DD^* Spectrum in Lattice QCD from Coordinate-Space Two-Hadron Operators and Long-Range Force Analysis

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第五届中国格点量子色动力学研讨会

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3. University of Cyprus.

Outline

- Background
- Lattice calculation
- Lattice spectrum analysis
- Summary

Background

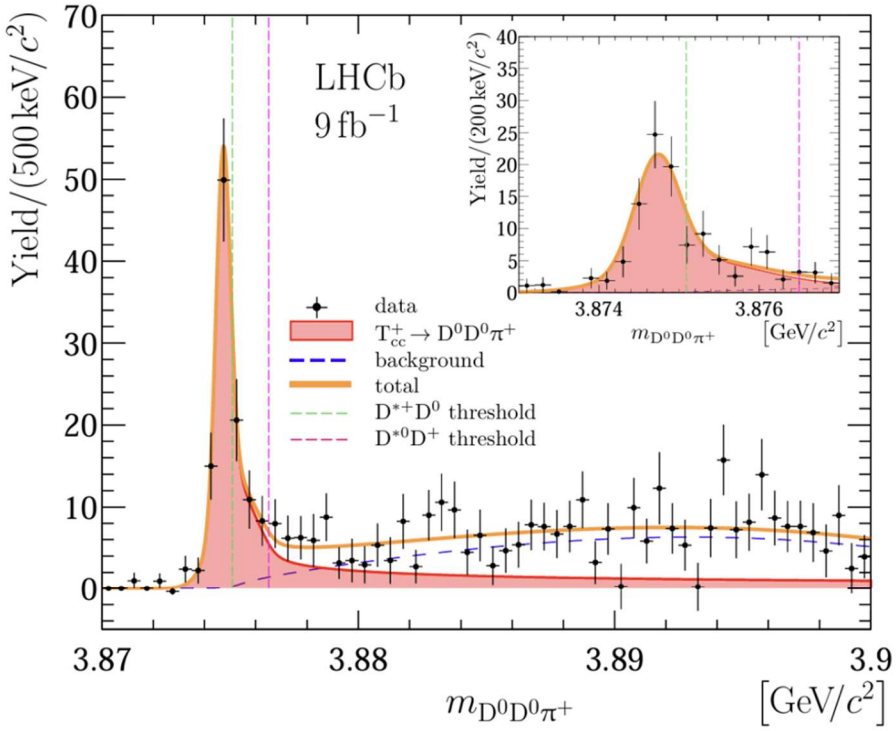

Figure 1: Distribution of $D^0D^0\pi^+$ mass. Distribution of $D^0D^0\pi^+$ mass where the contribution of the non-D^0 background has been statistically subtracted. The result of the fit described in the text is overlaid. Uncertainties on the data points are statistical only and represent one standard deviation, calculated as a sum in quadrature of the assigned weights from the background-subtraction procedure.

The LHCb collaboration reported the doubly charm tetraquark T_{cc} with flavor $cc\bar{u}\bar{d}$ and likely isospin $I = 0$ and spin-parity $J^P = 1^+$.

R. Aaij et al. (LHCb Collaboration), Nature Physics 18, 751 (2022).

R. Aaij et al. (LHCb Collaboration), Nature Communications 13, 3351 (2022).

Theoretical phenomenological models for Tcc still lack a definitive explanation, with a prevailing preference for interpreting it as a shallow bound state of DD^*

Guang-Juan Wang, Zhi Yang, Jia-Jun Wu, Makoto Oka, Shi-Lin Zhu *Sci.Bull.* 69 (2024) 3036-3041

Background

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Related lattice calculations

CLS	P. Junnarkar, N. Mathur, and M. Padmanath, Phys. Rev. D 99, 034507 (2019), I=0 for $qq\bar{Q}\bar{Q}$ system with J=0,1, an energy level -23 ± 11 MeV relative to the DD threshold, m_π>260 MeV M. Padmanath and S. Prelovsek, Phys. Rev. Lett. 129, 032002 (2022), virtual bound state pole, with ERE and Lüscher, m_π ~280 MeV S. Collins, A. Nefediev, M. Padmanath, and S. Prelovsek, Phys. Rev. D 109, 094509 (2024), virtual bound state pole, with ERE/LSE and Lüscher, m_π ~280 MeV, study the quark mass dependence L. Meng, E. Ortiz-Pacheco, V. Baru, E. Epelbaum, M. Padmanath, and S. Prelovsek Phys.Rev.D 111 (2025) 3, 3, 2411.06266, the standard Lüscher method and the recently proposed effective-field-theory-based approach in the plane-wave basis for I=1 case L. Meng, V. Baru, E. Epelbaum, A. A. Filin, and A. M. Gasparyan, Phys. Rev. D 109, L071506 (2024) effective-field-theory-based approach in the plane-wave basis for I=0 case
HSC	HSC Hadron Spectrum Collaboration, Phys. Rev. D 111, 034511 (2025), 2405.15741, I=0 for DD*, D*D* coupled-channel, virtual bound state pole, with ERE and Lüscher, $m_\pi \sim 391$ MeV Hadron Spectrum Collaboration arXiv:2505.01363, I=1 for DD, DD*, D*D* coupled-channel
CLQCD	CLQCD S. Chen, C. Shi, Y. Chen, M. Gong, Z. Liu, W. Sun, and R. Zhang, Phys. Lett. B 833, 137391 (2022), I = 0 attractive and I=1 replusive, m_π ~350 MeV, with ERE and Lüscher P.-P. Shi, F.-K. Guo, C. Liu, L. Liu, P. Sun, J.-J. Wu, and H. Xing (2025), *DD* scattering in lattice QCD.
HAL QCD	Jlab Y. Ikeda, B. Charron, S. Aoki, T. Doi, T. Hatsuda, T. In oue, N. Ishii, K. Murano, H. Nemura, and K. Sasaki, Phys. Lett. B 729, 85 (2014), The phase shifts in the isospin triplet (I = 1) channels indicate repulsive interactions, while those in the I = 0 channels suggest attraction, growing as m_π decreases. HAL QCD method Y. Lyu, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda, and J. Meng (2023), *Phys.Rev.Lett.* 131 (2023) 16, 161901, virtual state, and the virtual state is shown to evolve into a loosely bound state as m_π decreases, ERE expansion.

Lattice calculation

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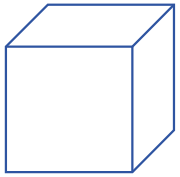
Symmetry

SO(3)

$L^P = 0^+, 1^-, 2^+, \ldots\ldots$

O_h



$A_1^{\pm}, A_2^{\pm}, E^{\pm}, T_1^{\pm}, T_2^{\pm}$

SO(3)	O_h
0^+	A_1^+
1^-	T_1^-
2^+	$E^+ \oplus T_2^+$
3^-	$A_2^- \oplus T_1^- \oplus T_2^-$
4^+	$A_1^+ \oplus E^+ \oplus T_1^+ \oplus T_2^+$
5^-	$E^- \oplus 2T_1^- \oplus T_2^-$
6^+	$A_1^+ \oplus A_2^+ \oplus E^+ \oplus T_1^+ \oplus 2T_2^+$
7^-	$A_2^- \oplus E^- \oplus 2T_1^- \oplus 2T_2^-$
8^+	$A_1^+ \oplus 2E^+ \oplus 2T_1^+ \oplus 2T_2^+$
9^-	$A_1^- \oplus A_2^- \oplus E^- \oplus 3T_1^- \oplus 2T_2^-$

O_h	SO(3)
A_1^+	$0^+, 4^+, 8^+, \ldots.$
A_1^-	$9^-, \ldots.$
A_2^+	$6^+, \ldots.$
A_2^-	$3^-, 7^-, 9^-, \ldots.$
E^+	$2^+, 4^+, 6^+, 8^+, \ldots.$
E^-	$5^-, 7^-, 9^-, \ldots.$
T_1^+	$4^+, 6^+, 8^+, \ldots.$
T_1^-	$1^-, 3^-, 5^-, 7^-, 9^-, \ldots.$
T_2^+	$2^+, 4^+, 6^+, 8^+, \ldots.$
T_2^-	$3^-, 5^-, 7^-, 9^-, \ldots.$

In the finite volume, an irreducible representation contains contributions from different partial waves.

How to project the correlation function into a specific irreducible representation ?

- Momentum space
- Coordinate space

Lattice calculation

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Symmetry

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$L^P = 0^+, 1^-, 2^+, \ldots\ldots$

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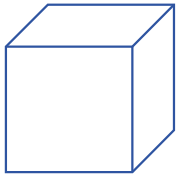

$A_1^{\pm}, A_2^{\pm}, E^{\pm}, T_1^{\pm}, T_2^{\pm}$

Momentum space: It requires substantial storage and computational resources, for instance, $\pi\pi$ system:

$$(\pi\pi)^{[\vec{k}_1,\vec{k}_2]\dagger}_{\vec{P},\Lambda,\mu} = \sum_{\substack{\vec{k}_1\in\{\vec{k}_1\}^\star \\ \vec{k}_2\in\{\vec{k}_2\}^\star \\ \vec{k}_1+\vec{k}_2=\vec{P}}} \mathcal{C}(\vec{P},\Lambda,\mu;\ \vec{k}_1;\ \vec{k}_2)\ \pi^\dagger(\vec{k}_1)\ \pi^\dagger(\vec{k}_2)$$

$$\pi^\dagger(\vec{k},t) = \sum_{\vec{x}} e^{i\vec{k}\cdot\vec{x}} \left[\bar{\psi}\Box_\sigma \mathbf{\Gamma}_t^\dagger \Box_\sigma \psi\right](\vec{x},t)$$

Advantage: It allows for precise control over the scattering energy levels for each lattice momentum.

Lattice calculation

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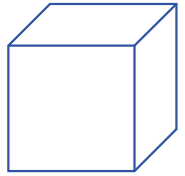
How to project the correlation function into a specific irreducible representation ?

- Momentum space
- Coordinate space

O_h



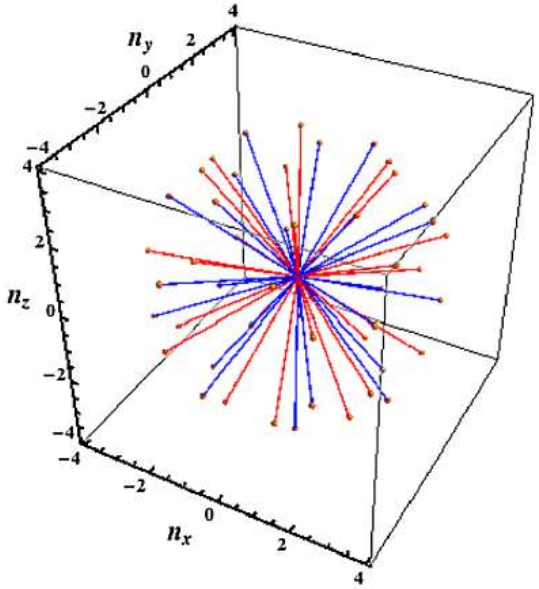
Coordinate space: It significantly reduces storage and computational requirements.

Disadvantage: We can only calculate the ground state for each irreducible representation.

Key method: Using rotations to generate the regular representation space of the spatial symmetry group which contains all the irreducible representation, then performing a projection onto the regular representation space to obtain the operators and correlation functions corresponding to specific irreducible representations.

$\hat{R}$	Rotation Axis	Rotation Angle	(x_1,x_2,x_3)
$\hat{R}_1$	Any	0°	(x_1,x_2,x_3)
$\hat{R}_2$	(1,1,1)	-120°	(x_2,x_3,x_1)
…	…	…	…
$\hat{R}_{24}$	(0,0,1)	-180°	$(-x_1,-x_2,x_3)$
$\hat{R}_{25}=\hat{\pi}\hat{R}_1$	-	-	$(-x_1,-x_2,-x_3)$
$\hat{R}_{26}=\hat{\pi}\hat{R}_2$	-	-	$(-x_2,-x_3,-x_1)$
…	…	…	…
$\hat{R}_{48}=\hat{\pi}\hat{R}_{24}$	-	-	$(x_1,x_2,-x_3)$

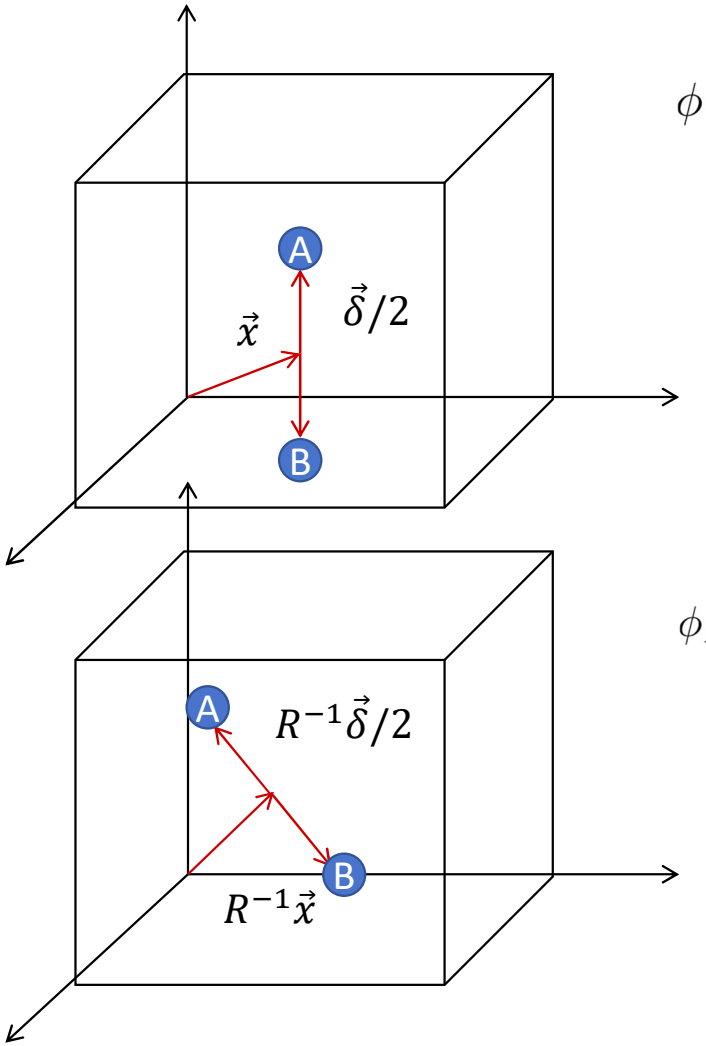
$A_1^{\pm}, A_2^{\pm}, E^{\pm}, T_1^{\pm}, T_2^{\pm}$



T. Luu and M. J. Savage, PRD 83 114508(2011)

Lattice calculation

Key method: Using rotations to generate the regular representation space of the spatial symmetry group which contains all the irreducible representation, then performing a projection onto the regular representation space to obtain the operators and correlation functions corresponding to specific irreducible representations.



$$\phi(\vec{x},\vec{\delta},t)=\chi_A(\vec{x}+\vec{\delta}/2,t)\chi_B(\vec{x}-\vec{\delta}/2,t)$$

Rotation Elements: $\hat{R}=\{\hat{R}_1,\hat{R}_2,\dots,\hat{R}_{48}\}$

$$\vec{\delta}=(\delta_x,\delta_y,\delta_z)$$

$$\delta_x\neq\delta_y\neq\delta_z$$

$$\vec{x}\pm\vec{\delta}/2=\vec{n}\in \mathrm{Z}^3$$

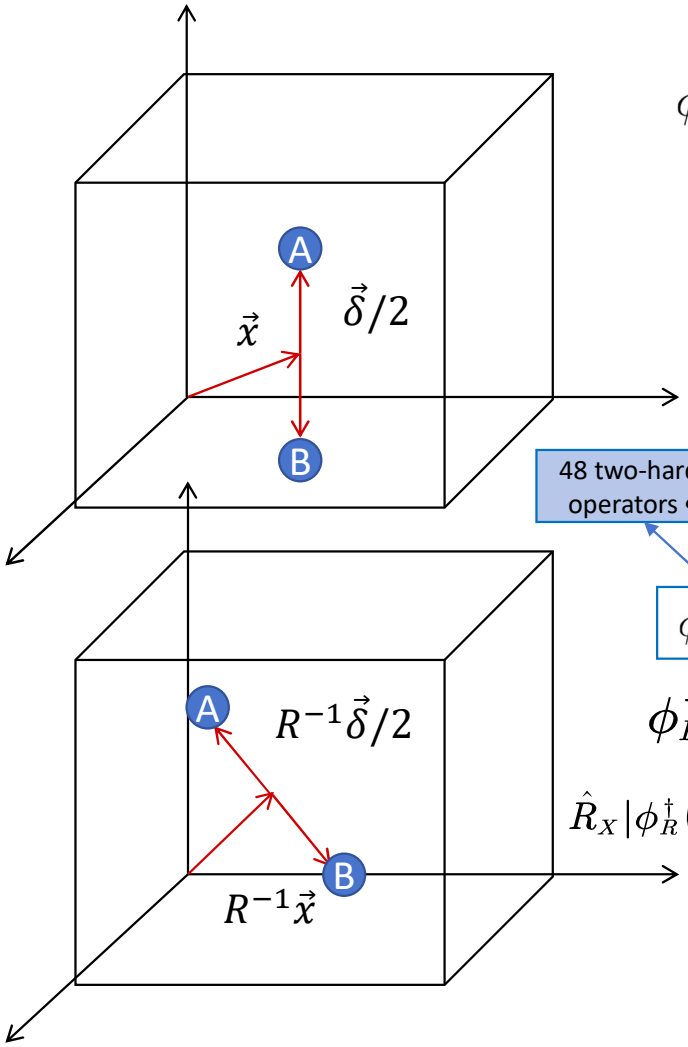
$$\vec{x}=(\frac{1}{2},\frac{1}{2},\frac{1}{2})\ \text{and}\ \vec{\delta}=(1,3,5)$$

Simplifies continuum rotation symmetry

$$\phi_R(\vec{x},\vec{\delta},t)=\chi_A(\hat{R}^{-1}(\vec{x}+\vec{\delta}/2),t)\chi_B(\hat{R}^{-1}(\vec{x}-\vec{\delta}/2),t)$$

Lattice calculation

Key method: Using rotations to generate the regular representation space of the spatial symmetry group which contains all the irreducible representation, then performing a projection onto the regular representation space to obtain the operators and correlation functions corresponding to specific irreducible representations.



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Rotation Elements: $\hat{R}=\{\hat{R}_1,\hat{R}_2,\ldots,\hat{R}_{48}\}$

$$\vec{\delta}=(\delta_x,\delta_y,\delta_z)$$
$$\delta_x\neq\delta_y\neq\delta_z$$
$$\vec{x}\pm\vec{\delta}/2=\vec{n}\in Z^3$$
$$\vec{x}=(\frac{1}{2},\frac{1}{2},\frac{1}{2})\ \text{and}\ \vec{\delta}=(1,3,5)$$

Simplifies continuum rotation symmetry

48 two-hardon operators Φ_R

$$\phi_R(\vec{x},\vec{\delta},t)=\chi_A(\hat{R}^{-1}(\vec{x}+\vec{\delta}/2),t)\chi_B(\hat{R}^{-1}(\vec{x}-\vec{\delta}/2),t)$$

$$\phi_R^\dagger\left(\vec{x},\vec{\delta},t\right)|\Omega\rangle=|\phi_R^\dagger\left(\vec{x},\vec{\delta},t\right)\rangle\sim|\phi_R^\dagger\left(\vec{x},\vec{\delta}\right)\rangle$$

$$\hat{R}_X|\phi_R^\dagger\left(\vec{x},\vec{\delta},t\right)\rangle=|\phi_{R_XR}^\dagger\left(\vec{x},\vec{\delta},t\right)\rangle=\sum_{R'}|\phi_{R'}^\dagger\left(\vec{x},\vec{\delta},t\right)\rangle\left[\bar{\Gamma}_{\text{reg}}(R_X)\right]_{R',R}$$

$$S^{-1}\bar{\Gamma}_{\text{reg}}(R_X)S=A_1^+\oplus A_2^+\oplus 2E^+\oplus 3T_1^+\oplus 3T_2^+$$
$$\oplus A_1^-\oplus A_2^-\oplus 2E^-\oplus 3T_1^-\oplus 3T_2^-$$

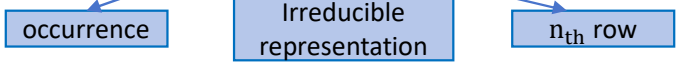
Lattice calculation

Key method: Using rotations to generate the regular representation space of the spatial symmetry group which contains all the irreducible representation, then performing a projection onto the regular representation space to obtain the operators and correlation functions corresponding to specific irreducible representations.

$$\hat{R}_X|\phi^\dagger_R(\bar{x},\bar{\delta},t)\rangle = |\phi^\dagger_{R_x R}(\bar{x},\bar{\delta},t)\rangle = \sum_{R'}|\phi^\dagger_{R'}(\bar{x},\bar{\delta},t)\rangle\left[\bar{\Gamma}_{\text{reg}}(R_X)\right]_{R',R}$$

$$S^{-1}\bar{\Gamma}_{\text{reg}}(R_X)S = A_1^+ \oplus A_2^+ \oplus 2E^+ \oplus 3T_1^+ \oplus 3T_2^+ \oplus A_1^- \oplus A_2^- \oplus 2E^- \oplus 3T_1^- \oplus 3T_2^-$$

$$= \bar{\Gamma}_{i,\Gamma,n;i',\Gamma',n'}$$



This block-diagonal matrices will lead to a new set of basis states: $|\Phi^\dagger_{i,\Gamma,n}(\vec{x},\vec{\delta})\rangle$

$$\hat{R}_X|\Phi^\dagger_{i,\Gamma,n}(\vec{x},\vec{\delta})\rangle = \sum_{i',\Gamma',n'}|\Phi^\dagger_{i',\Gamma',n'}(\vec{x},\vec{\delta})\rangle\left[S^{-1}\bar{\Gamma}_{\text{reg}}(R_X)S\right]_{i',\Gamma',n';i,\Gamma,n}$$

$$|\Phi^\dagger_R(\vec{x},\vec{\delta})\rangle \xrightarrow{S} |\Phi^\dagger_{i,\Gamma,n}(\vec{x},\vec{\delta})\rangle$$

$$|\Phi^\dagger_{i,\Gamma,n}(\vec{x},\vec{\delta})\rangle = \sum_R |\Phi^\dagger_R(\vec{x},\vec{\delta})\rangle[S]_{R;i,\Gamma,n}$$

Lattice calculation

Key method: Using rotations to generate the regular representation space of the spatial symmetry group which contains all the irreducible representation, then performing a projection onto the regular representation space to obtain the operators and correlation functions corresponding to specific irreducible representations.

$$\sum_{(\vec{y}-\vec{x})\in\mathbb{Z}^3} e^{-i\vec{p}\cdot(\vec{x}-\vec{y})}\langle\Omega|\Phi_{i,\Gamma,n}(t,\vec{y},\vec{\delta})\Phi^{\dagger}_{i',\Gamma',n'}(0,\vec{x},\vec{\delta})|\Omega\rangle$$

$$=\delta_{i,i'}\delta_{\Gamma,\Gamma'}\delta_{n,n'}\sum_{(\vec{y}-\vec{x})\in\mathbb{Z}^3} e^{-i\vec{p}\cdot(\vec{x}-\vec{y})}\frac{h}{l_\Gamma}\sum_R \chi_\Gamma(R)\langle\Omega|\phi_R(t,\vec{y},\vec{\delta})\phi^{\dagger}(0,\vec{x},\vec{\delta})|\Omega\rangle$$

- $\delta_{i,i'}\delta_{\Gamma,\Gamma'}\delta_{n,n'}$: Diagonal and Just rely on Γ
- $\frac{h}{l_\Gamma}$: h and l_Γ are the orders of O_h group and the dimension of irreducible representation Γ.
- $\chi_\Gamma(R)$: Character Number of Γ
- $\phi_R(t,\vec{y},\vec{\delta})$: "Shell" Sink
- $\phi^{\dagger}(0,\vec{x},\vec{\delta})$: Two source locations as $\vec{x}+\vec{\delta}/2$ and $\vec{x}-\vec{\delta}/2$

$$\sum_R \left\langle \; R_1\left(\vec{y}+\frac{\vec{\delta}}{2}\right),\ R_2\left(\vec{y}+\frac{\vec{\delta}}{2}\right),\ R_2\left(\vec{y}-\frac{\vec{\delta}}{2}\right),\ R_1\left(\vec{y}-\frac{\vec{\delta}}{2}\right) \;\middle|\; \vec{x}+\frac{\vec{\delta}}{2},\ \vec{x}-\frac{\vec{\delta}}{2} \right\rangle$$

Coordinate space: It significantly reduces storage and computational requirements.

CSSM-QCDSF Collaboration Jia-Jun Wu et al.*PRD* 105 (2022) 7, 074506

Lattice calculation DD^* system

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$$D^0:\phi(\vec{x})\equiv\sum_a \bar{u}^a(\vec{x})\gamma_5 c^a(\vec{x})$$

$$D^{*+}:\phi_\mu(\vec{x})\equiv\sum_a \bar{d}^a(\vec{x})\gamma_\mu c^a(\vec{x}),\quad \mu=x,y,z$$

$$\Phi_{\mu\hat{R}}(\vec{x},\vec{\delta})\equiv\hat{P}_{\hat{R}}\phi(\vec{x}+\vec{\delta}/2)\phi_\mu(\vec{x}-\vec{\delta}/2)\hat{P}_{\hat{R}^{-1}}$$

$$=\phi(\hat{R}^{-1}(\vec{x}+\vec{\delta}/2))\sum_\nu \bar{D}_{\nu\mu}\left(\hat{R}_{\text{in}}\right)\phi_\nu(\hat{R}^{-1}(\vec{x}-\vec{\delta}/2))$$

$$|\Phi_{\mu\hat{R}}(\vec{x},\vec{\delta})\rangle \xrightarrow{\bar{S}} |\Phi_{\mu,i\Gamma' n'}(\vec{x},\vec{\delta})\rangle \xrightarrow{\bar{S}'} |\Phi_{i\Gamma n}(\vec{x},\vec{\delta})\rangle$$

Transformation matrix for regular representation.

Transformation matrix for the direct product of irreducible representation matrix obtained by the reduction of regular representation and the representation matrix of spin 1.

DD^* rest frame, $|P|^2=0$

Γ	$T_1^+\otimes\Gamma'(\ell)$
A_1^+	$T_1^+\otimes T_1^+(4)$
A_2^+	$T_1^+\otimes T_2^+(2)$
E^+	$T_1^+\otimes T_1^+(2),\ T_1^+\otimes T_2^+(2)$
T_1^+	$T_1^+\otimes A_1^+(0),\ T_1^+\otimes E^+(2),\ T_1^+\otimes T_1^+(4),\ T_1^+\otimes T_2^+(2)$
T_2^+	$T_1^+\otimes A_2^+(6),\ T_1^+\otimes E^+(2),\ T_1^+\otimes T_1^+(4),\ T_1^+\otimes T_2^+(2)$
A_1^-	$T_1^+\otimes T_1^-(1)$
A_2^-	$T_1^+\otimes T_2^-(3)$
E^-	$T_1^+\otimes T_1^-(1),\ T_1^+\otimes T_2^-(3)$
T_1^-	$T_1^+\otimes A_1^-(9),\ T_1^+\otimes E^-(5),\ T_1^+\otimes T_1^-(1),\ T_1^+\otimes T_2^-(3)$
T_2^-	$T_1^+\otimes A_2^-(9),\ T_1^+\otimes E^-(5),\ T_1^+\otimes T_1^-(1),\ T_1^+\otimes T_2^-(3)$

Γ	$(A_2\oplus E)\otimes\Gamma'(\ell)$
A_1	$A_2\otimes A_2(4), E\otimes E(1)$
A_2	$A_2\otimes A_1(0), E\otimes E(1)$
B_1	$A_2\otimes B_2(2), E\otimes E(1)$
B_2	$A_2\otimes B_1(2), E\otimes E(1)$
E	$E\otimes A_1(0), A_2\otimes E(1)$

DD^* moving frame, $|P|^2=1$, in this case, T_1^+ is reducible and can be decomposed into $A_2\oplus E$.

Lattice calculation DD^* system

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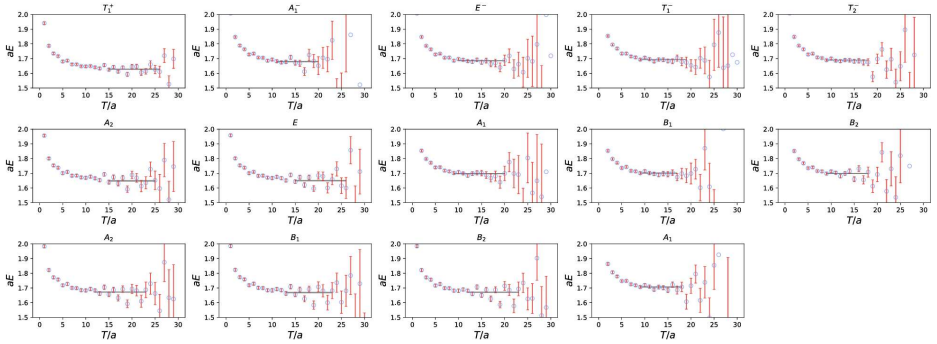
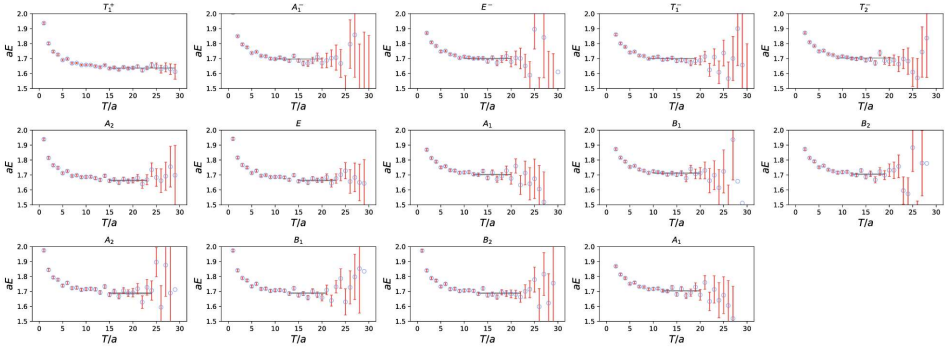
I=0

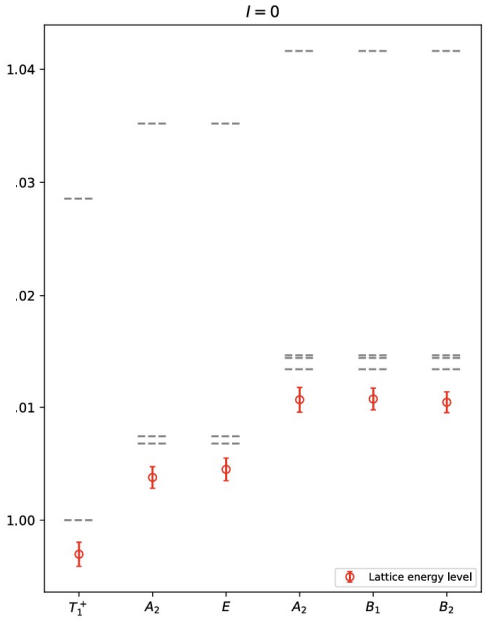


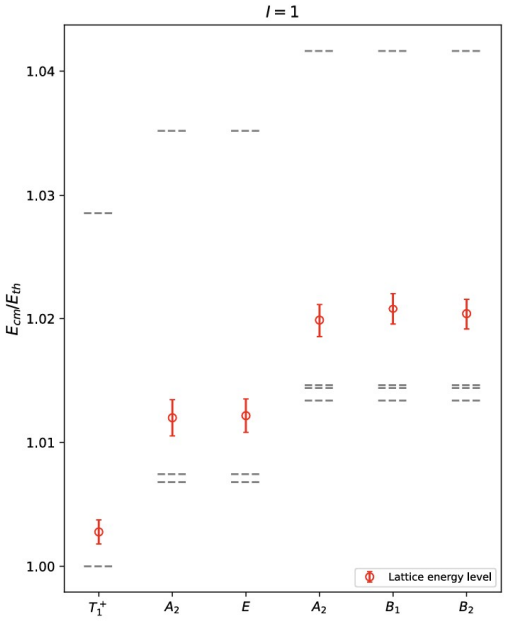
I=1



$L^3 \times T$	$a[fm]$	$m_\pi\,[MeV]$	$m_D\,[MeV]$	$m_{D^*}[MeV]$
$32^3 \times 64$	0.074	359.4	2125.2	2225.4

from the QCDSF Collaboration G. S. Bali et al., Nucl. Phys. B 866, 1 (2013)





$|P|^2 = 0$ 1 1 2 2 2

Lattice spectrum analysis DD^* system

Lüscher formula and ERE

$$\det\left(M^{\Gamma,\vec{p}}_{Jln,J'l'n'}(E)-\delta_{JJ'}\delta_{ll'}\delta_{nn'}\cot\delta_{Jl}\right)=0$$

$$p\cot\delta_0=\frac{1}{a_0}+\frac{1}{2}r_0p^2$$

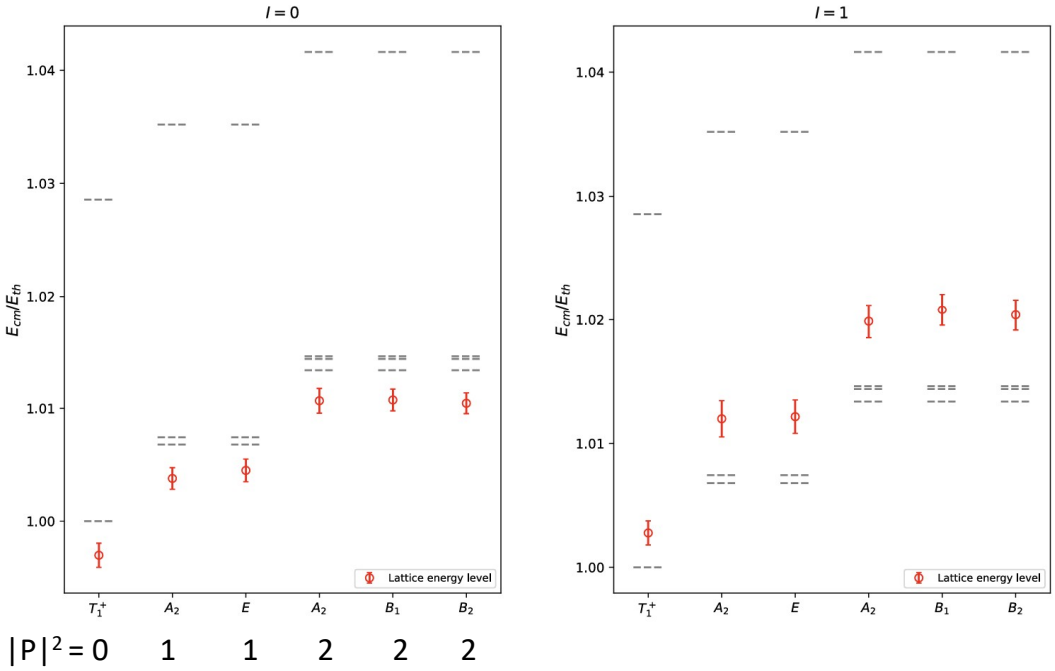
But this method is limited by short-range interactions.
Long-range interactions will bring two problems.

1.In the presence of a nearby lhc, the ERE is valid only in a very narrow energy range.

Du, Filin, Baru, Dong, Epel-baum, Guo, Hanhart, Nefediev, Nieves,and Wang, PRL 131, 131903 (2023)

2.Standard Lüscher formula also encounters difficulties due to the lhc.

Raposo and Hansen, JHEP 08, 075 (2024)
Bubna, Hammer, Müller, Pang, Rusetsky, and Wu, JHEP 05, 168 (2024)
Meng, Baru, Epelbaum, Filin, and Gasparyan, PRD 109, L071506 (2024)
Yu, Wang, Wu, and Yang, JHEP 04 108(2025)



To incorporate the long range contribution from one-pion exchange, we adopt HEFT method to anlysis lattice spectrum.

Lattice spectrum analysis DD^* system

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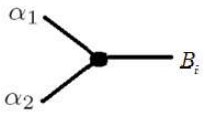
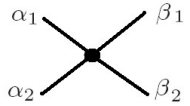
$$H = H_0 + H_I \qquad H_I = \hat{g} + \hat{v}$$



$$\hat{g} = \sum_{\alpha} \sum_{i=1,n} \left[\left|\alpha(k_\alpha)\right\rangle g^{+}_{i,\alpha} \left\langle B_i \right| + \left| B_i \right\rangle g_{i,\alpha} \left\langle \alpha(k_\alpha) \right| \right]$$

$$\hat{v} = \sum_{\alpha,\beta} \left|\alpha(k_\alpha)\right\rangle v_{\alpha,\beta} \left\langle \beta(k_\beta) \right|$$



Continuous → Discrete

$\int d\vec{k}$	and	$\left\|\alpha(\vec{k}_\alpha)\right\rangle$	and	$\left\langle \beta(\vec{k}_\beta) \middle\| \alpha(\vec{k}_\alpha) \right\rangle = \delta_{\alpha\beta}\delta(\vec{k}_\alpha - \vec{k}_\beta)$
$\sum_{\mathrm{i}} \left(2\pi/\mathrm{L}\right)^3$	and	$\left(2\pi/\mathrm{L}\right)^{-3/2} \left\|\vec{k}_{\mathrm{i}}, -\vec{k}_{\mathrm{i}}\right\rangle_\alpha$	and	$_\beta\left\langle \vec{k}_{\mathrm{j}}, -\vec{k}_{\mathrm{j}} \middle\| \vec{k}_{\mathrm{i}}, -\vec{k}_{\mathrm{i}} \right\rangle_\alpha = \delta_{\alpha\beta}\delta_{\mathrm{ij}}$

$$[H_0]_{N_c+1} = \begin{pmatrix} m_0 & 0 & 0 & \cdots & 0 & 0 & \cdots \\ 0 & \epsilon_1(k_0) & 0 & \cdots & 0 & 0 & \cdots \\ 0 & 0 & \epsilon_2(k_0) & \cdots & 0 & 0 & \cdots \\ 0 & 0 & 0 & \ddots & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots & \epsilon_{n_c}(k_0) & 0 & \cdots \\ 0 & 0 & 0 & \cdots & 0 & \epsilon_1(k_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$[H_I]_{N_c+1} = \begin{pmatrix} 0 & g_1^V(k_0) & g_2^V(k_0) & \cdots & g_{n_c}^V(k_0) & g_1^V(k_1) & \cdots \\ g_1^V(k_0) & v_{1,1}^V(k_0,k_0) & v_{1,2}^V(k_0,k_0) & \cdots & v_{1,n_c}^V(k_0,k_0) & v_{1,1}^V(k_0,k_1) & \cdots \\ g_2^V(k_0) & v_{2,1}^V(k_0,k_0) & v_{2,2}^V(k_0,k_0) & \cdots & v_{2,n_c}^V(k_0,k_0) & v_{2,1}^V(k_0,k_1) & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \cdots \\ g_{n_c}^V(k_0) & v_{n_c,1}^V(k_0,k_0) & v_{n_c,2}^V(k_0,k_0) & \cdots & v_{n_c,n_c}^V(k_0,k_0) & v_{n_c,1}^V(k_0,k_1) & \cdots \\ g_1^V(k_1) & v_{1,1}^V(k_1,k_0) & v_{1,2}^V(k_1,k_0) & \cdots & v_{1,n_c}^V(k_1,k_0) & v_{1,1}^V(k_1,k_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Resonance
(Mass, width, pole position, coupling)

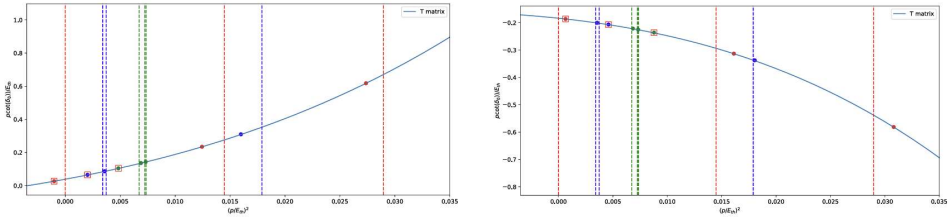
HEFT

T matrix
(phase shift, Inelascity)

Lattice spectrum

$$(H_0 + H_I)|\Psi\rangle = E|\Psi\rangle$$

Eigen-value ⇔ Lattice spectrum



Incorporatin long range force, the difference between Lüscher formula and HEFT was discussed in the report of Kangyu yesterday.

Lattice spectrum analysis DD^* system

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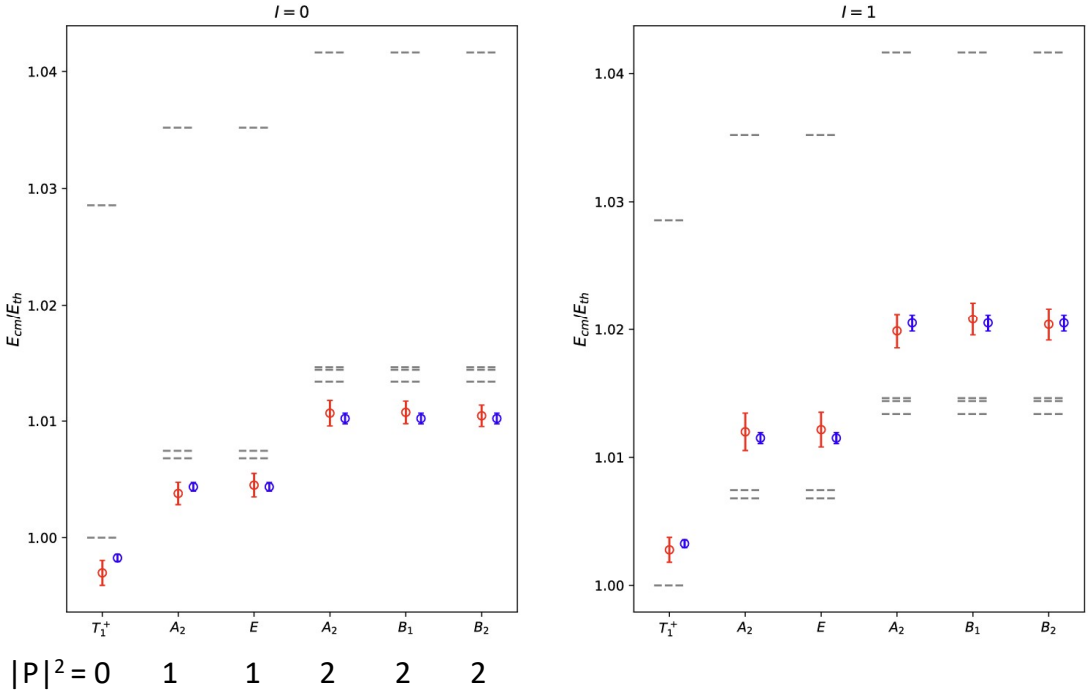
$$H = H_0 + H_I$$

$$H_I = V_{\text{Short}} + V_{\text{Long}}$$

Single parameter fitting

$$V^I_{\text{Short}}(\vec{p},\vec{k}) \equiv c_S^I \left(\frac{\Lambda^2}{\vec{k}^2+\Lambda^2}\right)^2 \left(\frac{\Lambda^2}{\vec{p}^2+\Lambda^2}\right)^2$$

$$V_{\text{Long}}(|\vec{p}|,|\vec{k}|) = \frac{g_c}{8|\vec{p}||\vec{k}|f_\pi^2(2\pi)^2} \ln\left[\frac{\left(|\vec{p}|+|\vec{k}|\right)^2+m_{\text{eff}}^2}{\left(|\vec{p}|-|\vec{k}|\right)^2+m_{\text{eff}}^2}\right]$$



$|P|^2 = 0$ 1 1 2 2 2

f_π=112.3 MeV and g_c=0.550 with lattice spacing a = 0.074 fm and $m_\pi \simeq$ 359.4 MeV .

D. Becirevic and F. Sanfilippo, Phys. Lett. B 721, 94(2013)

Meng, Baru, Epelbaum, Filin, and Gasparyan, PRD 109, L071506 (2024)

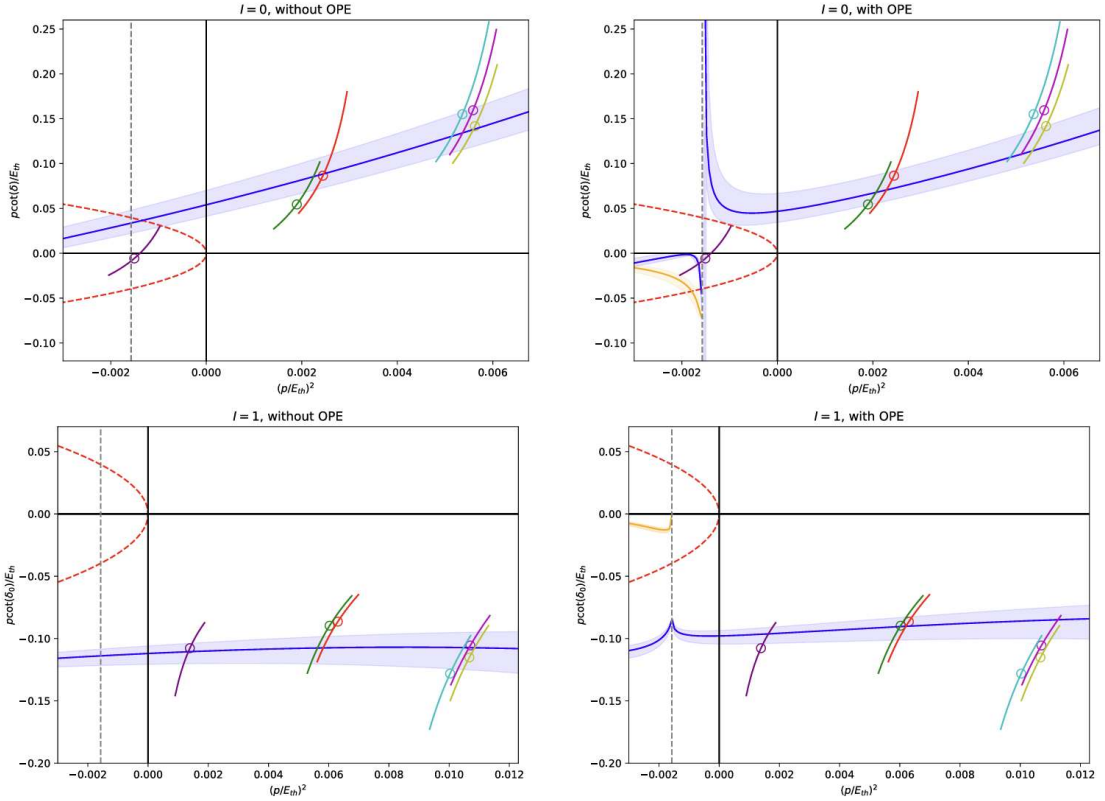
Lattice spectrum analysis DD^* system

University of Chinese Academy of Sciences

$$V^I_{\text{Short}}(\vec{p},\vec{k}) \equiv c_S^I \left(\frac{\Lambda^2}{\vec{k}^2+\Lambda^2}\right)^2 \left(\frac{\Lambda^2}{\vec{p}^2+\Lambda^2}\right)^2$$

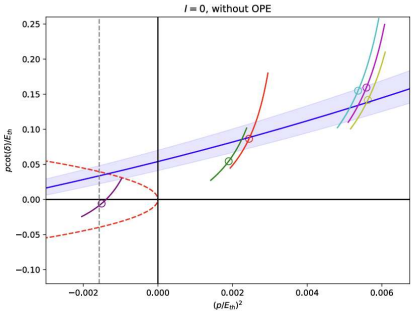
$$V_{\text{Long}}(|\vec{p}|,|\vec{k}|) = \frac{g_c}{8|\vec{p}||\vec{k}|f_\pi^2(2\pi)^2} \ln\left[\frac{\left(|\vec{p}|+|\vec{k}|\right)^2+m_{\text{eff}}^2}{\left(|\vec{p}|-|\vec{k}|\right)^2+m_{\text{eff}}^2}\right]$$

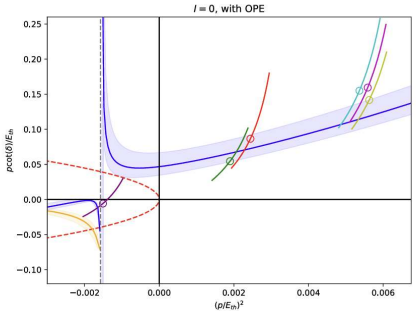
Isopsin	Potential	$c_S^I[10^{-5}\text{MeV}^2]$	χ^2/dof	$E^{\text{Pole}}-E_{\text{th}}[\text{MeV}]$
0	without OPE	-1.05(0.12)	2.75/5	-12.2(4.1)
0	with OPE	-1.55(0.15)	2.35/5	$-10.2(2.4) \pm 5.1(3.2)i$
1	without OPE	3.37(0.61)	6.15/5	$41.9(2.7) - 2.3(0.2)i$
1	with OPE	4.42(0.90)	5.55/5	$42.5(4.1) - 2.7(0.3)i$

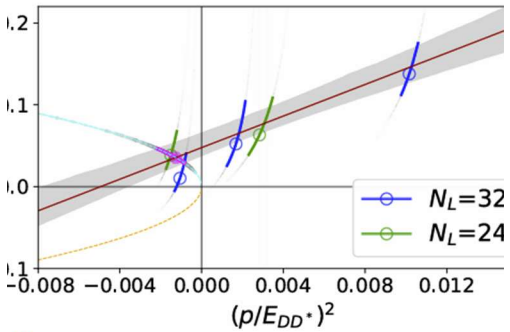


- For I=0 case, a singularity with an imaginary part emerges below the threshold.
- For I=1 case, results are consistent with the short range potential.
- The attraction and repulsion of the short-range potential determine the behavior near the left-hand cut generated by the long-range potentials.

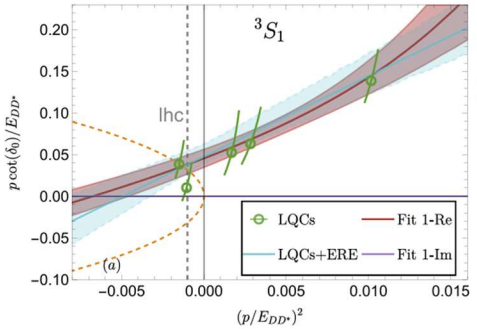
Lattice spectrum analysis DD^* system

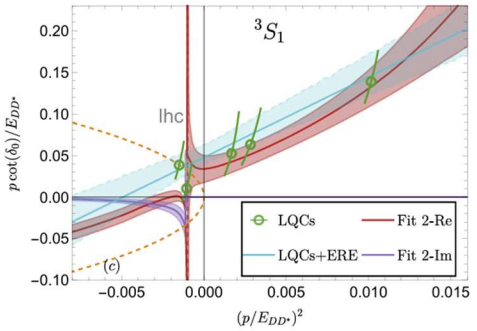




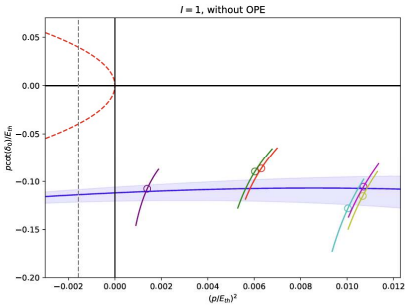


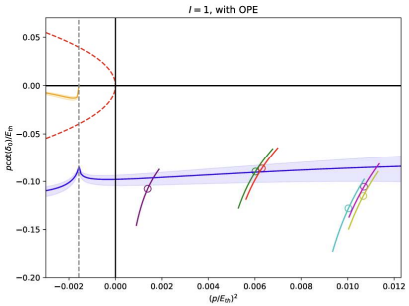
M. Padmanath and S. Prelovsek, Phys. Rev. Lett. 129, 032002 (2022)

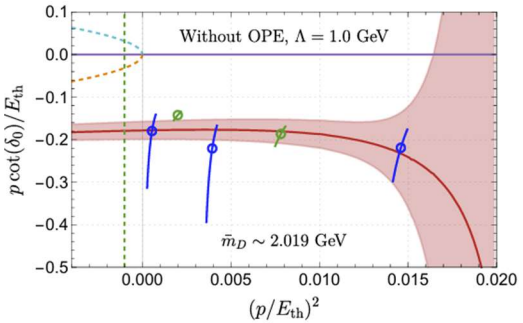


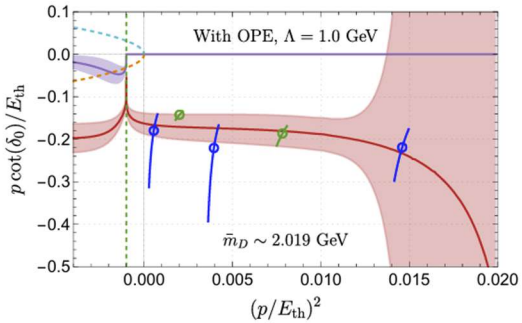


L. Meng, V. Baru, E. Epelbaum, A. A. Filin, and A. M. Gasparyan, Phys. Rev. D 109, L071506 (2024)









L. Meng, E. Ortiz-Pacheco, V. Baru, E. Epelbaum, M. Padmanath, and S. Prelovsek Phys.Rev.D 111 (2025) 3, 3, 2411.06266

Summary

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- Utilizing a new method which is based on the projection of rotation in coordinate space , we construct the operator referring to different irreps and calculate corresponding lattice energy spectrum.

- Using HEFT method, the impact of including long-range potentials on the lattice data fitting is analyzed. It was found that for the I=0 case, the presence of long-range potentials alters the position and behavior of the poles, whereas for I=1, the results remain largely unaffected.

- The lattice analysis indicates that T_{cc} is not a bound state in this case, but a resonance below the DD^* threshold.