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利用国产格点QCD组态抽取ρ介子质量和宽度

Zheng-Li Wang, Derek B. Leinweber, Chuan Liu, Liuming Liu, Peng Sun, Anthony W. Thomas, Jia-jun Wu, Hanyang Xing and Kang Yu

Zheng-Li Wang, et al., *JHEP* 08 (2025) 064

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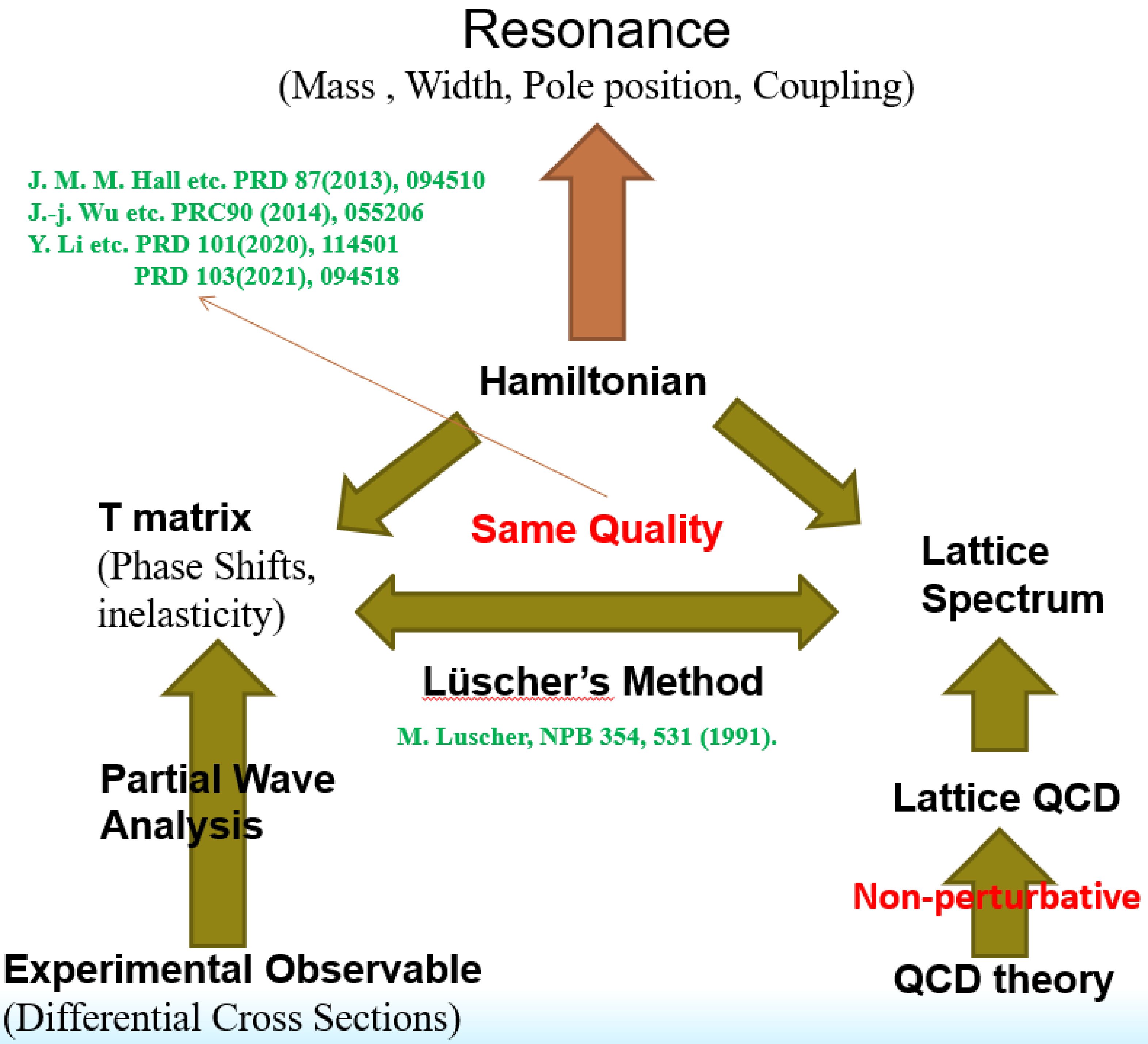
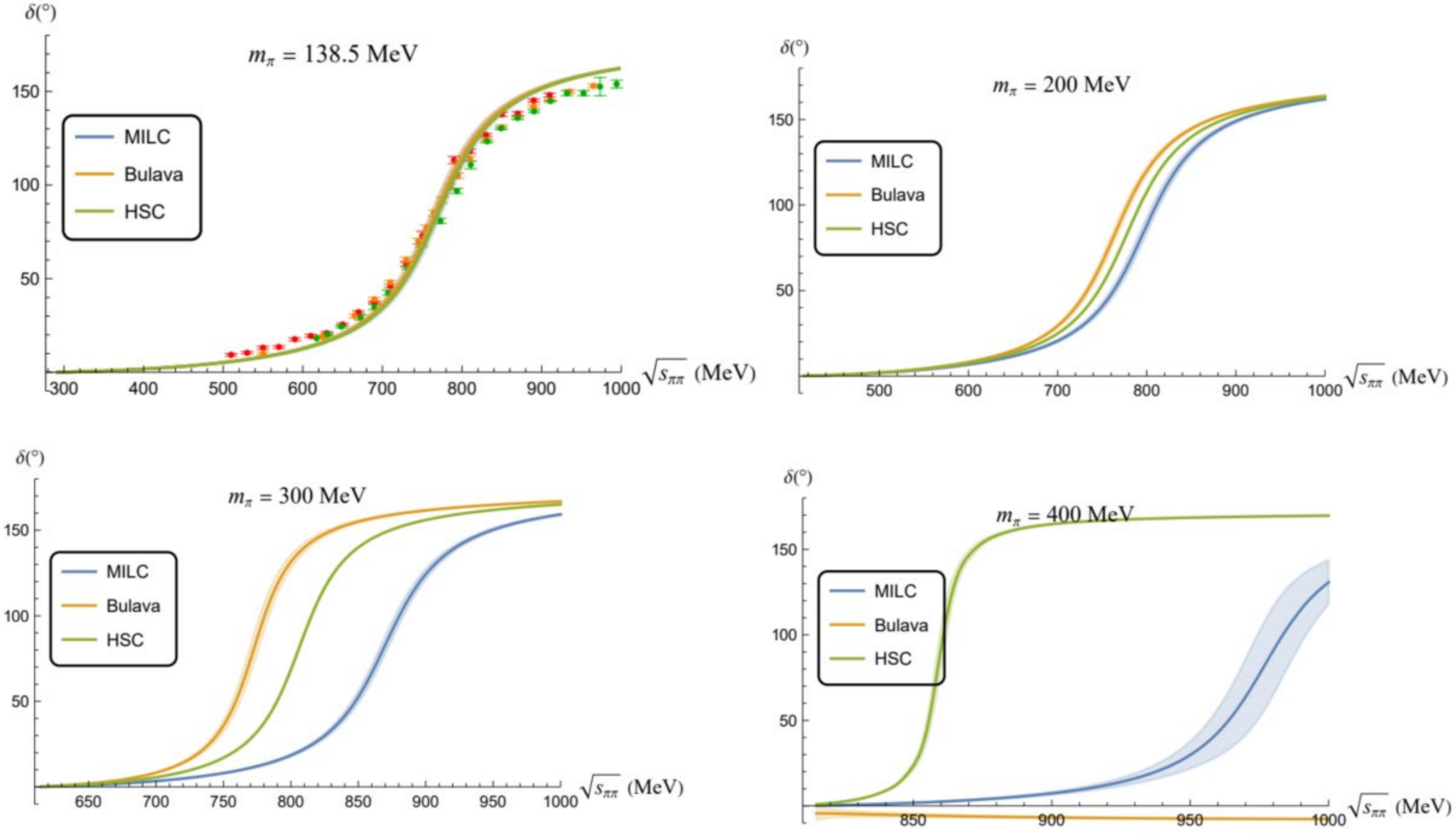


Motivation

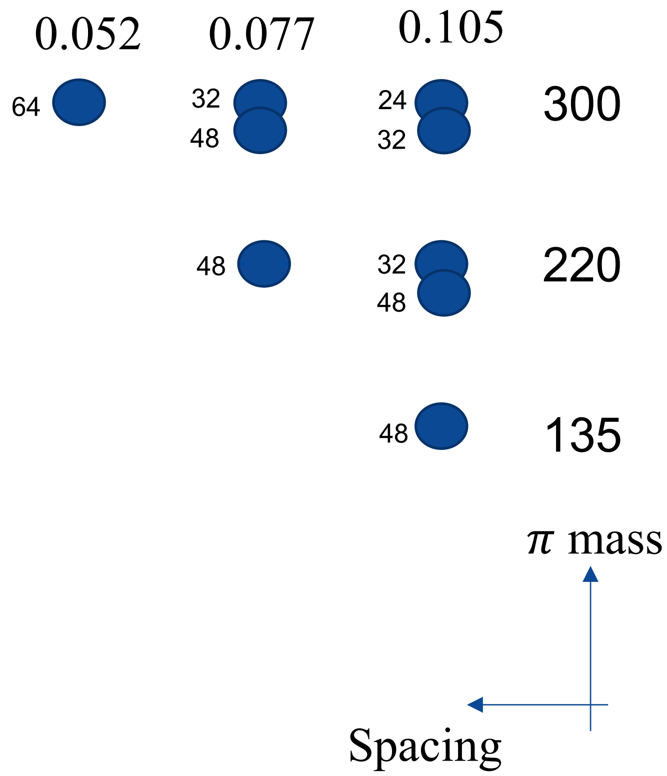
- 然而，不同的费米作用量，不同的格距，…
- 我们需要一套系统的ρ的数据！

CLQCD



Configurations

Name	Volume	Spacing	β	m_π/MeV	$m_\pi L$
C24P29	$24^3 \times 72$	$0.10530 fm$	6.20	292	3.75
C32P29	$32^3 \times 64$			292	5.01
C32P23	$32^3 \times 64$			228	3.91
C48P23	$48^3 \times 96$			225	5.79
C48P14	$48^3 \times 96$			135	3.56
F32P30	$32^3 \times 96$	$0.07746 fm$	6.41	303	3.81
F48P30	$48^3 \times 96$			303	5.72
F48P21	$48^3 \times 96$			207	3.91
H48P32	$48^3 \times 144$	$0.05187 fm$	6.72	321	4.06



Zhi-Cheng Hu, et al., Phys. Rev. D 109 (2024) 5, 054507

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Spectroscopy on lattice

- Build large basis of operators $\{\mathcal{O}_1, \mathcal{O}_2, \ldots\}$ with desired quantum numbers, construct the matrix of correlation function:

$$C_{ij} = \left\langle 0 \middle| \mathcal{O}_i \mathcal{O}_j^\dagger \middle| 0 \right\rangle = \sum_n Z_i^n Z_j^{n^*} e^{-E_n t}$$

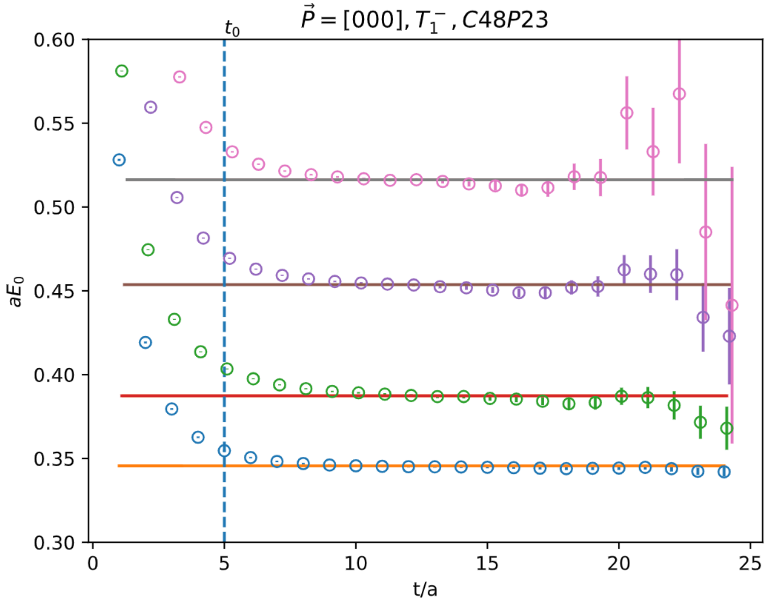
$\rho, \pi\pi, \pi\omega, K\bar{K}, \ldots$

- Solve the generalized eigenvalue problem(GEVP):

$$C_{ij} v_j^n(t) = \lambda_n(t) C_{ij}^0 v_j^n(t)$$

- Eigenvalues:

$$\lambda_n(t) \sim e^{-E_n t}\left(1 + e^{-\Delta E t}\right)$$



Operator

$$\rho^0 = \frac{1}{\sqrt{2}}(\bar{u}\Gamma^P u - \bar{d}\Gamma^P d), \quad \pi^+ = \bar{d}\Gamma^k u, \quad \pi^- = \bar{u}\Gamma^k d$$

$$\begin{aligned} \pi\pi &= \frac{1}{\sqrt{2}}(\pi^+(k_1)\pi^-(k_2) - \pi^-(k_1)\pi^+(k_2)) \\ &= \frac{1}{\sqrt{2}}(\bar{d}\Gamma^{k_1}u\bar{u}\Gamma^{k_2}d - \bar{u}\Gamma^{k_1}d\bar{d}\Gamma^{k_2}u) \end{aligned}$$

$$\langle \rho(y)\rho(x)^\dagger \rangle = -Tr[\Gamma^P M^{-1}(y,x)\bar{\Gamma}^P M^{-1}(x,y)]$$

State	J^{PC}	Γ	Particles
Scalar	0^{++}	$\mathbb{1}$, γ_4	f_0, a_0, K_0^*, ...
Pseudoscalar	0^{-+}	γ_5 , $\gamma_4\gamma_5$	$\pi^\pm, \pi^0, \eta, K^\pm, K^0$, ...
Vector	1^{--}	γ_i , $\gamma_4\gamma_i$	$\rho^\pm, \rho^0, \omega, K^*, \phi$, ...
Axial vector	1^{++}	$\gamma_i\gamma_5$	a_1, f_1, ...
Tensor	1^{+-}	$\gamma_i\gamma_j$	h_1, b_1, ...

LapH smearing and distillation

$$q^s_x(t) = \sum_{k=1}^{N_v} v_x^{(k)}(t) v_y^{(k)\dagger}(t) q_y(t) \equiv \Box_{xy}(t) q_y(t)$$

$$\nabla^2(t) v^{(k)}(t) = \lambda^{(k)}(t) v^{(k)}(t)$$

$$-\nabla^2_{xy}(t) = 6\delta_{xy} - \sum_{j=1}^{3} (\tilde{U}_j(x,t)\delta_{x+\hat{j},y} + \tilde{U}^\dagger_j(x-\hat{j},t)\delta_{x-\hat{j},,y})$$

$$\pi^-(\vec{p},t) = \bar{u}_x(t)\Box_{xy}(t)\cdot e^{-ip\cdot y}\Gamma_{yz}(t)\cdot\Box_{zw}(t) d_w(t)$$

$$\langle \pi^+_{t'} (\pi^+)^\dagger_t \rangle = -Tr[\phi^k_{t'} \tau^u_{t't} \bar{\phi}^k_t \tau^d_{tt'}]$$

$$\tau_{\alpha\beta}(t',t) = V^\dagger(t') M^{-1}_{\alpha\beta} V(t)$$

1. 可以计算非连通图
2. 节省大量时间

$$M^{-1} \rightarrow \tau$$

$$N_c N_L^3 \rightarrow N_v$$

HSC, *Phys.Rev.D* 80 (2009) 054506

Symmetries

$[000]T_1^-$	$[001]A_1$	$[001]E_2$	$[011]A_1$
$\rho[000]$	$\rho[001]$	$\rho[001]$	$\rho[011]$
$\pi[001]\pi[00-1]$	$\pi[000]\pi[001]$	$\pi[0-10]\pi[011]$	$\pi[000]\pi[011]$
$\pi[011]\pi[0-1-1]$	$\pi[0-10]\pi[011]$	$\pi[0-1-1]\pi[111]$	$\pi[-100]\pi[111]$
$\pi[111]\pi[-1-1-1]$	$\pi[-1-10]\pi[111]$		$\pi[01-1]\pi[002]$
	$\pi[00-1]\pi[002]$		

$[011]B_1$	$[011]B_2$	$[111]A_1$	$[002]A_1$
$\rho[011]$	$\rho[011]$	$\rho[111]$	$\rho[002]$
$\pi[010]\pi[001]$	$\pi[-100]\pi[111]$	$\pi[000]\pi[111]$	$\pi[000]\pi[002]$
$\pi[110]\pi[-101]$	$\pi[110]\pi[-101]$	$\pi[100]\pi[011]$	
$\pi[0-11]\pi[002]$		$\pi[200]\pi[-111]$	

$$C_{ij}(t)v_j^n(t) = \lambda_n(t)C_{ij}(t_0)v_j^n(t)$$

Thermal state

$$C^w_{\pi'\pi}(t) = e^{(E_{\pi'}-E_\pi)t} C_{\pi'\pi}(t)$$
$$\tilde{C}^w_{\pi'\pi}(t) = C^w_{\pi'\pi}(t) - C^w_{\pi'\pi}(t+\delta t)$$
$$C^E_{\pi'\pi}(t) = e^{-(E_{\pi'}-E_\pi)t} \tilde{C}^w_{\pi'\pi}(t)$$

$$\begin{aligned} C_{\pi'}(t)C_\pi(t) \approx\ & (e^{-(E_{\pi'}+E_\pi)t} + e^{-(E_{\pi'}+E_\pi)(T-t)}) \\ & + (e^{-E_\pi T} e^{(E_\pi - E_{\pi'})t} + e^{-E_{\pi'} T} e^{(E_{\pi'}-E_\pi)t}) \end{aligned}$$

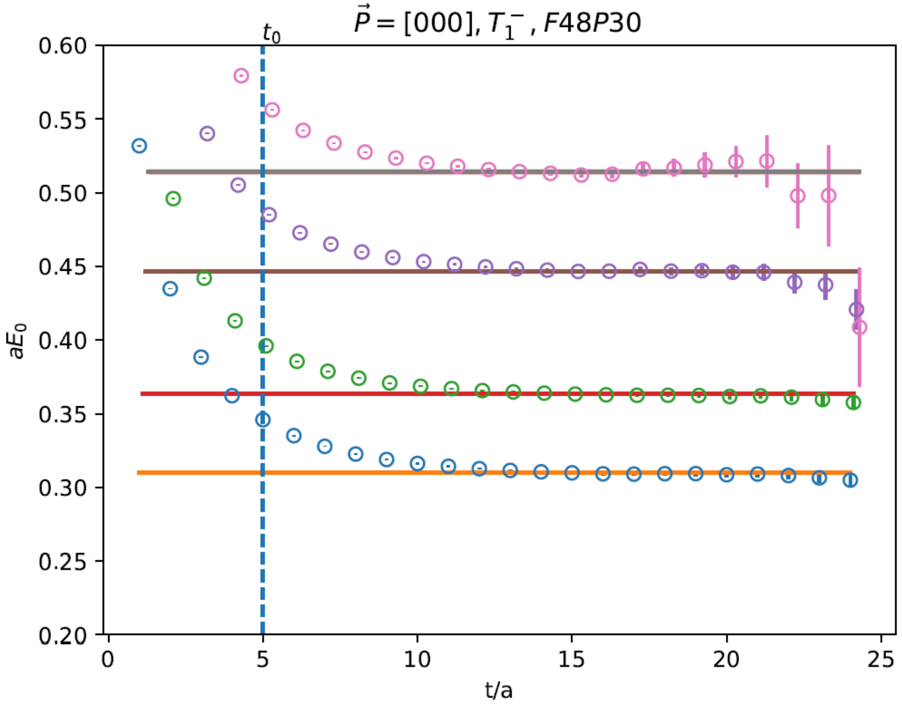
$$\begin{aligned} C_{\pi'\pi}(t) \approx\ & (e^{-E_{\pi'\pi}t} + e^{-E_{\pi'\pi}(T-t)}) \\ & + A(e^{-E_\pi T} e^{(E_\pi - E_{\pi'})t} + e^{-E_{\pi'} T} e^{(E_{\pi'}-E_\pi)t}) \end{aligned}$$

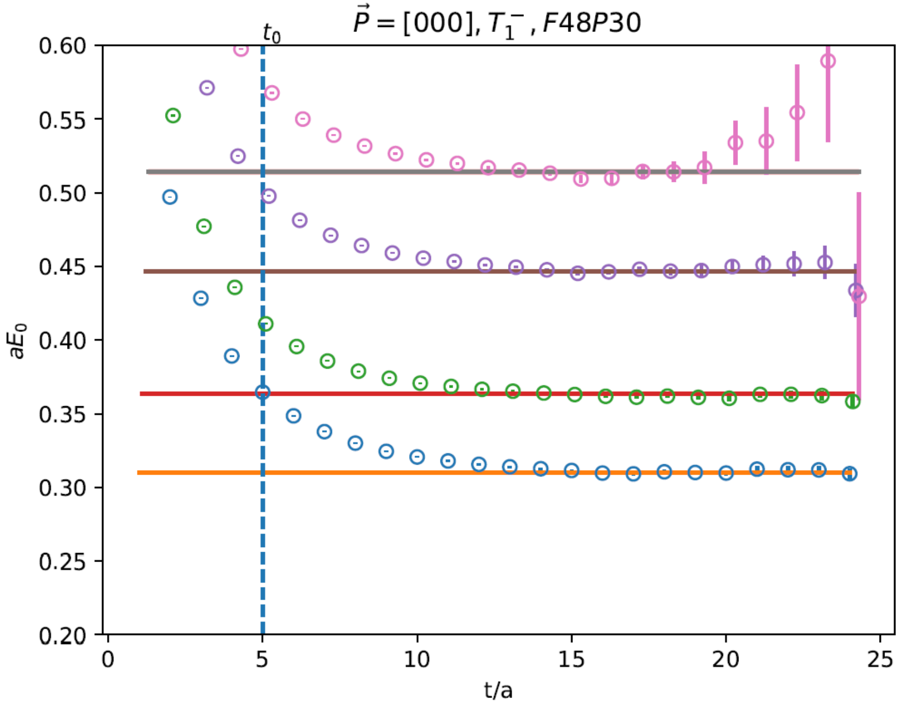
$$\begin{aligned} C^E_{\pi'\pi}(t) =\ & (1 - e^{(E_{\pi'}-E_\pi-E_{\pi'\pi})\delta t})(e^{-E_{\pi'\pi}t} + e^{-E_{\pi'\pi}(T-t)}) \\ & + (e^{(E_{\pi'}-E_\pi-E_{\pi'\pi})\delta t} - e^{(E_{\pi'}-E_\pi+E_{\pi'\pi})\delta t}) e^{-E_{\pi'\pi}T} e^{E_{\pi'\pi}t} \\ & + A(1 - e^{(E_{\pi'}-E_\pi)2\delta t}) e^{-E_{\pi'}T} e^{(E_{\pi'}-E_\pi)t} \end{aligned}$$

反向传播

w/

w/o





GEVP

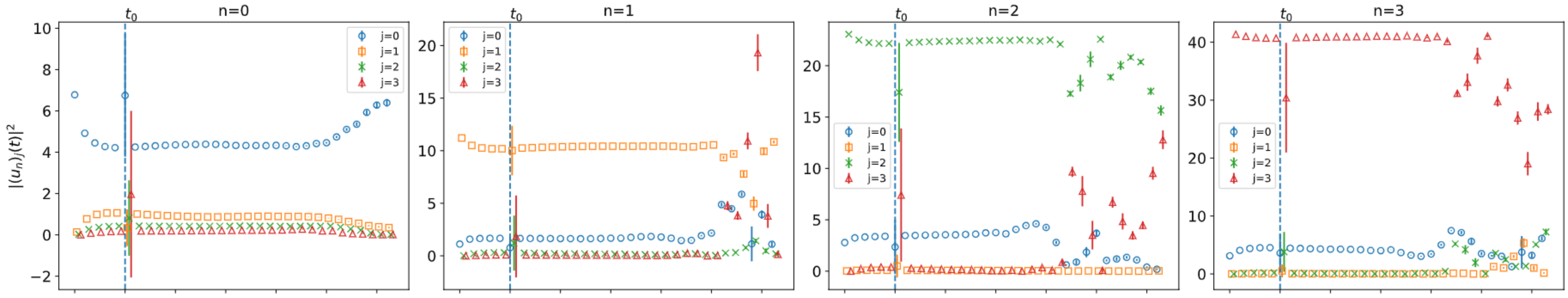
- 排序：本征值，本征矢
- Fixed-eigenvector method

$$C(t)u_n(t,t_0) = \lambda_n(t,t_0)C(t_0)u_n(t,t_0)$$

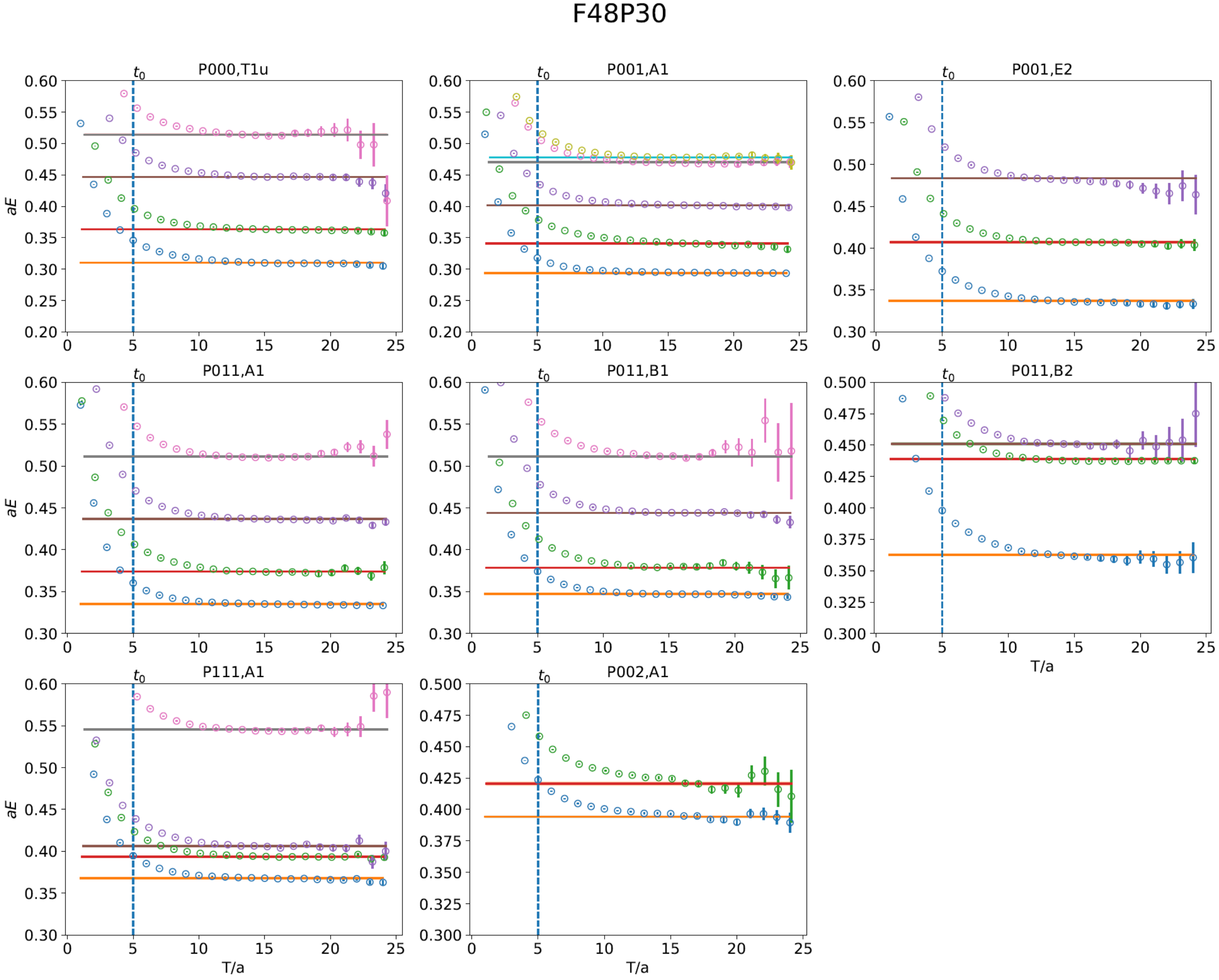
$$\lambda_n(t,t_0) = u_i^{n*}(t,t_0)C_{ij}(t)u_j^n(t,t_0)$$

$$\lambda_n(t,t_0) = u_i^{n*}(\tilde{t},t_0)C_{ij}(t)u_j^n(\tilde{t},t_0)$$

John M. Bulava, et al., *Phys.Rev.D* 79 (2009) 034505
M. Selim Mahbub, et al., *Phys.Rev.D* 87 (2013) 9, 094506
Adrian L. Kiratidis, et al., *Phys.Rev.D* 91 (2015) 094509
Adrian L. Kiratidis, et al., *Phys.Rev.D* 95 (2017) 7, 074507



Spectra



Including $|p|^2 = 0$ only T_1^-, $|p|^2 = 1$ with A_1 and E_2, $|p|^2 = 2$ with A_1, B_1 and B_2, and $|p|^2 = 3$ and 4 only with A_1 irreps.

We should mention that since the operator basis we have used does not sample well the inelastic spectrum, we will restrict our analysis to the elastic region below $K\bar{K}$ and $\pi\omega$

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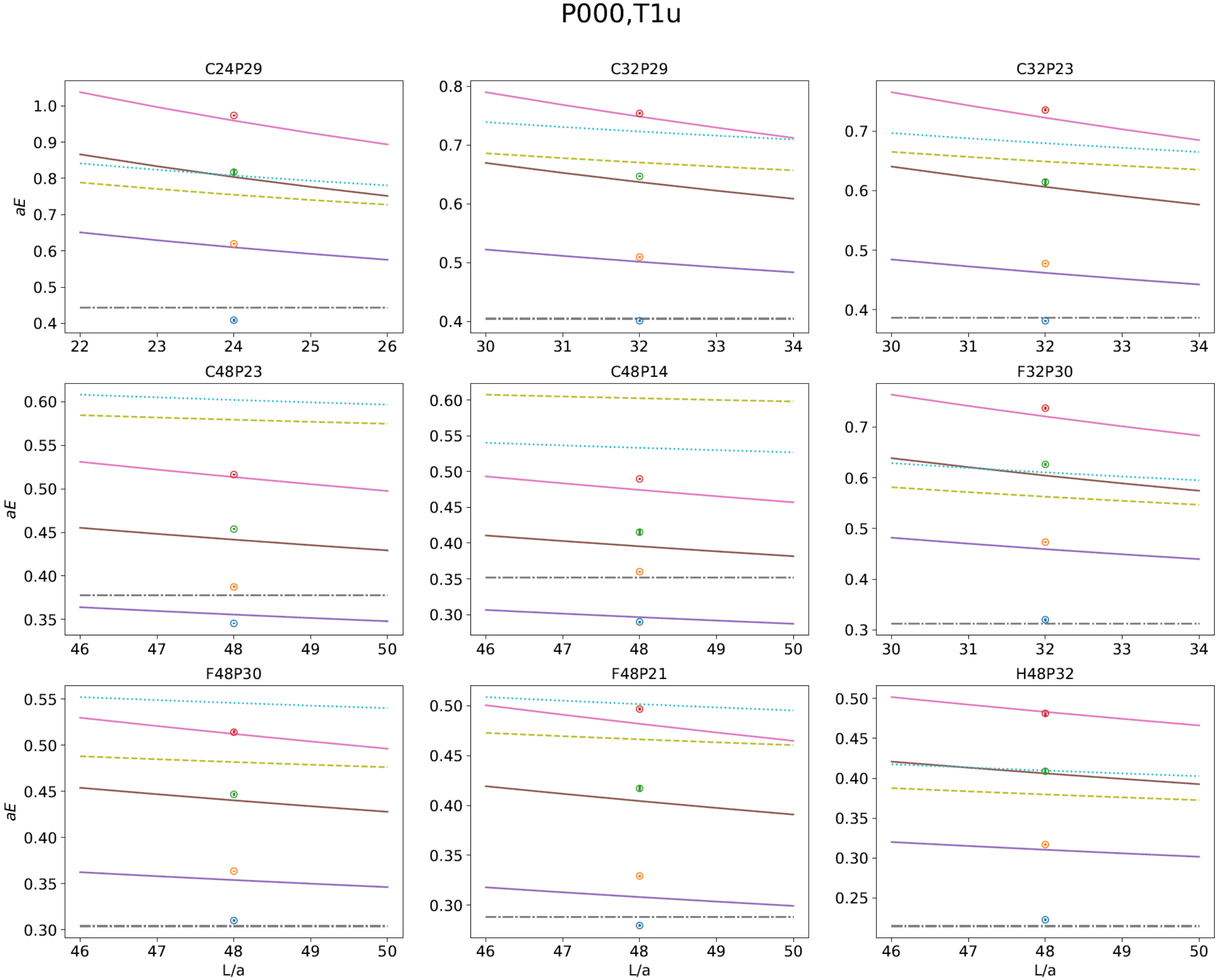
至此，我们得到的了一套系统的ρ的格点能谱，即同一作用量下不同格距和不同pion质量的数据！

8*9=72组！

Finite volume spectra: P000/T1u



1. The ρ can decay in some lattice
2. The spectra above ρ lie above the free energy levels of $\pi\pi$, spectra below ρ lie below the free energy levels of $\pi\pi$
3. There is only one energy level between the two free levels of $\pi\pi$
4. Energy levels decrease as the size of the lattice L increases
5. The ρ is more likely to decay in moving systems
6. The energy levels farther from the mass of the ρ being near the free energy levels of $\pi\pi$

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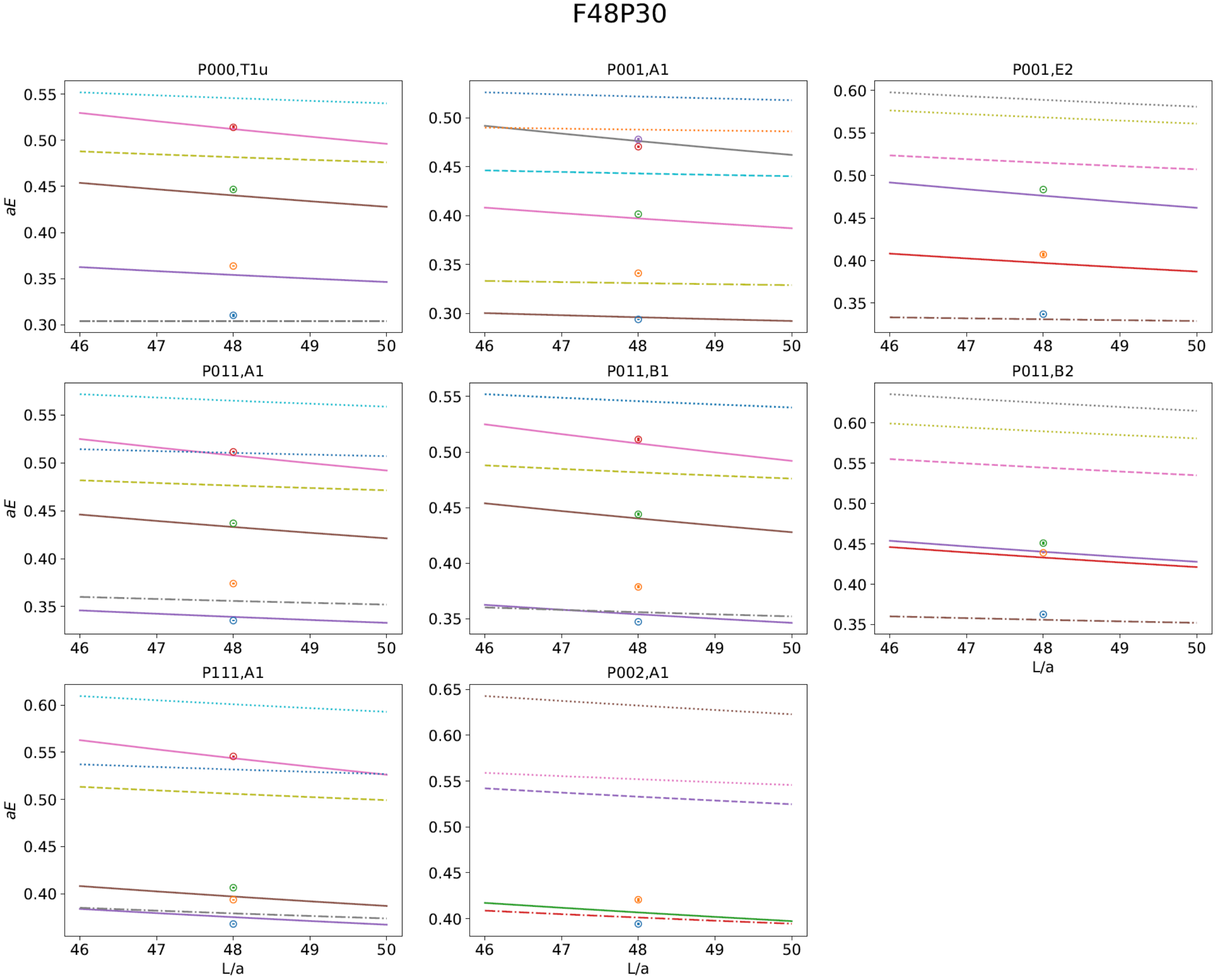
Finite volume spectra: F48P30



1. The ρ can decay in some lattice
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3. There is only one energy level between the two free levels of $\pi\pi$
4. Energy levels decrease as the size of the lattice L increases
5. The ρ is more likely to decay in moving systems, (0,0,1/2)
6. The energy levels farther from the mass of the ρ being near the free energy levels of $\pi\pi$

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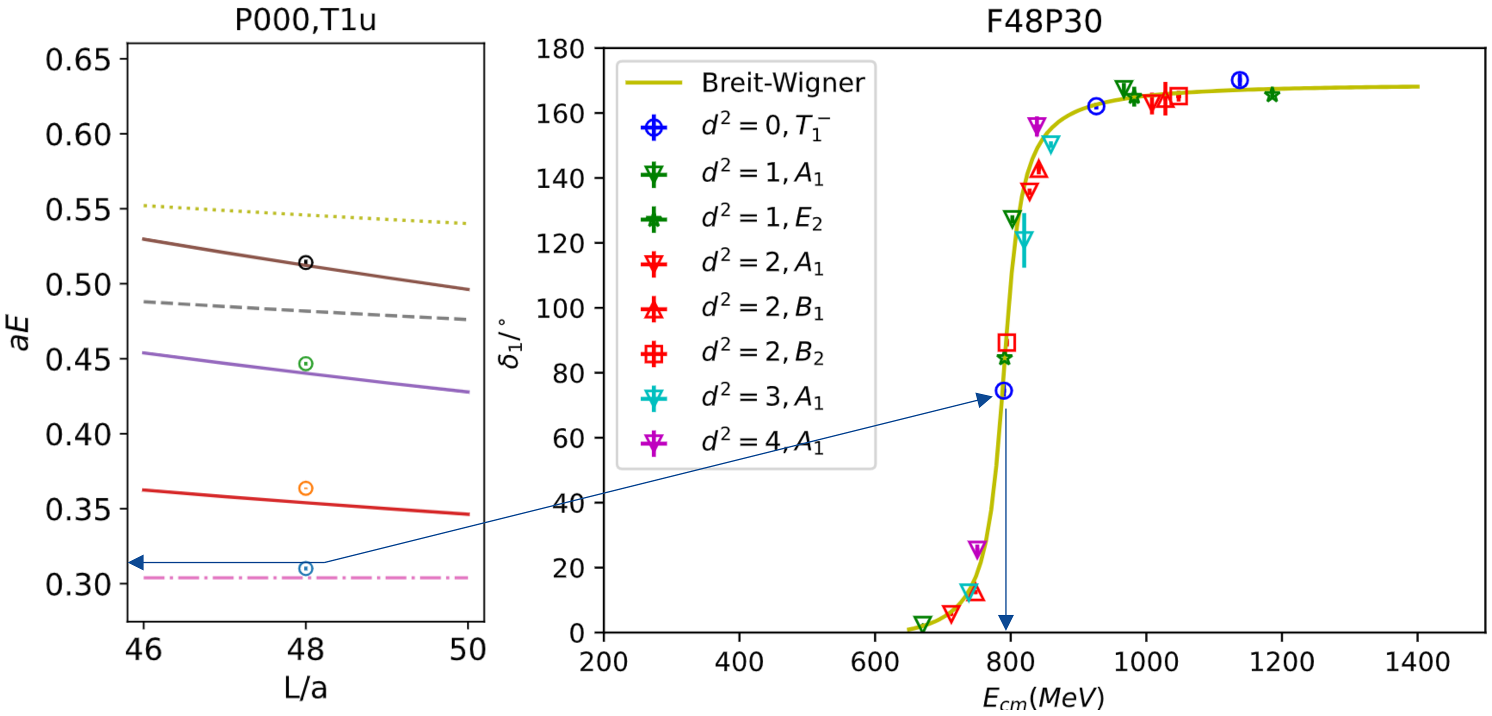
Luescher Formalisms

$$\det\left(M_{ln,l'n'}(k) - \delta_{ll'}\delta_{nn'}\cot(\delta_l)\right) = 0,$$

$$\delta_1 = \operatorname{arccot} M_{11,11}^{(\vec{P},\Lambda,\mu)},$$

$$w_{j,s} = \frac{\mathcal{Z}_{js}\left(1,q^2\right)}{\pi^{3/2}\sqrt{2j+1}\gamma q^{j+1}}, \qquad q = \frac{kL}{2\pi},$$

d^2	Λ, μ	$M_{11,11}^{(\vec{P},\Lambda,\mu)}$
0	$T_1^-, 2$	$w_{0,0}$
1	$A_1, 1$	$w_{0,0} + 2w_{2,0}$
1	$E_2, 1$	$w_{0,0} - w_{2,0}$
1	$E_2, 2$	$w_{0,0} - w_{2,0}$
2	$A_1, 1$	$w_{0,0} + \frac{1}{2}w_{2,0} + i\sqrt{6}w_{2,1} - \frac{\sqrt{6}}{2}w_{2,2}$
2	$B_1, 1$	$w_{0,0} + \frac{1}{2}w_{2,0} - i\sqrt{6}w_{2,1} - \frac{\sqrt{6}}{2}w_{2,2}$
2	$B_2, 1$	$w_{0,0} - w_{2,0} + \sqrt{6}w_{2,2}$
3	$A_1, 1$	$w_{0,0} - i2\sqrt{6}w_{2,2}$



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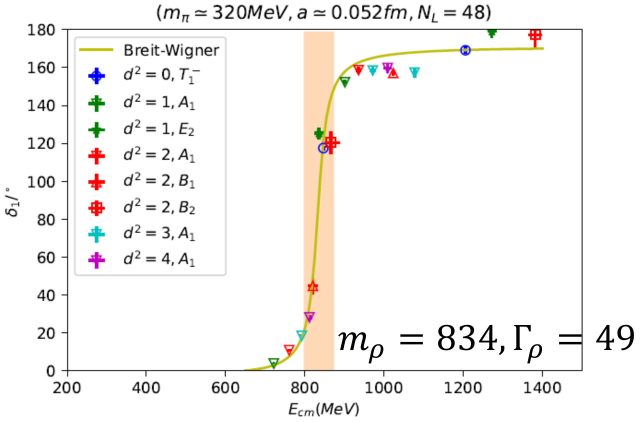
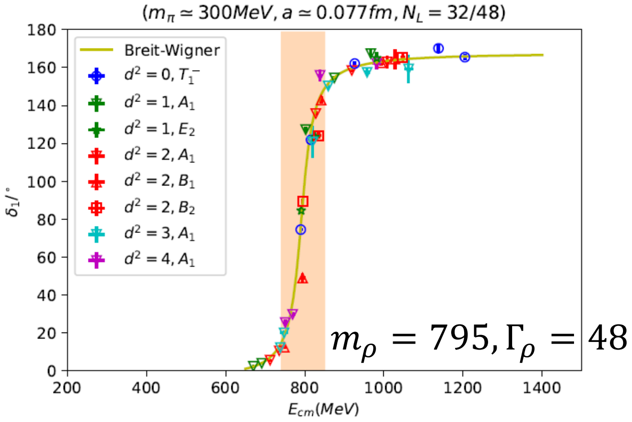
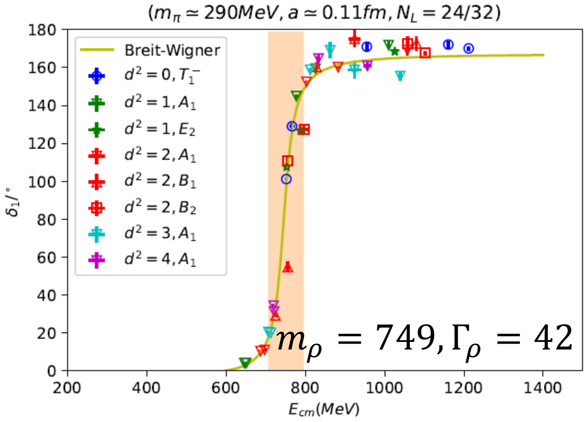
$$a_1 = \frac{-\sqrt{s}\Gamma(s)}{s - m_\rho^2 + \mathrm{i}\sqrt{s}\Gamma(s)} = \mathrm{e}^{\mathrm{i}\delta(s)}\sin\delta(s),$$

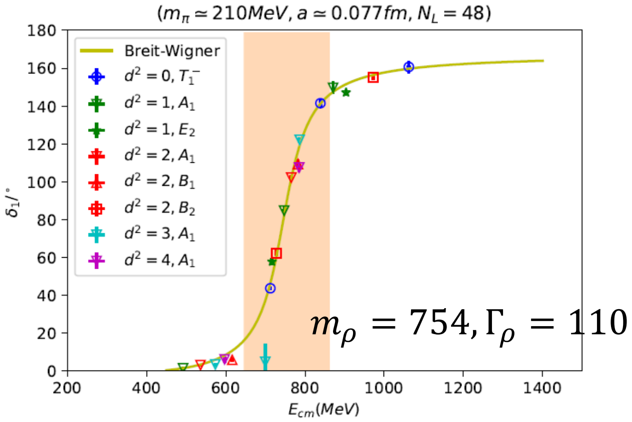
$$\Gamma(s) = \frac{p^{*3}}{s}\frac{g_{\rho\pi\pi}^2}{6\pi}.$$

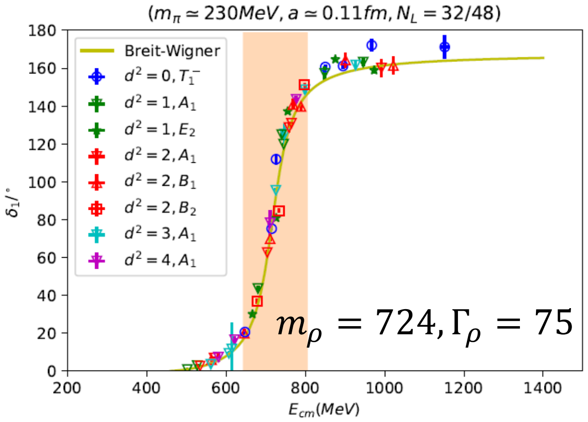




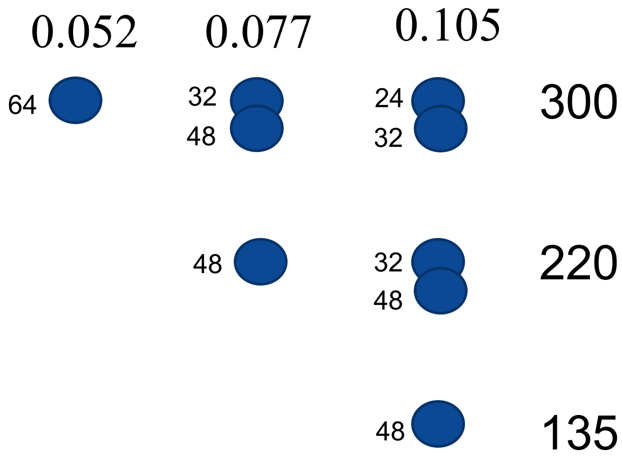


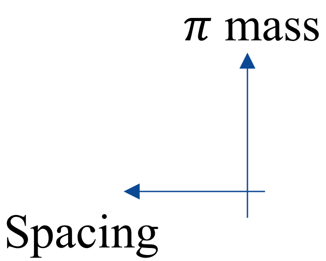
	m_ρ (MeV)	$g_{\rho\pi\pi}$	$Z_{\rm pole}$(MeV)
$(m_\pi \simeq 290MeV, a \simeq 0.11fm, N_L = 24/32)$	749.0(6.0)	5.859(79)	$745.7(6.0) - i\,19.7(1.5)$
$(m_\pi \simeq 230MeV, a \simeq 0.11fm, N_L = 32/48)$	723.9(5.6)	5.827(14)	$717.1(5.6) - i\,37.2(2.8)$
$(m_\pi \simeq 140MeV, a \simeq 0.11fm, N_L = 48)$	714.0(34.5)	6.300(440)	$699.2(34.4) - i\,71.4(22.5)$
$(m_\pi \simeq 300MeV, a \simeq 0.077fm, N_L = 32/48)$	795.5(4.2)	5.775(58)	$791.7(4.3) - i\,22.6(1.0)$
$(m_\pi \simeq 210MeV, a \simeq 0.077fm, N_L = 48)$	754.2(3.0)	5.954(38)	$744.7(3.0) - i\,49.5(1.7)$
$(m_\pi \simeq 320MeV, a \simeq 0.052fm, N_L = 48)$	834.8(7.4)	4.916(45)	$832.5(7.4) - i\,18.1(0.9)$

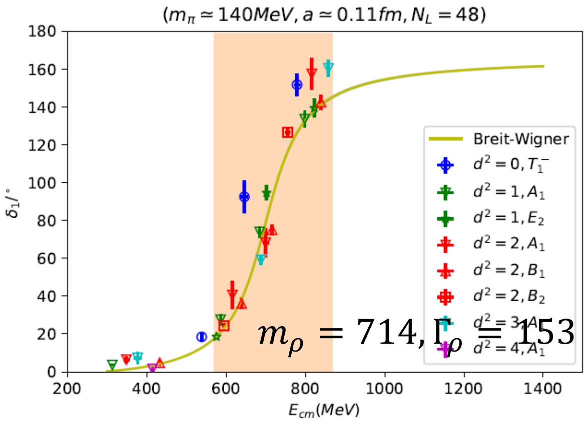




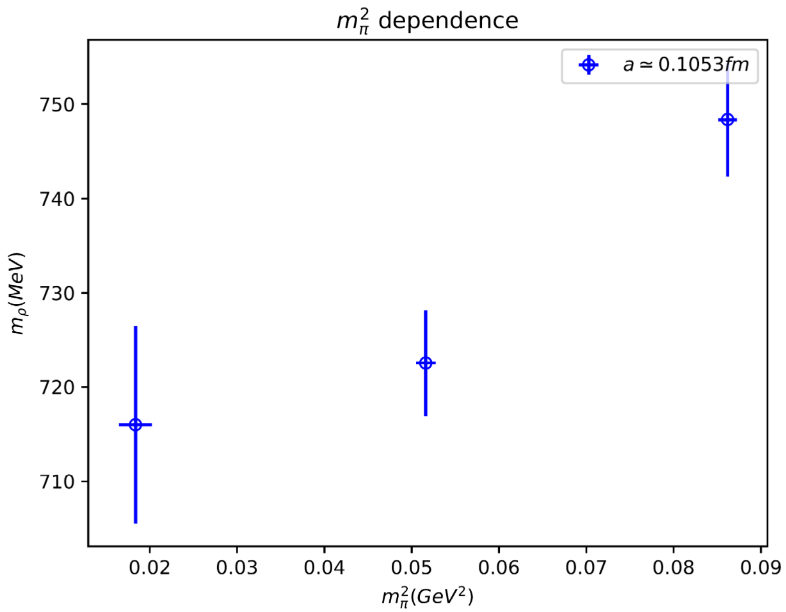
1. The lineshape of the phase shift from large lattice size are even more smooth since $m_\pi L$ is larger
2. There exists the jump of phase shift from 0 degree to 180 degrees

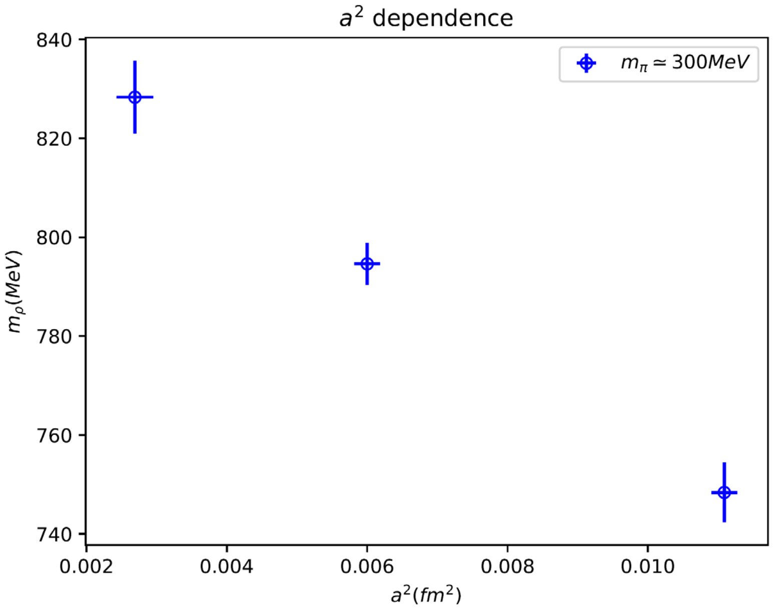






Extrapolation





0.052	0.077	0.105	
834/49	795/48	794/42	300
	754/110	724/75	220
		714/153	135

$$m_\rho = c_0 + c_1 m_\pi^2 + c_2 a^2,$$

$$g_{\rho\pi\pi} = \tilde{c}_0 + \tilde{c}_1 m_\pi^2 + \tilde{c}_2 a^2,$$

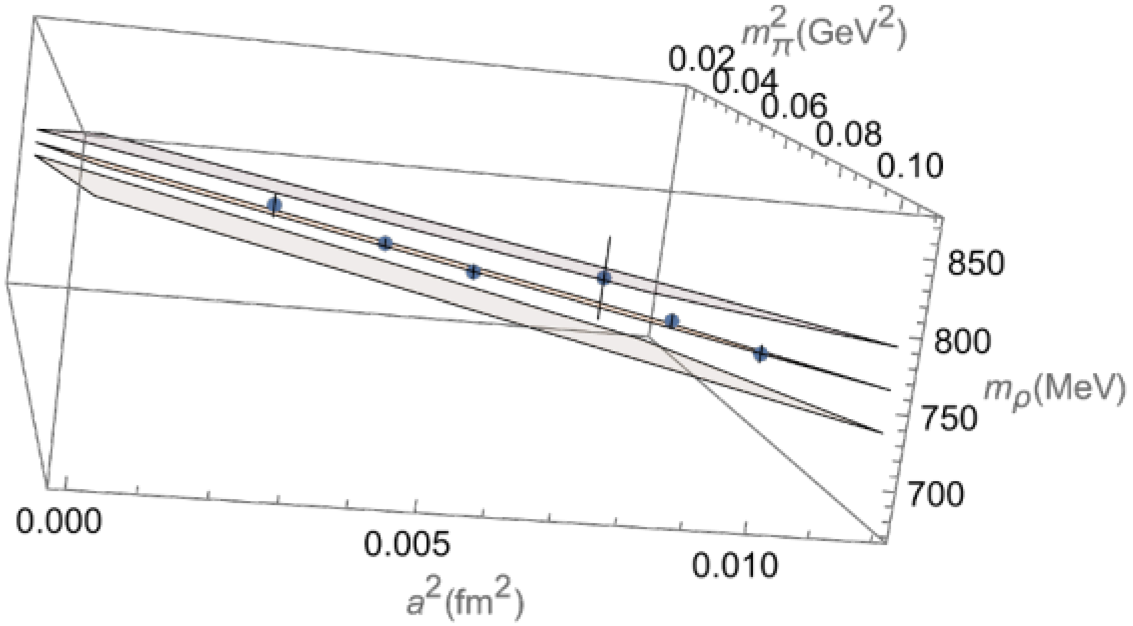
π mass

Spacing

$$c_0 = 766.2(8.5)MeV$$
$$c_1 = 0.84(9)GeV^{-1}$$
$$c_2 = -7.97(81)GeV \cdot fm^{-2}$$

$$\tilde{c}_0 = 5.74(67)$$
$$\tilde{c}_1 = -5.3(6.4)GeV^{-2}$$
$$\tilde{c}_2 = 46(48)fm^{-2}$$

Extrapolation



$$m_\rho = c_0 + c_1 m_\pi^2 + c_2 a^2, \qquad g_{\rho\pi\pi} = \tilde{c}_0 + \tilde{c}_1 m_\pi^2 + \tilde{c}_2 a^2,$$

$$m_\rho = 781.6(10.0)\,\text{MeV}, \quad \Gamma_\rho(m_\rho) = 146.5(9.9)\,\text{MeV},$$

$$Z_{\text{pole}} = 768.1\,(10.0) - i\,70.5\,(4.9)\,\text{MeV}.$$

$\rho(770)$ T-MATRIX POLE $\sqrt{s}$		$(761 - 765) - i(71 - 74)$ MeV
▾ $\rho(770)$ **MASS**		
NEUTRAL ONLY, e^+e^-	[1]	775.26 ± 0.23 MeV
CHARGED ONLY, τ DECAYS and e^+e^-	[2]	775.11 ± 0.34 MeV
MIXED CHARGES, OTHER REACTIONS		763.0 ± 1.2 MeV
CHARGED ONLY, HADROPRODUCED		766.5 ± 1.1 MeV
NEUTRAL ONLY, PHOTOPRODUCED		769.2 ± 0.9 MeV
NEUTRAL ONLY, OTHER REACTIONS		769.0 ± 0.9 MeV (S = 1.4)
$m_{\rho(770)^0} - m_{\rho(770)^\pm}$		-0.7 ± 0.8 MeV (S = 1.5)
$m_{\rho(770)^+} - m_{\rho(770)^-}$		
$\rho(770)$ RANGE PARAMETER		$5.3^{+0.9}_{-0.7}$ GeV^{-1}
▾ $\rho(770)$ **WIDTH**		
NEUTRAL ONLY, e^+e^-	[1]	147.4 ± 0.8 MeV (S = 2.0)
CHARGED ONLY, τ DECAYS and e^+e^-	[2]	149.1 ± 0.8 MeV
MIXED CHARGES, OTHER REACTIONS		149.5 ± 1.3 MeV
CHARGED ONLY, HADROPRODUCED		150.2 ± 2.4 MeV
NEUTRAL ONLY, PHOTOPRODUCED		$151.5^{+1.9}_{-2.1}$ MeV
NEUTRAL ONLY, OTHER REACTIONS		150.9 ± 1.7 MeV (S = 1.1)
$\Gamma_{\rho(770)^0} - \Gamma_{\rho(770)^\pm}$		0.3 ± 1.3 MeV (S = 1.4)
$\Gamma_{\rho(770)^+} - \Gamma_{\rho(770)^-}$		1.8 ± 2.1

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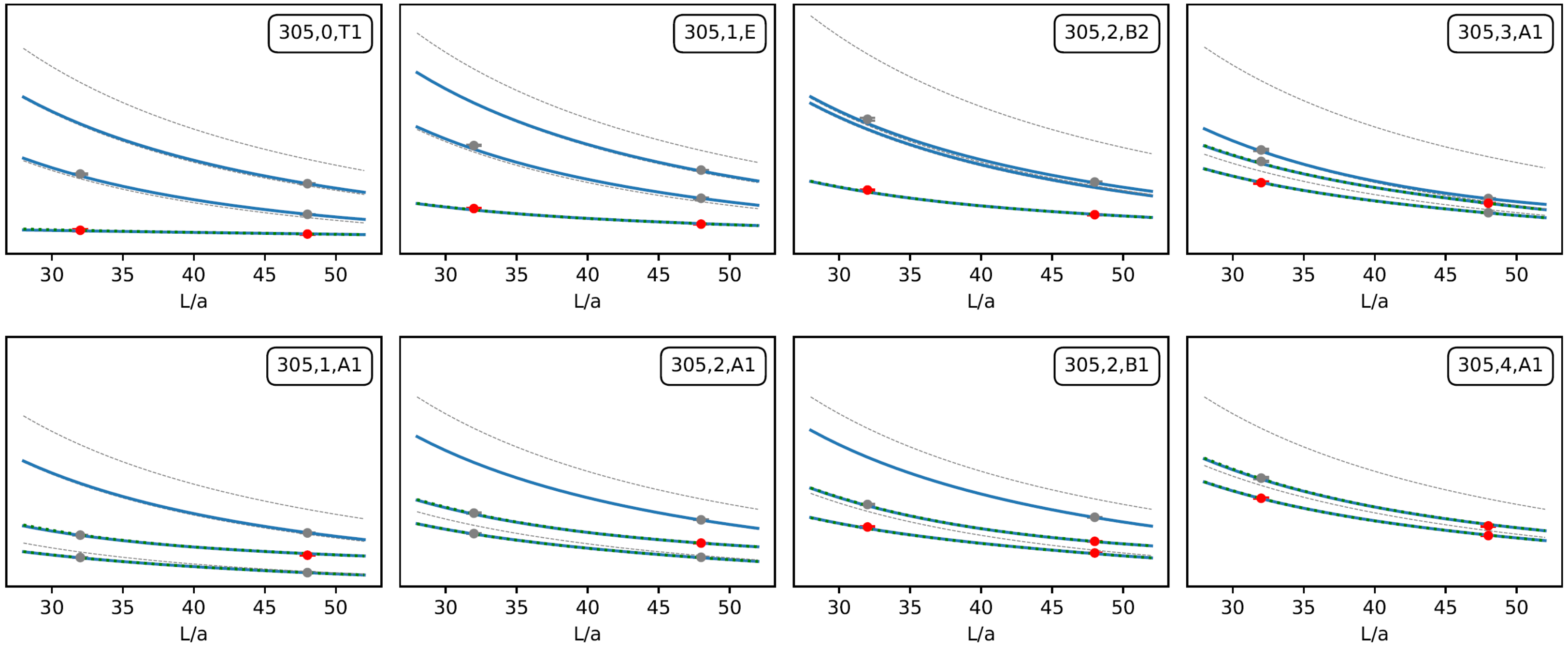
HEFT

$$T^{l=1}_{\pi\pi\to\pi\pi}(z) = \frac{\left|V_{\rho\pi\pi}(\bar{k}(z))\right|^2}{z - m_\rho^B - \Sigma(z)},$$

$$\Sigma_{\pi\pi}(z) = \int q^2 dq \frac{|V_{\rho\pi\pi}(q)|^2}{z - 2\sqrt{q^2 + m_\pi^2} + i\epsilon},$$

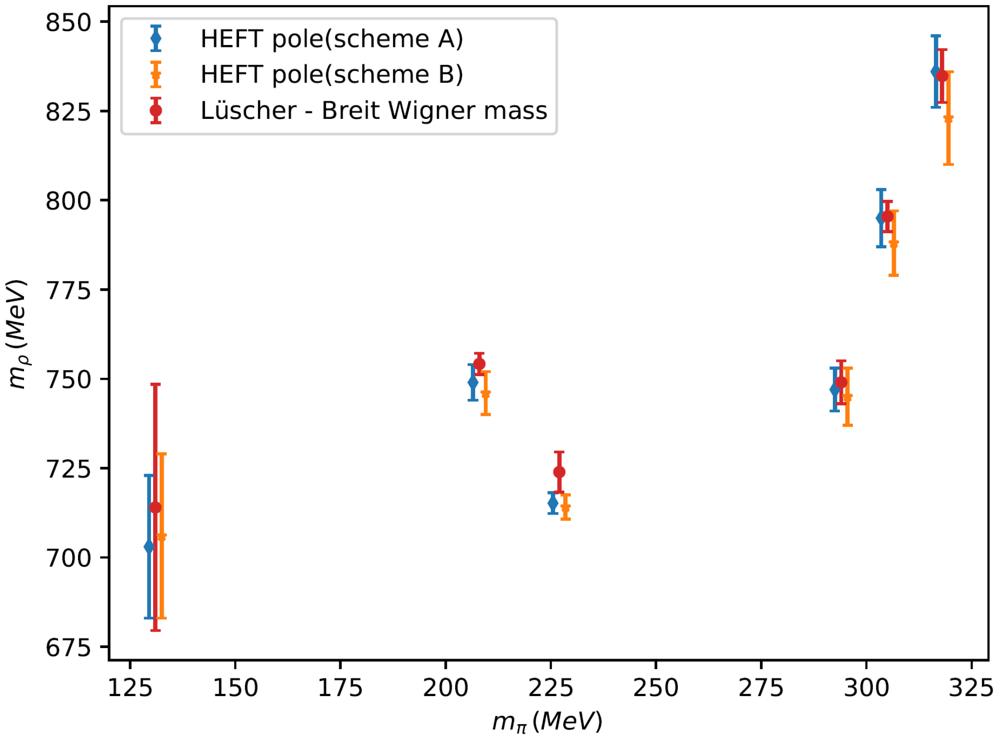
$$\Sigma_{\omega\pi}(z) = \int q^2 dq \frac{|V_{\rho\omega\pi}(q)|^2}{z - \sqrt{q^2 + m_\pi^2} - \sqrt{q^2 + m_\omega^2} + i\epsilon}$$



$$V_{\rho\pi\pi}(k) = \frac{g_{\rho\pi\pi} k}{\sqrt{m_\rho^B E_\pi(k)}} \left(\frac{\Lambda_{\rho\pi\pi}^2}{k^2 + \Lambda_{\rho\pi\pi}^2}\right)^2,$$

$$V_{\rho\omega\pi}(k) = \frac{g_{\rho\omega\pi} k \sqrt{m_\rho^B}}{\sqrt{2E_\pi(k)\, E_\omega(k)}} \left(\frac{\Lambda_{\rho\omega\pi}^2 - \mu_\pi^2}{k^2 + \Lambda_{\rho\omega\pi}^2}\right)^2,$$

- Three free parameters, $m_\rho^B, g_{\rho\pi\pi}, \Lambda_{\rho\pi\pi}$
- Here we take $g_{\rho\pi\pi}, \Lambda_{\rho\pi\pi}$ as constant for different m_π and a.
- But m_ρ^B is pion mass dependence.



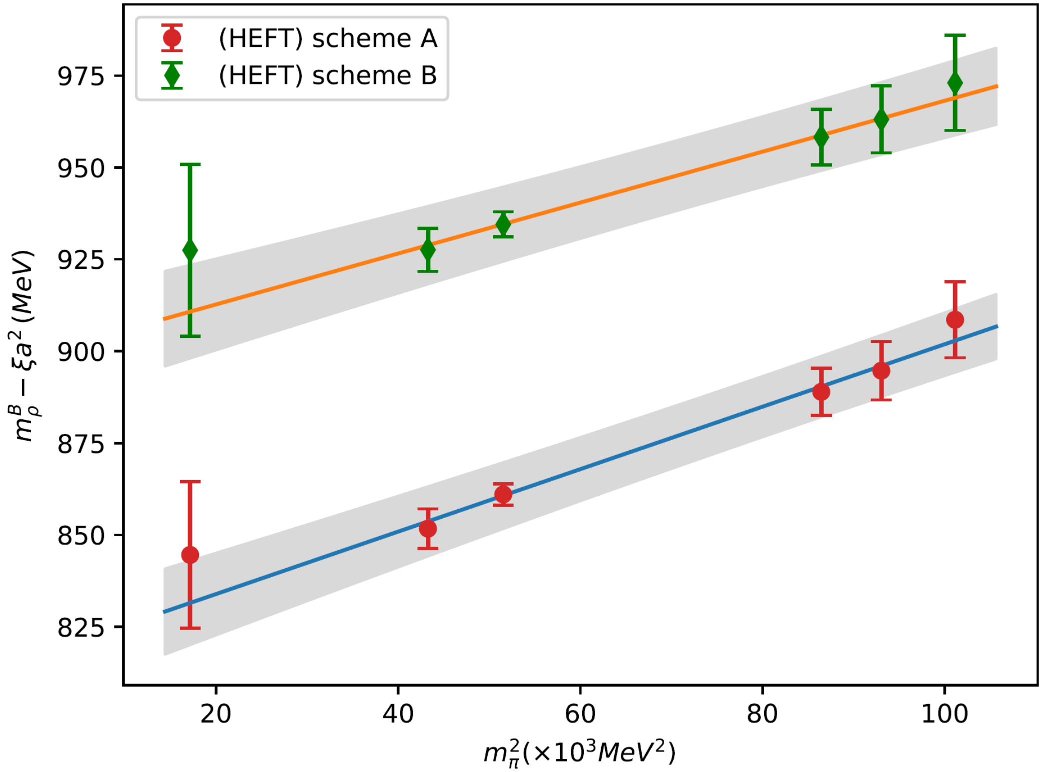
HEFT

$$m_\rho^B(m_\pi, a) = c_0^B + c_1^B m_\pi^2 + c_2^B a^2,$$

$$(c_0^B, c_1^B, c_2^B)^{\rm A} = (\,817.0\,(13.0),\ 0.85\,(12),\ -7.61\,(89)\,),$$
$$(c_0^B, c_1^B, c_2^B)^{\rm B} = (\,840.0\,(11.2),\ 0.72\,(11),\ -7.91\,(86)\,).$$

$$m_\rho^{\rm pole,ext(A)} = 787.0\,(15.0) - i\,59.0\,(6.0)\mathrm{MeV},$$
$$m_\rho^{\rm pole,ext(B)} = 777.0\,(15.0) - i\,60.0\,(6.0)\mathrm{MeV}.$$

$$m_\rho = 782.0(13.5)\,\mathrm{MeV}, \quad \Gamma_\rho(m_\rho) = 155.0(12.0)\,\mathrm{MeV}.$$



	m_ρ (MeV)	Γ_ρ(MeV)	$Z_{\rm pole}$(MeV)
Exp.	769.0(0.9)	$151.5^{+1.9}_{-2.1}$	$(761-765) - i(71-74)$
Lüscher	781.6(10.0)	146.5(9.9)	$768.1(10.0) - i70.5(4.9)$
HEFT, scheme B	782.0(13.5)	155.0(12.0)	$777.0(15.0) - i60.0(6.0)$

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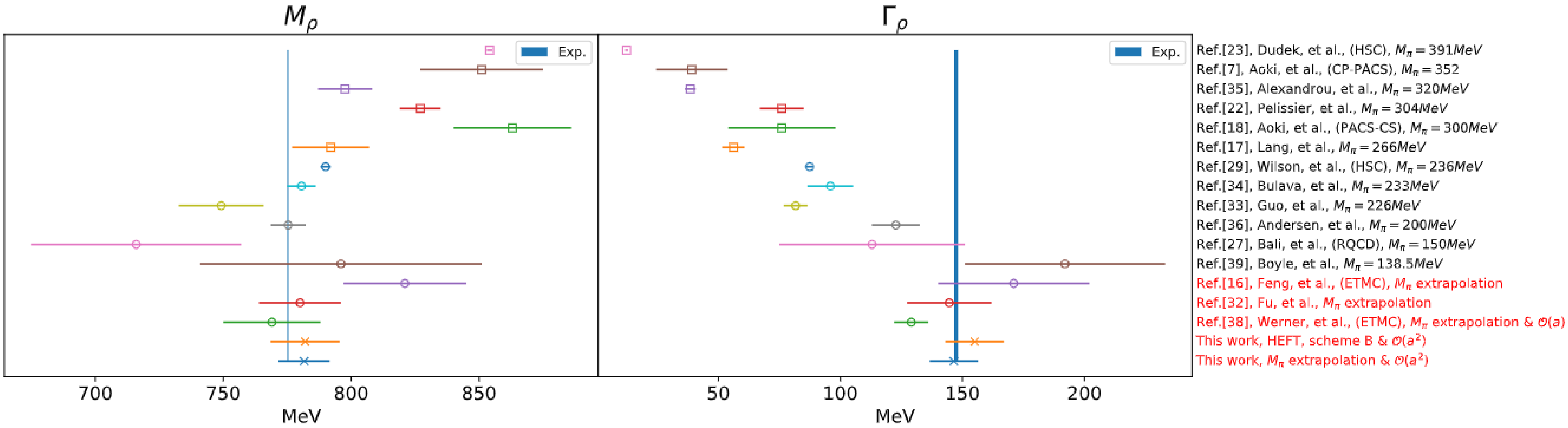
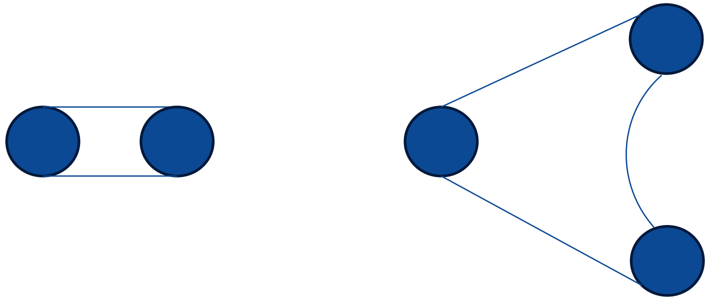
Summary

- We using CLQCD configuration measure the rho meson mass and width at the physical pion mass and continuum limit with the highest precision up to now!



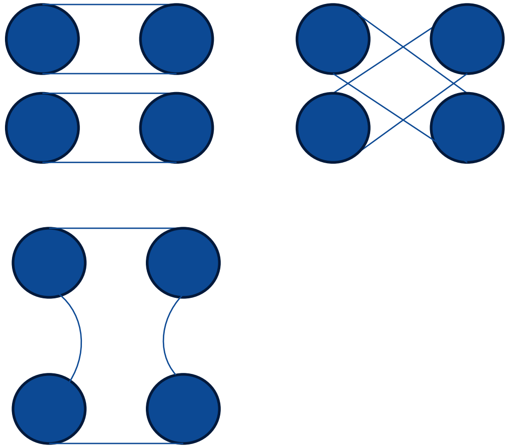
ρ结构

- 夸克模型
- 强子分子态模型
- 紧致夸克模型
- …

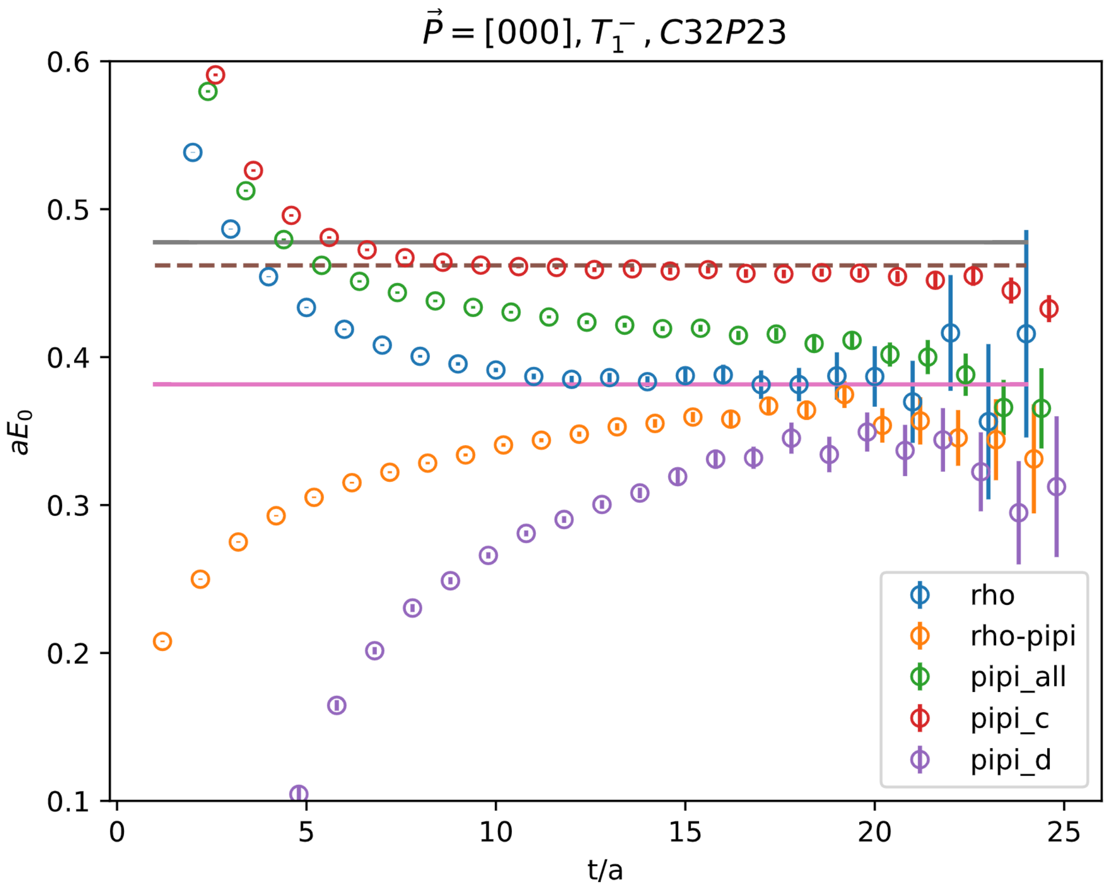


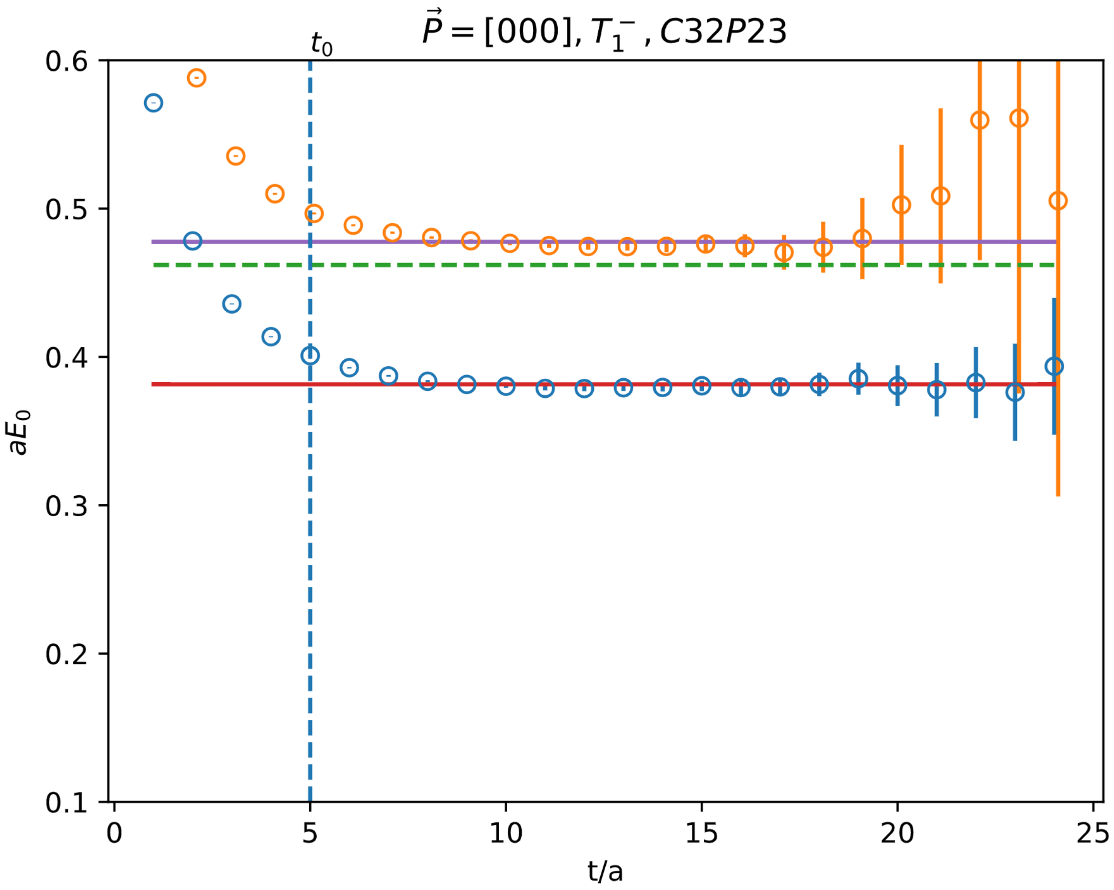
$$\rho \to \rho \qquad \rho \to \pi\pi$$

$$\pi\pi \to \pi\pi$$



ρ结构


$\vec{P}=[000], T_1^-, C32P23$
0.6
0.5
0.4
0.3
0.2
0.1
aE_0
0
5
10
15
20
25
t/a
rho
rho-pipi
pipi_all
pipi_c
pipi_d


$\vec{P}=[000], T_1^-, C32P23$
t_0
0.6
0.5
0.4
0.3
0.2
0.1
aE_0
0
5
10
15
20
25
t/a

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感谢！