Unpolarized Gluon PDF from Lattice QCD

CLQCD

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in collaboration with

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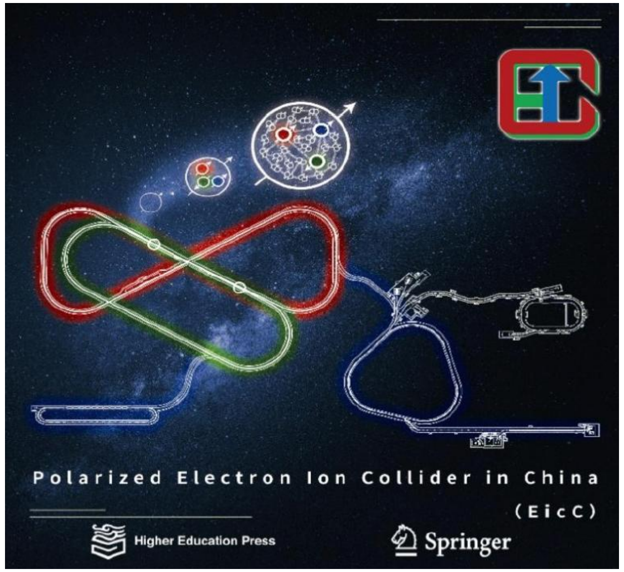
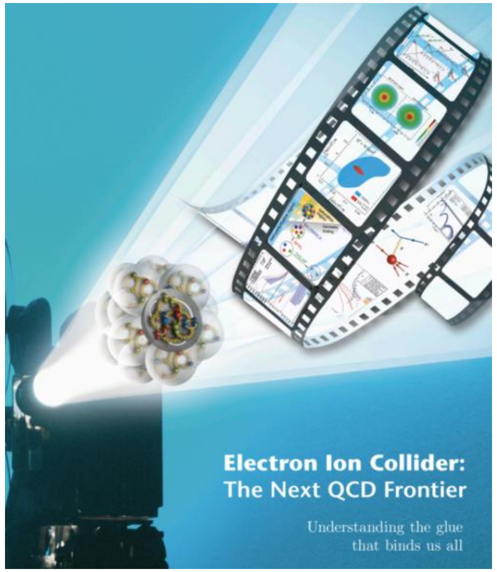
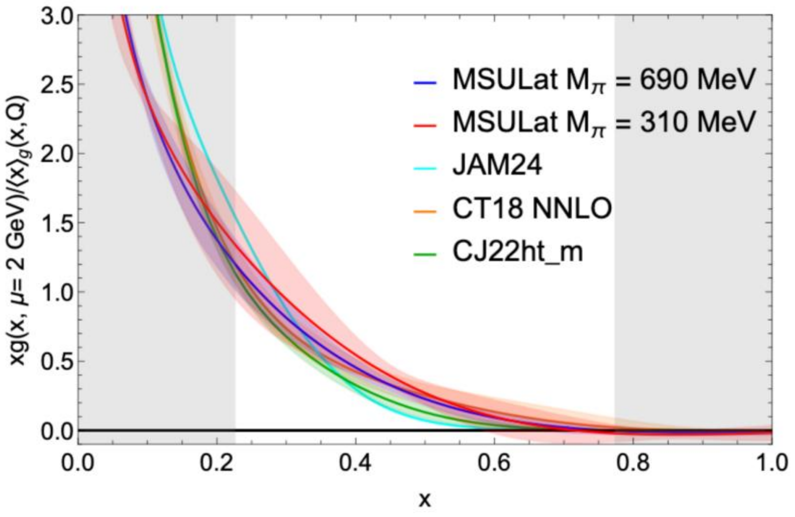
Introduction & Motivation

- Gluons: meditators of strong interaction
- Gluon PDFs : necessary input for high energy scattering experiments
- Phenomenological studies : model dependent, hard to constrain the PDF in large-x region
- Lattice QCD: large momentum effective theory (LaMET), pseudo-PDF methods

We calculate the unpolarized gluon PDF of proton using LaMET theory.





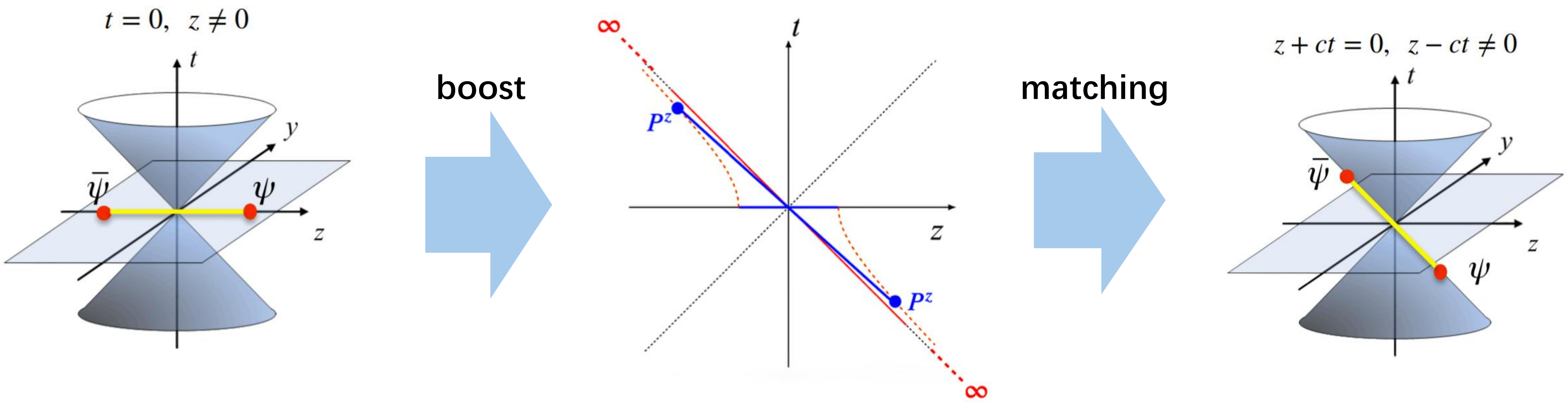


arXiv:2505.13321

Introduction & Motivation

Large momentum effective theory(LaMET)

Phys.Rev.Lett. 110 (2013) 262002



- Light-cone quantities cannot be calculated directly using Lattice QCD approach.
- The difference is in UV physics, perturbative matching can connect the quasi-PDFs (lattice) and the light-cone PDFs.

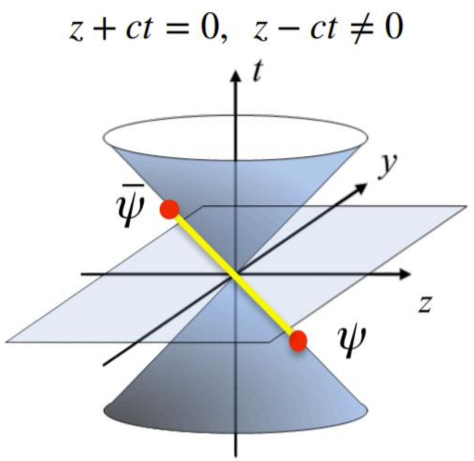
CLQCD Configurations

Name	Volume	Lattice Spacing (fm)	Beta	Pion mass (MeV)	conf
C24P29	$24^3 \times 72$	0.105	6.200	293	879
E32P29	$32^3 \times 64$	0.0897	6.308	290	900
F32P30	$32^3 \times 96$	0.0775	6.410	300	777

- 2+1 flavor ensembles with Wilson-clover quark action

PRD109(2024)5,054507

Bare matrix element

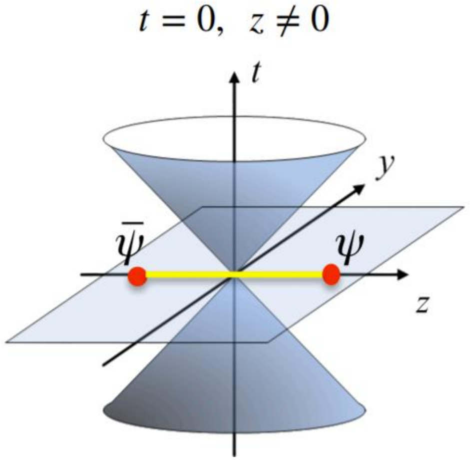


The light-cone PDF of gluon is defined as:

$$xg(x,\mu) = \int \frac{dz^-}{2\pi P^+} e^{-ixP^+z^-} \langle P|F^a_{+\mu}(z^-)U^{ab}(z^-,0)F^{b\mu}{}_{+}(0)(\mu)|P\rangle$$

According to LaMET, light-cone PDF can be extracted from quasi-PDF

$$x\tilde{g}(x,P_z) = \int \frac{dz}{2\pi P_z} e^{-ixzP_z} \langle P|O(z)|P\rangle$$



Where $\langle P|O(\mathrm{z})|P\rangle$ is quasi-matrix element which can be extracted from two-point and three-point correlators (calculated on lattice).

Quasi-matrix element

Two-point correlator is defined as

$$C_{2\mathrm{pt}}(P_z; t_{\mathrm{sep}}) = \sum_{t_0} \langle 0 | N(P_z; t_0 + t_{\mathrm{sep}}) N^{\dagger}(P_z; t_0) | 0 \rangle$$

Three-point correlator is defined as

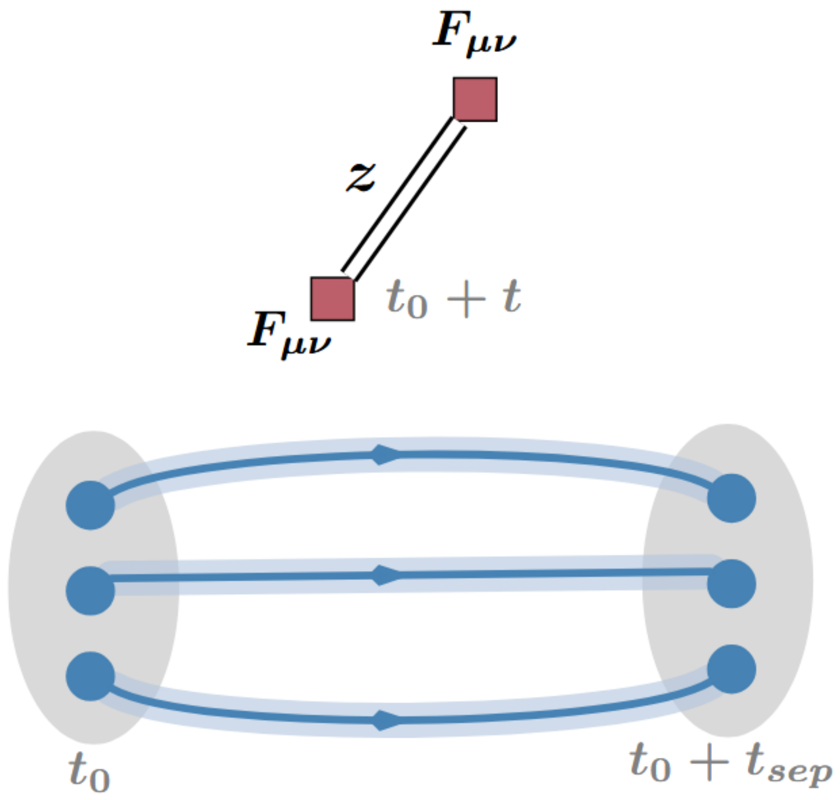
$$C_{3\mathrm{pt}}(P_z; z; t_{\mathrm{sep}}, t) = \sum_{t_0} \langle 0 | N(P_z; t_0 + t_{\mathrm{sep}}) O(z; t_0 + t) N^{\dagger}(P_z; t_0) | 0 \rangle$$

where $O(\mathrm{z};\ t)$ is gluon operator

$$O(z; \mathrm{t}) = M_{tx;tx}(z; \mathrm{t}) + M_{ty;ty}(z; \mathrm{t}) - 2M_{xy;xy}(z; \mathrm{t})$$

with $M_{\mu\lambda;\nu\rho}(z; t)$

$$= \sum_{\vec{x}} Tr\left[F_{\mu\lambda}(\vec{x} + z\hat{z}; t) U(\vec{x} + z\hat{z}, \vec{x}; t)\ F_{\nu\rho}(\vec{x}; \mathrm{t}) U(\vec{x}, \vec{x} + z\hat{z}; t)\right]$$



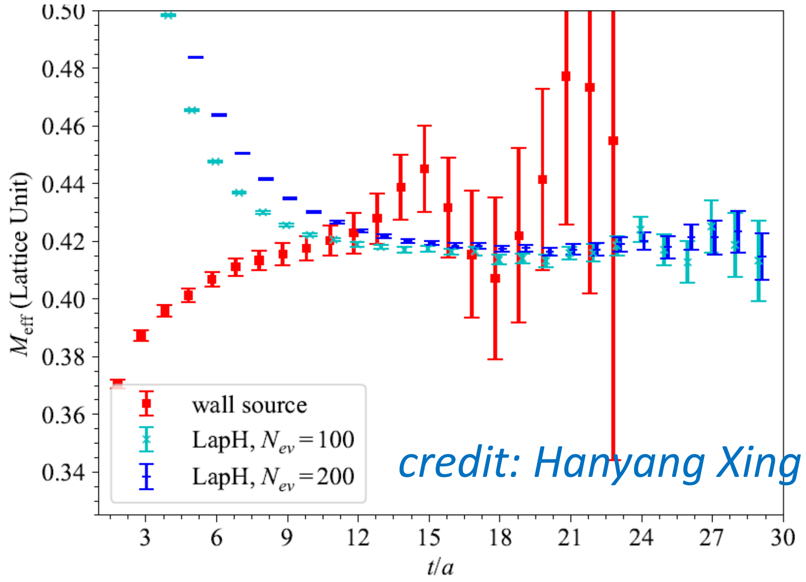
An illustration of gluon's three-point correlator

Noise reduction technique

➢ Laplacian Heavside (LaPH) smearing technique

- Smearing: an operator that effectively projects onto the hadronic states of interest.
- Applying LaPH operator on quark field: $\psi \longrightarrow VV^{\dagger}\psi$

V is made up of a small number (N_{eV}) of eigenvector of Laplacian operator ∇^2, corresponding to the lowest eigen values.

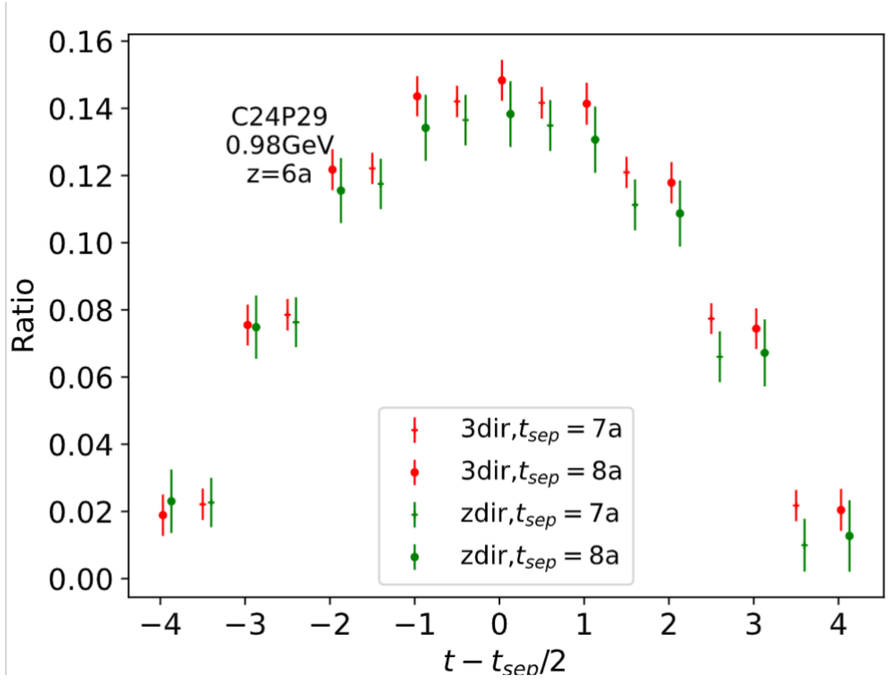


credit: Hanyang Xing (SCNU)

Effective mass of nucleon of ensemble F48P30 at $P_z = 0$ (Nconf=100)

➢ Use the rotational symmetry:

$$C_{3\mathrm{pt}} = \frac{1}{3}\left[C_{3\mathrm{pt}}(P_{\mathrm{x}}; \mathrm{x}; t_{\mathrm{sep}}, t) + C_{3\mathrm{pt}}(P_{\mathrm{y}}; \mathrm{y}; t_{\mathrm{sep}}, t) + C_{3\mathrm{pt}}(P_{z}; z; t_{\mathrm{sep}}, t)\right]$$



Ratio of 3pt to 2pt of ensemble C24P29

Bare matrix element

The three-point and two-point correlator can be decomposed as:

$$C_{\mathrm{2pt}}(P_z; t_{\mathrm{sep}}) = |\mathcal{A}_0|^2 e^{-E_0 t_{\mathrm{sep}}} + |\mathcal{A}_1|^2 e^{-E_1 t_{\mathrm{sep}}} \ldots$$

bare matrix elements

$$C_{\mathrm{3pt}}(P_z; z; t, t_{\mathrm{sep}}) = |\mathcal{A}_0|^2 \langle 0|O(z;t)|0\rangle e^{-E_0 t_{\mathrm{sep}}} + |\mathcal{A}_1|^2 \langle 1|O(z;t)|1\rangle e^{-E_1 t_{\mathrm{sep}}}$$
$$+ \mathcal{A}_1 \mathcal{A}_0^* \langle 1|O(z;t)|0\rangle e^{-E_1(t_{sep}-t)} e^{-E_0 t} + \mathcal{A}_0 \mathcal{A}_1^* \langle 0|O(z;t)|1\rangle e^{-E_0(t_{sep}-t)} e^{-E_1 t} \ \ldots$$

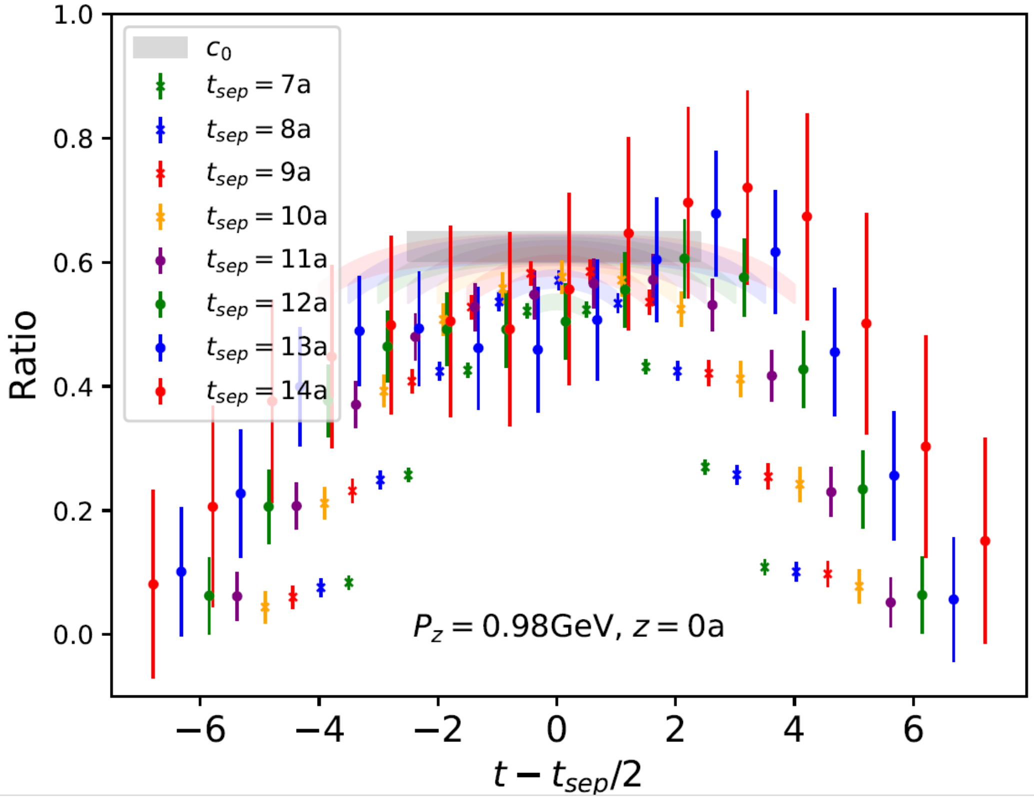
Bare matrix elements can be extracted by fitting the ratio of three- to two-point correlator:

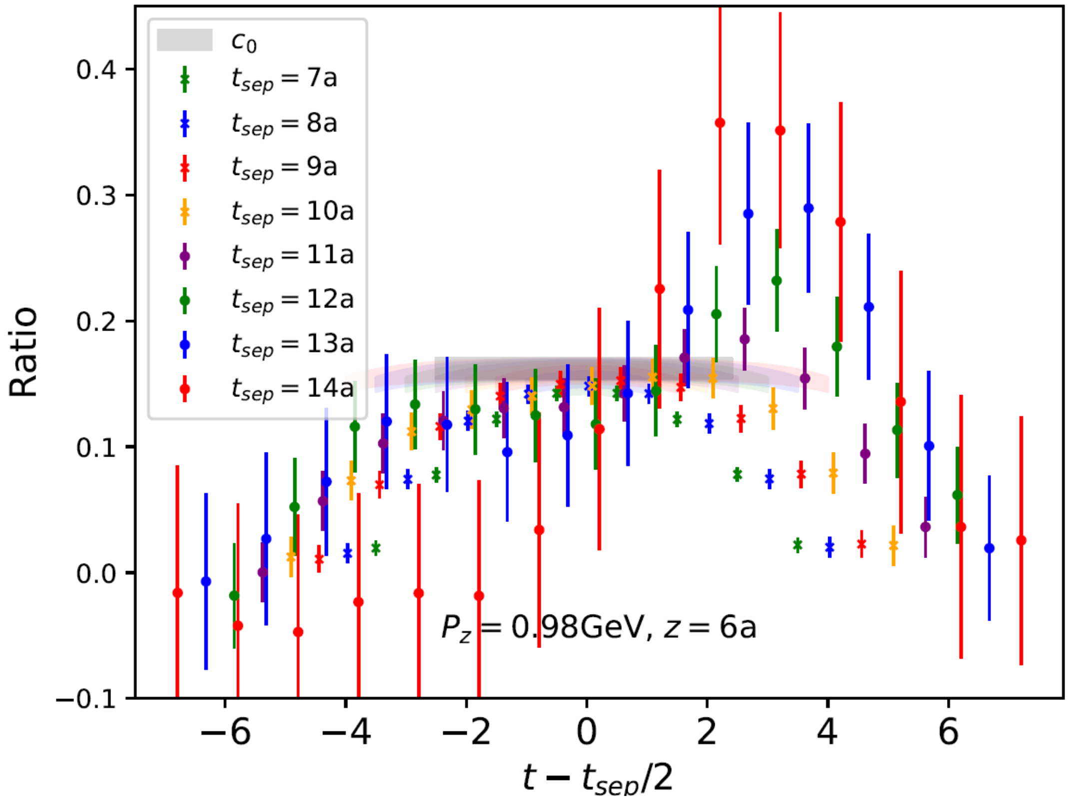
bare matrix elements

$$R(z, t, t_{sep}) = \frac{C_{3pt}}{C_{2pt}} = c_0(z) + c_1(z) e^{-\Delta E (t_{sep}-t)} + c_1(z) e^{-\Delta E t} \cancel{+ c_2(z) e^{-\Delta E t_{sep}}}$$

Bare matrix element

$$R(z,t,t_{sep}) = \frac{C_{3pt}}{C_{2pt}} = c_0(z) + c_1(z)e^{-\Delta E(t_{sep}-t)} + c_1(z)e^{-\Delta E t} \cancel{+ c_2(z)e^{-\Delta E t_{sep}}}, \qquad z = 0, na$$



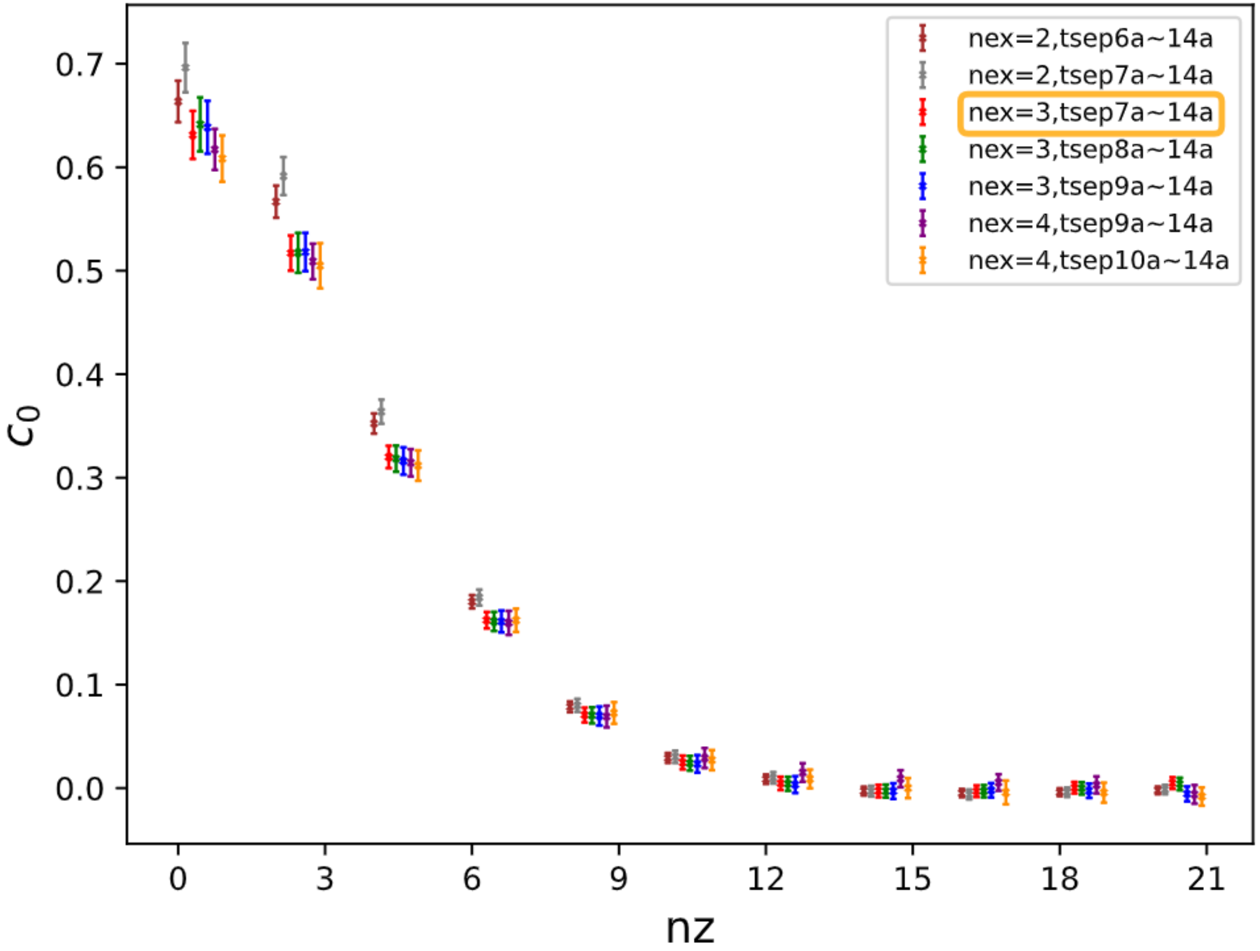


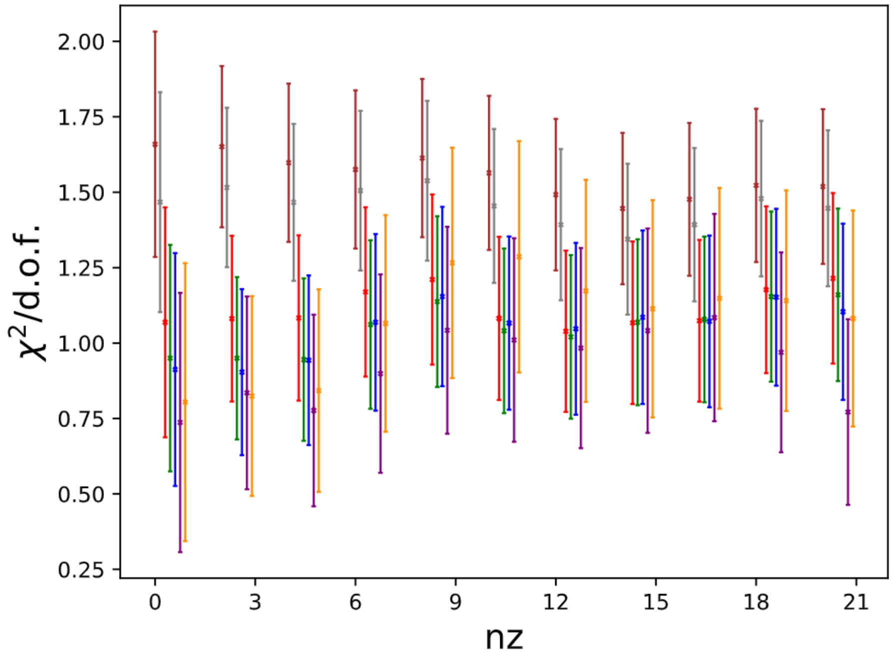
C24P29

Bare matrix element

Determine the fit range：

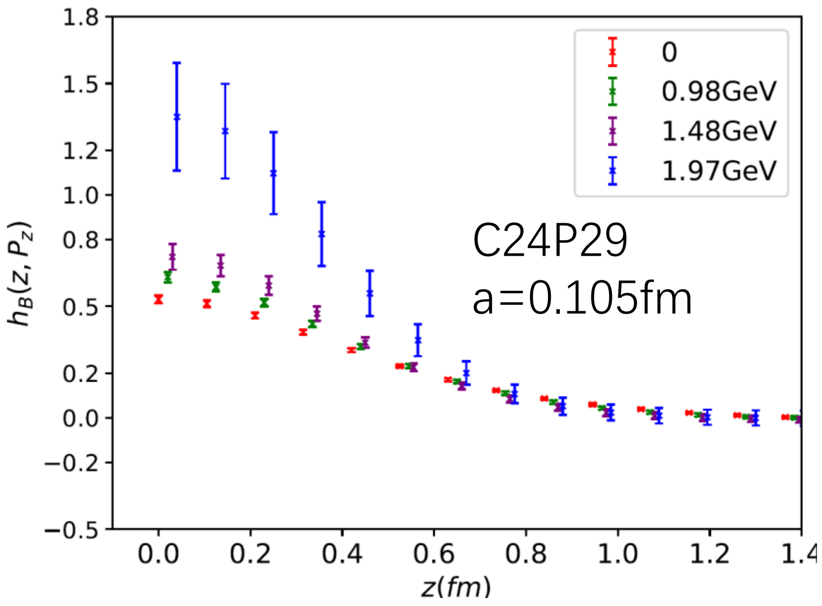
C24P29, $P_z = 0.98\text{GeV}$

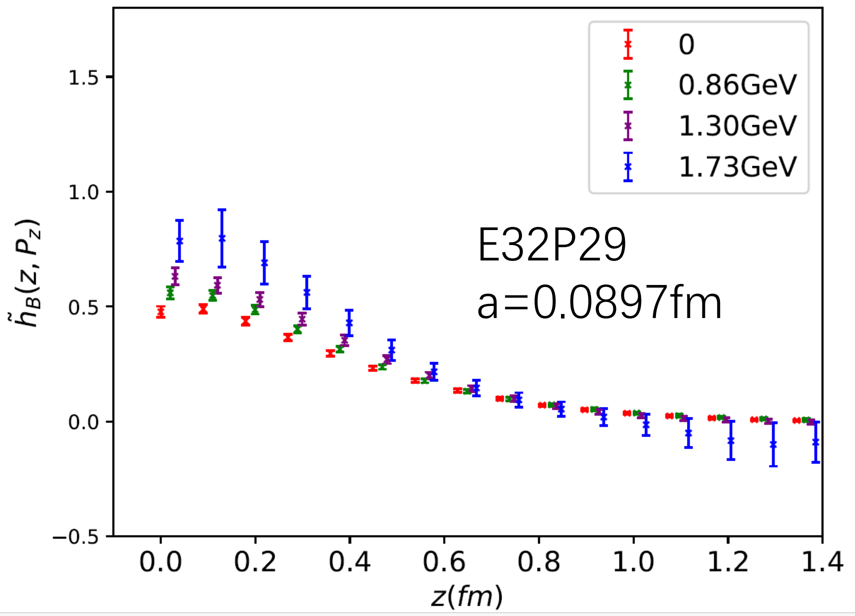


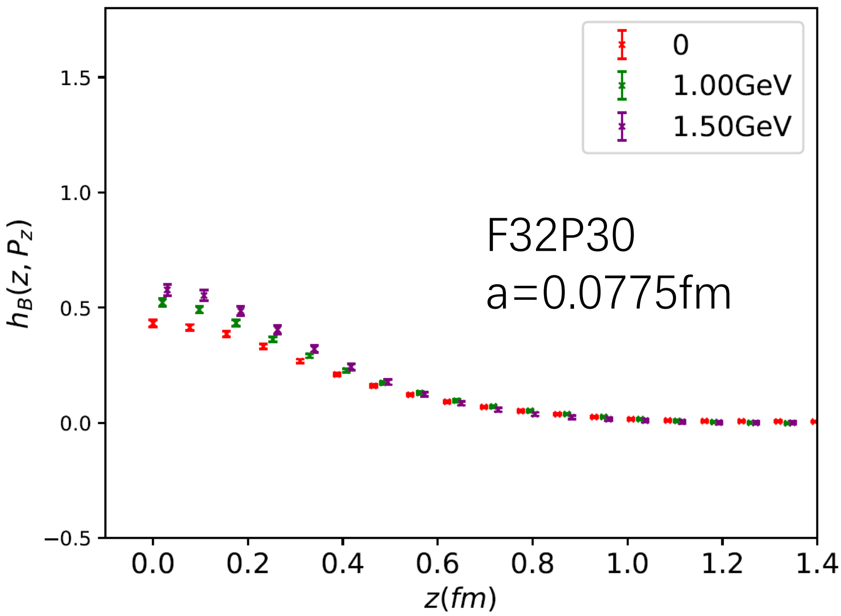


Bare matrix element

Fitting result for bare matrix elements







Renormalization

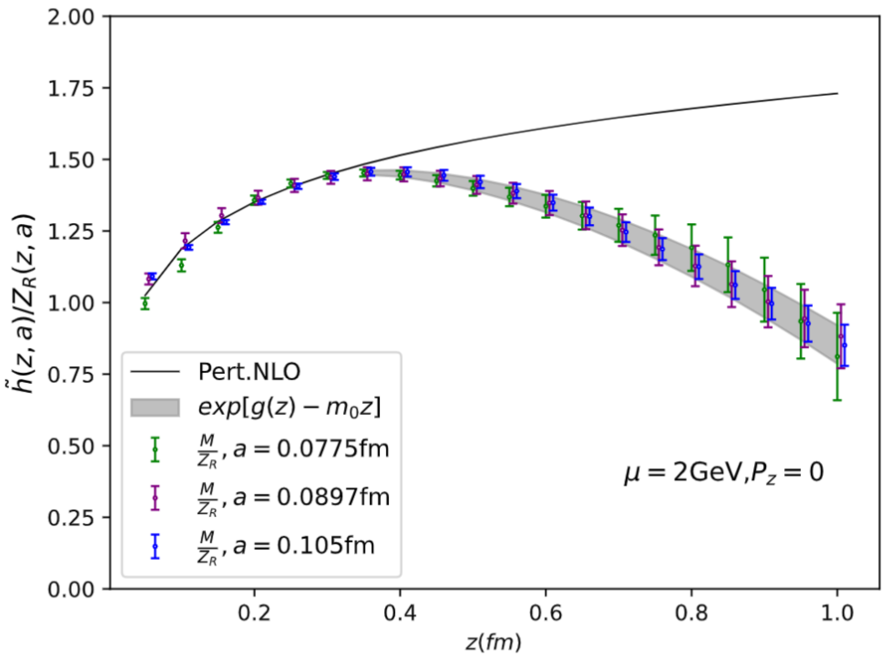
Hybrid renormalization:

$$\tilde{h}_R(z,P_z) = \frac{\tilde{h}_B^n(z,P_z,1/a)}{\tilde{h}_B^n(z,P_z=0,1/a)}\theta(z_s-|z|) + \eta_s \frac{\tilde{h}_B^n(z,P_z,1/a)}{Z_R(z,1/a)}\theta(|z|-z_s),\, z_s = 0.3\text{fm}$$

ratio scheme

self renormalization scheme

$$Z_R(z,1/a,\mu) = \exp\left\{\frac{kz}{a\ln[a\Lambda_{\text{QCD}}]} + m_0 z + \frac{5C_A}{3b_0}\ln\left[\frac{\ln(1/a\Lambda_{\text{QCD}})}{\ln(\mu/\Lambda_{\text{QCD}})}\right] + \frac{1}{2}\ln\left[1+\frac{d}{\ln[a\Lambda_{\text{QCD}}]}\right]^2\right\}$$

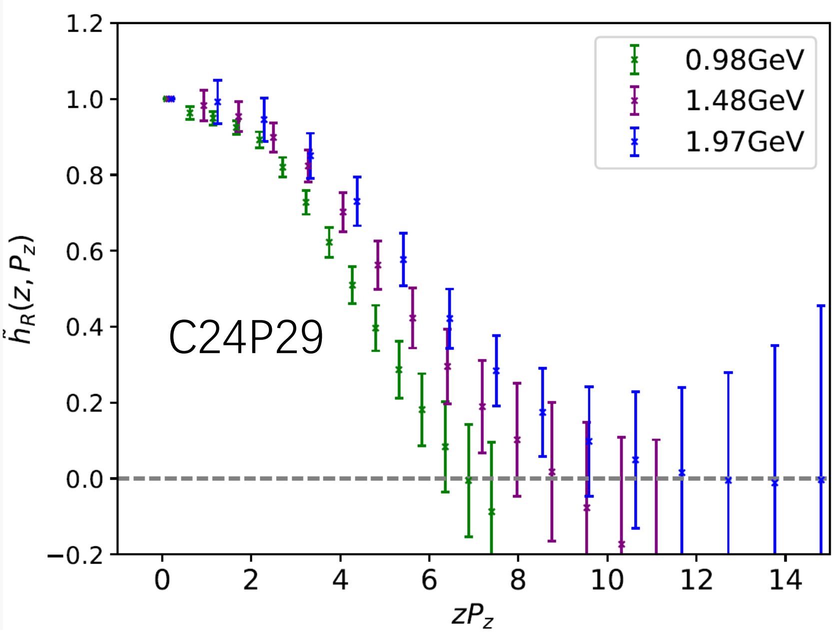


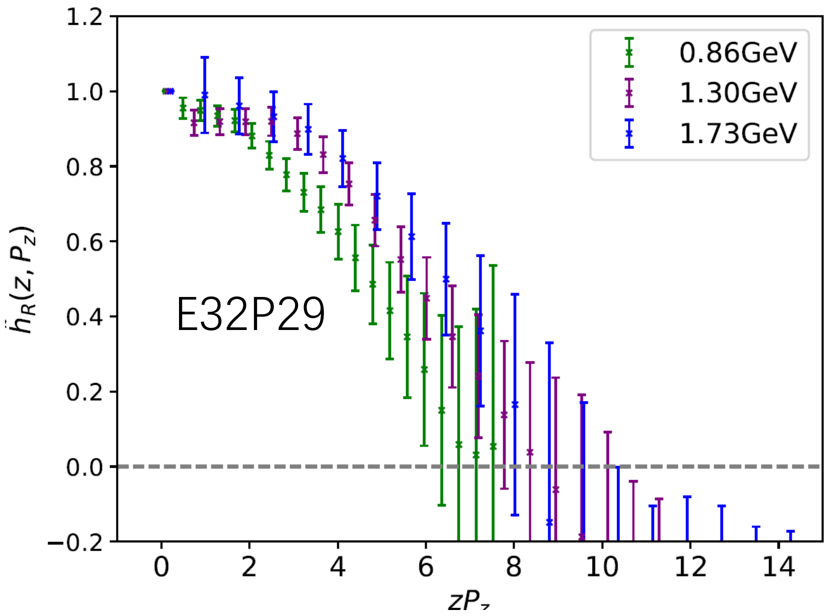
Self-renormalized matrix elements at $P_z = 0$

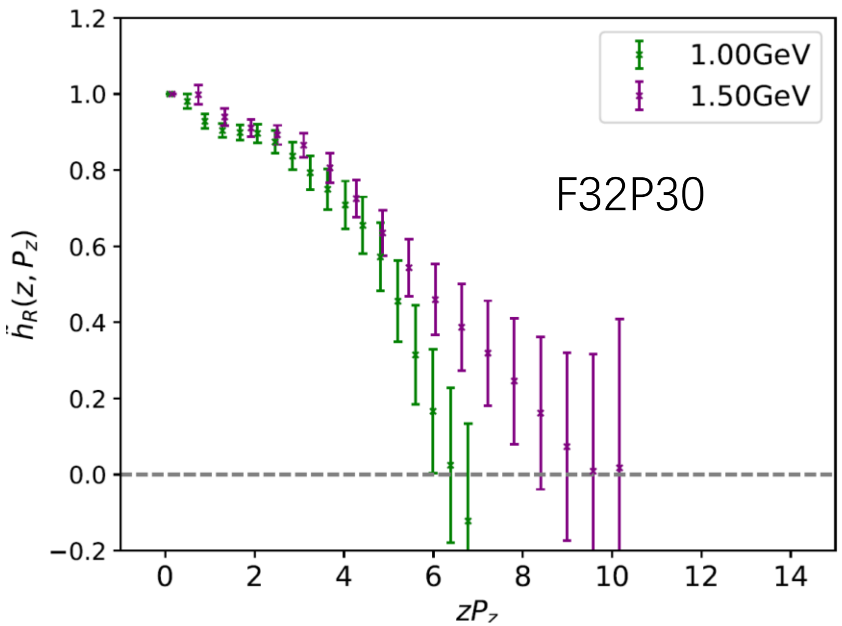
parameter	fit result
k	1.92(1.18)
Λ_{QCD}(GeV)	0.59(0.15)
m_0(GeV)	2.78(1.51)
d	$-0.079(0.403)$
χ^2/d.o.f.	0.09

Renormalization

Renormalized matrix elements:







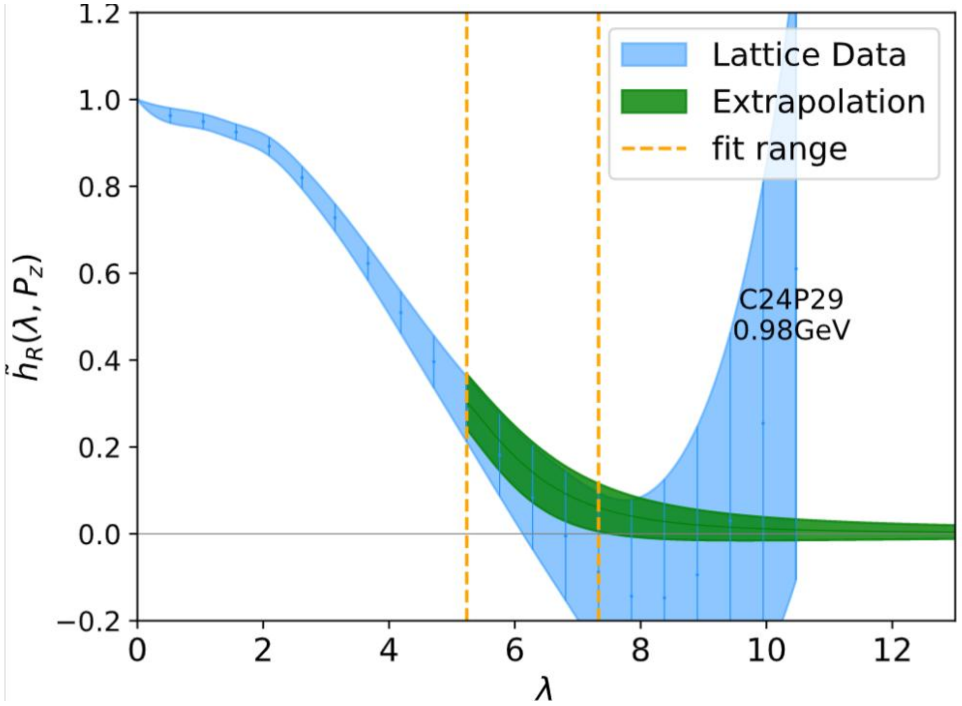
Quasi-PDF

- Large lambda extrapolation

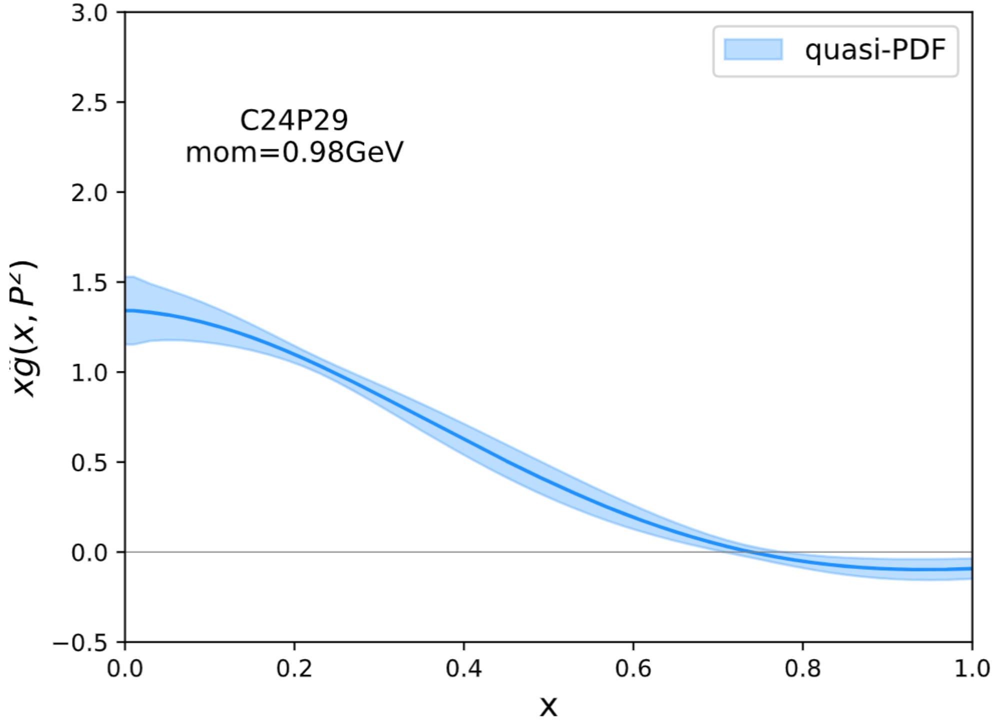
$$\tilde{h}_R(\lambda) = l_1 \lambda^{-a_1} e^{-\lambda/\lambda_0}, \lambda = zP_z$$

- Fourier transform

$$x\tilde{g}(x) = \frac{1}{2\pi}\int \tilde{h}_R(\lambda) e^{ix\lambda} \mathrm{d}\lambda$$



Large lambda extrapolation



Quasi-PDF

Matching & extrapolation

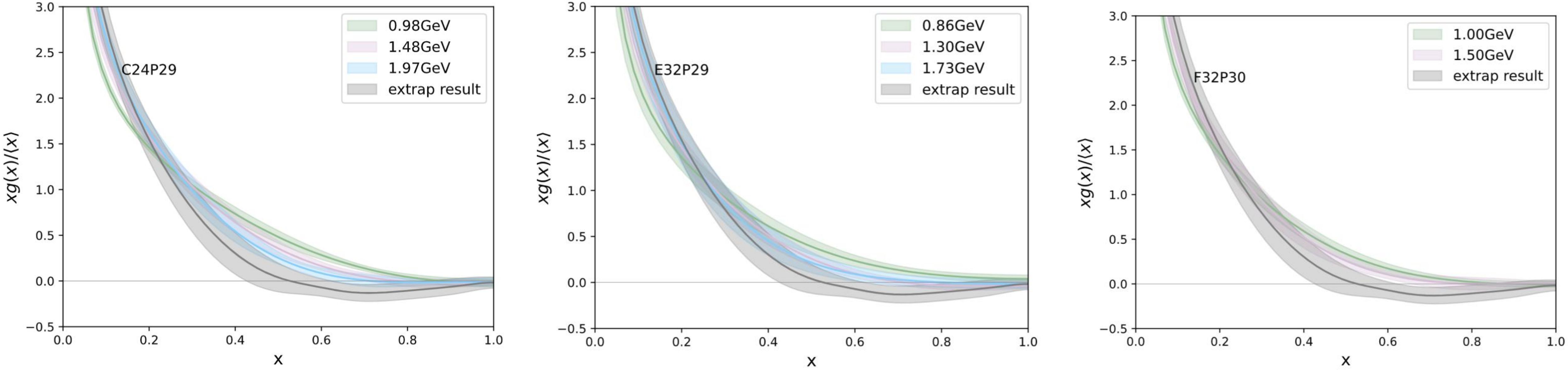
- matching： $$x\tilde{g}(x,P_z) = \int_{-1}^{1}\frac{\mathrm{d}y}{|y|} C_{hybrid}\left(\frac{x}{y},\lambda_s,\frac{\mu}{yP_z}\right) yg(y,\mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{(xP_z)^2},\frac{\Lambda_{QCD}^2}{((1-x)P_z)^2}\right)$$

Quasi-PDF (Lattice)

Matching kernel (Perturbative)

light-cone PDF

- Momentum & continuum extrapolation： $$xg(x,P_z,a) = xg_{inf}(x) + a^2 f(x) + a^2 P_z^2 h(x) + \frac{d(x)}{P_z^2}$$



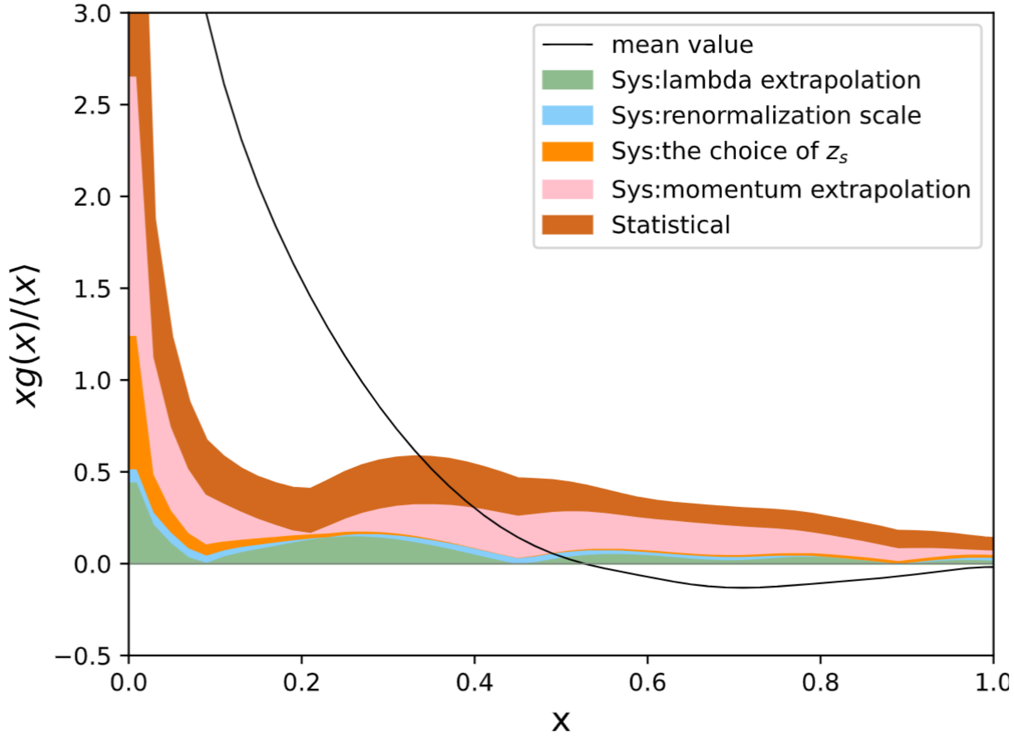
a=0.105fm a=0.0897fm a=0.0775fm

Light-cone PDF

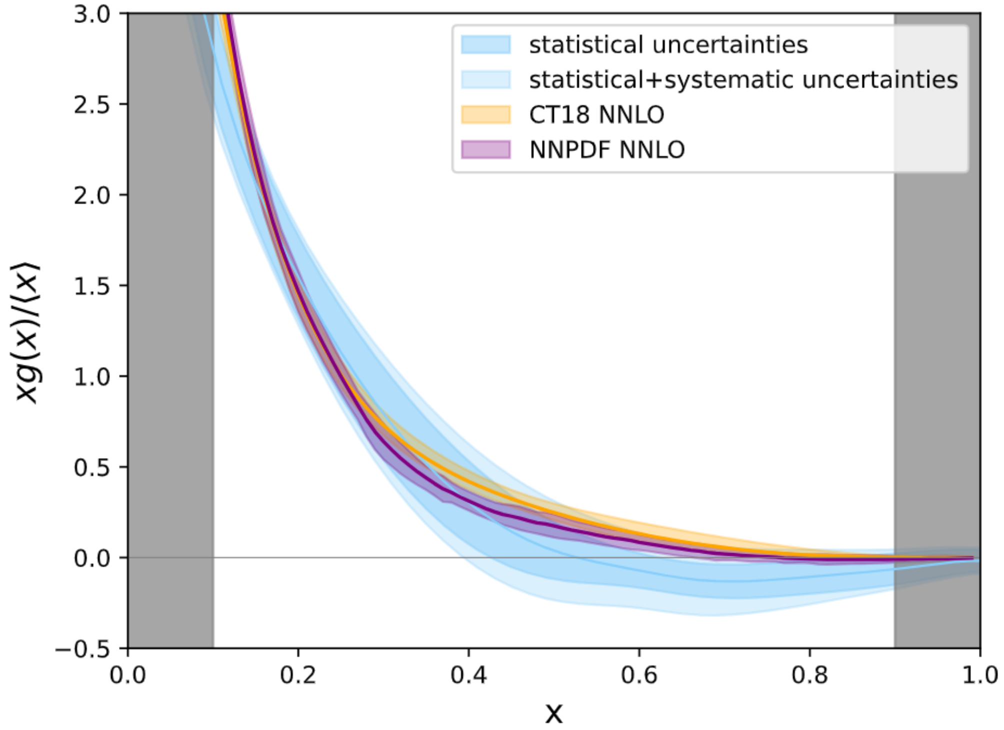
Numerical Result

Systematic uncertainties：

- $\boldsymbol{\lambda}$ extrapolation
- Renormalization scale $\boldsymbol{\mu}$ dependence
- Infinite momentum and continuum extrapolation
- Hybrid scheme z_s dependence



Estimation of statistical & systematic uncertainties



Lattice results in comparison with global fit results

Summary and outlook

- We calculated the unpolarized gluon PDF of the nucleon at $m_\pi \sim 300\mathrm{MeV}$ in LaMET framework. The result agrees well with the global fit result
- To be improved:

- Mixing with quarks is ignored
- Use momentum smearing to further improve the signal of large momentum
- More lattice spacings, physical pion mass.

Thankyou!

Backups

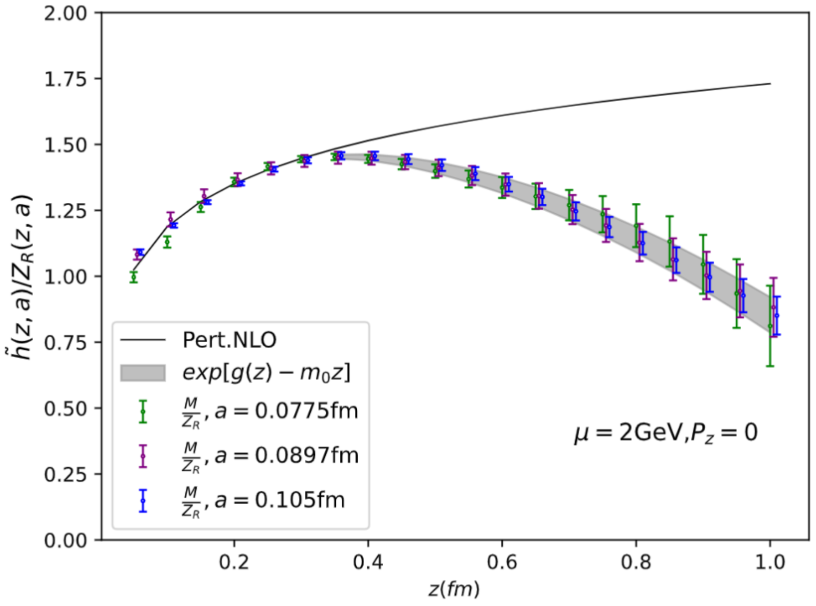
Renormalization

- For gluon the fitting formula is :

$$\ln\tilde{h}_B(z,P_z=0,1/a)=\frac{kz}{a\ln[a\Lambda_{\rm QCD}]}+\cancel{f(z)a^2}+\frac{5C_A}{3b_0}\ln\left[\frac{\ln(1/a\Lambda_{\rm QCD})}{\ln(\mu/\Lambda_{\rm QCD})}\right]+\frac{1}{2}\ln\left[1+\frac{d}{\ln[a\Lambda_{\rm QCD}]}\right]^2 + \begin{cases}\ln[Z_{\overline{\rm MS}}(z,\mu)]+m_0z & \text{if } z_0\le z\le z_1\\ g(z) & \text{if } z_1<z\end{cases},$$

- Self renormalization factor can be given by:

$$Z_R(z,1/a,\mu)=\exp\left\{\frac{kz}{a\ln[a\Lambda_{\rm QCD}]}+m_0z+\cancel{f(z)a^2}+\frac{5C_A}{3b_0}\ln\left[\frac{\ln(1/a\Lambda_{\rm QCD})}{\ln(\mu/\Lambda_{\rm QCD})}\right]+\frac{1}{2}\ln\left[1+\frac{d}{\ln[a\Lambda_{\rm QCD}]}\right]^2\right\},$$



Cg5gt operator

z0=0.15fm,z1=0.3fm

parameter	fit result
k	1.92(1.18)
$\Lambda_{\rm QCD}$(GeV)	0.59(0.15)
m_0(GeV)	2.78(1.51)
d	−0.079(0.403)
χ^2/d.o.f.	0.09

Estimation of systematic and statistical uncertainties

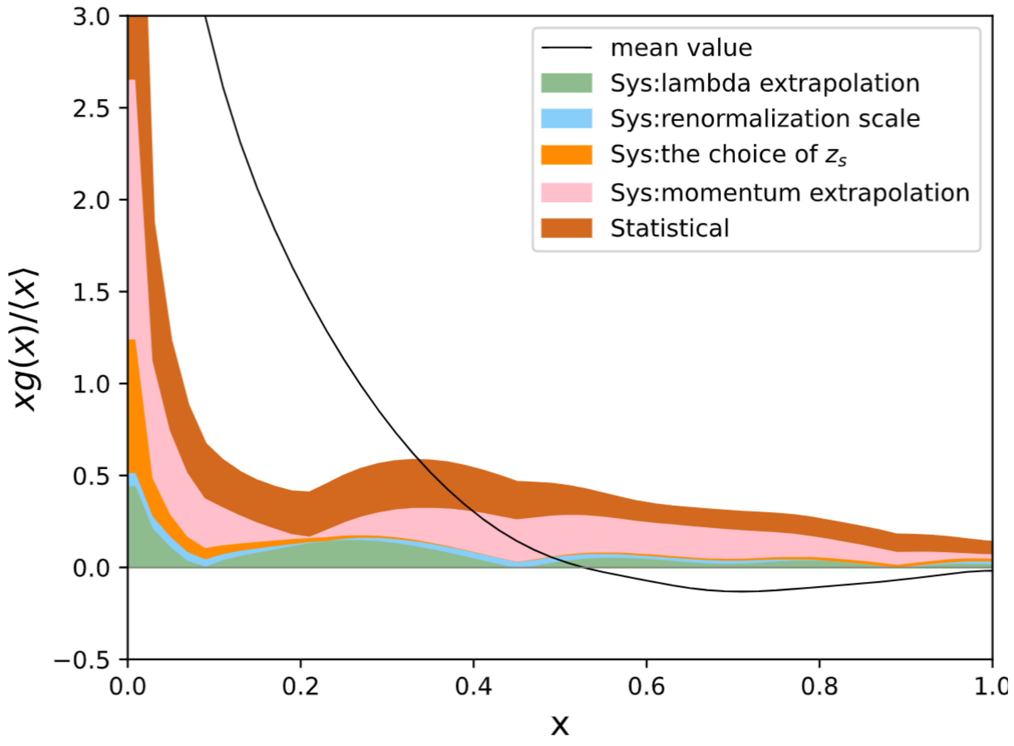
- $\boldsymbol{\lambda}$ **extrapolation:** taking the difference between the results from two fitting ranges

Ensemble	P_z (GeV)	default range (aP_z)	new range (aP_z)
C24P29	0.98	[10, 14]	[8, 14]
	1.48	[7, 11]	[5, 11]
	1.97	[6, 10]	[4, 10]
E32P29	0.86	[12, 16]	[10, 16]
	1.3	[9, 13]	[7, 13]
	1.73	[7, 11]	[5, 11]
F32P30	1.00	[10, 15]	[8, 15]
	1.50	[8, 15]	[7, 15]

- **Renormalization scale $\boldsymbol{\mu}$ dependence:**

$\mu = 2\ \mathrm{GeV}, \alpha_s(\mu) = 0.296$ (default) & $\mu = 4\ \mathrm{GeV}, \alpha_s(\mu) = 0.23$

- **Infinite momentum and continuum-limit extrapolation:** combined extrapolation result & the result at $a = 0.0775\ \mathrm{fm}$; $P_z = 1.5\ \mathrm{GeV}$
- **Choice of $\boldsymbol{z_s}$** : $z_s = 0.3\mathrm{fm}$ (default) & $z_s = 0.2\mathrm{fm}$



Matching kernel

$$C_{\mathrm{ratio}}\left(\xi,\frac{\mu}{yP_z}\right)=\delta\left(1-\xi\right)+\frac{\alpha_s C_A}{2\pi}\begin{cases}\left[\frac{2(1-\xi+\xi^2)^2}{1-\xi}\ln\frac{\xi}{\xi-1}+\frac{11-28\xi+18\xi^2-12\xi^3}{6(1-\xi)}\right]_+ & \xi>1\\ \left[\frac{2(1-\xi+\xi^2)^2}{1-\xi}\left(-\ln\frac{\mu^2}{4y^2P_z^2}+\ln(\xi(1-\xi))\right)-\frac{15-56\xi+102\xi^2-96\xi^3+48\xi^4}{6(1-\xi)}\right]_+ & 0<\xi<1\\ \left[\frac{-2(1-\xi+\xi^2)^2}{1-\xi}\ln\frac{\xi}{\xi-1}-\frac{11-28\xi+18\xi^2-12\xi^3}{6(1-\xi)}\right]_+ & \xi<0,\end{cases}$$

$$C_{\mathrm{hybrid}}(\xi,\lambda_s,\frac{\mu}{yP_z})=C_{\mathrm{ratio}}\left(\xi,\frac{\mu}{yP_z}\right)+\frac{\alpha_s C_A}{2\pi}\frac{5}{6}\left(-\frac{1}{|1-\xi|}+\frac{2\mathrm{Si}((1-\xi)|y|\lambda_s)}{\pi(1-\xi)}\right)_+$$