Direct calculation of trace anomaly on CLQCD ensembles

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基础理论

- 带夸克场ψ和胶子场强张量$F_{\mu\nu}$的QCD能量动量张量，在重整化中不可避免地会引入无法被消除的迹[2]
- 强子质量与其能动量张量的迹的矩阵元直接相关[3]-[5]

$$T^{\mu}_{\mu} = H_m + (\gamma_m H_m + \frac{\beta}{2g} F^2)$$

$$m_H = \left\langle T^{\mu}_{\mu} \right\rangle_H = (1+\gamma_m)\left\langle H_m \right\rangle_H + \frac{\beta}{2g}\left\langle F^2 \right\rangle_H = (1+\gamma_m)\left\langle H_m \right\rangle_H + \hat{\beta}\left\langle Tr[g^2 F^2] \right\rangle_H$$

- Anomalous dimension

$$\begin{cases} \gamma_m = -\frac{\mu}{m}\frac{\partial m}{\partial \mu} = \frac{2}{\pi}\alpha_s + 0\ (\alpha_s^2) \\ \widehat{\beta} = -\frac{1}{4\pi}\frac{\mu^2}{{\alpha_s}^2}\frac{\partial \alpha_s}{\partial \mu^2} = -(\frac{1}{4\pi})^2(11 - \frac{2n_f}{3}) + 0\ (\alpha_s) \end{cases}$$

[2]Y. Hatta, A. Rajan, and K. Tanaka, JHEP 12, 008 (2018), arXiv:1810.05116 [hep-ph].
[3]M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov,Phys. Lett. B78, 443 (1978).
[4]X.-D. Ji, Phys. Rev. Lett. 74, 1071 (1995), arXiv:hep_x0002_ph/9410274 [hep-ph].
[5]X. Ji and Y. Liu, (2021), arXiv:2101.04483 [hep-ph].

数值计算

- 强子的两点函数和三点函数[1]：

$$C_2(t_f;\mathcal{H}) = \langle \sum_{\vec{y}} \mathcal{H}(t_f,\vec{y}) \sum_{\vec{x}} \mathcal{H}^{\dagger}(0,\vec{x}) \rangle$$

$$SC_3{}^{q_s,g}(t_f;\mathcal{H}) = \langle \sum_{\vec{y}} \mathcal{H}(t_f,\vec{y}) \sum_{t\in(t_f,0),\vec{z}} O^g(t,\vec{z}) \sum_{\vec{x}} \mathcal{H}^{\dagger}(0,\vec{x}) \rangle = \sum_{0<t<t_f} \sum_{\vec{y}} \sum_{\vec{x}} \langle \left[C_2(t_f,\vec{y};\mathcal{M}) - \langle C_2(t_f,\vec{y};\mathcal{M}) \rangle \right] [O^g(t,\vec{x}) - \langle O^{q_s,g}(t,\vec{x}) \rangle] \rangle$$

- Trace anomaly通过差分形式算出[1]，Feynman-Hellman inspired method：

$$Tr[\langle g^2F^2 \rangle]_H = \Delta R^g(t_f;H) + \mathcal{O}(e^{-\delta m t_f})$$

$$\Delta R^g(t_f) \equiv \frac{SC_3{}^g(t_f;\mathcal{H})}{C_2(t_f;\mathcal{H})} - \frac{SC_3{}^g(t_f-1;\mathcal{H})}{C_2(t_f-1;\mathcal{H})}$$

[1]F. He, P. Sun, and Y.-B. Yang (χQCD), Phys. Rev. D 104, 074507 (2021), arXiv:2101.04942 [hep-lat].

构造胶子算符

- 格点上一个基本Wilson圈的近似泰勒展开：

$$\mathcal{P}_{\mu,\nu}(x) = U_\mu(x)U_\nu(x+\mu)U_\mu^\dagger(x+\nu)U_\nu^\dagger(x)$$
$$= 1 + ig_0a^2F_{\mu\nu} - \frac{1}{2}a^4g_0^2F_{\mu\nu}F_{\mu\nu} + i\mathcal{O}(a^4)$$

- $Tr[\langle g^2F^2\rangle]_H$：

$$\begin{cases} clo(clover): Tr\{[\mathcal{P}_{\mu,\nu}(x) - \mathcal{P}_{\mu,\nu}^\dagger(x)]/2i\}^2 \\ pla(plaquette): 2Re[1 - Tr(\mathcal{P}_{\mu,\nu}(x))] \end{cases}$$

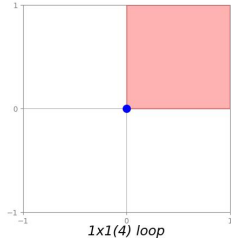
- $U_\mu(x)U_\nu(x+\mu)U_\mu^\dagger(x+\nu)U_\nu^\dagger(x)$：

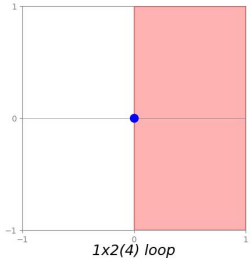
- 除了常用的1x1圈，可以有更多种Wilson圈的形式，用以探究UO的贡献

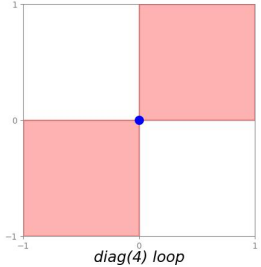
[6]M. Lüscher, Properties and uses of the Wilson flow in lattice QCD, JHEP 08 (2010) 071 [1006.4518]

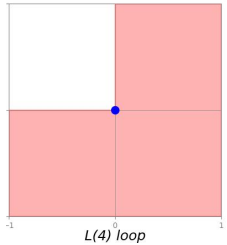
构造胶子算符

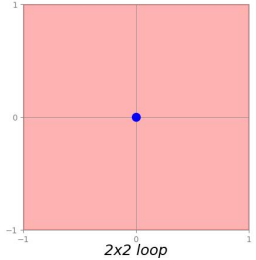
- $r < \sqrt{2}a$











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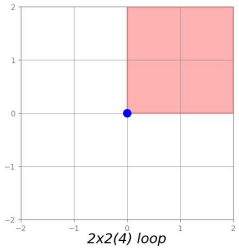
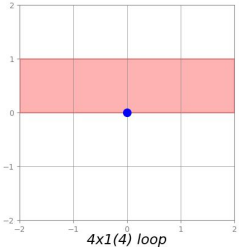
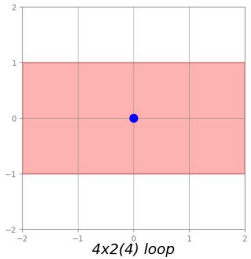
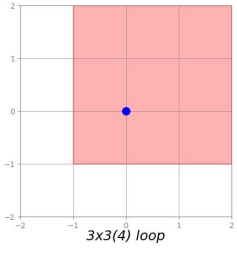
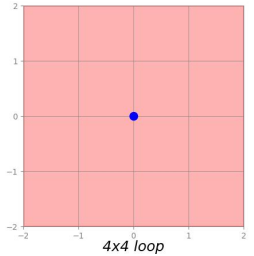


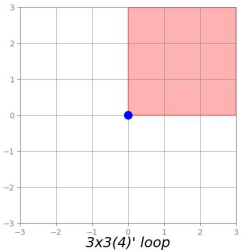


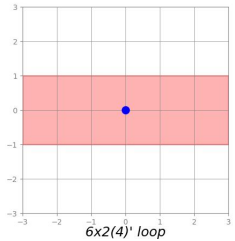


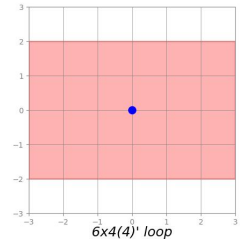


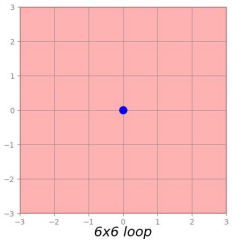


- $r < 3\sqrt{2}a$









- 不同构造形式

1x1(4)_pla

4x4_clo

.....

不同能标下构造胶子算符

- 对规范场作用Wilson flow(WF)(等效于无穷小的stout link smearing):

$$\dot{V}_t(x,\mu) = -g_0^2\{\partial_{x,\mu}S_w(V_t)\}V_t(x,\mu) \qquad V_t(x,\mu)|_{t=0} = U(x,\mu)$$

- $\dot{S_w} \leqslant 0 \Rightarrow$ WF会使场变得更平滑[10]
- 在H48P32上组态计算不同WF能标、不同胶子构造方式下Omega_ccc粒子的trace anomaly

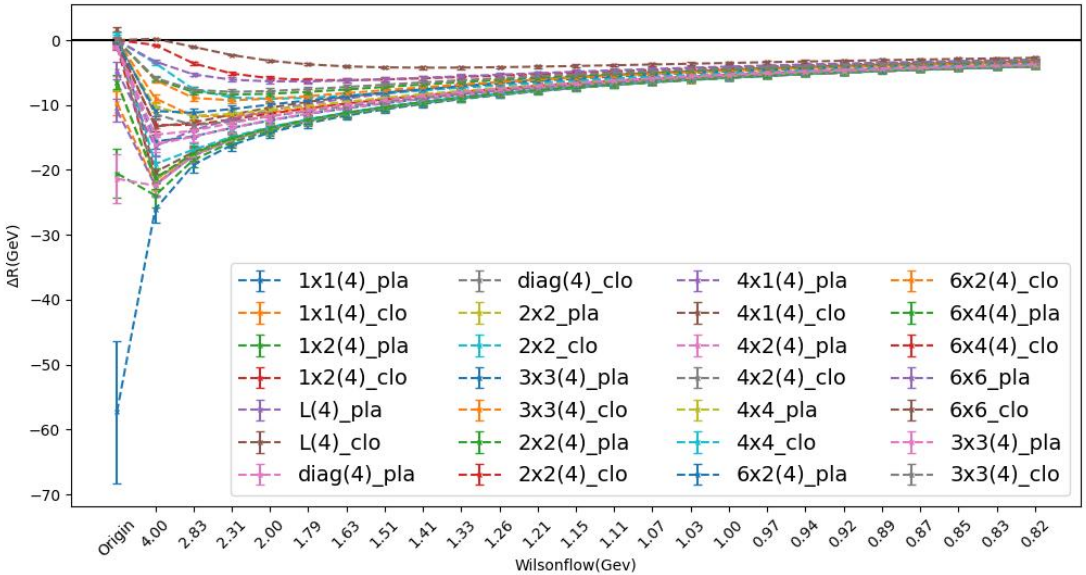


图1：不同胶子算符几何构型计算结果随WF能标变化

- Origin指不对规范场做WF
- 随着WF能标的降低，各种计算结果逐渐收敛，但仍然存在一定差别

[6]M. Lüscher, Properties and uses of the Wilson flow in lattice QCD, JHEP 08 (2010) 071 [1006.4518]
[10]Martin Lüscher, "Topology, the Wilson flow and the HMC algorithm," PoS LATTICE2010, 015 (2010), arXiv:1009.5877 [hep-lat].

不同能标下构造胶子算符

- 不同胶子算符构造方式在不同WF能标下对比

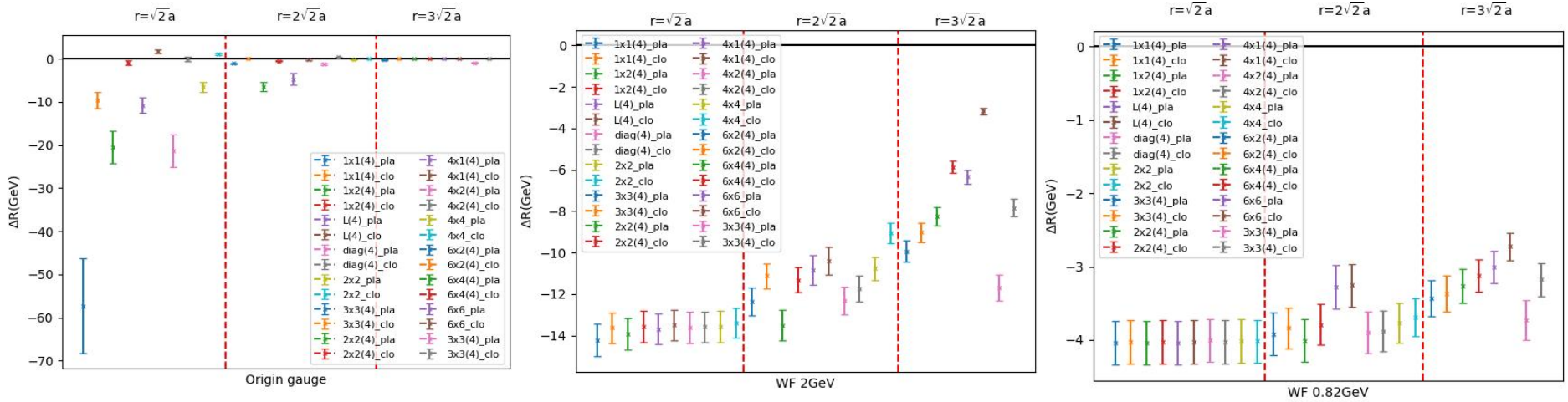


图2：WF 2GeV

- Origin gauge下计算结果差异很大，且存在正值，这意味着负值的trace anomaly
- WF 2GeV和WF 0.82GeV下，在$r = \sqrt{2}a$时各种构型收敛性良好。而随着圈圈越来越大，结果也有不同程度的压低。
- 在DWF@Overlap、HISQ@HISQ等均存在类似结果

不同能标下构造胶子算符

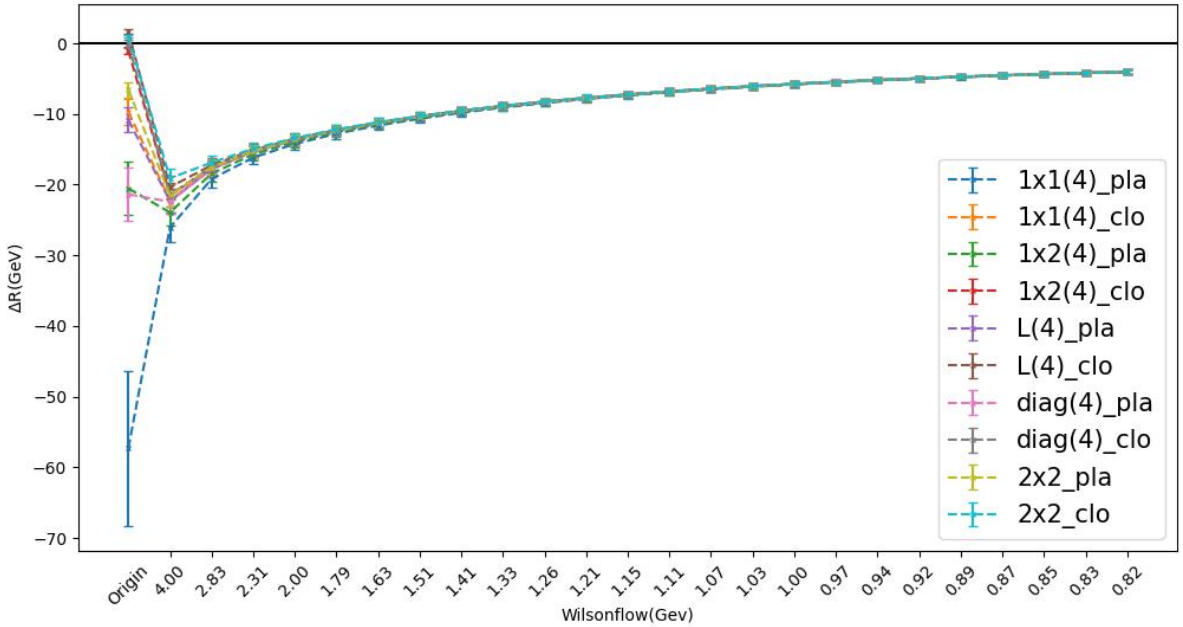


图3：$r<\sqrt{2}a$

- 只观察$r=\sqrt{2}a$结果，收敛性良好，不同构型在WF足够低(2GeV)能标下已基本无差别
- 后续计算我们将采用1x1(4)_clo作为代表，这也是最常用的clover形式胶子算符

Anomalous dimension

- 求和规则the sum rule：$m_H = \left\langle T_\mu^\mu \right\rangle_H = (1+\gamma_m)\sigma_H + \hat{\beta}\left\langle Tr[g^2F^2] \right\rangle_H$
- 在[7]中精确计算了30种强子的m_H、σ_H
- 我们在CLQCD H48P32组态上直接计算各种强子的 $\left\langle Tr[g^2F^2] \right\rangle_H$

Symbol	$\tilde{L}^3 \times \tilde{T}$	$\hat{\beta}$	a (fm)	$\tilde{m}_l^b$	$\tilde{m}_s^b$	m_π(MeV)	m_{η_s}(MeV)	$m_\pi L$	n_{cfg}	n_{src}^l	$n_{\text{src}}^{s,c}$
H48P32	$48^3 \times 144$	6.72	0.05198(20)(31)	-0.1850	-0.1700	316.6(1.0)	691.88(65)	4.06	157	12	12

- 将计算结果代回求和规则进行联合拟合，我们得到了anomalous dimension,并和$\overline{MS}$方案下的值进行比较

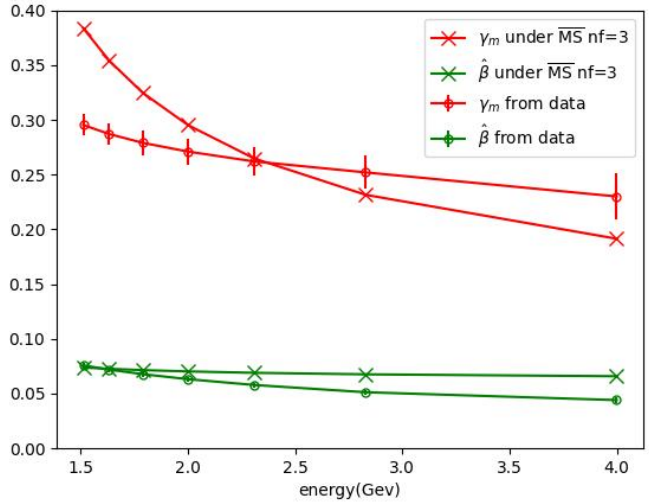


图4：Anomalous dimension

- $\overline{MS}$(叉点)WF(空心点)拟合结果进行比较，二者趋势相同，且数值接近
- 将得到的anomalous dimension代回sumrule进行自洽性检验

[7]B. Hu, X. Jiang, K.-F. Liu, P. Sun, and Y.-B. Yang, Trace anomaly contributions to baryon masses from Lattice QCD, arXiv:2411.18402

Anomalous dimension

• 求和规则the sum rule： $m_H = \left\langle T^{\mu}_{\mu} \right\rangle_H = (1+\gamma_m)\sigma_H + \hat{\beta}\left\langle Tr[g^2F^2] \right\rangle_H$

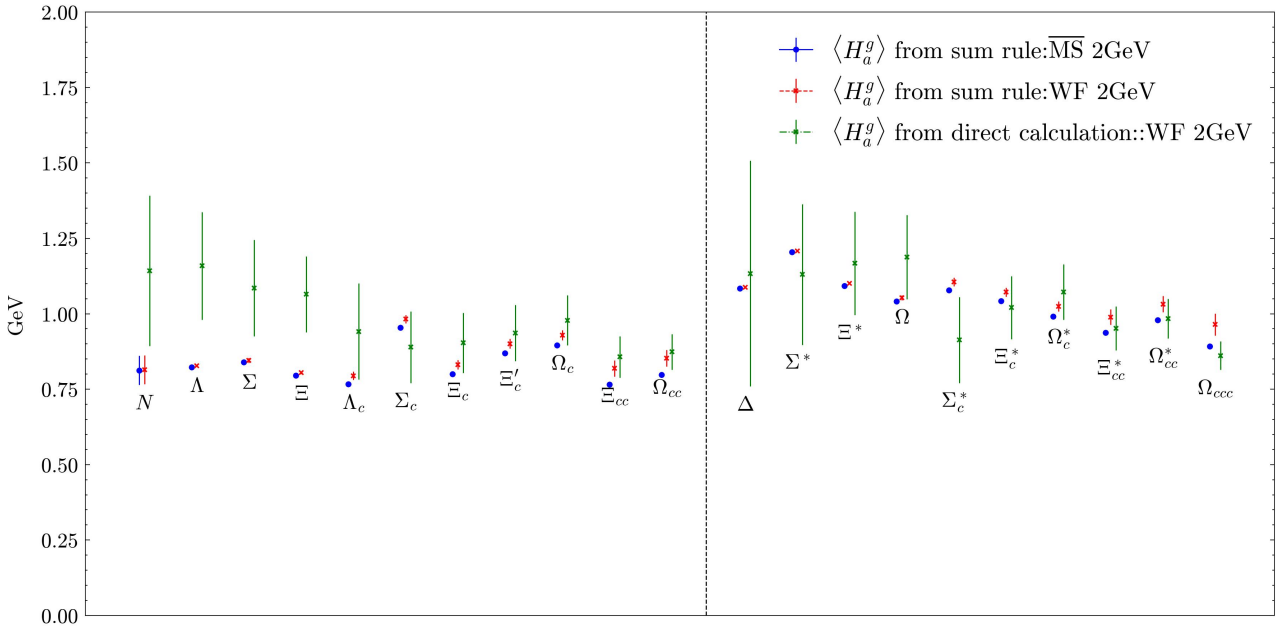


图5：自洽性检验

• 蓝色点和红色点为通过求和规则间接计算结果，参数anomalous dimension分别采用$\overline{MS}$(蓝)和WF(红)结果

• 绿色点为直接计算结果，anomalous dimension采用WF拟合结果

• 三者在2 σ 区间内自洽

参考文献

[1]F. He, P. Sun, and Y.-B. Yang (χQCD), Phys. Rev. D 104, 074507 (2021), arXiv:2101.04942 [hep-lat].

[2]Y. Hatta, A. Rajan, and K. Tanaka, JHEP 12, 008 (2018), arXiv:1810.05116 [hep-ph].

[3]M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov,Phys. Lett. B78, 443 (1978).

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[7]B. Hu, X. Jiang, K.-F. Liu, P. Sun, and Y.-B. Yang, Trace anomaly contributions to baryon masses from Lattice QCD, arXiv:2411.18402

[8]E. Follana, Q. Mason, C. Davies, K. Hornbostel, G. P. Lepage, J. Shigemitsu, H. Trottier, and K. Wong (HPQCD, UKQCD), Highly improved staggered quarks on the lattice, with applications to charm physics, Phys. Rev. D 75, 054502 (2007), arXiv:hep-lat/0610092.

[9]X.-D. Ji, Phys. Rev. D52, 271 (1995), arXiv:hep-ph/9502213 [hep-ph].

[10]Martin Lüscher, “Topology, the Wilson flow and the HMC algorithm,” PoS LATTICE2010, 015 (2010), arXiv:1009.5877 [hep-lat].

Thanks

补充材料

$$\partial^a_{x,\mu} f(U) = \frac{\mathrm{d}}{\mathrm{d}s} f(\mathrm{e}^{sX} U)\Big|_{s=0}, \qquad X(y,\nu) = \begin{cases} T^a & \text{if } (y,\nu) = (x,\mu), \\ 0 & \text{otherwise.} \end{cases} \tag{A.3}$$

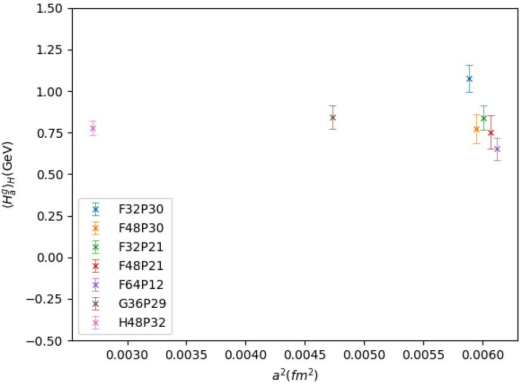
While these depend on the choice of the generators T^a, the combination

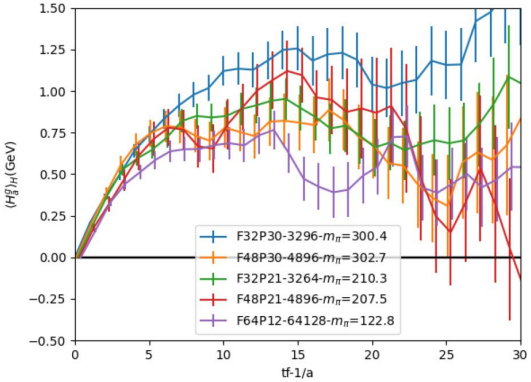
$$\partial_{x,\mu} f(U) = T^a \partial^a_{x,\mu} f(U) \tag{A.4}$$

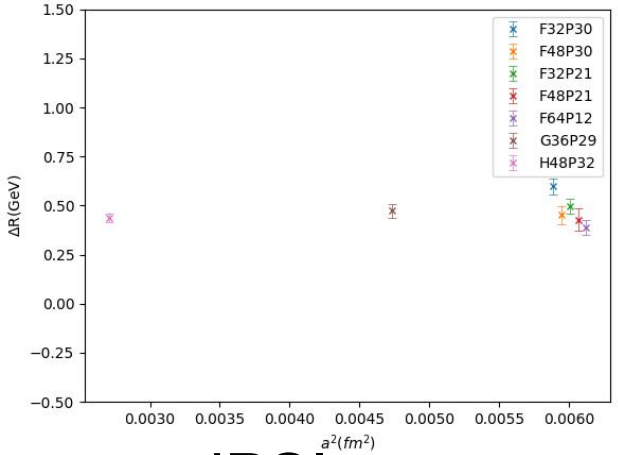
can be shown to be basis-independent.

Symbol	$\tilde{L}^3 \times \tilde{T}$	$\hat{\beta}$	a (fm)	$\tilde{m}^b_l$	$\tilde{m}^b_s$	m_π(MeV)	m_{η_s}(MeV)	$m_\pi L$	n_{cfg}	n^l_{src}	$n^{s,c}_{\mathrm{src}}$
F32P30	$32^3 \times 96$	6.41	0.07751(14)(45)	-0.2295	-0.2050	300.4(1.2)	675.98(97)	3.81	250	3	3
F48P30	$48^3 \times 96$			-0.2295	-0.2050	302.7(0.9)	674.76(58)	5.72	99	3	3
F32P21	$32^3 \times 64$			-0.2320	-0.2050	210.3(2.3)	658.79(94)	2.67	194	3	3
F48P21	$48^3 \times 96$			-0.2320	-0.2050	207.5(1.1)	661.94(64)	3.91	75	3	3
F64P12	$64^3 \times 128$			-0.2336	-0.2030	122.8(0.9)	679.90(30)	3.09	109	4	4
G36P29	$36^3 \times 108$	6.498	0.06884(18)(41)	-0.2150	-0.1926	297.2(0.9)	693.05(46)	3.68	270	4	4
H48P32	$48^3 \times 144$	6.72	0.05198(20)(31)	-0.1850	-0.1700	316.6(1.0)	691.88(65)	4.06	157	12	12

Omega_ccc







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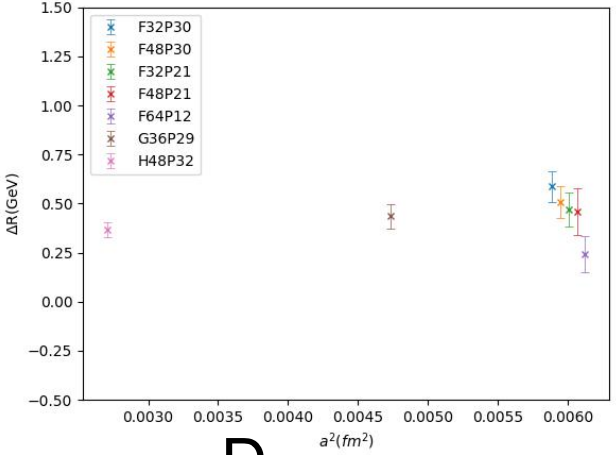

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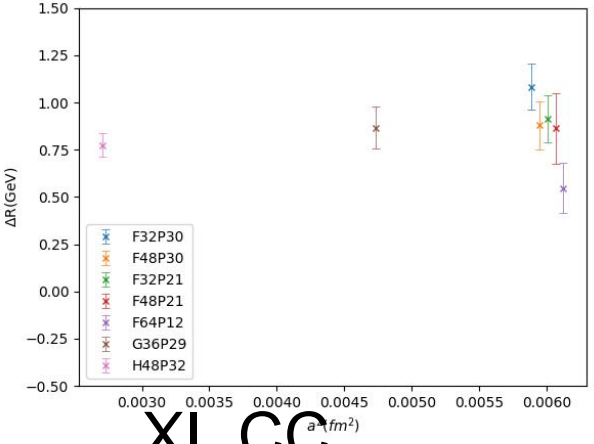

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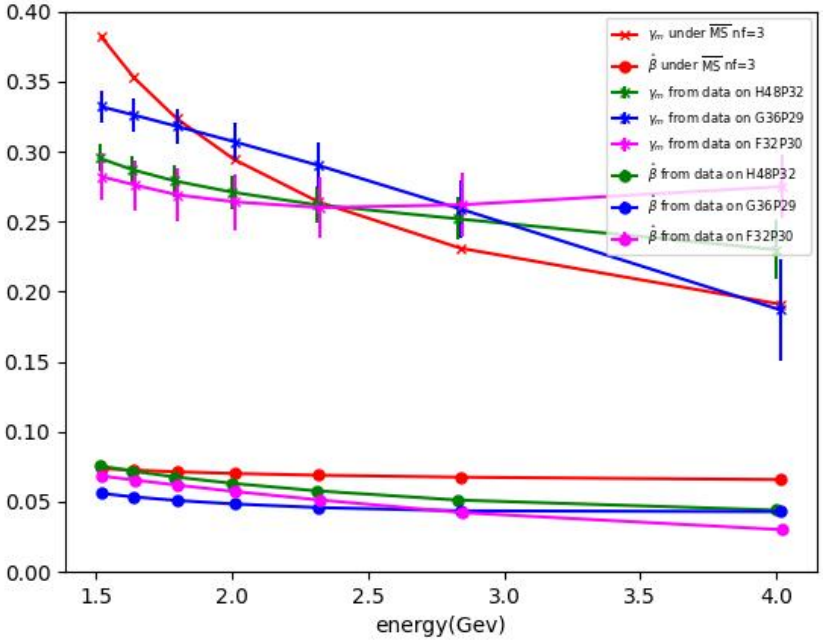
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0.35
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0.00
1.5
2.0
2.5
3.0
3.5
4.0
energy(Gev)
γ_m under $\overline{MS}$ nf=3
$\hat{\beta}$ under $\overline{MS}$ nf=3
γ_m from data on H48P32
γ_m from data on G36P29
γ_m from data on F32P30
$\hat{\beta}$ from data on H48P32
$\hat{\beta}$ from data on G36P29
$\hat{\beta}$ from data on F32P30