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Lattice QCD towards TMDPDF: Boer-Mulders function

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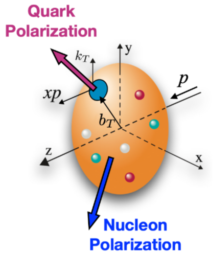
Outline

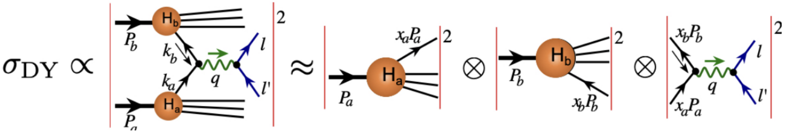
Overview

- Nucleons, like proton and neutron, has dynamical internal structures being described by distribution functions.
- Colinear PDF $f(x)$. Distr. of parton carrying longitudinal momentum fraction x.
- GPD $f(x, \Delta, \xi)$. Distr. involved in non-forward processes like DVCS.
- TMDPDF $f(x, k_\perp)$. Distr. of parton carrying longitudinal momentum x and transversal momentum $k_\perp$.



TMDPDF

Drell-Yan process



$q_\perp \gg \Lambda_{\rm QCD}$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q^4} = \sum_{i,j} \int_{x_a}^{1} \mathrm{d}\xi_a \int_{x_b}^{1} \mathrm{d}\xi_b f_{i/H_a}(\xi_a) f_{j/H_b}(\xi_b) \frac{\mathrm{d}\hat{\sigma}_{ij}(\xi_a,\xi_b)}{\mathrm{d}Q^2 \mathrm{d}Y} [1 + \mathcal{O}(\frac{\Lambda^2_{\rm QCD}}{q^2_\perp}, \frac{\Lambda^2_{\rm QCD}}{Q^2})]$$

$q_\perp \sim \Lambda_{\rm QCD}$

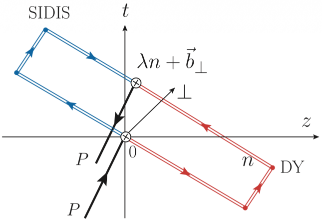
$$\frac{\mathrm{d}\sigma}{\mathrm{d}q^4} = \frac{1}{s} \sum_{i} \hat{\sigma}^{\rm TMD}_{i\bar{i}}(Q) \int \mathrm{d}^2\vec{k}_\perp \, f_{i/H_a}(x_a, \vec{k}_\perp) f_{\bar{i}/H_b}(x_b, \vec{q}_\perp - \vec{k}_\perp) [1 + \mathcal{O}(\frac{q^2_\perp}{Q^2}, \frac{\Lambda^2_{\rm QCD}}{Q^2})]$$

light-cone TMDPDF

The general definition of light-cone TMDPDF is

$$f_\Gamma(x, b_\perp) = \int \frac{\mathrm{d}b^-}{2\pi} e^{-ib^-(xP^+)} \langle P, S|\bar{\psi}(\lambda n + \vec{b}_\perp)\frac{\Gamma}{2}\mathcal{W}_\sqsubset(\lambda n + \vec{b}_\perp)\psi(0)|P, S\rangle$$

where $\Gamma \in \{\gamma^+, \gamma^+\gamma_5, i\sigma^{\alpha+}\gamma_5\}$. S denotes the spin polarization of hadron state. $\mathcal{W}_\sqsubset$ is the staple-shaped gauge link.



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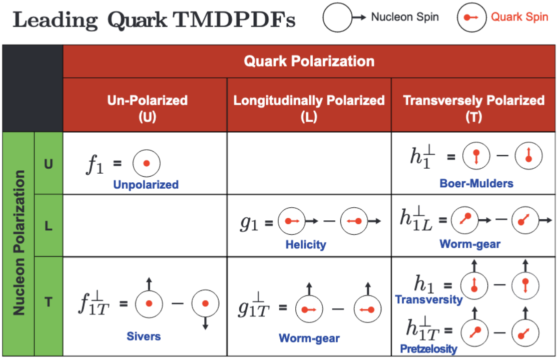
Decomposition of TMDPDF

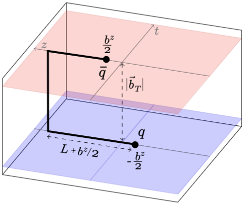


$$f_{i\sigma^{\alpha+}\gamma_5}(x, b_\perp) = S_T^\alpha h_1(x, b_\perp) - iS_L b_\perp^\alpha M h_{1L}^\perp(x, b_\perp) + i\epsilon^{\alpha\rho} b_{\perp\rho} M h_1^\perp(x, b_T)$$
$$+ \frac{1}{2} b_\perp^2 M^2 \left(\frac{1}{2} g_T^{\alpha\rho} + \frac{b_T^\alpha b_T^\rho}{b_T^2} S_{\perp\rho}\right) h_{1T}^\perp(x, b_\perp)$$

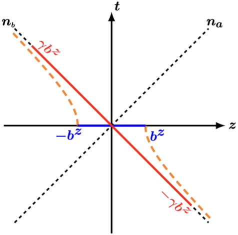
quasi-TMDPDF

How to calculate TMDPDF on lattice?

- Moments from OPE – not direct. Higher order, worse signal.
- LaMET – match light-cone TMDPDF with quasi-TMDPDF.



(a)



(b)

Definition of quasi-TMDPDF

$$\tilde{f}_\Gamma(x, b_\perp, P^z, \mu) = \lim_{L\to\infty} \int \frac{\mathrm{d}z}{2\pi} e^{-izxP^z} \frac{\langle P|\bar{\psi}(b_\perp \hat{n}_\perp)\Gamma\mathcal{W}(z, b_\perp, L)\psi(z\hat{n}_z)|P\rangle}{\sqrt{Z_E(2L, b_\perp, \mu)}Z_O(\mu, 1/a)}$$

- bare matrix element
  $\tilde{f}^0_\Gamma(P^z, z, L, b_\perp) = \langle P|\bar{\psi}(b_\perp \hat{n}_\perp)\Gamma\mathcal{W}(z, b_\perp, L)\psi(z\hat{n}_z)|P\rangle$
- Z_E rectangular spacelike Wilson loop.
- logrithmic factor
  $Z_O = \lim_{L\to\infty} \tilde{f}^0_\Gamma(0, z_0, L, b_{\perp,0})/(\sqrt{Z_E(2L+z_0, b_\perp, \mu)}\tilde{h}^{\overline{\mathrm{MS}}}_\Gamma(z_0, b_{\perp,0}, \mu))$
- Z_E is introduced to eliminate linear divergence and pinch-pole divegence which are related to staple link.
- Z_O eliminates UV divergence and relates lattice scheme to $\overline{\mathrm{MS}}$ scheme.

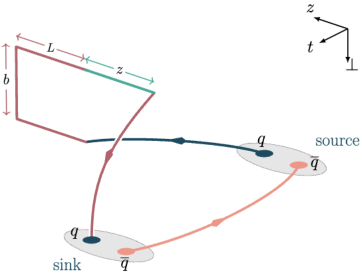
Matching formular

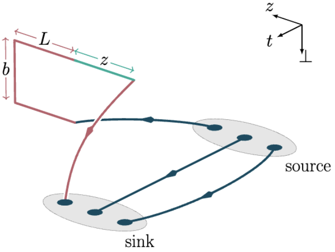
$$\tilde{f}(x, b_\perp, \mu, \zeta_z)\sqrt{S_r(b_\perp, \mu)} = H(\frac{\zeta_z}{\mu^2})e^{\frac{1}{2}K(b_\perp, \mu)\ln(\frac{\zeta_z}{\zeta})}f^{\mathrm{TMD}}(x, b_\perp, \mu, \zeta) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{\zeta_z}, \frac{M^2}{(\zeta_z)^2}, \frac{1}{b_\perp^2\zeta_z}\right)$$

- ζ is rapidity scale, $\zeta_z = (2xP^z)^2$.
- $K(b_\perp, \mu)$ Collins-Soper kernel
- $S(b_\perp, \mu)$ soft function, JHEP 08 (2023) 172
- $H(\frac{\zeta_z}{\mu^2})$ matching kernel with form of $H = e^h$

Lattice Calculation Setup

- Methods used to improve signal
  1. momentum smearing on sources.
  2. HYP smearing for gauge links.
  3. multiple sources along temporal direction.
- Sequential sources are used in calculation.





Lattice Calculation Setup

- pion

ensemble	a(fm)	$L^3 \times T$	m_π(MeV)	$m_\pi L$	$N_{\mathrm{conf.}}$
X650	0.098	$48^3 \times 48$	338	8.1	1892
H102	0.085	$32^3 \times 96$	354	4.9	1008
N203	0.064	$48^3 \times 128$	348	5.4	500/1543

- Nucleon

ensemble	a(fm)	$L^3 \times T$	m_π(MeV)	$m_\pi L$	$N_{\mathrm{conf.}}$
X650	0.098	$48^3 \times 48$	338	8.1	1250

Two-state fit

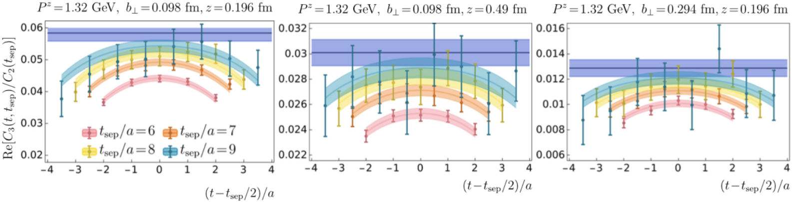
We calculate two-point function (2pt) and three-point function (3pt). Then extract the bare matrix element by combined fitting both 2pt and 3pt/2pt.

$$C_2(P^z, t_{\mathrm{sep}}) = c_0 e^{-E_0 t_{\mathrm{sep}}}(1 + c_1 e^{-\Delta E t_{\mathrm{sep}}})$$

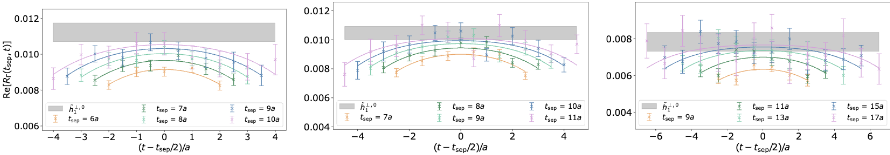
$$R(t, t_{\mathrm{sep}}) = \frac{\tilde{h}_0 + c_2 e^{-\Delta E t} + c_2 e^{-\Delta E (t_{\mathrm{sep}} - t)} + c_3 e^{-\Delta E t_{\mathrm{sep}}}}{1 + c_1 e^{-\Delta E t_{\mathrm{sep}}}}$$

$\tilde{h}_0$ corresponds to TMDPDF bare matrix element.

$R_\Gamma(t, t_{\rm sep})$ fitting for nucleon



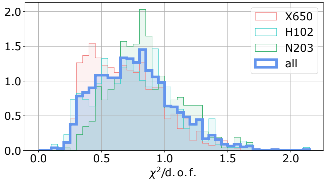
$R_\Gamma(t, t_{\rm sep})$ fitting for pion



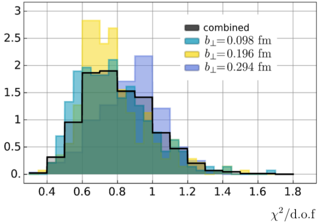
(a) X650, $P^z = 0.79$ GeV, $z = 5a$, $b_\perp = 3a$ (b) H102, $P^z = 0.91$ GeV, $z = a$, $b_\perp = 3a$ (c) N203, $P^z = 0.81$ GeV, $z = a$, $b_\perp = 3a$

χ^2/d.o.f histogram

Pion:

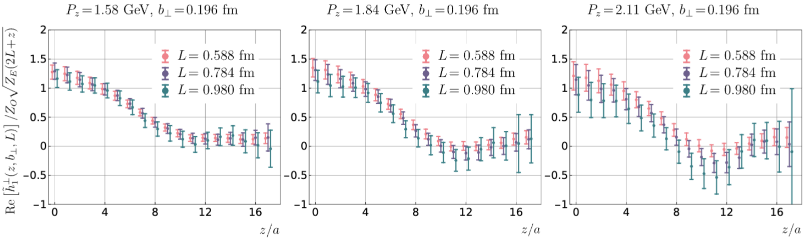


Nucleon:



Renormalization

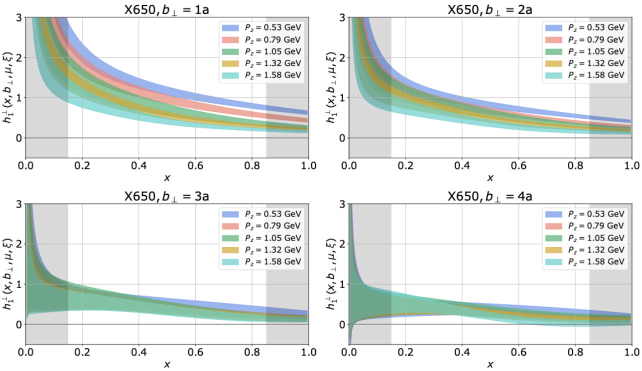
Take nucleon as example, with $L = \{6, 8, 10\}a$.



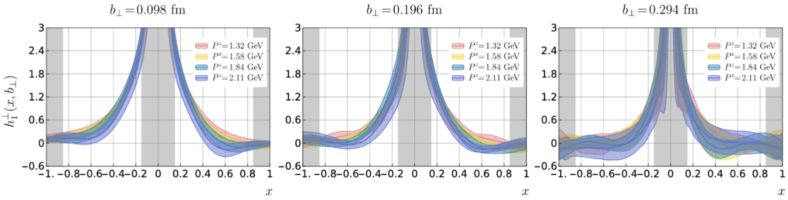
It shows that results of various L are convergent with each other in 1σ. We take results of $L = 8a$ as estimates of $L \to \infty$, while estimate systematics error by results of $L = 10a$.

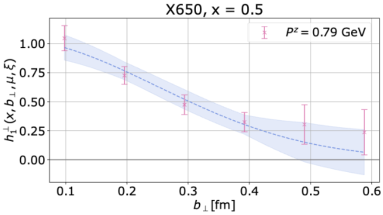
Matching Results

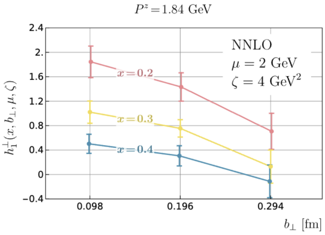
Pion ($\mu = 2\text{GeV}, \zeta = 4\text{Gev}^2$), with NNLO matching kernel, taking X650 as an example



Nucleon ($\mu = 2\text{GeV}$, $\zeta = 4\text{GeV}^2$)

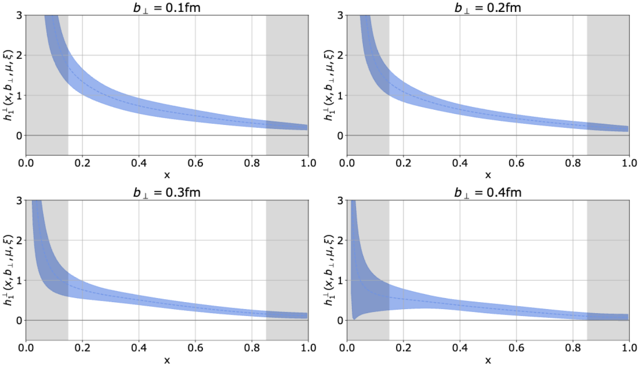






Continuum and Infinite Momentum Extrapolation

For pion case, collecting matched data of X650, N203, H102 ensembles with large P^z's, an extrapolation to infinite momentum and continuum limit is performed.



Conclusions and Outlooks

- In the framework of LaMET, we calculate and analyze pion and nucleon Boer-Mulders functions.
- Smearings and multiple sources are used to improve the signal.
- We perform continuum and infinite momentum extrapolation for pion case.
- We observed obviously decreasing $b_\perp$ dependence of both pion and nucleon Boer-Mulders functions.
- Future works would foucus on nucleon case with the other two ensembles.