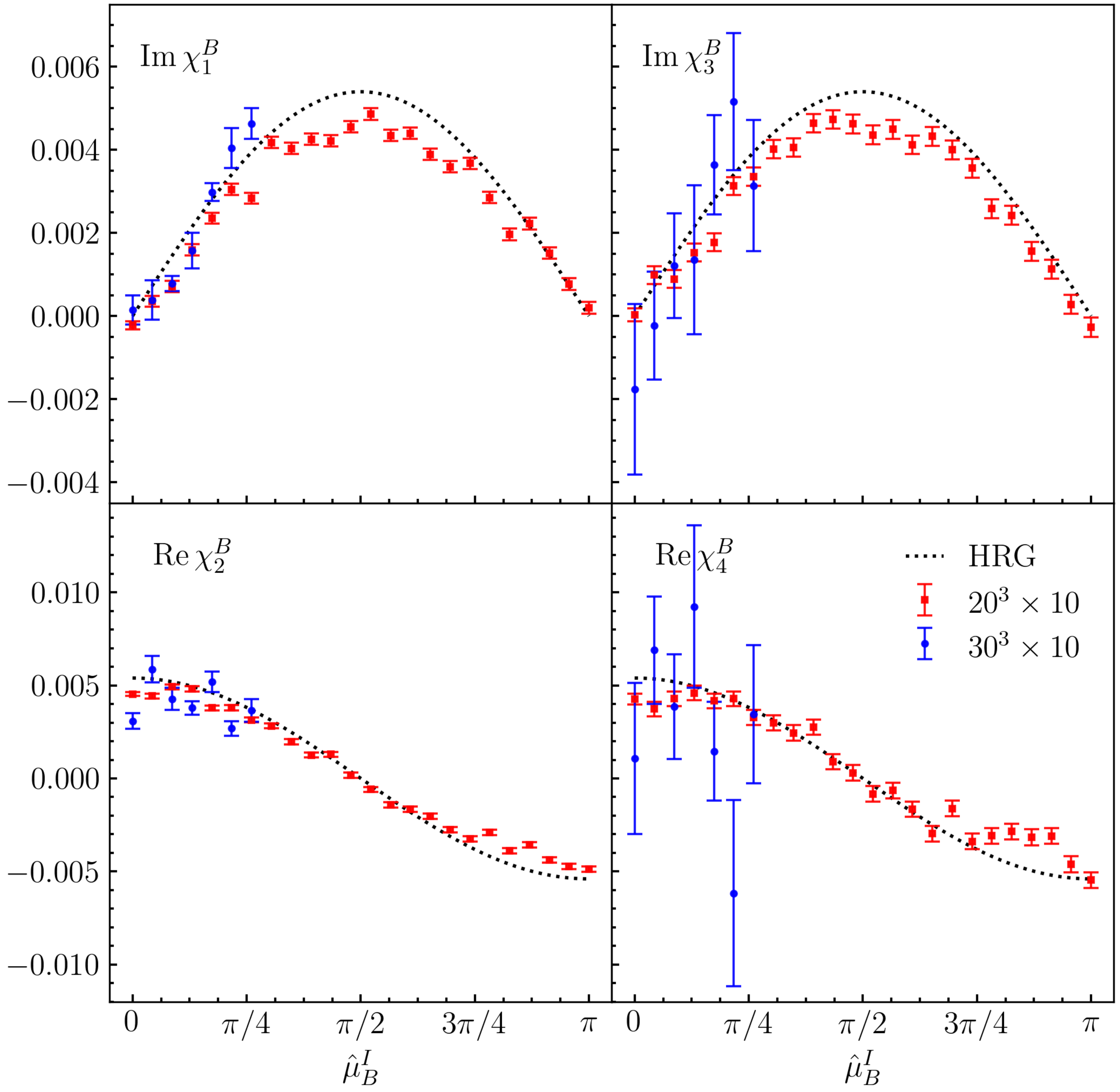
Lattice

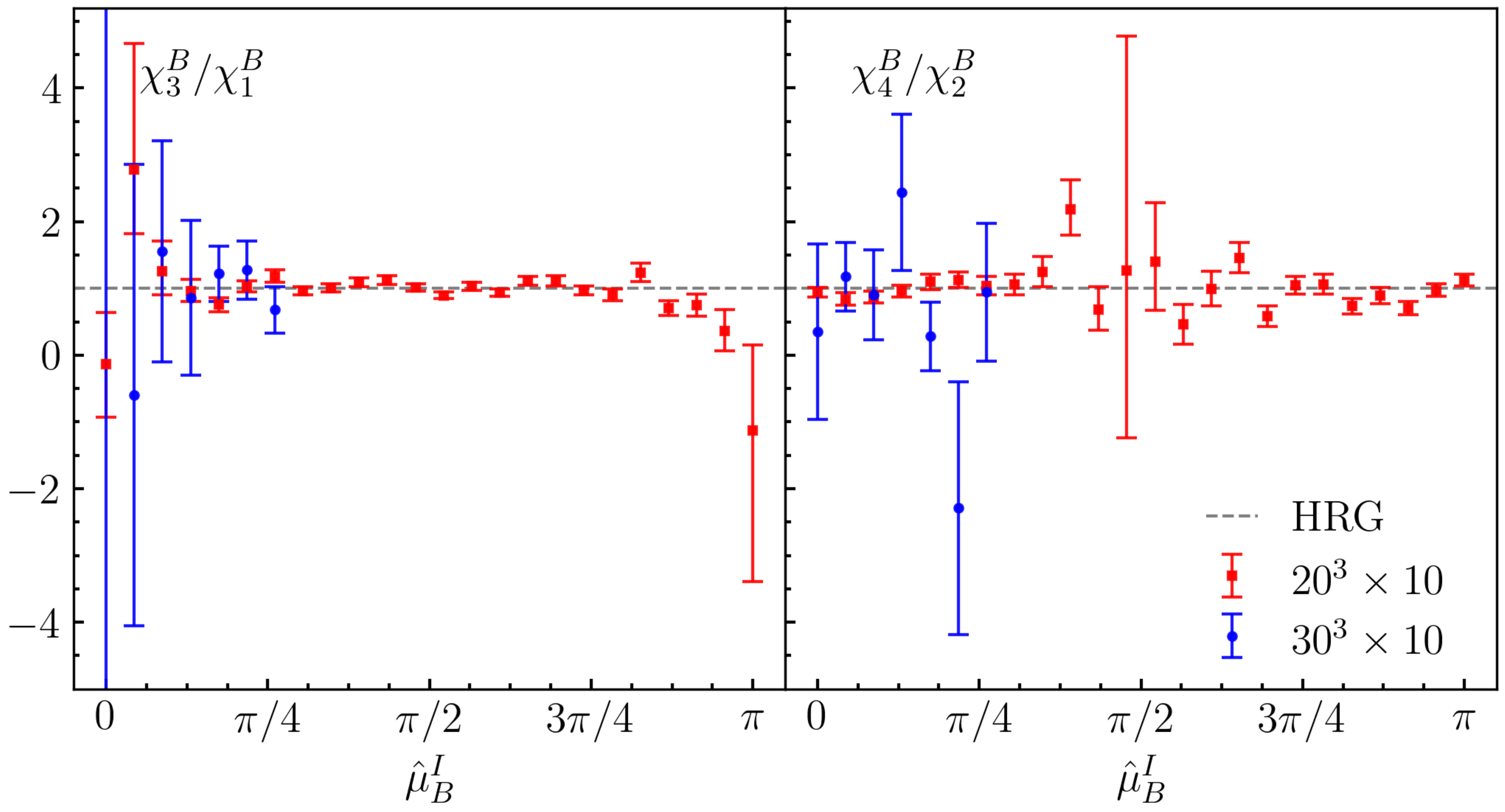
Region $\mathcal{R}$ (MeV)	$D(\mathcal{R})$ All 24 points 350	360	380	400	Random 12 points 350	360	380	400
387–502	0.092	0.092	0.092	-	0.124	0.124	0.124	-
420–750	0.073	0.073	0.073	0.073	0.133	0.133	0.132	0.133
420–632	0.071	0.071	0.071	0.071	0.126	0.126	0.126	0.126
540.0–664.2	0.0085	0.0085	0.0085	0.0085	0.023	0.023	0.022	0.023

HRG

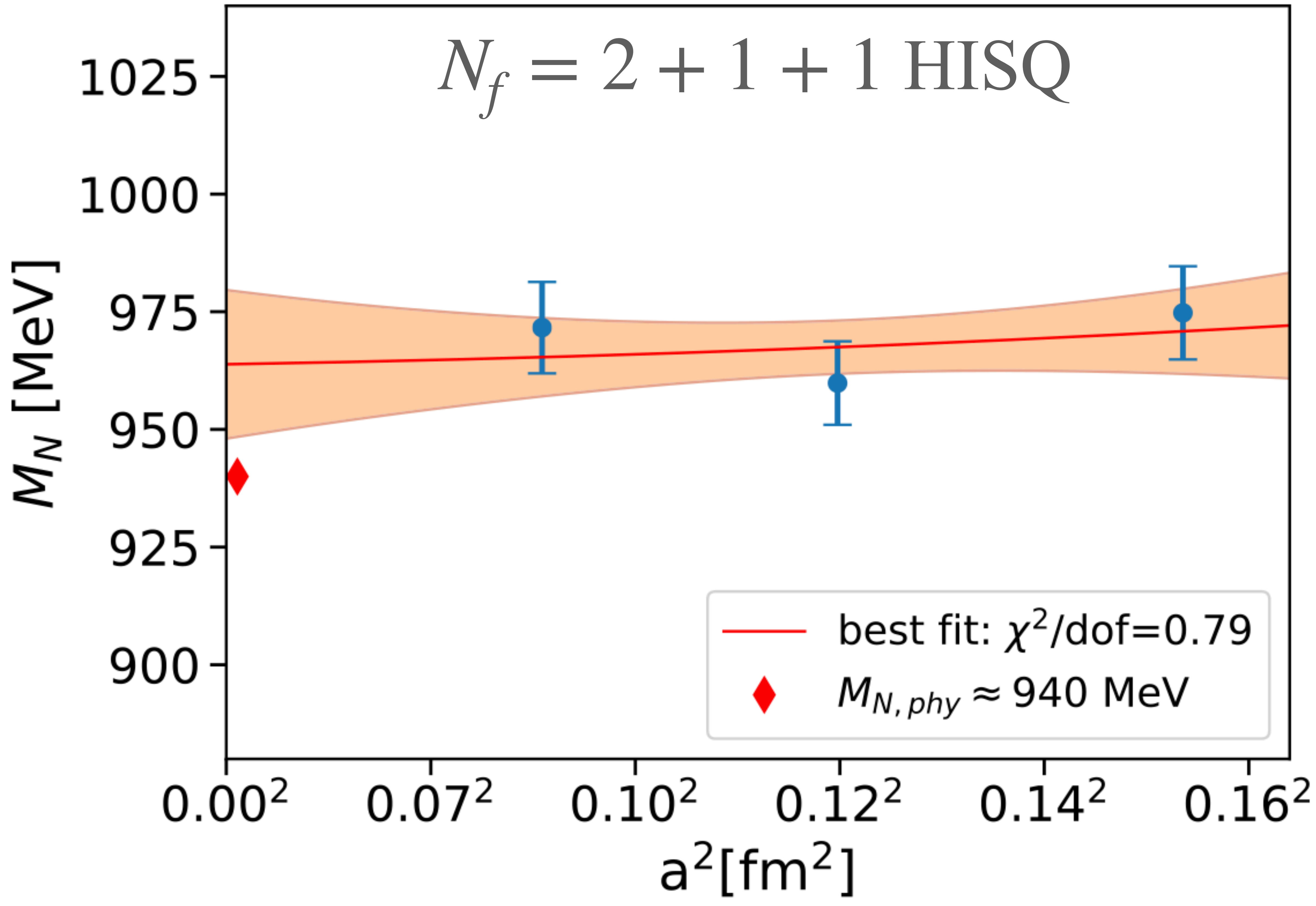
Region $\mathcal{R}$ (MeV)	$D(\mathcal{R})$ All 24 points 350	360	380	400	Random 12 points 350	360	380	400
387–502	0.204	0.204	0.204	-	0.263	0.225	0.197	-
420–750	0.194	0.194	0.194	0.194	0.301	0.218	0.175	0.184
420–632	0.187	0.187	0.187	0.187	0.293	0.211	0.168	0.177
540.0–664.2	0.0258	0.0259	0.0258	0.0258	0.0327	0.0266	0.0233	0.0247

Volume dependences





Lattice cutoff effects



Yin Lin et al., *Phys. Rev. D* 103 (2021) 3, 034501

- $N_\tau = 10$ at $T = 108$ MeV $\Rightarrow a \approx 0.18$ fm
- proton mass is inflated by about 4-5%, ~980MeV at $a \approx 0.18$ *fm*, as estimated from the difference in A fit parameter

$$A_{\mathrm{LAT}} = 4.584(35) \times 10^{-3}$$
$$A_{\mathrm{HRG}} = 5.397(42) \times 10^{-3}$$

- Consistent with direct computation in $N_f = 2+1+1$ QCD