Low-energy interactions of doubly charmed baryons and Goldstone bosons from lattice QCD

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Outline

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- Simulation in lattice QCD

Lattice setup

- Ensembles
- Distillation quark smearing

Results

- Energy levels
- Scattering lengths

Summary and Outlook

Doubly charmed baryons

Observation of doubly charmed baryons

- SELEX M. Mattson and others, Phys. Rev. Lett. 89, 112001 (2002).
- FOCUS S. P. Ratti, Nucl. Phys. B Proc. Suppl. 115, 33 (2003).
- Belle R. Chistov and others, Phys. Rev. Lett. 97, 162001 (2006).
- BABAR B. Aubert and others, Phys. Rev. D 74, 011103 (2006).
- LHCb R. Aaij and others, Phys. Rev. Lett. 119, 112001 (2017).

$$m_{\Xi_{cc}^{++}} = 3621.40 \pm 0.72(\text{stat.}) \pm 0.27(\text{syst.}) \pm 0.14(\Lambda_c^+)\ \text{MeV}/c^2$$

Observations in other decay channels

- $\Xi_{cc}^{++} \to \Xi_c^+\pi^+$ R. Aaij and others, Phys. Rev. Lett. **121**, 162002 (2018).
- $\Xi_{cc}^{++} \to \Xi_c^{\prime +}\pi^+$ R. Aaij and others, JHEP **05**, 038 (2022).
- $\Xi_{cc}^{++} \to \Xi_c^0\pi^+\pi^+$ R. Aaij and others, arXiv:2504.05063 [hep-ex] (2025).

More precise measurements

- Mass: R. Aaij and others, JHEP 02, 049 (2020)
- Lifetime: R. Aaij and others, Phys. Rev. Lett. **121**, 052002 (2018).
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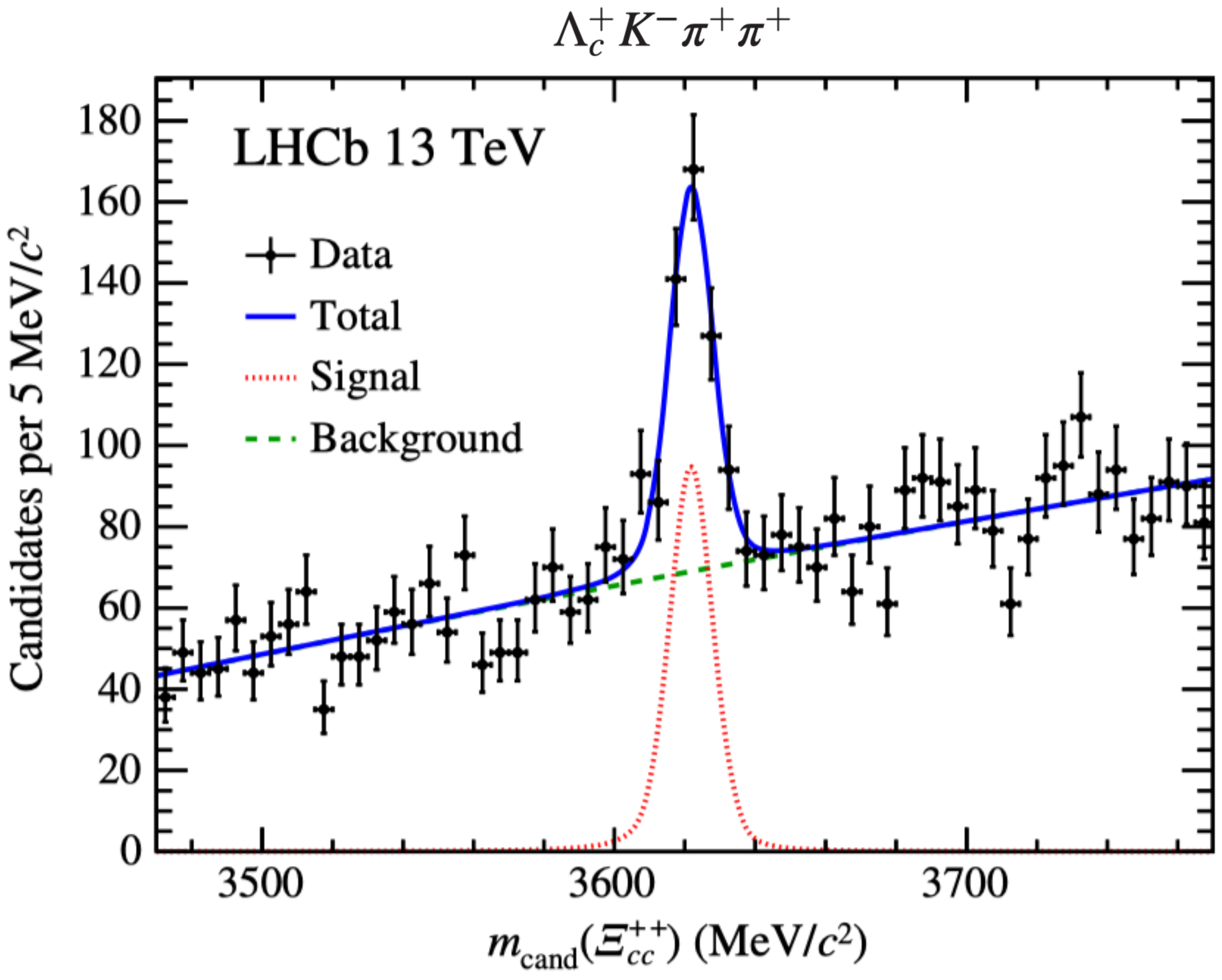

R. Aaij and others, Phys. Rev. Lett. 119, 112001 (2017)

Recent searches for Ξ_{cc}^+ and Ω_{cc}^+ at LHCb

- R. Aaij and others, JHEP **12**, 107 (2021).
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Doubly charmed baryons

Lattice QCD calculations

Mass

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- C. Alexandrou and C. Kallidonis, Phys. Rev. D **96**, 034511 (2017).
- S. Mondal, M. Padmanath, and N. Mathur, EPJ Web Conf. **175**, 05021 (2018).
- K. U. Can, H. Bahtiyar, G. Erkol, P. Gubler, M. Oka, and T. T. Takahashi, JPS Conf. Proc. **26**, 022028 (2019).
- N. Mathur and M. Padmanath, Phys. Rev. D 99, 031501 (2019).
- M. Padmanath, Heavy Baryon Spectroscopy from Lattice QCD (2019).
- H. Bahtiyar, K. U. Can, G. Erkol, P. Gubler, M. Oka, and T. T. Takahashi, Phys. Rev. D **102**, 054513 (2020).
- C. Alexandrou, S. Bacchio, G. Christou, and J. Finkenrath, Phys. Rev. D **108**, 094510 (2023).

Other properties (electromagnetic form factors, radiative transitions, …)

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- K. U. Can, G. Erkol, B. Isildak, M. Oka, and T. T. Takahashi, JHEP **05**, 125 (2014).
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Doubly charmed baryons

Chiral perturbation theory

- Z.-H. Guo, Phys. Rev. D 96, 074004 (2017).
- Z.-F. Sun and M. J. Vicente Vacas, Phys. Rev. D 93, 094002 (2016).
- A. N. Hiller Blin, Z.-F. Sun, and M. J. Vicente Vacas, Phys. Rev. D 98, 054025 (2018).
- M.-J. Yan, X.-H. Liu, S. Gonzàlez-Solís, F.-K. Guo, C. Hanhart, U.-G. Meißner, and B.-S. Zou, Phys. Rev. D 98, 091502 (2018).
- M.-Z. Liu, Y. Xiao, and L.-S. Geng, Phys. Rev. D 98, 014040 (2018).
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- M.-J. Yan, X.-H. Liu, S. Gonzàlez-Solís, F.-K. Guo, C. Hanhart, U.-G. Meißner, and B.-S. Zou, Phys. Rev. D 98, 091502 (2018).
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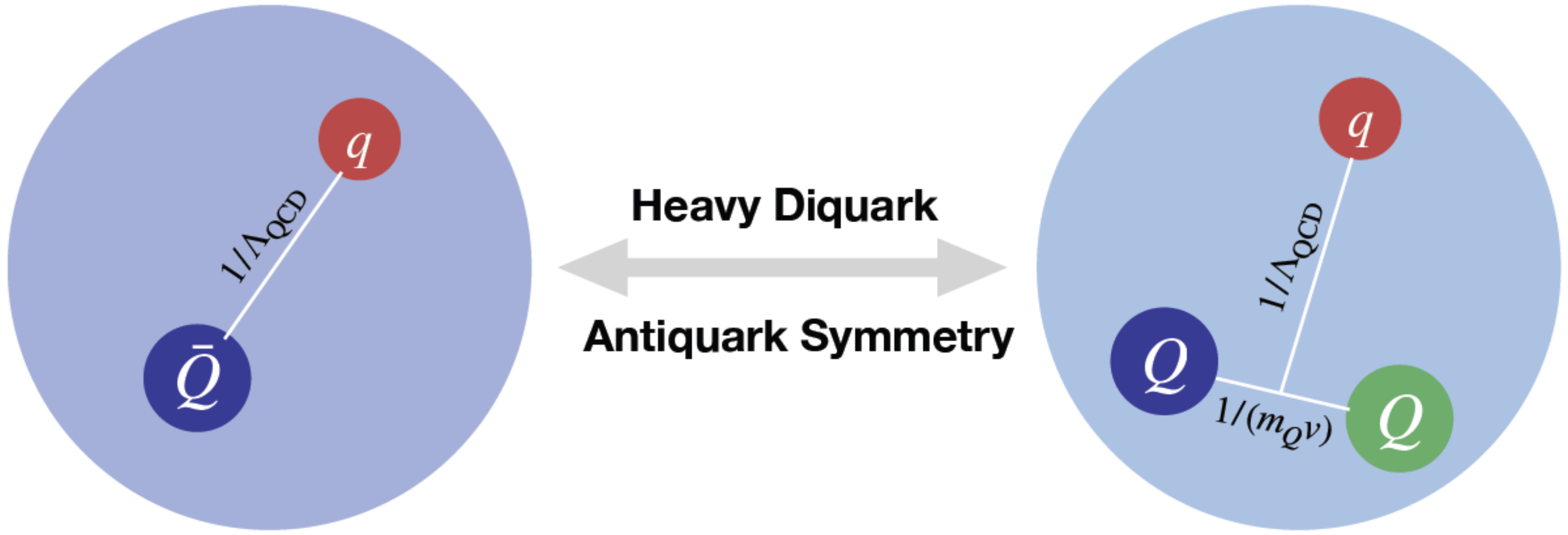
- Z.-R. Liang, P.-C. Qiu, and D.-L. Yao, JHEP 07, 124 (2023).
- ……

Baryon chiral perturbation theory (BChPT)

$$\mathcal{L}^{(1)}_{\psi\phi} = \bar{\psi}\left(i\not{D} - m\right)\psi + \frac{g}{2}\bar{\psi}\not{u}\gamma_5\psi\ ,$$

$$\begin{aligned}
\mathcal{L}^{(2)}_{\psi\phi} = {} & b_1\bar{\psi}\langle\chi_+\rangle\psi + b_2\bar{\psi}\widetilde{\chi}_+\psi + b_3\bar{\psi}u^2\psi + b_4\bar{\psi}\left\langle u^2\right\rangle\psi + \frac{b_5}{m^2}\bar{\psi}\left(\{u^\mu, u^\nu\}D_{\mu\nu} + H.c.\right)\psi \\
& + \frac{b_6}{m^2}\bar{\psi}\left(\langle u^\mu u^\nu\rangle D_{\mu\nu} + H.c.\right)\psi + ib_7\bar{\psi}\left[u^\mu, u^\nu\right]\sigma_{\mu\nu}\psi\ ,
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}^{(3)}_{\psi\phi} = {} & ic_{11}\bar{\psi}\left[u_\mu, h^{\mu\nu}\right]\gamma_\nu\psi + \frac{c_{12}}{m^2}\bar{\psi}\left(i\left[u^\mu, h^{\nu\rho}\right]\gamma_\mu D_{\nu\rho} + H.c.\right)\psi + \frac{c_{13}}{m}\bar{\psi}\left(i\left\{u^\mu, h^{\nu\rho}\right\}\right. \\
& \left.\times\sigma_{\mu\nu}D_\rho + H.c.\right)\psi + \frac{c_{14}}{m}\bar{\psi}\left(i\sigma_{\mu\nu}\left\langle u^\mu h^{\nu\rho}\right\rangle D_\rho + H.c.\right)\psi + c_{15}\bar{\psi}\left\{u^\mu, \widetilde{\chi}_+\right\}\gamma_5\gamma_\mu\psi \\
& + c_{16}\bar{\psi}u^\mu\gamma_5\gamma_\mu\langle\chi_+\rangle\psi + c_{17}\bar{\psi}\gamma_5\gamma_\mu\left\langle u^\mu\widetilde{\chi}_+\right\rangle\psi + ic_{18}\bar{\psi}\gamma_5\gamma_\mu\left[D^\mu, \widetilde{\chi}_-\right]\psi \\
& + ic_{19}\bar{\psi}\gamma_5\gamma_\mu\left\langle\left[D^\mu, \chi_-\right]\right\rangle\psi + c_{20}\bar{\psi}\left[\widetilde{\chi}_-, u^\mu\right]\gamma_\mu\psi\ .
\end{aligned}$$



Doubly charmed baryons

Scattering length in BChPT

Z.-R. Liang, P.-C. Qiu, and D.-L. Yao, JHEP 07, 124 (2023).

$$a_{0+}^{(S,I)} = \frac{m_\psi}{4\pi\left(m_\psi + m_\phi\right)}\left\{\left[A^{(S,I)}(s,0)\right]_{\mathbf{q}^2=0} + m_\phi\left[B^{(S,I)}(s,0)\right]_{\mathbf{q}^2=0}\right\}$$

$$a_{1+}^{(S,I)} = \frac{m_\psi}{6\pi\left(m_\psi + m_\phi\right)}\left\{\left[\partial_t A^{(S,I)}(s,t)\right]_{t=0,\mathbf{q}^2=0} + m_\phi\left[\partial_t B^{(S,I)}(s,t)\right]_{t=0,\mathbf{q}^2=0}\right\}$$

$$a_{1-}^{(S,I)} = a_{1+}^{(S,I)} - \frac{1}{16\pi m_\psi\left(m_\psi + m_\phi\right)}\left\{\left[A^{(S,I)}(s,0)\right]_{\mathbf{q}^2=0} - \left(2m_\psi + m_\phi\right)\left[B^{(S,I)}(s,0)\right]_{\mathbf{q}^2=0}\right\}$$

Scattering length in Lattice QCD

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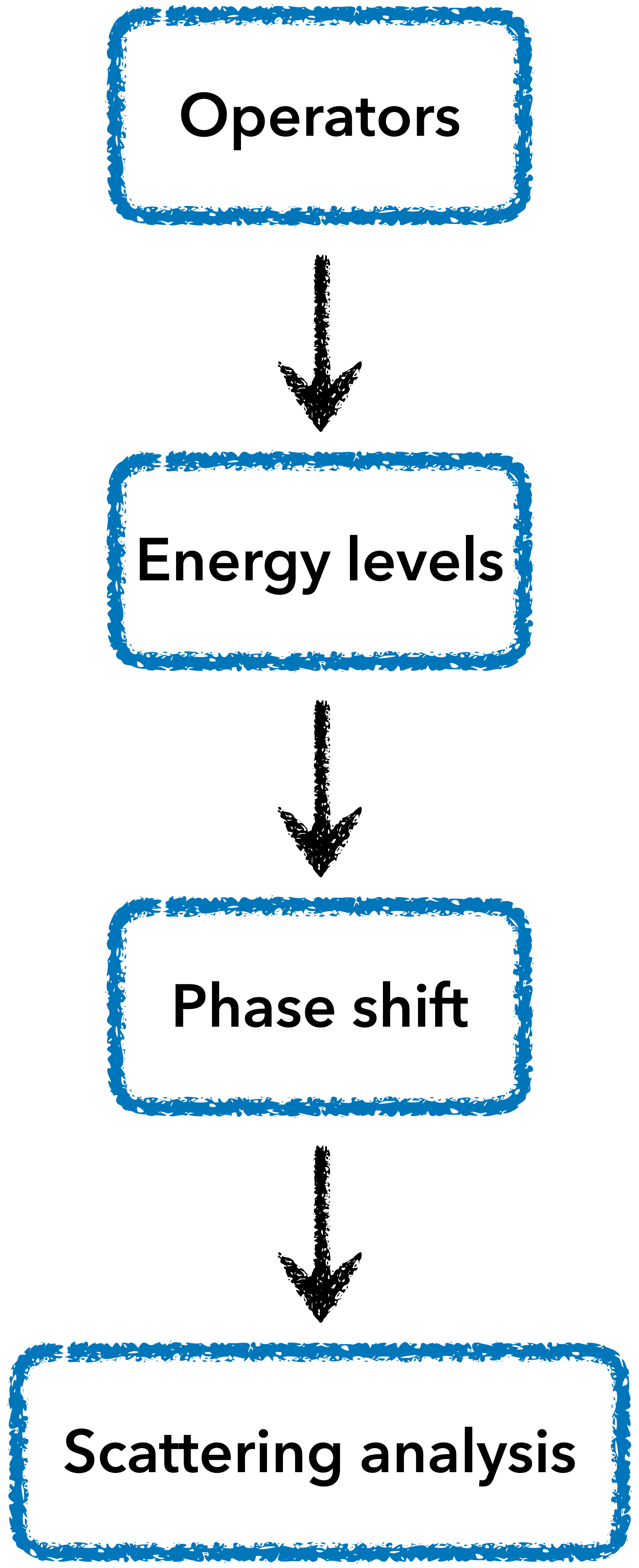
- Xing et al., "First Observation of the Hidden-Charm Pentaquarks on Lattice."
- Padmanath and Prelovsek, "Signature of a Doubly Charm Tetraquark Pole in DD* Scattering on the Lattice."
- Bulava et al., "The Two-Pole Nature of the $\Lambda(1405)$ from Lattice QCD."
- ……



Operators

Energy levels

Phase shift

Scattering analysis

Simulation in lattice QCD

Operators

↓

Energy levels

↓

Phase shift

↓

Scattering analysis

Single-particle operators

$$\mathcal{O}_{\pi^+}(x) = \bar{d}(x)^a_\alpha \left(\gamma_5\right)_{\alpha\beta} u(x)^a_\beta \qquad \mathcal{O}_{\Xi_{cc}^{++}(ccu)}(x) = \epsilon^{ijk} P_+ \left[Q_c^{iT}(x) C\gamma_5 q_u^j(x)\right] Q_c^k(x)$$

$$\mathcal{O}_{K^+}(x) = \bar{s}(x)^a_\alpha \left(\gamma_5\right)_{\alpha\beta} u(x)^a_\beta \qquad \mathcal{O}_{\Xi_{cc}^{+}(ccd)}(x) = \epsilon^{ijk} P_+ \left[Q_c^{iT}(x) C\gamma_5 q_d^j(x)\right] Q_c^k(x)$$

$$\mathcal{O}_{K^0}(x) = \bar{s}(x)^a_\alpha \left(\gamma_5\right)_{\alpha\beta} d(x)^a_\beta \qquad \mathcal{O}_{\Omega_{cc}^{+}(ccs)}(x) = \epsilon^{ijk} P_+ \left[Q_c^{iT}(x) C\gamma_5 q_s^j(x)\right] Q_c^k(x)$$

Two-particle operators for S-wave scattering $J^P = (1/2)^- \longrightarrow G_1^-$

$$\mathcal{O}^{\Xi_{cc}K,\ I=0}_{\mathbf{p}_1,\mathbf{p}_2} = \sum_{\alpha,\mathbf{p}_1,\mathbf{p}_2} C_{\alpha,\mathbf{p}_1,\mathbf{p}_2} \left(\frac{1}{\sqrt{2}} \mathcal{O}_{\Xi_{cc}^{++},\alpha}(\mathbf{p}_1)\mathcal{O}_{K^0}(\mathbf{p}_2) - \frac{1}{\sqrt{2}} \mathcal{O}_{\Xi_{cc}^{+},\alpha}(\mathbf{p}_1)\mathcal{O}_{K^+}(\mathbf{p}_2) \right)$$

$$\mathcal{O}^{\Xi_{cc}K,\ I=1}_{\mathbf{p}_1,\mathbf{p}_2} = \sum_{\alpha,\mathbf{p}_1,\mathbf{p}_2} C_{\alpha,\mathbf{p}_1,\mathbf{p}_2} \left(\mathcal{O}_{\Xi_{cc}^{++},\alpha}(\mathbf{p}_1)\mathcal{O}_{K^+}(\mathbf{p}_2) \right) \qquad \mathcal{O}^{\Xi_{cc}\pi,\ I=3/2}_{\mathbf{p}_1,\mathbf{p}_2} = \sum_{\alpha,\mathbf{p}_1,\mathbf{p}_2} C_{\alpha,\mathbf{p}_1,\mathbf{p}_2} \left(\mathcal{O}_{\Xi_{cc}^{++},\alpha}(\mathbf{p}_1)\mathcal{O}_{\pi^+}(\mathbf{p}_2) \right)$$

$$\mathcal{O}^{\Omega_{cc}\bar{K},\ I=1/2}_{\mathbf{p}_1,\mathbf{p}_2} = \sum_{\alpha,\mathbf{p}_1,\mathbf{p}_2} C_{\alpha,\mathbf{p}_1,\mathbf{p}_2} \left(\mathcal{O}_{\Omega_{cc}^{+},\alpha}(\mathbf{p}_1)\mathcal{O}_{\bar{K}^0}(\mathbf{p}_2) \right)$$

Simulation in lattice QCD

Operators

↓

Energy levels

↓

Phase shift

↓

Scattering analysis

Two-point correlation function and single-particle effective mass

Mesons: $C_0 e^{-\frac{T}{2}E_0}\cosh\left[E_0\left(\frac{T}{2}-t\right)\right]$ Baryons : $A_0 e^{-E_0 t}$

Four-point correlation function and two-particle effective mass

- Solve generalized eigenvalue problem (GEVP)

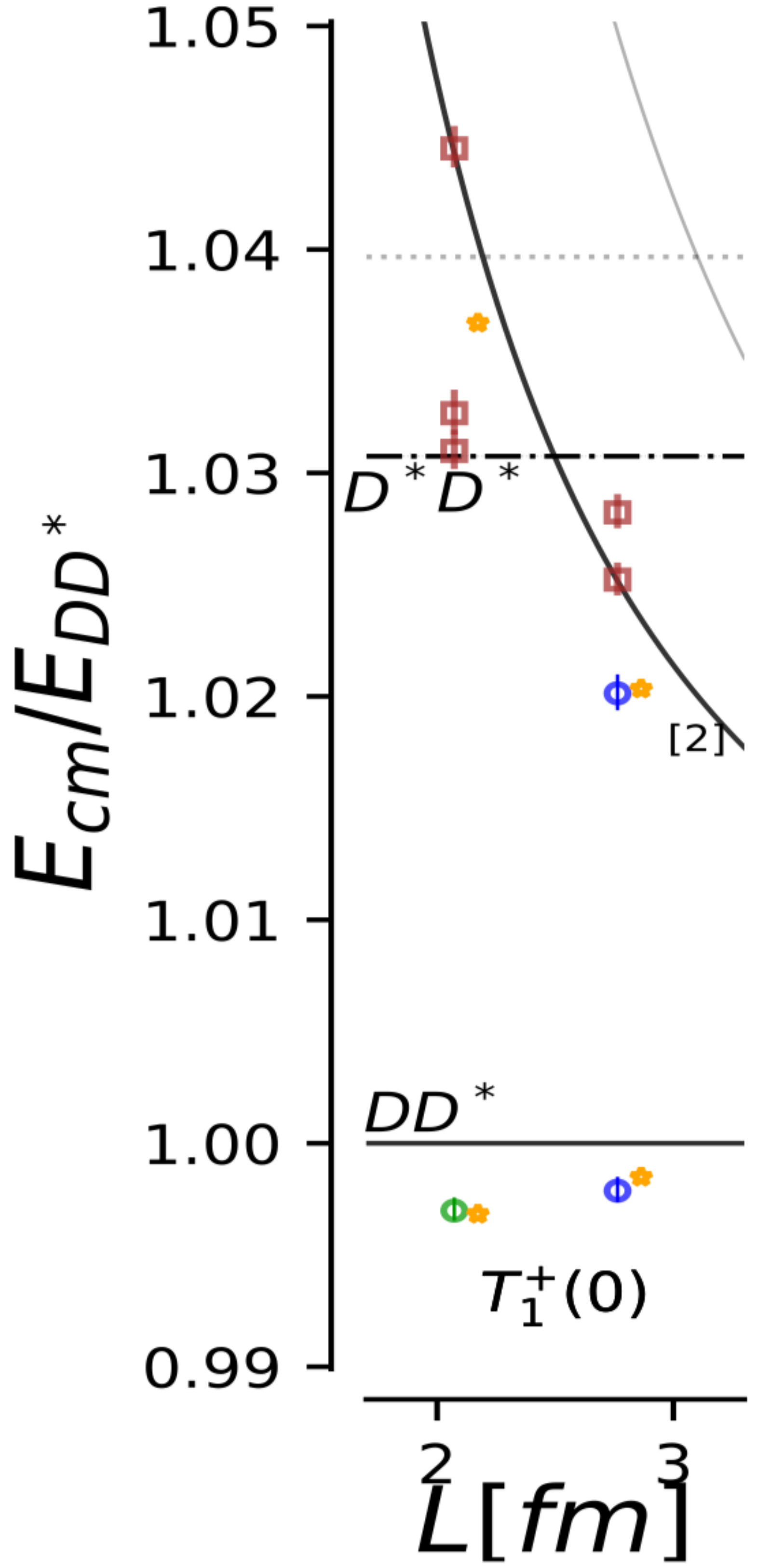
$$C_{ij}(t)=\sum_{t_{src}}\left\langle \mathcal{O}_i\left(t+t_{src}\right)\mathcal{O}_j^{\dagger}\left(t_{src}\right)\right\rangle \qquad C(t)v^n(t)=\lambda^n(t)C\left(t_0\right)v^n(t)$$

M. Luscher and U. Wolff, Nucl. Phys. B 339, 222 (1990).

- Extract effective mass by two-state fit

$$\lambda^n(t)=\left(1-A_n\right)e^{-E_n\left(t-t_0\right)}+A_n e^{-E_n'\left(t-t_0\right)}$$

$\longrightarrow$ Energy shift ΔE



M. Padmanath and S. Prelovsek, Phys. Rev. Lett. 129, 032002 (2022).

Simulation in lattice QCD

Operators

Energy levels

Phase shift

Scattering analysis

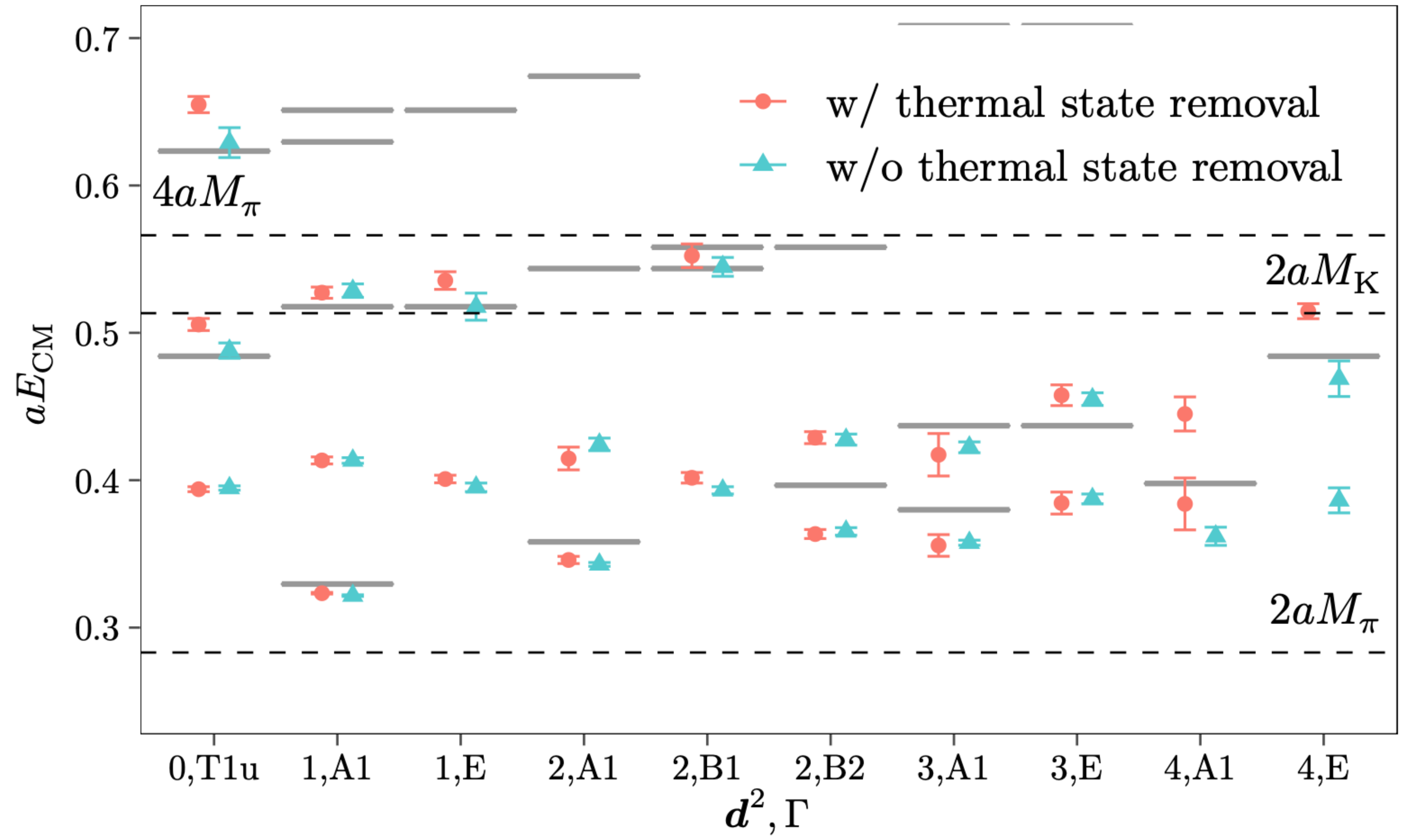
Luscher's formula for two-body scattering M. Luscher, Nucl. Phys. B 354, 531 (1991).

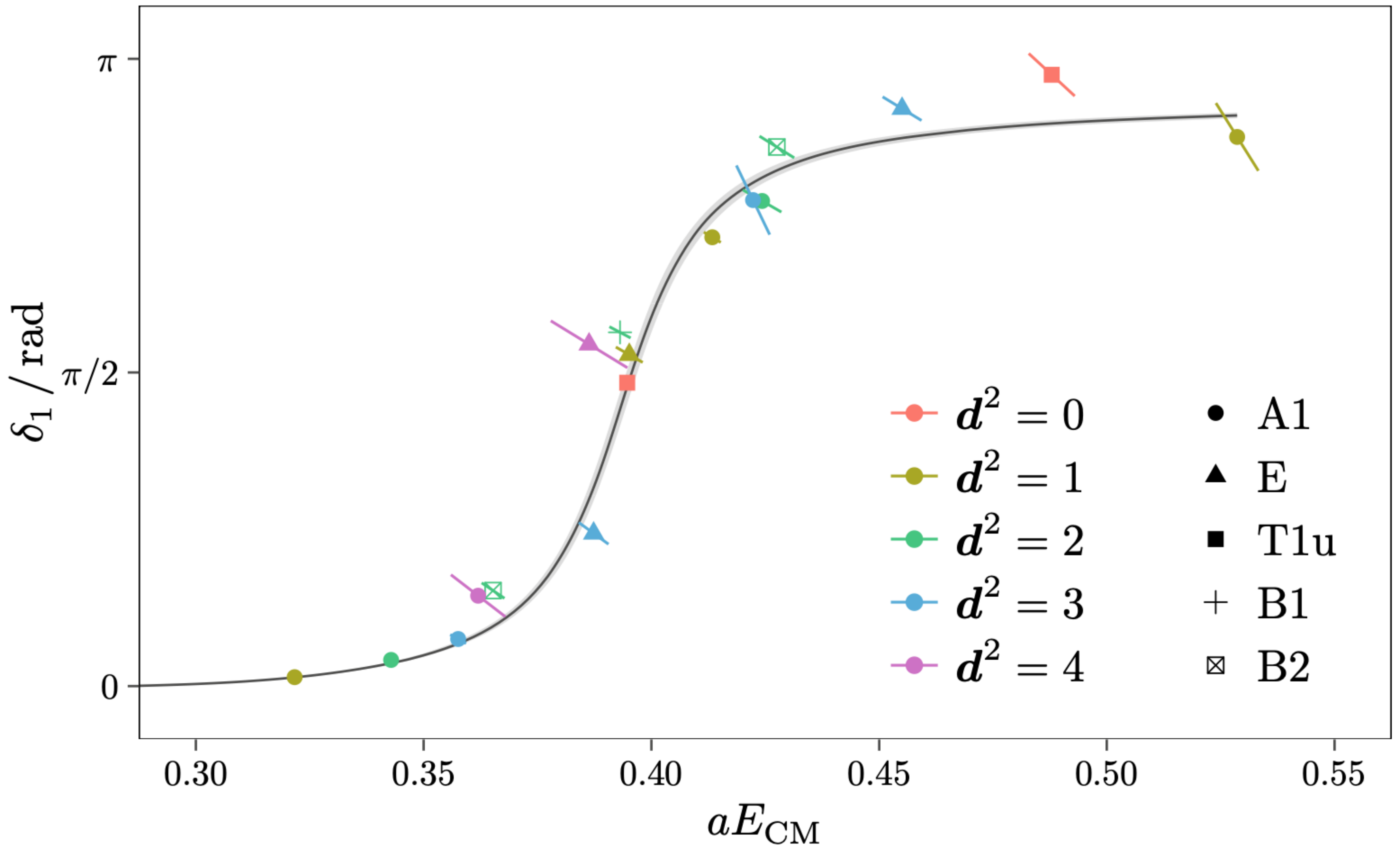
$$\det[\mathbf{1} + i\rho \cdot \mathbf{t} \cdot (1 + i\mathbf{M})] = 0$$

For single-channel S-wave scattering

$$p \cot \delta_0(p) = \frac{2}{L\sqrt{\pi}} \mathscr{Z}_{00}\left(1; q^2\right)$$

$$E = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2} \qquad q = \frac{pL}{2\pi}$$





M. Werner and others, Eur. Phys. J. A 56, 61 (2020).

Simulation in lattice QCD

Operators

Energy levels

Phase shift

Scattering analysis

Effective range expansion

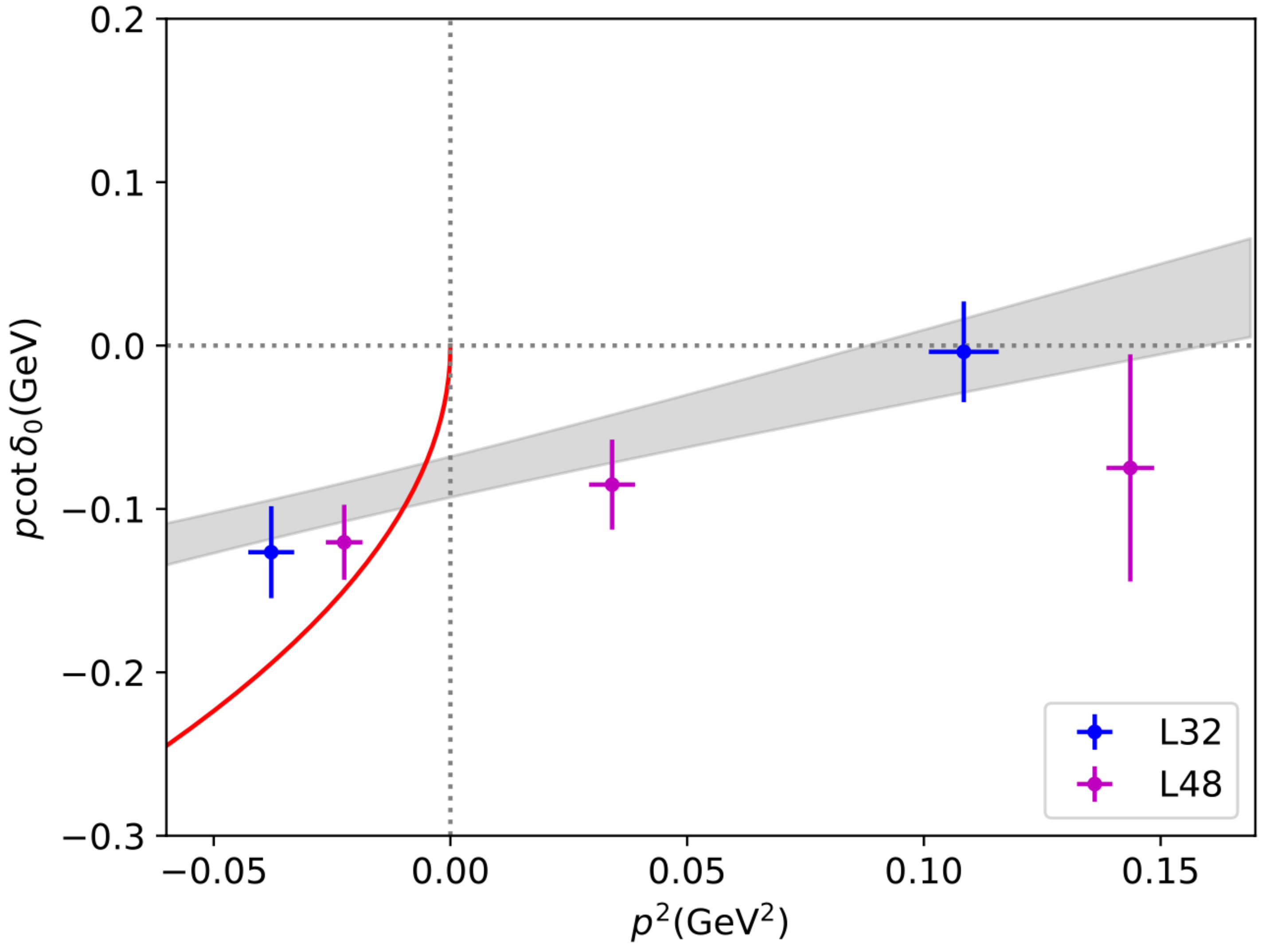
$$\color{red}{p\cot\delta_0} = \frac{1}{a_0} + \frac{1}{2} r_0 p^2 + \mathcal{O}(p^4)$$

Scattering amplitude

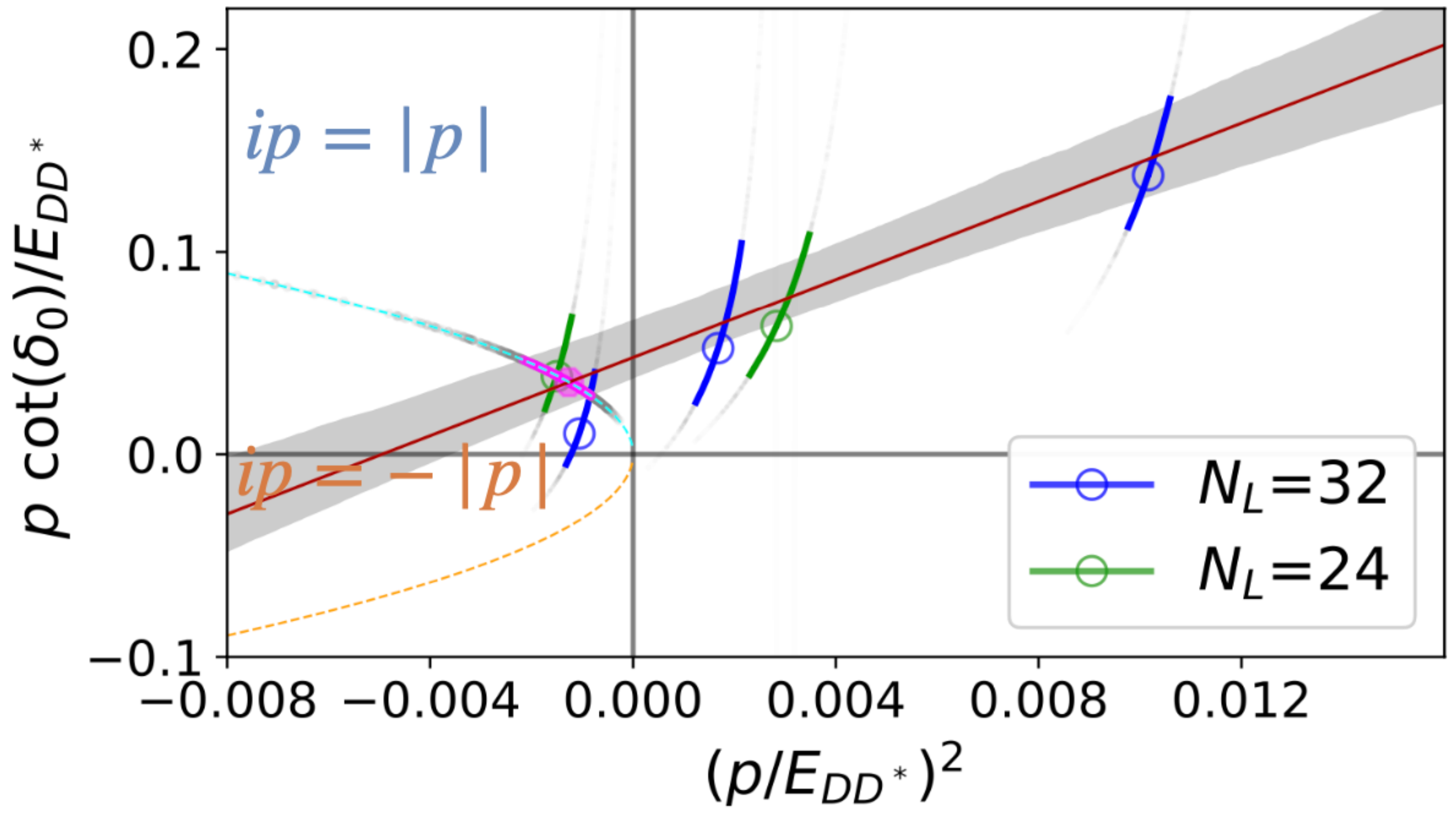
$$t_l^{(J)} \sim \frac{1}{\color{red}{p\cot\delta_l^{(J)}} - ip}$$

For partial wave l and total angular momentum J

- L. Liu, K. Orginos, F.-K. Guo, C. Hanhart, and U.-G. Meissner, Phys. Rev. D 87, 014508 (2013).
- Y. Lyu, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda, and J. Meng, arXiv:2302.04505 [hep-lat] (2023).
- H. Xing, J. Liang, L. Liu, P. Sun, and Y.-B. Yang, arXiv:2210.08555 [hep-lat] (2022).
- M. Padmanath and S. Prelovsek, Phys. Rev. Lett. 129, 032002 (2022).
- ……



H. Xing, J. Liang, L. Liu, P. Sun, and Y.-B. Yang, arXiv:2210.08555 [hep-lat] (2022).



M. Padmanath and S. Prelovsek, Phys. Rev. Lett. 129, 032002 (2022).

Lattice setup

Gauge configurations from CLQCD

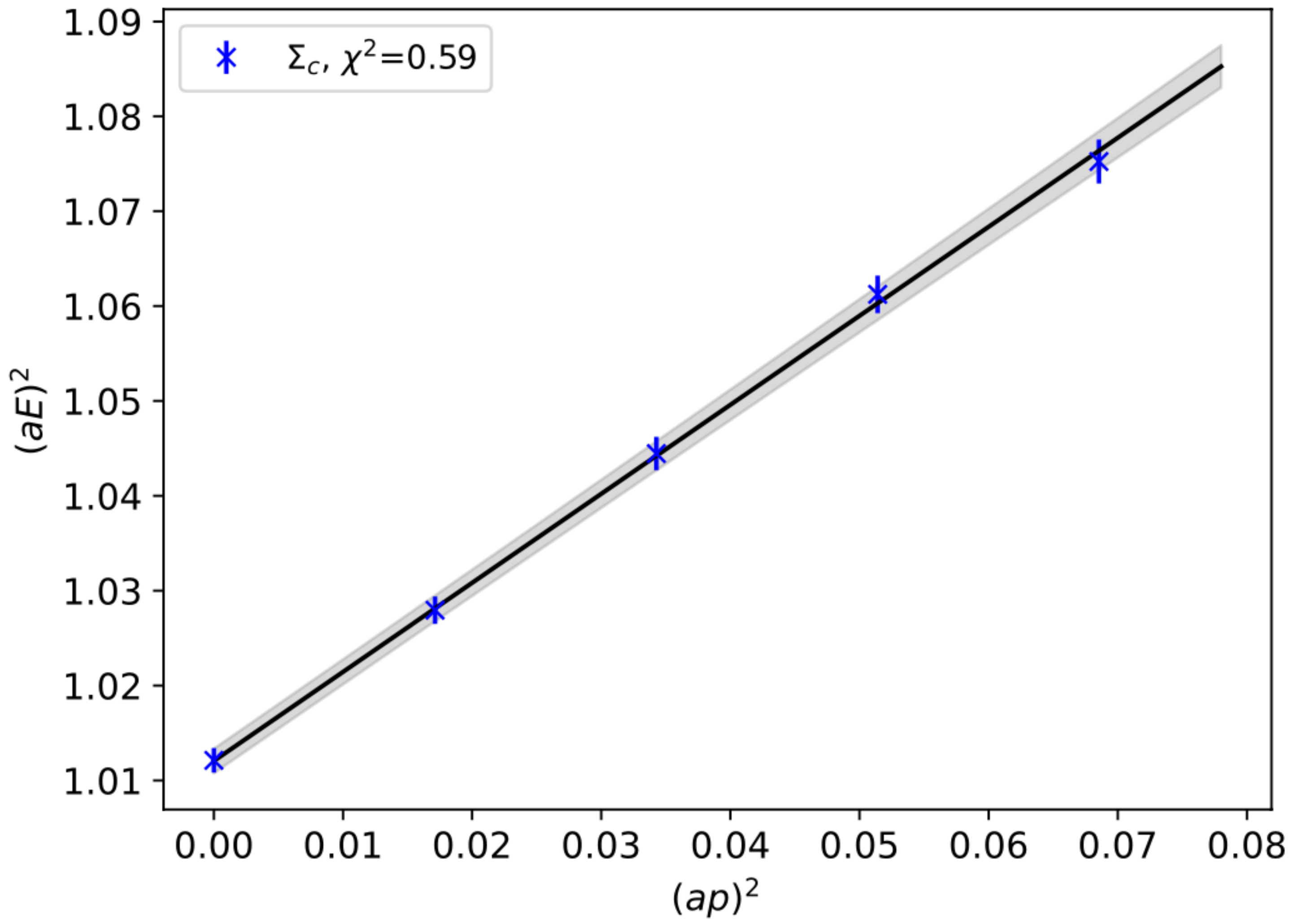
ID	β	a(fm)	am_l	am_s	M_π(MeV)	$L^3 \times T$	$N_{cfgs.}$
F32P30	6.41	0.07746(18)	-0.2295	-0.2050	304.0(0.6)	$32^3 \times 96$	750
F48P30	6.41	0.07746(18)	-0.2295	-0.2050	304.9(0.4)	$48^3 \times 96$	359
F32P21	6.41	0.07746(18)	-0.2320	-0.2050	208.1(1.9)	$32^3 \times 64$	459
F48P21	6.41	0.07746(18)	-0.2320	-0.2050	207.7(0.7)	$48^3 \times 96$	222

The results presented in this work are based on

- Four ensembles of gauge configurations with 2+1 dynamical quark flavors.
- tree-level Symanzik-improved gauge action.
- Shekholeslami-Wohlert fermion action with tree-level tadpole improvement.
- Pion mass ~ 300 and 210 MeV.
- Distillation quark smearing ($N_{ev} = 100$).

Charm quark mass

- Determined by physical J/ψ.
- DCBs mass deviate from continuum dispersion relation due to lattice artifacts.



H. Xing, J. Liang, L. Liu, P. Sun, and Y.-B. Yang, arXiv:2210.08555 [hep-lat] (2022).

Numerical results

Effective masses and Energy levels

Mesons: $C_0 e^{-\frac{T}{2}E_0} \cosh\left[E_0\left(\frac{T}{2}-t\right)\right]$

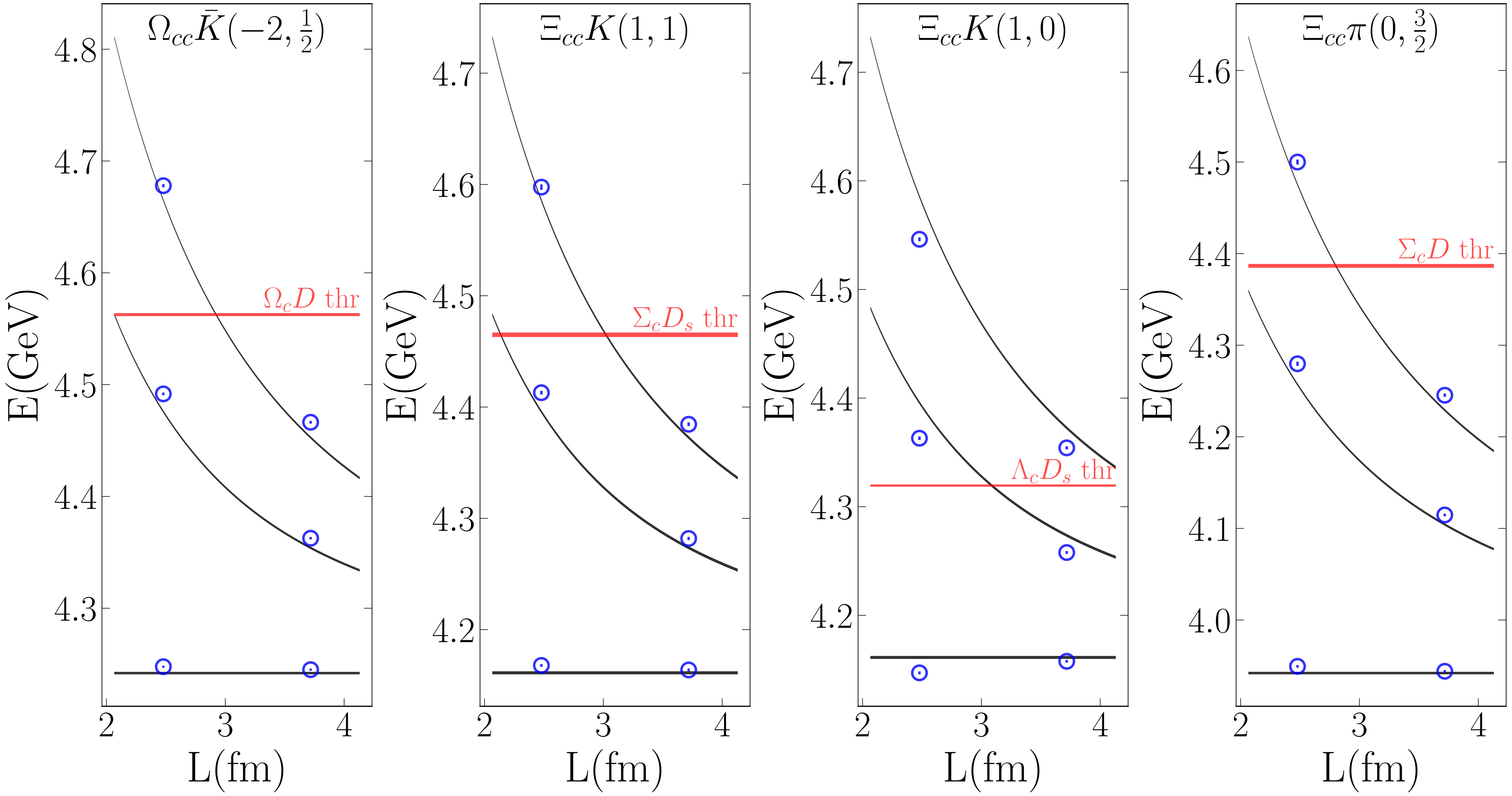
Baryons : $A_0 e^{-E_0 t}$

Two-particle system :

$$C_{ij}(t) = \sum_{t_{src}} \left\langle \mathcal{O}_i\left(t+t_{src}\right) \mathcal{O}_j^\dagger\left(t_{src}\right)\right\rangle$$

$$C(t)v^n(t) = \lambda^n(t) C\left(t_0\right) v^n(t)$$

$$\lambda^n(t) = \left(1-A_n\right) e^{-E_n\left(t-t_0\right)} + A_n e^{-E_n'\left(t-t_0\right)}$$



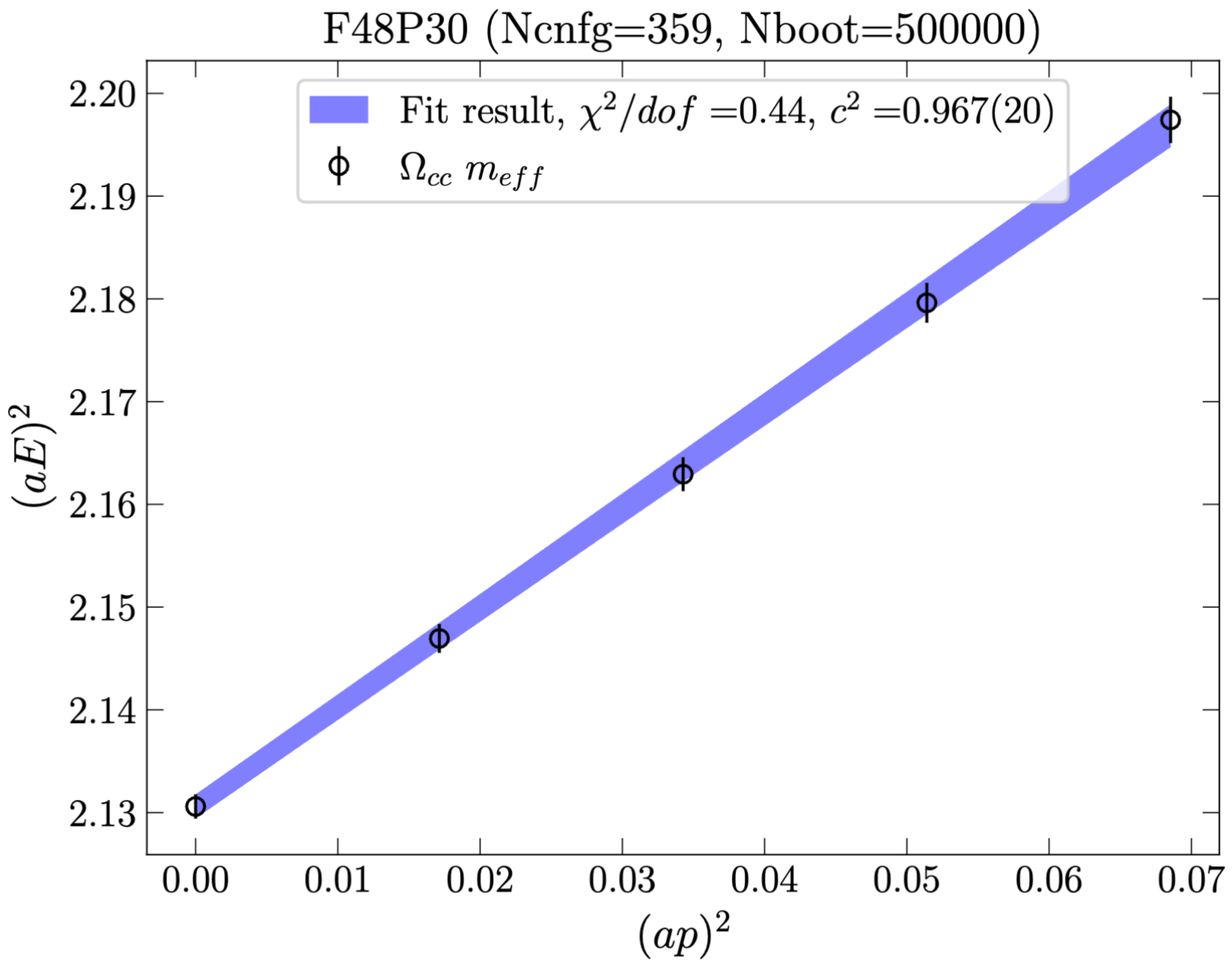
	π	K	Ξ_{cc}	Ω_{cc}
F32P30	303.9(6)	523.0(4)	3632.4(9)	3713.7(6)
F48P30	304.9(4)	524.1(3)	3637.2(1.3)	3717.9(1.0)
F32P21	208.1(1.9)	491.7(7)	3605.6(1.4)	3693.8(8)
F48P21	207.4(7)	491.1(3)	3607.6(1.8)	3699.7(1.0)

	D	D_s	Λ_c	Σ_c	Ω_c
F48P30	1.8923(4)	1.9706(2)	2.3488(9)	2.4943(19)	2.6703(11)
F48P21	1.8736(4)	1.9595(3)	2.2842(18)	2.4438(18)	2.6420(10)

Numerical results

Single-particle effective masses and dispersion relation $E^2 = m_0{}^2 + c^2 p^2$

Single particles	Ensembles	m_0 (GeV)	c^2	χ^2/dof
π	F32P30	0.30396(60)	1.0069(42)	0.69
	F48P30	0.30487(39)	1.0075(22)	1.02
	F32P21	0.2085(19)	1.0582(94)	0.73
	F48P21	0.20774(69)	1.0157(37)	1.58
K	F32P30	0.52300(43)	1.0026(23)	1.47
	F48P30	0.52407(28)	1.0034(15)	0.53
	F32P21	0.49173(69)	1.0097(52)	0.26
	F48P21	0.49109(33)	1.0081(25)	0.63
Ξ_{cc}	F32P30	3.63302(84)	0.9915(61)	1.70
	F48P30	3.6369(12)	0.964(19)	0.38
	F32P21	3.6055(14)	1.006(10)	0.48
	F48P21	3.6080(16)	1.052(22)	0.56
Ω_{cc}	F32P30	3.71393(61)	0.9747(37)	0.92
	F48P30	3.7179(10)	0.967(20)	0.44
	F32P21	3.69380(81)	0.9801(51)	0.11
	F48P21	3.69950(96)	0.998(13)	1.16



$$E'_p = E_p + \left(M^{cont.}_{\phi(p)} + M^{cont.}_{\psi_{cc}(p)}\right) - \left(M^{lat.}_{\phi(p)} + M^{lat.}_{\psi_{cc}(p)}\right).$$

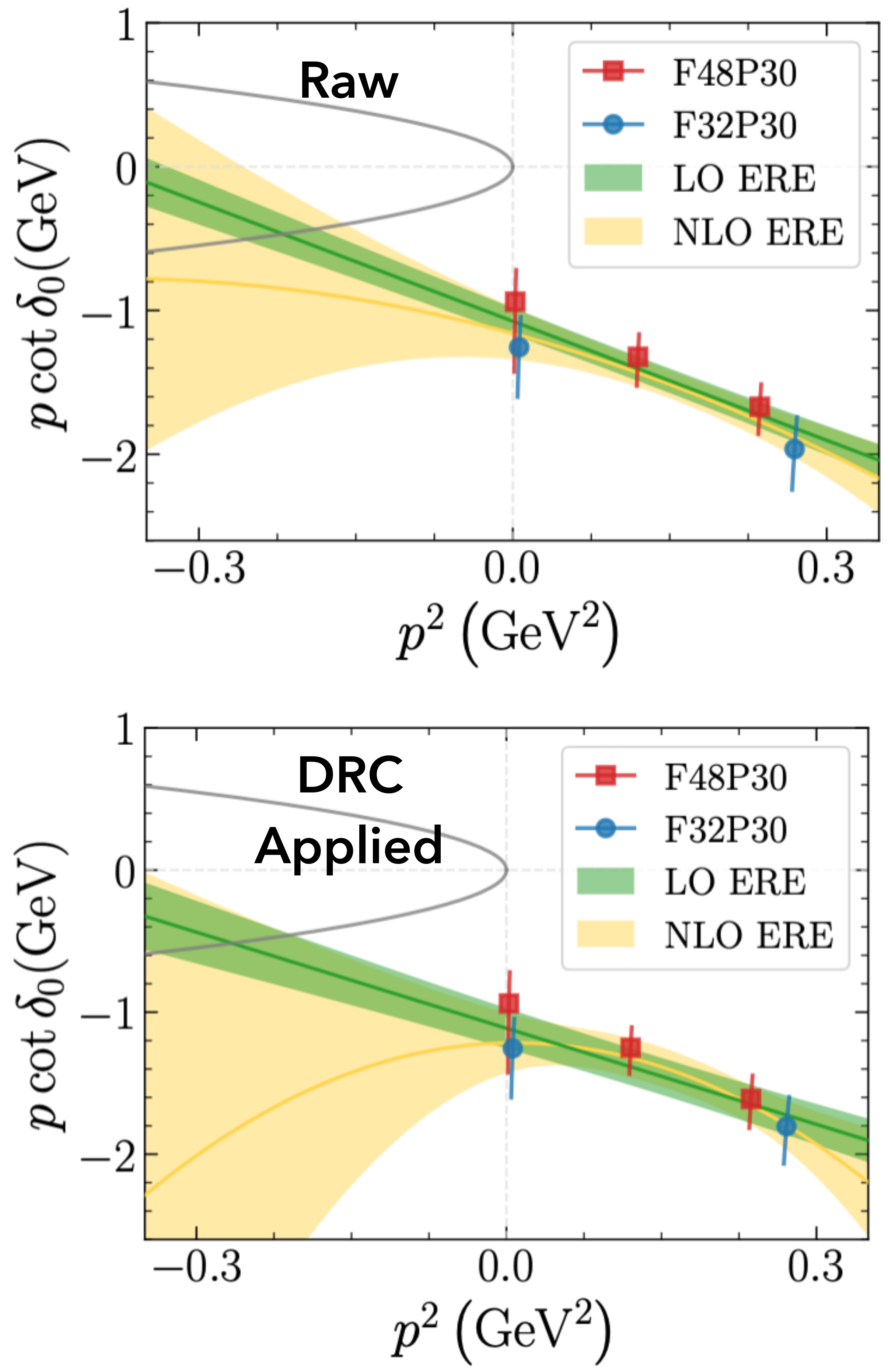
Numerical results

Scattering parameters and dispersion relation correction (DRC)

$$p\cot\delta_0 = \frac{1}{a_0} + \frac{1}{2}r_0 p^2 + \mathcal{O}(p^4) \qquad E^2 = m_0^2 + c^2p^2$$

(S, I)	Processes	M_π (MeV)	DRC	a_0 (fm)	r_0 (fm)	χ^2/dof
$(-2, \frac{1}{2})$	$\Omega_{cc}\bar{K} \to \Omega_{cc}\bar{K}$	300	Raw	-0.172(13)	-1.05(10)	1.11
			Applied	-0.161(20)	-0.79(15)	1.14
		210	Raw	-0.125(12)	-0.33(22)	1.71
			Applied	-0.131(12)	-0.62(26)	0.82
$(1, 1)$	$\Xi_{cc}K \to \Xi_{cc}K$	300	Raw	-0.184(16)	-1.09(12)	0.29
			Applied	-0.177(23)	-0.89(16)	0.47
		210	Raw	-0.211(14)	-0.88(14)	0.45
			Applied	-0.212(14)	-1.42(28)	0.52
$(1, 0)$	$\Xi_{cc}K \to \Xi_{cc}K$	300	Raw	0.63(11)	1.00(15)	1.07
			Applied	0.63(10)	1.11(19)	1.03
		210	Raw	0.686(90)	1.299(97)	0.67
			Applied	0.694(90)	1.20(20)	0.69
$(0, \frac{3}{2})$	$\Xi_{cc}\pi \to \Xi_{cc}\pi$	300	Raw	-0.166(12)	-0.23(12)	4.32
			Applied	-0.140(15)	-0.04(20)	1.56
		210	Raw	-0.150(22)	0.48(27)	1.17
			Applied	-0.143(24)	-0.08(40)	1.02

$\Xi_{cc}K(1{,}1)$ ERE fit at $M_\pi \sim 300$ MeV

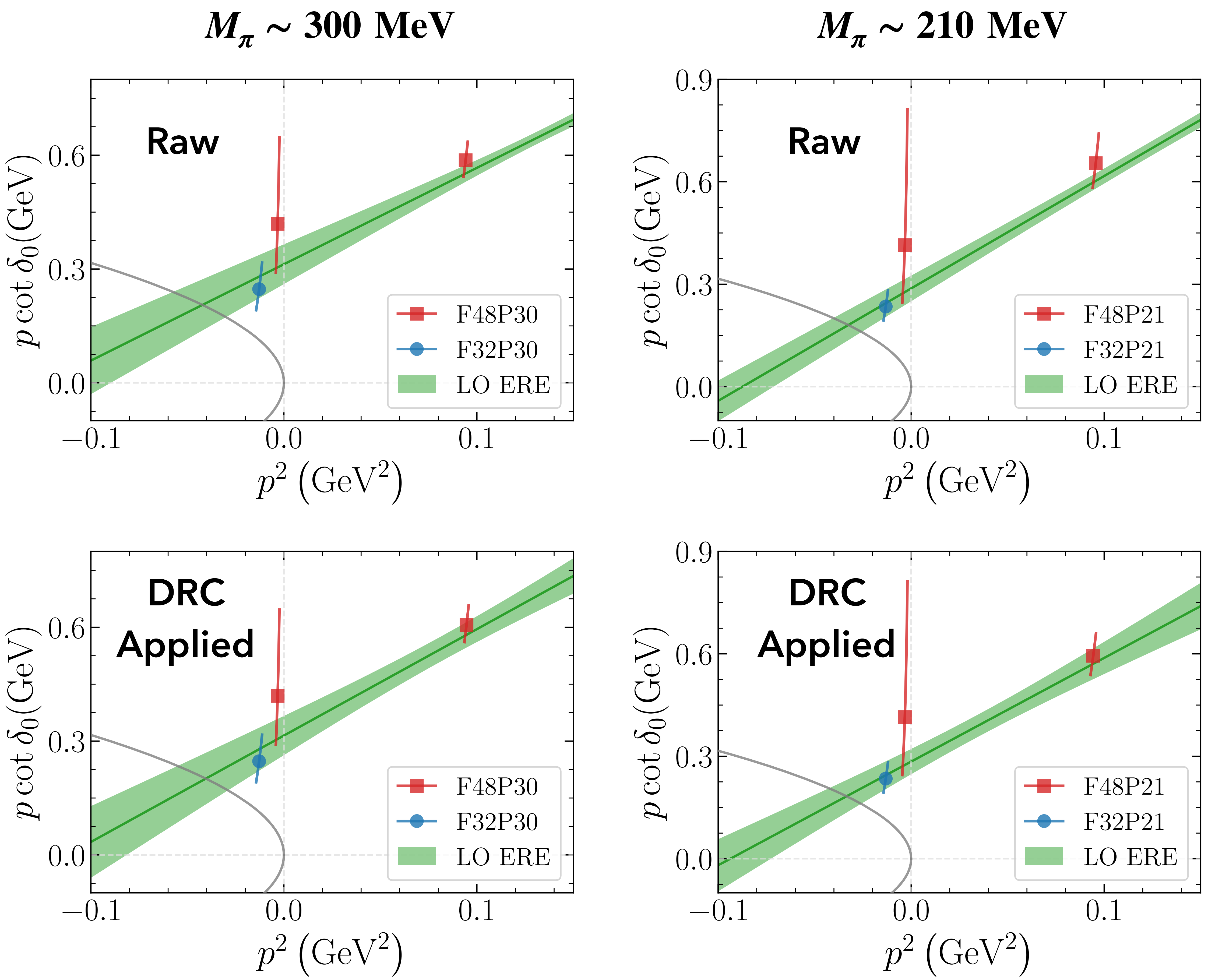


Numerical results

Scattering parameters and dispersion relation correction (DRC)

$$p\cot\delta_0 = \frac{1}{a_0} + \frac{1}{2}r_0 p^2 + \mathcal{O}(p^4) \qquad t \sim \frac{1}{p\cot\delta_0 - ip}$$

(S, I)	Processes	M_π (MeV)	DRC	a_0 (fm)	r_0 (fm)	χ^2/dof
$(-2, \frac{1}{2})$	$\Omega_{cc}\bar{K} \to \Omega_{cc}\bar{K}$	300	Raw	-0.172(13)	-1.05(10)	1.11
			Applied	-0.161(20)	-0.79(15)	1.14
		210	Raw	-0.125(12)	-0.33(22)	1.71
			Applied	-0.131(12)	-0.62(26)	0.82
$(1, 1)$	$\Xi_{cc}K \to \Xi_{cc}K$	300	Raw	-0.184(16)	-1.09(12)	0.29
			Applied	-0.177(23)	-0.89(16)	0.47
		210	Raw	-0.211(14)	-0.88(14)	0.45
			Applied	-0.212(14)	-1.42(28)	0.52
$(1, 0)$	$\Xi_{cc}K \to \Xi_{cc}K$	300	Raw	0.63(11)	1.00(15)	1.07
			Applied	0.63(10)	1.11(19)	1.03
		210	Raw	0.686(90)	1.299(97)	0.67
			Applied	0.694(90)	1.20(20)	0.69
$(0, \frac{3}{2})$	$\Xi_{cc}\pi \to \Xi_{cc}\pi$	300	Raw	-0.166(12)	-0.23(12)	4.32
			Applied	-0.140(15)	-0.04(20)	1.56
		210	Raw	-0.150(22)	0.48(27)	1.17
			Applied	-0.143(24)	-0.08(40)	1.02



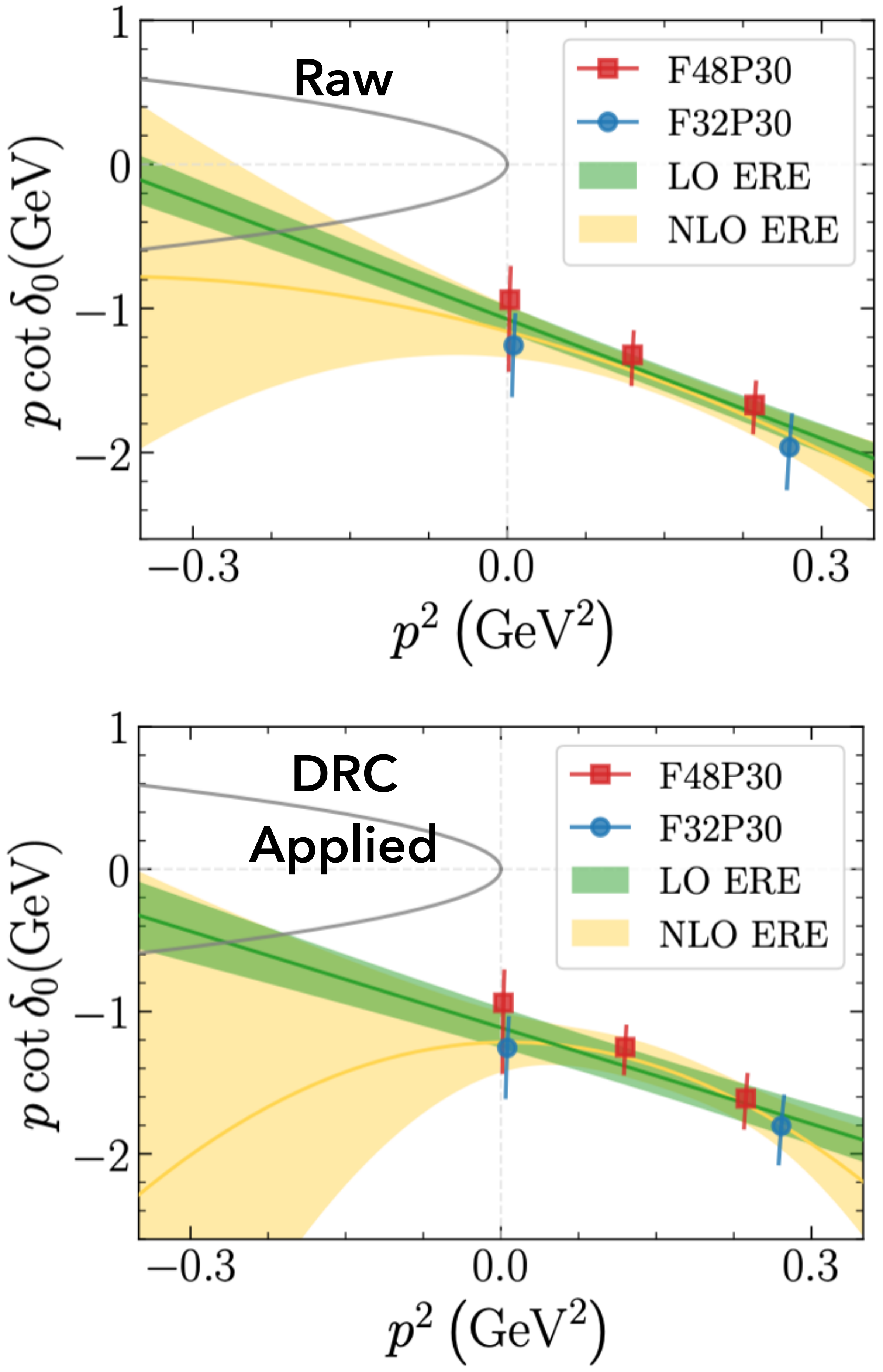
Numerical results

S-wave scattering lengths a_0 extracted at LO and NLO ERE

$$p\cot\delta_0 = \frac{1}{a_0} + \frac{1}{2}r_0 p^2 + \mathcal{O}(p^4)$$

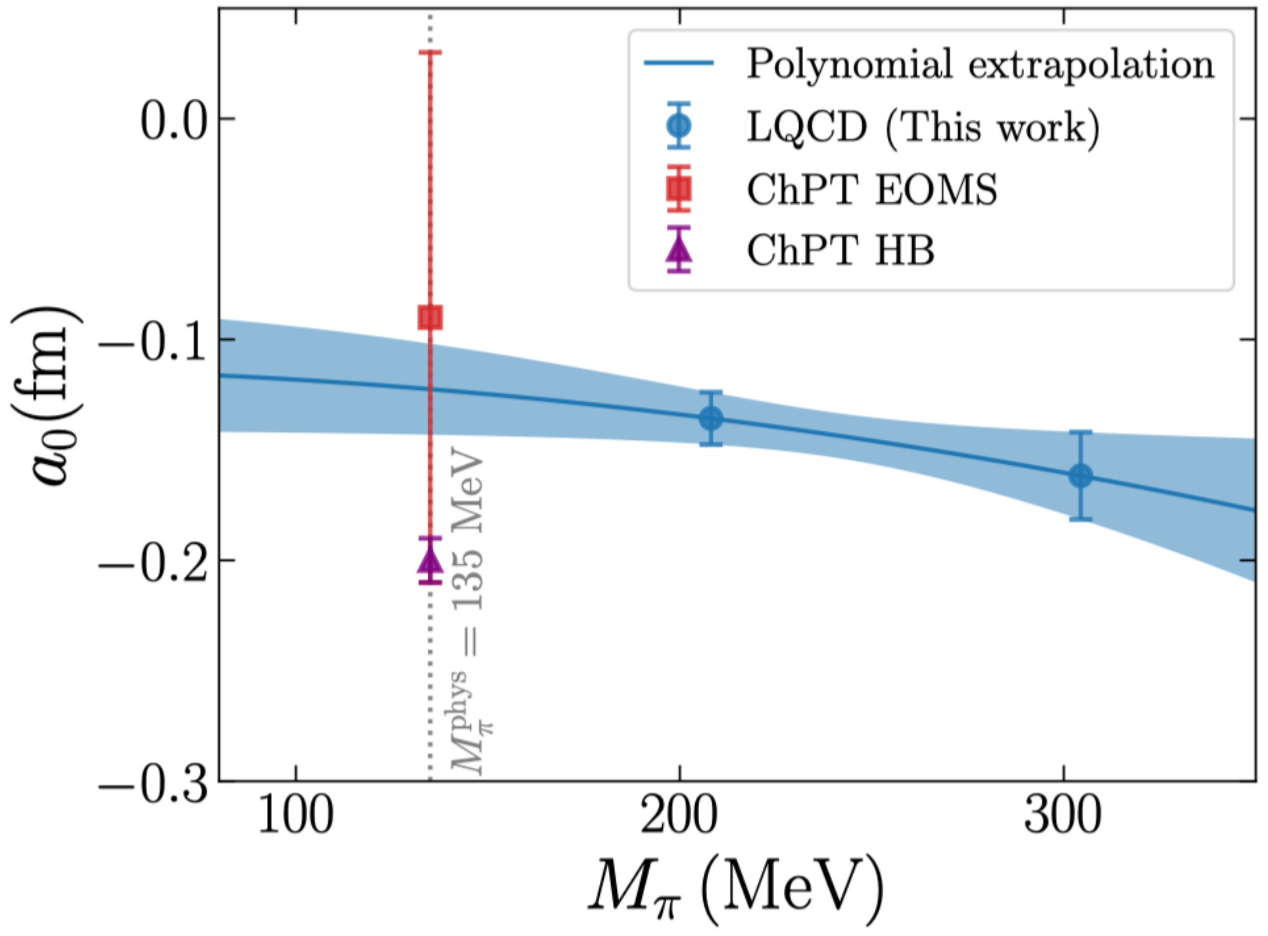
(S, I)	Processes	M_π (MeV)	DRC	a_0 (LO)	a_0 (NLO)	Deviations
$(-2, \frac{1}{2})$	$\Omega_{cc}\bar{K} \to \Omega_{cc}\bar{K}$	300	Raw	-0.172(13)	-0.164(22)	$< 1\sigma$
			Applied	-0.161(20)	-0.159(24)	$< 1\sigma$
		210	Raw	-0.125(12)	-0.135(13)	$< 1\sigma$
			Applied	-0.136(12)	-0.136(13)	$< 1\sigma$
$(1, 1)$	$\Xi_{cc}K \to \Xi_{cc}K$	300	Raw	-0.184(16)	-0.170(27)	$< 1\sigma$
			Applied	-0.177(23)	-0.162(28)	$< 1\sigma$
		210	Raw	-0.211(14)	-0.211(15)	$< 1\sigma$
			Applied	-0.212(14)	-0.209(17)	$< 1\sigma$
$(0, \frac{3}{2})$	$\Xi_{cc}\pi \to \Xi_{cc}\pi$	300	Raw	-0.166(12)	-0.112(22)	2.1σ
			Applied	-0.140(15)	-0.111(22)	1.1σ
		210	Raw	-0.150(22)	-0.139(26)	$< 1\sigma$
			Applied	-0.143(24)	-0.141(28)	$< 1\sigma$

$\Xi_{cc}K(1,1)$ ERE fit at $M_\pi \sim 300$ MeV

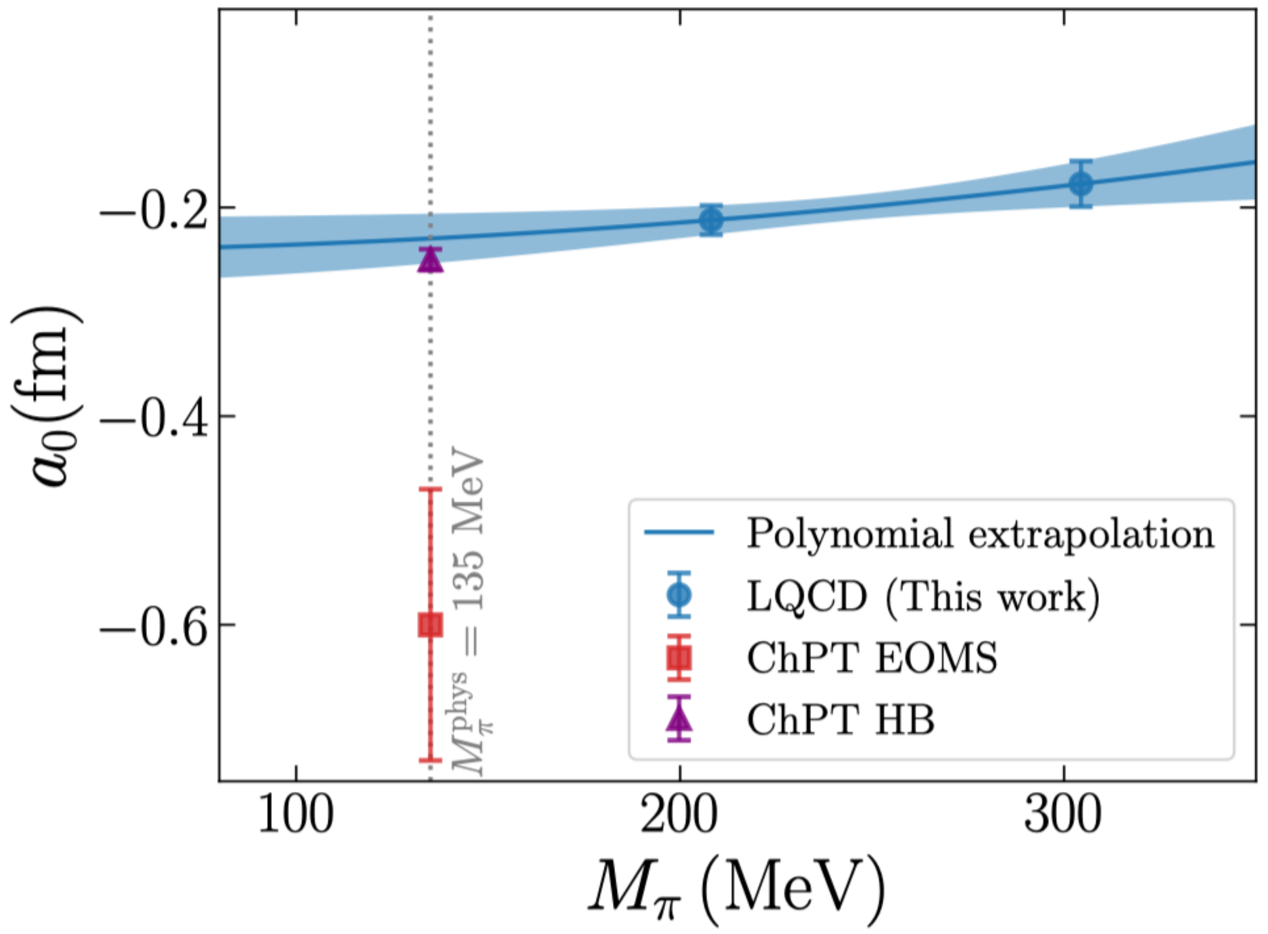


Numerical results

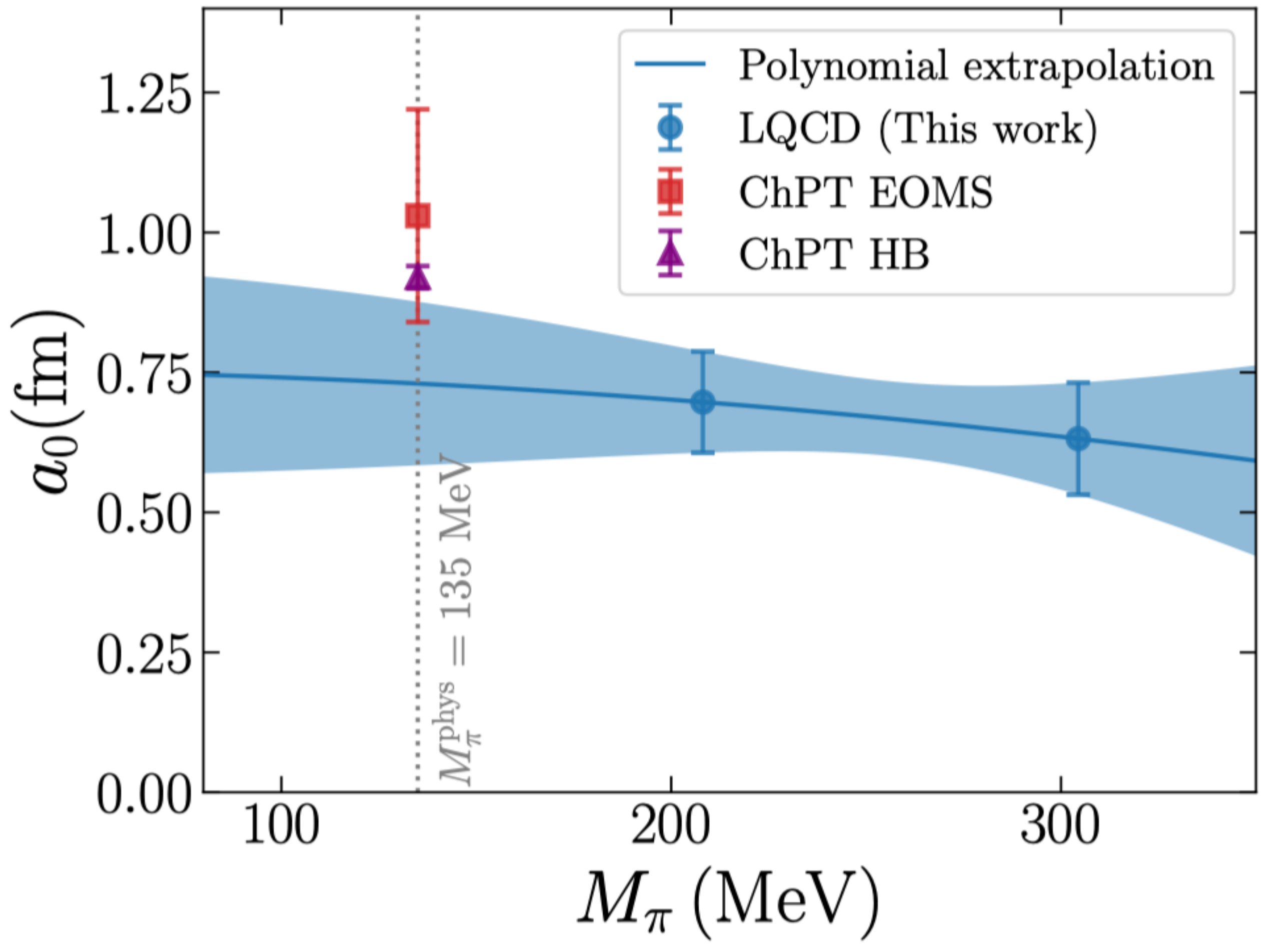
Extrapolation of S-wave scattering length a_0 to the physical pion mass



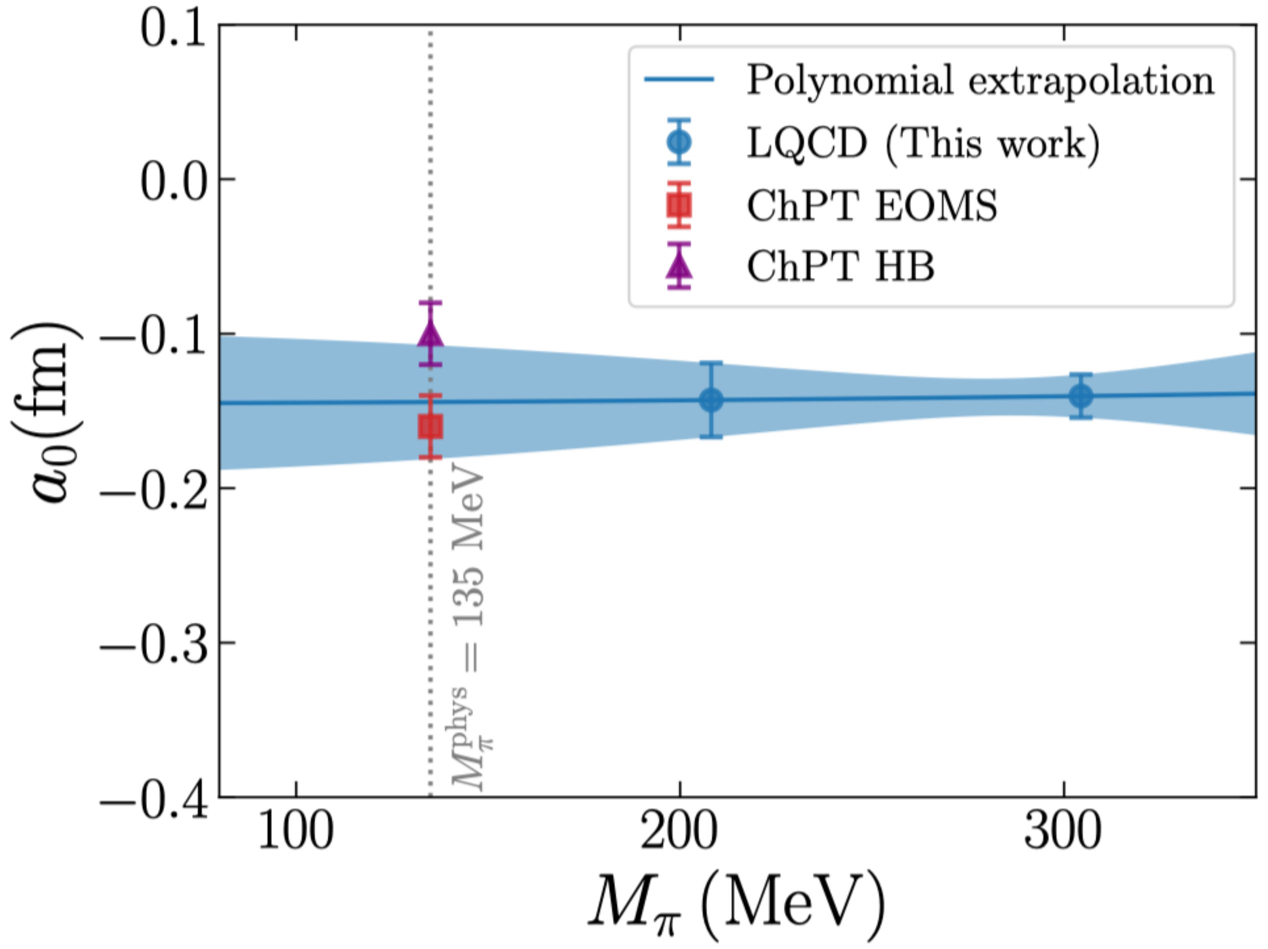
(a) $\Omega_{cc}\bar{K}^{(-2,\frac{1}{2})}$



(b) $\Xi_{cc}K(1,1)$



(c) $\Xi_{cc}K(1,0)$



(d) $\Xi_{cc}\pi(0,\frac{3}{2})$

To extrapolate them to the physical pion mass, we employ a polynomial in M_π^2 as

$$a_0(M_\pi) = c_0 + c_1 M_\pi^2 .$$

The extrapolated scattering lengths at the physical pion mass are determined as follows:

$$a_0^{\text{phy}}(\Omega_{cc}\bar{K}^{(-2,1/2)}) = -0.123(21)\ \text{fm}, \quad a_0^{\text{phy}}(\Xi_{cc}K^{(1,1)}) = -0.230(24)\ \text{fm}\ ,$$
$$a_0^{\text{phy}}(\Xi_{cc}K^{(1,0)}) = 0.73(15)\ \text{fm}, \quad a_0^{\text{phy}}(\Xi_{cc}\pi^{(0,3/2)}) = -0.144(37)\ \text{fm}\ .$$

Overall, our results are in good agreement with ChPT predictions at the physical point.

L. Meng and S.-L. Zhu, Phys. Rev. D 100, 014006 (2019)

Z.-R. Liang, P.-C. Qiu, and D.-L. Yao, JHEP 07, 124 (2023)

Summary and outlook

Summary

- S-wave scattering lengths of four single channels.
- A possible virtual state in $(S, I) = (1,0)$ channel.
- Determination of LECs in BChPT.
- Examination of the validity of HDA symmetry.
- Better understanding of hadron spectroscopy.

Outlook

- Coupled channels
- More ensembles
- Chiral extrapolation

(S, I)	Processes	Connected(C)/ Disconnected(D)	(S, I)	Relation to $D\phi$ scattering
✓ $(-2, \frac{1}{2})$	$\Omega_{cc}\bar{K} \to \Omega_{cc}\bar{K}$	C	$(2, \frac{1}{2})$	$D_sK \to D_sK$
$(-1, 1)$	$\Omega_{cc}\pi \to \Omega_{cc}\pi$ $\Xi_{cc}\bar{K} \to \Xi_{cc}\bar{K}$ $\Omega_{cc}\pi \to \Xi_{cc}\bar{K}$	C	$(1, 1)$	$D_s\pi \to D_s\pi$ $DK \to DK$ $D_s\pi \to DK$
$(-1, 0)$	$\Xi_{cc}\bar{K} \to \Xi_{cc}\bar{K}$ $\Omega_{cc}\eta \to \Omega_{cc}\eta$ $\Xi_{cc}\bar{K} \to \Omega_{cc}\eta$	D	$(1, 0)$	$DK \to DK$ $D_s\eta \to D_s\eta$ $DK \to D_s\eta$
✓ $(1, 0)$	$\Xi_{cc}K \to \Xi_{cc}K$	C	$(-1, 0)$	$D\bar{K} \to D\bar{K}$
✓ $(1, 1)$	$\Xi_{cc}K \to \Xi_{cc}K$	C	$(-1, 1)$	$D\bar{K} \to D\bar{K}$
$(0, \frac{1}{2})$	$\Xi_{cc}\pi \to \Xi_{cc}\pi$ $\Xi_{cc}\eta \to \Xi_{cc}\eta$ $\Omega_{cc}K \to \Omega_{cc}K$ $\Xi_{cc}\pi \to \Xi_{cc}\eta$ $\Xi_{cc}\pi \to \Omega_{cc}K$ $\Xi_{cc}\eta \to \Omega_{cc}K$	D	$(0, \frac{1}{2})$	$D\pi \to D\pi$ $D\eta \to D\eta$ $D_s\bar{K} \to D_s\bar{K}$ $D\pi \to D\eta$ $D\pi \to D_s\bar{K}$ $D\eta \to D_s\bar{K}$
✓ $(0, \frac{3}{2})$	$\Xi_{cc}\pi \to \Xi_{cc}\pi$	C	$(0, \frac{3}{2})$	$D\pi \to D\pi$