Development on precision calculation using CLQCD ensembles

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For CLQCD collaboration

CLQCD

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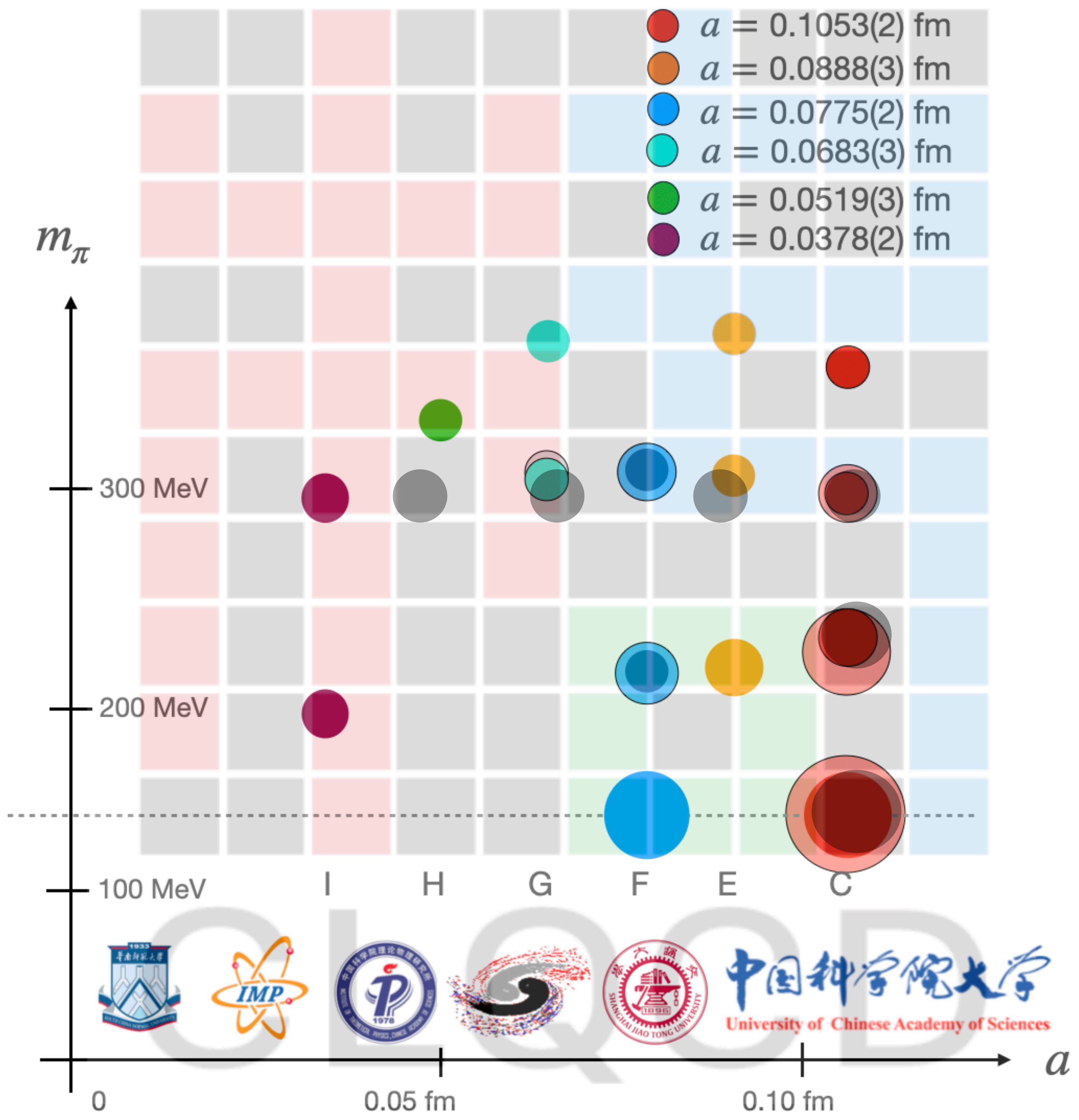
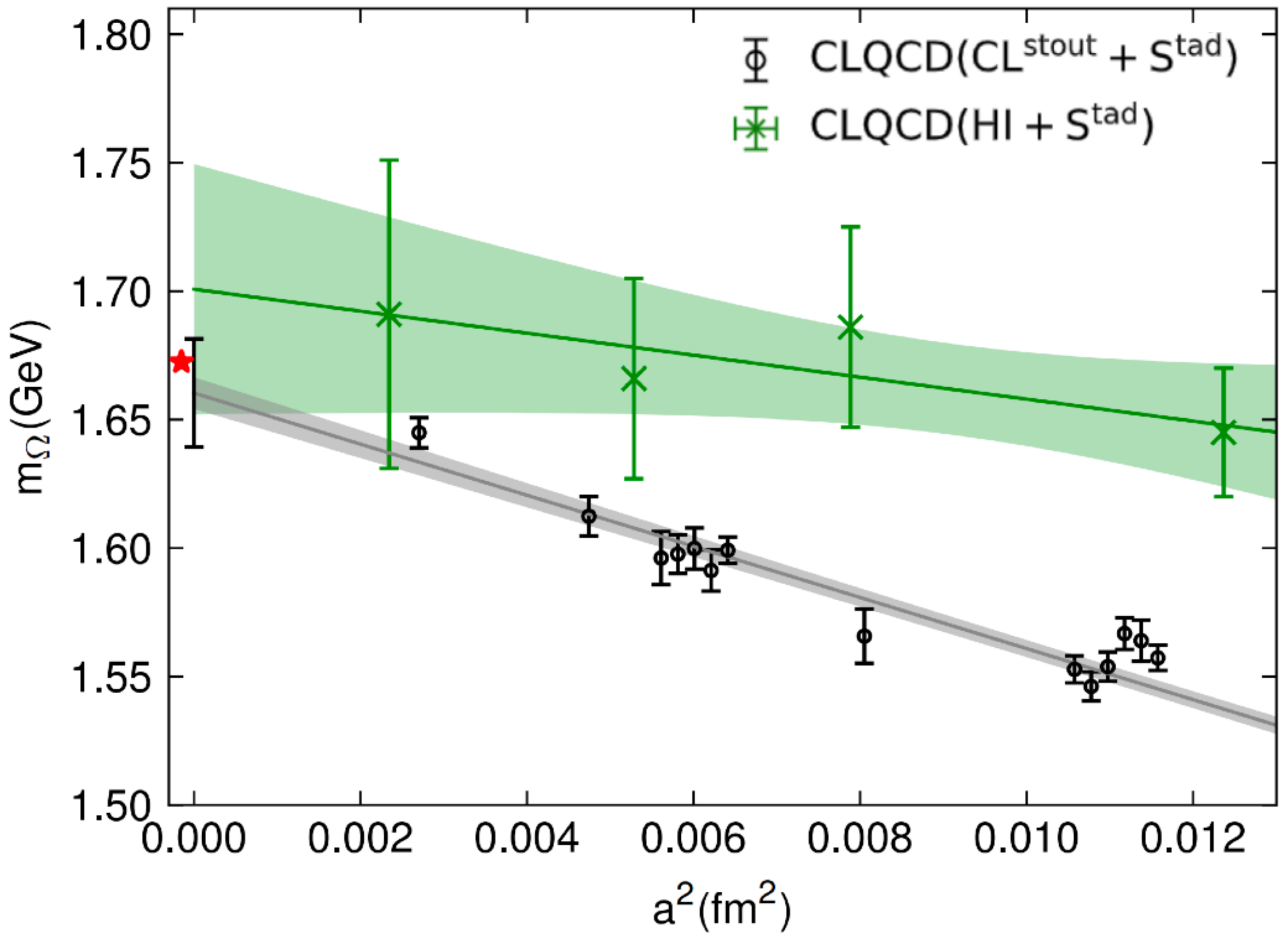
CLQCD ensembles

	Country/ Region	Smallest lattice spacing	No. of physical point ensembles	Largest spacial size	No. of fermion discretization
MILC	US	0.03 fm	5	5.8 fm	1
RBC	US	0.06 fm	3	5.5 fm	1
BMW	EN	0.05 fm	15	10 fm	2
CLS	EN	0.04 fm	2	5.5 fm	1
ETM	EN	0.05 fm	5	6.3 fm	1
PACS	JP	0.06 fm	3	10 fm	1
CLQCD	**CN**	**0.04 fm**	**3**	**6.7 fm**	**2**



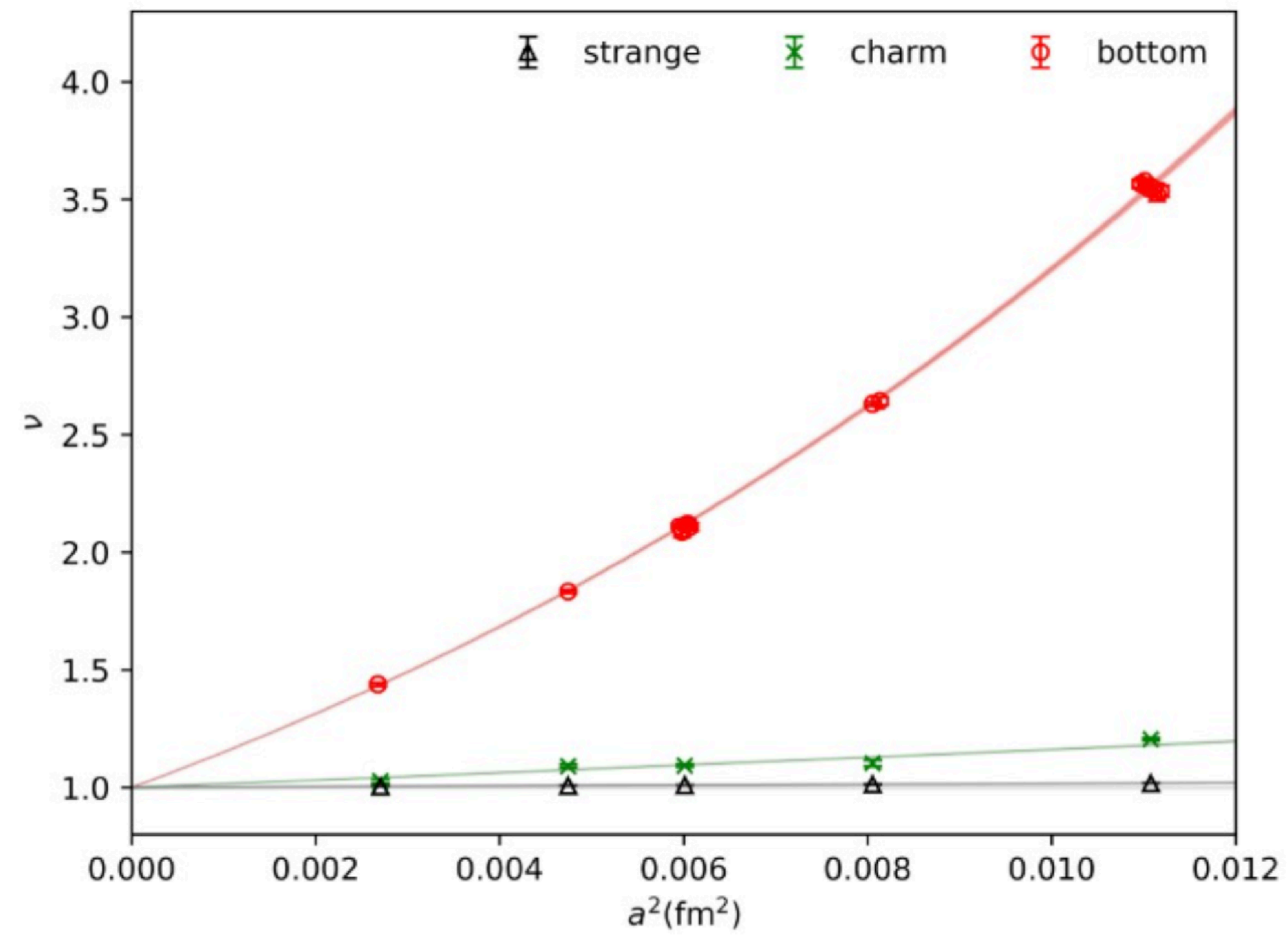
- The first ensemble set from China which can control most of the systematic uncertainties；
- Unique advantage on finite volume studies.



- New ensembles $(\mathrm{HI}+\mathrm{S}^{\mathrm{tad}})$ with 2+1+1 flavor HISQ fermion can provide proper estimate of the charm sea effects;
- Compared to the current 2+1 flavor Clover fermion ensembles $(\mathrm{CL}^{\mathrm{stout}}+\mathrm{S}^{\mathrm{tad}})$, the discretization errors are also suppressed in kinds of the cases.

Oct.11th 16:40-17:00 分会2 张遵贤
Oct.11th 17:00-17:20 分会2 林彤巍

CLQCD ensembles

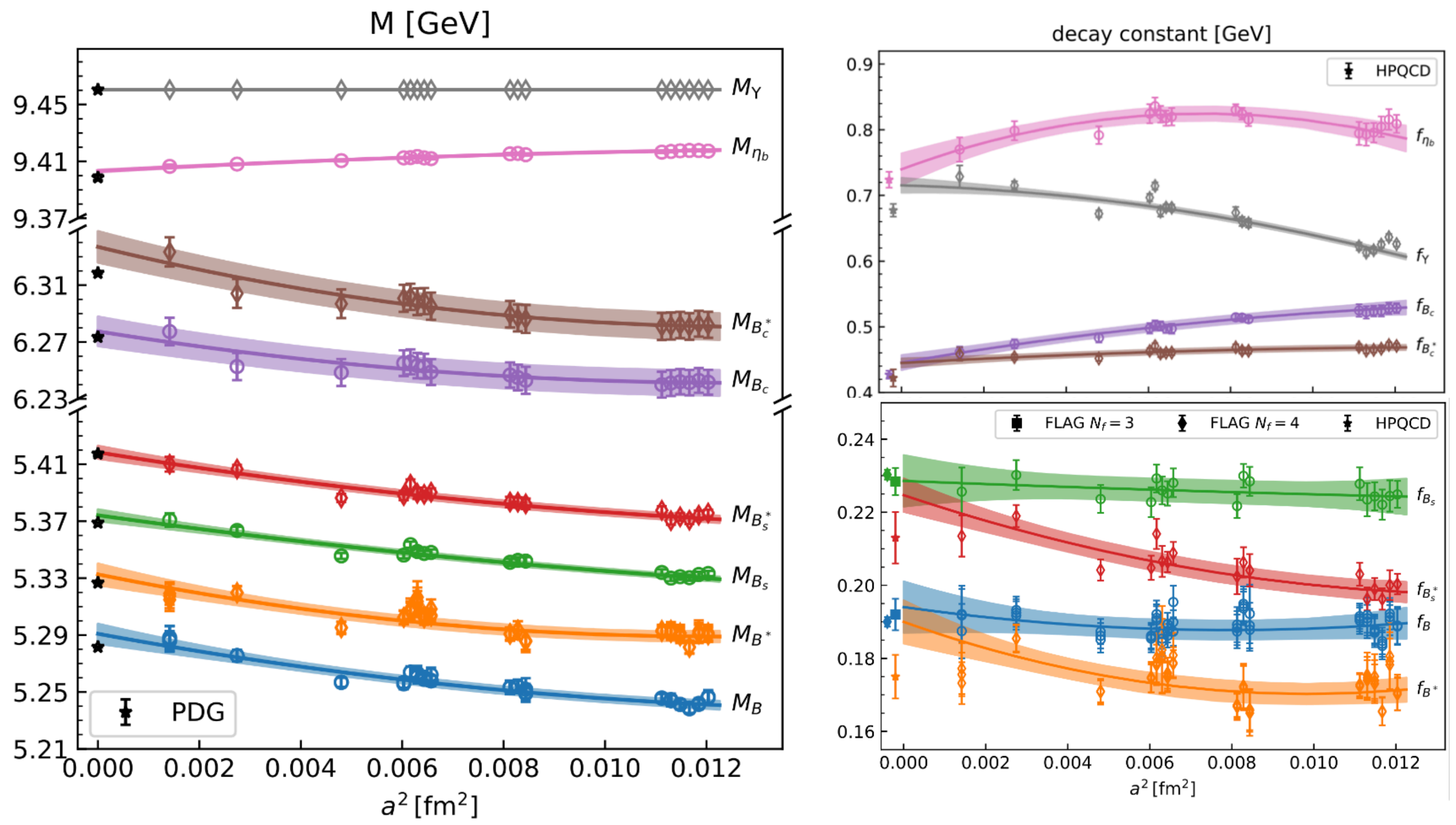


- Absorb most of the discretization errors of the bottom quark into an rescale factor along the temporal direction;

Oct.12th 14:50-15:10 分会1 杜海洋

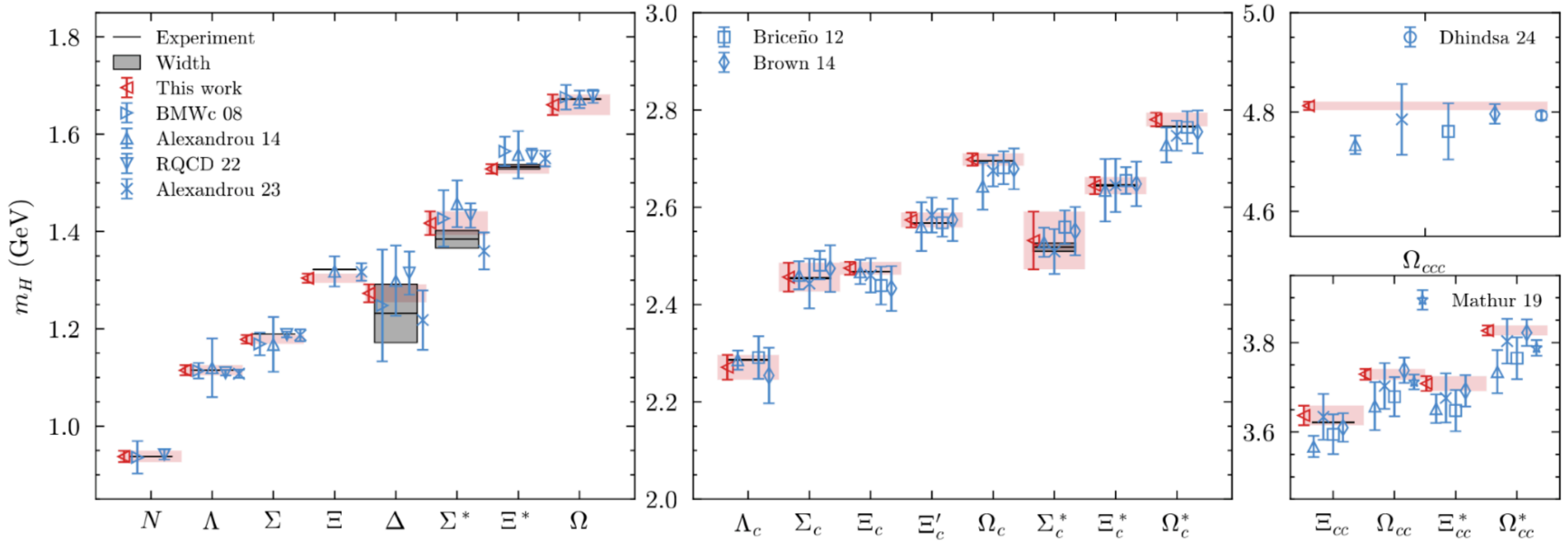
- And developed the corresponding non-perturbative renormalization.

Oct.11th 11:20-11:40 蔡孟初



Hadron mass

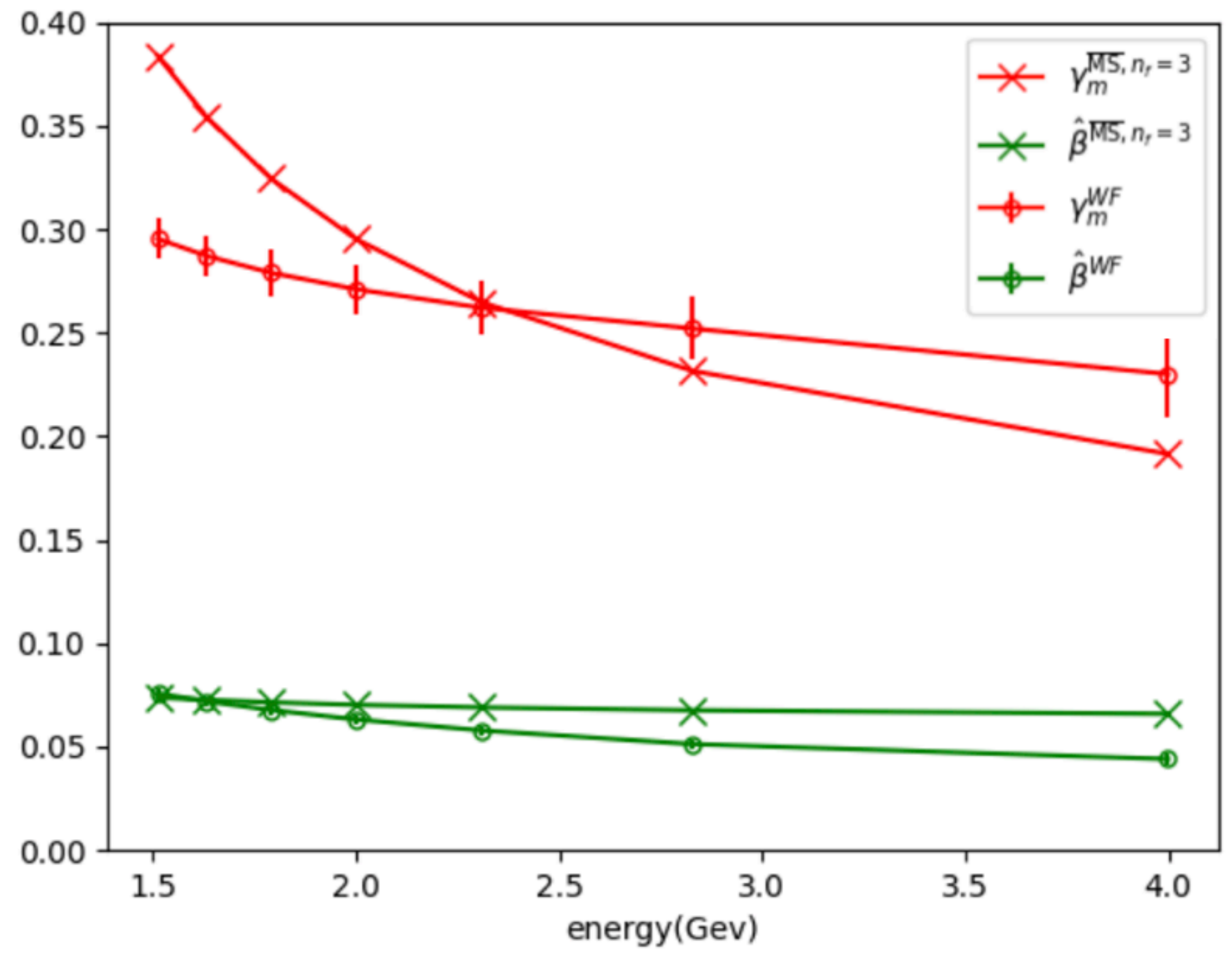
Pure QCD without ISB

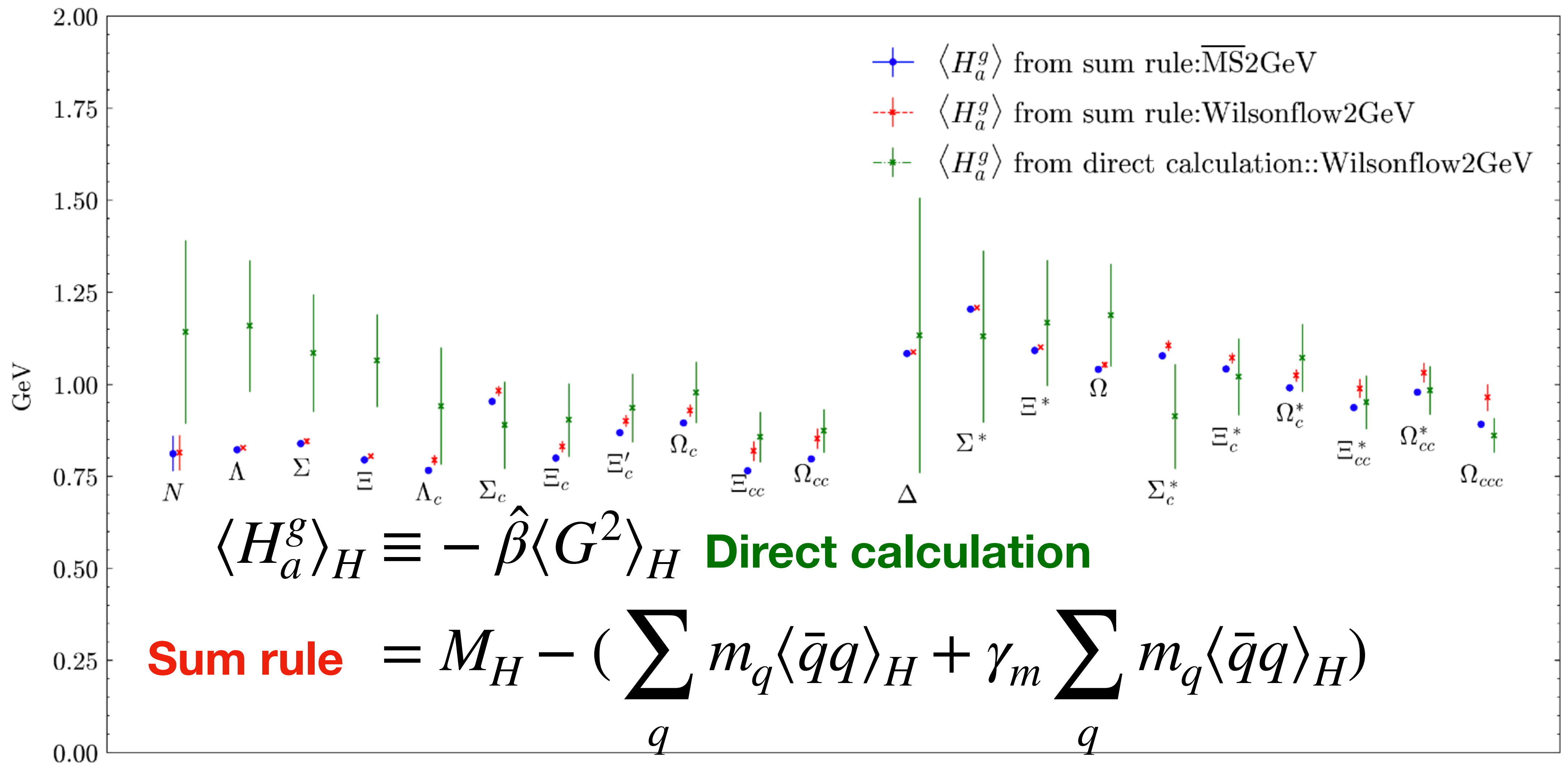


B.-L. Hu, et. al., [CLQCD], arXiv: 2411.18402

$$\gamma_m \equiv -\frac{2\mu^2}{m}\frac{\partial m}{\partial \mu^2} = [\frac{2}{\pi}\alpha_s + \mathcal{O}(\alpha_s^2)]$$

$$\hat{\beta} \equiv -\frac{1}{4\pi}\frac{\mu^2}{\alpha_s^2}\frac{\partial \alpha_s}{\partial \mu^2} = [(\frac{11}{8\pi} - \frac{N_f}{12\pi})\alpha_s + \mathcal{O}(\alpha_s^2)]$$





$\langle H_a^g \rangle_H \equiv -\hat{\beta}\langle G^2 \rangle_H$ **Direct calculation**

Sum rule $= M_H - (\sum_q m_q \langle \bar{q}q \rangle_H + \gamma_m \sum_q m_q \langle \bar{q}q \rangle_H)$

- The gluon trace anomaly from baryons with different flavors are close, from both the direct calculation of $\langle H_a^g \rangle_H$ at $a = 0.052$ fm and $m_\pi = 320$ MeV and also that from sum rule;
- The $\hat{\beta}$ and γ_m under the Wilson flow and $\overline{\mathrm{MS}}$ scheme are close;
- Deviations would come from perturbative matching and kinds of systematics, including iso-spin breaking (ISB) effects and QED corrections.

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C.Y. Zhang, et. al., [CLQCD], in preparation

QCD+QED

- The hadron mass and matrix elements in the real world require the full QCD+QED calculation. But since the lattice calculation can only reach an $\mathcal{O}(1\%)$ ($\mathcal{O}(0.1\%)$ in the heavy quark case) precision, one can expand the prediction in term of the polynomial of α and also $\delta_{\rm ISB} \equiv (m_d - m_u)/\Lambda_{\rm QCD}$:

$$\mathcal{M}^{\rm QCD+QED} = \mathcal{M}^{\rm isoQCD} + \alpha\mathcal{M}^{(0,1)} + \delta_{\rm ISB}\mathcal{M}^{(1,0)} + \mathcal{O}(\alpha^2, \alpha\delta_{\rm ISB}, \delta_{\rm ISB}^2).$$

- Naive power counting suggests that both ISB and QED corrections are 1%;

- There are kinds of known results for the ISB and QED corrections:

$$m_n - m_p = 2.52_{\rm ISB}(29)\ \text{MeV} - 1.00_{\rm QED}(16)\ \text{MeV},$$

$$m_{D^+} - m_{D^0} = 2.54_{\rm ISB}(13)\ \text{MeV} + 2.14_{\rm QED}(13)\ \text{MeV}, \qquad m_{B^+} - m_{B^0} = -1.88_{\rm ISB}(60)\ \text{MeV} + 1.58_{\rm QED}(24)\ \text{MeV}\,.$$

BMWc, Science 347(2015)1452

M. Rowe, R. Zwicky, JHEP(2023)089

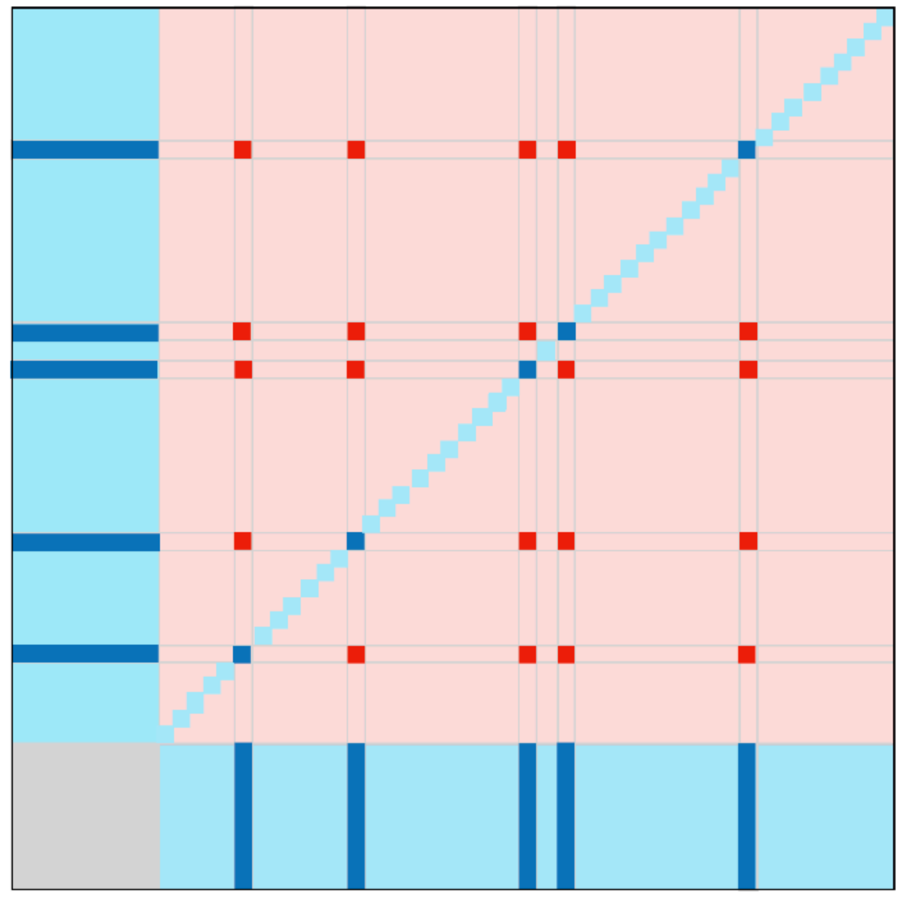
- ISB effect is only 0.1% for the charmed hadron, how about the QED effect?

- How to understand the sizable QED correction for the charged pion mass in the chiral limit?

$$m_{\pi^+}|_{m_q\to 0} = 0_{m_q\bar{q}q} + 0_{\mathcal{O}(\alpha_s)G^2} + 32_{\mathcal{O}(\alpha)}\ \text{MeV} \qquad \text{given } (m_{\pi^+}^2 - m_{\pi^0}^2)|_{m_q\to 0} \simeq\ 1000\ \text{MeV}^2$$

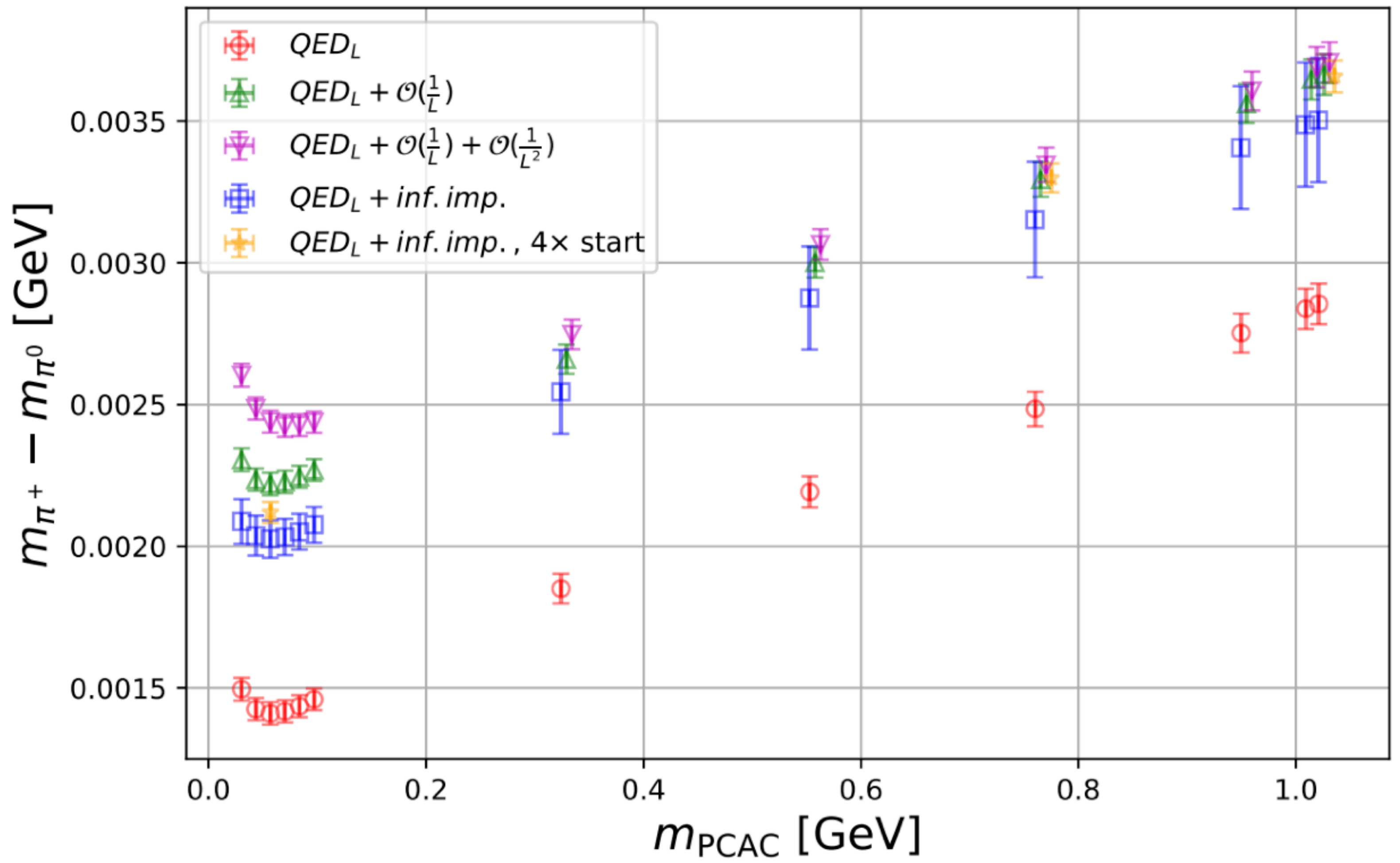
Outline

- **Blending method for statistical uncertainty**

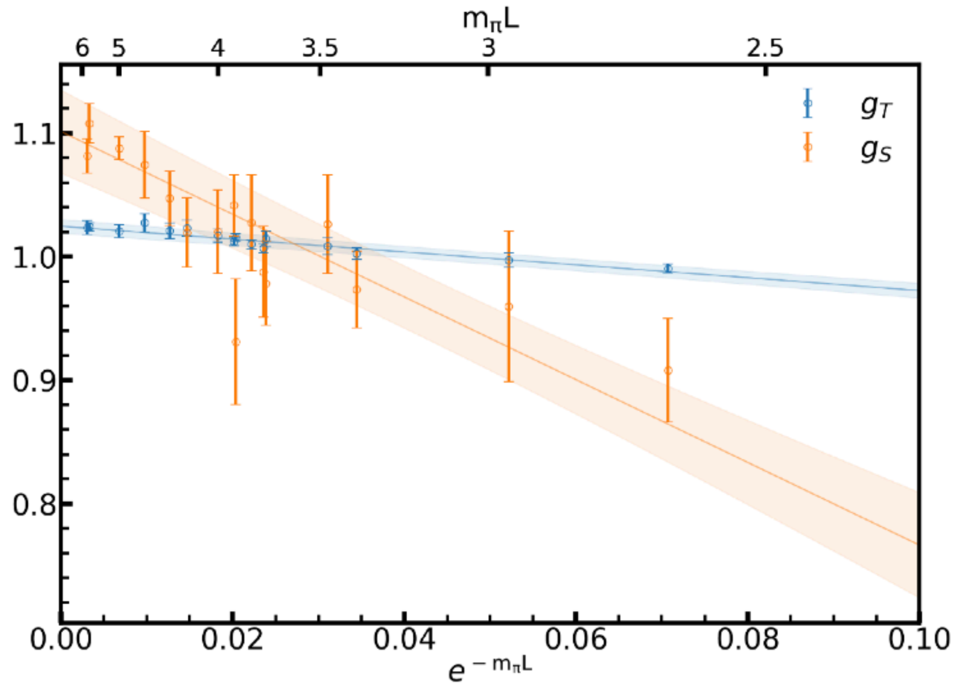


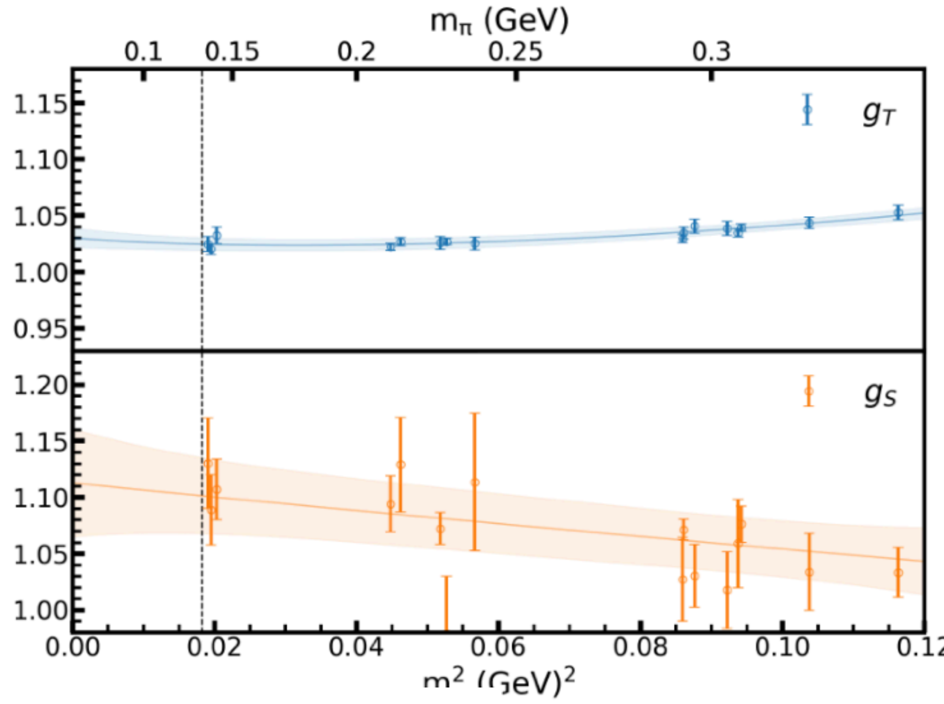
$$\Omega^{(2)}_{ij} = \begin{cases} 1 & \text{for } i, j \le N_e, \\ \Pi_{i=0}^{1} \frac{V - N_e - i}{N_{st} - i} & \text{for } i, j > N_e,\ i \ne j \\ \frac{V - N_e}{N_{st}} & \text{for the other cases,} \end{cases}$$

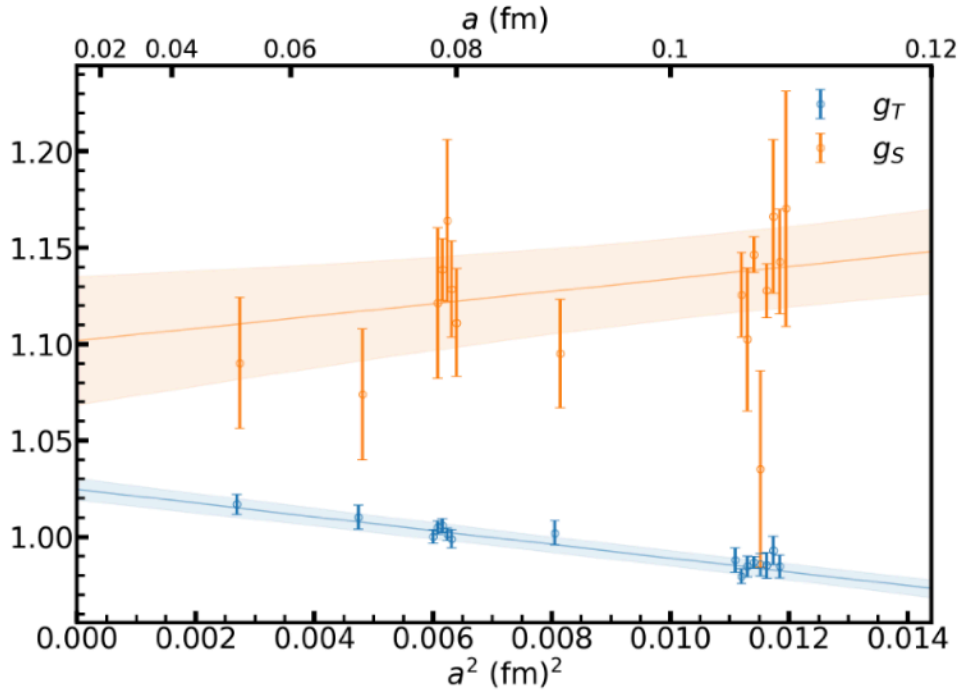
- QED and ISB corrections



- Systematic uncertainties of IsoQCD



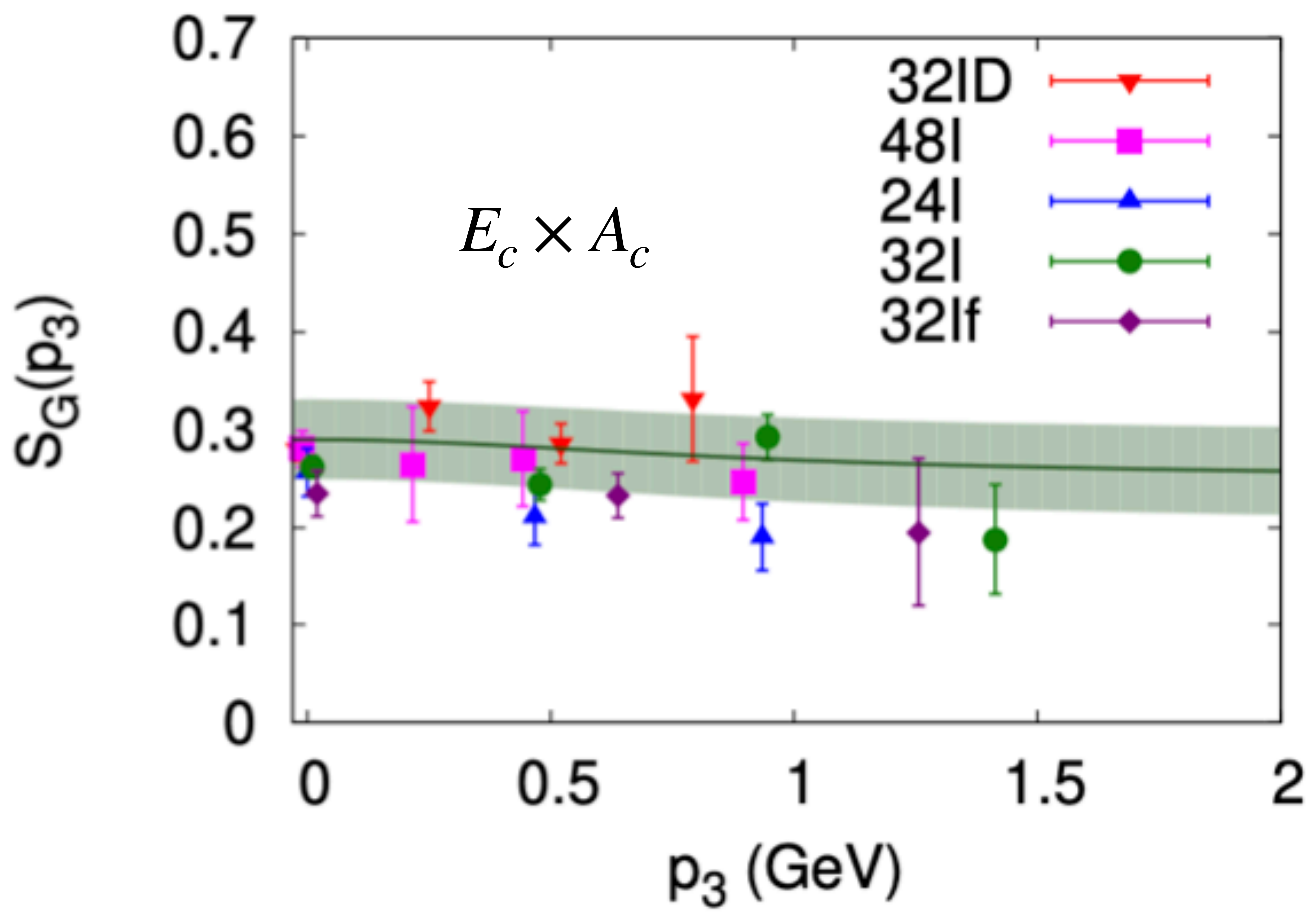




- Outlook

Distillation method

Application in ΔG



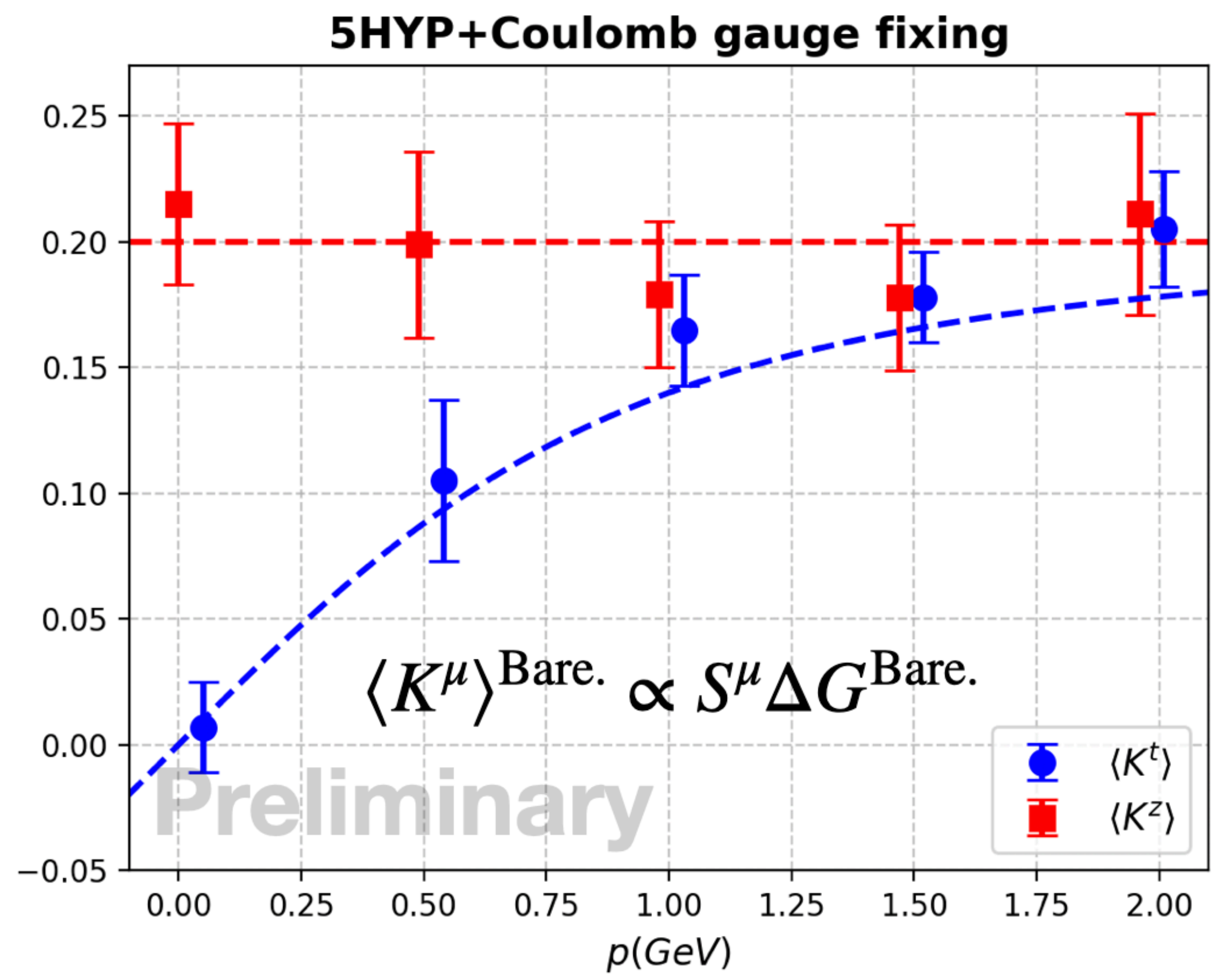
YBY et al. [χQCD], PRL118(2017)102001

$$K^\mu(x) = \epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}[A_\nu F_{\rho\sigma} - 2ig_s A_\nu A_\rho A_\sigma/3](x)$$

Matching-free

Non-perturbative renormalizable

Distillation + momentum smearing



Oct.12th 9:30-9:50 赵殿君

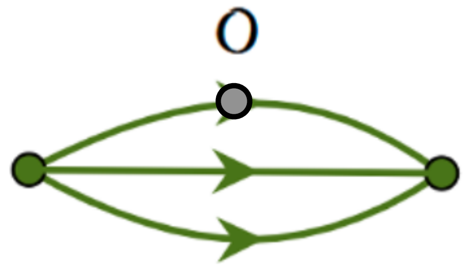
D.J. Zhao, et. al. [LPC], In preparation

$$\hat{I}_{x,y} \to \sum_{i=1}^{N_e}\sum_{j=1}^{N_e} |\phi_i(x)\rangle\langle\phi_j(y)|$$

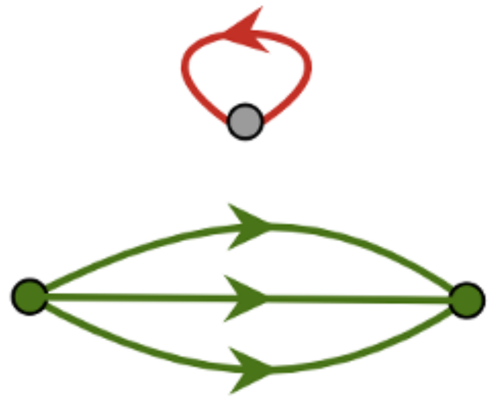
$$\nabla^2|\phi_i(x)\rangle = \lambda_i|\phi_i(x)\rangle, \lambda_i < \lambda_{i+1}$$

- Distillation method projected the quark propagator into its low momentum modes and suppress the excited state contaminations efficiently;
- It has been widely used in the hadron spectrum calculation and can also be applied to the gluon matrix elements;
- But incapable with the connected insertion quark diagrams since the high momentum modes are missing.

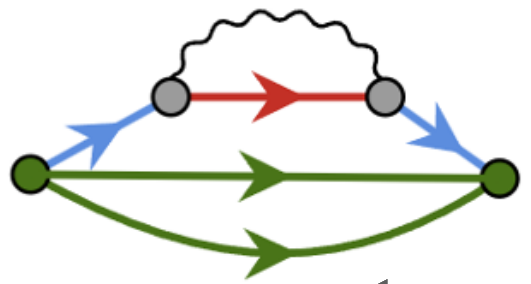
Blending method



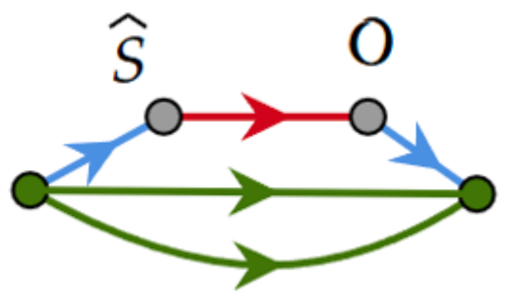
$\propto 1$



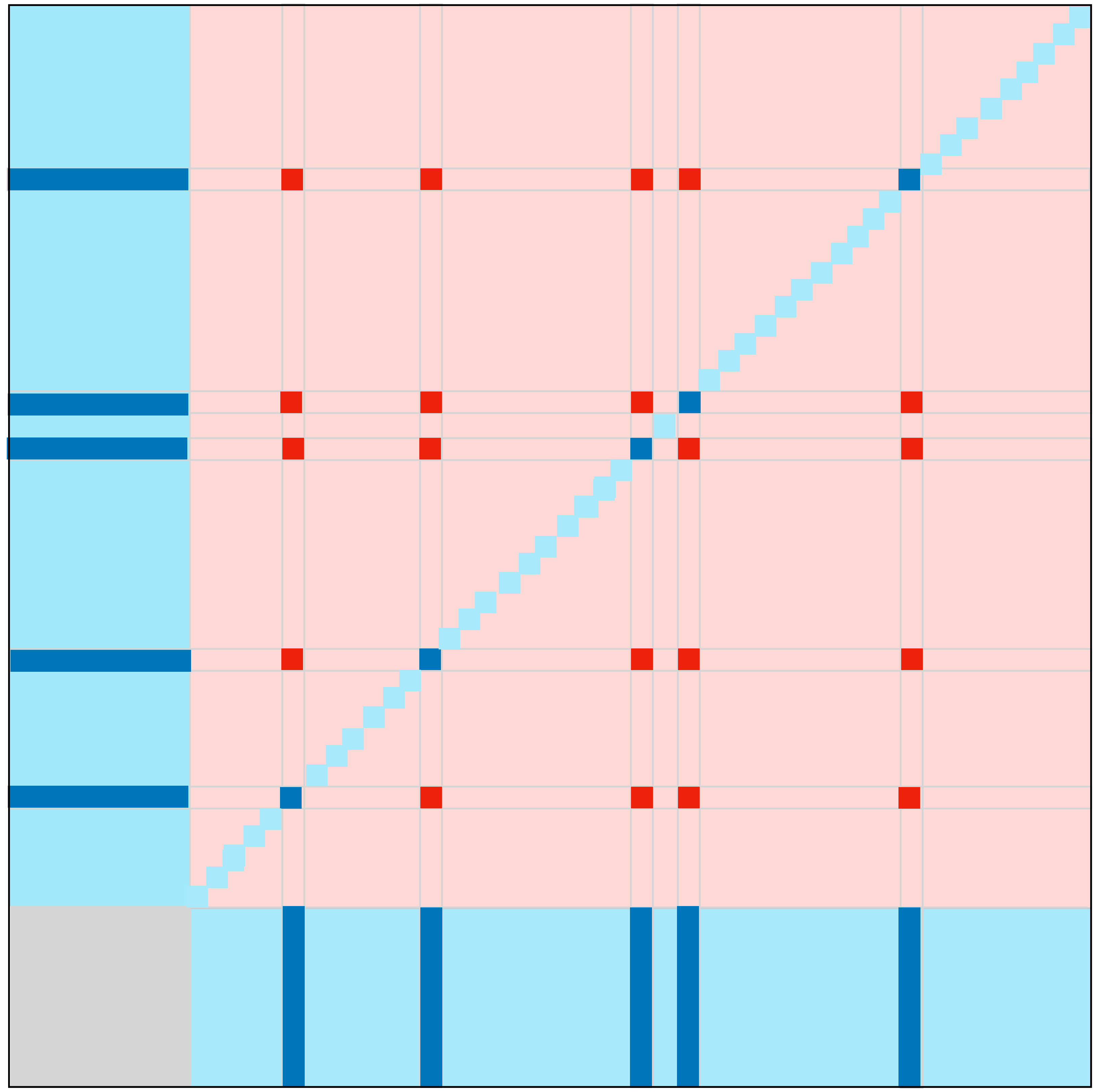
$\propto \alpha_s^2$



$\propto \alpha_{\mathrm{QED}} = \frac{1}{137}$



$\propto \frac{m_d - m_u}{\Lambda_{\mathrm{QCD}}} \sim 1\,\%$



- The "blending" method projects the all-to-all propagators into exact low momentum modes plus stochastic samples of high momentum modes;
- And allow us to calculate the hadron matrix element efficiently with kinds of high order diagrams (disconnected quark diagram, QED correction, iso-spin breaking effect et. al.).

$$M_{\mathcal{O}}(x,y) = \int \mathrm{d}^3z \mathrm{d}^3w \hat{I}_{x,z} \mathcal{O}(z,w) \hat{I}_{w,y} = \sum_{i=1}^{N_e+N_{st}} \sum_{j=1}^{N_e+N_{st}} |\phi_i(x)\rangle \mathcal{O}_{ij} \langle \phi_j(y)| + \mathcal{O}(\frac{1}{N_{st}}),$$

$$\mathcal{O}_{ij} \equiv \Omega^{(2)}_{ij} \int \mathrm{d}^3z \mathrm{d}^3w \langle \phi_i(z) | M_{\mathcal{O}}(z,w) | \phi_j(w) \rangle$$

$$\Omega^{(2)}_{ij} = \begin{cases} 1 & \text{for } i,j \le N_e, \\ \Pi^1_{i=0} \frac{V - N_e - i}{N_{st} - i} & \text{for } i,j > N_e,\ i \ne j \\ \frac{V - N_e}{N_{st}} & \text{for the other cases,} \end{cases}$$

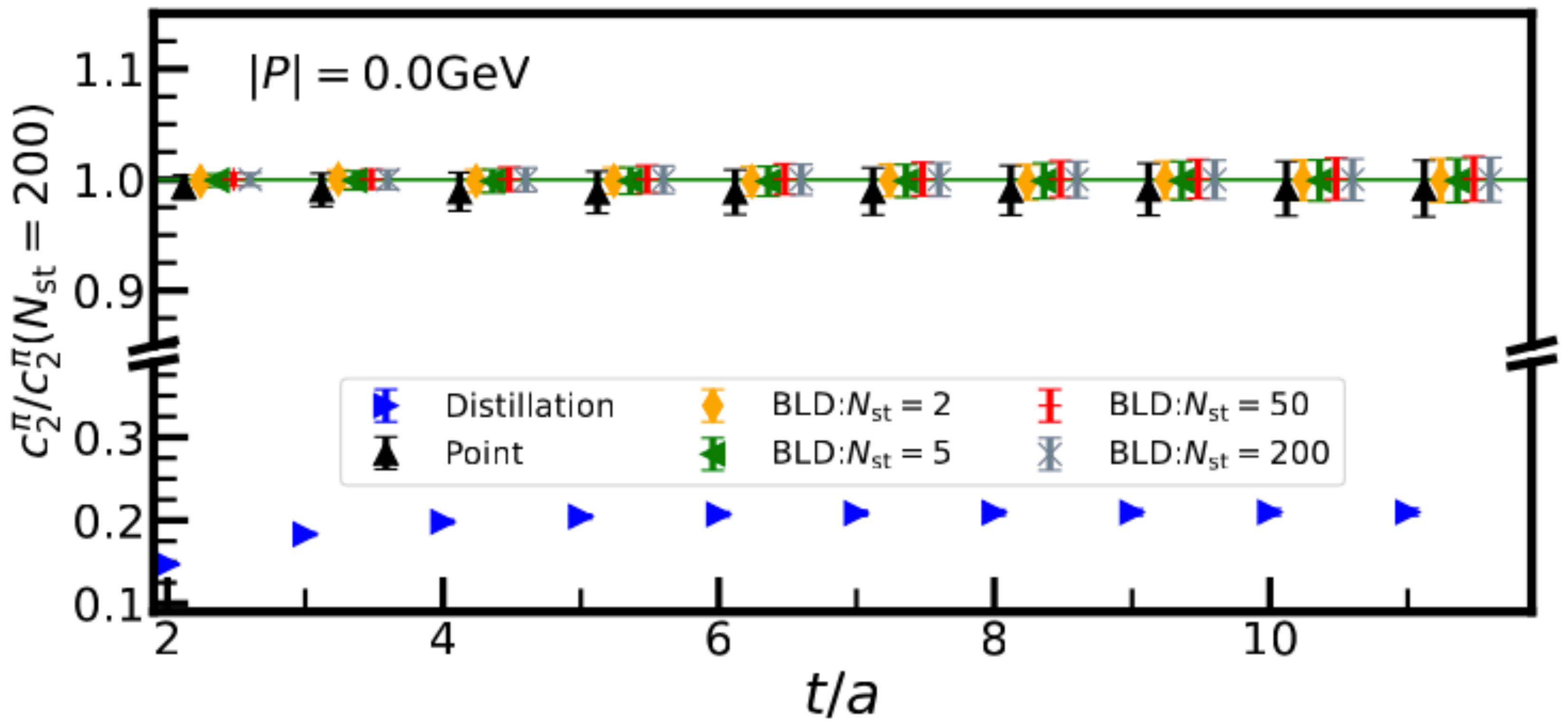
Oct.11th 16:40-17:00 分会1 胡志成

Z.C.Hu, J.H. Wang et. al., arXiv:2505.01719

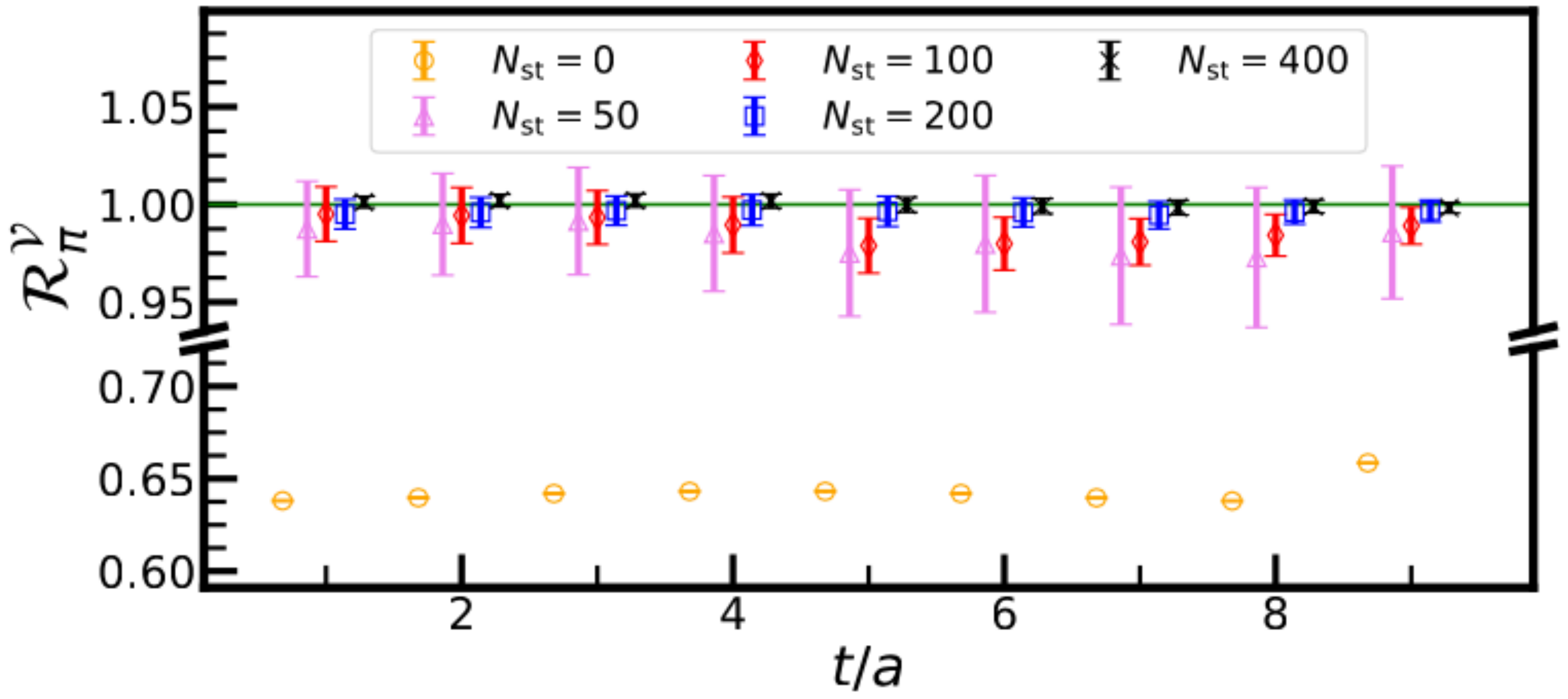
Blending method

Verification

$C_2^{\mathrm{BLD}}(t)/C_2^{\mathrm{pt}}(t) = 1$

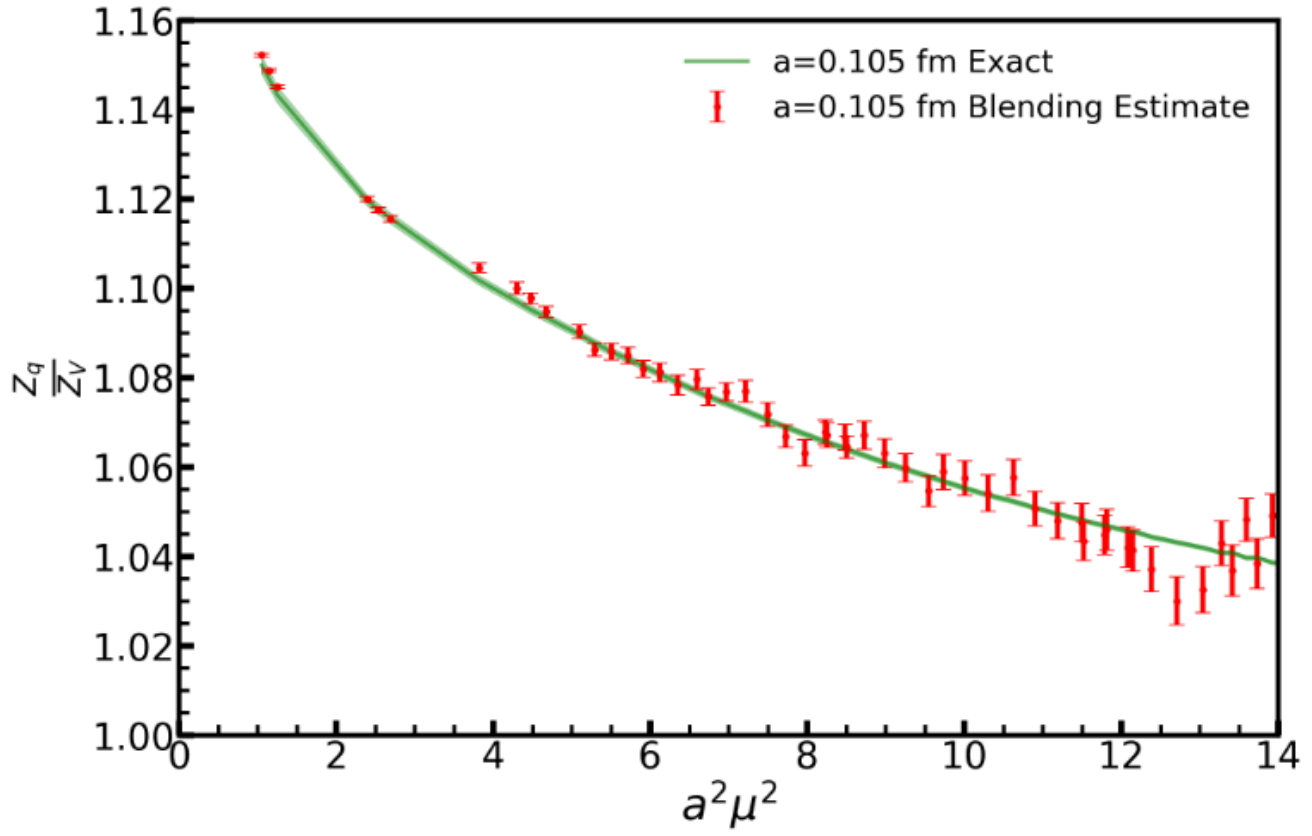


$\langle \mathcal{V}_4^{\mathrm{conserved}} \rangle_\pi = 1$

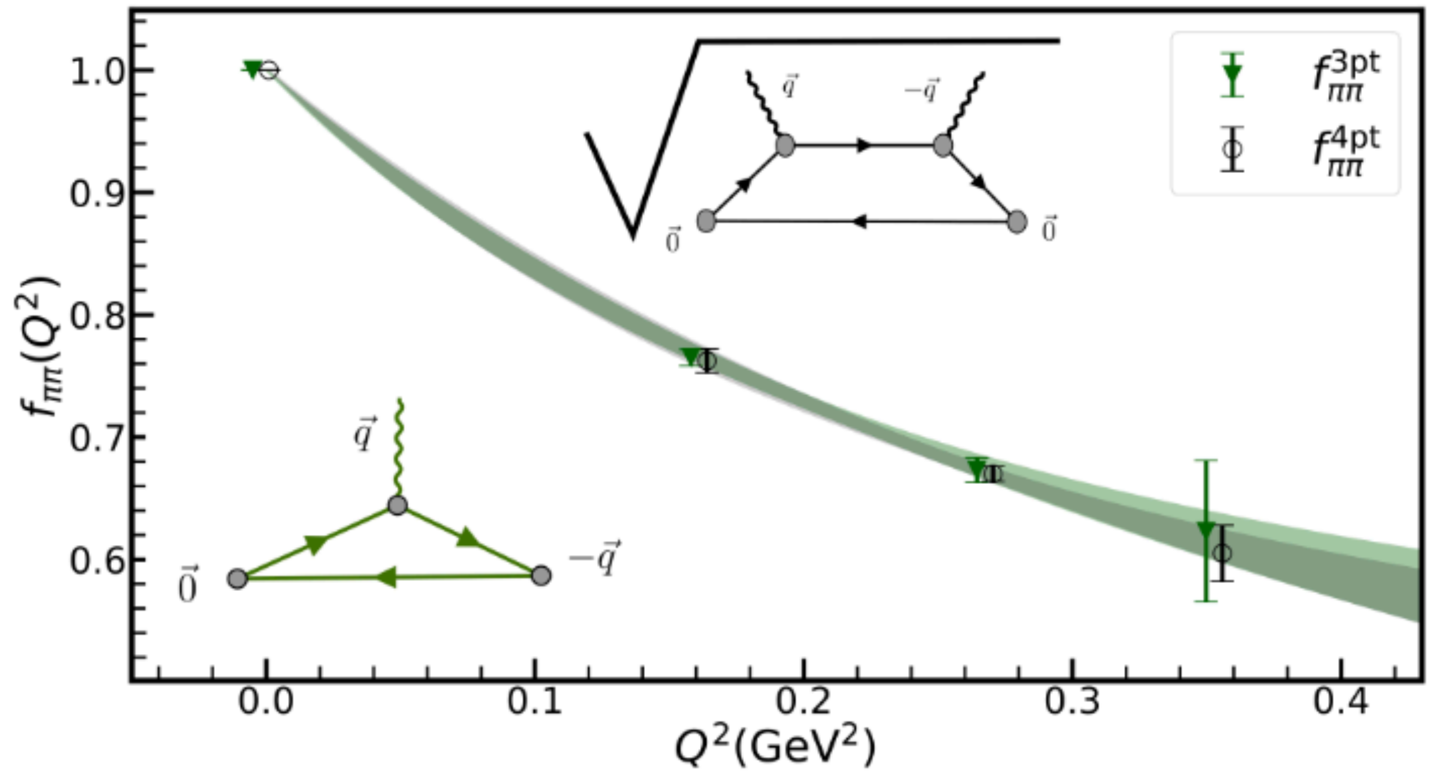


- All-to-all propagator can be approximated unbiasedly with $\mathcal{O}(1\%)$ inversions;
- The high momentum mode turns out to be important in the full correlation functions and can not be ignored.
- Unbiasedness has been verified using two-, three-, and four-point functions.

$Z_q^{\mathrm{BLD}}(a^2\mu^2)/Z_q^{\mathrm{vol}}(a^2\mu^2) = 1$



$f_{\pi\pi}^{\mathrm{3pt,BLD}}(Q^2) = f_{\pi\pi}^{\mathrm{4pt,BLD}}(Q^2)$

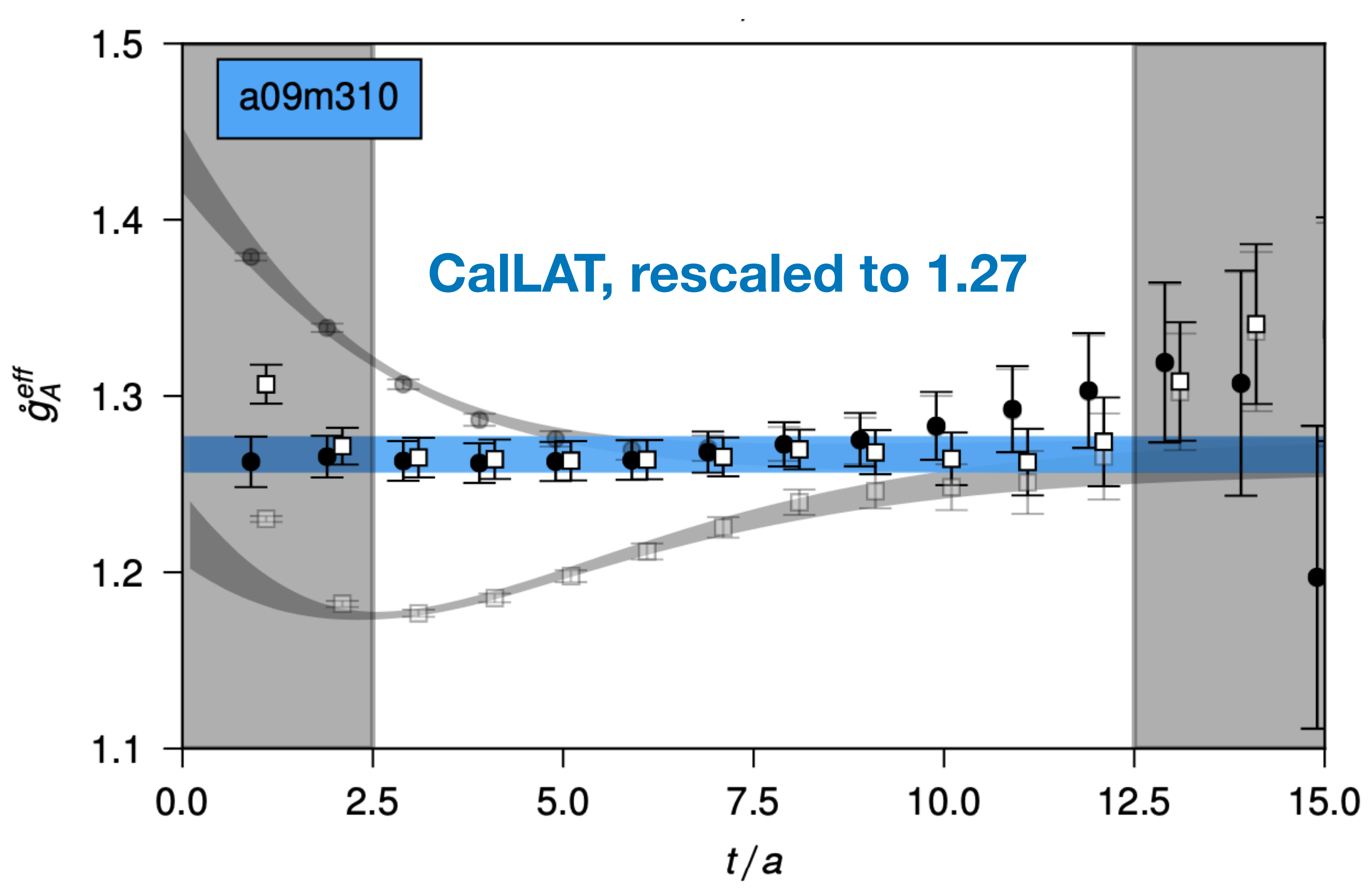


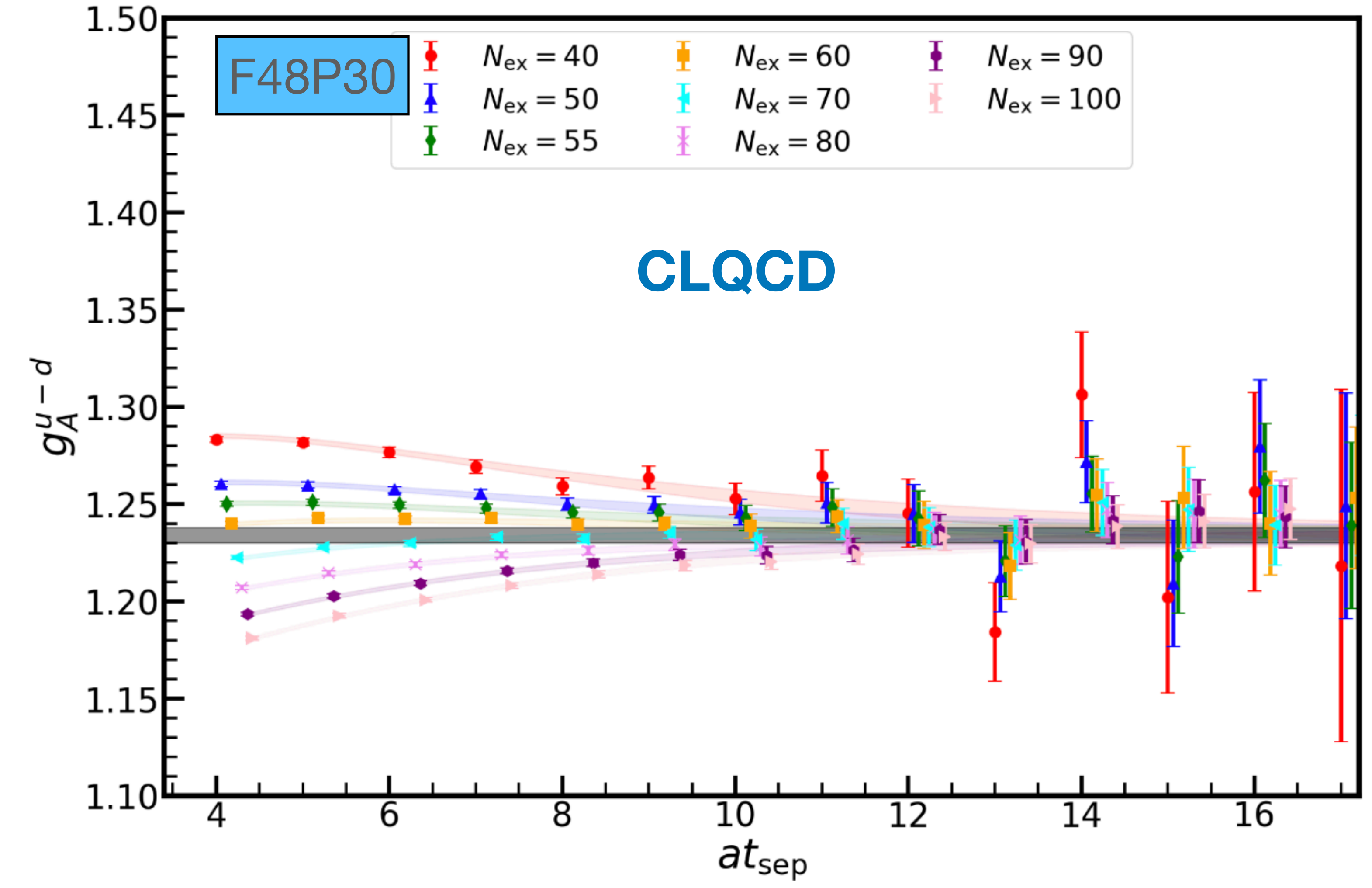
Oct.11th 16:40-17:00 分会1 胡志成

Z.C.Hu, J.H. Wang et. al., arXiv:2505.01719

Blending method

Cost comparison





	Ensemble	L	T	a(fm)	m_π	n_{cfg}	g_A^{u-d}	Propagators	Propagators for 0.35% error
CLQCD	F48P30	48	96	0.078	300	**40**	**1.234(04)**	**0.19M**	**0.19M**
CalLAT	a09m310	32	96	0.090	310	784	1.235(11)	0.06M	0.37M
RQCD	N450	48	128	0.076	287	1132	1.238(24)	0.02M	0.44M

C.C.Chang et. al., [CalLAT], Nature 558(2018)91
G.S.Bali et. al., [RQCD], PRD108(2023)094512

- Blending method can reach the same precision with **50%** cost of either FH-inspired (CalLAT) or stochastic (RQCD) method, and also

1. Provide the information from arbitrary source-current-sink separations;
2. Reuse data in any other calculation which needs light quark propagators, including disconnected insertions.

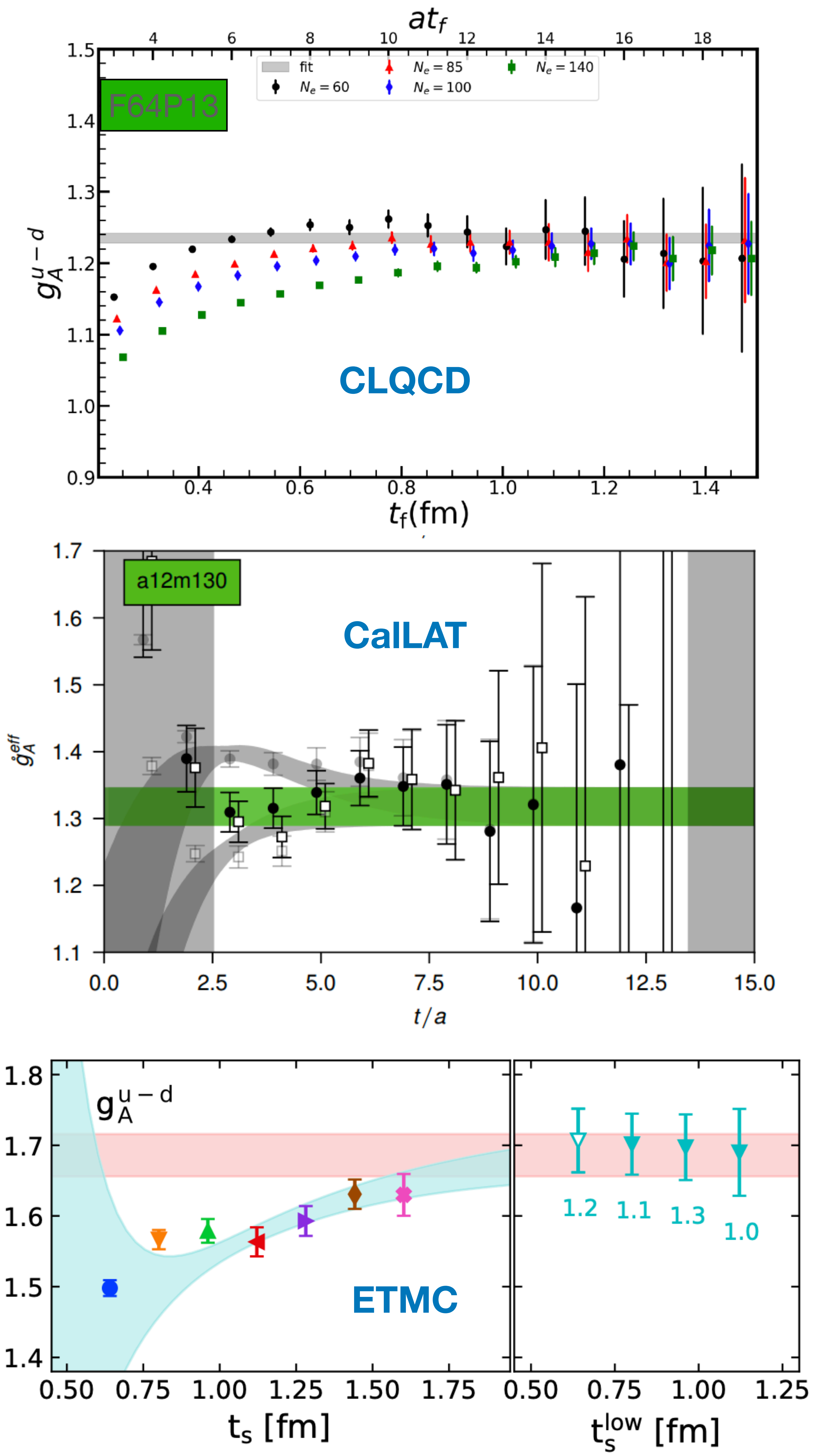
	Ensemble	L	T	a(fm)	m_π	n_{cfg}	$g_A^{u,\mathrm{DI}}$	Propagators	Propagators for 17% error
CLQCD	F48P30	48	96	0.078	300	**40**	**-0.052(9)**	$0^\dagger$	$0^\dagger$
PNDME	a09m310	32	96	0.090	310	1081	-0.053(6)	4.0M	1.8M

† Shared with that for g_A^{u-d}

Z.C.Hu, J.H. Wang et. al., arXiv:2505.01719

Blending method

Cost comparison at physical pion mass



	Ensemble	L	T	a(fm)	m_π	$n_{\rm cfg}$	g_A^{u-d}	Propagators	Propagators for 1% error
CLQCD	F64P13	64	128	0.078	134	40	1.24(01)	0.34M	0.11M
CalLAT	a12m130	48	64	0.121	131	1000	1.29(03)	0.03M	0.15M
ETMC	cB211.072.64	64	128	0.080	139	750	1.29(02)	1.71M	5.5M
RQCD	D452	64	128	0.076	156	1000	1.19(25)	0.01M	5.2M
PNDME	a09m130	64	96	0.090	138	1290	1.32(03)	1.69M	11.2M

Z.C.Hu, et. al.,[CLQCD] in preparation

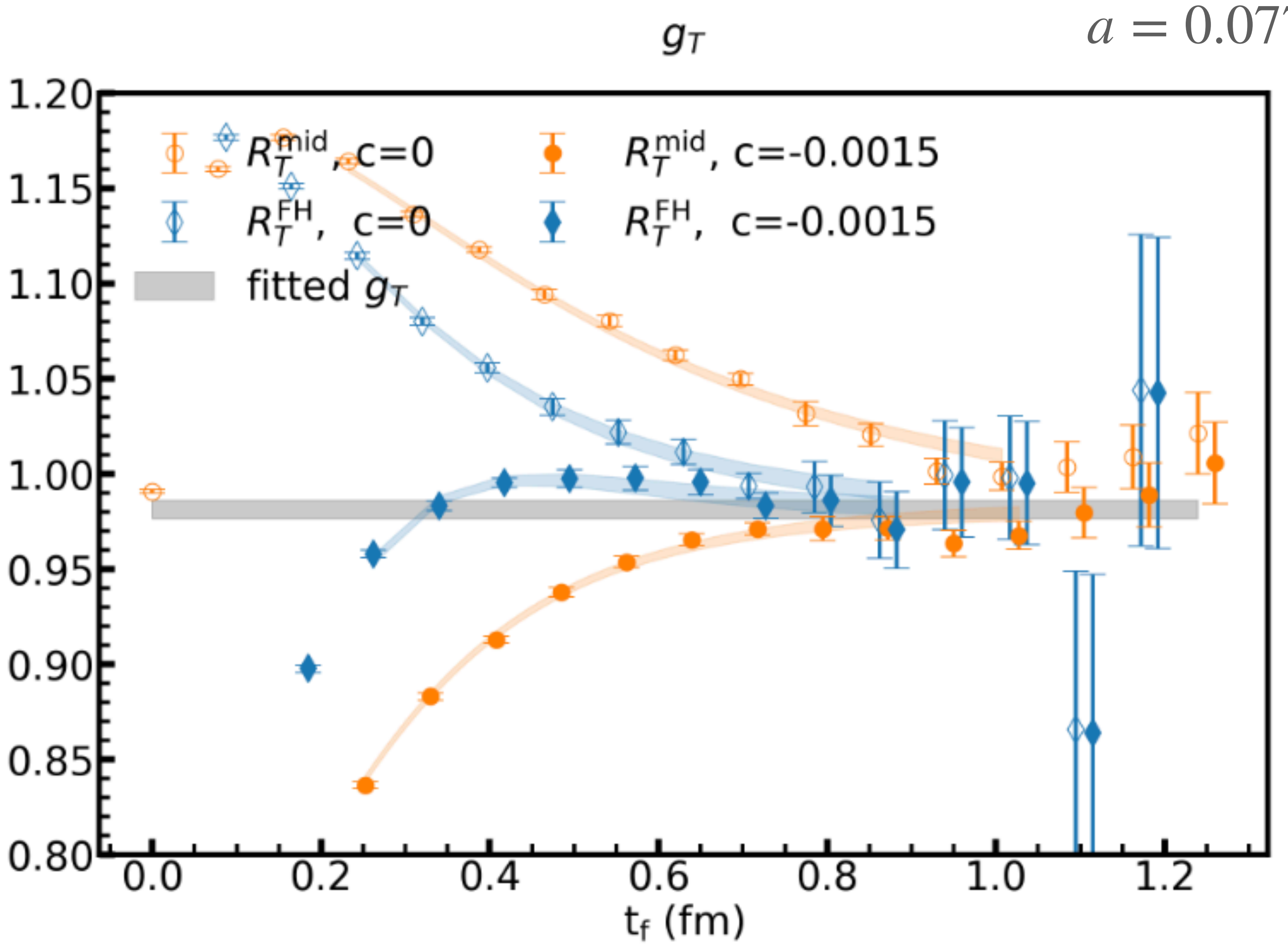
C.C.Chang et. al., [CalLAT], Nature 558(2018)91
C.Alexandrou et. al., [ETMC], PRD102(2020)054517
G.S.Bali et. al., [RQCD], PRD108(2023)094512
Y.C.Jang et. al., [PNDME], PRD109(2023)014503

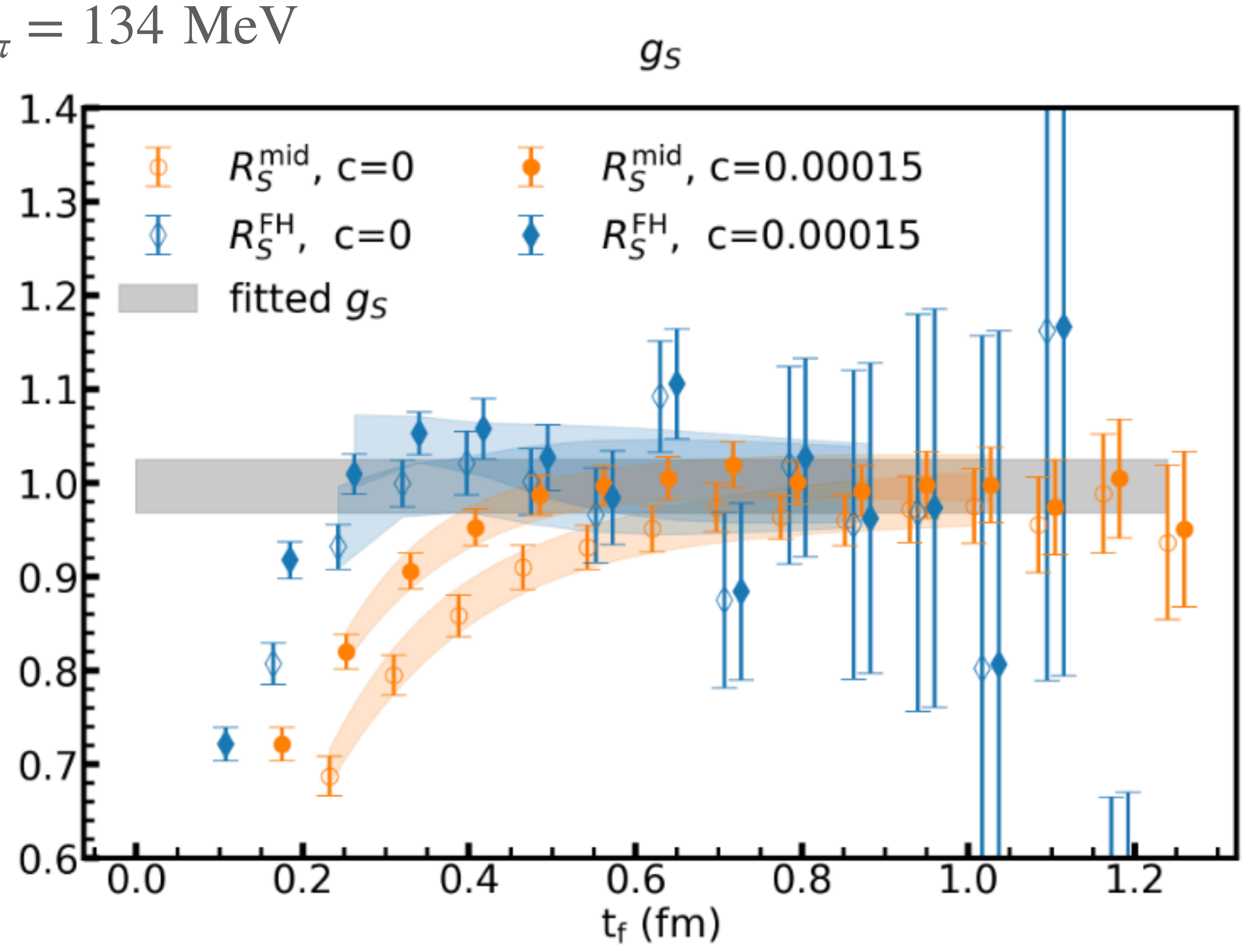
- Advantage becomes much more significant at the physical pion mass, except the CalLAT results which is only available at much larger lattice spacing;
- And also provide much more information on different source-current-sink separations and nucleon interpolation fields, which allow us to have much better control on the excited state contaminations.

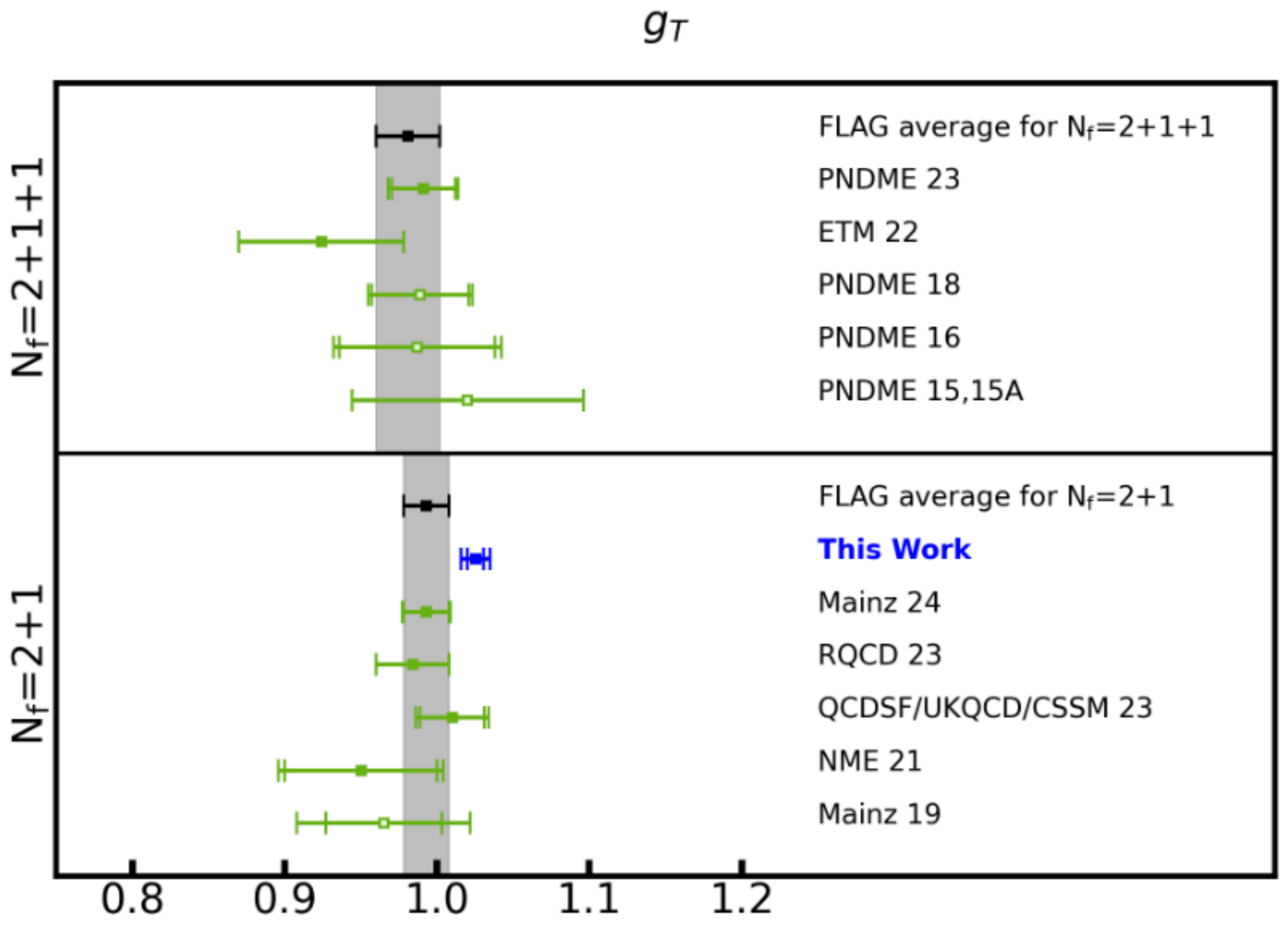
Blending method

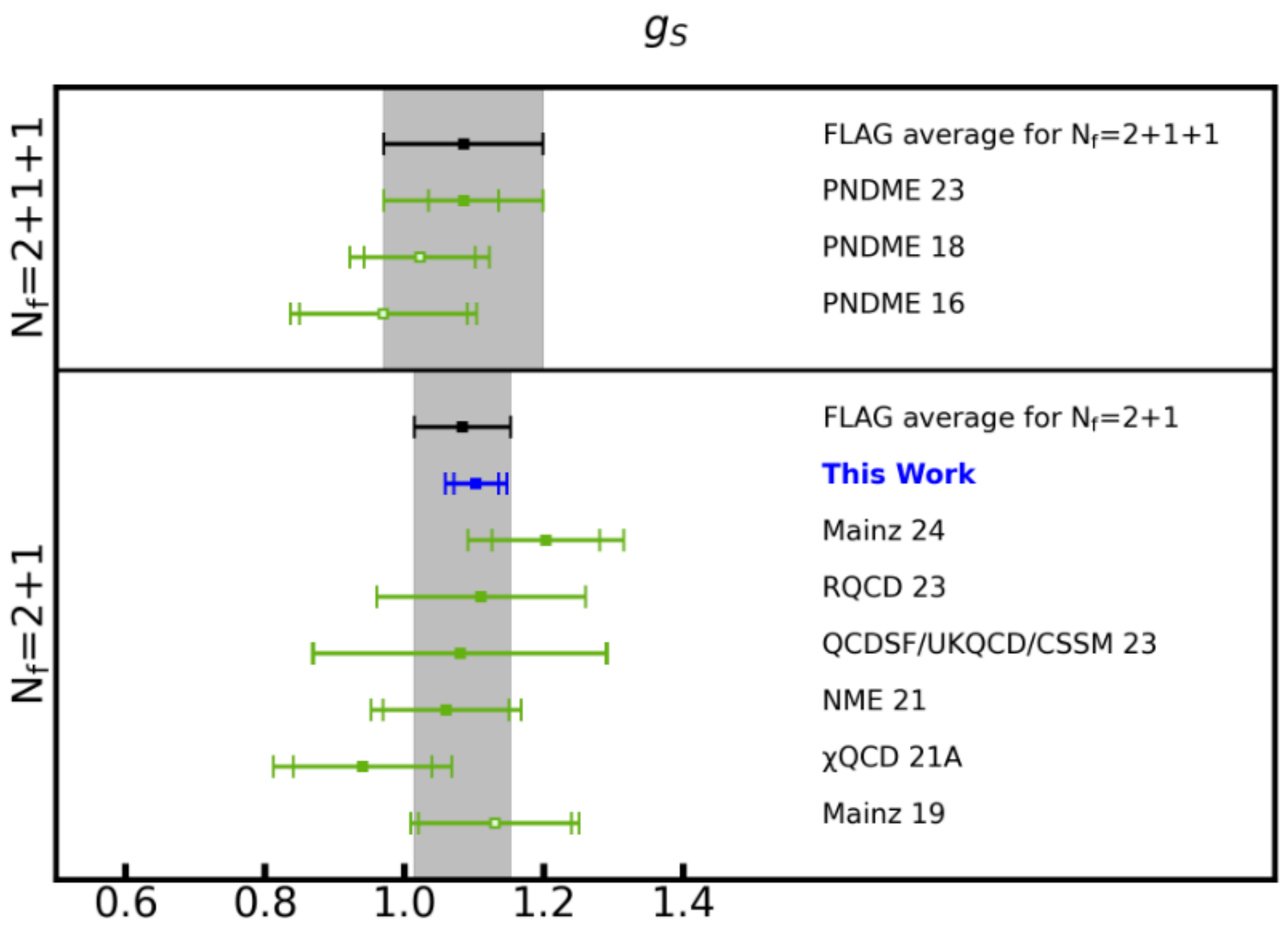
Application in $g_{S,T}^{u-d}$

$a = 0.077$ fm, $m_\pi = 134$ MeV









- Using the blending method, the signal at the physical pion mass can be improved by an order of magnitude;
- With the FLAG average of $m_d - m_u$ and QED correction from BMWc, we predict:

$$m_n - m_p = 1.59[0.23]_{\text{tot}}(0.10)_{g_S}(0.13)_{\text{ISB}}(0.16)_{\text{QED}}\ \text{MeV},$$

which agrees with the experimental value 1.293 MeV within 1.3σ.

- Direct QCD+QED calculation using CLQCD ensemble can suppress the systematic uncertainties from ISB and QED, and is in progress.

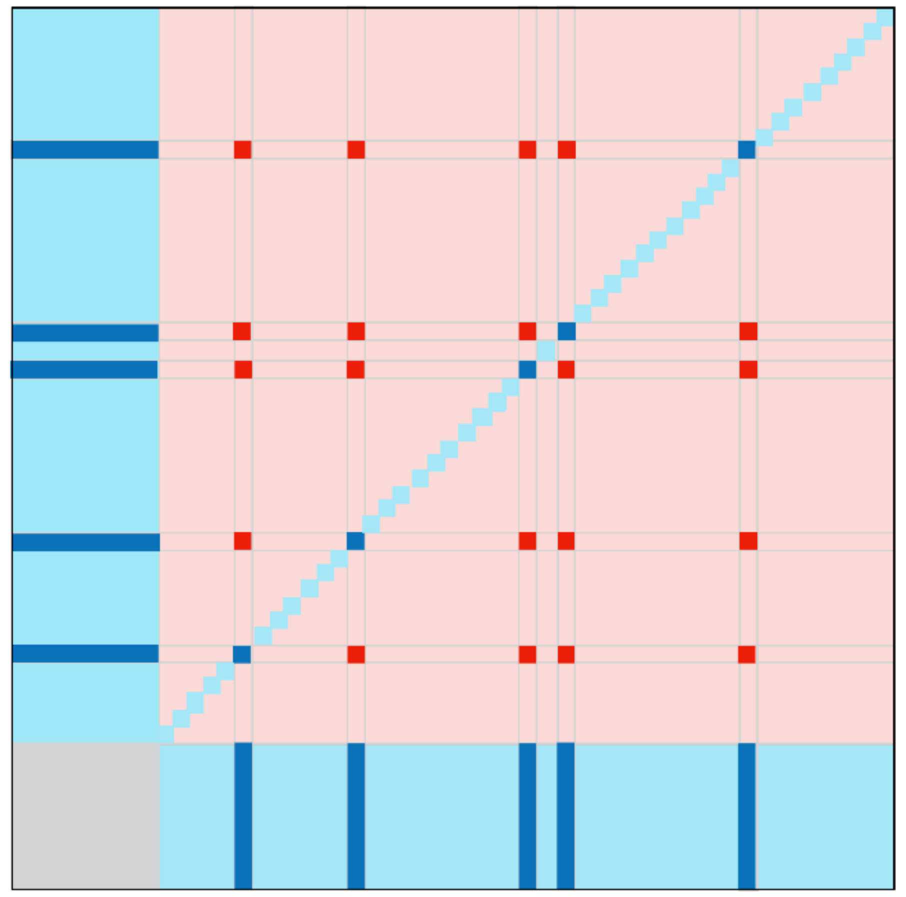
Flavor lattice average group, 2411.04268

Oct.11th 17:00-17:20 分会1 王积昊

J.H. Wang, et. al. [CLQCD], In preparation

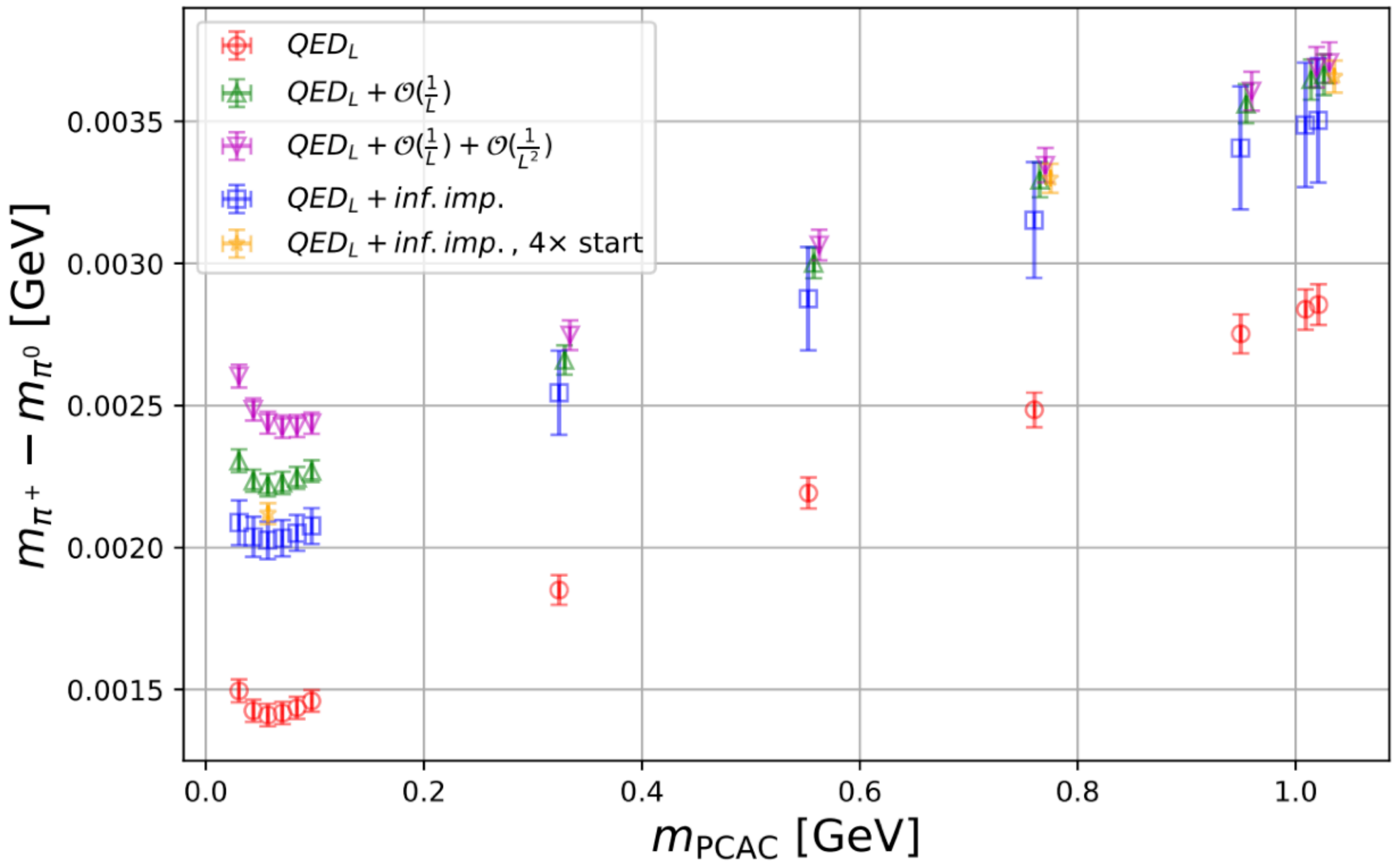
Outline

- Blending method for statistical uncertainty

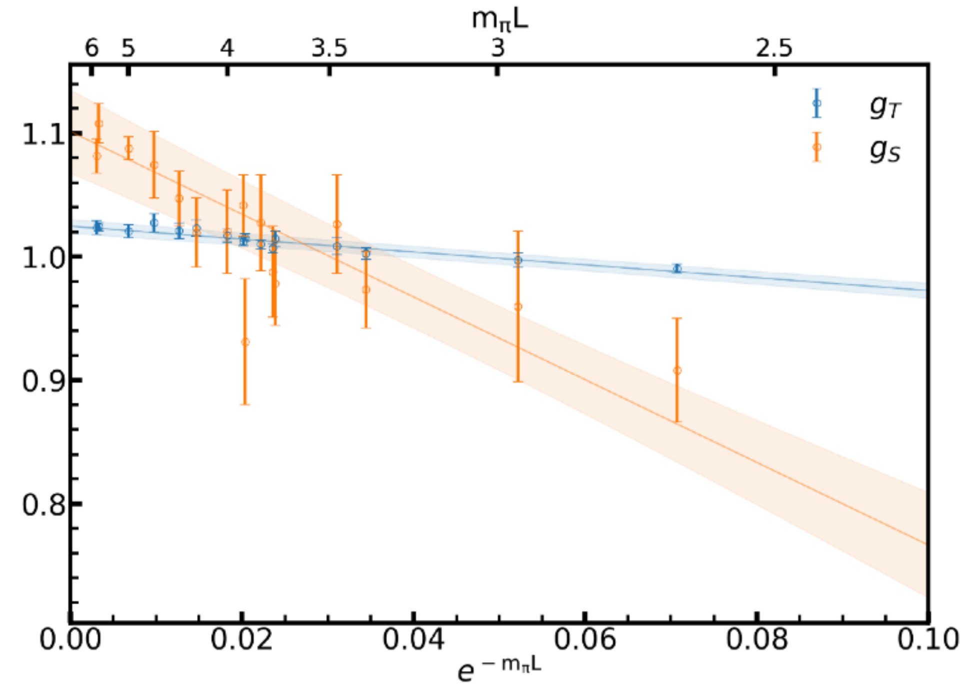


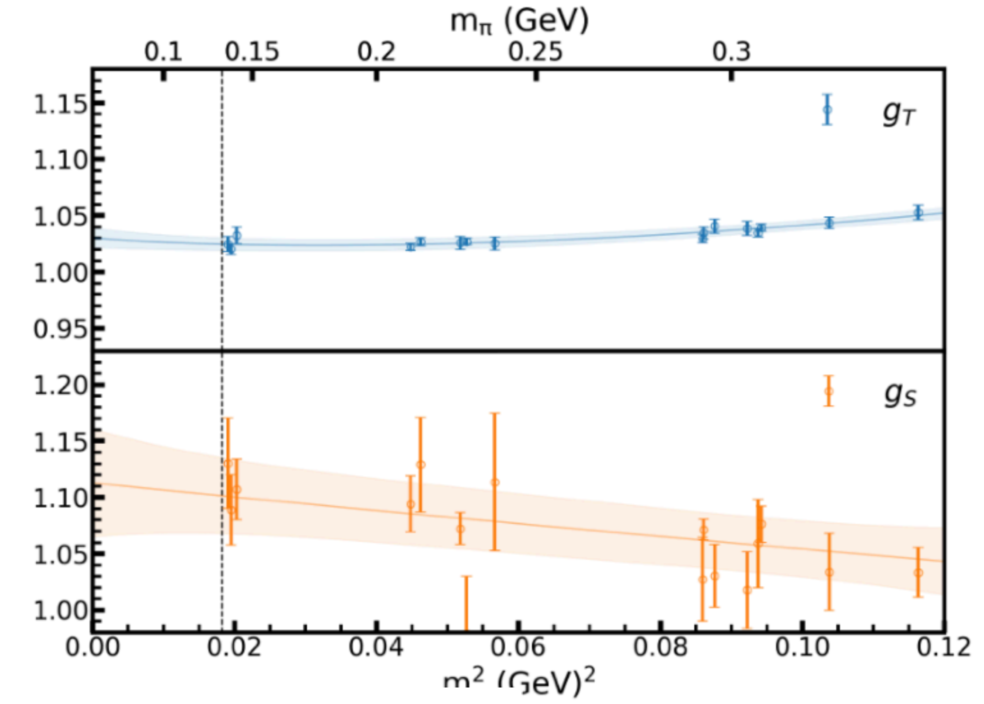
$$\Omega^{(2)}_{ij} = \begin{cases} 1 & \text{for } i,j \le N_{\rm e}, \\ \Pi_{i=0}^{1} \frac{V-N_{\rm e}-i}{N_{\rm st}-i} & \text{for } i,j > N_{\rm e},\ i \neq j \\ \frac{V-N_{\rm e}}{N_{\rm st}} & \text{for the other cases,} \end{cases}$$

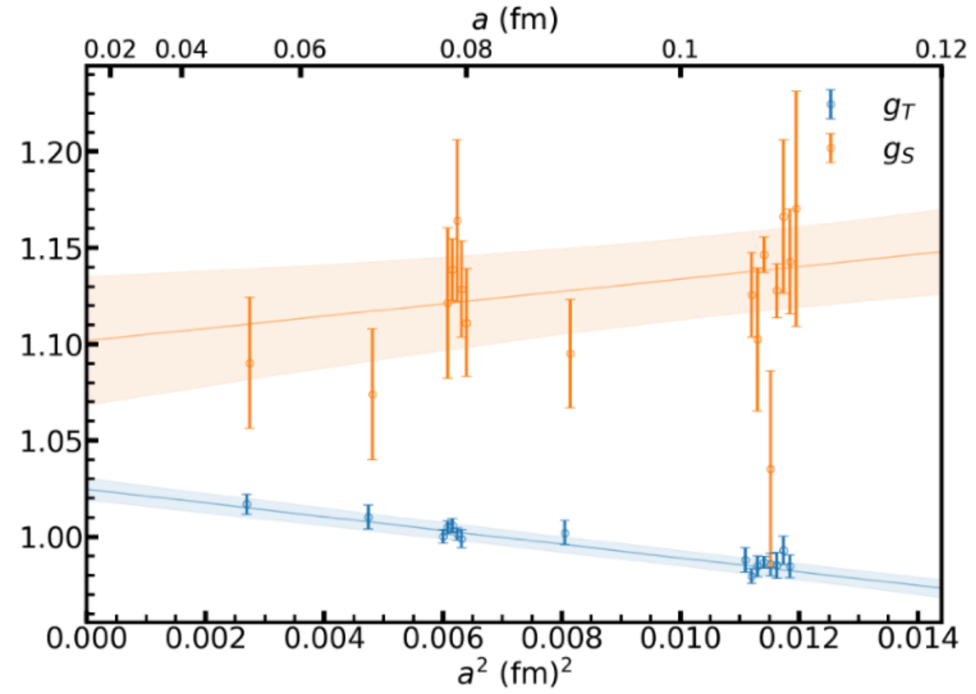
- QED and ISB corrections



- **Systematic uncertainties of IsoQCD**



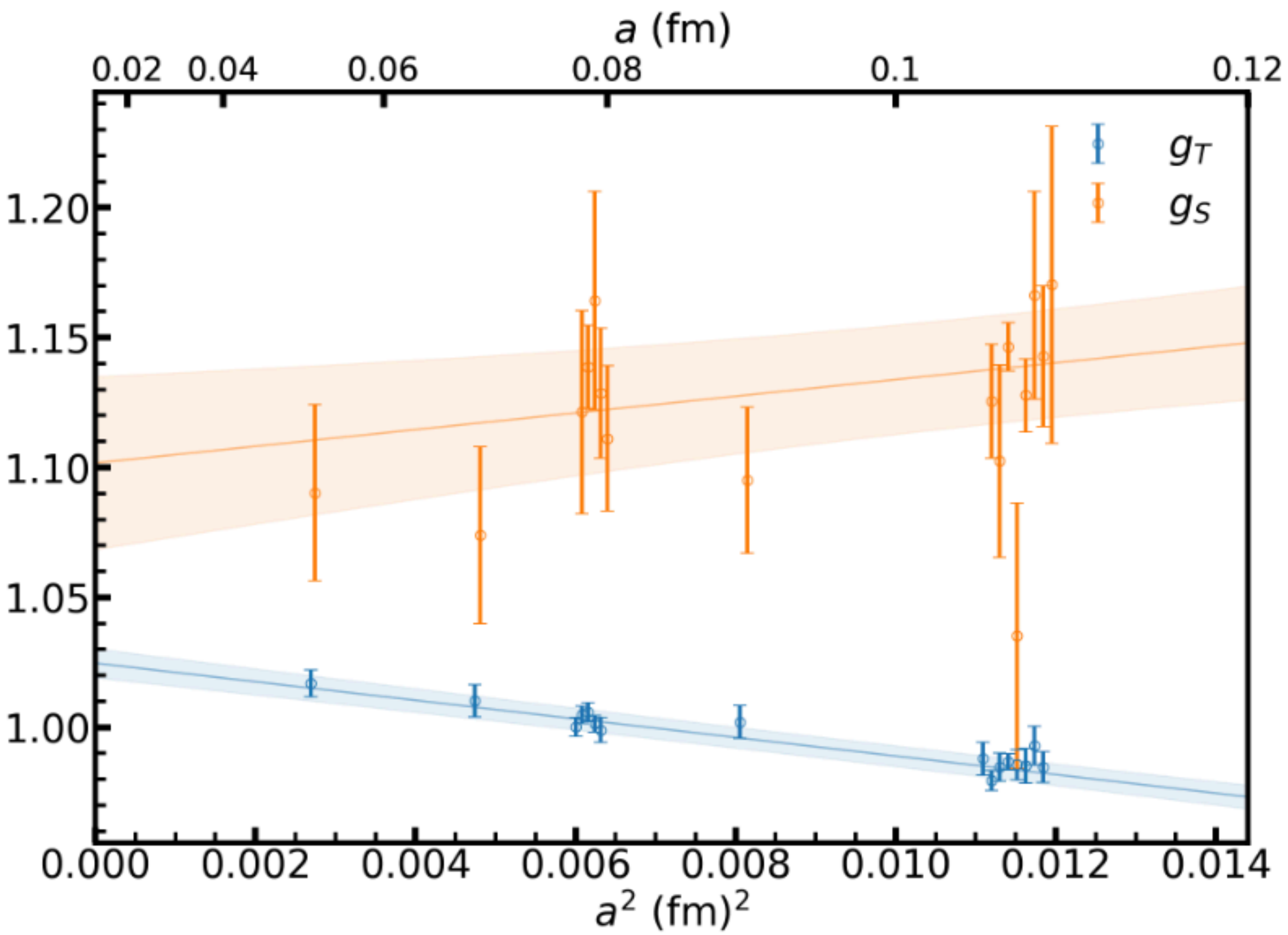


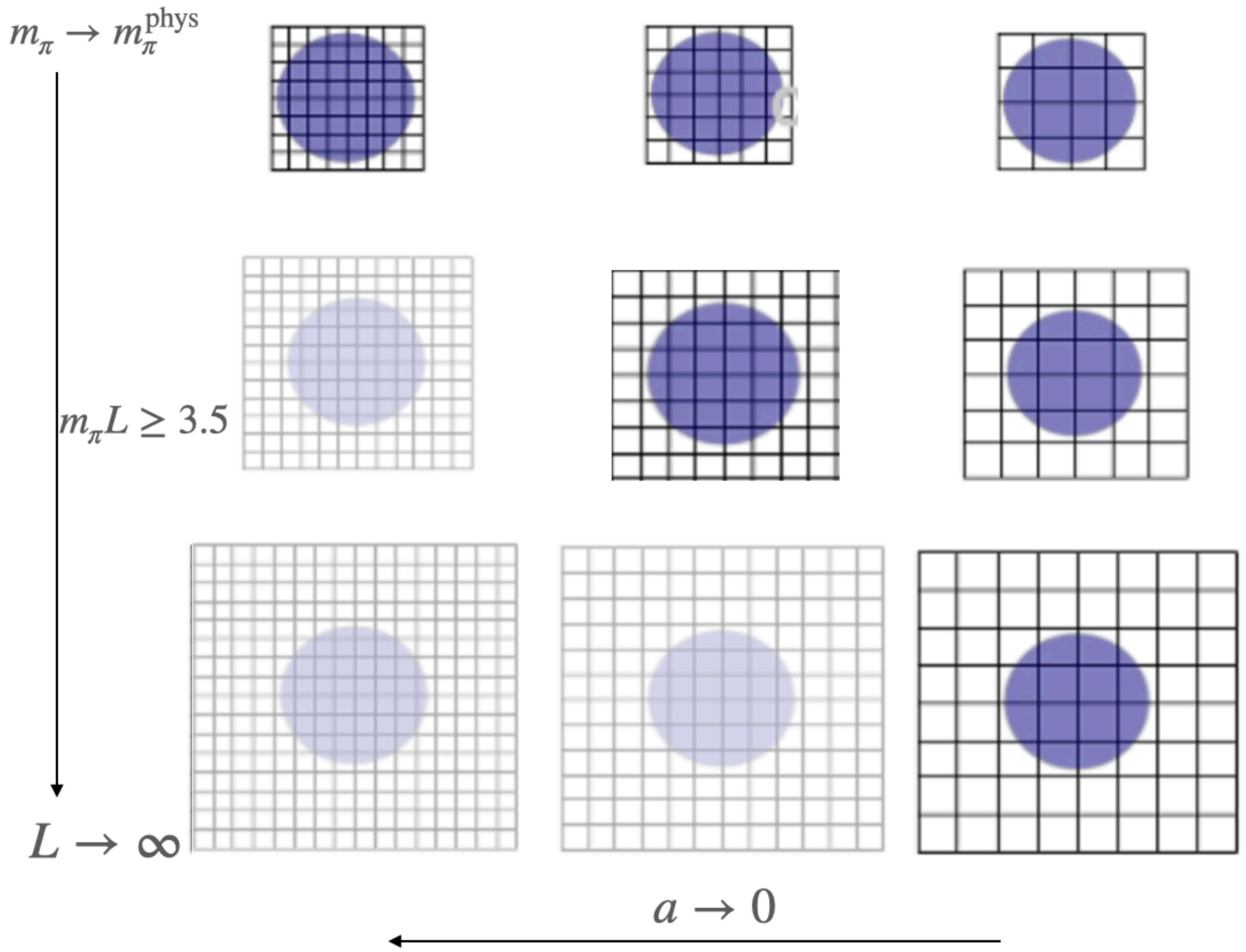


- Outlook

Systematic uncertainties

Continuum extrapolation



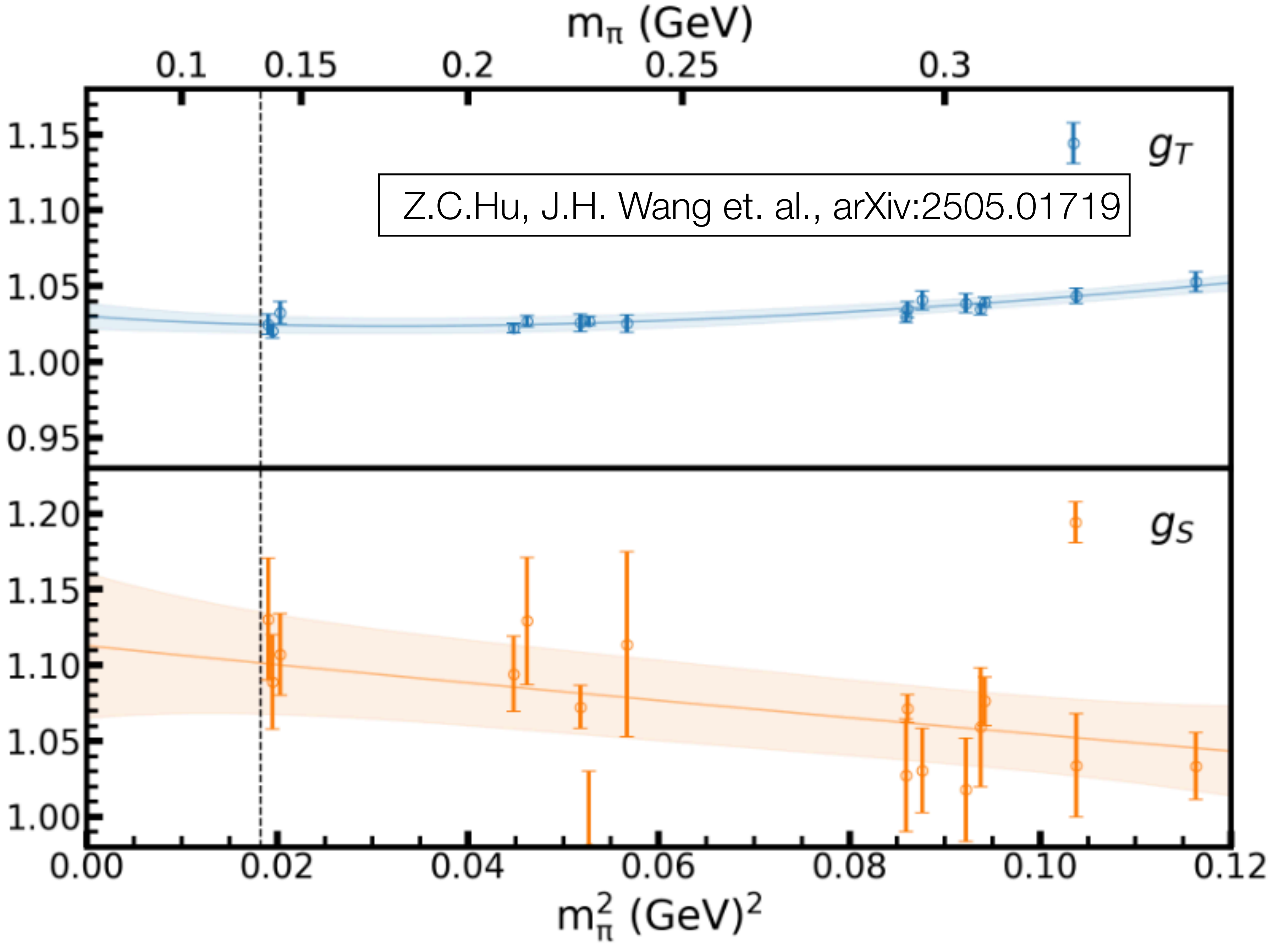


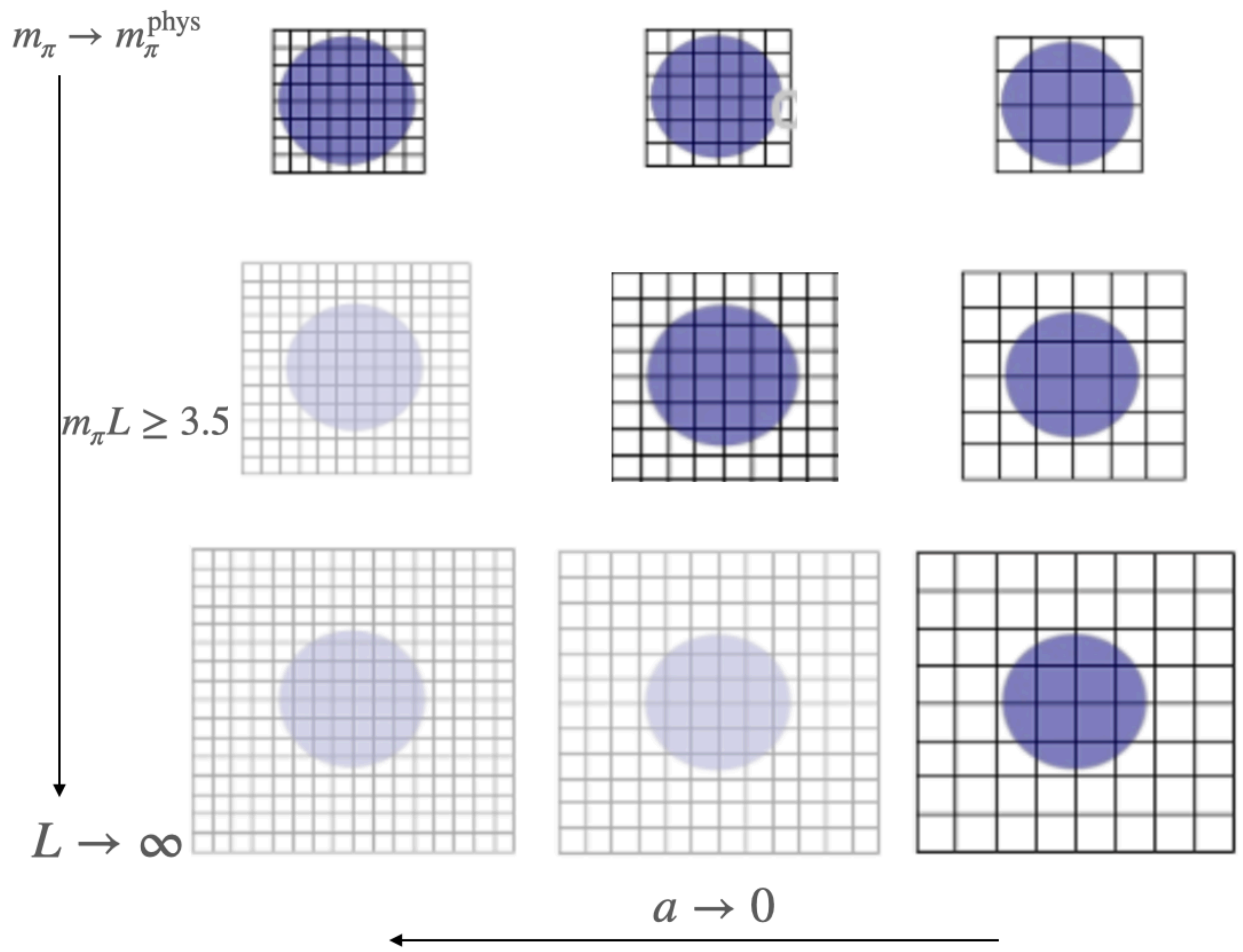
Ensemble	L	T	a(fm)	m_π
H48P32	48	96	0.052	320
I64P29	64	128	0.038	290

- The cost at $a \sim 0.05$ fm is ~16x of that at $a \sim 0.10$ fm with the same V;
- Different discretized fermion and gauge action can only have consistent prediction after the continuum extrapolation.
- Current FLAG **"green star"** grade requires 3 different lattice spacing a with two of them smaller than 0.1 fm, and $a^2_{\rm max}/a^2_{\rm min} \geq 2$.
- **Such a requirement can be satisfied efficiently using the ensembles at relatively heavy $m_\pi \simeq 300$ MeV.**

J.H. Wang, et. al. [CLQCD], In preparation

Systematic uncertainties





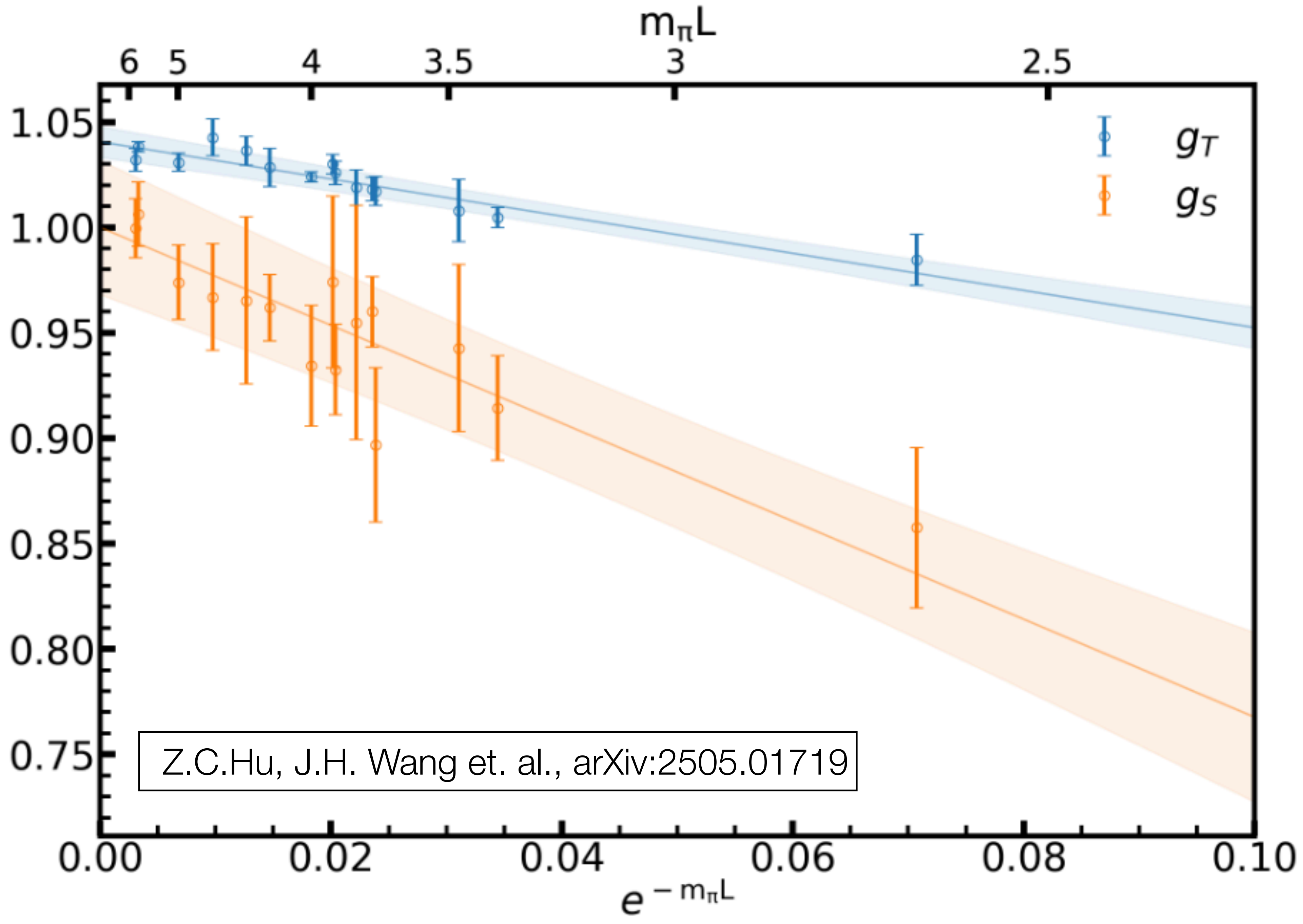
Ensemble	L	T	a(fm)	m_π
C48P14	48	96	0.105	135
C64P14	64	96	0.105	135
F64P13	64	128	0.077	134

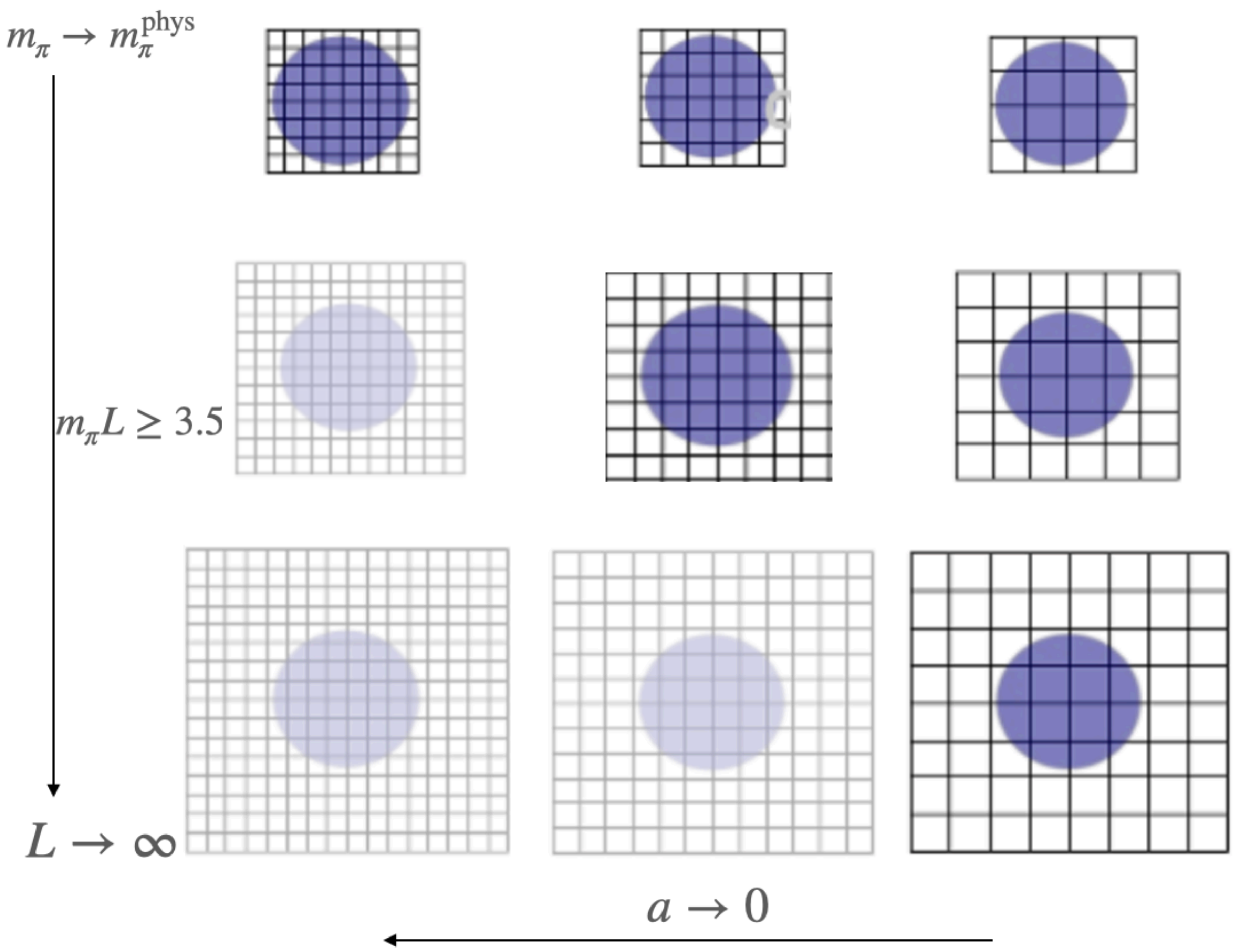
- The cost at $m_\pi \simeq 135$ MeV is ~4x of that at $m_\pi \simeq 310$ MeV, and requires additional 4x statistics to reach similar precision;
- Current FLAG **"green star"** grade requires 3 different m_π with $m_{\pi,\rm min} < 200$ MeV in the chiral extrapolation, or $m_{\pi,\rm\ case1} = 135 \pm 10$ MeV and $m_{\pi,\rm\ case2} < 200$ MeV.
- **Such a requirement can be satisfied efficiently using the ensembles at the coarsest lattice spacing.**

J.H. Wang, et. al. [CLQCD], In preparation

Systematic uncertainties

Infinite volume extrapolation





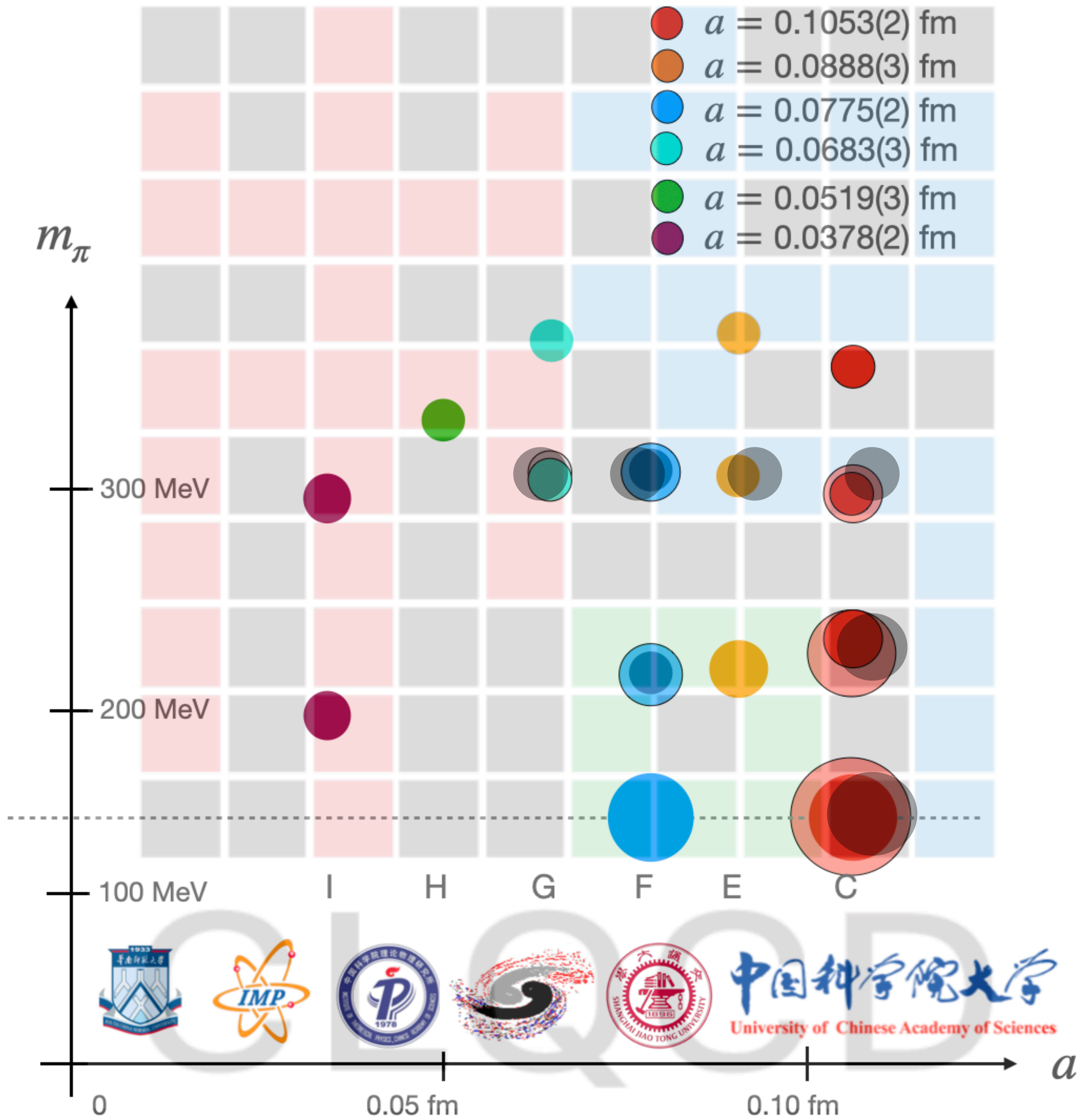
Ensemble	L	T	a(fm)	
C48P14/C64P14	48/64	96	0.105	135
C24P23/C32P23/C48P23	24/32/48	64/96	0.105	227
C24P29/C32P29/C48P29	24/32/48	64/72	0.105	290
F32P21/F48P21	32/48	96	0.077	210
F32P30/F48P30	32/48	64/96	0.077	300

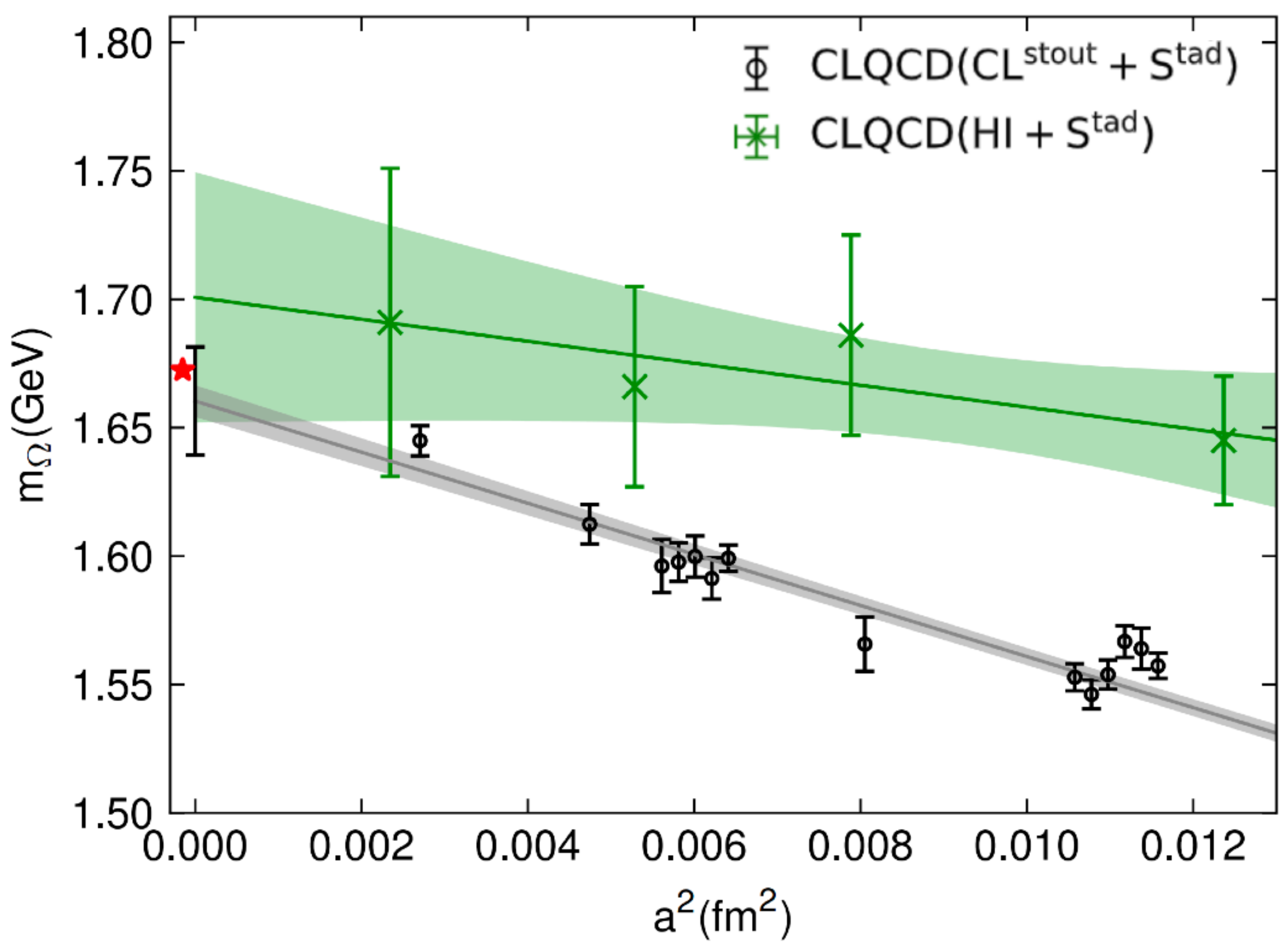
- The cost at $L \sim 5$ fm is 8x of that at $L \sim 2.5$ fm, and χPT suggests an $e^{-m_\pi L}$ behavior with $m_\pi L \geq 3$;
- Current FLAG **"green star"** grade requires $m_\pi L \sim 3.2$ with $m_\pi \sim$ 135 MeV, or at least three L in the infinite volume extrapolation.
- **Such a requirement can be satisfied efficiently using the ensembles at the coarsest lattice spacing and or heavier m_π;**

J.H. Wang, et. al. [CLQCD], In preparation

Systematic uncertainties

Charmed sea effects

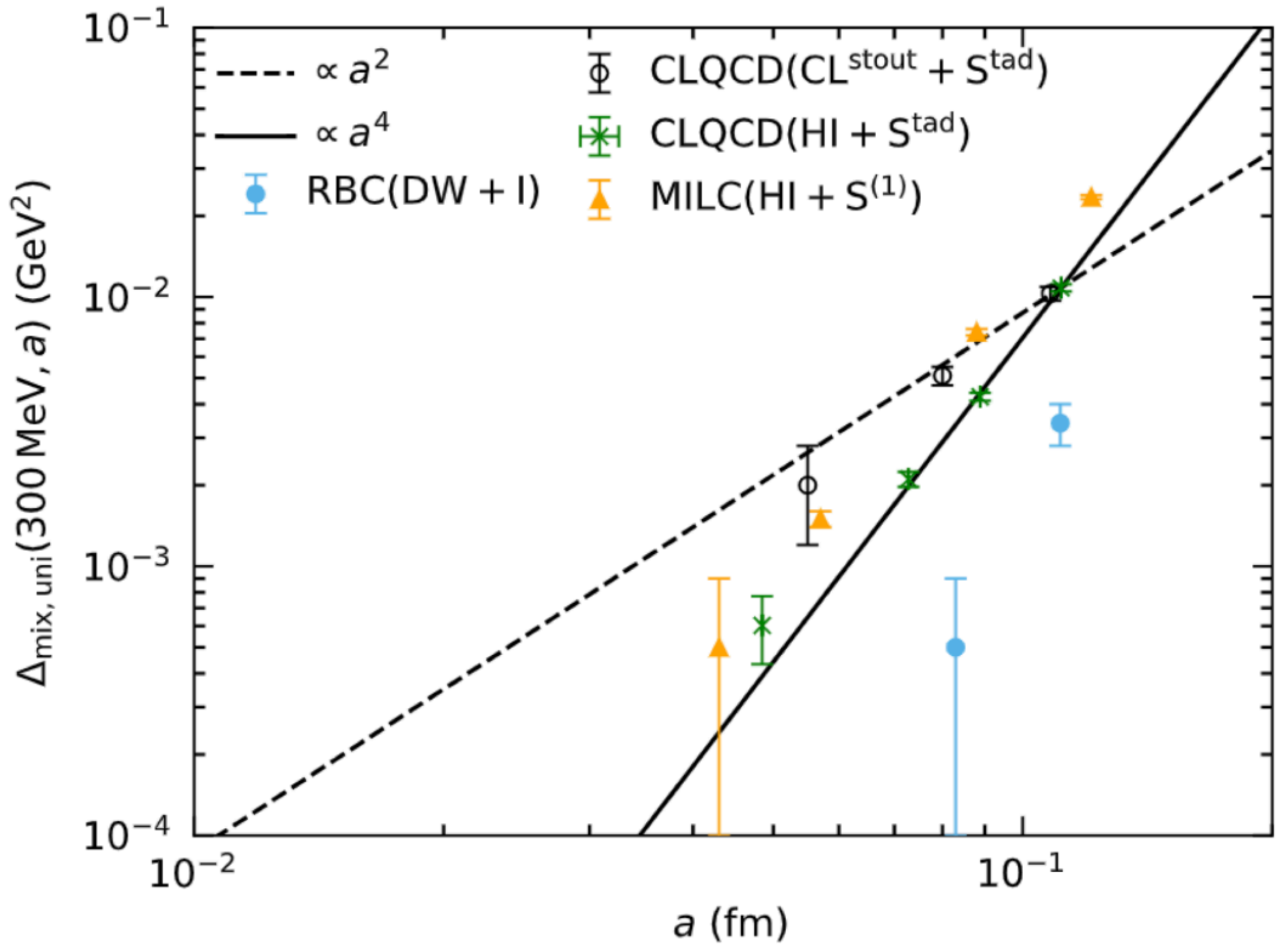




$$O_H(m_q, a, 1/L)$$

$$= \sum c_{1,q=l,s,c,b..}(m_q - m_q^{\rm phys}) + c_2 e^{-m_\pi L}$$

$$+ \begin{cases} O_H^{N_f=2+1}(m_\pi^{\rm phys}, 0) + c_{3,\rm CL}^1 a^2, & \text{CL on CL} \\ O_H^{N_f=2+1+1}(m_\pi^{\rm phys}, 0) + c_{3,\rm HI} a^2, & \text{CL on HI} \end{cases}$$



- The mixed action effect of using clover-valence and HISQ-sea is $\mathcal{O}(a^4)$ and smaller than the similar MILC ensembles.

- New ensembles (HI + S^{tad}) with 2+1+1 flavor HISQ fermion can provide proper estimate of the charm sea effects with much lower cost;

- Joint fit using both the 2+1 flavor (CL^{stout} + S^{tad}) and 2+1+1 (HI + S^{tad}) ensembles can provide proper estimate of the charmed sea effects;

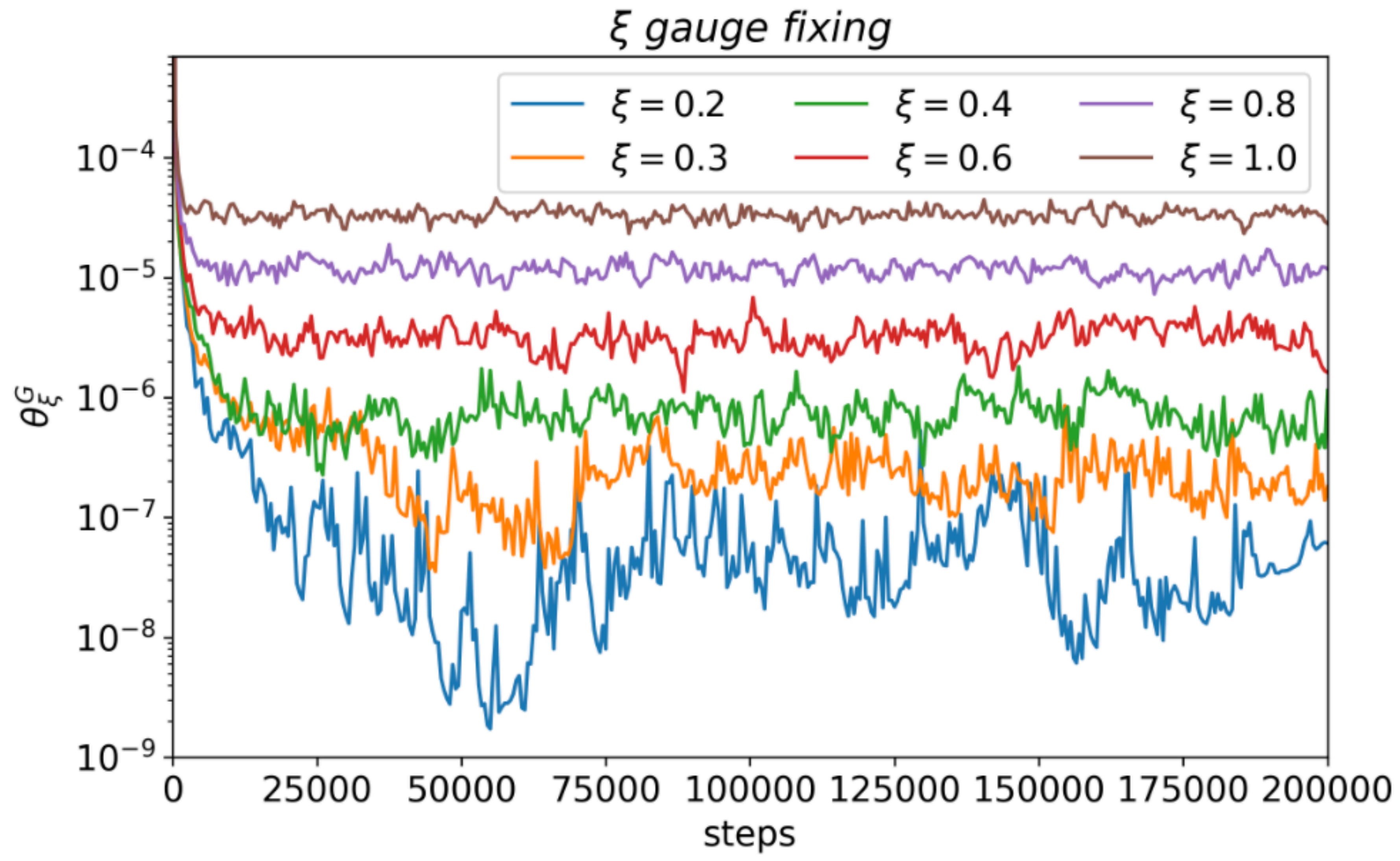
Oct.11th 16:40-17:00 分会2 张遵贤
Oct.11th 17:00-17:20 分会2 林彤巍

Z.X. Zhang et al. [CLQCD], in preparation

T.W. Lin et al. [CLQCD], in preparation

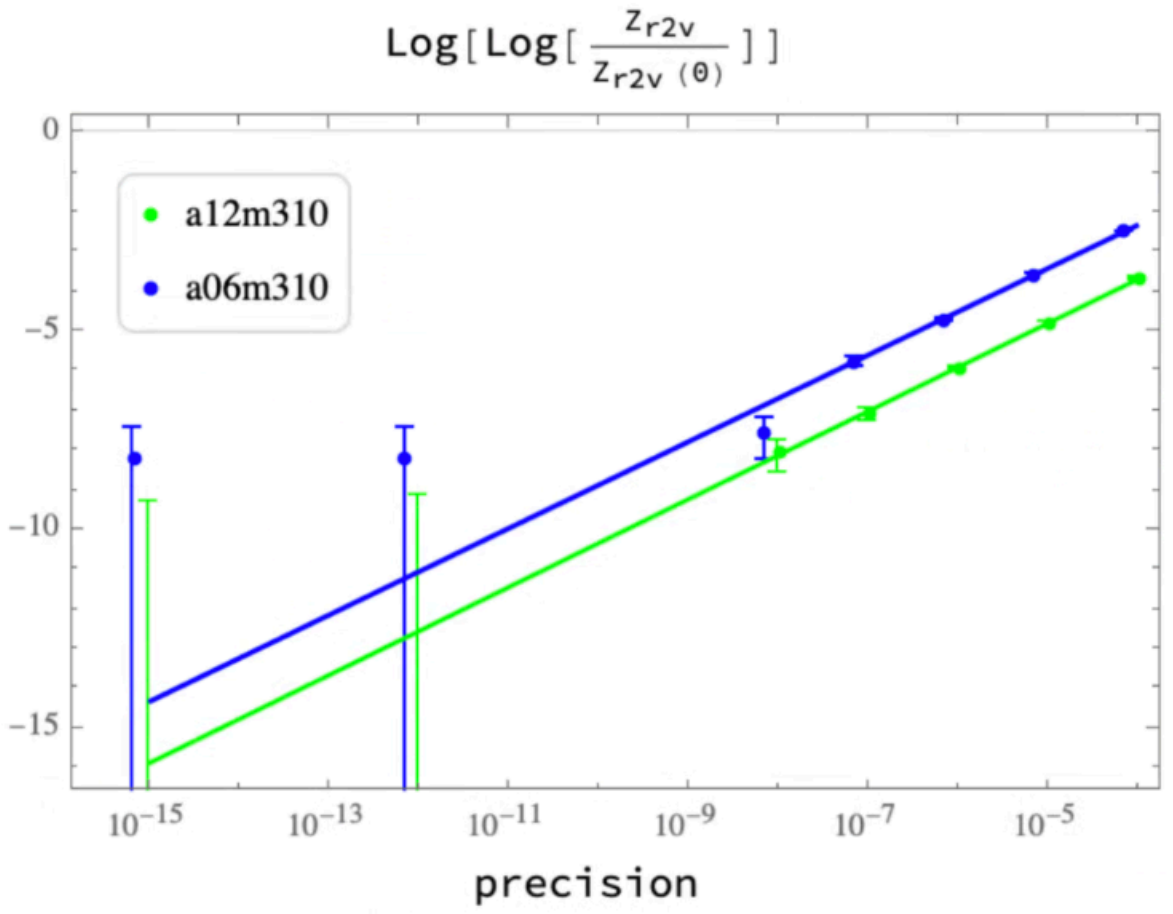
Systematic uncertainties:

Non-perturbative renormalization



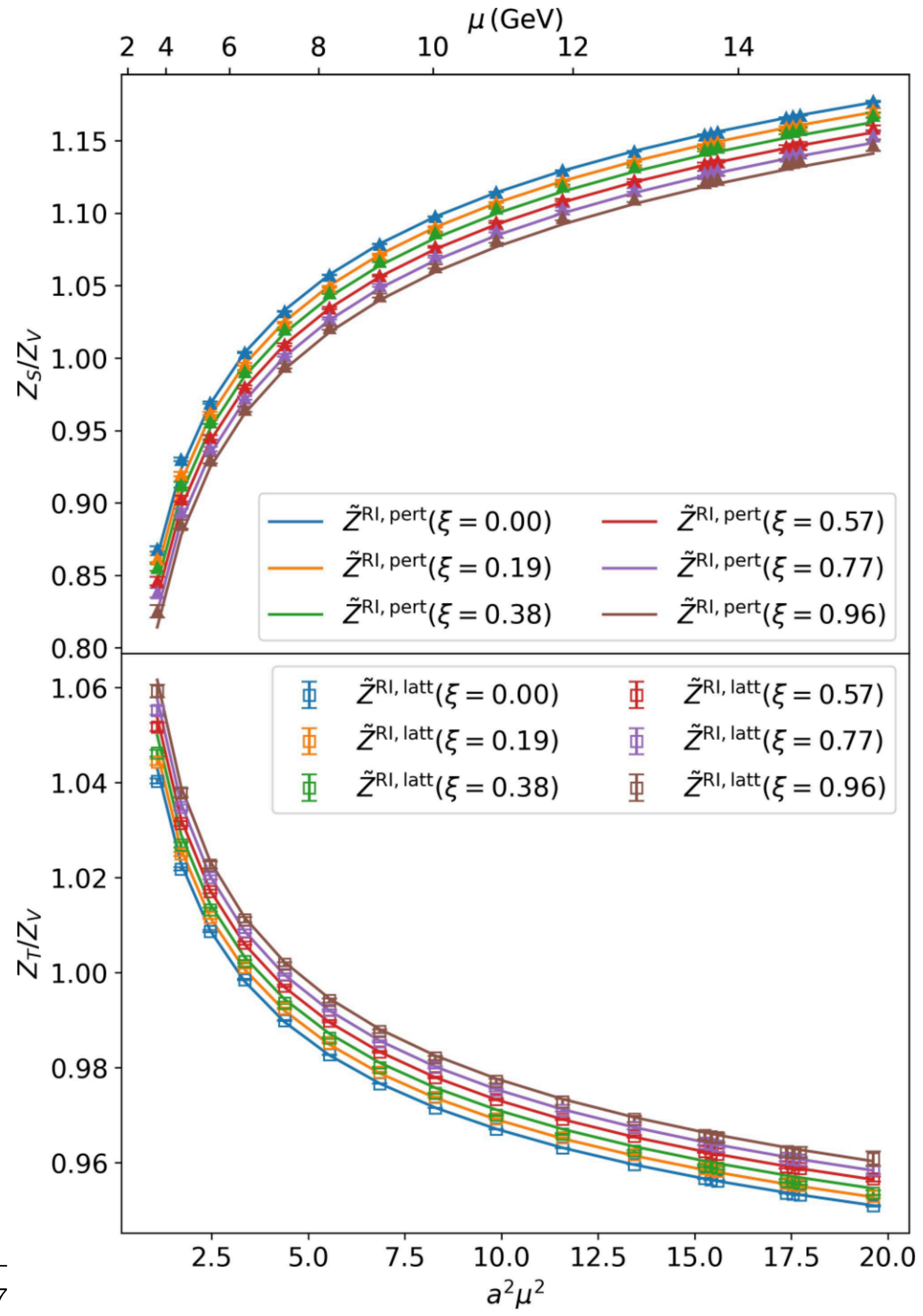
- Most of the non-perturbative renormalizations are limited to the Landau gauge.

- The "plateau" of the gauge fixing residual of the ξ gauge fixing increases rapidly on ξ, roughly $10^{-7+2.5\xi}$.



- The quantities with precise ξ gauge fixing can be estimated using an empirical form
$$X(\theta) = X(0)e^{-c(X)\theta^{n(X)}},$$
with $X(\theta)$ at different residual θ.

- ξ dependence of the RI-MOM renormalization constants can be reproduced at 0.2% level after the continuum extrapolation, up to $\xi \sim 1$.

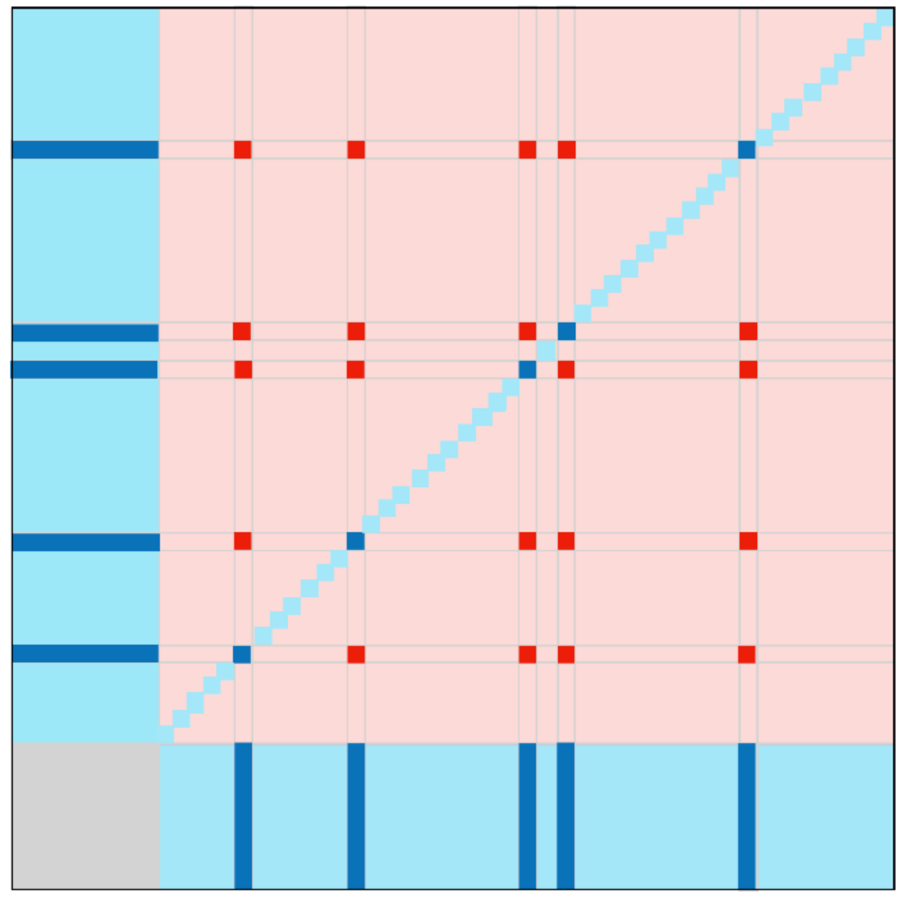


Oct.11th 16:20-16:40 分会2 周立军

L.J. Zhou et. al., [χQCD], 2509.09367

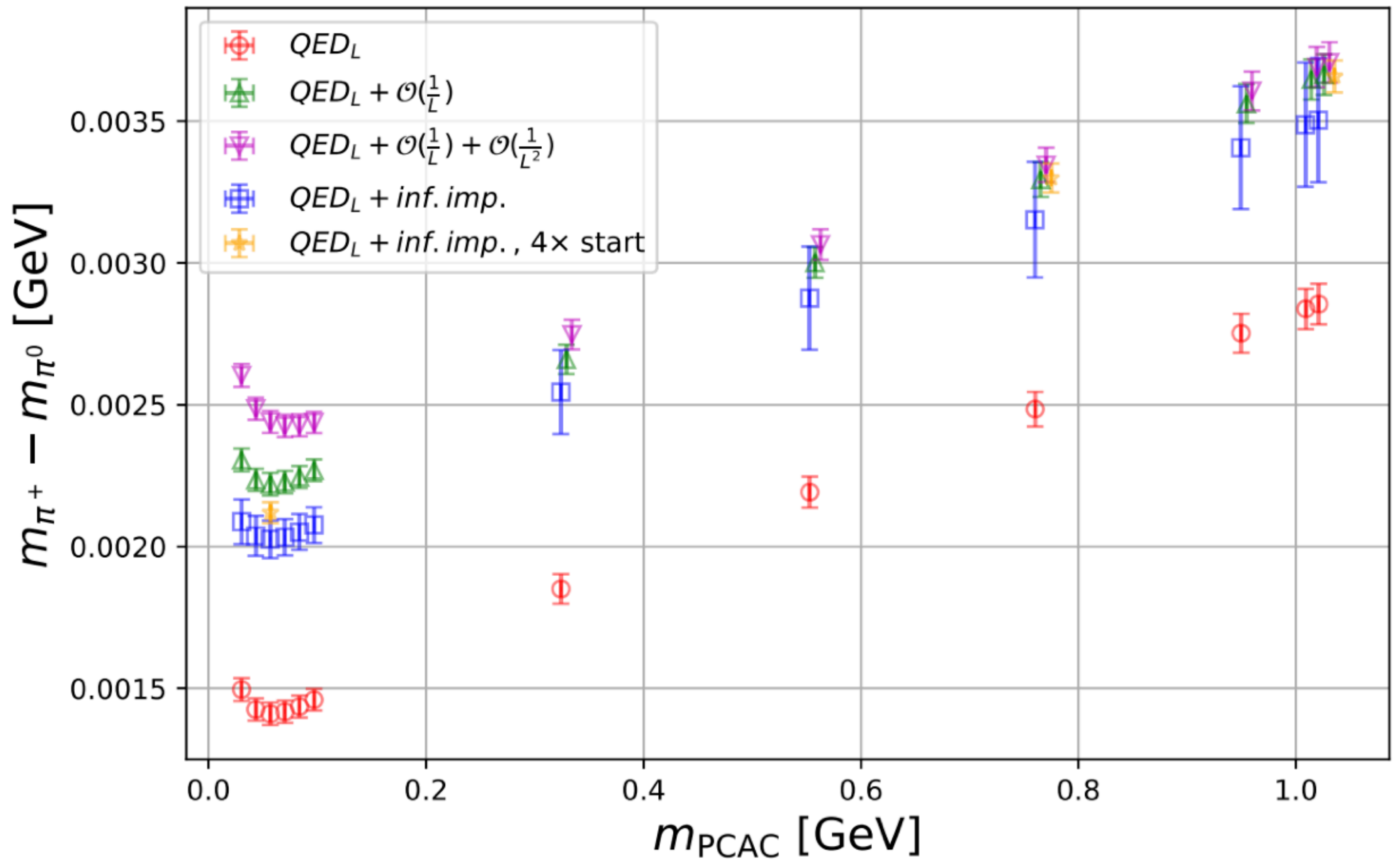
Outline

- Blending method for statistical uncertainty

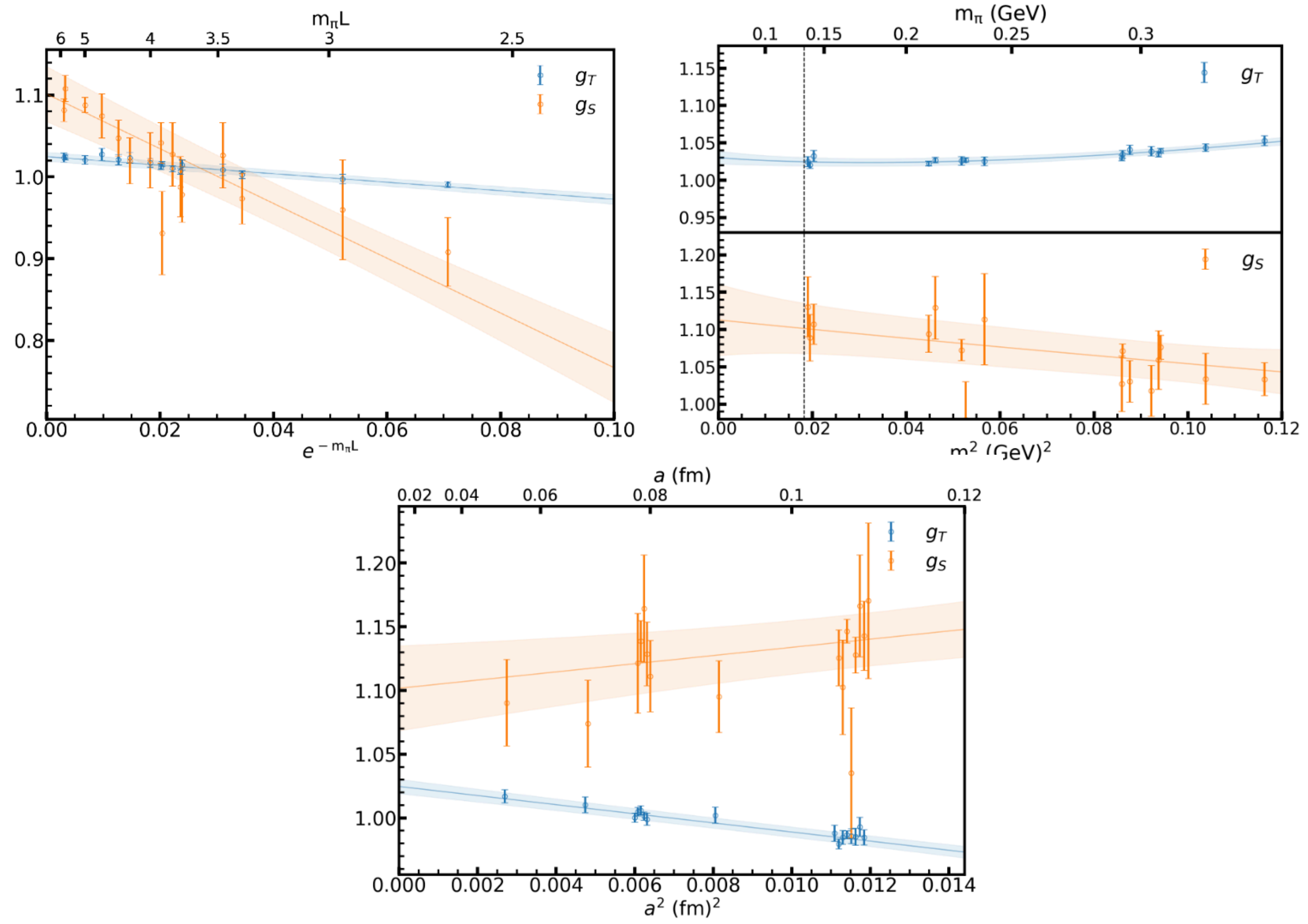


$$\Omega_{ij}^{(2)} = \begin{cases} 1 & \text{for } i, j \leq N_e, \\ \Pi_{i=0}^{1} \frac{V - N_e - i}{N_{st} - i} & \text{for } i, j > N_e,\ i \neq j \\ \frac{V - N_e}{N_{st}} & \text{for the other cases,} \end{cases}$$

- **QED and ISB corrections**



- Systematic uncertainties of IsoQCD



- Outlook

QED corrections

QED_L

- The QCD+QED calculation can be done under the quenched QED approximation using QED_L for the valence fermion:

$$U_\mu^{\mathrm{QCD+QED}} = U_\mu^{\mathrm{QCD}} e^{-\mathrm{i} e e_q A_\mu},\ A_\mu(x) = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} e^{-\mathrm{i} p\cdot x} A_\mu(p), P_{A_\mu(p)}|_{\vec{p}\neq 0} \propto e^{-\frac{1}{2N_V}\hat{p}^2 A_\mu^2(p)}.$$

- The QED finite volume correction (FVC) of hadron masses using QED_L is independent of the hadron structure until $\mathcal{O}(1/(m_H L)^3)$:

$$\delta_{\mathrm{QED-FVC}} m_H = e_H^2 \frac{c_1}{L}(1 + \frac{2}{m_H L} + \mathcal{O}(\frac{1}{(m_H L)^2})).$$

BMWc, Science 347(2015)1452

Thus the QED-FVC of neutral particles using QED_L are highly suppressed, likes that using QED_∞.

- One can further improve QED_L by enlarging the weights of the near-zero momentum modes:

$$\delta_{\mathrm{QED-FVC}}^{\mathrm{inf.imp.}} m_H = \mathcal{O}(\frac{e_H^2}{(m_H L)^2}),\ P_{A_\mu(p)}^{\mathrm{inf.imp.}}|_{\vec{p}\neq 0} \propto e^{-\frac{1+1.4856\delta(p^2-\frac{4\pi^2}{L^2})}{2N_V}\hat{p}^2 A_\mu^2(p)}.$$

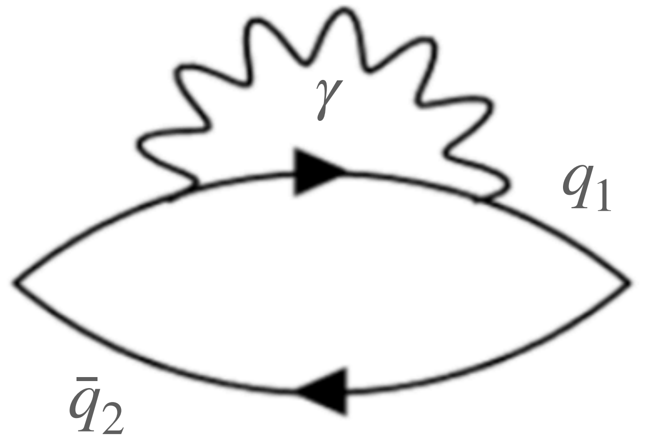
Z. Davoudi et.al., PRD99(2019)034510

QED corrections

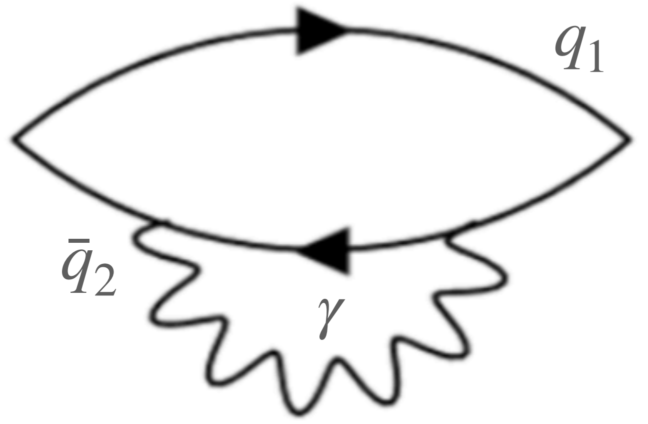
NPU Scheme

- One can define the neutral pion uncorrected (NPU) scheme by tuning the bare mass $m_q^b(e_q)$ of the quark with a QED charge e_q, to ensure the neutral iso-vector pseudoscalar meson mass to be the same as that using the QED-neutral quark with bare mass $m_q^b(0)$:

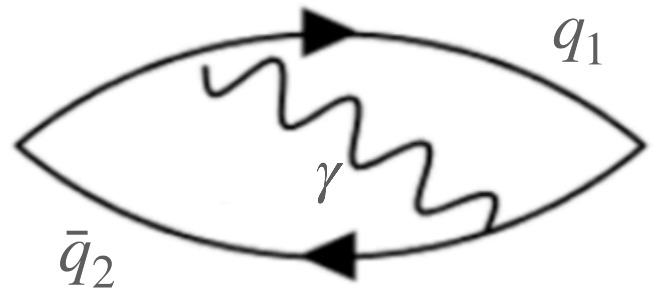
$$m_{\eta_q}^{\mathrm{QCD+QED}}(m_q^b(e_q)) = m_{\eta_q}^{\mathrm{QCD}}(m_q^b(0)).$$



$e_{q_1}^2 \delta^{\mathrm{self}} m$



$e_{q_2}^2 \delta^{\mathrm{self}} m$



$-2e_{q_1}e_{q_2}\delta^{\mathrm{int}} m$

- Using the QED quark diagram decomposition, we have ($\delta^{\mathrm{self}} m = m^{\mathrm{QCD+QED}} - m^{\mathrm{QCD}}$):

$$\delta m_{\pi^0} = \frac{5}{18}\delta^{\mathrm{self}} m_{\pi^0} - \frac{5}{18}\delta^{\mathrm{int}} m_{\pi^0} + \mathcal{O}(\alpha\alpha_s^2, \alpha^2),$$

$$\delta m_{\pi^+} = \frac{5}{18}\delta^{\mathrm{self}} m_{\pi^+} + \frac{4}{18}\delta^{\mathrm{int}} m_{\pi^+} + \mathcal{O}(\alpha\alpha_s^2, \alpha^2),$$

$$\delta m_{\eta_c} = \frac{4}{9}\delta^{\mathrm{self}} m_{\eta_c} - \frac{4}{9}\delta^{\mathrm{int}} m_{\eta_c} + \mathcal{O}(\alpha\alpha_s^2, \alpha^2)$$

$$\delta m_{\eta_b} = \frac{1}{9}\delta^{\mathrm{self}} m_{\eta_b} - \frac{1}{9}\delta^{\mathrm{int}} m_{\eta_b} + \mathcal{O}(\alpha\alpha_s^2, \alpha^2)$$

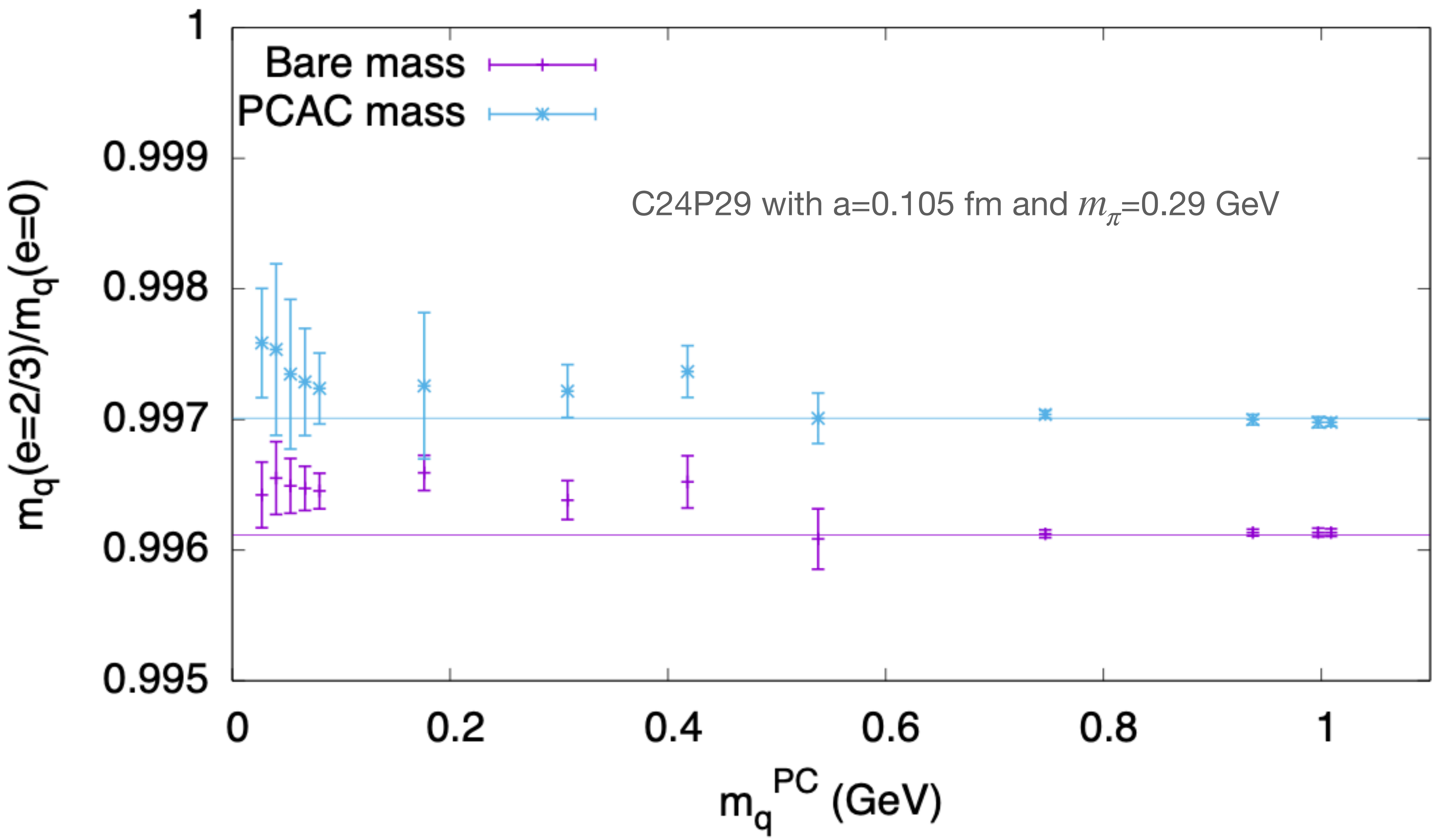
- $\delta^{\mathrm{self}} m_{\eta_q}$ requires from the QED UV renormalization and a matching condition.

- The NPU scheme defines $\delta^{\mathrm{self}} m_{\eta_q} = \delta^{\mathrm{int}} m_{\eta_q}$, and then

$$\delta m_{\pi^+} = \frac{1}{2}\delta^{\mathrm{int}} m_{\pi^+} + \mathcal{O}(\alpha\alpha_s^2, \alpha^2).$$

QED corrections

Quark mass renormalization



- For the u-type quarks:

1. PCAC quark mass is changed by 0.30(5)%,
2. Bare quark mass $m_q^b - m_q^{\mathrm{crti}}$ is changed by 0.39(5)%,

   using the NPU scheme, with their difference coming from the additive chiral symmetry breaking of the clover fermion;

- The correction would be quark mass independent and more statistics is necessary to verify it.
- The perturbative calculation shows that the QED UV scale dependence is 0.12% from a=0.105 fm to a=0.052 fm.
- That of the d-type quarks will be suppressed by a factor of 4.

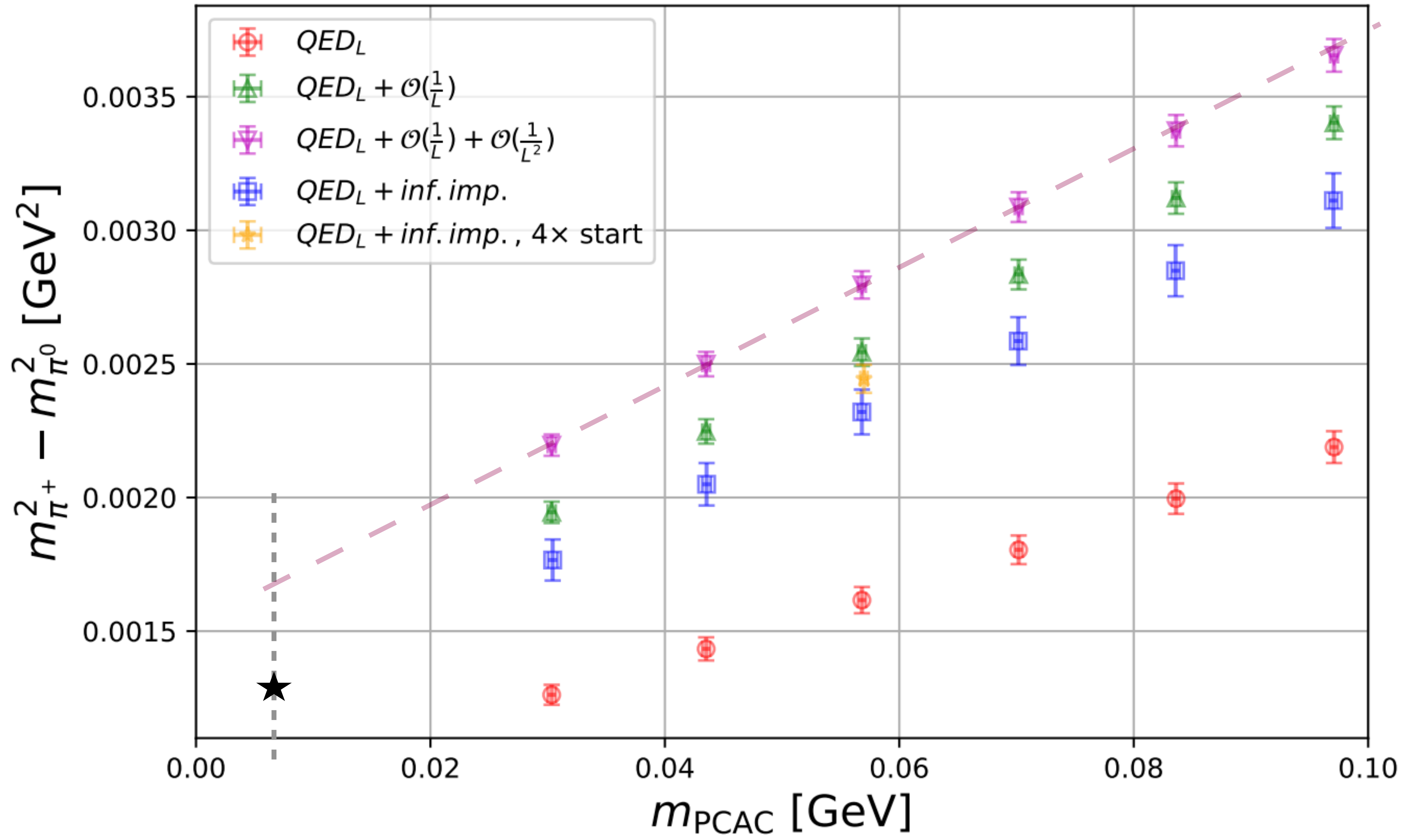
Y.Y. Liu et al. [CLQCD], in preparation

QED corrections

m_π^+ correction

$$\delta_{\mathrm{QED-FVE}} m_H^2 = e_H^2 m_H \frac{2c_1}{L}\left(1 + \frac{2}{m_H L} + \mathcal{O}\left(\frac{1}{(m_H L)^2}\right)\right)$$

$\mathcal{O}(\frac{1}{L})$ $\mathcal{O}(\frac{1}{L^2})$

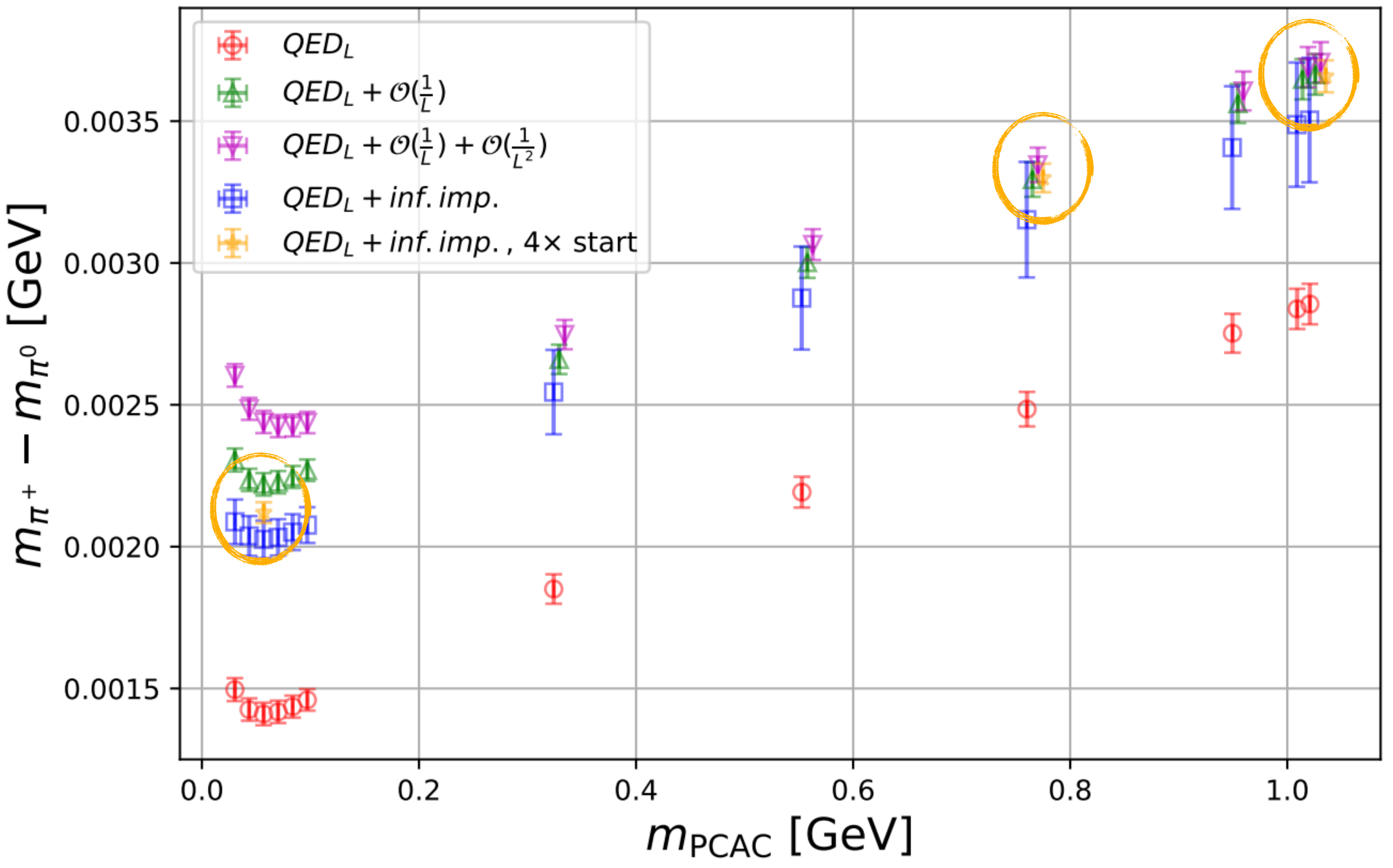


C24P29 with a=0.105 fm and m_π=0.29 GeV

- Ignoring the iso-spin breaking (ISB) correction, the mass difference between $m_{\pi^+}[\bar{q}(e_q)\gamma_5 q(e_q)]$ and $m_{\pi^0}[\bar{q}(-e_q)\gamma_5 q(e_q)]$ is a pure QED correction;
- With the NNLO QED-FVC, we have $m^2_{\pi^+} - m^2_{\pi^0} = 1.5 \times 10^3\ \mathrm{MeV}^2$ after the chiral extrapolation of the valence quark mass, at a=0.105 fm and m_π^{sea}=0.29 GeV, which is not far away from the **physical value.**
- The inferred improved QED_L result can include the $\mathcal{O}(1/L)$ FVC automatically and closes to the NLO QED_L result.

Y.Y. Liu et al. [CLQCD], in preparation

QED corrections



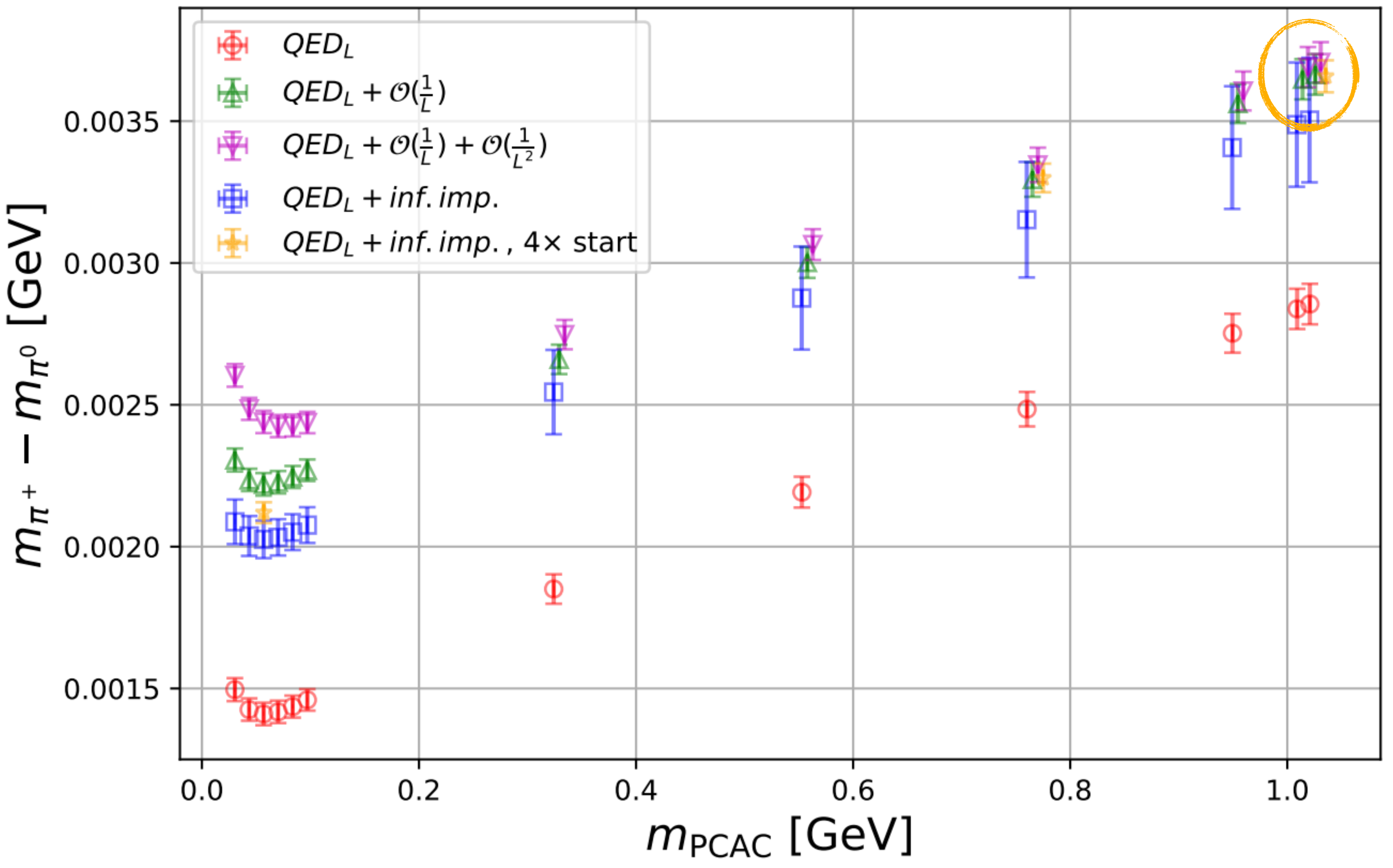
Y.Y. Liu et al. [CLQCD], in preparation

C24P29 with a=0.105 fm and m_π=0.29 GeV

- With heavier quark mass, the NNLO QED-FVC becomes negligible and then only the NLO QED-FVE matters;
- The inferred improved QED_L result becomes closer to the NLO QED-FVE result with heavier quark mass, while the statistical uncertainty is larger;
- The agreement becomes even better after the statistics of the inferred improved QED_L result is improved.

QED corrections

Impact on the charm physics



Y.Y. Liu et al. [CLQCD], in preparation

C24P29 with a=0.105 fm and m_π=0.29 GeV

- For the charm quark:

1. QED interaction correction $-3.7 * 2 \times (2/3)^2 \simeq -3.2(1)$ MeV of m_{η_c} is also similar with -3.0(1) MeV obtained in 2009.07667.

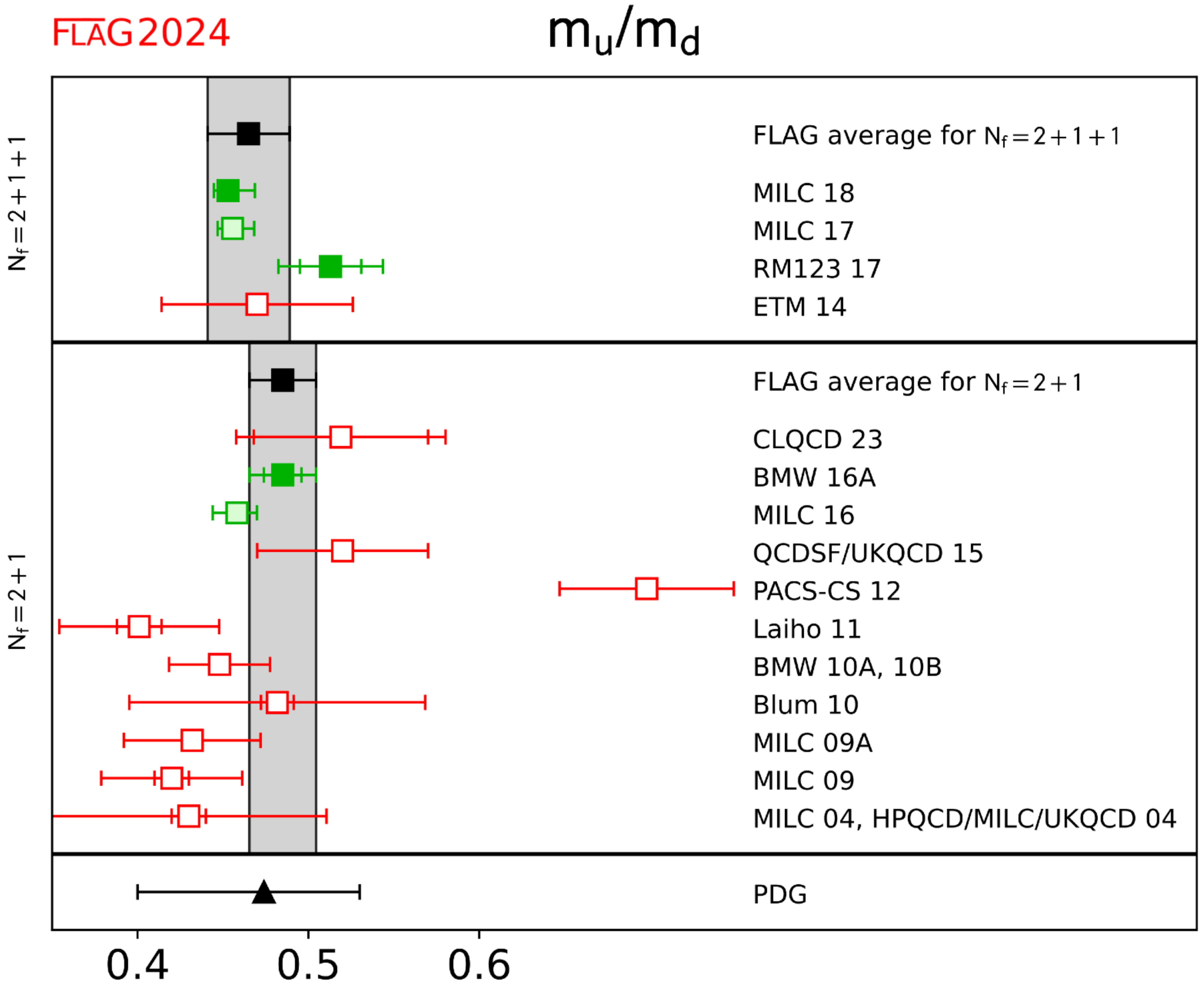
2. QED self energy correction will be $3.7 \times (2/3)^2 \simeq 1.6(1)$ MeV which is 0.15% of the charm quark mass;

3. Combining the QED interaction correction $-4.7(5)e_c e_l$ MeV of m_{D^0} obtained in 2009.07667 and ignoring the light quark self energy correction, we have $\delta_{\rm QED} m_{D^0} = -0.5(2)$ MeV and $\delta_{\rm QED} m_{D^{+(s)}} = 2.6(2)$ MeV which also agree with those from literature.

ISB corrections

- Proton-neutron mass difference:

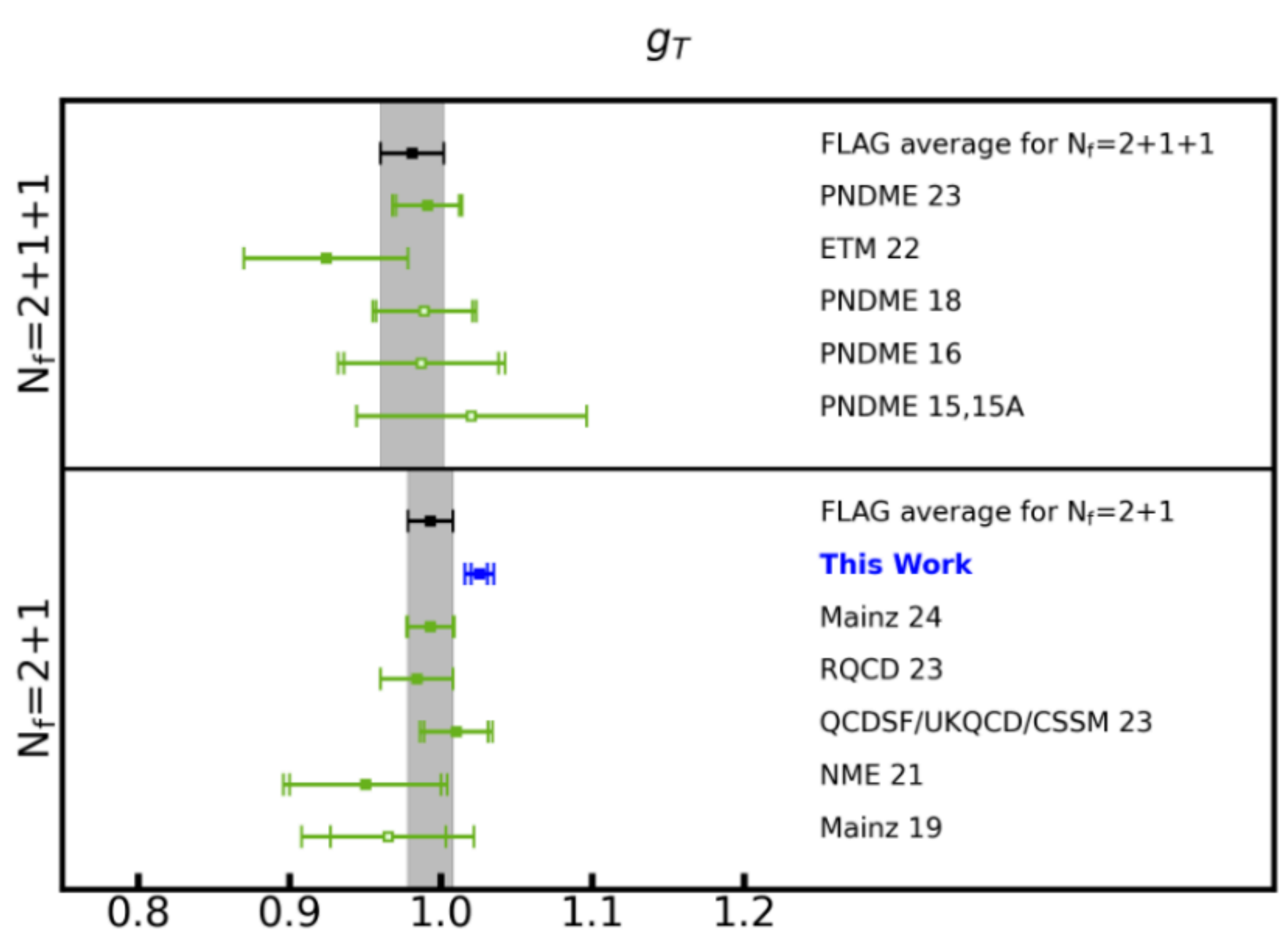
$$m_n - m_p = m_u(\frac{\partial m_n}{\partial m_u} - \frac{\partial m_p}{\partial m_u}) + m_d(\frac{\partial m_n}{\partial m_d} - \frac{\partial m_p}{\partial m_d}) + \delta^{\text{QED}} m_p^{\text{isoQCD}} = (m_d - m_u) g_S^{u-d} + \delta^{\text{QED}} m_p^{\text{isoQCD}}.$$

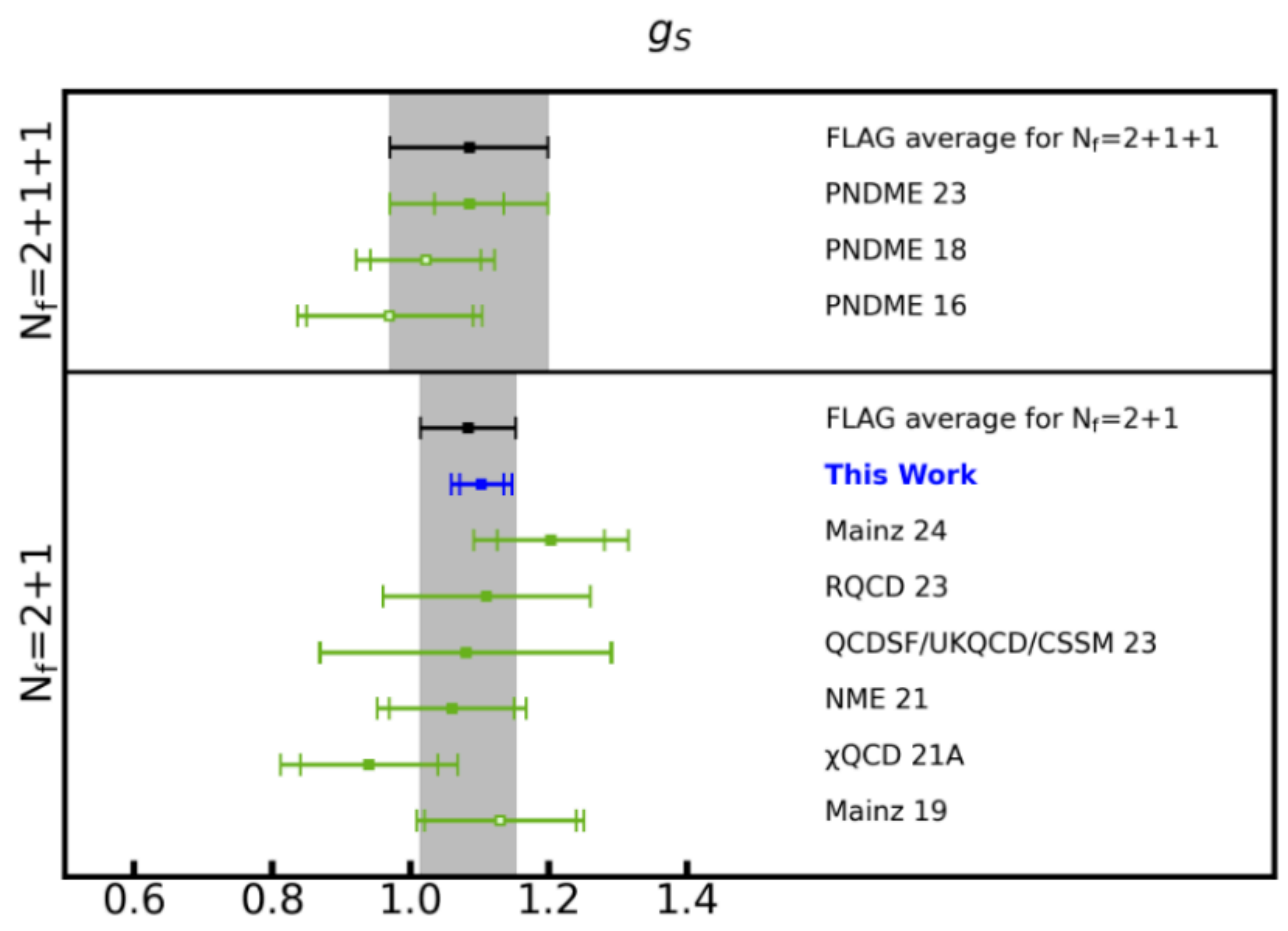


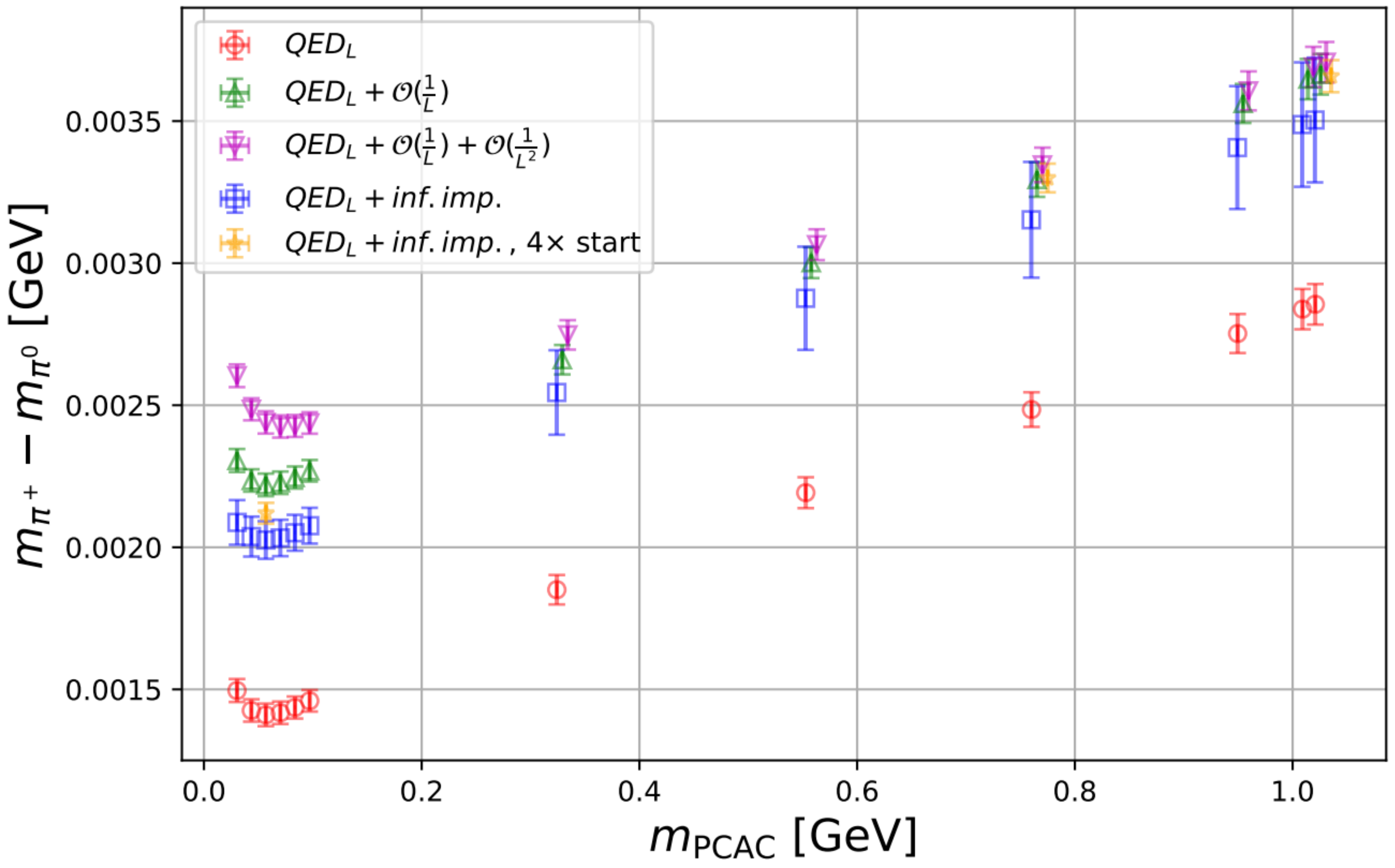
- Determination of $m_d - m_u$ requires the QED correction of $m_{K^+} - m_{K^0} = g^l_{S,K}(m_d - m_u) + \delta^{\text{QED}} m_K^{\text{isoQCD}}$;
- CLQCD 23 used the QED corrections $\delta^{\text{QED}} m_K^{\text{isoQCD}} = 2.07(15)$ MeV from literature and then our
  D.Giusti et al. [RM123], PRD95(2017)114504
  prediction of m_u/m_d is NOT used for the FLAG average;
  Flavor lattice average group, 2411.04268
- Direct QCD+QED correction on the CLQCD ensembles using the NPU scheme could provide more precise determination of the QED and ISB corrections.

Summary

- CLQCD ensembles can now provide high precision hadron matrix elements through the blending method;
- And also control all the systematic uncertainties of the iso-symmetric QCD.







- The QCD+QED simulation on the CLQCD ensemble C24P29 (a=0.105 fm and m_π=0.29 GeV) agrees with that in the literature reasonably;
- The QED corrections of the charm and bottom quark masses would be around 3-4 MeV.