Bottom baryon mass and trace anomaly contribution

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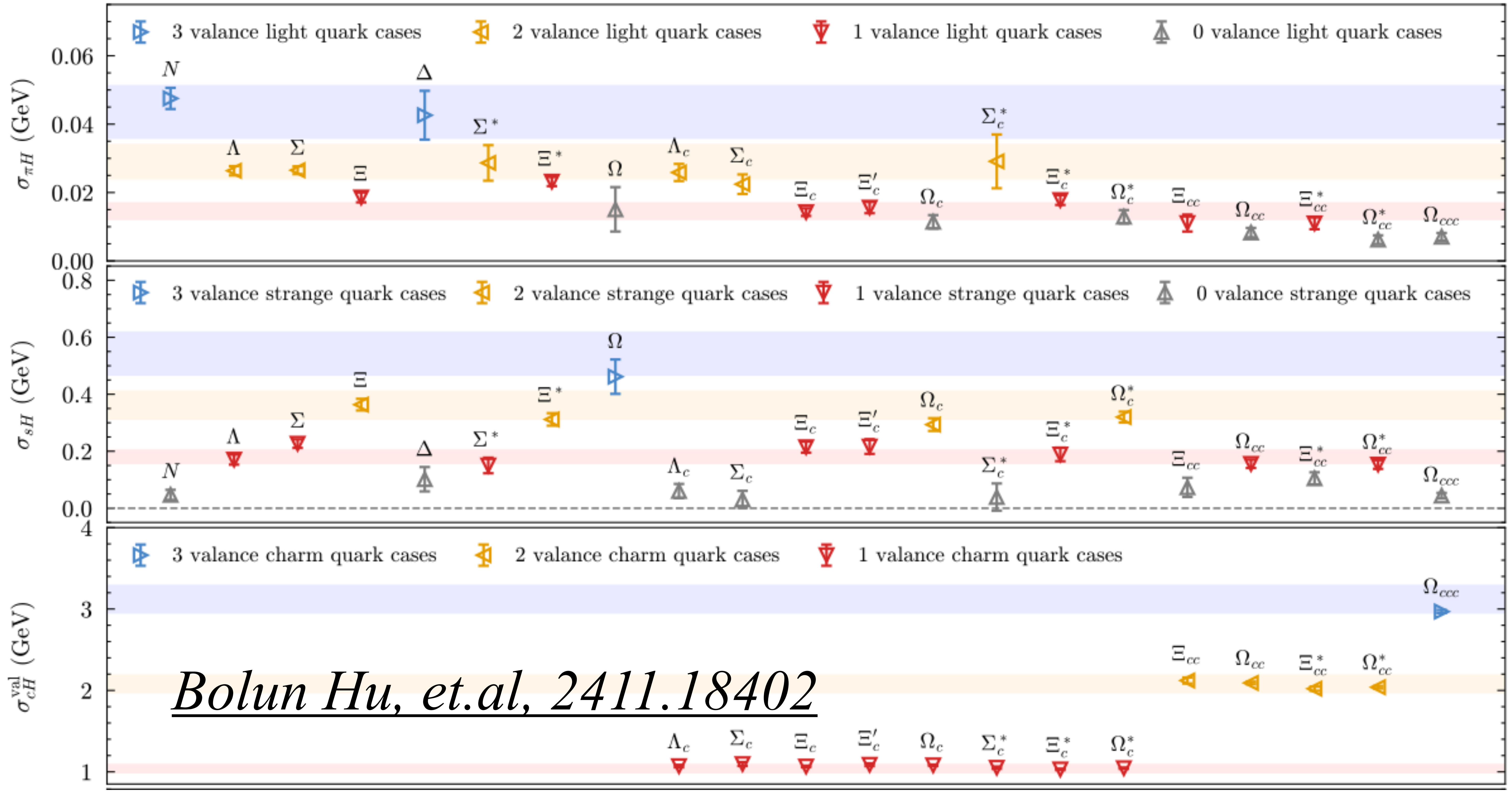
2025.10.12

THEORETICAL FRAMEWORK

- Hadron mass decomposition: $m_H = \sum_q \sigma_{q,H} + \gamma_m \sum_q \sigma_{q,H} + \frac{\beta(\alpha_s)}{2\alpha_s}\langle G^{\mu\nu}G_{\mu\nu}\rangle_H$
- Sigma term: quark flavor contribution $\sigma_{q,H} \equiv m_q\langle \bar{q}q\rangle_H$, where $\langle \bar{q}q\rangle_H$ with $\langle O\rangle_H \equiv \langle H|O|H\rangle/\langle H|H\rangle$ can be understood as the condensate of quark and antiquark within the hadron H.
- Scale violation of quark mass dimension $\gamma_m = \frac{2}{\pi}\alpha_s + O(\alpha_s^2)$
- Gluon trace anomaly part: $\frac{\beta(\alpha_s)}{2\alpha_s} = (-\frac{11}{8\pi} + \frac{N_f}{12\pi})\alpha_s + O(\alpha_s^2)$
- Extract sigma term: Feynman-Hellman theorem $\sigma_{q,H}^{\mathrm{FH}} = m_q^{\mathrm{PC}}\frac{\partial m_H}{\partial m_q^{\mathrm{PC}}}$ or directly 3-pt calculation.

THEORETICAL FRAMEWORK

- Bottom quark sigma term contribution: proportional to the number of valence bottom quark, just like charm quark case.



Bolun Hu, et.al, 2411.18402

NUMERICAL SETUP

Ensemble information

- CLQCD 2+1 clover action, 6 lattice spacing, several sea π and η_s mass and different volume.

0.105fm: C24P34 C24P29 C32P29 C32P23 C48P23 C48P14

0.090fm: E28P35 E32P29 E32P22

0.077fm: F32P30 F48P30 F32P21 F48P21 F64P14

0.069fm: G36P29

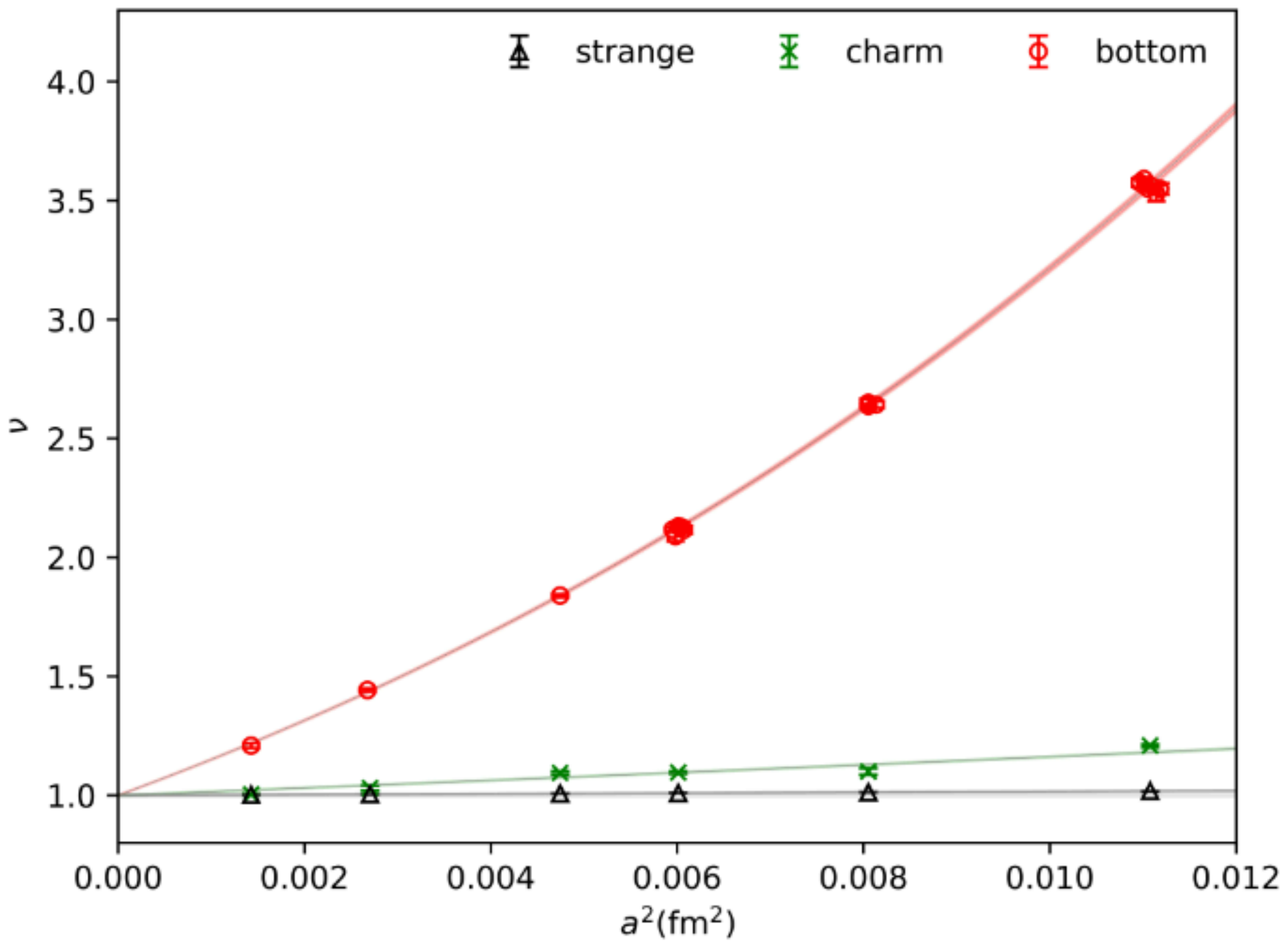
0.052fm: H48P32

0.038fm: I64P30

NUMERICAL SETUP

Bottom quark action

- Anisotropic clover action for bottom quark
$$S_Q = a^4 \sum_x \bar{Q}[m_Q + \gamma_4 \nabla_4 - \frac{a}{2}\nabla_4^2 + \nu \sum_{i=1}^{3} (\gamma_i \nabla_i - \frac{a}{2}\nabla_i^2) - c_E \frac{a}{2} \sum_{i=1}^{3} \sigma_{0i} F_{0i} - c_B \frac{a}{4} \sum_{i,j=1}^{3} \sigma_{ij} F_{ij}]Q$$
Z. S. Brown, et.al *Phys.Rev.D*, 90(9) 094507. L. Liu, et.al *Phys.Rev.D*, 81 094507
- 0-step stout-link smearing
- Clover coefficients $c_E = \frac{1+\nu}{2u_0^3}, c_B = \frac{\nu}{u_0^3}$ P. Chen, *Phys.Rev.D*, 64 034509
- Turning the values of m_Q and ν to satisfy dispersion relation and physical Υ (1S vector bottomonium) mass: $E_V(\vec{p})^2 = m_V^2 + c^2 p^2, c = 1, \;\; m_V = m_\Upsilon = 9460.40(10)$ MeV
- Empirical formula of anisotropy ν for bottom case
$$\nu = \frac{\mathrm{Sinh}(cm_V a)}{cm_V a} \times [1 + c_1(m_\pi^2 - m_{\pi,phy}^2) + c_2(m_{\eta_s}^2 - m_{\eta_s,phy}^2) + c_3 e^{-m_\pi L}] \text{ with}$$
$$c = 0.6130(11),\; c_1 = 0.247(39),\; c_2 = -0.008(35),\; c_3 = 0.063(54)$$



See, Meng-Chu Cai's talk

NUMERICAL SETUP

Interpolating field of bottom baryon

★single heavy quark:

- Spin-1/2, Σ-type: $\Sigma : \epsilon^{abc} P^+ (Q_a^T C\gamma^5 q_b) q_c$. For Σ_Q, q is u or d and for Ω_Q, q is s.
- Spin-1/2, Ξ′-type: $\Xi' : \frac{1}{\sqrt{2}} \epsilon^{abc} P^+ [(q_a^T C\gamma^5 Q_b) q'_c + (q_a'^T C\gamma^5 Q_b) q_c]$ where q is u or d, $q' = s$.
- Spin-1/2, Λ-type: octet lambda $\Lambda_o : \frac{1}{\sqrt{6}} \epsilon^{abc} P^+ [2(q_a^T C\gamma^5 q'_b) Q_c + (q_a^T C\gamma^5 Q_b) q'_c - (q_a'^T C\gamma^5 Q_b) q_c]$ where $q = u, q' = d$ for Λ_Q, and $q = u, q' = s$ for Ξ_Q.
- Spin-3/2, $\Sigma^* : \frac{1}{\sqrt{3}} \epsilon^{abc} P^+ [2(Q_a^T C\gamma^\mu q_b) q_c + (q_a^T C\gamma^\mu q_b) Q_c]$ $\Xi_Q^* : \frac{2}{\sqrt{3}} \epsilon^{abc} P^+ [(q_a^T C\gamma^\mu q'_b) Q_c + (q_a'^T C\gamma^\mu Q_b) q_c + (Q_a^T C\gamma^\mu q_b) q'_c]$

★double heavy quark:

- Two heavy quark have the same mass: spin-1/2 only Σ-type: interchange the role of light and heavy fields
- Two heavy quark have the same mass: spin-3/2 similar to Σ*, for Ξ_{QQ}^*, $Q \to q = u/d, q \to Q$ and for Ω_{QQ}^*, $Q \to s, q \to Q$.
- Two unlike heavy flavors: spin-3/2 corresponding baryon $\Xi_{QQ'}^*$ and $\Omega_{QQ'}^*$ can be obtained by letting $q, q' \to Q, Q'$ and $Q \to u/d$ or $Q \to s$ in Ξ_Q^*
- Two unlike heavy flavors: spin-1/2 $\Xi_{QQ'}$ and $\Omega_{QQ'}$ from Ξ'_Q, $\Xi'_{QQ'}$ and $\Omega'_{QQ'}$ from Λ

★Baryon with three heavy quark:

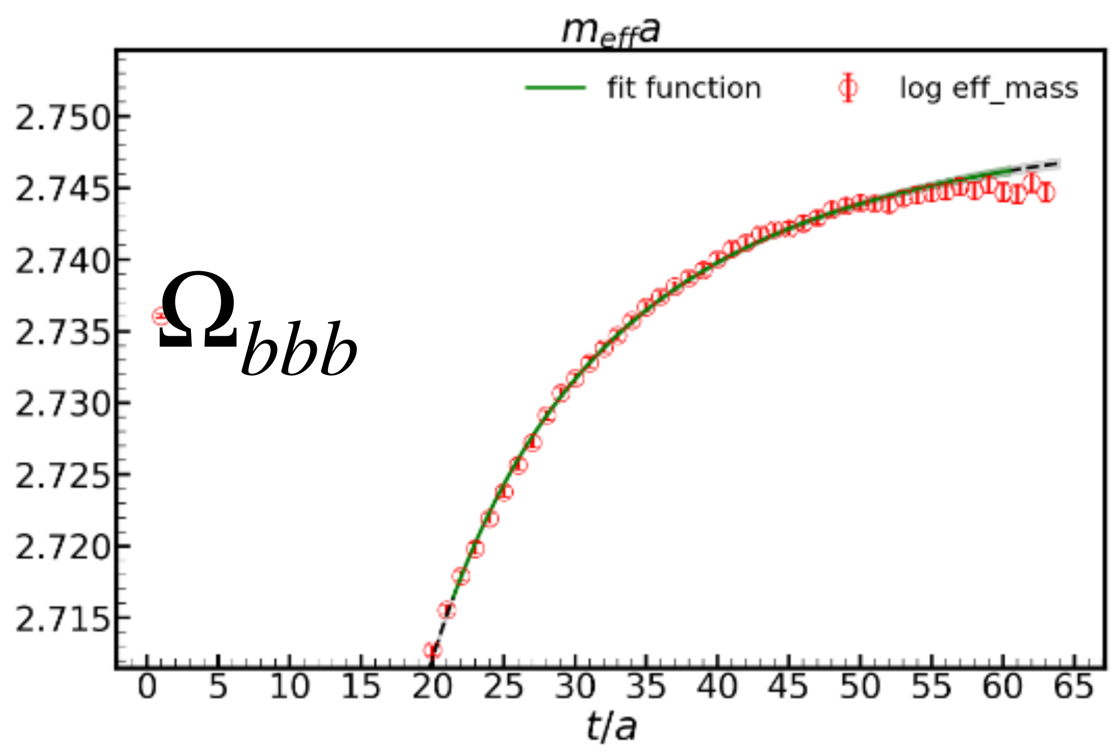
- $\Omega_{QQQ'}$ and $\Omega_{QQQ'}^*$: similar to Ω_{QQ} and Ω_{QQ}^*
- Ω_{QQQ}: $\epsilon^{abc} P^+ (Q_a^T C\gamma^\mu Q_b) Q_c$

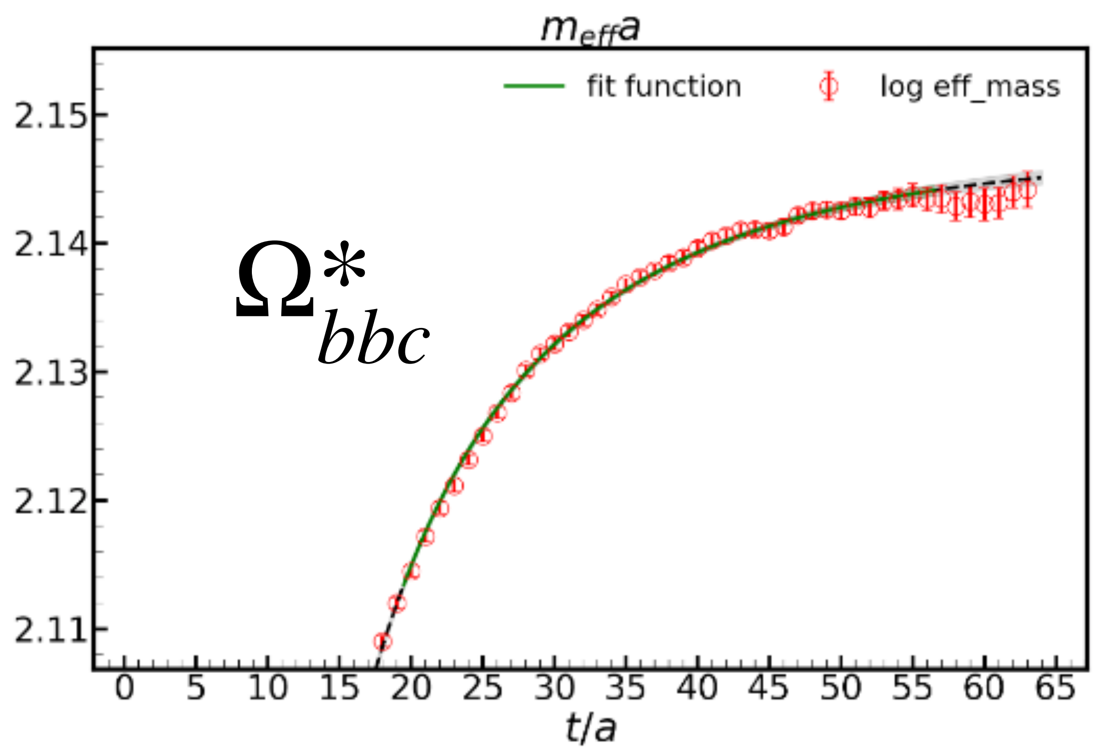
PRELIMINARY RESULTS

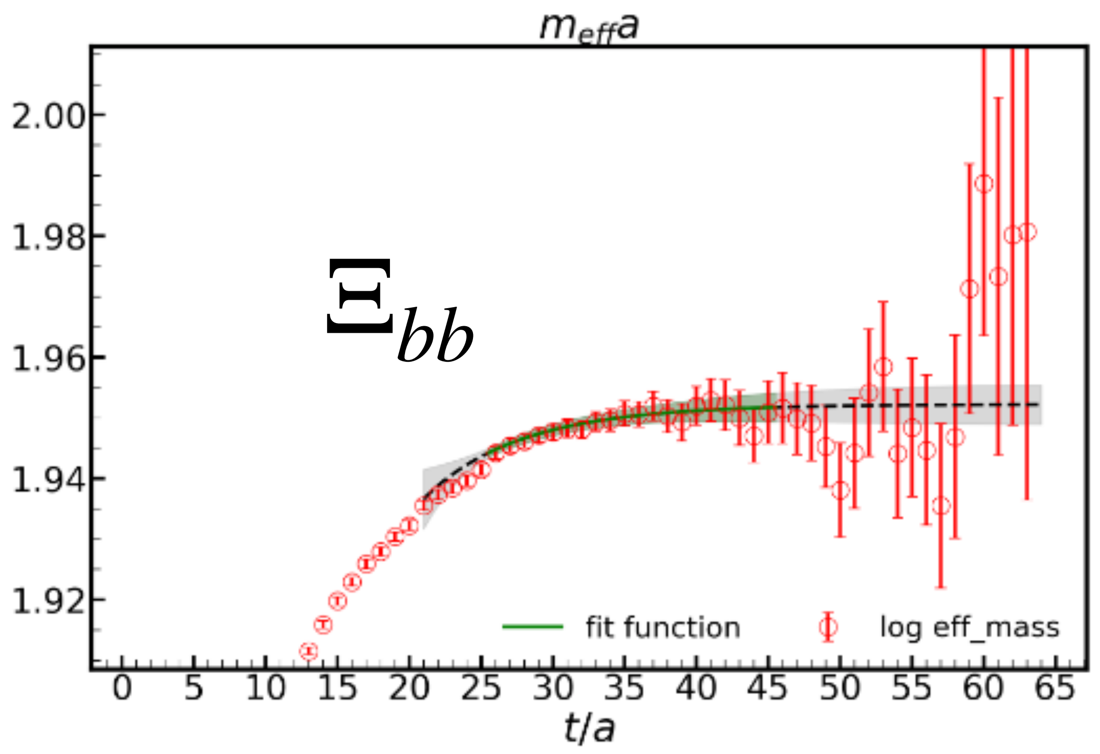
Two-state fit for baryon mass

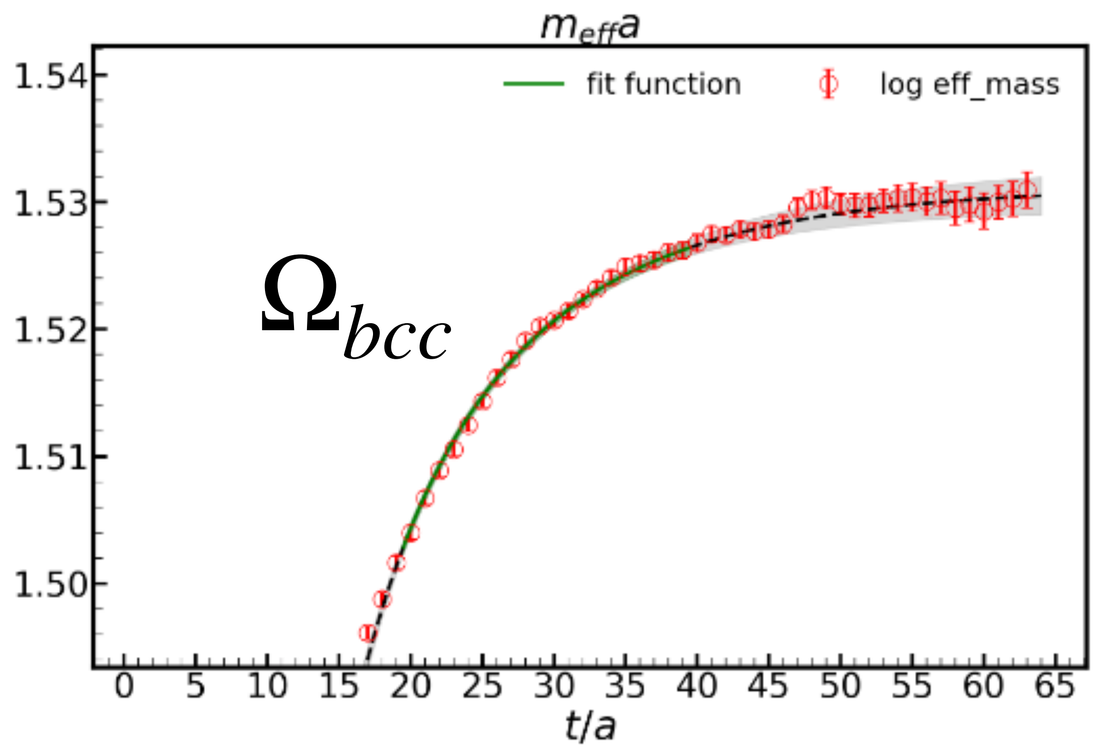
- $m_H = c_0 e^{-m_H t}(1 + c_1 e^{-Et})$

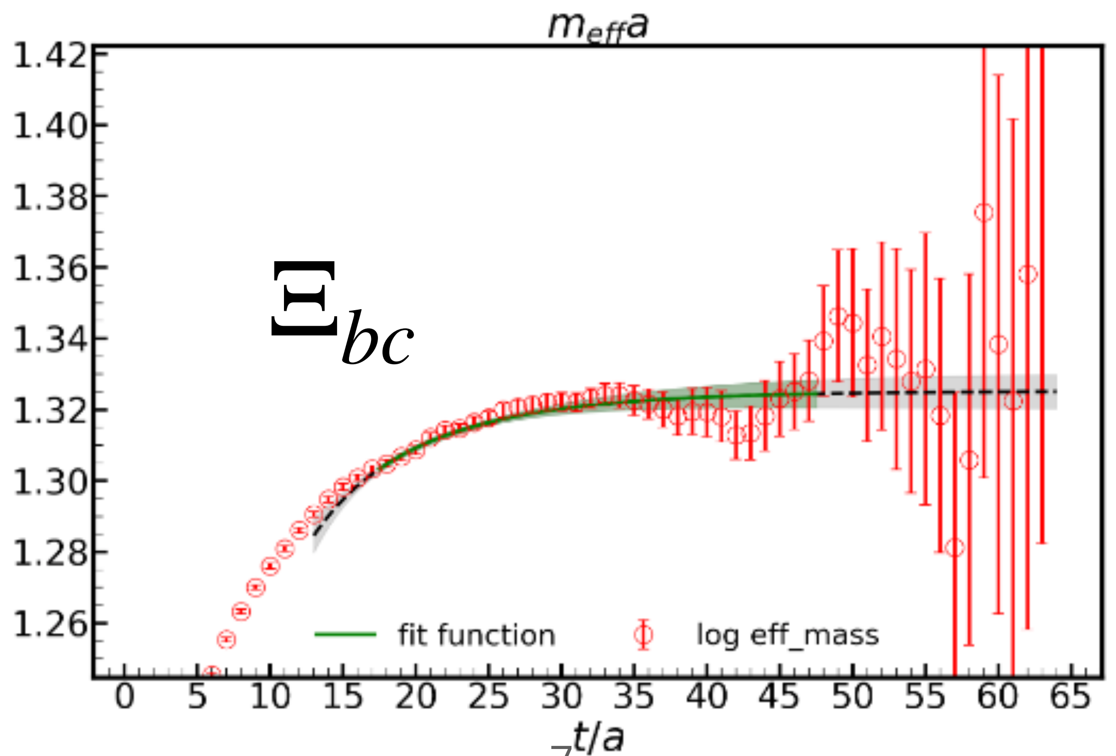
I64P30

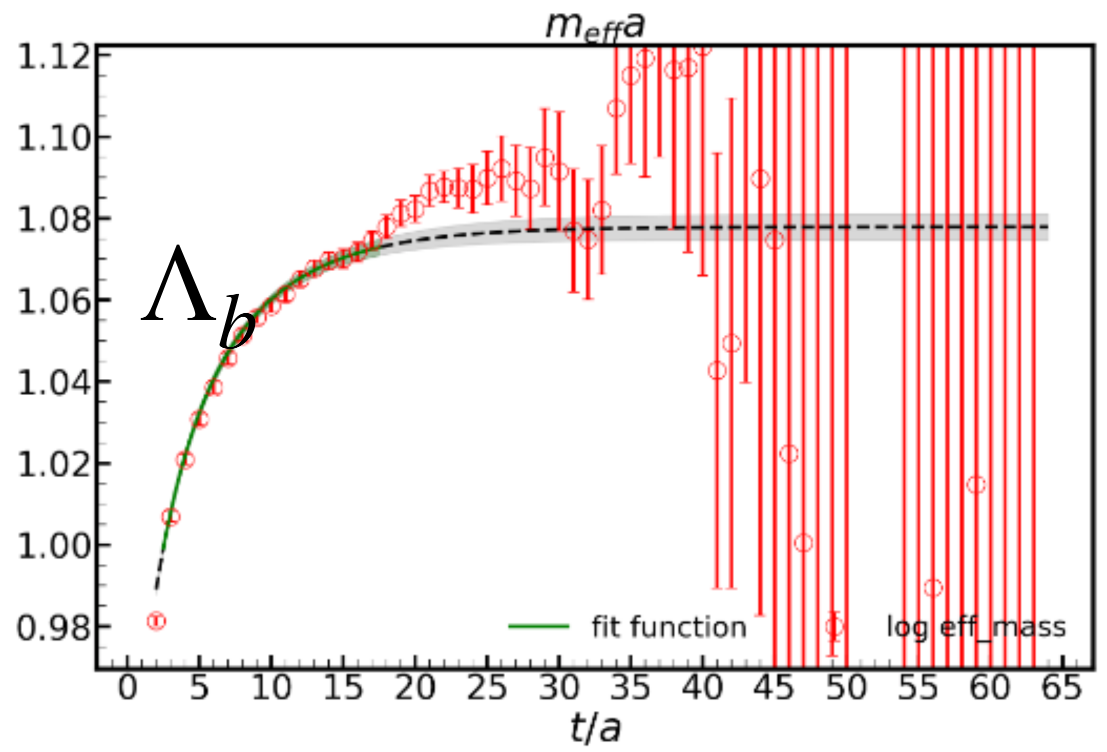








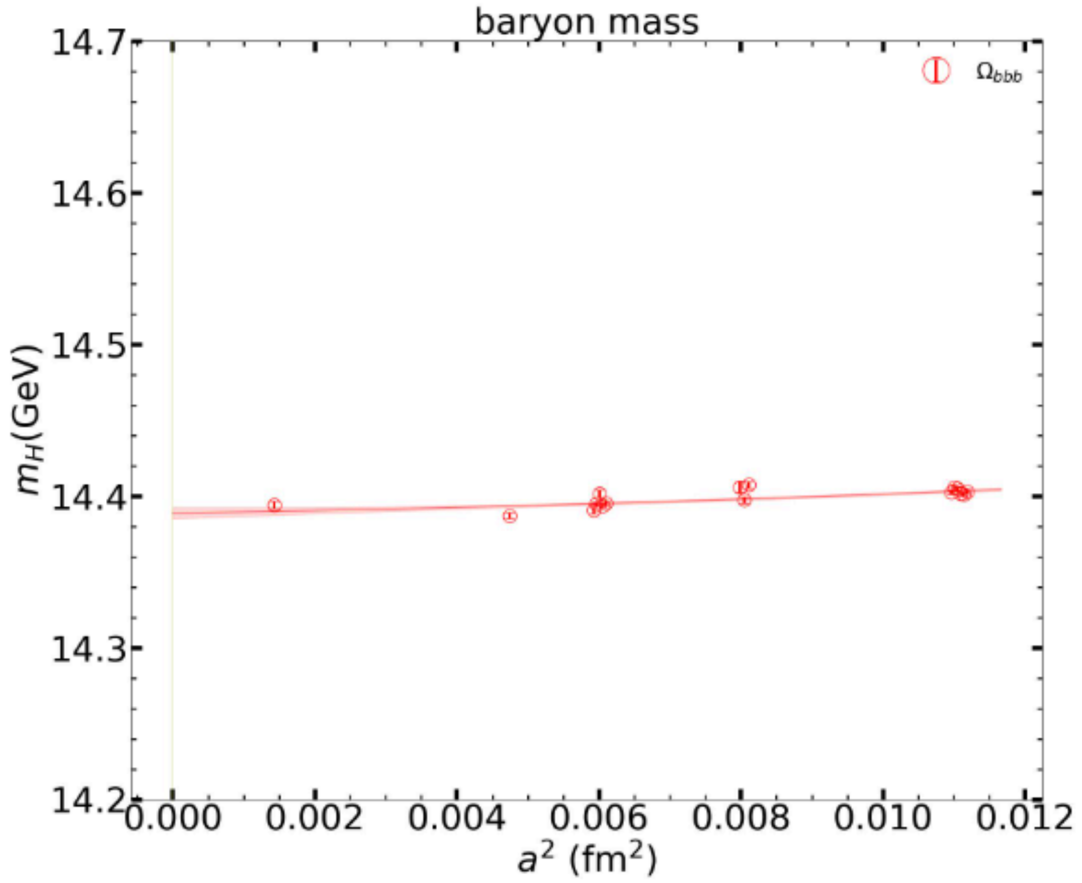


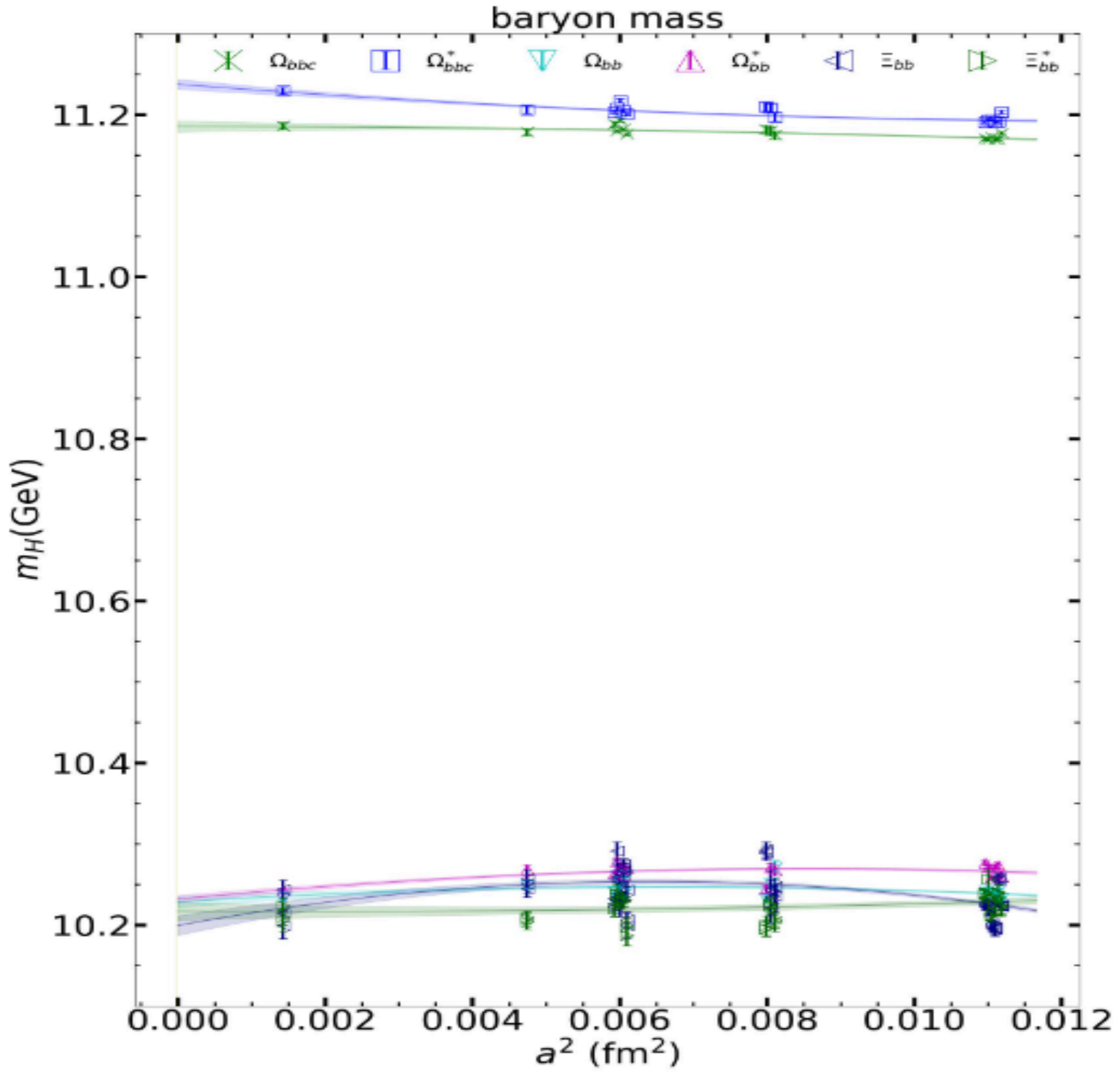


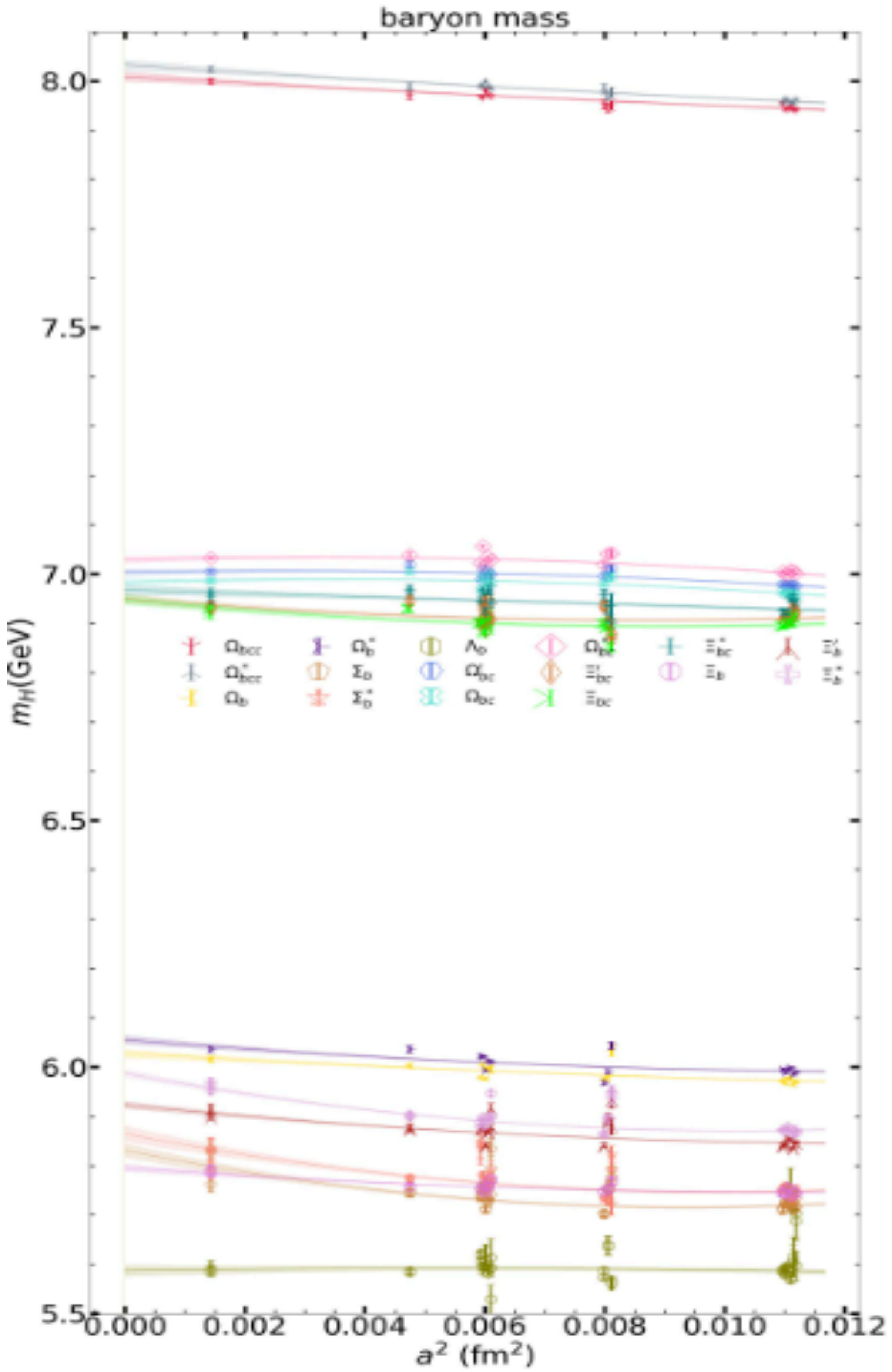
PRELIMINARY RESULTS

Global fit for baryon mass

- Global fit: $m_H(m_{\pi,\mathrm{val}}, m_{\pi,\mathrm{sea}}, m_{\eta_s,\mathrm{sea}}, a) = m_H^{\mathrm{phy}} + c_1(m_{\pi,\mathrm{val}}^2 + m_{\pi,\mathrm{sea}}^2 - m_{\pi,\mathrm{phy}}^2) + c_2(m_{\eta_s,\mathrm{sea}}^2 - m_{\eta_s,\mathrm{phy}}^2) + c_3a^2 + c_4a^4$
- Valence strange: interpolate to "physical" η_s mass 689.89(49) MeV
- Valence charm: interpolate to pure QCD D_s mass 1966.7(1.5) MeV



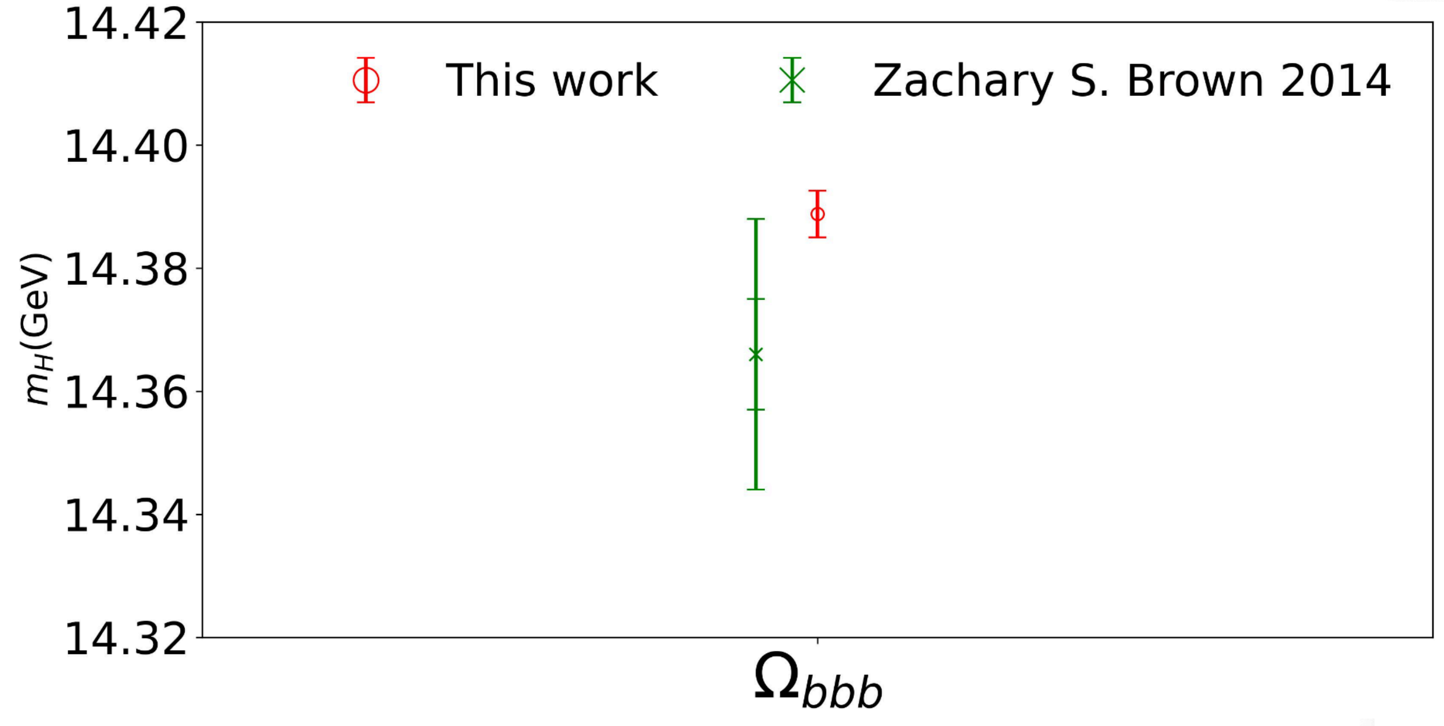


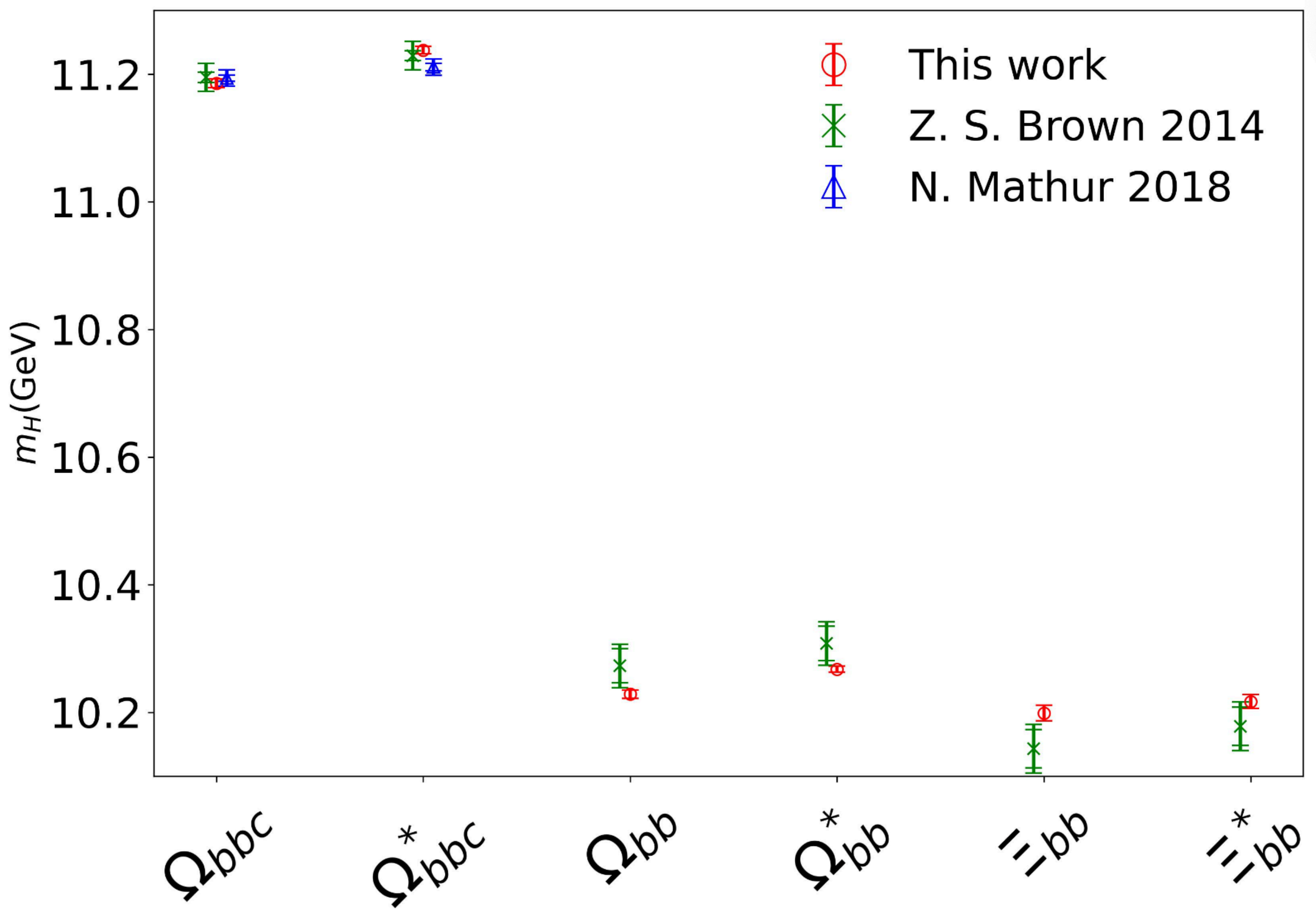


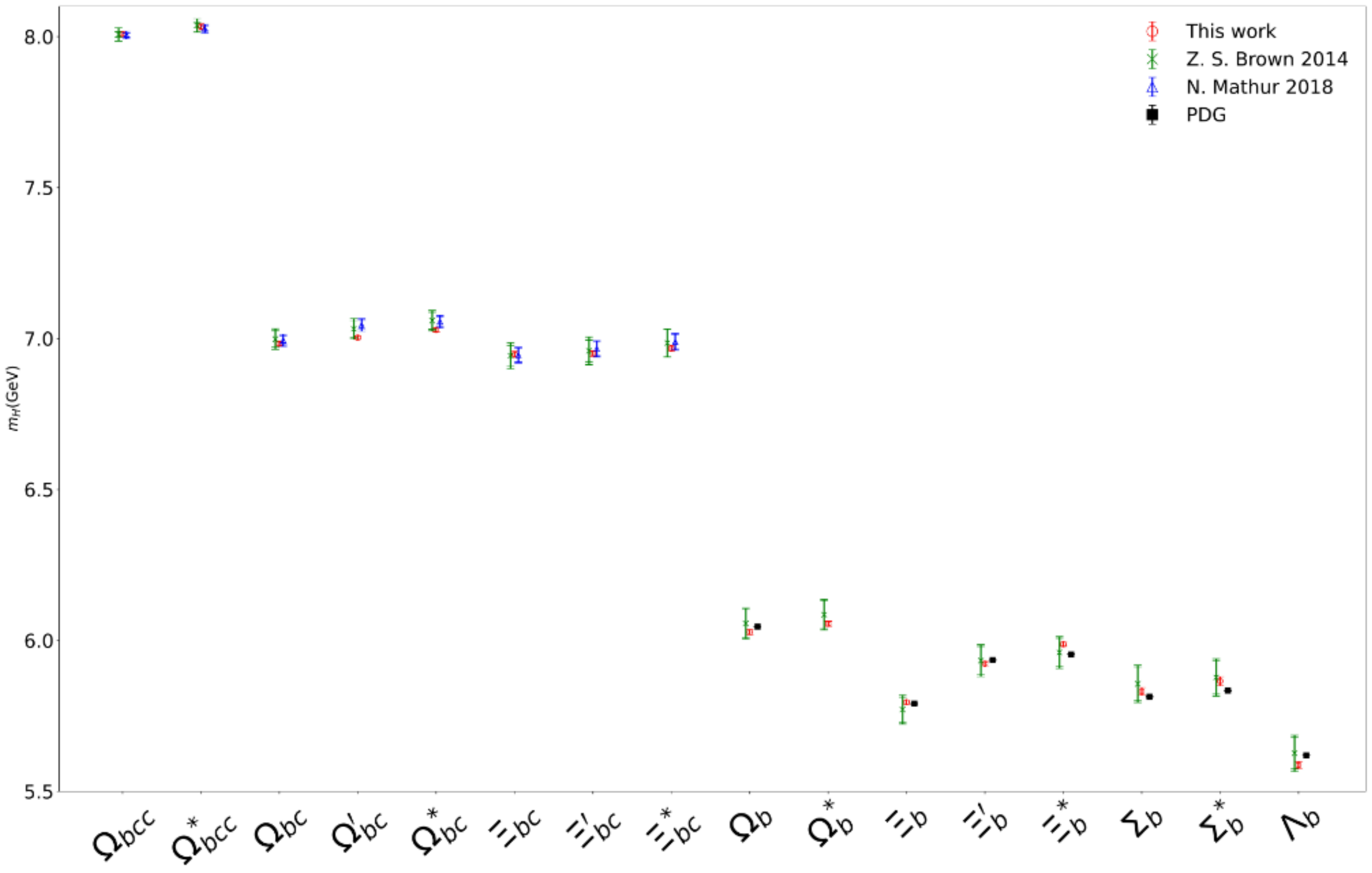
PRELIMINARY RESULTS

Global fit for baryon mass

- Prediction for bottom baryon mass: only statistical uncertainty





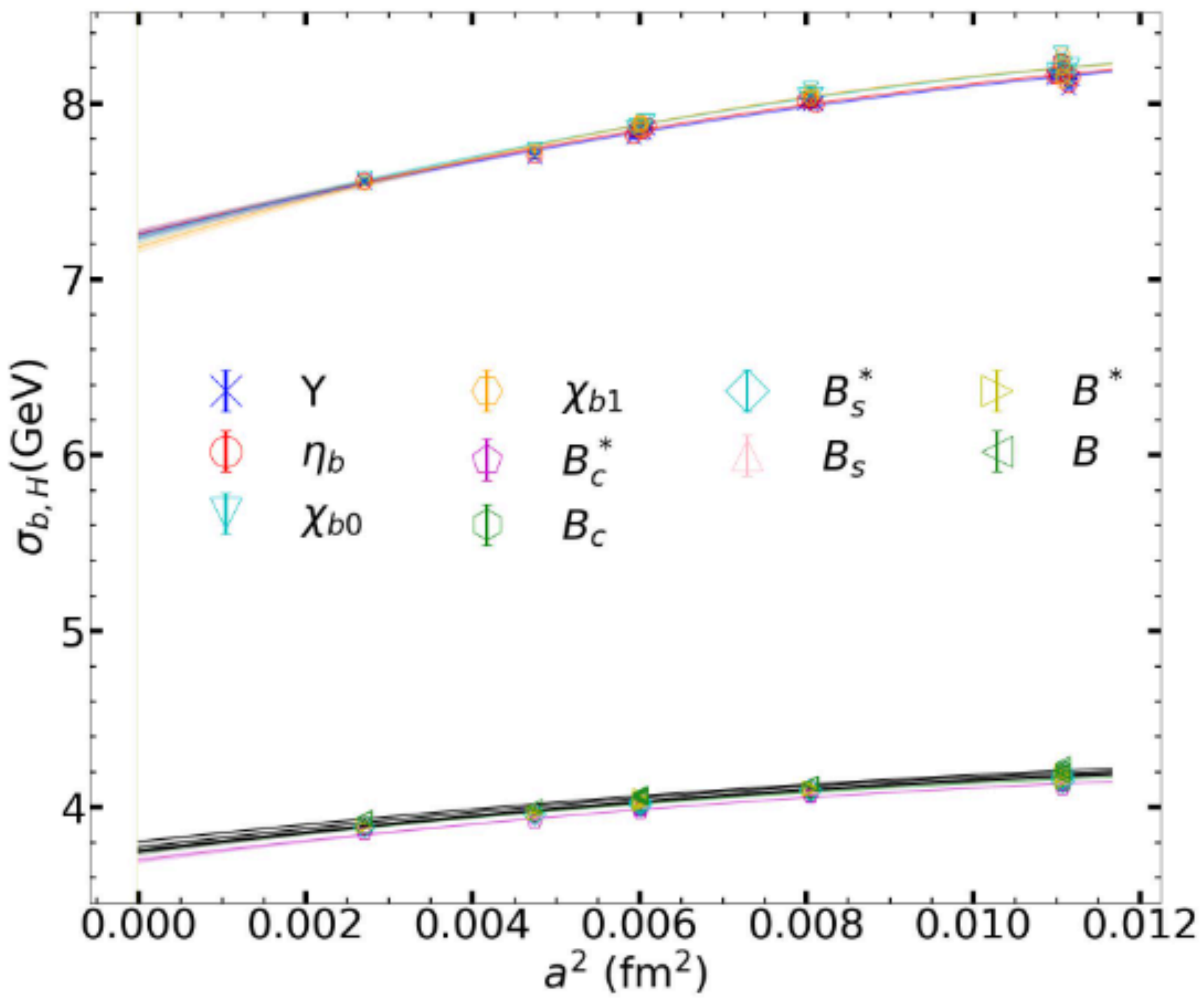


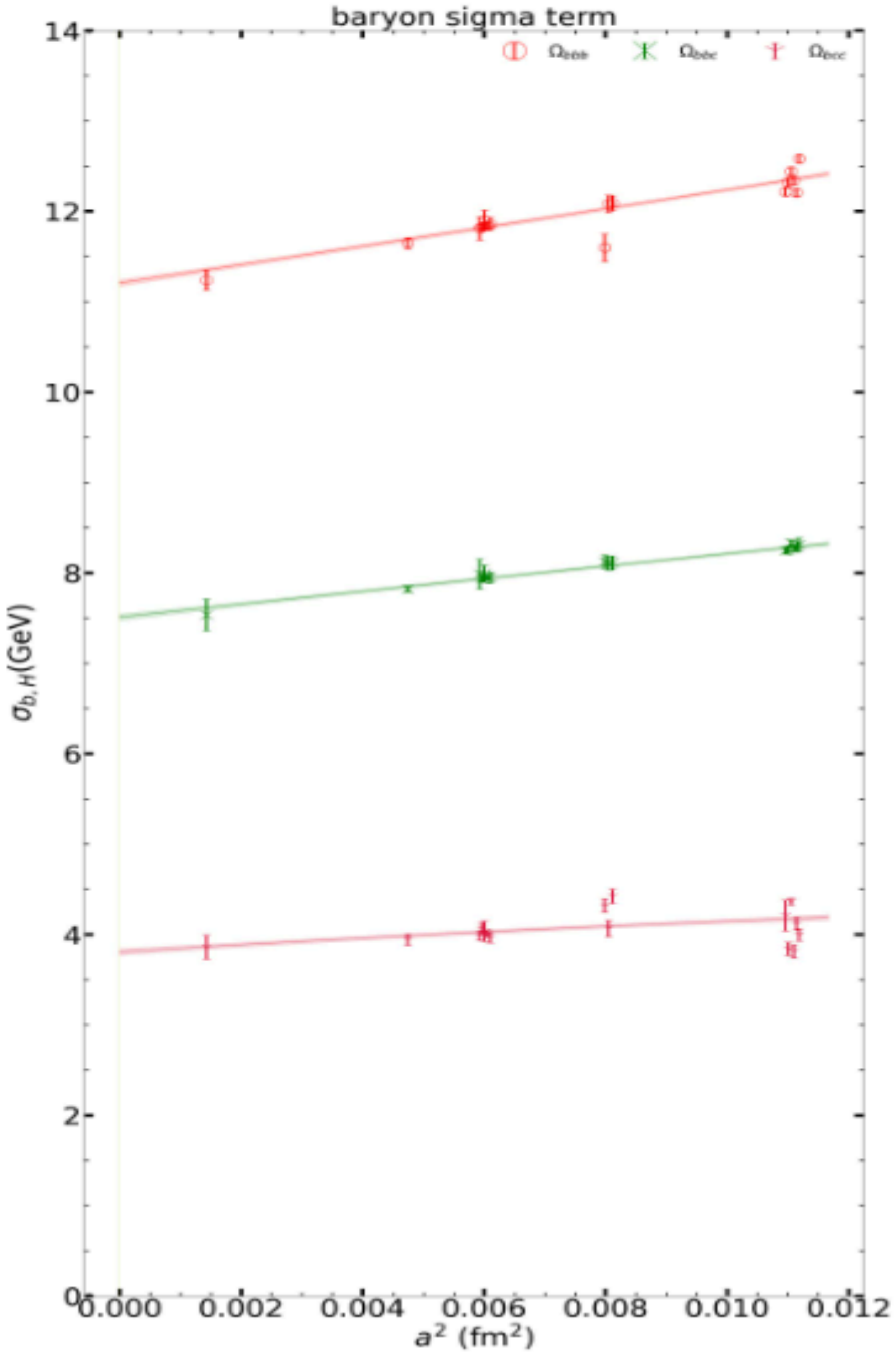
PRELIMINARY RESULTS

Sigma term extraction: Feynman-Hellman theorem

- Extract sigma term: Feynman-Hellman theorem
  $\sigma_{q,H}^{\mathrm{FH}} = m_q^{\mathrm{PC}} \frac{\partial m_H}{\partial m_q^{\mathrm{PC}}}$
- Global fit:

$$\sigma_{b,H}(m_{\pi,\mathrm{val}}, m_{\pi,\mathrm{sea}}, m_{\eta_s,\mathrm{sea}}, a) = \sigma_{b,H}^{\mathrm{phy}} + c_1(m_{\pi,\mathrm{val}}^2 + m_{\pi,\mathrm{sea}}^2 - m_{\pi,\mathrm{phy}}^2) + c_2(m_{\eta_s,\mathrm{sea}}^2 - m_{\eta_s,\mathrm{phy}}^2) + c_3 a^2 + c_4 a^4$$

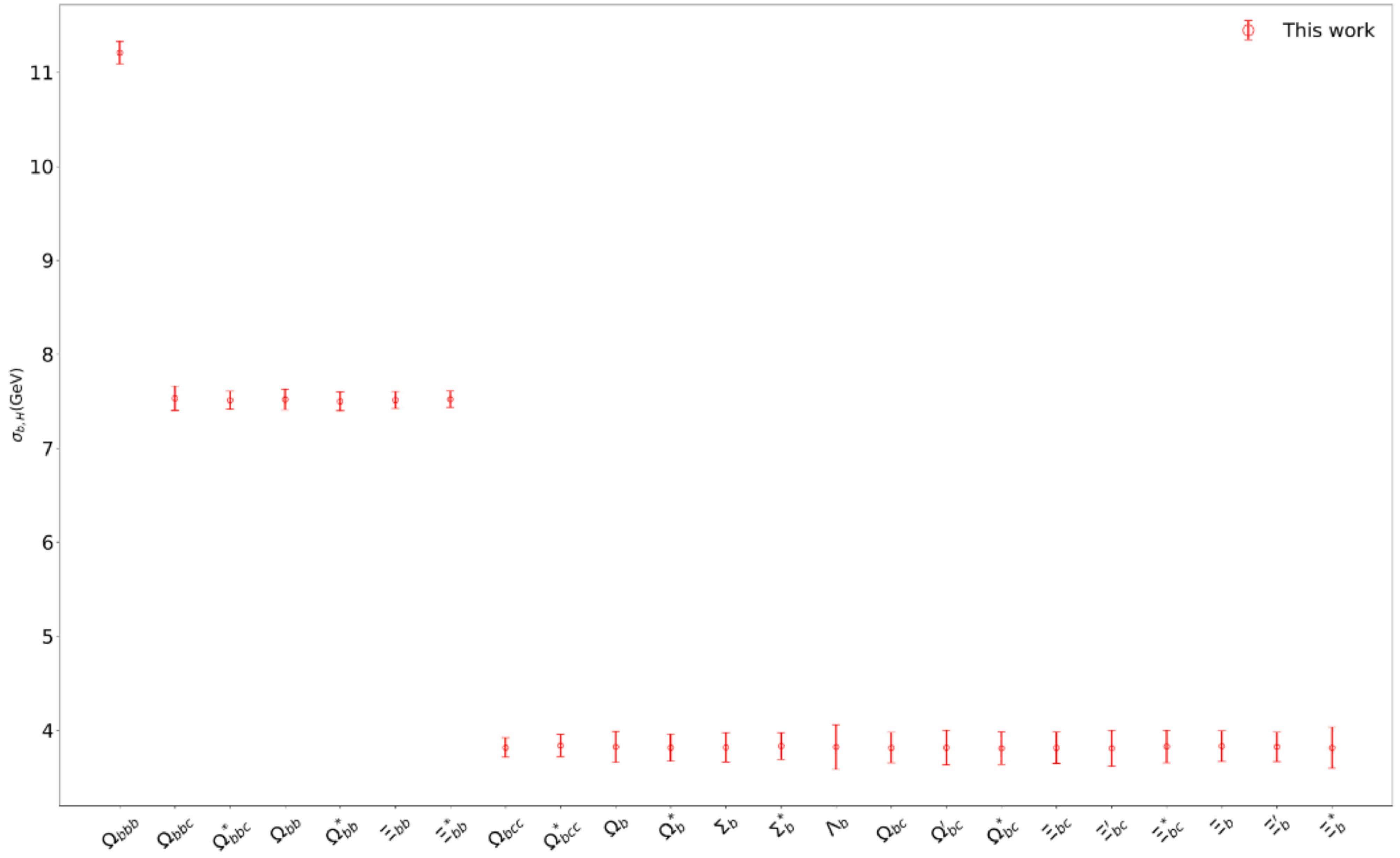




PRELIMINARY RESULTS

Sigma term extraction: Feynman-Hellman theorem

- Only valence bottom contribution.
- $\sigma_{b,H}/n_b$: ~3.74(4) GeV for three bottom quarks (Ω_{bbb}), ~3.75(5) GeV for double bottom quarks and ~3.82(17) GeV for single bottom quarks



PRELIMINARY RESULTS

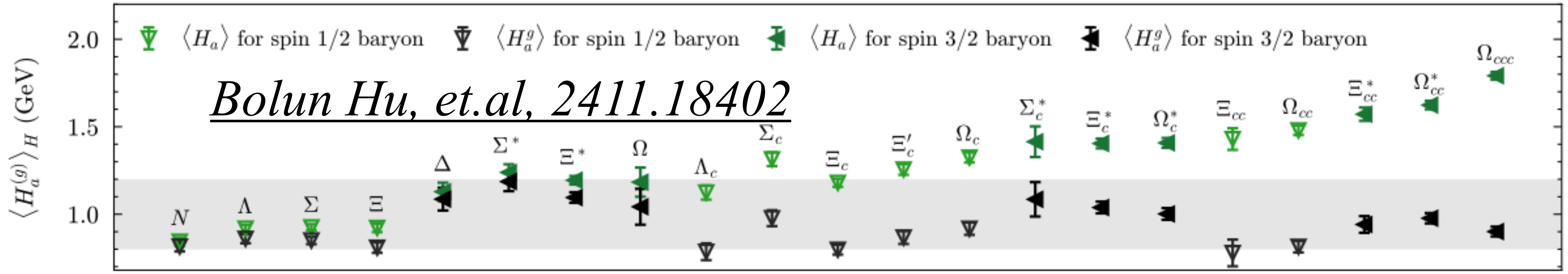
Discussion

- Hadron mass decomposition: $m_H = \sum_q \sigma_{q,H} + \gamma_m \sum_q \sigma_{q,H} + \frac{\beta(\alpha_s)}{2\alpha_s}\langle G^{\mu\nu}G_{\mu\nu}\rangle_H$
- Sigma term: scale independent $\sigma_{q,H} \equiv m_q\langle \bar{q}q\rangle_H$, where $\langle \bar{q}q\rangle_H$ with $\langle O\rangle_H \equiv \langle H|O|H\rangle/\langle H|H\rangle$, m_q can be m_q^{PC} or m_q^R
- Bottom quark condensate in baryon: scale dependence
- $\langle \bar{b}b\rangle_H/n_b \sim 0.76$ at $m_b^{\overline{\mathrm{MS}}\ 2\ (\mathrm{GeV})} \sim 5.0$ GeV
- $\langle \bar{b}b\rangle_H/n_b \sim 0.90$ at $m_b(m_b) \sim 4.2$ GeV
- $\gamma_m = 0.295$ at $\overline{\mathrm{MS}}$ 2 GeV: about gluon trace anomaly contribution$\langle H_a\rangle_H = m_H^{\mathrm{phy}} - \sigma_H$, $\langle H_a^g\rangle_H = \langle H_a\rangle_H - \gamma_m\sigma_H = m_H^{\mathrm{phy}} - (1+\gamma_m)\sigma_H$
- $(1+\gamma_m)\sigma_{b,H}$ at $\overline{\mathrm{MS}}$ 2 GeV: ~ 14.4 GeV for Ω_{bbb}, ~ 9.7 GeV for Ξ_{bb}, ~ 4.9 GeV for Σ_b ($\sigma_{l/s,H}$ can be ignored compared to bottom quark)

PRELIMINARY RESULTS

Discussion

- Hadron mass decomposition: $m_H = \sum_q \sigma_{q,H} + \gamma_m \sum_q \sigma_{q,H} + \frac{\beta(\alpha_s)}{2\alpha_s}\langle G^{\mu\nu}G_{\mu\nu}\rangle_H$
- $\gamma_m = 0.295$ at $\overline{\text{MS}}$ 2 GeV: $\langle H_a\rangle_H = m_H^{\text{phy}} - \sigma_H$, $\langle H_a^g\rangle_H = m_H^{\text{phy}} - (1+\gamma_m)\sigma_H$
- $(1+\gamma_m)\sigma_{b,H}$ at $\overline{\text{MS}}$ 2 GeV: ~ 14.4 GeV for Ω_{bbb}, ~ 9.7 GeV for Ξ_{bb}, ~ 4.9 GeV for Σ_b.
- $\langle H_a^g\rangle_H$ at $\overline{\text{MS}}$ 2 GeV: ~ 0 GeV for Ω_{bbb}, ~ 0.4 GeV for Ξ_{bb}, ~ 0.9 GeV for Σ_b. Unlike light and charmed baryon (~ 1 GeV)
- $\langle H_a^g\rangle_H$: need larger scale to reach 1 GeV for double bottom baryon.



<u>*Bolun Hu, et.al, 2411.18402*</u>

Thank you