Mixed-Action Effects on HISQ Ensembles

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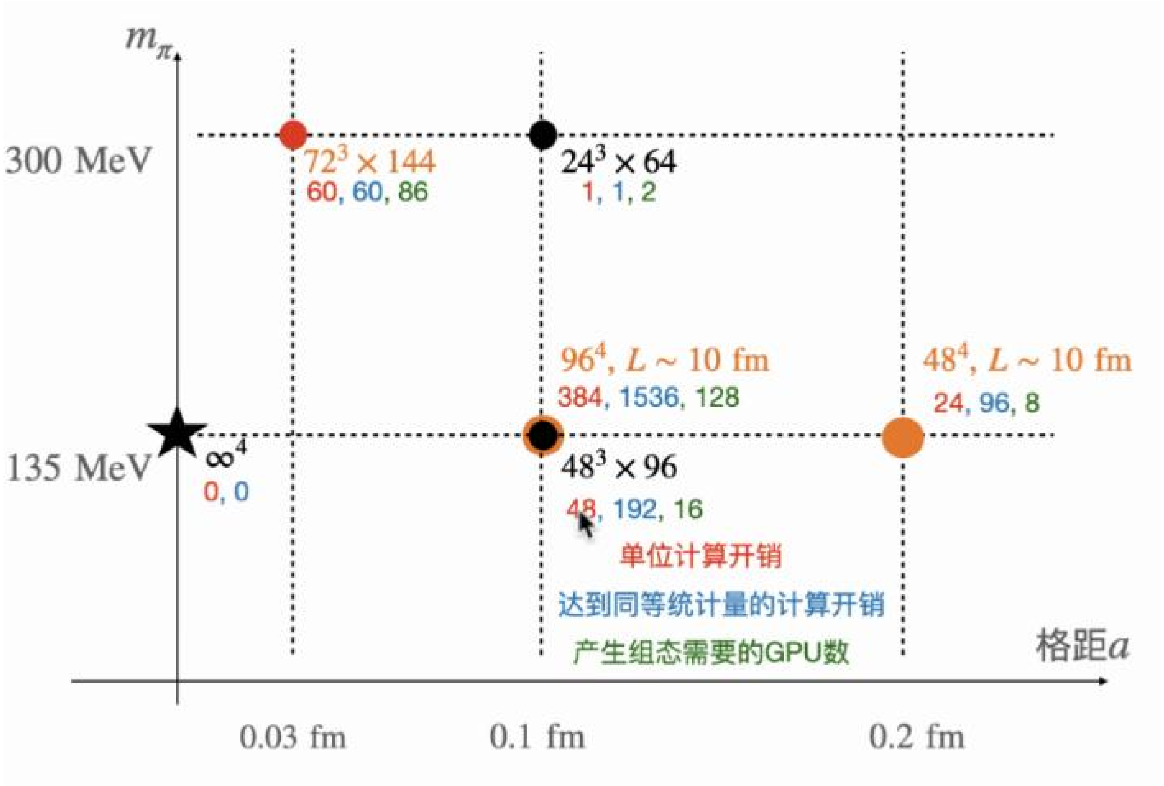
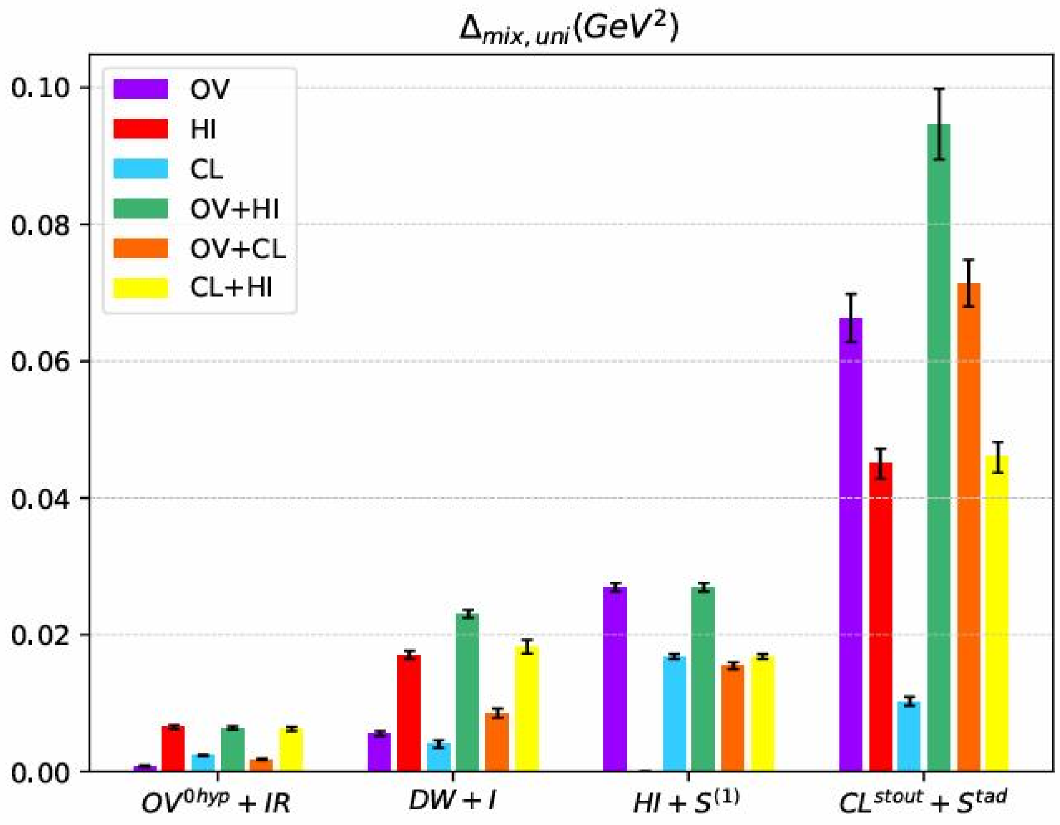
• 1. INTRODUCTION





Hai-Yang Du et al. (CLQCD Collaboration), Phys. Rev. D 109, 054507 (2024), arXiv:2310.00814 [hep-lat].

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- Generating configurations with finer lattice spacings or larger volumes using the current CLQCD approach demands significantly greater computational resources.
- Significant discretization errors are observed at a = 0.1 fm.

- Employing type-B valence fermions on type-A ensembles leads to additional discretization errors scaling as $\mathcal{O}(a^4)$.

Lower-Cost Staggered Ensembles for High-Complexity Fermion Calculations

2.1 HISQ Ensemble Generation and Scale Setting

1.gauge action

Consistent with the current CLQCD ensemble with clover fermion, we adopt the tadpole improved tree level Symanzik gauge action.

$$S_g = \beta\left(C_p \underbrace{\sum_P \left(1 - \frac{1}{3} ReTr(P)\right)}^{1\times 1\ \text{loop, plaquette}\ P} + C_R \underbrace{\sum_R \left(1 - \frac{1}{3} ReTr(R)\right)}^{2\times 1\ \text{loop, rectangle}\ R}\right)$$

Coefficient: $C_p = 1.0$

$$C_R = \frac{-1}{20u_0^2}$$

The tadpole factor is extracted from the plaquette.

$$u_0 = (\langle Re\, TrP\rangle/3)^{1/4}$$

The effective gauge coupling $\beta = \frac{10}{g^2}$.

2.1 HISQ Ensemble Generation and Scale Setting

2.Fermion action

The Highly Improved Staggered Quark (HISQ) action reduces both taste-symmetry breaking and discretization errors of staggered fermions and, compared to the clover fermion action, better preserves chiral symmetry.

$$S_{HISQ} = \sum_x \bar{\psi}(x)(\gamma \cdot D^{HISQ} + m)\psi(x)$$

$$U_\mu(x) \xrightarrow{\text{First-level Fat7 smearing}(\mathcal{F}_\mu)} V_\mu(x) = \mathcal{F}_\mu U_\mu(x) \xrightarrow{\text{Projection onto SU(3)}(\mathcal{U})} U_\mu^{(V)}(x) = \mathcal{U} V_\mu(x) \xrightarrow{\text{Second-level Fat7 smearing}(\mathcal{F}_\mu)} W_\mu(x) = \mathcal{U} U_\mu^{(V)}(x)$$

Suppresses ultraviolet fluctuations and tree-level taste exchange.

Ensures gauge invariance

Further suppresses taste mixing

$$D_\mu^{HISQ} \equiv \Delta_\mu(W) - \frac{a^2}{6}(1+\epsilon)\Delta_\mu^3(X)$$

with X representing the unsmeared, original gauge link $U_\mu(x)$

The ϵ parameter is introduced in order to correct the Naik term, thereby suppressing $(am)^4$ errors.

HPQCD Collaboration, Phys. Rev. D 75, 054502 (2007)

2.2 Currently Generated HISQ Ensembles

Nf =2+1+1

Symbol	$\hat{\beta}$	a (fm)	u_0	$\tilde{m}_l^{\rm b}$	$\tilde{m}_s^{\rm b}$	$\tilde{m}_c^{\rm b}$	$\tilde{L}^3 \times \tilde{T}$	m_π (MeV)	m_{η_s} (MeV)	m_{η_c} (GeV)
c24P29	7.29	0.111254(54)	0.8794	0.00944	0.04721	0.5555	$24^3 \times 48$	298(04)	667(03)	2.897(02)
c24P30	7.29	0.110932(53)	0.8794	0.00944	0.055	0.5555	$24^3 \times 48$	301(03)	725(02)	2.906(01)
c24P21	7.29	0.109418(56)	0.8794	0.00472	0.04721	0.5555	$24^3 \times 48$	217(04)	677(03)	2.942(01)
c32P30	7.29	0.110981(33)	0.8794	0.00944	0.04721	0.5555	$32^3 \times 48$	305(04)	673(03)	2.905(01)
c32P21	7.29	0.109466(36)	0.8794	0.00472	0.04721	0.5555	$32^3 \times 48$	215(05)	676(03)	2.941(01)
c48P13	7.29	0.108471(20)	0.8795	0.00174	0.04721	0.5555	$48^3 \times 48$	134(06)	683(03)	2.941(01)
e32P30	7.54	0.088830(39)	0.8863	0.00743	0.03715	0.4371	$32^3 \times 64$	304(04)	678(03)	2.944(02)
g32P31	7.75	0.072789(50)	0.8914	0.00579	0.02895	0.34	$32^3 \times 64$	307(05)	673(05)	2.912(03)
g48P30	7.75	0.072805(24)	0.8914	0.00579	0.02895	0.34	$48^3 \times 64$	300(07)	675(04)	2.912(02)
h48P32	8.2	0.048468(95)	0.9005	0.003526	0.01763	0.207	$48^3 \times 96$	320(11)	679(08)	2.869(08)

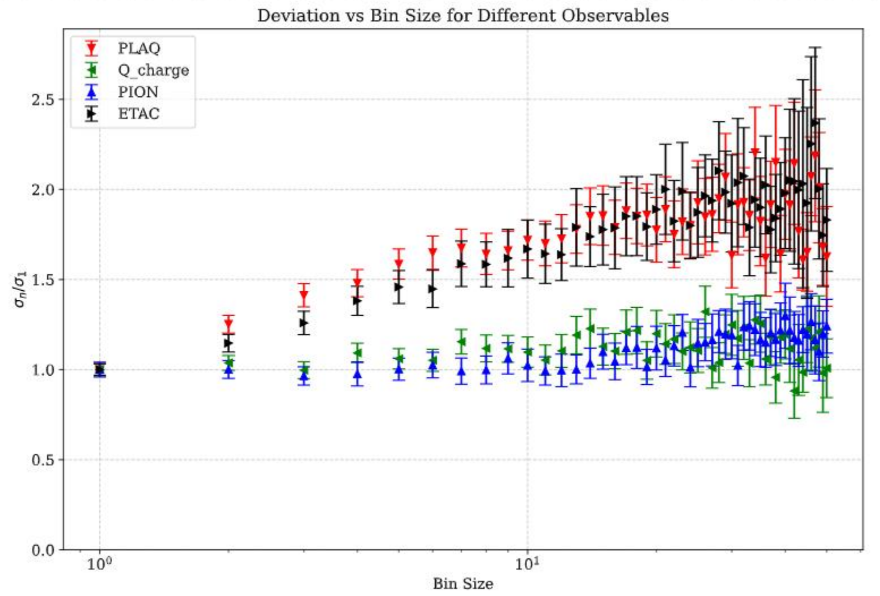
Informations

- 4 lattice spacings from 0.048 fm to 0.111 fm;
- 3 pion masses from 134 MeV to 320 MeV;
- Volumes from 2.3 fm to 5.2 fm

Nf =2+1+1

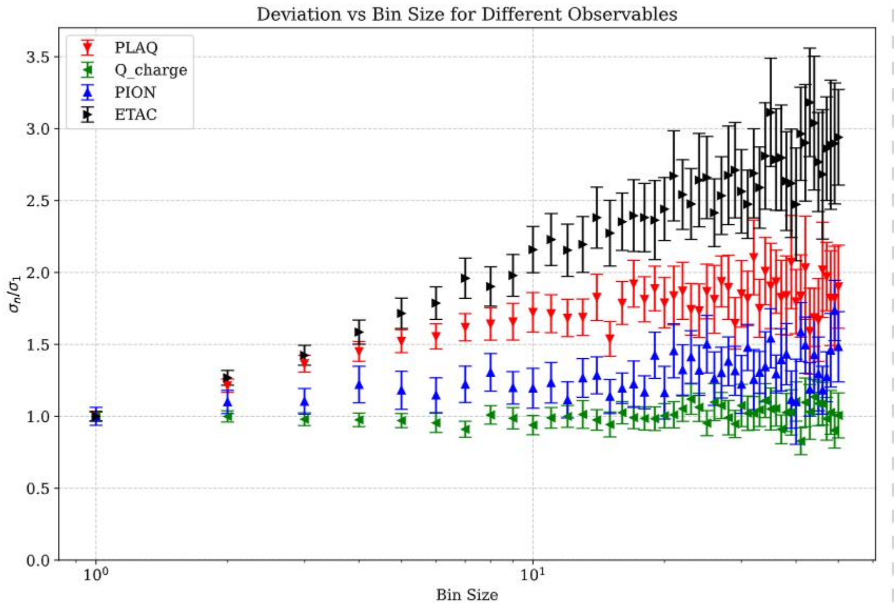
- w_0=0.17256 fm

2.2 Currently Generated HISQ Ensembles

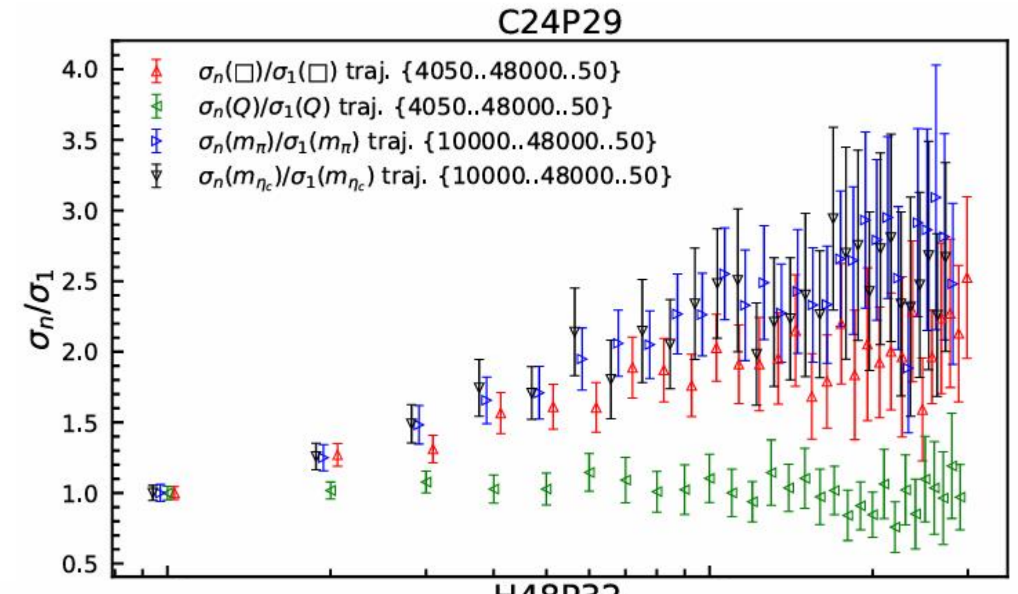


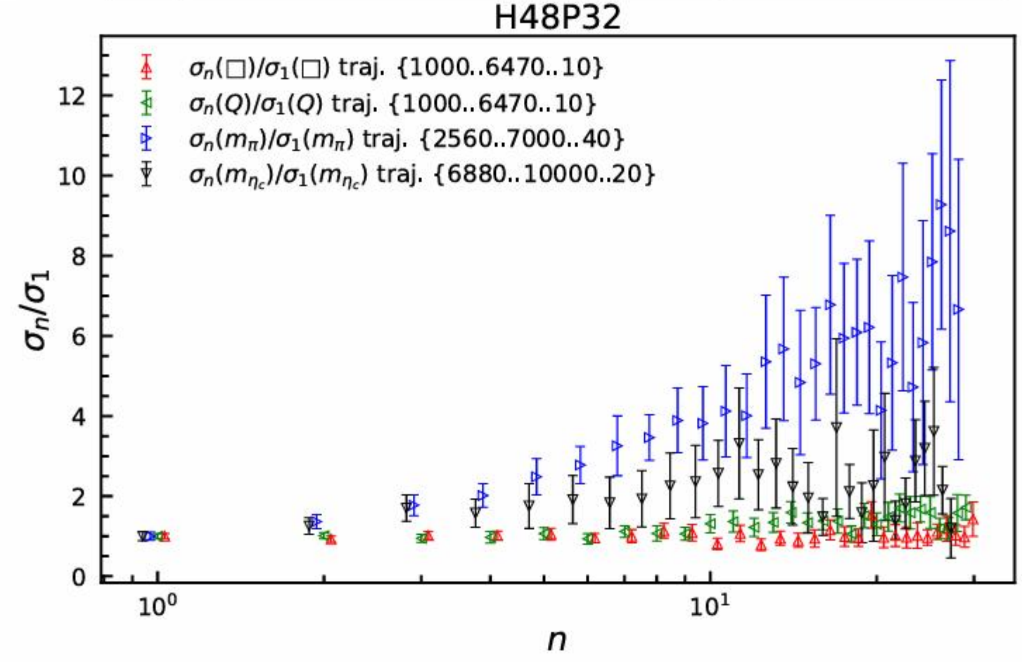
c24p29

The autocorrelation is weaker on the HISQ ensembles thanks to larger MC time （$tau=2$） for each trajectory



h48p32





3.1 Compute mixed action effect definition

Mixed action partially quenched chiral perturbation theory (MAPQχPT)

$$\Delta_{mix}^{v/s}(m_{\pi,vv}, m_{\pi,ss}, a) \equiv m_{\pi,vs}^2 - \frac{m_{\pi,vv}^2 + m_{\pi,ss}^2}{2}$$

Val_Val pion mass : $\boldsymbol{m_{\pi,vv}}$
Sea_Sea pion mass : $\boldsymbol{m_{\pi,ss}}$
Val_Sea pion mass : $\boldsymbol{m_{\pi,vs}}$

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$$\Delta_{mix}^{B/A}(m_\pi, a) \equiv m_{\pi,AB}^2 - \frac{m_{\pi,AA}^2 + m_{\pi,BB}^2}{2} \Big|_{m_{\pi,BB}=m_{\pi,AA}=m_\pi}$$

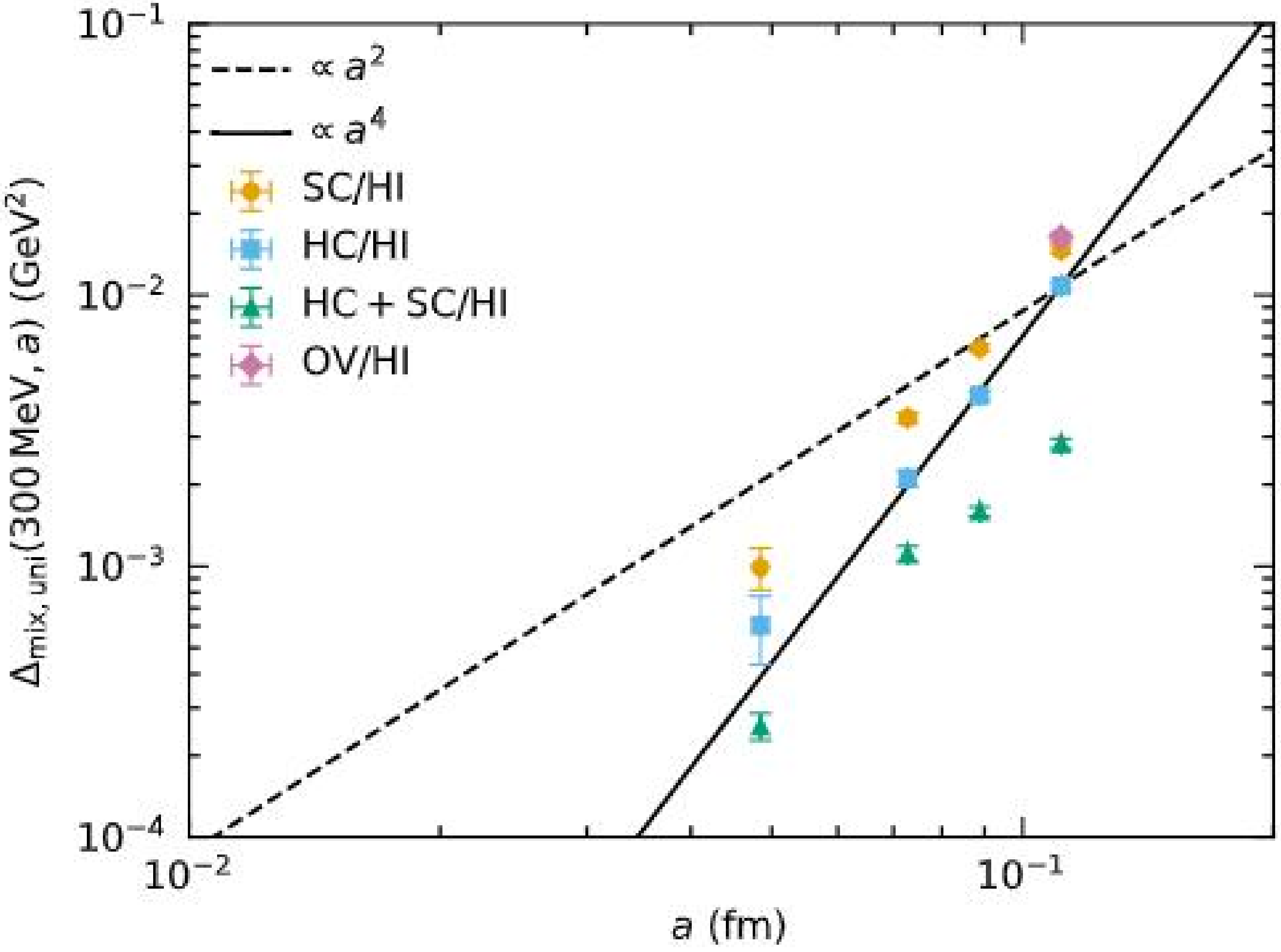
$$\overline{\Delta}_{mix,uni}^{B+C/A}(m_\pi, a) \equiv m_{\pi,BC}^2 - \frac{m_{\pi,BB}^2 + m_{\pi,CC}^2}{2} \Big|_{m_{\pi,BB}=m_{\pi,AA}=m_{\pi,AA}=m_\pi}$$

3.1 Compute mixed action effect definition

Action	Symbol	a(fm)	$m_{\pi,ss}$ (MeV)
DW+I	24I	0.111	340
DW+I	32I	0.083	302
HI+$S^{(1)}$	a12m310	0.121	310
HI+$S^{(1)}$	a09m310	0.088	310
HI+$S^{(1)}$	a06m310	0.057	310
HI+$S^{(1)}$	a04m310	0.043	310
HI+S^{tad}	c24P29	0.111254	298
HI+S^{tad}	e32P30	0.088830	304
HI+S^{tad}	g32P31	0.072789	307
HI+S^{tad}	h48P32	0.048468	320
$CL^{stout}+S^{tad}$	C11	0.108	290
$CL^{stout}+S^{tad}$	C08	0.080	300
$CL^{stout}+S^{tad}$	C06	0.055	300

3.2 Distance between various discretized fermion actions

Delta mix under HISQ sea



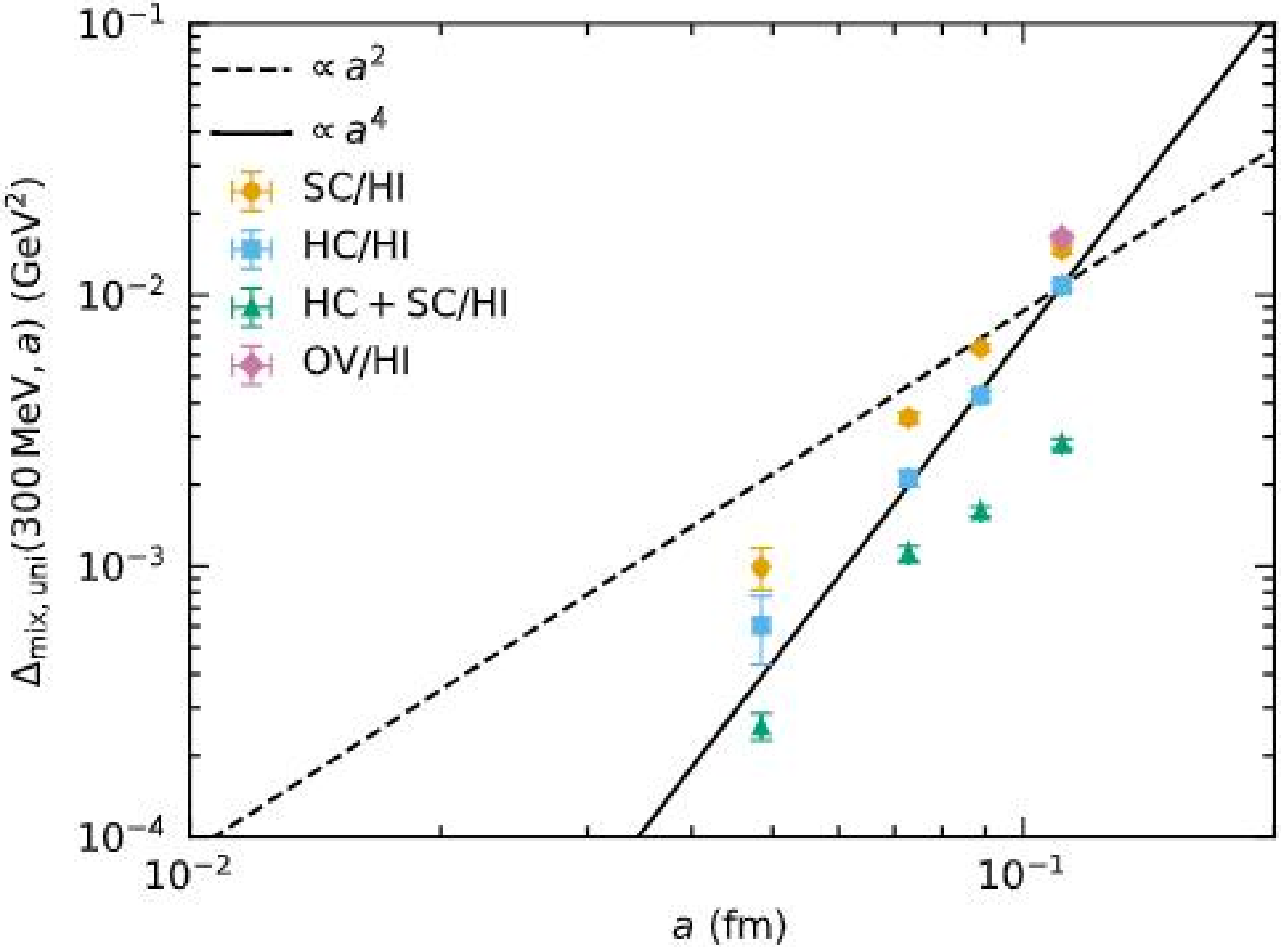
4 kinds of the valence fermion actions to calculate:

SC : Clover fermion with 1-step STOUT smearing and tree level tadpole improved clover coefficient c_{sw};
HC : Clover fermion with 1-step HYP smearing and tree level tadpole improved clover coefficient c_{sw};
OV : Overlap fermion with 1-step HYP smearing and ρ=1.5.

For the sea fermion actions:
use the 2+1+1 flavor HISQ fermion action with the tadpole improved tree level Symanzik gauge action at four lattice spacings and m=300MeV (HI for short);

3.2 Distance between various discretized fermion actions

Delta mix under HISQ sea



$$|\Delta^{B/A}_{mix,uni} - \Delta^{C/A}_{mix,uni}| \le \overline{\Delta}^{B+C/A}_{mix,uni} \le \Delta^{B/A}_{mix,uni} + \Delta^{C/A}_{mix,uni}$$

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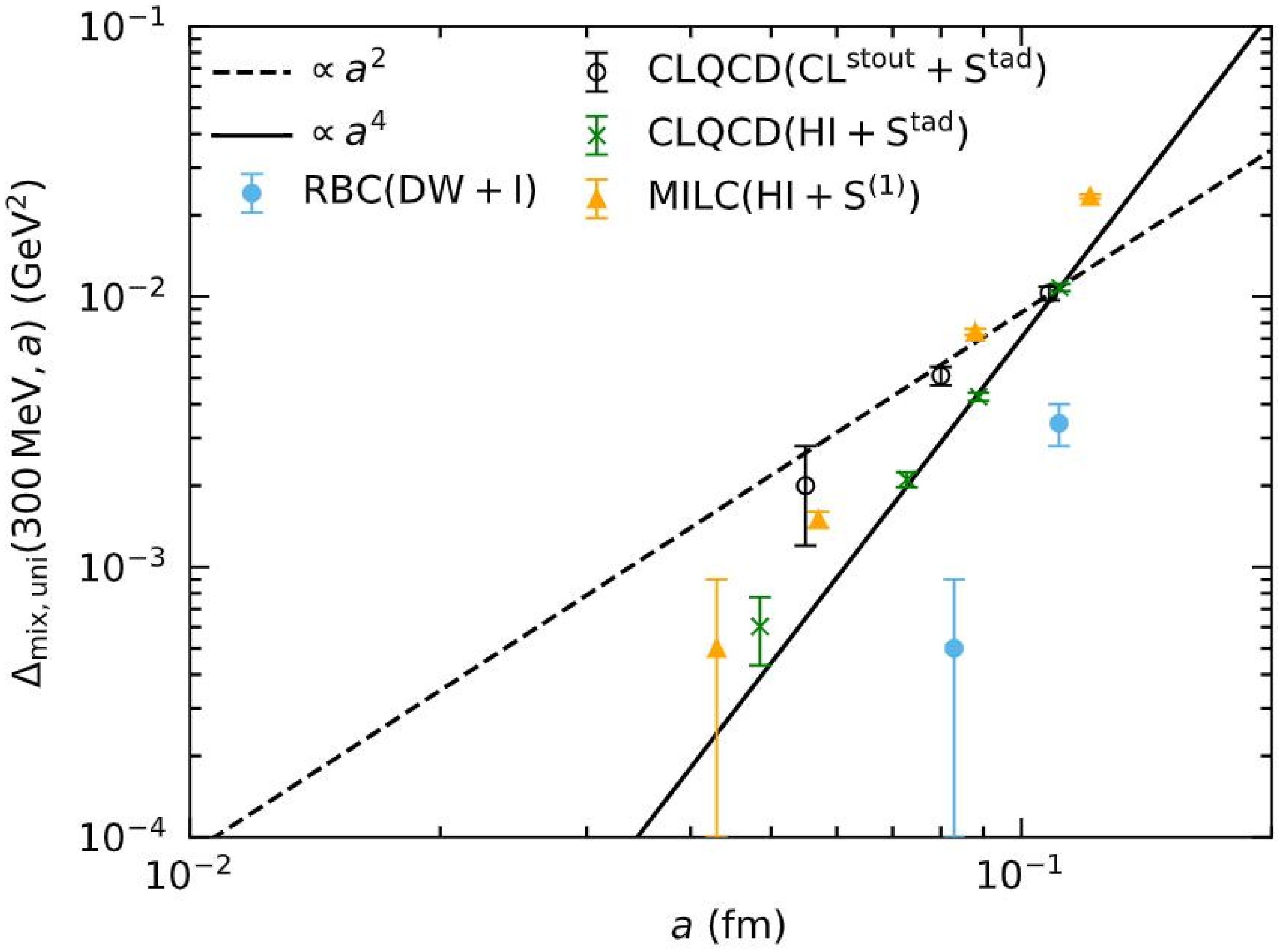
Δ_{mix} become less significant than other discretization errors at smaller lattice spacing which is $\mathcal{O}(a^2)$.

3.2 Distance between various discretized fermion actions

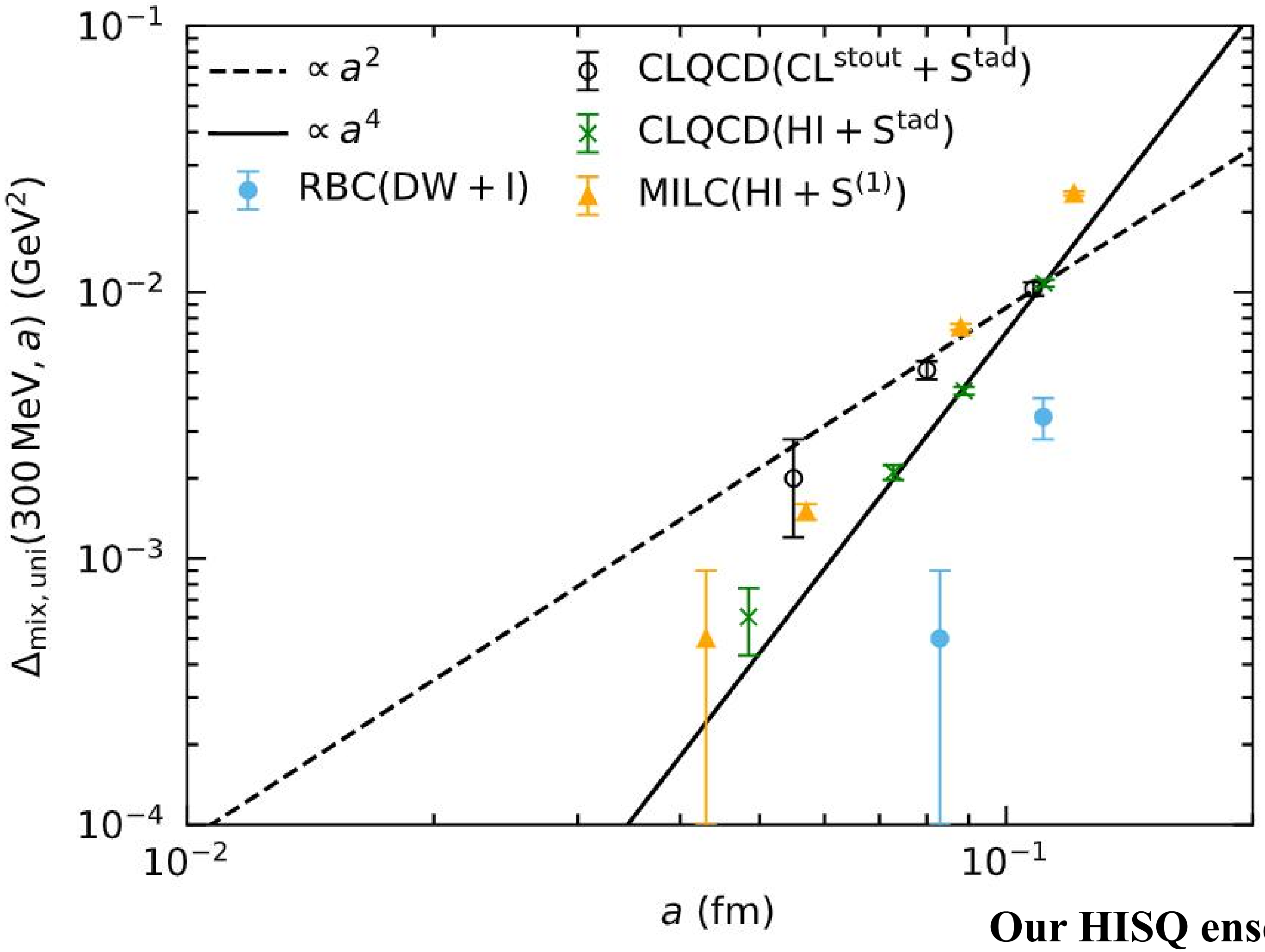


If the valence quark action is fixed to be HC, one can compare the Δ_{mix} on ensembles using different fermion and gauge actions.

CLQCD(CL^{stout}+S^{tad})
CL^{stout} : the 2+1 flavor tadpole improved clover fermion action (with 1-step stout link smearing on the gauge link with smearing parameter 0.125)
S^{tad} : tadpole improved Symanzik gauge action

CLQCD(HI+S^{tad})
HI : HISQ fermion action
S^{tad} : tadpole improved Symanzik gauge action

3.2 Distance between various discretized fermion actions



RBC(DW+I)
DW : the Domain Wall fermion action
I : Iwasaki gauge action

MILC(HI+S^1)
HI : HISQ fermion action
$S^{(1)}$: one-loop Symanzik improved gauge action

CLQCD(HI+S^{tad})
HI : HISQ fermion action
S^{tad} : tadpole improved Symanzik gauge action

Our HISQ ensemble (HI+S^{tad}) has smaller Δ_{mix} than either current CLQCD ensembles (same gauge different fermion) or MILC ensembles (same fermion different gauge).

Summary and Outlook

- Comparing to the current CLQCD ensembles with clover fermion, the new CLQCD HISQ ensembles:
  1) include the charm sea;
  2) smaller HMC cost and autocorrelation;
  3) smaller mixed action effects.
- Can be combined with the current CLQCD ensembles to provide better control on kinds of systematic uncertainties.

THANKS FOR YOUR ATTENTION !