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LQCD Determination of Quark Spin in Octet Baryons and SU(3)-Flavor Symmetry

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Context

Context

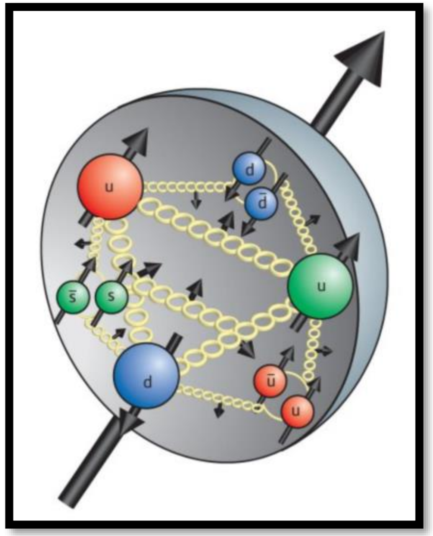
◆ **Chapter 1: Introduction**

◆ **Chapter 2: Theoretical Framework**

◆ **Chapter 3: Numerical result of Quark Spin and Discussion**

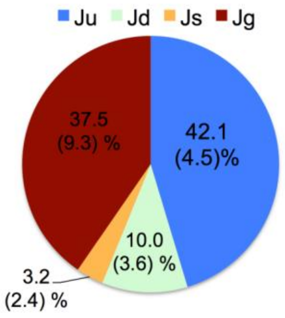
◆ **Chapter 4: Conclusion and Outlook**

1. Introduction: Decomposition of Proton Spin



Ji's sum rule: $\frac{1}{2} = \sum_{q=u,d,s,\cdots} (\frac{1}{2}\Delta q + L_q) + J_g$

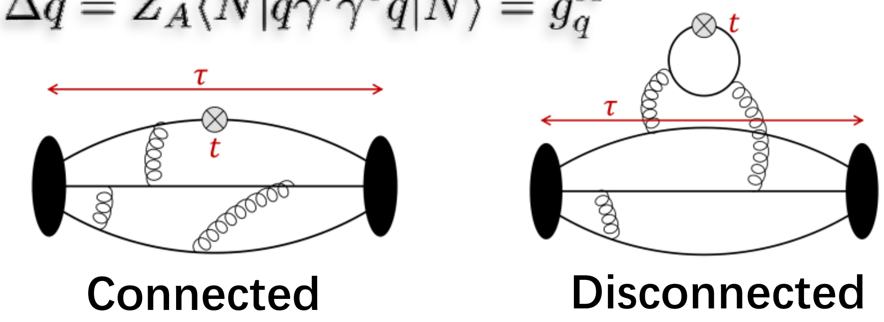
*X.D. Ji, PRL **78**, 610(1997)*



*Nature Rev.Phys. **3**, 27-38(2021)*

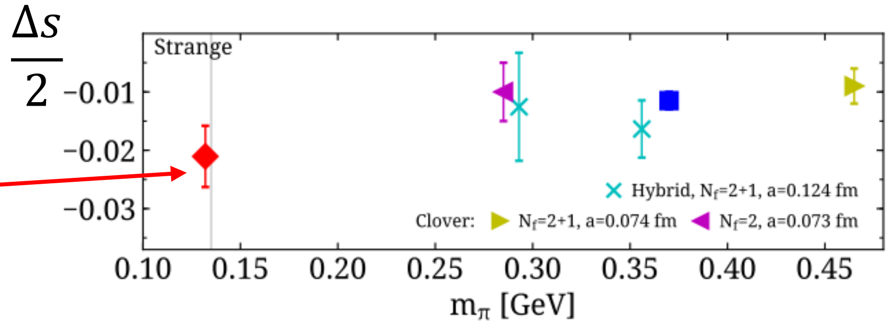
- Total intrinsic quark spin contribute ~30-40% to nucleon;

$$\Delta q = Z_A \langle N|\bar{q}\gamma^i\gamma^5 q|N\rangle = g_q^A$$



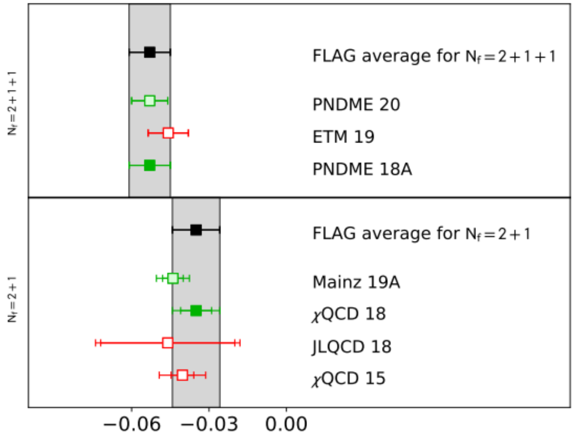
- LQCD faces large challenge on calculate quark-disconnected insertion contribution;

$$\left.\begin{aligned} n_{\text{cfg}} &= 2153 \\ n_{\text{src}} &= 100 \\ n_{stoc} &= 1087 \end{aligned}\right\}$$



*C. Alexandrou et al., PRL **119**, 142002 (2017)*

g_A^s

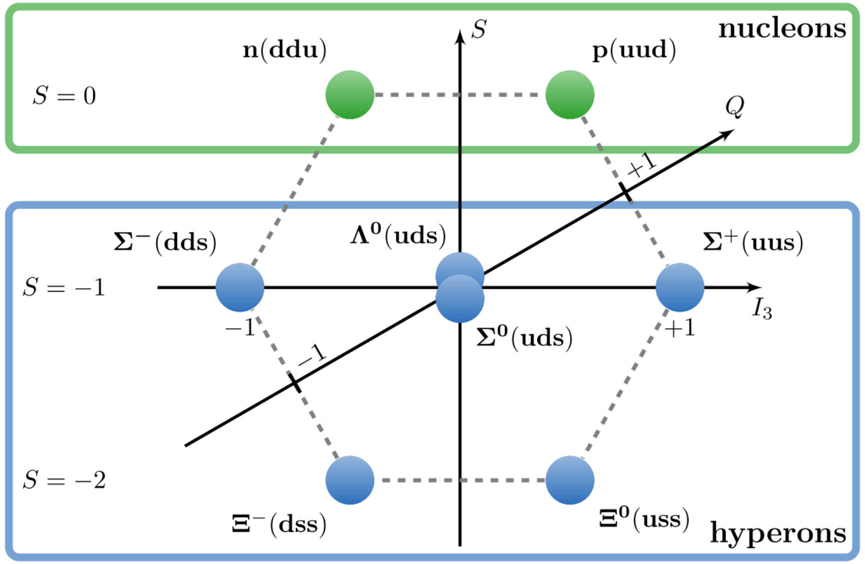


*PNDME, PRD **98**, 094512 (2018)*

*χQCD, PRD **98**, 074505 (2018)*

Mainz, arXiv 1911.01177 (2019)

1. Introduction: SU(3) flavour symmetry

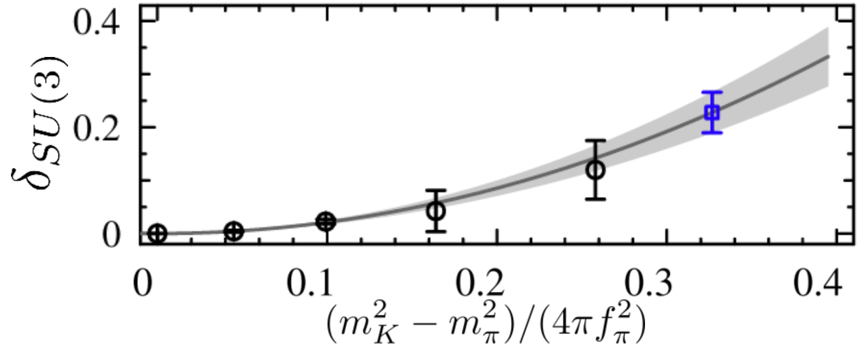


$$m_u^{\overline{\text{MS}}(2\text{GeV})} \simeq 2.1\text{MeV}$$
$$m_d^{\overline{\text{MS}}(2\text{GeV})} \simeq 4.7\text{MeV}$$
$$m_s^{\overline{\text{MS}}(2\text{GeV})} \simeq 93\text{MeV}$$

FLAG(2024), arXiv:2411.04268

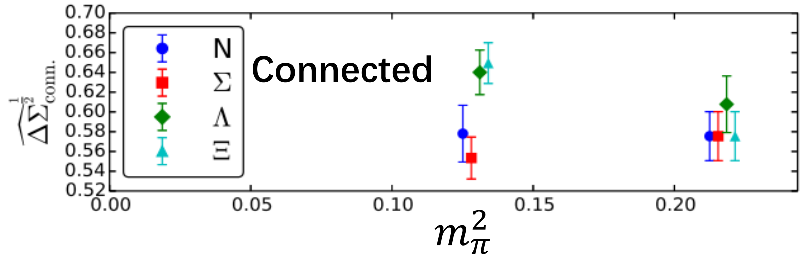
- There is moderate SU(3) symmetry breaking effects for the iso-vector axial charges, $\delta_{SU(3)}/g_A^n \sim 9\%$;

$$\delta_{SU(3)} = g^n_{A(u-d)} - g^{\Sigma}_{A(u-d)} + g^{\Xi}_{A(u-d)}$$



*H.W. Lin et al, PRD **79**, 034507 (2009)*
*A. Savanur, PRD **102**, 014501 (2020)*
*RQCD, PRD **108**, 034512 (2023)*

- How does SU(3) flavour symmetry and breaking affect Hyperons Spin Structure? *A. J. Chambers et al, PRD **90**, 014510 (2014)*



✓ In this study we use a novel ***Blending method*** to study **"Quark Spin in Octet Baryons"** (including quark DI contribution)at 4 ensembles(include 3 pion mass and 2 lattice spacing).

Context

◆ Chapter 1: Introduction

◆ **Chapter 2: Theoretical Framework**

◆ Chapter 3: Numerical result of Quark Spin and Discussion

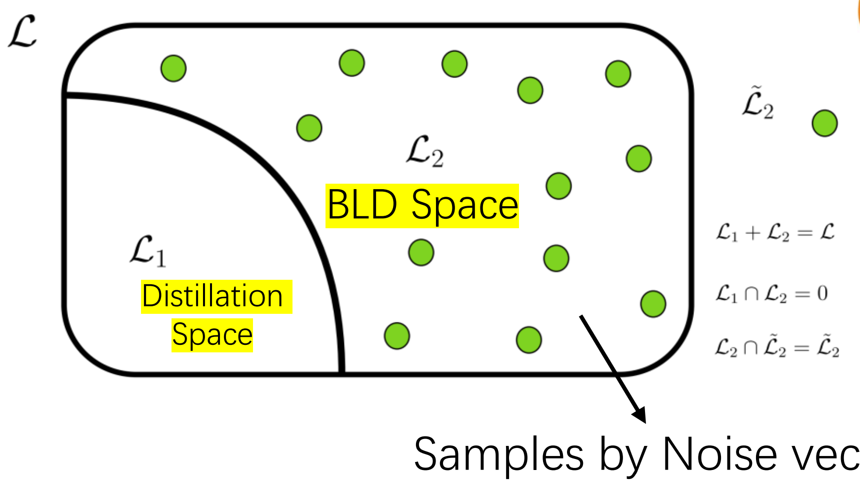
◆ Chapter 4: Conclusion and Outlook

2: Theoretical Framework: Blending Method

- *Distillation* can significantly improve the signal-to-noise ratio and suppress excited-state contamination;

$$-\nabla^2|\phi\rangle = \lambda|\phi\rangle,$$

$$L = \{|\lambda\rangle; -\nabla^2|\phi\rangle = \lambda|\phi\rangle\}.$$ *PRD. **80**, 054506 (2009)*



- Distillation space $\mathcal{L}_1$ is not complete, resulting in bias of local current ME;
- *Blending* method(BLD) can provide an unbiased estimation of identity matrix, based on samples in $\mathcal{L}_2$;

Z.C. Hu et al. , arXiv: 2505.01719(2025)

✓ All-to-All fermion Propagator could be stored and reused.

$$S_{ij}(t_1,t_2) = \int \mathrm{d}^3x\mathrm{d}^3y\langle\phi_i(\vec{x},t_1)|S(\vec{x},t_1;\vec{y},t_2)|\phi_j(\vec{y},t_2)\rangle,$$

All-to-all propagator size: $(4N_cN_TN_V)^2$ ✗ Unacceptable!

1. Vectors → $(N_cN_TN_V)(N_e+N_{st})$

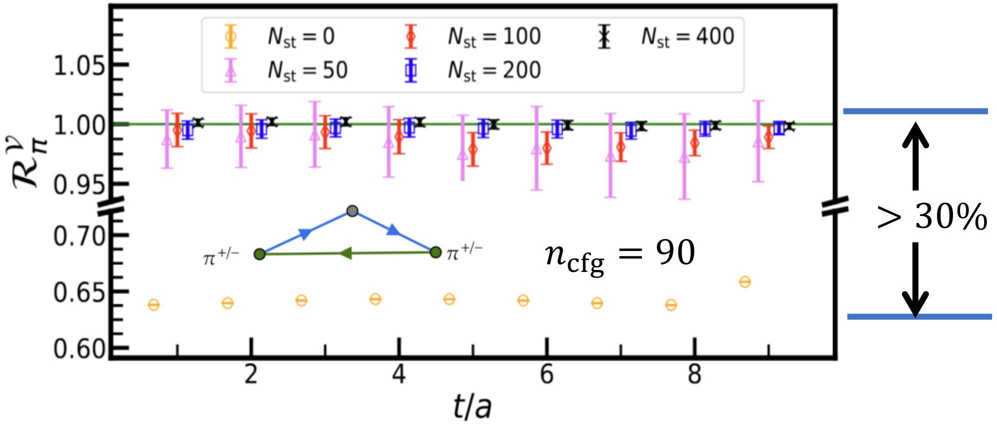
2.Perambulator → $[4N_T(N_e+N_{st})]^2$

$$\frac{N_e+N_{st}}{[\mathcal{L}]} \approx 1\%$$

✓ Acceptable!

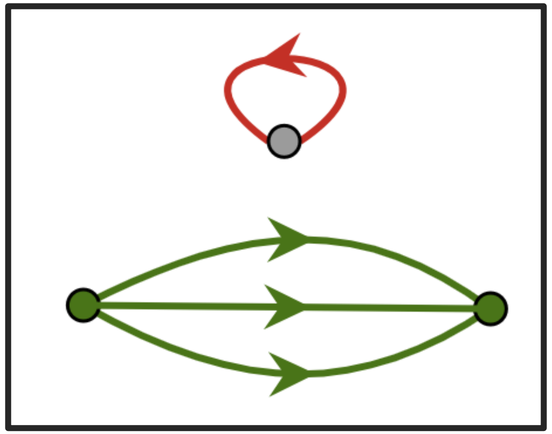
Conserved Current Test:

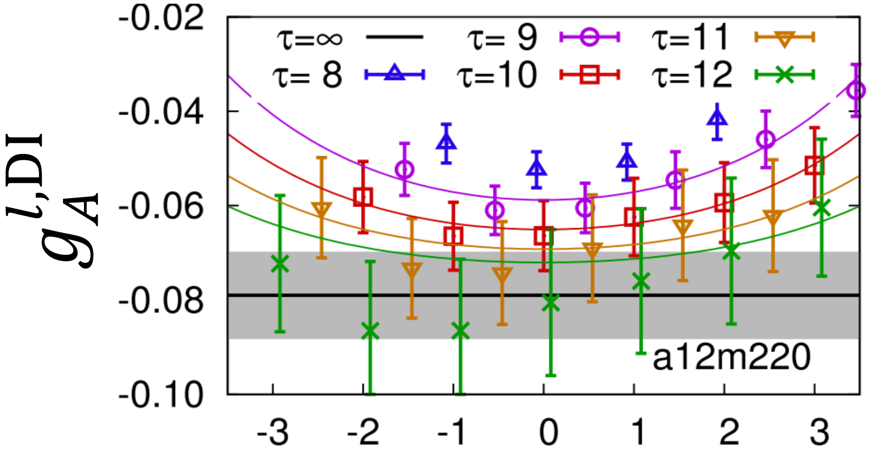
$$\langle H|V^c|H\rangle/\langle H|H\rangle = 1$$



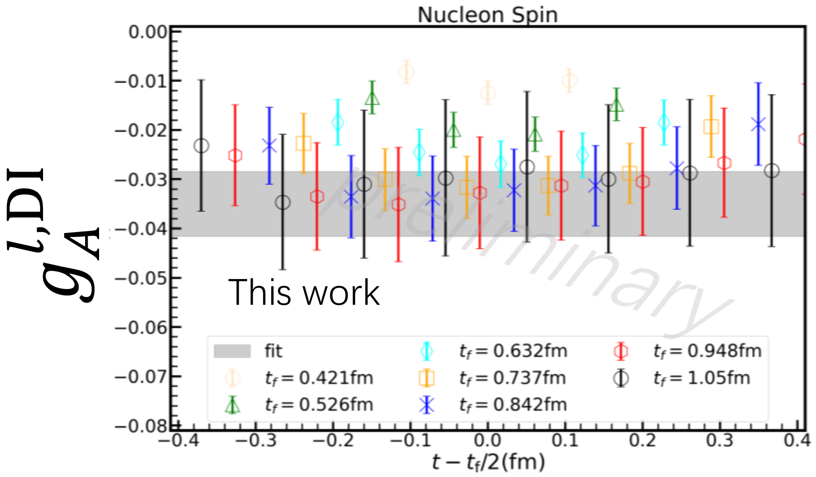
✓ $\langle\pi^+|V^c|\pi^+\rangle = 1.0002(29)$ with $N_{st} = 400$, $m_\pi \approx 290\mathrm{MeV}$;

2: Theoretical Framework: Blending Method





PNDME, P.R.D 98, 094512 (2018)



a) A better ground-state convergence!

b) The cost could be control very well.

c) Data of fermion Propagator could be stored and reused.

	Ensemble	m_π	$L^3 \times T$	n_{cfg}	n_{inv}	$g_A^{l,\mathrm{DI}}$	$g_A^{s,\mathrm{DI}}$
PNDME Collaboration	a12m220	228MeV	32x64	958	11000	-0.075(9)	-0.037(6)
CLQCD(This work)	C32P23	227MeV	32x64	80	$3200^\dagger$	-0.035(7)	-0.021(2)

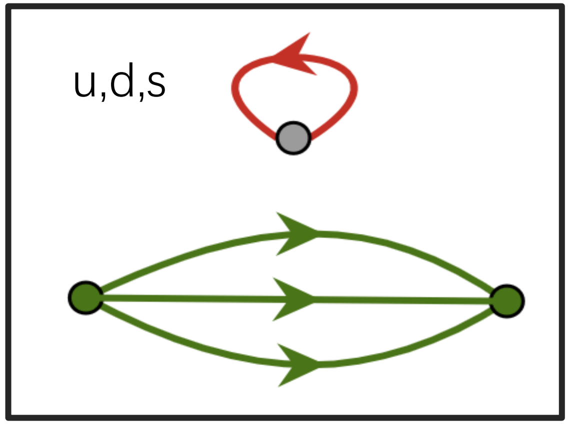
- 2+1 flavor tadpole-improved **Clover Fermion** with Symanzik gauge action;

	$a(\mathrm{fm})$	$m_\pi(\mathrm{MeV})$	$L_x \times L_t$	$m_\pi L$	n_{cfg}
C32P29	0.105	292	32x64	5.0	200
C32P23	0.105	227	32x64	3.9	80
C48P14	0.105	136	48x96	3.8	45
F48P30	0.077	303	48x96	5.7	40

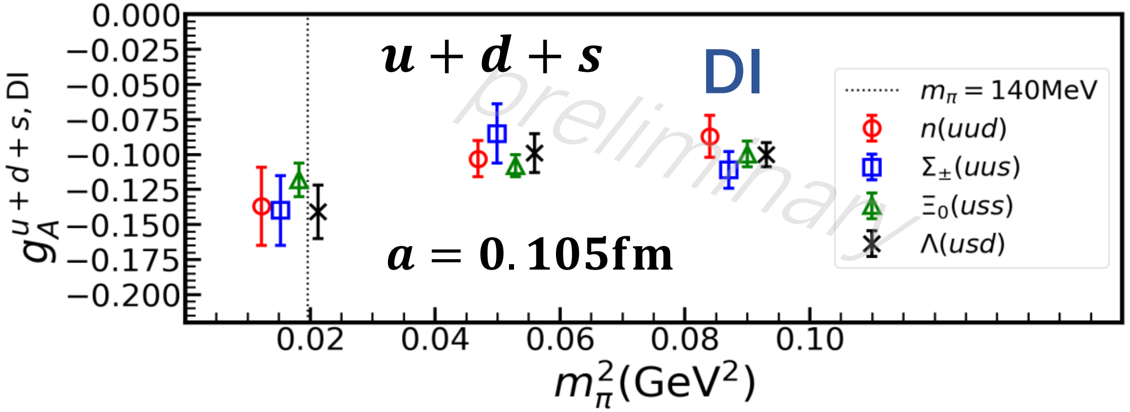
*CLQCD, PRD **109**, 054507 (2024)*
*CLQCD, PRD **111**, 054504 (2025)*

Context

3.1 Disconnected Insertion Contribution



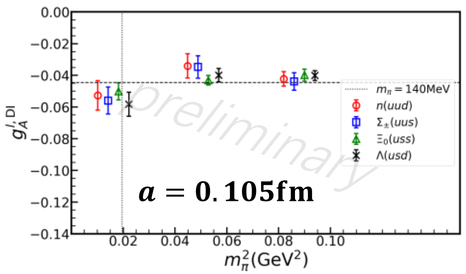
✓ The contribution of sea quarks is basically consistent in all octet baryons;

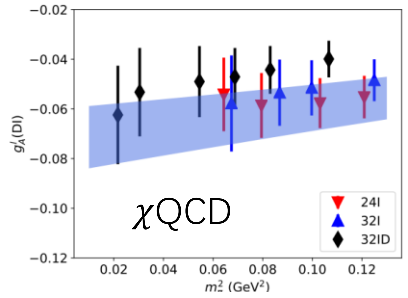


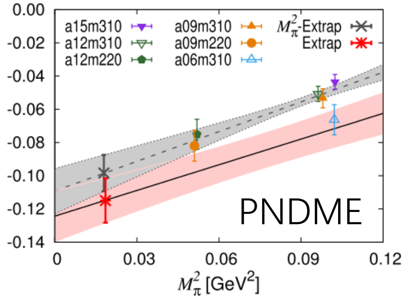
■ How about SU(3) symmetry for gluon helicity ΔG ?

➢ We didn't see remarkable m_π dependence at for disconnected quark insertion contribution;

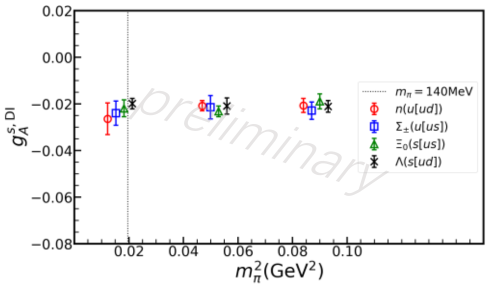
up/down

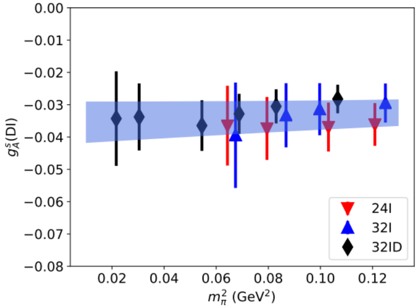


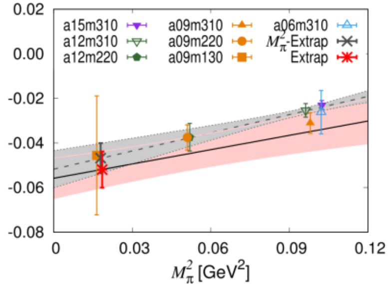




strange







■ CLQCD result is more closed χQCD collaboration, which is support for weaker m_π dependence.

*PNDME, PRD **98**, 094512 (2018)*

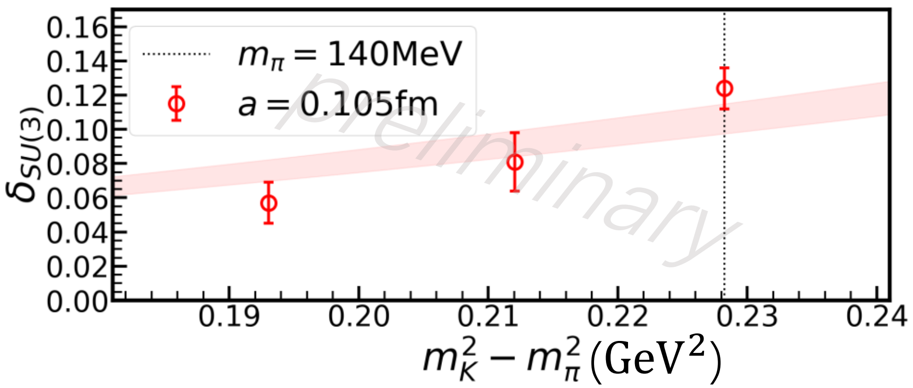
*χQCD, PRD **98**, 074505 (2018)*

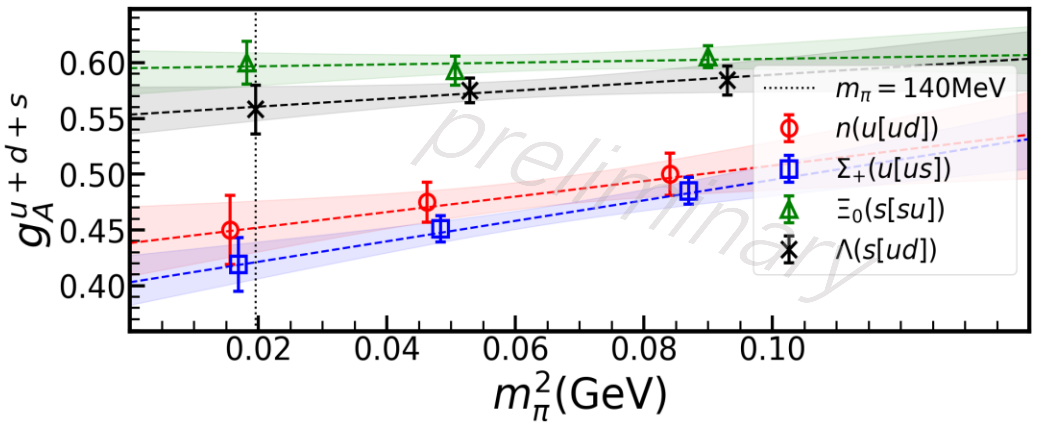
3.2 The total singlet quark spin and SU(3)-flavour symmetry

up/down dominant $\begin{cases} n = u(u^T C\gamma^5 d) \\ \Sigma^+ = u(u^T C\gamma^5 s) \end{cases}$

strange dominant $\begin{cases} \Xi_0 = s(s^T C\gamma^5 u) \\ \Lambda_0 = 2s[u^T d] + d[u^T s] - u[d^T s] \end{cases}$

$$\delta_{SU(3)} = g^n_{A(u-d)} - g^\Sigma_{A(u-d)} + g^\Xi_{A(u-d)}$$





- There is moderate SU(3) symmetry breaking effects for the iso-vector axial charges, $\delta_{SU(3)}/g_A^n \sim 10\%$;

- There are SU(3) symmetry breaking in quark spin at $a = 0.105$fm ;
- SU(3) symmetry keep well inside di-quark operator;

$$n(u[ud]) \underset{\mathrm{d}}{\overset{\mathrm{s}}{\rightleftarrows}} \Sigma(u[us]) \qquad \Xi(s[su]) \underset{\mathrm{s}}{\overset{\mathrm{d}}{\rightleftarrows}} \Lambda(s[ud])$$

3.2 The total singlet quark spin and SU(3)-flavour symmetry

◆ After continuum extrapolation, there is a tendency for the SU(3) symmetry to partially recover.

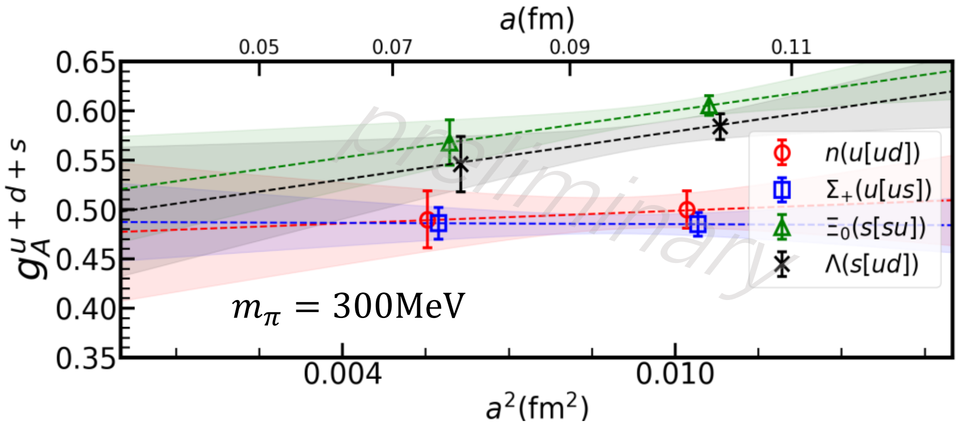
n: $\frac{1}{2}\Delta q \sim 0.20 - 0.25$

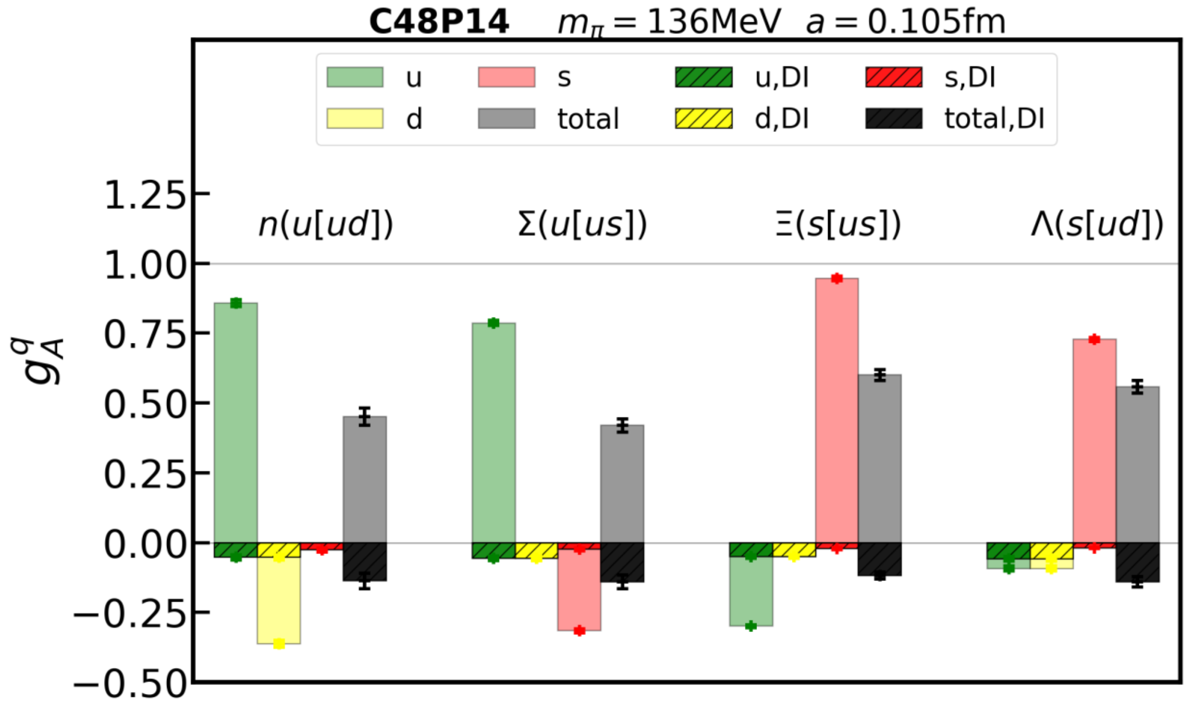
Σ: $\frac{1}{2}\Delta q \sim 0.20 - 0.25$

Ξ: $\frac{1}{2}\Delta q \sim 0.25 - 0.30$

Λ: $\frac{1}{2}\Delta q \sim 0.25 - 0.30$

■ There is moderate SU(3) symmetry breaking effects for the quark spin $\sim 10(5)\%$;





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Conclusion

1) The sea quark contribution is consistent in Octet baryons ranging $m_\pi \in [135, 220{,}300]\mathrm{MeV}$;

2) SU(3)-flavour symmetry breaking has ~10% influence in quark spin;

3) The symmetry is better preserved within specific diquark structures, as seen in the pairs like, (n, Σ) (Λ, Ξ);

4) There is a tendency for the SU(3) symmetry to partially recover after continuum extrapolation.

Outlook

1) The chiral behavior of sea quark helicity should be checked carefully;

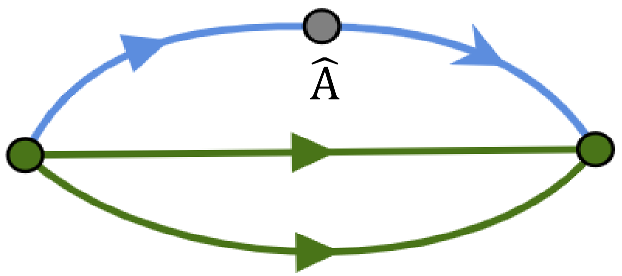
2) More systematical computation in different lattice spacing such as $a = 0.077$fm and $a = 0.052$fm will be done in the near future;

Thanks you!

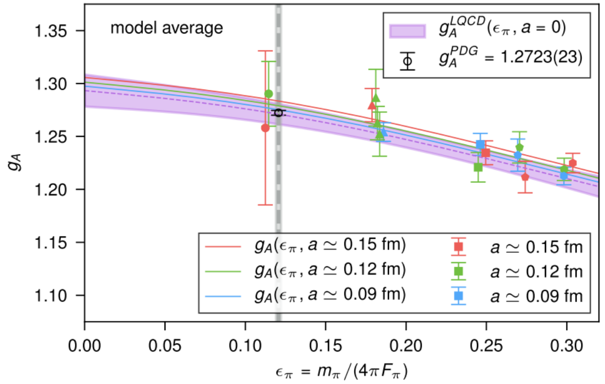
APP: Theoretical Framework: Blending Method

◆ **Low-lying space of Laplace (Distillation) can get high precision nucleon external state without momentum;**

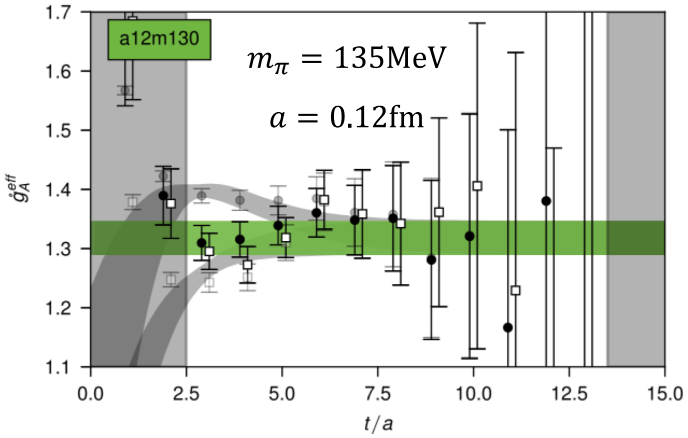
✓ **We observe substantially improved results at the physical point** ($m_\pi = 135\text{MeV}$);



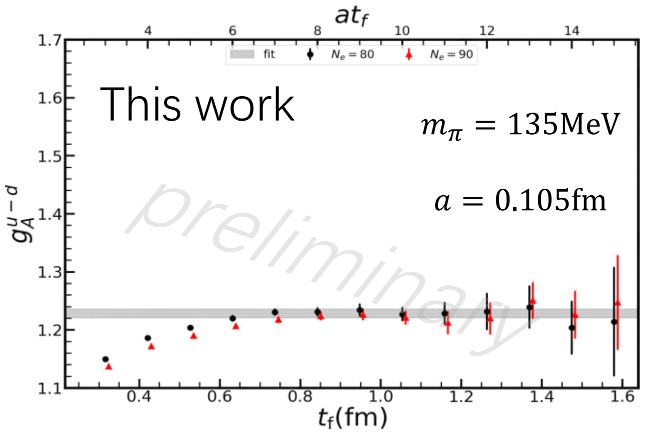
$$\langle P|\bar{u}\gamma^5\gamma^i u - \bar{d}\gamma^5\gamma^i d|P\rangle$$



Severe model dependent !!!



Nature 558,91 (2018)



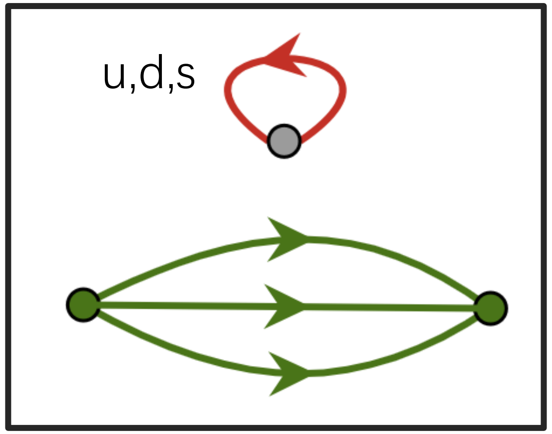
	Ensemble	n_{cfg}	g_A^{u-d}	$\delta x/\lvert x\rvert$
CLQCD(This work)	C48P14	46	1.221(07)	0.57%
CalLat	a15m130	1000	1.270(72)	5.66%
[Nature 558, 91(2018)]	a12m130	1000	1.292(30)	2.32%
RQCD *[PRD 108, 034512 (2023)]*	D452	1000	1.17(24)	20.5%

preliminary

✓ It is expected to achieve experimental-level precision of 0.1% with an internationally comparable level of statistics (e.g., 1000 configurations);

App. Renormalization and Flavour mixing

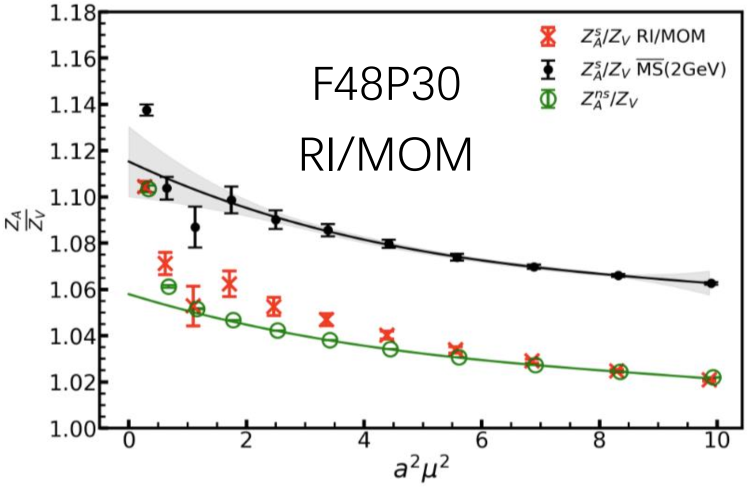
◆ Flavour Non-singlet and Singlet renormalized constant;

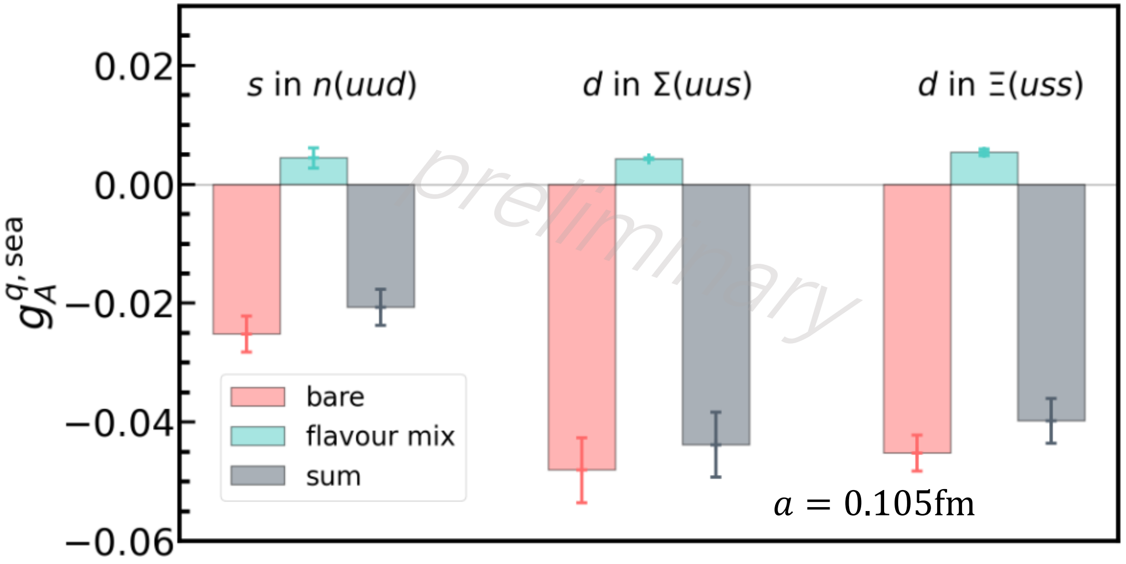


$$g_A^{q,\mathrm{bare}} \equiv \langle N|\bar{q}\gamma_i\gamma_5 q|N\rangle;$$

Flavor Mixing: $$g_A^{q,\mathrm{R}} = Z_A^{\mathrm{ns}} g_A^{q,\mathrm{bare}} + \frac{Z_A^{\mathrm{s}} - Z_A^{\mathrm{ns}}}{N_f} \sum_{q=u,d,s} g_A^{q,\mathrm{bare}},$$

A. J. Chambers et al. , PRD 92, 114517 (2015)





m_π = 290MeV

◆ Flavour mixing will lead to 1σ shift at current precision globally;

◆ $\delta g_A^{\mathrm{mix}} = \frac{Z_A^S - Z_A^{ns}}{Nf} \sum_q g_A^{q,B}$ couldn't be ignored for strange quark in nucleon, which has 20%~30% contribution;

App. Ensemble information