Charmed $P \to V$ semileptonic decay from lattice QCD

孟 雨（郑州大学）

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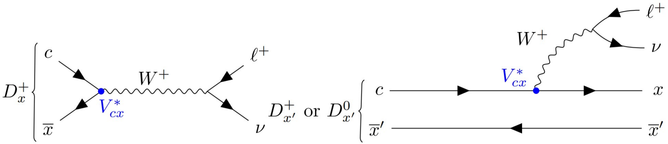
Charmed high-precision frontier

"November Revolution"— The discovery of J/ψ particle in 1974, greatly facilitated the establishment of the Standard Model.

- Precise test for Standard Model and search for new physics
  — High-precison measurements(BESIII,etc) and theoretical calculations
    — CKM matrix element $V_{cs(d)}$



- Test the interplay of pert and non-pert QCD
  — intermediate energy scale

Charmed high-precision frontier

- $D \to \pi l \nu$ and $D \to K l \nu$

Collaboration	$f_+^{D\to\pi}(0)$	$f_+^{D\to K}(0)$	Refs
FNAL/MILC 22	0.6300 ± 0.0051	0.7452 ± 0.0031	PRD107,094516(2023)
ETM 17	0.612 ± 0.035	0.765 ± 0.031	PRD96,054514(2017)
HPQCD 22	–	0.7441 ± 0.0040	PRD107,014510(2023)
LQCD-AVER	0.6296 ± 0.0050	0.7449 ± 0.0025	HFLAV2023
	$f_+^{D\to\pi}(0)\lvert V_{cd}\rvert$	$f_+^{D\to K}(0)\lvert V_{cs}\rvert$	
EXP-AVER	0.1426 ± 0.0018	0.7180 ± 0.0033	HFLAV2023

$$|V_{cs}| = 0.9639 \pm 0.0044_{\text{EXP}} \pm 0.0032_{\text{LQCD}}$$
$$|V_{cd}| = 0.2265 \pm 0.0029_{\text{EXP}} \pm 0.0018_{\text{LQCD}}$$

- The QED correction is a main source of lattice error

$P \to V$ semileptonic decay

- For $P' \to P$:

$$\frac{d\Gamma(P' \to Pe\nu_e)}{dq^2 d\cos\theta_l} = \frac{G_F^2}{32\pi^3}|\vec{p}|^3|V_{xy}|^2 \sin^2\theta_l |f_+(q^2)|^2$$

  for example, $D \to \pi$ and $D \to K$

- For $P \to V$:

$$\frac{\mathrm{d}\Gamma(P \to Ve\nu_e)}{\mathrm{d}q^2\mathrm{d}\chi\mathrm{d}\cos\theta_\ell\mathrm{d}\cos\theta_V} = \frac{G_F^2|V_{xy}|^2|\vec{p}|q^2}{12(2\pi)^4 M^2} W\left(q^2, \theta_V, \theta_\ell, \chi\right)$$

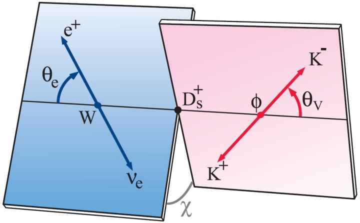
  - Independent determination of the CKM matrix element
  - Stronger test with abundant polarization information

$P \to V$ decay amplitude

$$
\begin{aligned}
W(\theta_K,\theta_\ell,\chi) = & \frac{9}{32}\left(1+\cos^2\theta_\ell\right)\sin^2\theta_K\left(|H_+|^2+|H_-|^2\right)+\frac{9}{8}\sin^2\theta_\ell\cos^2\theta_K|H_0|^2 \\
& +\frac{9}{16}\cos\theta_\ell\sin^2\theta_K\left(|H_+|^2-|H_-|^2\right)-\frac{9}{16}\sin^2\theta_\ell\sin^2\theta_K\cos 2\chi\left(H_+H_-\right) \\
& +\frac{9}{16}\sin\theta_\ell\sin 2\theta_K\cos\chi\left(H_+H_0-H_-H_0\right) \\
& +\frac{9}{32}\sin 2\theta_\ell\sin 2\theta_K\cos\chi\left(H_+H_0+H_-H_0\right) \\
& +\frac{m_\ell^2}{2q^2}\left[\frac{9}{4}\cos^2\theta_K|H_t|^2-\frac{9}{2}\cos\theta_\ell\cos^2\theta_K\left(H_0H_t\right)+\frac{9}{4}\cos^2\theta_\ell\cos^2\theta_K|H_0|^2\right. \\
& +\frac{9}{16}\sin^2\theta_\ell\sin^2\theta_K\left(|H_+|^2+|H_-|^2\right)+\frac{9}{8}\sin^2\theta_\ell\sin^2\theta_K\cos 2\chi\left(H_+H_-\right) \\
& +\frac{9}{8}\sin\theta_\ell\sin 2\theta_K\cos\chi\left(H_+H_t+H_-H_t\right) \\
& \left.-\frac{9}{16}\sin 2\theta_\ell\sin 2\theta_K\cos\chi\left(H_+H_0+H_-H_0\right)\right]
\end{aligned}
$$

and the helicity amplitudes are

$$
\begin{aligned}
H_\pm\left(q^2\right) &= \left(M+m\right)A_1\left(q^2\right)\mp\frac{2M|\vec{p}|}{M+m}V\left(q^2\right) \\
H_0\left(q^2\right) &= \frac{1}{2M\sqrt{q^2}}\times\left[\left(M^2-m^2-q^2\right)\left(M+m\right)A_1\left(q^2\right)-4\frac{M^2|\vec{p}|^2}{M+m}A_2\left(q^2\right)\right] \\
H_t\left(q^2\right) &= \frac{2M|\vec{p}|}{\sqrt{q^2}}A_0\left(q^2\right)
\end{aligned}
$$



$P \to V$ matrix element

- For $P' \to P$:

$$\langle P(p)|V_\mu|H(p')\rangle = f_+(q^2)\left[(p'+p)_\mu - \frac{M^2-m^2}{q^2}q_\mu\right] + f_0(q^2)\frac{M^2-m^2}{q^2}q_\mu$$

- For $P \to V$:

$$\begin{aligned}\langle h_V(p)|V_\mu - A_\mu|H(p')\rangle &= \varepsilon^*_\nu \epsilon_{\mu\nu\alpha\beta} p'_\alpha p_\beta \frac{2V}{m+M} + (M+m)\,\varepsilon^*_\mu A_1 \\ &+ \frac{\varepsilon^*\cdot q}{M+m}\left(p+p'\right)_\mu A_2 - 2m\frac{\varepsilon^*\cdot q}{q^2} q_\mu \left(A_0 - A_3\right)\end{aligned}$$

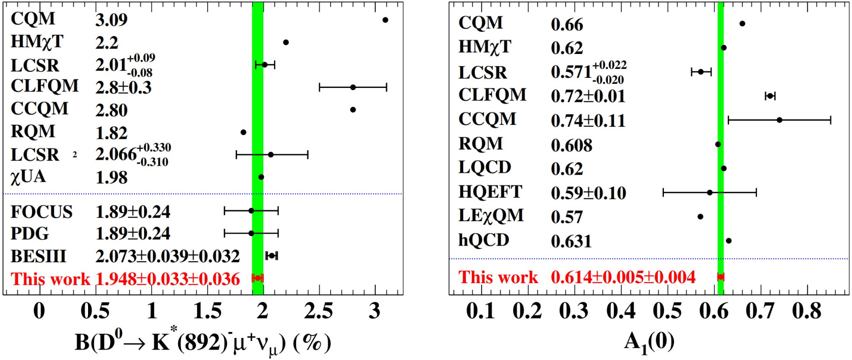
  - Two key physical quantities:

$$r_V \equiv V(0)/A_1(0), \qquad r_2 \equiv A_2(0)/A_1(0)$$

Charmed $P \to V$ decay from experiments

Channels	r_V	r_2	Refs
$D^0 \to K^*(892)$			
(μ)	$1.37 \pm 0.09 \pm 0.03$	$0.76 \pm 0.06 \pm 0.02$	PRL134,011803(2025)
(μ)	$1.46 \pm 0.11 \pm 0.04$	$0.71 \pm 0.08 \pm 0.03$	PRL135,111803(2025)
(e)	$1.48 \pm 0.05 \pm 0.02$	$0.70 \pm 0.04 \pm 0.02$	JHEP03,197(2025)
Average	$1.456 \pm 0.040 \pm 0.016$	$0.715 \pm 0.031 \pm 0.014$	
$D_s \to K^{*,0} e\nu_e$	$1.67 \pm 0.34 \pm 0.16$	$0.77 \pm 0.28 \pm 0.07$	PRL122,061801(2019)
$D_s \to \phi$			
(e)	$1.807 \pm 0.046 \pm 0.065$	$0.816 \pm 0.036 \pm 0.030$	PRD78,051101(2008)
(μ)	$1.58 \pm 0.17 \pm 0.02$	$0.71 \pm 0.14 \pm 0.02$	JHEP12,072(2023)
$D^0 \to \rho^- e^+ \nu_e$	$1.695 \pm 0.083 \pm 0.051$	$0.845 \pm 0.056 \pm 0.039$	PRL122,062001(2019)

$D \to K^*(892)$ from experiment



BESIII,PRL135,111803(2025)

- No full lattice calculation.

$D_s \to \phi$ & $D \to K^*(892)$

$K^*(892)$ $\quad I(J^P) = 1/2(1^-)$

$K^*(892)$ T-Matrix Pole $\sqrt{s}$ (890+-14) - i (26+-6) MeV

$K^*(892)$ **DECAY MODES**

	Mode	Fraction (Γ_i/Γ)	Scale Factor/ Conf. Level	P(MeV/c)
Γ_1	$K\pi$	~ 100 %		289

$\phi(1020)$ $\quad I^G(J^{PC}) = 0^-(1^{--})$

Expand/Collapse All

$\phi(1020)$ MASS	1019.460 ± 0.016 MeV
$\phi(1020)$ WIDTH	4.249 ± 0.013 MeV (S = 1.1)

- Γ_{K^*} is not so small, $K^* \to K\pi$ may have visible effect
- Γ_ϕ is only ~ 4 MeV, a reasonable approximation to regard ϕ as stable
  - (i) Heavier pion mass in our calculation, $m_\phi - 2m_K < 0$
  - (ii) m_π and f_ϕ agree well with current results

Our first target: $D_s \to \phi l \nu_l$

- Previous lattice calculation: HPQCD,PRD90,074506(2014)
  - Three ensembles with two coarse lattice spacings, ~ 0.9 fm and ~ 0.12 fm
  - Extrapolated to physical limit by z-expansion

$$F(z) = \sum_{n=0}^{3} B_n^F \left[1 + C_n^F a^2 + D_n^F a^4 + E_n^F (m_l/m_{s,\mathrm{phys}})\right] z^n$$

  leading to $r_V = 1.72(21)$, $r_2 = 0.74(12)$

- Experiment:

	r_V	r_2	Refs
BaBar(e)	$1.807 \pm 0.046 \pm 0.065$	$0.816 \pm 0.036 \pm 0.030$	PRD78,051101(2008)
BESIII(μ)	$1.58 \pm 0.17 \pm 0.02$	$0.71 \pm 0.14 \pm 0.02$	JHEP12,072(2023)

- A more precise and systematic lattice study is essential

Extraction of the form factor: scalar function method

- Matrix element parameterization

$$\langle h_\nu/J_\nu^{\text{em}}(p)|J_\mu^{\text{em}/W}(0)|H(p')\rangle = \sum_j \mathcal{Q}^j_{\mu\nu}(p,p')F_j(q^2)$$

  - $P \to 2\gamma$: $\pi_0/\eta/\eta'/\eta_c \to 2\gamma$
  - $V \to \gamma P$: $J/\psi \to \gamma\eta_c, D_s^* \to \gamma D_s, D^* \to \gamma D$
  - $P \to P' l\nu_l$, $P \to V l\nu_l$
  - $P \to \gamma l\nu_l$, $P \to l'\bar{l}' l\nu_l$

- Constructing scalar functions

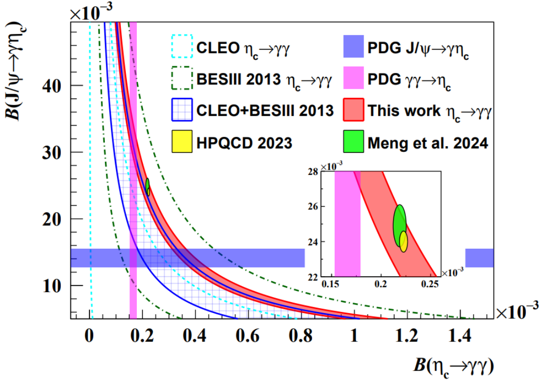
$$\begin{aligned} \tilde{I}_i(q^2) &\equiv \mathcal{Q}^i_{\mu\nu} \cdot \langle h_\nu/J_\nu^{\text{em}}(p)|J_\mu^{\text{em}/W}(0)|H(p')\rangle \\ &= \sum_j \omega_{ij}(p,p')F_j(q^2), \quad \omega_{ij} \equiv \mathcal{Q}^i_{\mu\nu} \cdot \mathcal{Q}^j_{\mu\nu} \end{aligned}$$

- Solving the linear equation

$$F = \omega^{-1} \cdot \tilde{I}$$

Scalar function method: applications

- $\Gamma(\eta_c \to 2\gamma) = 6.67(16)(6)$ keV
  Y.M,Xu Feng,Chuan Liu,Teng Wang,Zuoheng Zou, Sci Bull 68,1880(2023)
- $\Gamma(J/\psi \to \gamma\eta_c) = 2.30(10)$ keV
  Y.M,Chuan Liu,Teng Wang,Haobo Yan, PRD 111,014508(2025)
- $\Gamma(D_s^* \to \gamma D_s) = 0.0549(54)$ keV
  Y.M,Jinlong Dang,Chuan Liu,Zhaofeng Liu,Tinghong Shen,Haobo Yan,Kelong Zhang,PRD 109,074511(2024)
- $\mathrm{Br}(J/\psi \to D_s/De\nu_e) = \begin{cases} 1.90(8) \times 10^{-10} \\ 1.21(11) \times 10^{-10} \end{cases}$
  Y.M,Jinlong Dang,Chuan Liu,Xinyu Tuo,Haobo Yan,Yibo Yang,Kelong Zhang, PRD 110,074510(2024)



BESIII,PRL134,181901(2025)

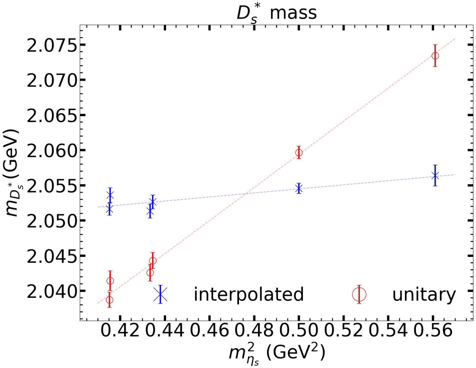
Lattice setup

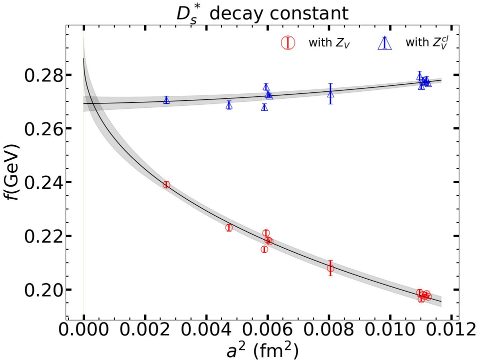
	C24P29	C32P23	C32P29	F32P30	F48P21	G36P29	H48P32
a (fm)	0.10524(05)(62)			0.07753(03)(45)		0.06887(12)(41)	0.05199(08)(31)
am_l	-0.2770	-0.2790	-0.2770	-0.2295	-0.2320	-0.2150	-0.1850
am_s	-0.2400	-0.2400	-0.2400	-0.2050	-0.2050	-0.1926	-0.1700
am_s^{V}	$-0.2356(1)$	$-0.2337(1)$	$-0.2358(1)$	$-0.2038(1)$	$-0.2025(1)$	$-0.1928(1)$	$-0.1701(1)$
am_c^{V}	0.4159(07)	0.4190(07)	0.4150(06)	0.1974(05)	0.1997(04)	0.1433(12)	0.0551(07)
L (fm)	2.53	3.37	3.37	2.48	3.72	2.48	2.50
$L^3 \times T$	$24^3 \times 72$	$32^3 \times 64$	$32^3 \times 64$	$32^3 \times 96$	$48^3 \times 96$	$36^3 \times 108$	$48^3 \times 144$
N_{mea}	$450 \times 72 \times 2$	$333 \times 64 \times 3$	$397 \times 64 \times 2$	$360 \times 96 \times 2$	$241 \times 48 \times 4$	$300 \times 54 \times 2$	$300 \times 72 \times 2$
m_π (MeV/c^2)	292.3(1.0)	227.9(1.2)	293.1(0.8)	300.4(1.2)	207.5(1.1)	297.2(0.9)	316.6(1.0)
t	$2-17$	$2-20$	$2-20$	$4-22$	$4-26$	$2-32$	$8-30$
Z_V^s	0.85184(06)	0.85350(04)	0.85167(04)	0.86900(03)	0.86880(02)	0.87473(05)	0.88780(01)
Z_V^c	1.57353(18)	1.57644(12)	1.57163(14)	1.30566(07)	1.30673(04)	1.23990(13)	1.12882(11)
Z_A/Z_V	1.07244(70)	1.07375(40)	1.07648(63)	1.05549(54)	1.05434(88)	1.04500(22)	1.03802(28)

- Four lattice spacings $[0.052 - 0.105]$ fm $\rightarrow$ continuum limit
- $m_\pi = 208 - 317$ MeV $\rightarrow$ physical pion msss
- Same lattice spacings and pion mass: C24P29 vs C32P29 $\rightarrow$ finite-volume effect

Scale setting and renormalization

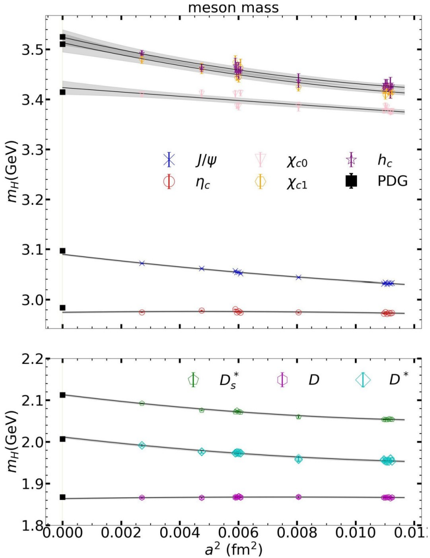
- Interpolated strange quark mass determined by $\eta_s = 689.89(49)$ MeV on each ensemble
- Optimised vector renormalization constant $Z_V^{c,s}$ and $Z_A^{c,s} = Z_A/Z_V \times Z_V^{c,s}$



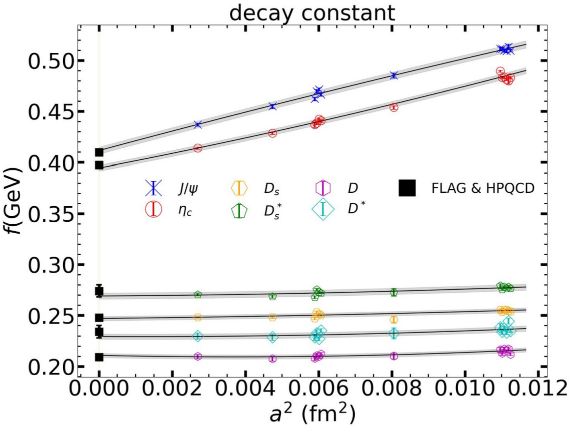


CLQCD,PRD111,054504(2025)

Mass and decay constant-I



CLQCD,PRD111,054504(2025)



Mass and decay constant-II

- m_ϕ and f_ϕ in physical extrapolation

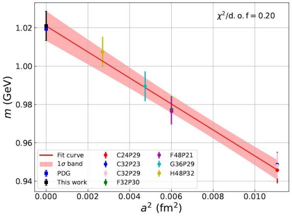
$$m_\phi, f_\phi = c_0 + c_a a^2 + c_m \left(m_\pi^2 - m_{\pi,\text{phys}}^2\right)$$

- Our result

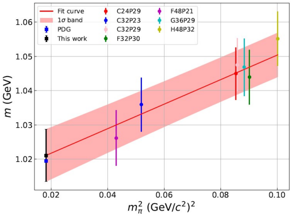
$$m_\phi = 1.0211(76)\ \text{GeV}$$
$$f_\phi = 0.2462(41)\ \text{GeV}$$

while HPQCD(14)

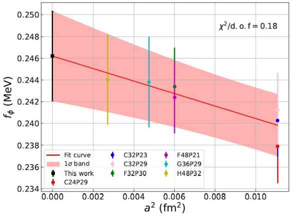
$$m_\phi = 1.0320(160)\ \text{GeV}$$
$$f_\phi = 0.2410(180)\ \text{GeV}$$



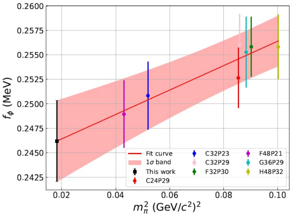
(i) ϕ mass at $m_\pi = 0.135\ \text{GeV}/c^2$.



(ii) ϕ mass at $a = 0.0$ fm.

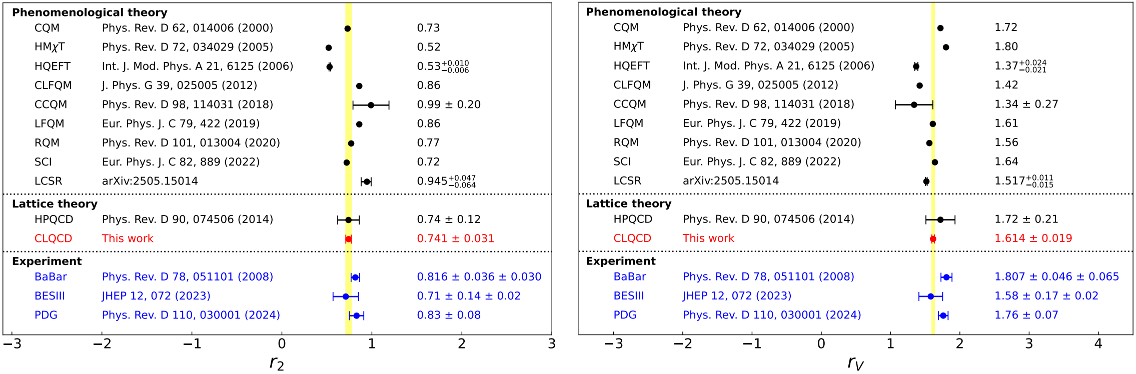


(iii) ϕ decay constant at $m_\pi = 0.135\ \text{GeV}/c^2$.

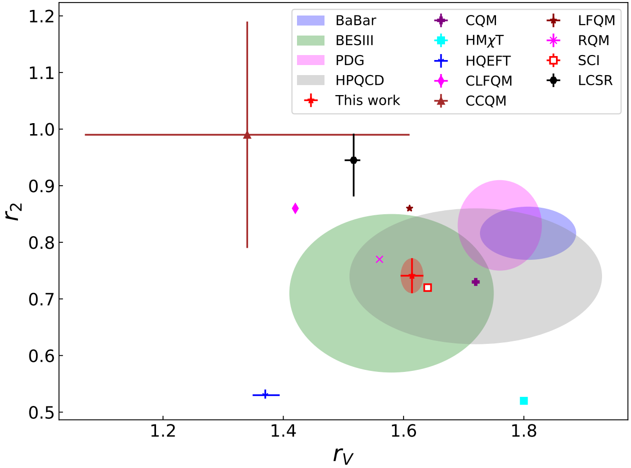


(iv) ϕ decay constant at $a = 0.0$ fm.

$D_s \to \phi$ form factors



$D_s \to \phi$ form factors



Differential decay width

$\mathcal{B}(D_s \to \phi \ell \nu_\ell) \times 10^{-2}$	μ channel	e channel
This work	2.351(67)	2.493(73)
BaBar [7]	–	2.61(17)
CLEO [8]	–	2.14(19)
BESIII (2018) [9]	1.94(54)	2.26(46)
BESIII (2023) [10]	2.25(11)	–
PDG [39]	2.24(11)	2.34(12)

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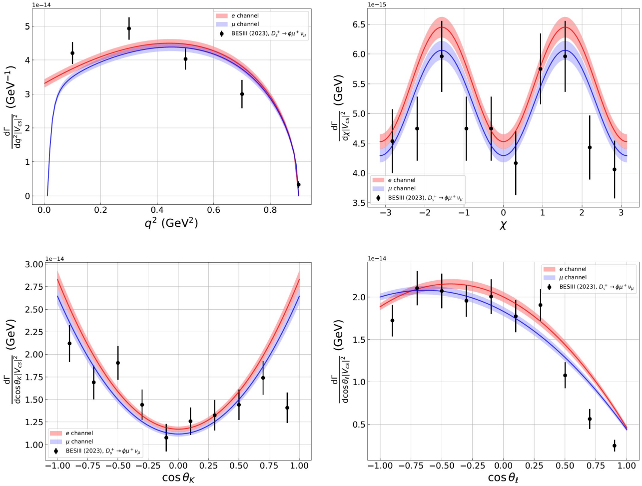
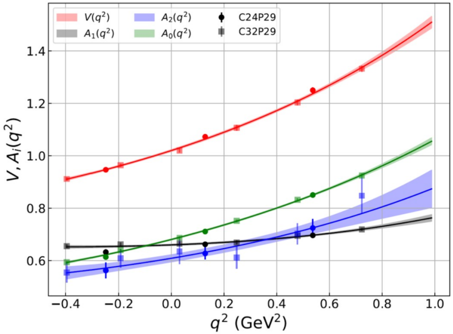


Finite-volume effect

C24P29, L=2.53 fm

C32P29, L=3.37 fm



	C24P29	χ^2/d.o.f	C32P29	χ^2/d.o.f	Combined	χ^2/d.o.f
$V(0)$	1.0271(66)	0.03	1.0167(43)	0.29	1.0202(36)	0.63
$A_1(0)$	0.6523(55)	0.01	0.6639(37)	0.24	0.6605(30)	0.82
$A_2(0)$	0.605(18)	0.01	0.616(20)	0.53	0.609(14)	0.38
$A_0(0)$	0.6759(45)	0.07	0.6835(29)	0.26	0.6811(24)	0.54

Conclusion and outlook

- Conclusion
  - Scalar function method for $P \to V$ semileptonic decay
  - Systematic study on $D_s \to \phi l \nu_l$ using seven CLQCD ensembles
  - Most precise form factors at physical point after continuum and chiral limit
- Outlook
  - Effect of the vector particle decay width, $P \to V(\to P_1 P_2) l \nu_l$
  - Charmed meson semileptonic decay: $D \to K^*$, $D \to \rho$

Thank you for attention!