Multi-hadron resonances

燕浩波（Haobo Yan）

With Maxim Mai, Marco Garofalo, Yuchuan Feng, Micheal Döring, Ulf-G. Meißner, Carsten Urbach,
孟雨（Yu Meng），林勇辉（Yonghui Lin），何丽萍（Liping He），刘柳明（Liuming Liu），刘川（Chuan Liu）

Based on:

1. Yan et al., arXiv:2510.xxxxx on $\pi(1300)$ **(appearing tomorrow morning)**
2. Yan et al., arXiv:25xx.xxxx on $D_0^*(2300)$
3. Yan et al., arXiv:2507.16070 on the OpTion package

北京大學
PEKING UNIVERSITY

2025.8.12 @ 惠州

Table of contents

Milestones of hadron scattering

- Two-body
  - 1991: Lüscher – the famous formula
  - 1992-2014 – development of Lüscher formalism
  - 2014: Briceño – the most general form
  - 2012: HadSpec – energy-dependent phase shift of $I = 2\ \pi\pi$
  - 2013: HadSpec – $I = 1\ \pi\pi \to \rho$
  - 2012-now: scattering with charm, bottom, baryons, notably the debate on NN

@ 吕岩/张宽's talk on NN
@ 王政力/谢嘉竣/贾宇航's talk on $\pi\pi/T_{cc}/KN$
@ 李更's talk on charmonia resonances
@ 李衢智's talk on $K\pi$
@ 邢瀚洋/义景宇's talk on baryonic systems

- Three-body
  - 2014-2017: Hansen/Sharpe, Mai/Döring, Hammer/Pang/Rusetsky – development of three-particle formalism
  - 2021: HadSpec – energy-dependent $I = 3\ \pi\pi\pi$
  - 2021: GWQCD – $I = 1\ \pi\pi\pi \to a_1(1260)$
  - 2024: CLQCD + Bonn – $I = 0\ \pi\pi\pi \to \omega(782)$
  - 2025-now: ongoing challenges: T_{cc}, Roper…

PHYSICAL REVIEW LETTERS **133,** 211906 (2024)

Editors' Suggestion

ω Meson from Lattice QCD

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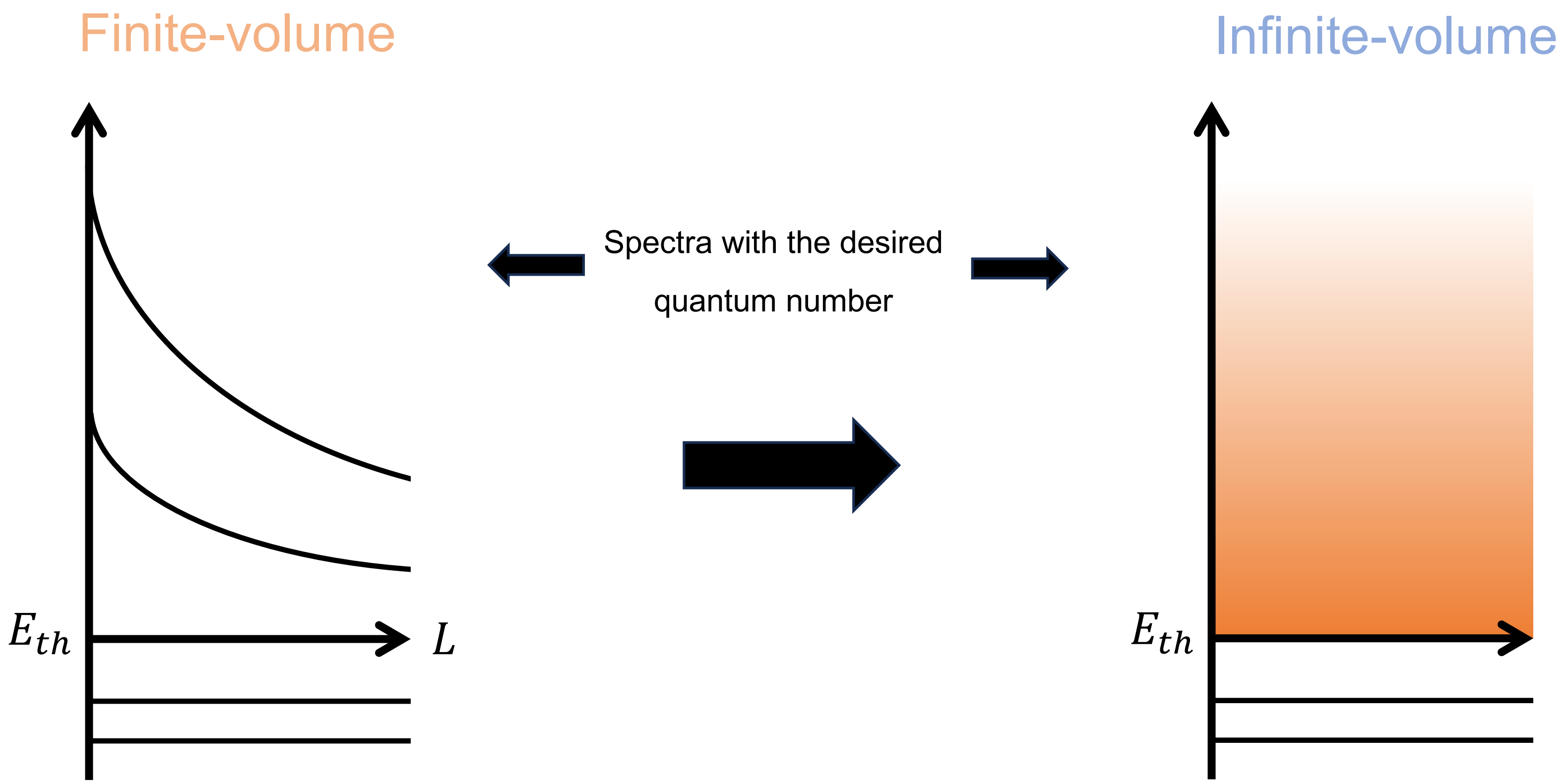
Routine



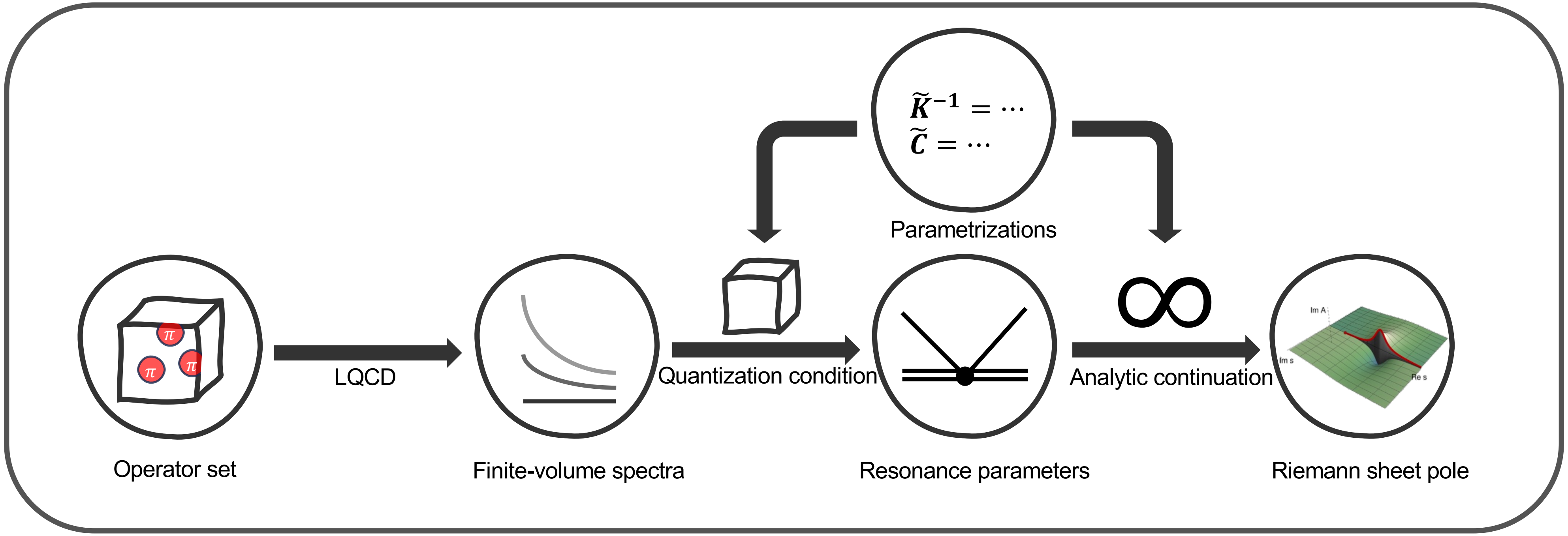
@ Dudek's talk in Hadron2025

- How to extract the infinite-volume information from finite-volume spectra?

  ➡ Quantization condition: $f\big(\mathcal{M}(E_n),\ \mathcal{F}(E_n, L)\big) = 0$

- More E_n and L give more constraints on $\mathcal{M}(E_n)$

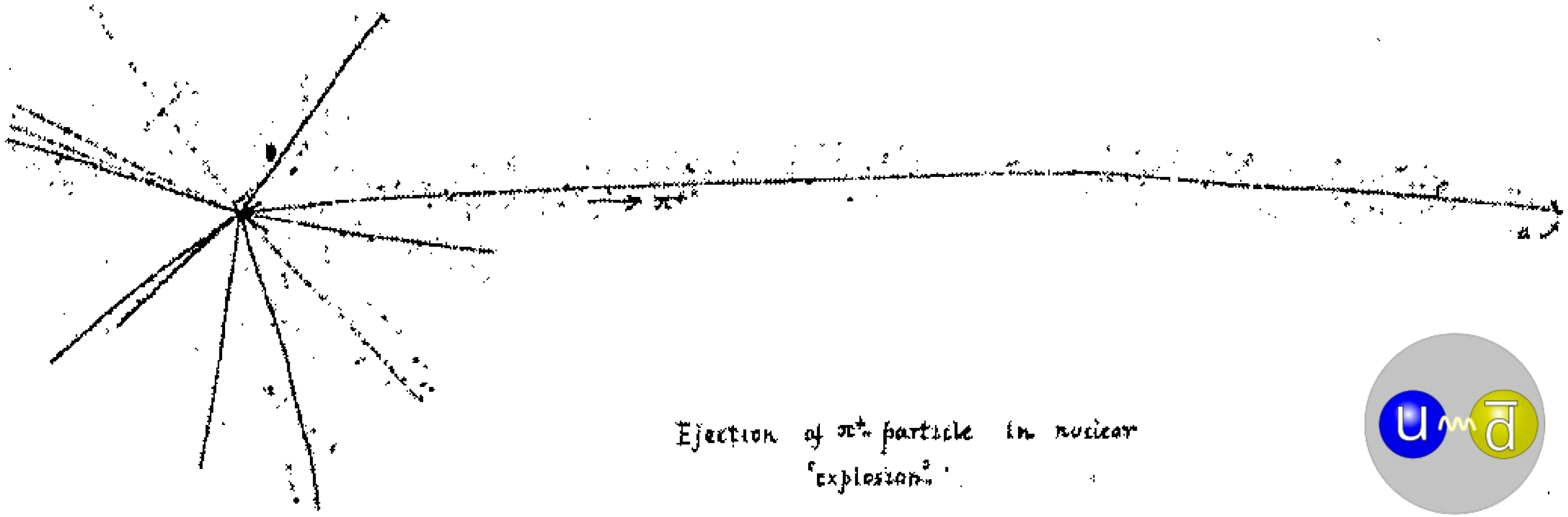
Routine



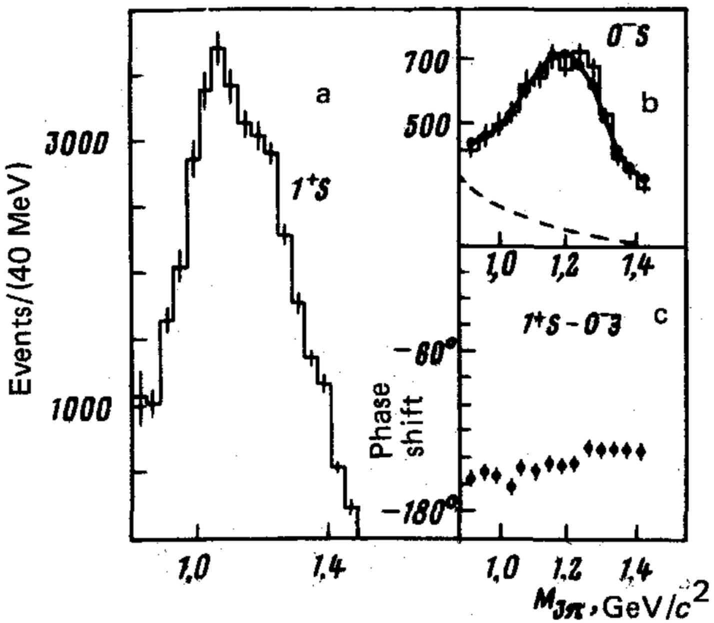
$\widetilde{K}^{-1} = \cdots$
$\widetilde{C} = \cdots$
Parametrizations
π
π
π
LQCD
Quantization condition
∞
Analytic continuation
Im A
Im s
Re s
Operator set
Finite-volume spectra
Resonance parameters
Riemann sheet pole

Table of contents

Bellini et al (1961)



Discovery of π



Evidence of $\pi(1300)$

- π: the lightest hadron, Goldstone boson
- $\pi(1300)$ very heavy, existence not even established
- Decay to $\pi\pi\pi$ without centrifugal barrier
- A second step towards Roper $\rightarrow N\pi\pi$

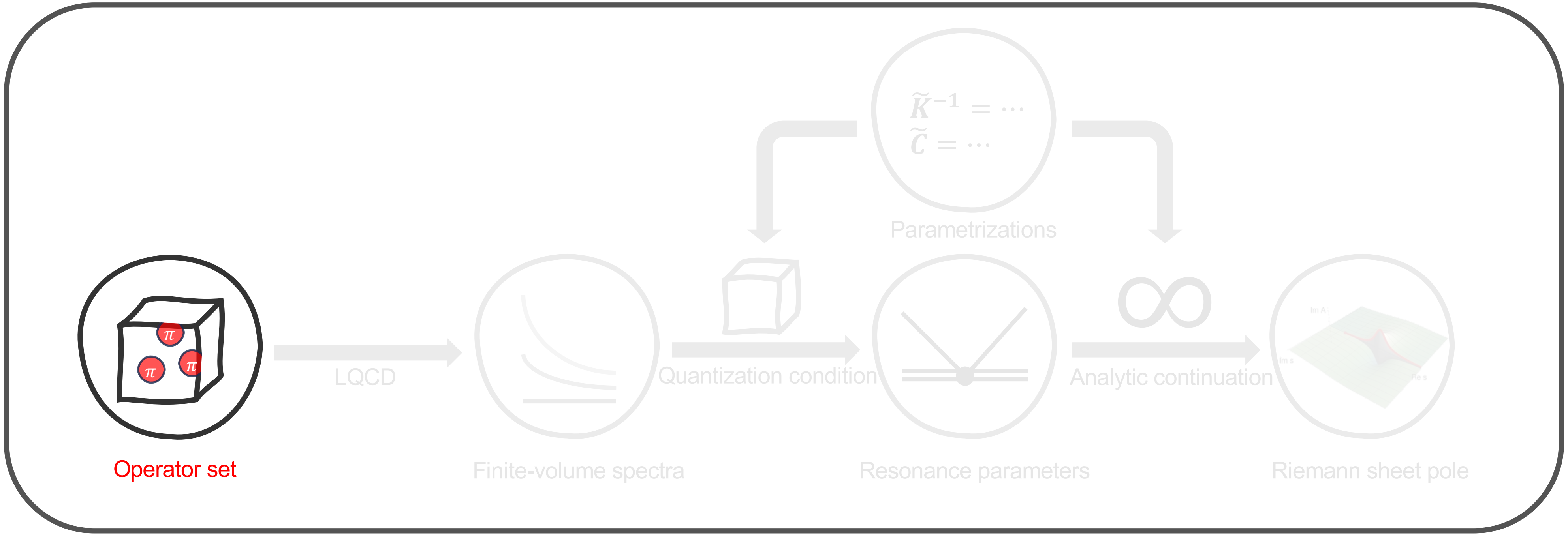
Lattice setup

configuration	volume	a/fm	m_π/MeV	N_{cfgs}
C48P14	$48^3 \times 96$	0.10530(18)	135.5(1.6)	259
F32P21	$32^3 \times 64$	0.07746(18)	210.9(2.2)	459
F48P21	$48^3 \times 96$	0.07746(18)	207.2(1.1)	222
F32P30	$32^3 \times 96$	0.07746(18)	303.2(1.3)	567
F48P30	$48^3 \times 96$	0.07746(18)	303.4(9)	201
H48P32	$48^3 \times 144$	0.05187(26)	317.2(0.9)	274

[CLQCD, 2024]

- CLQCD ensembles with $N_f = 2 + 1$ Clover fermions at the same lattice spacing
- Two pion masses

Routine



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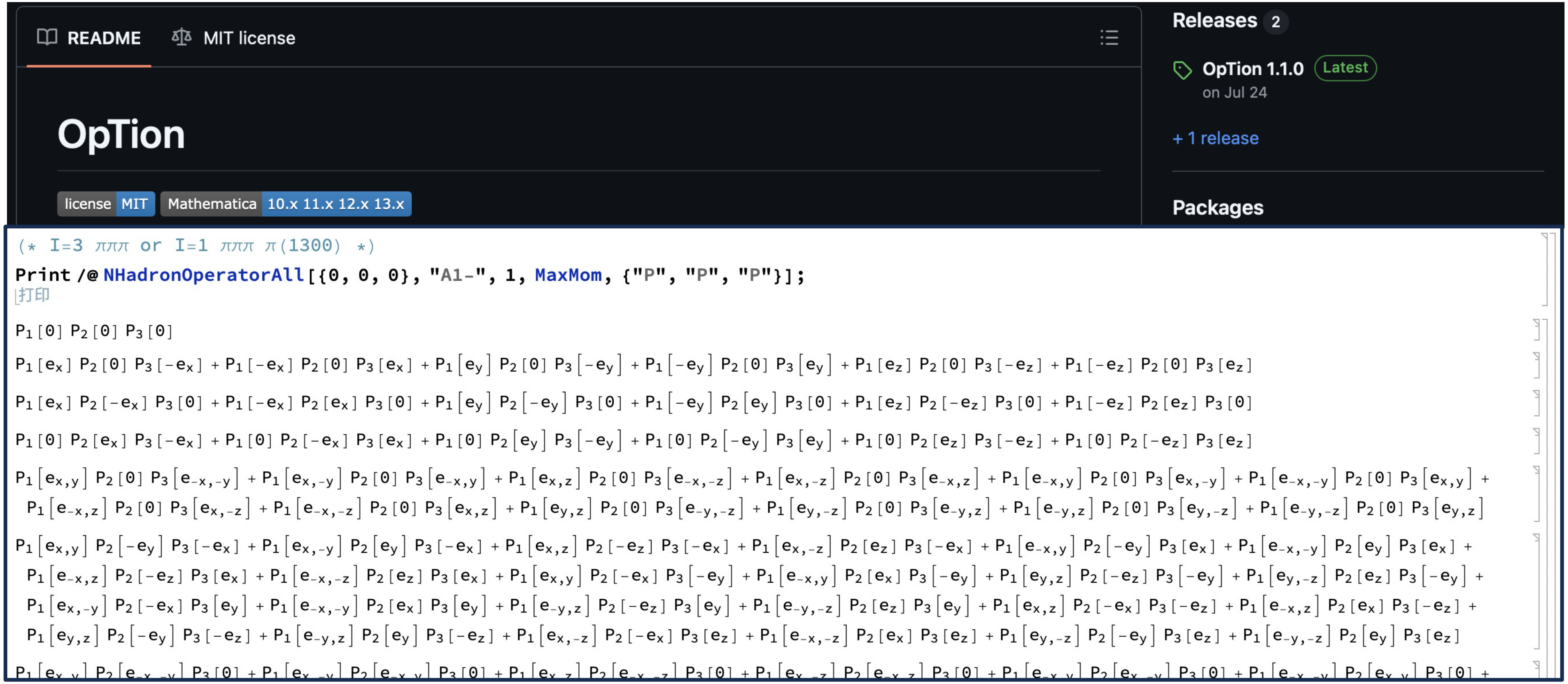
Operator construction

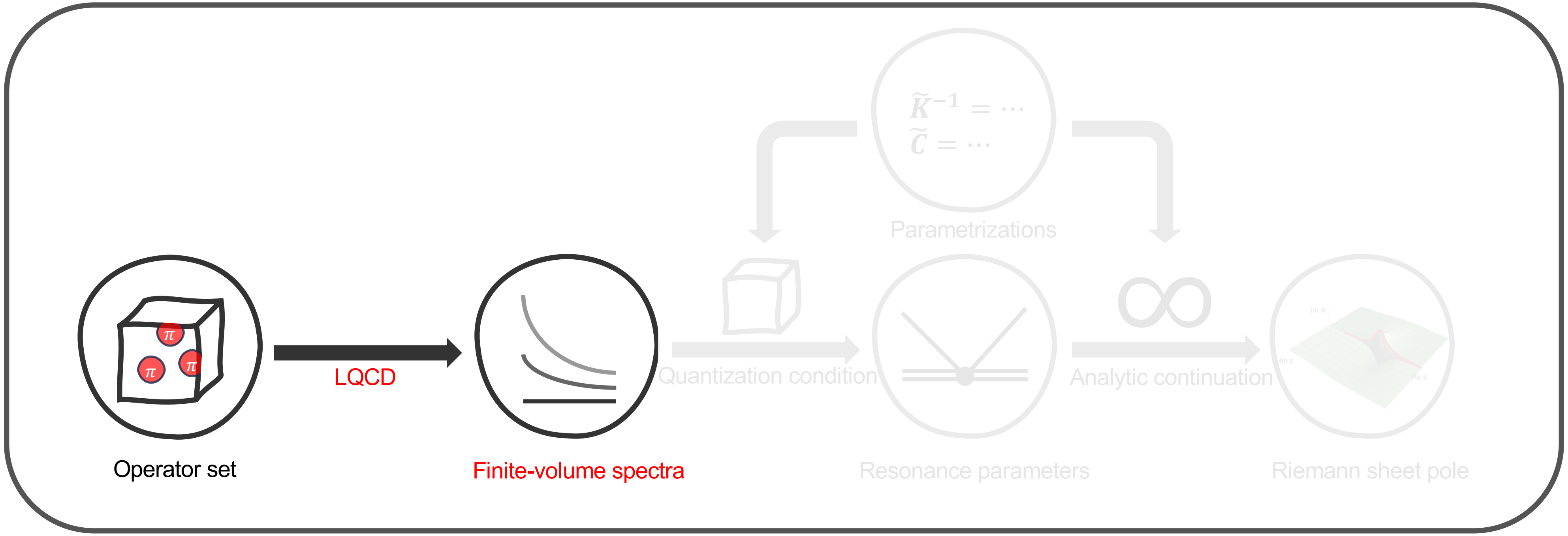
- **OpTion** is utilized to generate general N-hadron operators with arbitrary momenta



⋮

燕浩波（Haobo Yan），刘川（Chuan Liu），刘柳明（Liuming Liu），孟雨（Yu Meng），arXiv:2507.16070, JHEP (2025) (https://github.com/wittscien/OpTion)

Routine



π
π
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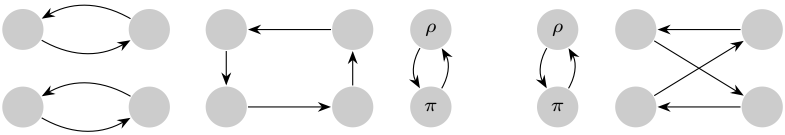
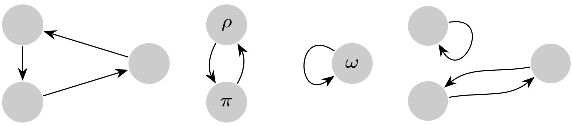
Contractions



D B A E

$\sim\mathcal{O}(10)$



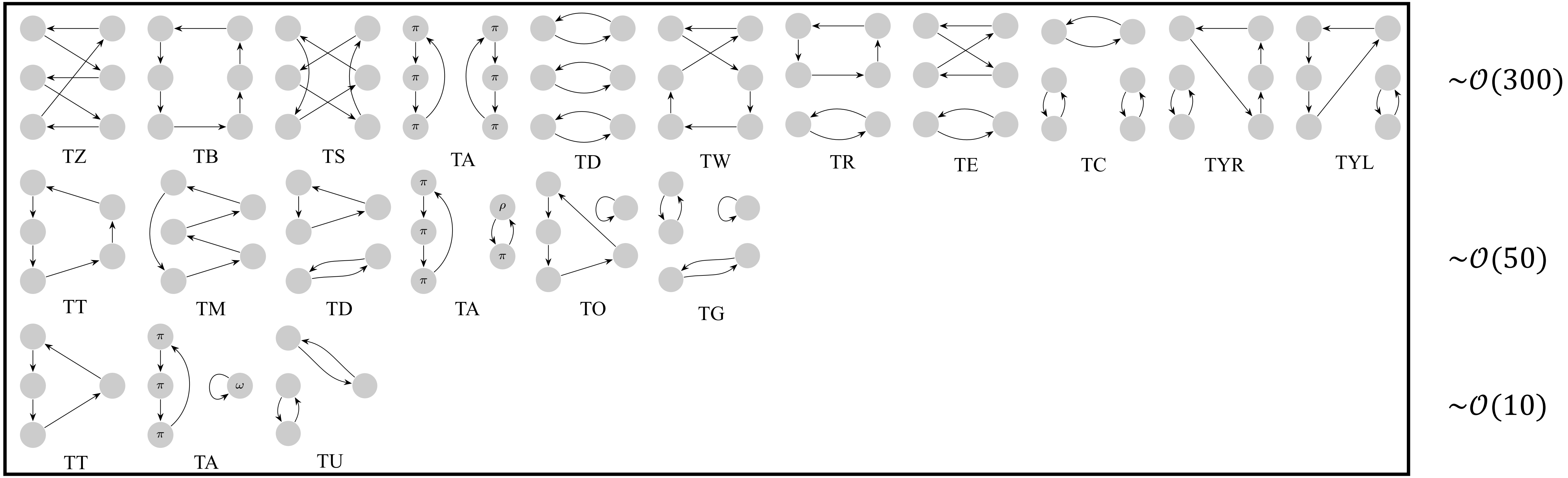
T A S

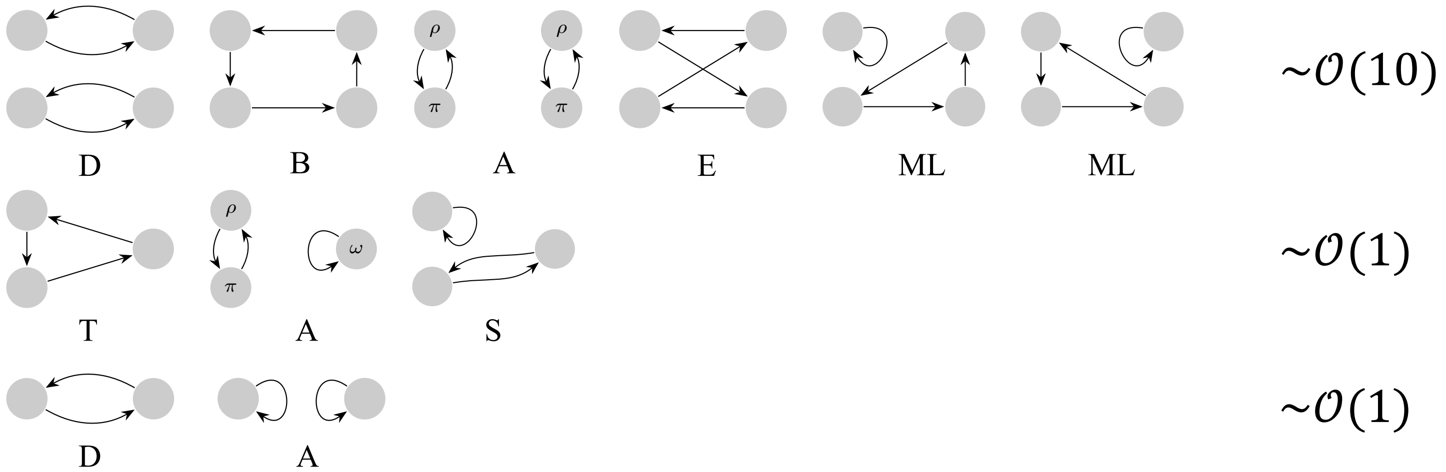
$\sim\mathcal{O}(1)$

D A

$\sim\mathcal{O}(1)$

Contractions

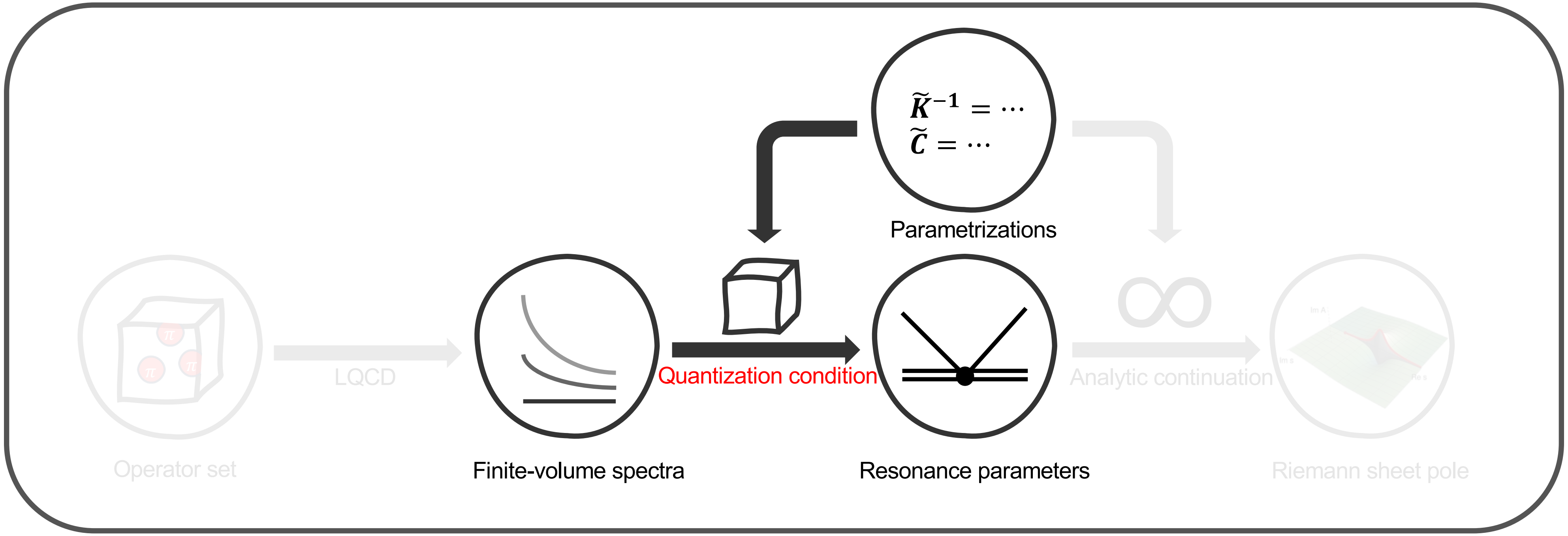




- A code is written to write code
- Of course, distillation is applied

Peardon et al (2009)

Routine



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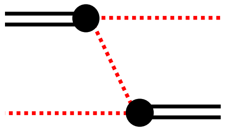
State-of-the-art formalisms

- FVU [Mai and Döring, 2017]
- RFT [Hansen and Sharpe, 2014]
- NREFT [Hammer, Pang, and Rusetsky, 2017]

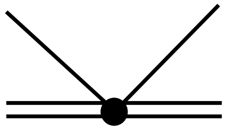
- In FVU (**F**inite-**V**olume **U**nitarity):

$$\begin{cases} \tilde{K}^{-1}(\sigma) - \Sigma^{FV}(\sigma) = 0 \\ \det[(\tilde{K}^{-1}(s) - \Sigma^{FV}(s))E_L - (\tilde{B}(s) + \tilde{C}(s))] = 0 \end{cases}$$



B

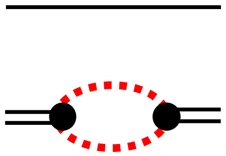
fixed by unitarity



C

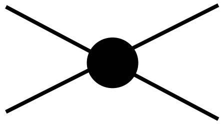
generic form

$$C^{\alpha\beta} = C_c^{\alpha\beta} + \frac{C_p^{\alpha\beta}}{s - m_0^2}$$



Σ

fixed by unitarity



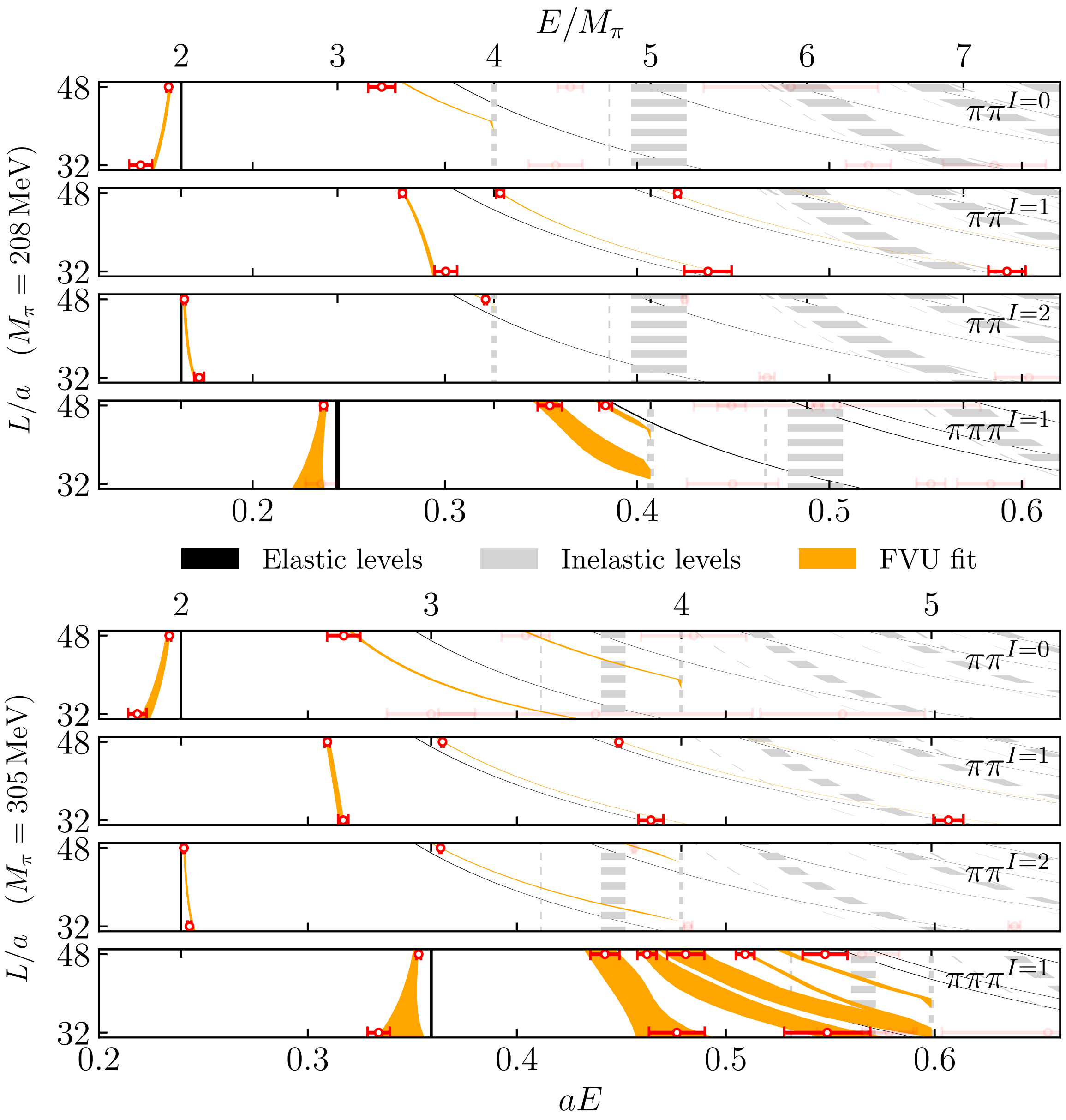
$\tilde{K}^{-1}$

Modified Inverse Amplitude Method (mIAM)

- Two-body is input to three-body
- Results not sensitive to Spectator momentum cutoff dependence
- $\tilde{K}^{-1} \to 0$ slowly below a cutoff

See Mai PRL (2019) for quantitative tests of smooth and hard cutoffs

Finite-volume spectra and fit

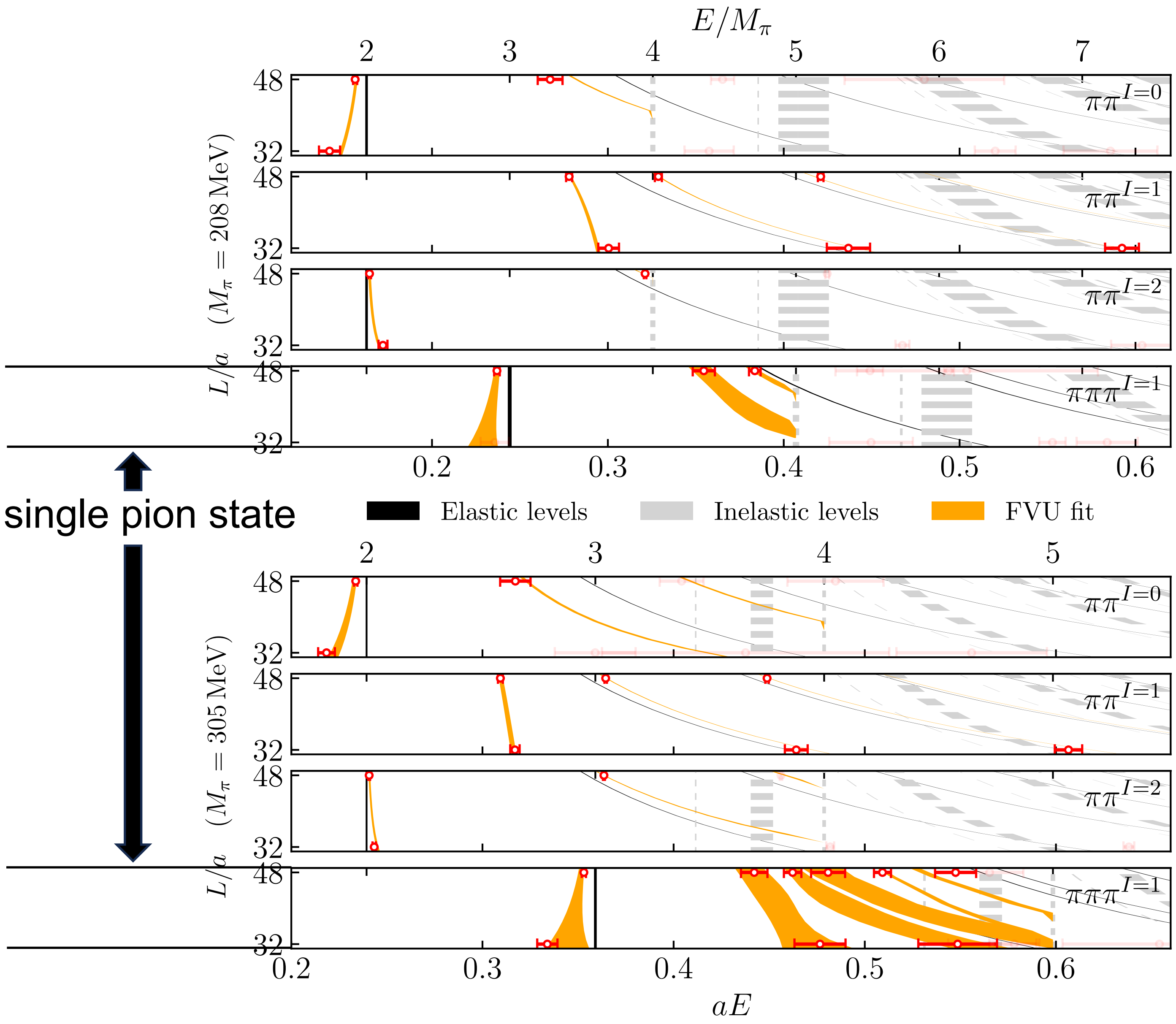


- $I = 0, 1, 2$ $\pi\pi$ levels as expected
- Aggregation of additional states in $\pi\pi\pi$

⬇

- Indication of a resonance in 0^-

Finite-volume spectra and fit



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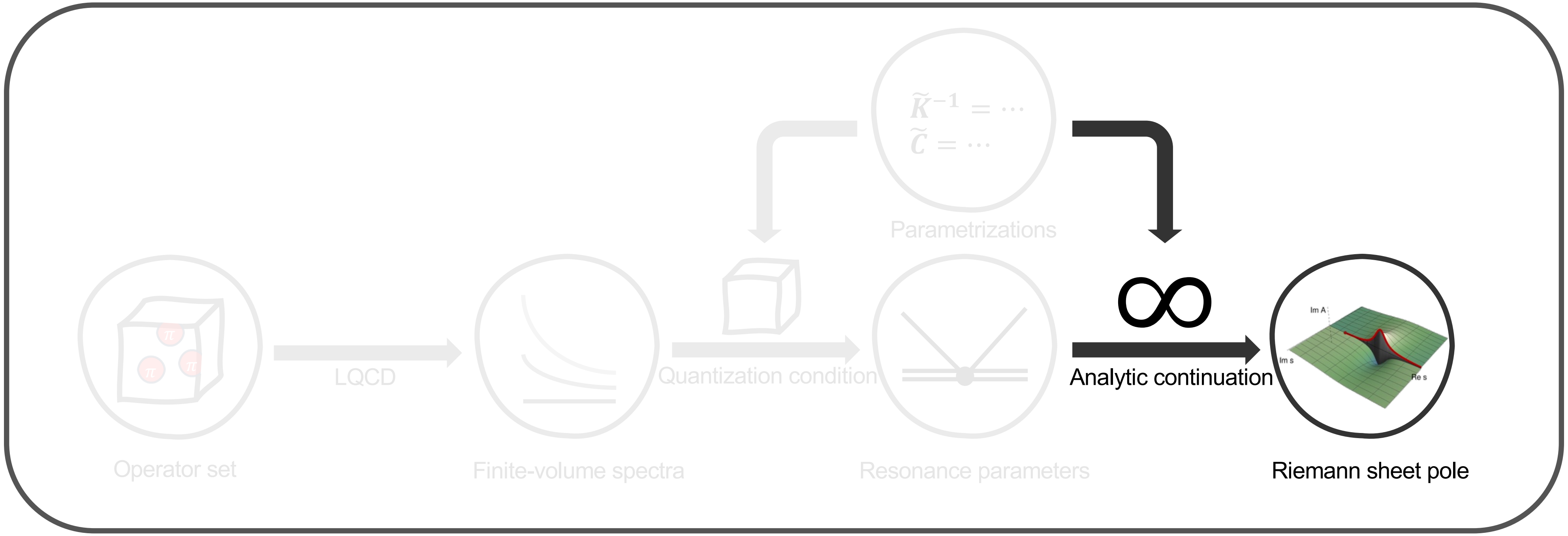
- Indication of a resonance in 0^-

Routine



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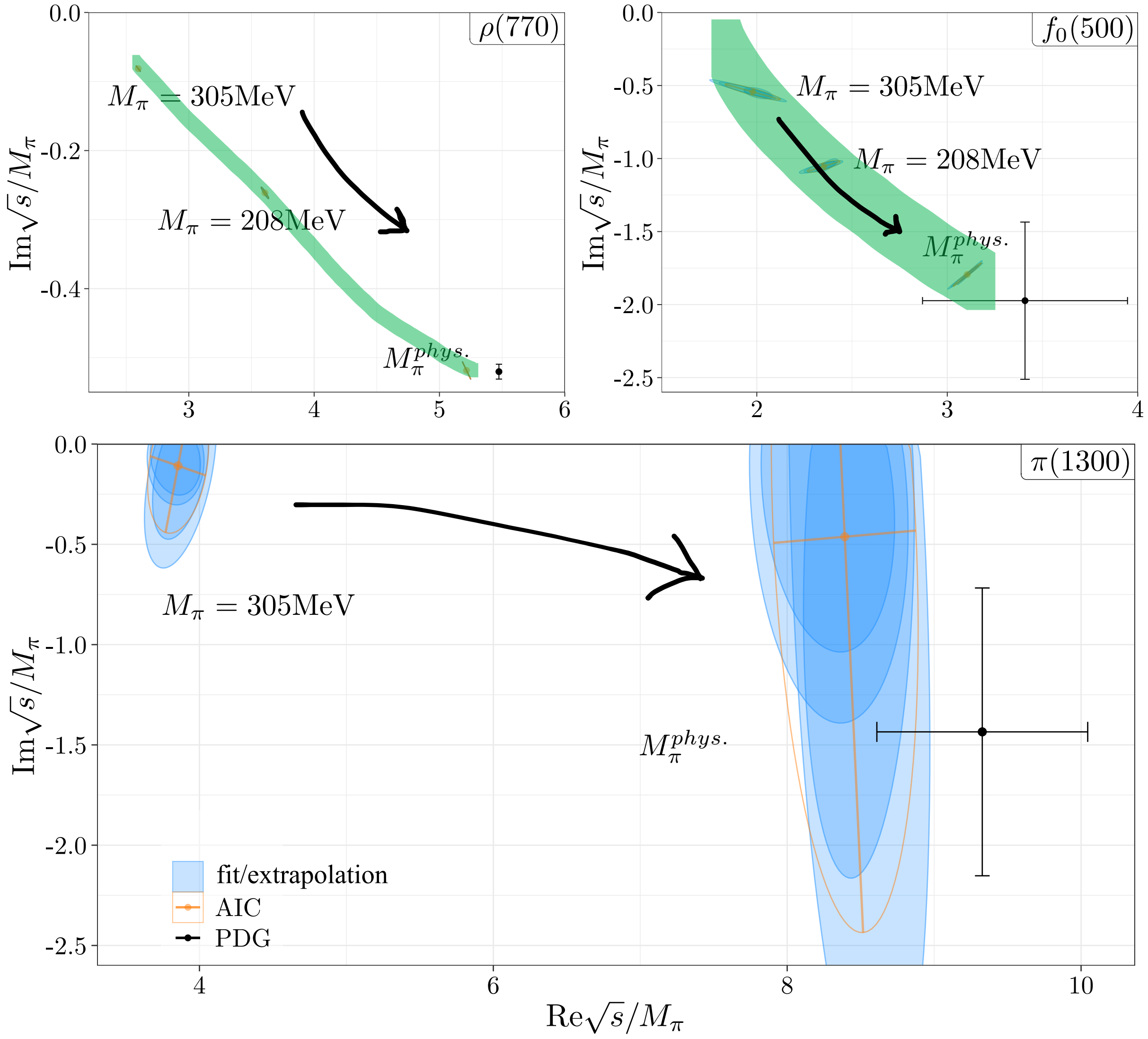
Pole positions



- $M_\pi = 305$ MeV: resonance
- $M_\pi = 208$ MeV: only few of the fits lead to poles
- Extrapolation to the physical pion mass
  - $\pi\pi$: mIAM (ChPT)
  - $\pi\pi\pi$: assume negligible M_π dependence

$$M_{\pi(1300)} = (1169 \pm 46) - i(62 \pm 169)\ \text{MeV}$$

$$M_{\rho(770)} = (727 \pm 3) - i(72 \pm 1)\ \text{MeV}$$

$$M_{f_0(500)} = (433 \pm 7) - i(250 \pm 7)\ \text{MeV}$$

- Limitations
  - Strangeness is not considered
  - mIAM is constraining
  - No continuum extrapolation

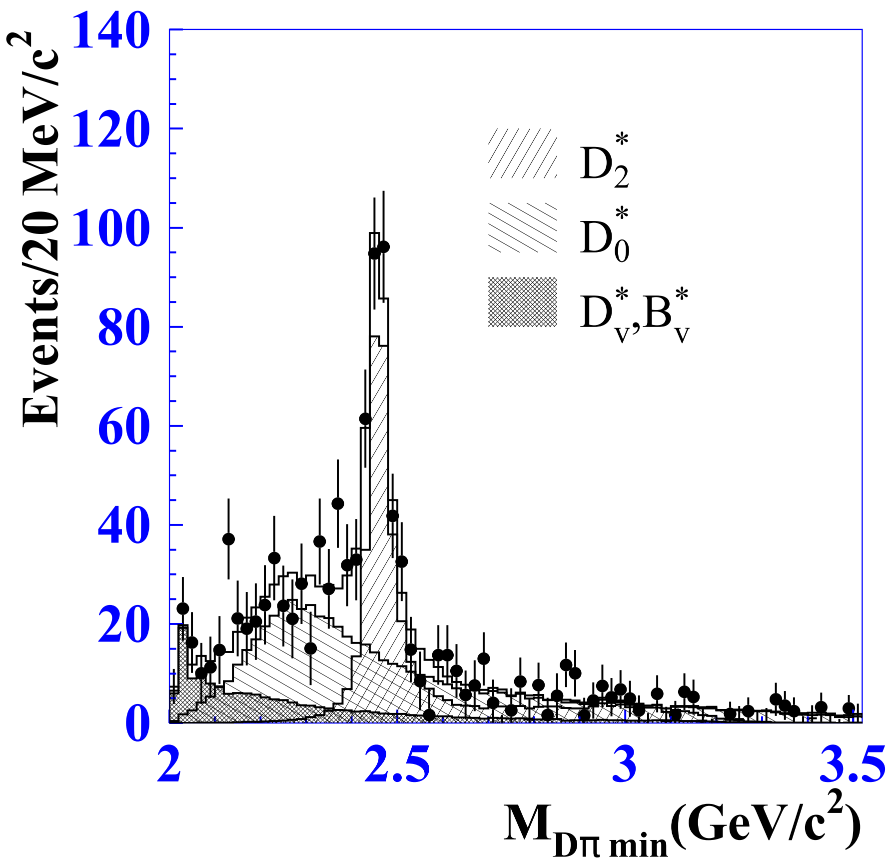
arXiv:2510:xxxxx – appearing tomorrow!

燕浩波（Haobo Yan）, Maxim Mai, Marco Garofalo, Yuchuan Feng, Micheal Döring, 刘川（Chuan Liu）, 刘柳明（Liuming Liu）, Ulf-G. Meißner, Carsten Urbach

Table of contents

Puzzles on $D_0^*(2300)$

- A broad resonance $D_0^* \to D\pi$ was found by Belle collaboration in 2004

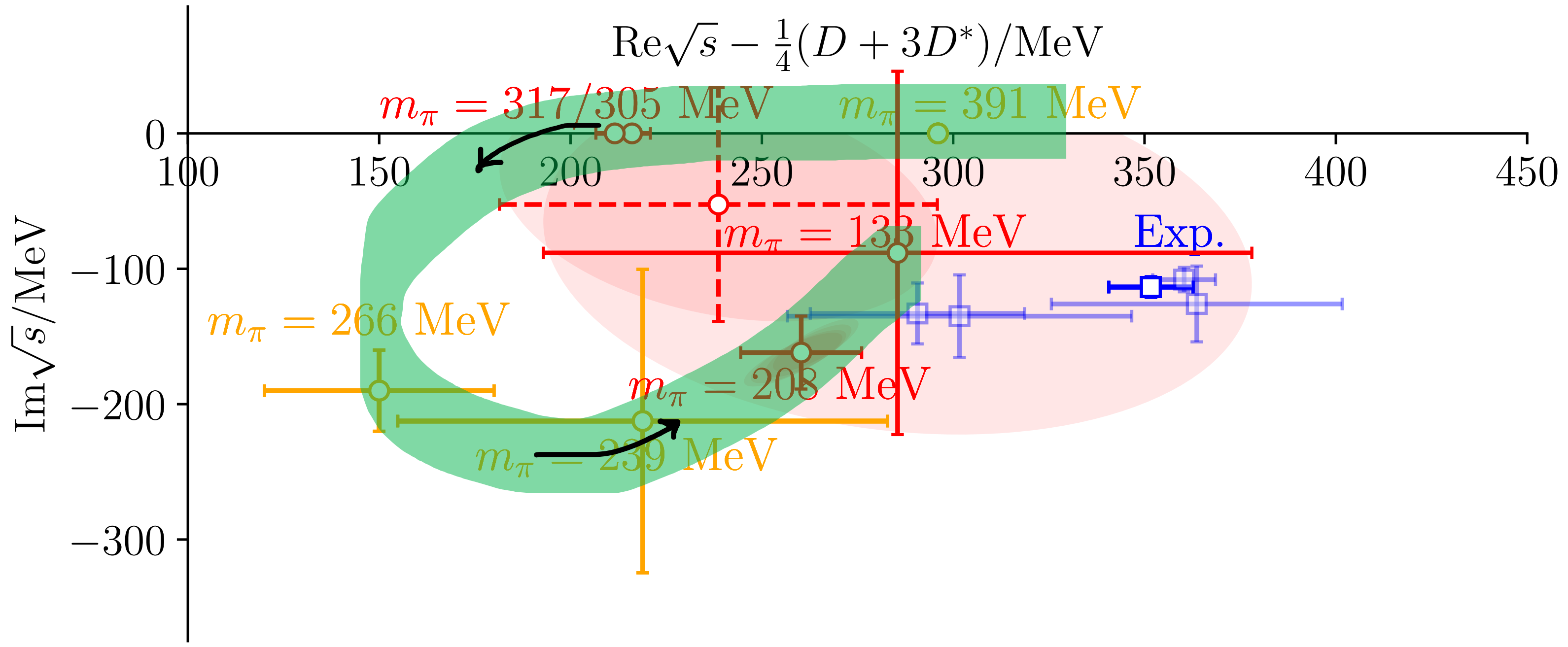


Satpathy et al, PRB 159 (2003) 553.

- $m_{D_0^*} \approx m_{D_{s0}^*}$
- $m_{D_0^*}^{ChPT} \ll m_{D_0^*}^{exp}$
- could be explained by UChPT where a **two-pole structure** is proposed Albaladejo et al, PLB 767 (2017) 465
- Should be tested on the lattice at a series of m_π
- Input for T_{cc}

@ Christoph Hanhart's talk on Exotic Hadron Spectroscopy 2024

What we know from single channel scattering



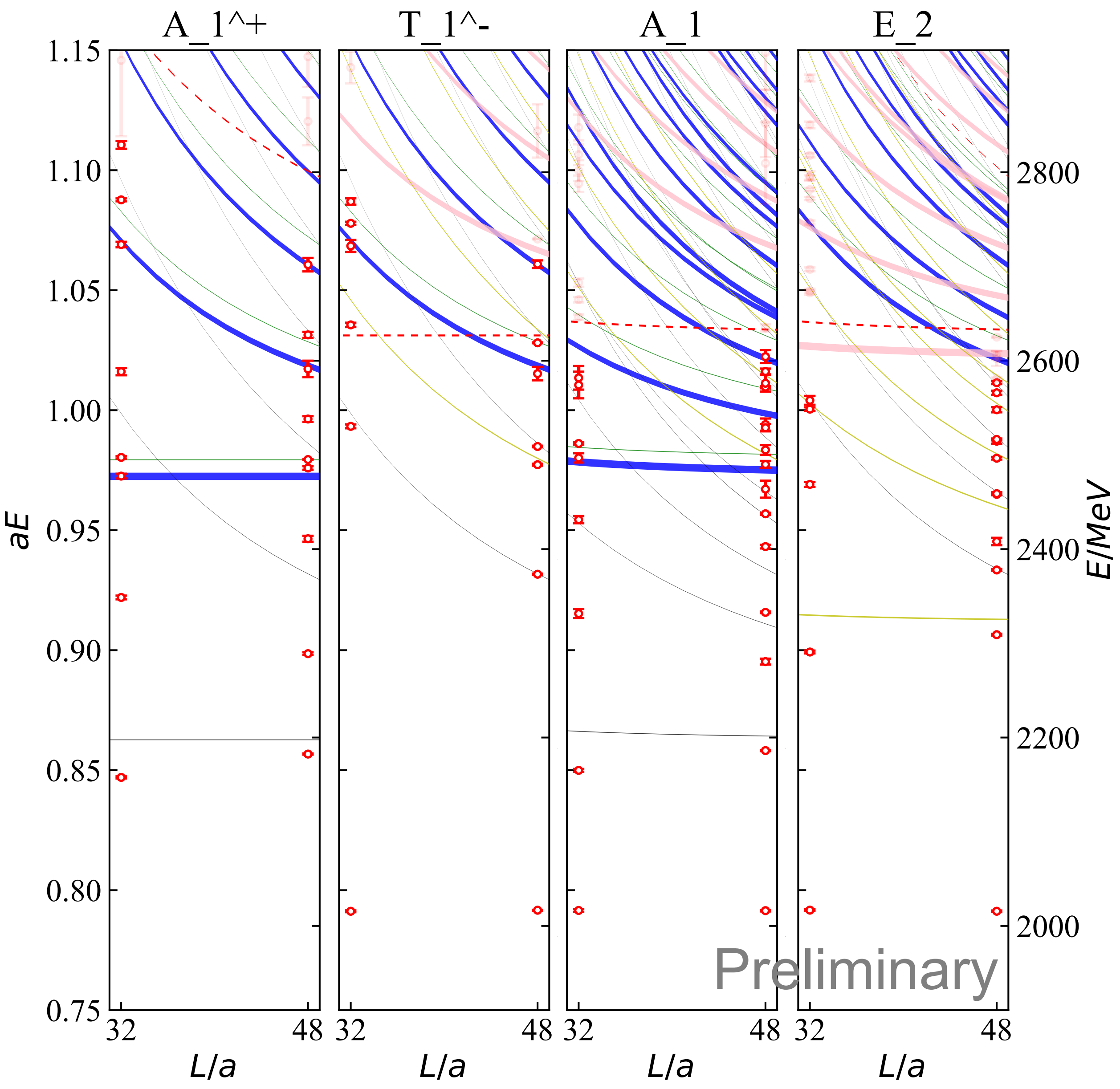
- Only **one pole** within the single-channel scattering picture
- Trajectory of the D_0^* pole is identified

bound state → virtual state → resonance

See related lattice works in
Mohler et al., PRD 87 ((2013)) 03450
HadSpec, JHEP 10 (2016) 011
HadSpec, JHEP 07 (2021) 123
HadSpec, JHEP 07 (2024) 012

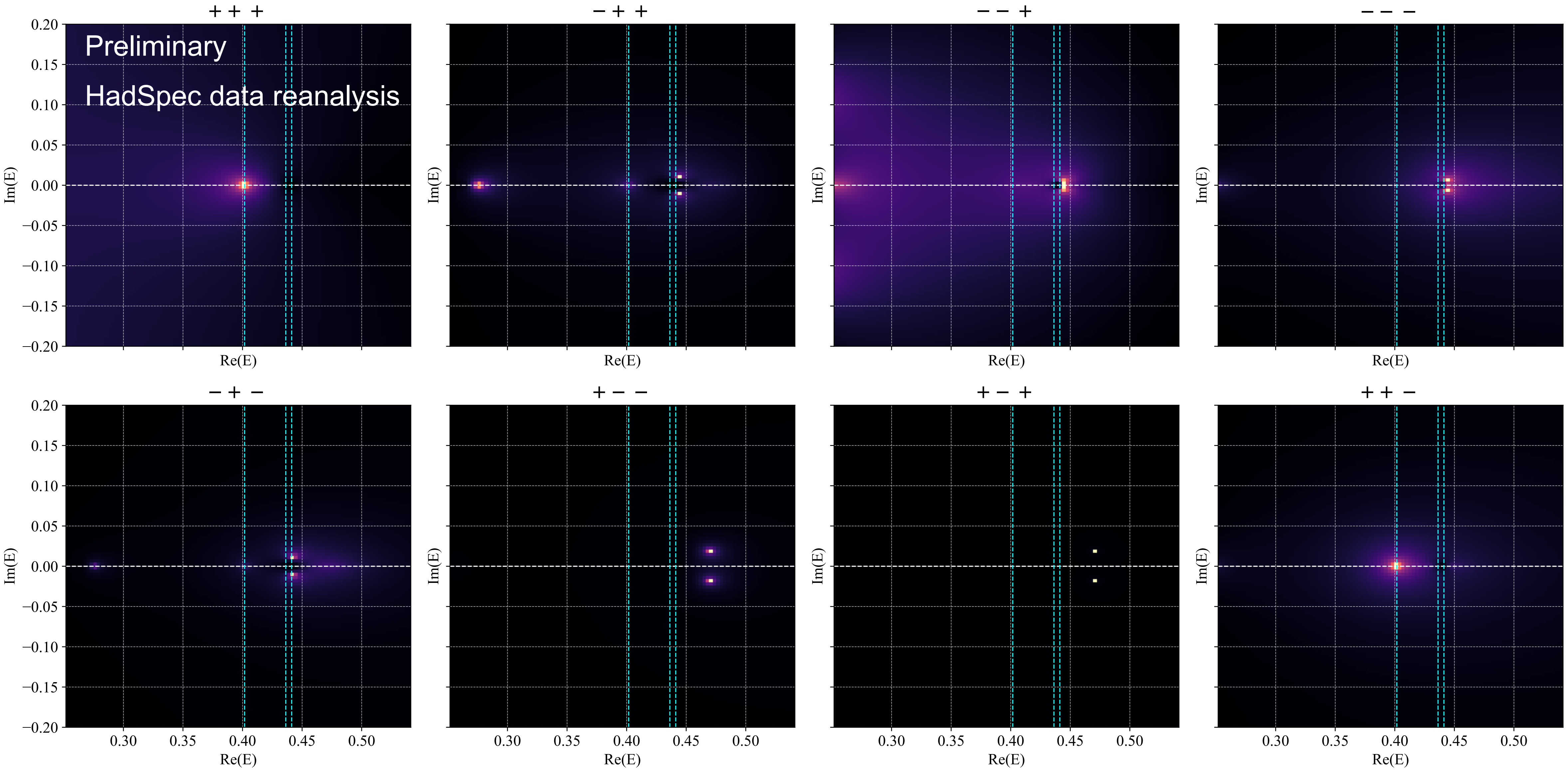
Haobo Yan, Chuan Liu, Liuming Liu, Yu Meng, Hanyang Xing, PRD 111 (2025) 014503

$D\pi - D\eta - D_s\bar{K} - D^*\pi$ coupled channel scattering

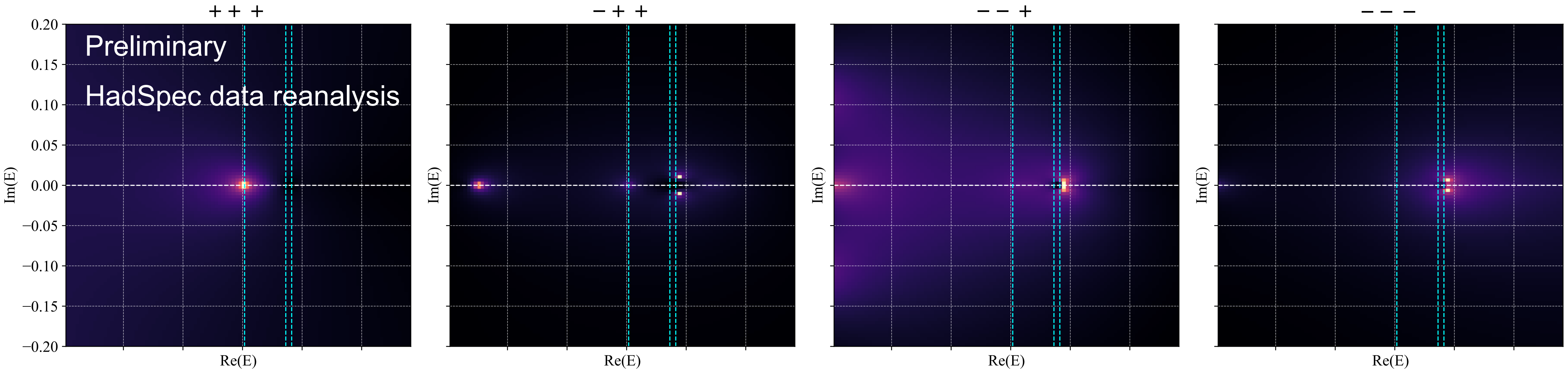


- $\mathcal{O}(1000)$ energy levels across irreps
- $D\pi - D\eta - D_s\bar{K}$ S and D wave
- $D^*\pi$ ${}^3S_1, {}^3P_0, {}^3P_1, {}^3P_2$ wave

$D\pi - D\eta - D_s\bar{K} - D^*\pi$ coupled channel scattering – pole



$D\pi - D\eta - D_s\bar{K} - D^*\pi$ coupled channel scattering – pole



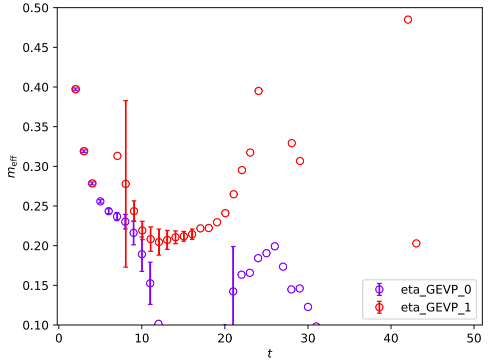
- CLQCD results expected soon -- **we do find 'two poles'**
- Combined picture
  - $M_\pi = 391$ MeV: bound + virtual + resonance
  - $M_\pi = 305$ MeV : virtual + virtual + resonance
  - $M_\pi = 208$ MeV : resonance + resonance

Interlude: the η, η' problem

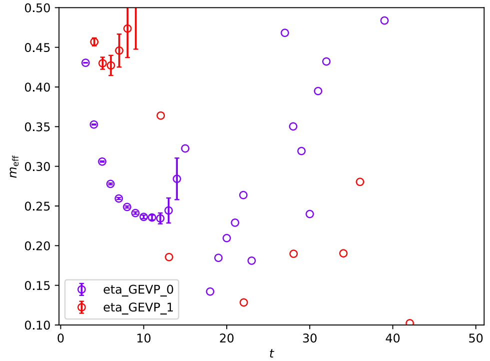
- η mixes with topology charge, leading to a constant pollution See, e.g. Ottnad et al, arXiv:2503.09895

$$\sim \frac{1}{V}\left(\chi_t - \frac{Q^2}{V} + \frac{c_4}{2V\chi_t}\right) + \mathcal{O}(V^{-2})$$

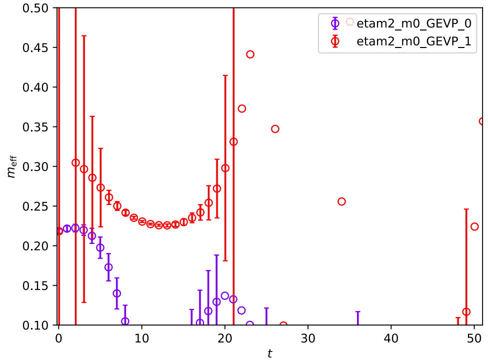
See @ 王楠's talk for details



$C(t) - C(t+1)$



- $C = \mathrm{conn} + \mathrm{disc} \rightarrow$ reconstruct the pion part to subtract off the excited state effect hep-lat/0106016



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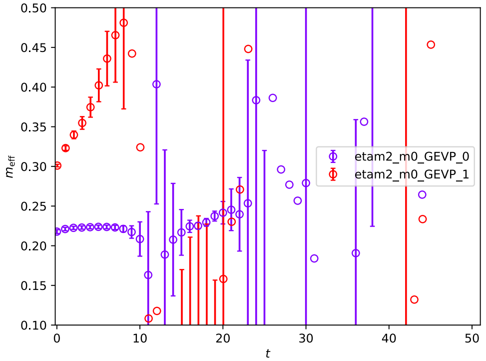

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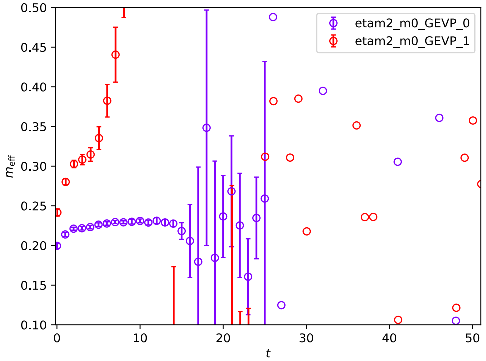
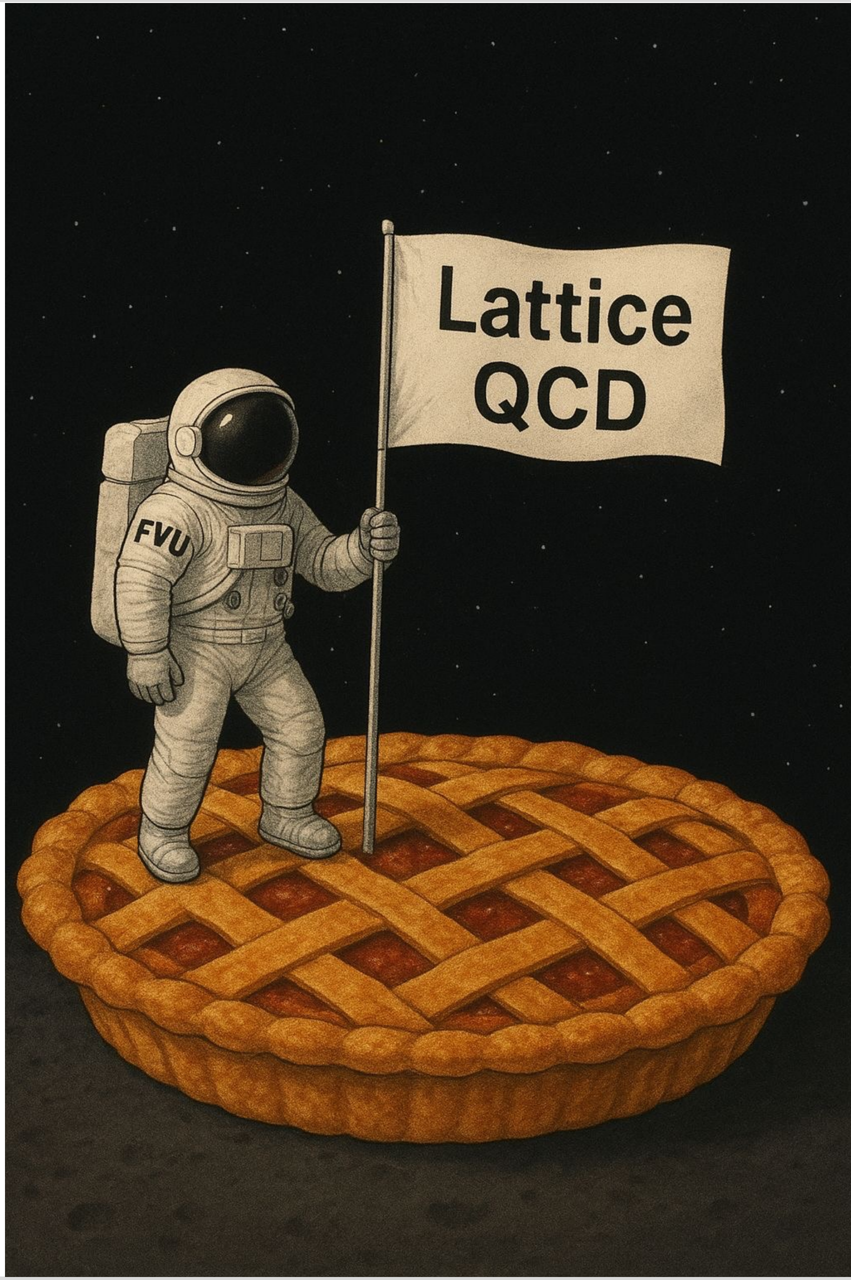


Summary

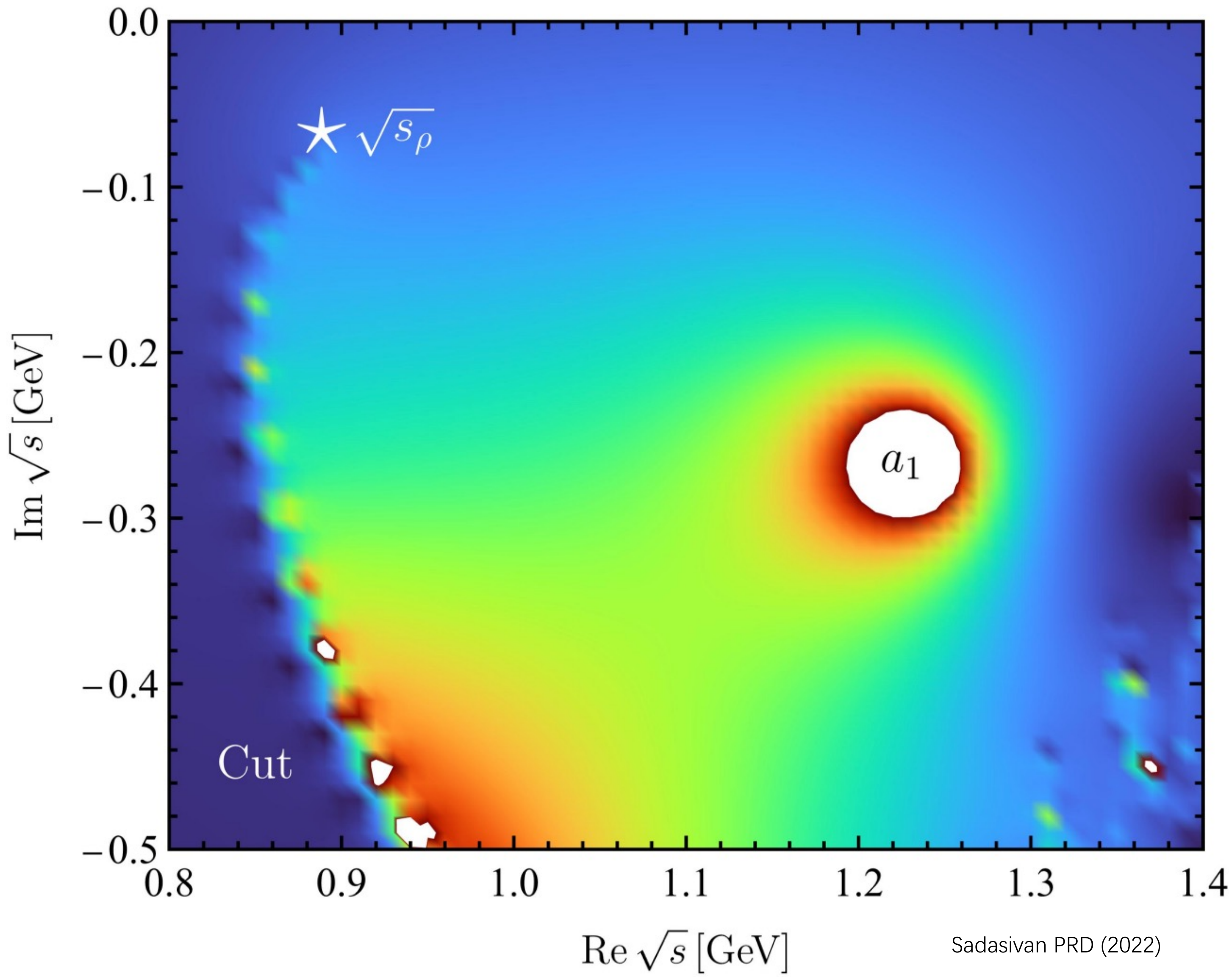
- Discovery of $\pi(1300)$ from QCD
- Two-pole structure of $D_0^*(2300)$ identified, results expected soon
- One step closer resolving the Roper resonance

Thank you!

$\boldsymbol{\pi}$



Analytic continuation example



Sadasivan PRD (2022)

- It is possible to do a smooth analytical continuation in our energy of interest
- Spectator momentum cutoff is chosen such that the analytical continuation covers the interested region of the Riemann sheet