η and η' mesons from $N_f = 2 + 1$ lattice QCD at physical point using topological charge operator

Nan Wang
South China Normal University

Quark model : η and η'

For exact SU(3) flavor symmetry one expects:

Flavor octet state: $|\eta_8\rangle = \dfrac{1}{\sqrt{6}}(|\bar{u}u\rangle + |\bar{d}d\rangle - 2|\bar{s}s\rangle)$

Flavor singlet state: $|\eta_1\rangle = \dfrac{1}{\sqrt{3}}(|\bar{u}u\rangle + |\bar{d}d\rangle + |\bar{s}s\rangle)$

However, SU(3) flavor symmetry is broken by large $m_s \gg m_u \approx m_d \equiv m_l$.

Physical η, η' states are not flavor eigenstates but mixtures,

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = M \begin{pmatrix} |\eta_8\rangle \\ |\eta_1\rangle \end{pmatrix}$$

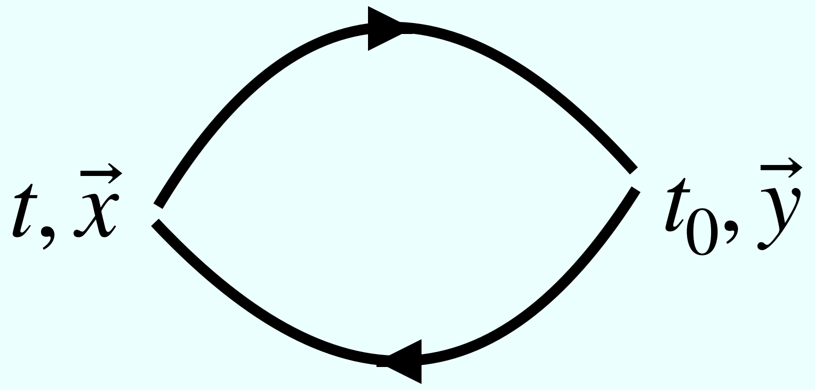
M is the mixing matrix of η and η'.

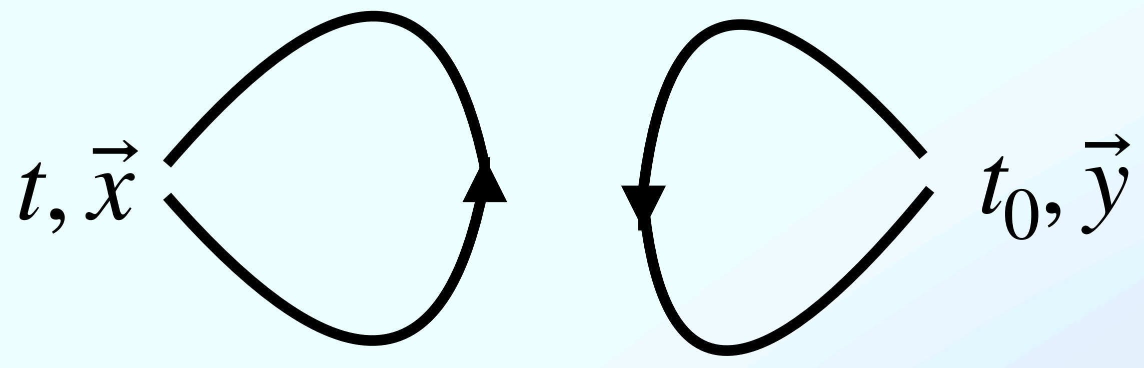
η and η' on the lattice

❖ Generally, η and η' are calculated using fermion fields on the lattice.

$$\eta(x) = \sum_{i}^{N_f} \bar{\psi}_i(x)\gamma_5\psi_i(x) \qquad C^{\eta}_{\vec{p}}(t, t_0) = \langle \tilde{\eta}(t, \vec{p})(\tilde{\eta}(t_0, \vec{p}))^{\dagger} \rangle$$

$$C^{\eta}_{\vec{p}}(t, t_0) = \frac{1}{V_3} \sum_{\vec{x},\vec{y}} e^{i\vec{p}(\vec{y}-\vec{x})} \langle \mathcal{Q}^{-1}_{\beta,b,t_0,\vec{y};\alpha,a,t,\vec{x}} \mathcal{Q}^{-1}_{\alpha,a,t,\vec{x};\beta,b,t_0,\vec{y}} - \mathcal{Q}^{-1}_{\beta,b,t_0,\vec{y};\alpha,b,t_0,\vec{y}} \mathcal{Q}^{-1}_{\alpha,a,t,\vec{x};\alpha,a,t,\vec{x}} \rangle$$





Disconnected & Require high statistics

❖ Using the topological charge operator avoids the difficulties of handling disconnected insertions.

$\eta \qquad I^G(J^{PC}) = 0^+(0^{-+})$

$\eta' \qquad I^G(J^{PC}) = 0^+(0^{-+})$

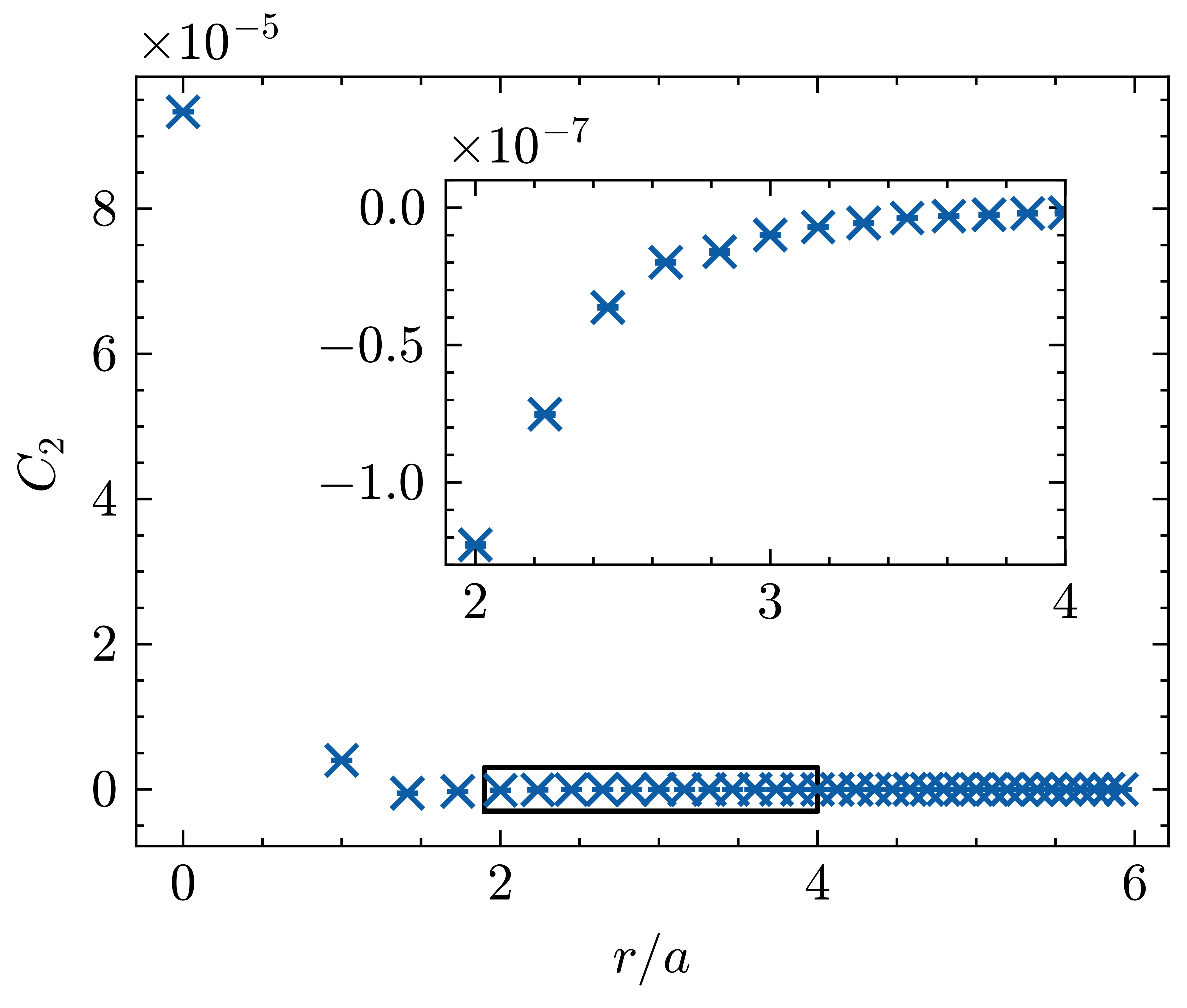
$$F_{\mu\nu}\tilde{F}^{\mu\nu} \propto \vec{E} \cdot \vec{B}$$

$$q(x) = \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}F^{\mu\nu}(x)F^{\rho\sigma}(x)$$

η and η' on the lattice

Constructing two-point correlation functions using topological charge density operators

$$C_2(r) = \frac{1}{VN_r} \sum_{y \in \{|y-x|=r|\}} \sum_x \langle q(y)\ q(x) \rangle$$

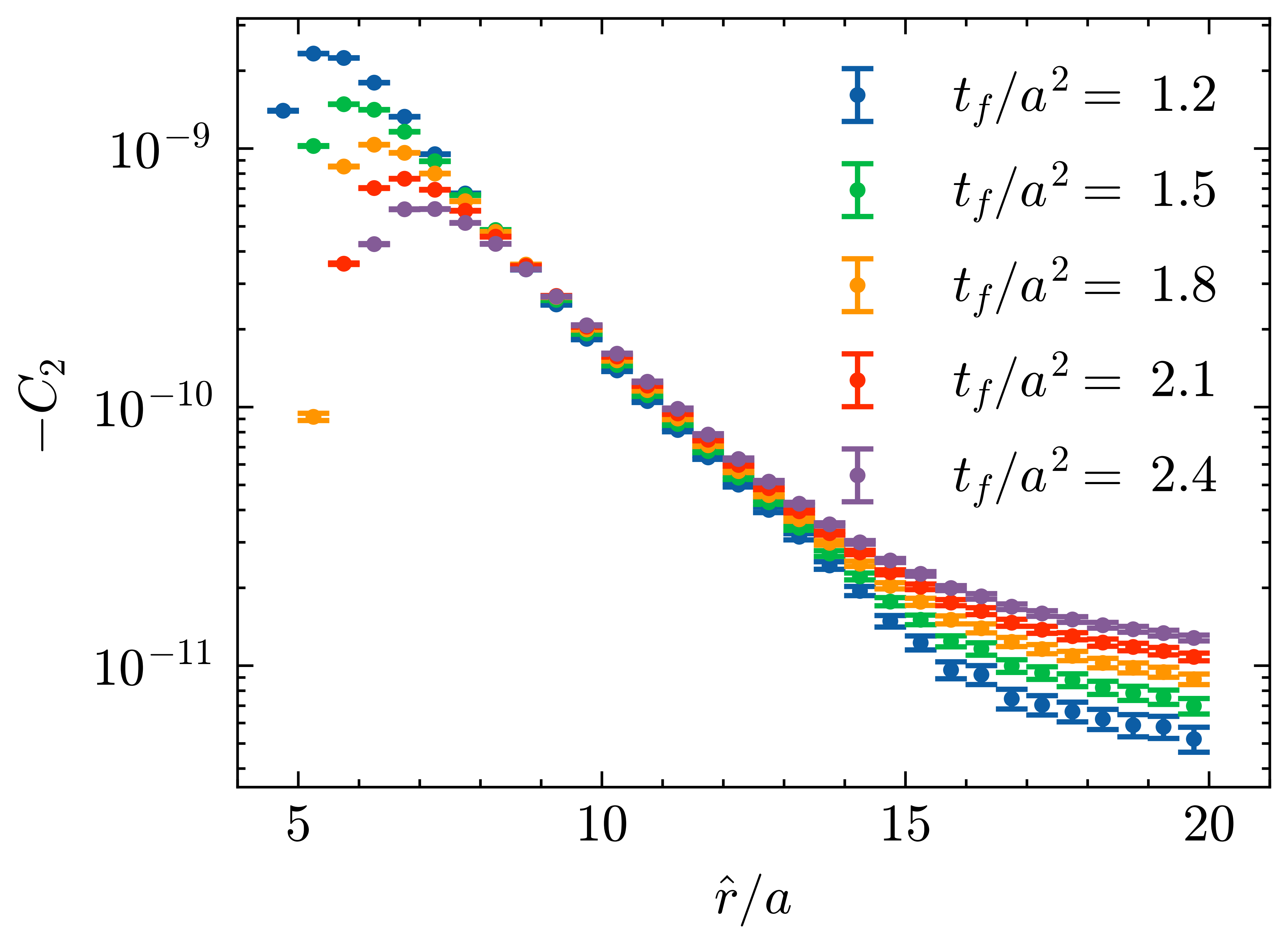


- ❖ The reflection positivity and the fact that the topological charge is reflection-odd ensure that the correlation functions are negative at non-zero distance.
- ❖ The positive contact contribution is a delta function at $r = 0$. However, the topological charges are not locally defined on the lattice, the positive contact contribution is smeared to cover several lattice spacings.
- ❖ The data points in the negative tail contain important physical information.

I. Horvath, et al. Phys. Rev. D 67, 011501(2003)
I. Horvath, et al. Phys. Lett. B 617, 49 (2005)

η and η' on the lattice

- ❖Bin data in $\delta r = 0.5a$ intervals to reduce strongly correlated between data points
- ❖Use Wilson flow to smear gauge fields for improved signal-to-noise ratio



Ensemble details

Symbol	L/T	Mpi (MeV)	M_K(MeV)	a (fm)	N_{cfg}
48I	48/96	139	499	0.1141(2)	356
64I	64/128	139	508	0.0837(2)	330

R. Arthur et al., PRD87, 094514 (2013)
T. Blum et al., PRD93, 074505 (2016)
P. Boyle et al., PRD 93, 054502 (2016)

- ❖ We employ two 2+1-flavor gauge ensembles of domain wall fermions generated by the RBC/UKQCD collaboration.
- ❖ Both of them are at the physical pion mass point.
- ❖ The discretization effects of our results are carefully estimated.

Effective mass

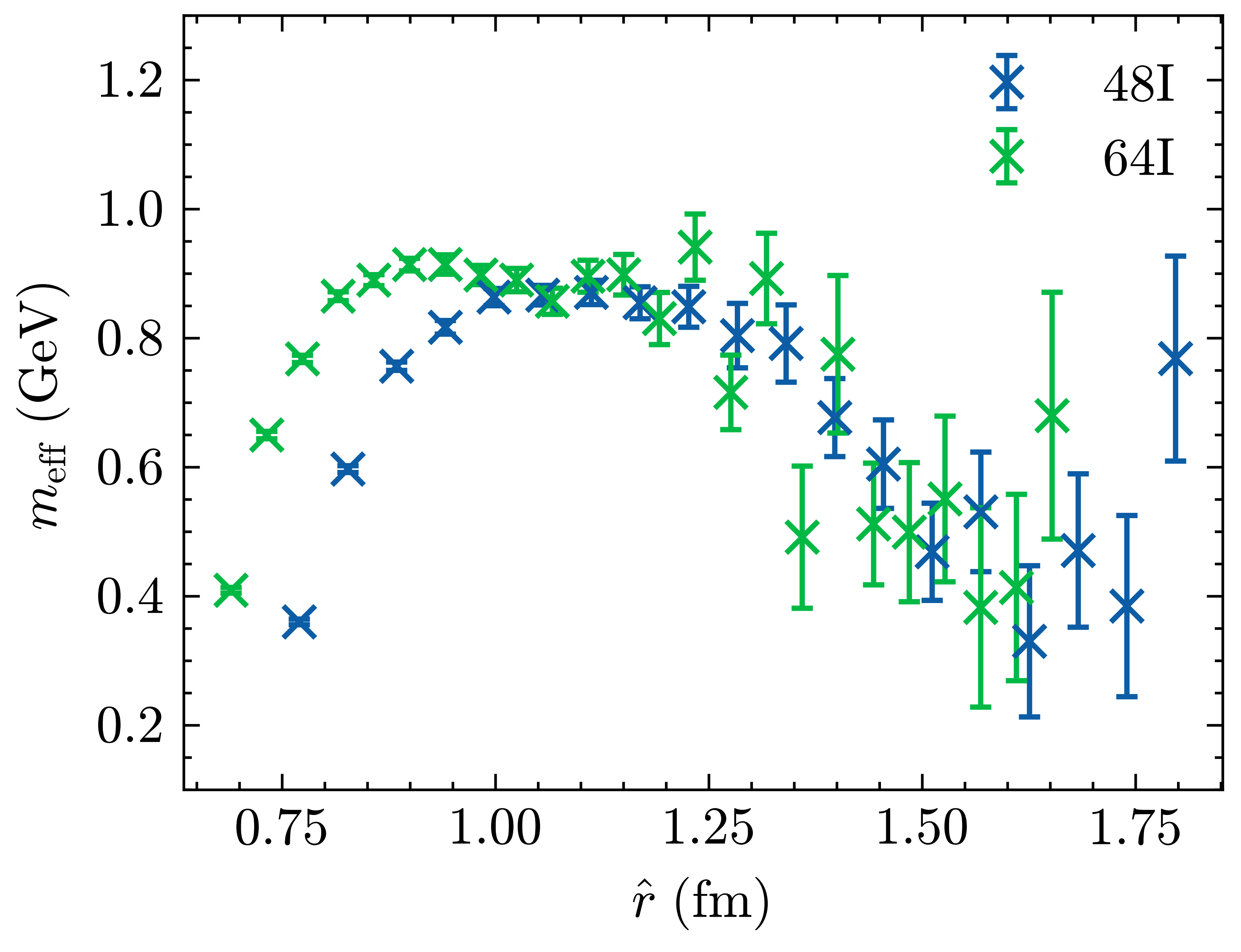
The correlation functions of topological charge density operators can be expressed as

$$C_2(\hat{r}) = \sum_n A_n \frac{m_n}{4\pi^2 \hat{r}} K_1(m_n \hat{r})$$

K_1 is the modified Bessel function of the second kind.

The effective mass at each $\hat{r}$ is obtained by solving the following equation numerically.

$$\frac{K_1(m_{\mathrm{eff}}\hat{r})}{K_1\left((m_{\mathrm{eff}}(\hat{r}-\delta_r)\right)} \frac{\hat{r}-\delta_r}{\hat{r}} = \frac{C_2(\hat{r})}{C_2(\hat{r}-\delta_r)}$$



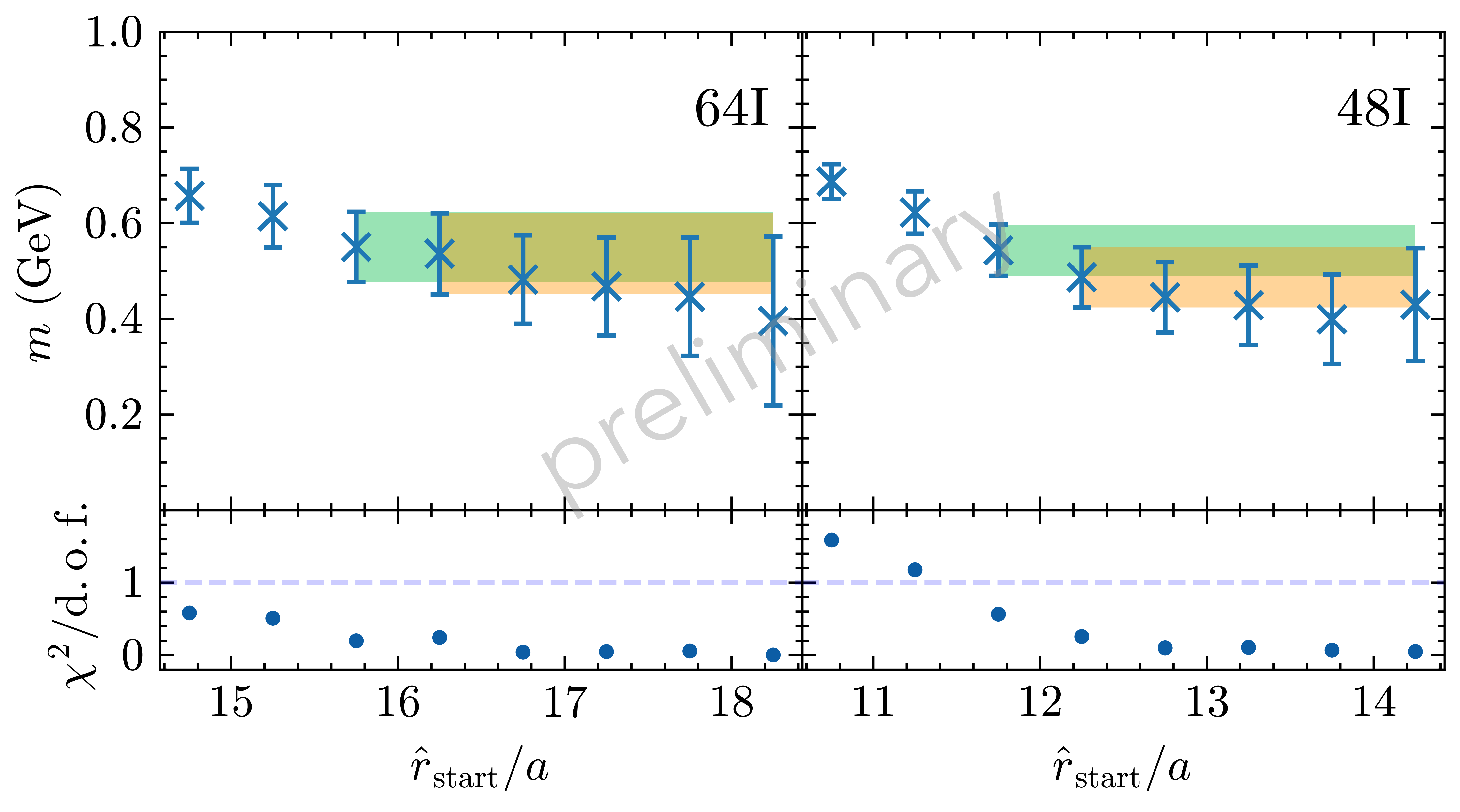
When $\hat{r}$ is larger than $\sim$1.4 fm, the effective masses are close to the η mass.

And in the middle $\hat{r}$ region, the effective masses are close to the η' mass.

Fittings and Results: mass of η

From two-point correlation functions to extracting the mass of η and η':

Single state fitting: $$C_2(\hat{r}) = A_\eta \frac{m_\eta}{4\pi^2 \hat{r}} K_1(m_\eta \hat{r})$$



$m_\eta^{64\mathrm{I}} = 0.550(73)$ GeV

$m_\eta^{48\mathrm{I}} = 0.543(53)$ GeV

Consistent within errors

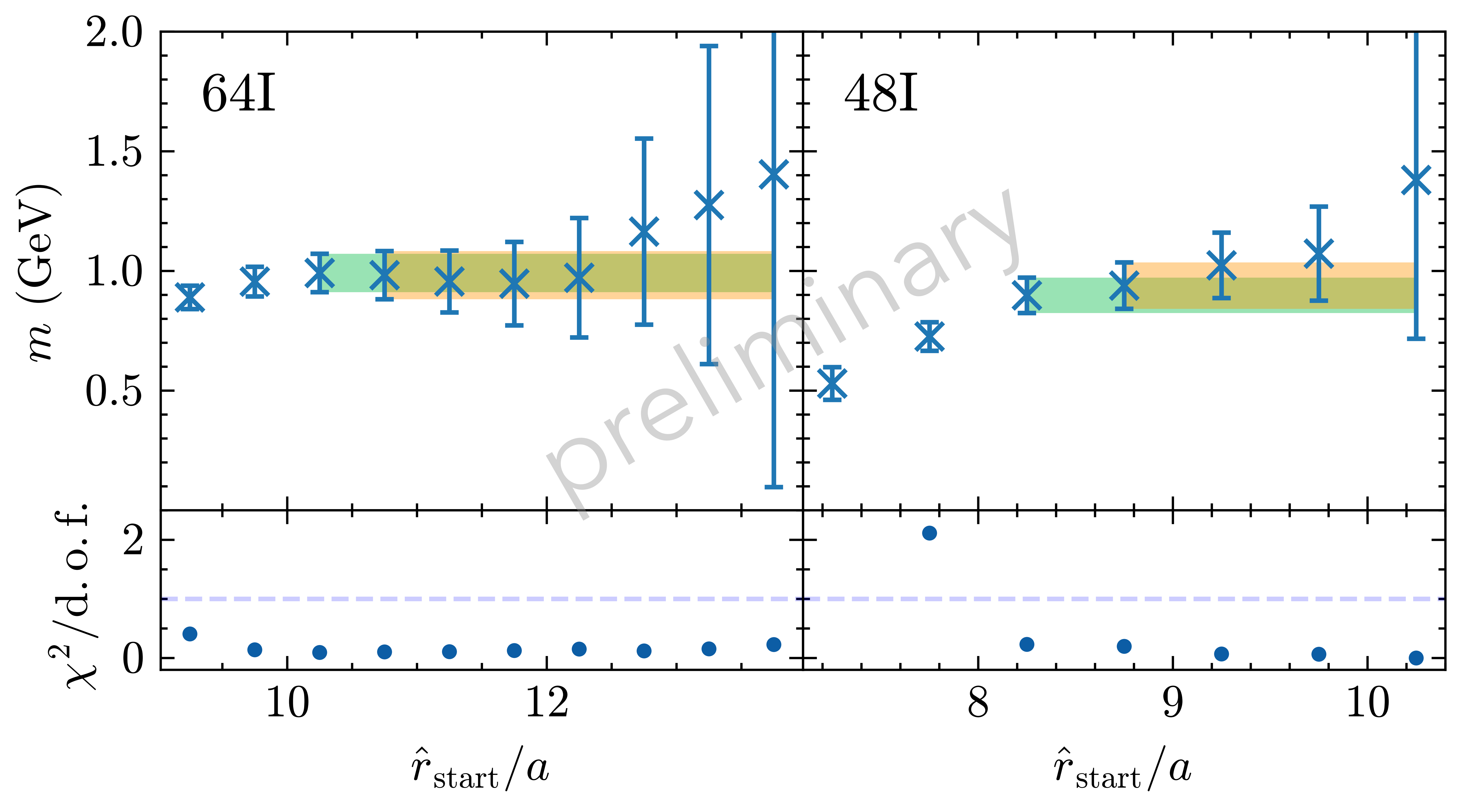
Our final results represent an average of the results from the two lattices.

$m_\eta = 0.546(43)(5)$ GeV

Fittings and Results: mass of η'

Double states fitting: $$C_2(\hat{r}) = A_{\eta'}\frac{m_{\eta'}}{4\pi^2\hat{r}}K_1\left(m_{\eta'}\hat{r}\right) + A_{\eta}\frac{m_{\eta}}{4\pi^2\hat{r}}K_1\left(m_{\eta}\hat{r}\right)$$

Use previously fitted η mass as priors in these fits



$m_{\eta'}^{64\mathrm{I}} = 0.992(80)$ GeV

$m_{\eta'}^{48\mathrm{I}} = 0.898(74)$ GeV

Following the same argument and strategy for the η case.

$m_{\eta'} = 0.941(54)(50)$ GeV

Fittings and Results: mixing angle

Double states fitting: $C_2(\hat{r}) = A_{\eta'}\frac{m_{\eta'}}{4\pi^2\hat{r}}K_1\left(m_{\eta'}\hat{r}\right) + A_{\eta}\frac{m_{\eta}}{4\pi^2\hat{r}}K_1\left(m_{\eta}\hat{r}\right)$

In the leading-order, the octet–singlet eigenstates (η_8, η_1) are related to the physical mass eigenstates (η, η') by:

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\eta_8\rangle \\ |\eta_1\rangle \end{pmatrix}$$

θ is the $\eta - \eta'$ mixing angle in the octet–singlet basis.

Modern treatment of the mixing is parametrized by the matrix elements using two mixing angles as:

$$\begin{cases} F^1_{\eta'} = F^1\cos\theta_1, F^8_{\eta'} = F^8\sin\theta_8 \\ F^1_{\eta} = -F^1\sin\theta_1, F^8_{\eta} = F^8\cos\theta_8 \end{cases}$$

L. Gan, et al. Phys.Rept. 945 (2022)

$$\begin{cases} F^1_{\eta/\eta'} = \langle 0|O_1|\eta/\eta'\rangle \\ F^8_{\eta/\eta'} = \langle 0|O_8|\eta/\eta'\rangle \end{cases}$$

O_8 and O_1 being the flavor octet and singlet operators.

Fittings and Results: mixing angle

Double states fitting: $C_2(\hat{r}) = A_{\eta'}\frac{m_{\eta'}}{4\pi^2\hat{r}}K_1\left(m_{\eta'}\hat{r}\right) + A_{\eta}\frac{m_{\eta}}{4\pi^2\hat{r}}K_1\left(m_{\eta}\hat{r}\right)$

Modern treatment of the mixing is parametrized by the matrix elements using two mixing angles as:

$$\begin{cases} F^1_{\eta'} = F^1 \cos\theta_1, F^8_{\eta'} = F^8 \sin\theta_8 \\ F^1_{\eta} = -F^1 \sin\theta_1, F^8_{\eta} = F^8 \cos\theta_8 \end{cases}$$

L. Gan, et al. Phys.Rept. 945 (2022)

$$\begin{cases} F^1_{\eta/\eta'} = \langle 0 | O_1 | \eta/\eta' \rangle \\ F^8_{\eta/\eta'} = \langle 0 | O_8 | \eta/\eta' \rangle \end{cases}$$

O_8 and O_1 being the flavor octet and singlet operators.

In our case, the topological charge operator is a flavor single operator, we use $O_1 = q$, and thus we arrive at:

$$\tan\theta_1 = -\frac{\langle 0 | q | \eta \rangle}{\langle 0 | q | \eta' \rangle}$$

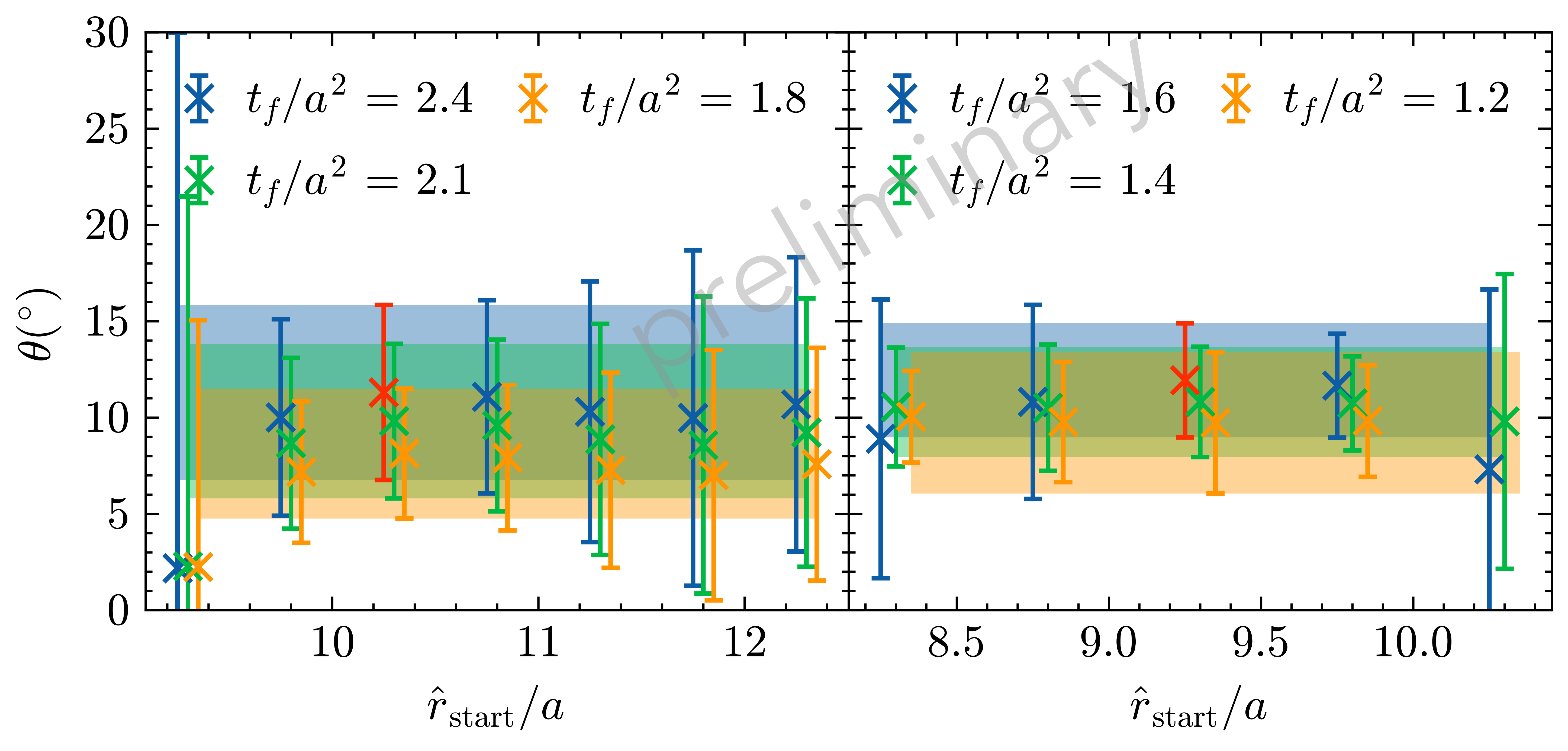
In the two-state fits:

$$A_{\eta/\eta'} = \left| \left\langle 0 \left| q \right| \eta/\eta' \right\rangle \right|^2$$

Mixing angle: $\theta_1 = -\arctan\sqrt{A_\eta/A_{\eta'}}$

Fittings and Results: mixing angle

Double states fitting: $C_2(\hat{r}) = A_{\eta'} \frac{m_{\eta'}}{4\pi^2 \hat{r}} K_1\left(m_{\eta'}\hat{r}\right) + A_{\eta} \frac{m_{\eta}}{4\pi^2 \hat{r}} K_1\left(m_{\eta}\hat{r}\right)$



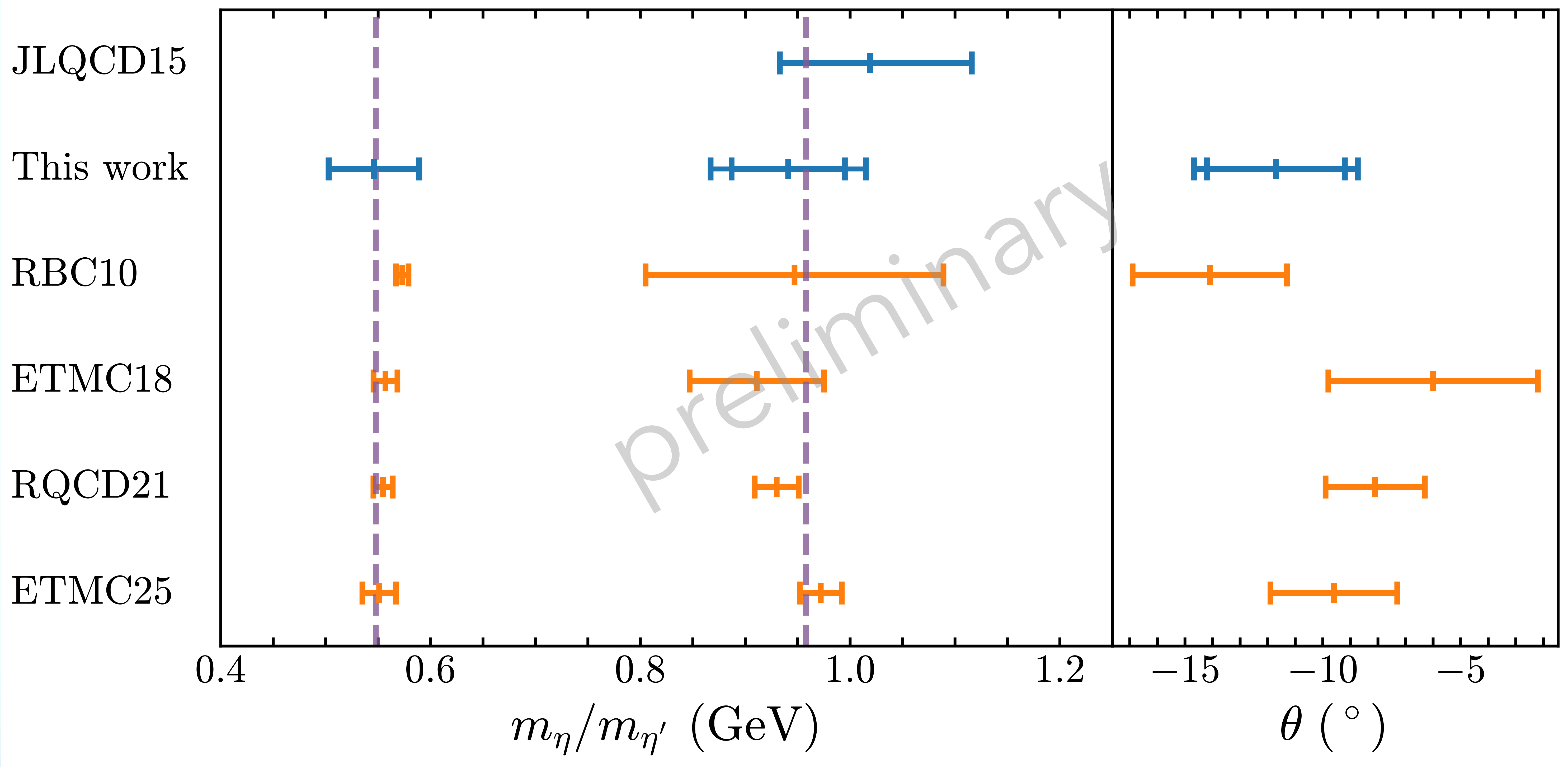
$\theta_1^{64\mathrm{I}} = -11.3(4.5)^\circ$

$\theta_1^{48\mathrm{I}} = -11.9(3.0)^\circ$

Consideration of continuum limit, and estimation of system uncertainty

$\theta_1 = -11.7(2.5)(1.6)^\circ$

Results



$$m_\eta = 0.546(43)(5)\ \text{GeV}$$

$$m_{\eta'} = 0.941(54)(50)\ \text{GeV}$$

$$\theta_1 = -11.7(2.5)(1.6)^\circ$$

H. Fukaya, et al., Phys. Rev. D 92, 111501 (2015)
N. H. Christ, et al., Phys. Rev. Lett. 105, 241601 (2010)
K. Ottnad, et al., Phys. Rev. D 97, 054508 (2018)
G. S. Bali, et al., JHEP 08, 137(2021)
K. Ottnad, et al., arXiv:2503.09895 [hep-lat]

- ❖ We first extract both the η and η' masses and also the mixing angle θ_1 by using topological charge operators.
- ❖ The topological charge operator is a flavor-singlet operator which couples primarily to the η' state ($|A_\eta/A_{\eta'}| = \tan^2\theta_1 \sim 0.036$)

Summary and outlook

❖ For the first time, based on gauge configurations with the physical pion mass, we extract both the η and η' masses and also the mixing angle θ_1 by using topological charge operators.

❖ The systematic uncertainties are carefully considered.

❖ Compared with quark bilinear operators, the topological charge operator achieves high precision while avoiding complex computations and reducing computational resource consumption.

Summary and outlook

- For the first time, based on gauge configurations with the physical pion mass, we extract both the η and η' masses and also the mixing angle θ_1 by using topological charge operators.
- The systematic uncertainties are carefully considered.
- Compared with quark bilinear operators, the topological charge operator achieves high precision while avoiding complex computations and reducing computational resource consumption.

Thank you!