Heavy meson light-cone distribution amplitudes from lattice QCD

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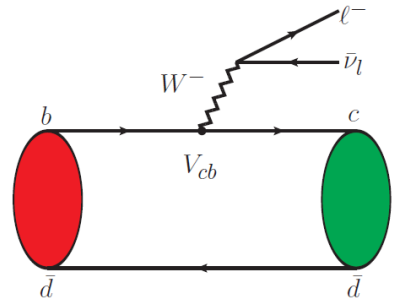
Outline

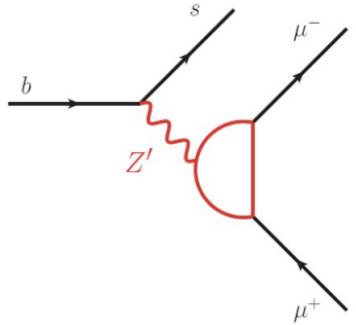
- Motivation
- Theory Framework
- Lattice QCD Calculation
- Summary and Outlook

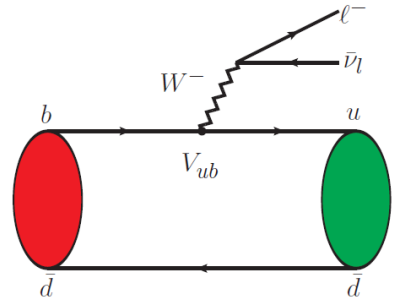
Why heavy meson LCDAs

➢Weak decays of heavy mesons play a crucial role in

- Precise tests of Standard Model
- Searching for New Physics







Gao, Lu, Shen, Wang, Wei, PRD 101 (2020) 074035

$$\mathcal{V}_{B\to K^*}(0) = 0.359^{+0.141}_{-0.085}\Big|_{\lambda_B}{}^{+0.019}_{-0.019}\Big|_{\sigma_1}{}^{+0.001}_{-0.062}\Big|_{\mu} {}^{+0.010}_{-0.004}\Big|_{M^2}{}^{+0.016}_{-0.017}\Big|_{s_0}{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$

λ_B, σ_1: Inverse moment of heavy meson LCDA

$\varphi_\pm$: systematic error from model uncertainty

Without reliable heavy meson LCDA, it is impossible to discuss precision calculation!

Why LQCD

- **In current theoretical predictions, the uncertainty originating from model selection play an important role:**

$$\mathcal{V}_{B\to K^*}(0) = 0.359^{+0.141}_{-0.085}\Big|_{\lambda_B}{}^{+0.019}_{-0.019}\Big|_{\sigma_1}{}^{+0.001}_{-0.062}\Big|_{\mu} {}^{+0.010}_{-0.004}\Big|_{M^2}{}^{+0.016}_{-0.017}\Big|_{s_0}{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$

λ_B: the first inverse moment; σ_1,: inverse-logarithmic moments;

$\varphi_\pm$: systematic error from model uncertainty

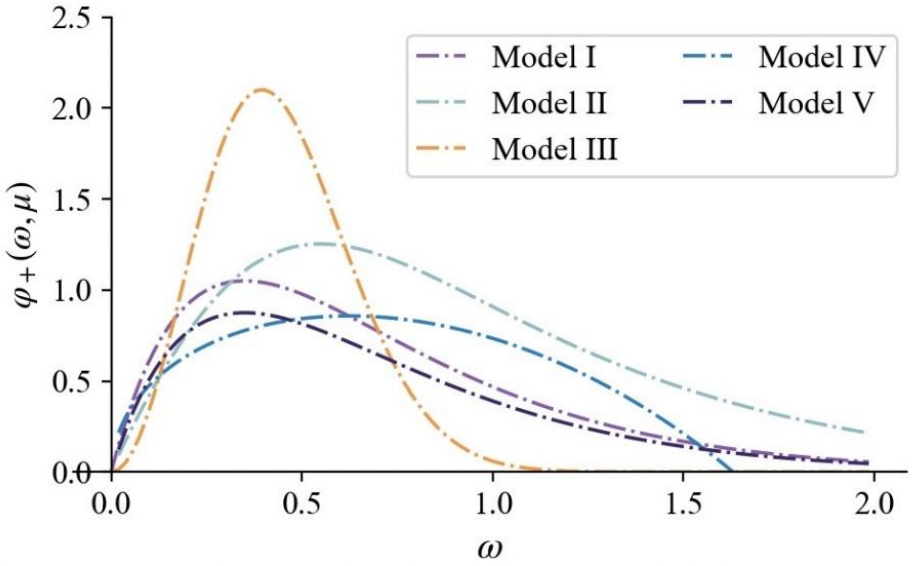
$$\varphi_{\rm I}^{+}(\omega,\mu_0) = \frac{\omega}{\omega_0^2}e^{-\omega/\omega_0},$$

$$\varphi_{\rm II}^{+}(\omega,\mu_0) = \frac{4}{\pi\omega_0}\frac{k}{k^2+1}\left[\frac{1}{k^2+1} - \frac{2\left(\sigma_B^{(1)}-1\right)}{\pi^2}\ln k\right],$$

$$\varphi_{\rm III}^{+}(\omega,\mu_0) = \frac{2\omega^2}{\omega_0\omega_1^2}e^{-(\omega/\omega_1)^2},$$

$$\varphi_{\rm IV}^{+}(\omega,\mu_0) = \frac{\omega}{\omega_0\omega_2}\frac{\omega_2-\omega}{\sqrt{\omega\,(2\omega_2-\omega)}}\theta(\omega_2-\omega),$$

$$\varphi_{\rm V}^{+}(\omega,\mu_0) = \frac{\Gamma(\beta)}{\Gamma(\alpha)}\frac{\omega}{\omega_0^2}e^{-\omega/\omega_0}U(\beta-\alpha,3-\alpha,\omega/\omega_0),$$



Gao, Lu, Shen, Wang, Wei, PRD 101 (2020) 074035

Calculating heavy meson LCDAs from first principles significantly improves the precision of theoretical predictions.

Heavy meson LCDAs

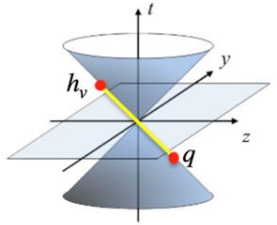
- LCDAs contribute to the exclusive decays:

$$\left\langle \pi\left(p'\right)\pi(q)\left|Q_i\right|\bar{B}(p)\right\rangle = f^{B\to\pi}\left(q^2\right)\int_0^1 dx T_i^{\mathrm{I}}(x)\phi_\pi(x) + \int_0^1 d\xi dx dy T_i^{\mathrm{II}}(\xi,x,y)\varphi_B^+(\xi)\phi_\pi(x)\phi_\pi(y)$$

Beneke,Buchalla,Neubert, Sachrajda3 PRL 83, 1914 (1999)

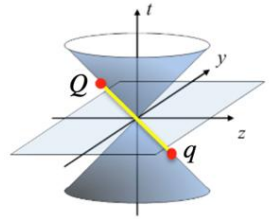
- **HQET LCDA**:

$$\varphi^+(\omega,\mu) = \frac{1}{i\tilde{f}_{H_Q}m_{H_Q}n_+\cdot v}\int\frac{dt}{2\pi}e^{-i\omega t\cdot n_+\cdot v}\left\langle 0\mid \bar{q}(tn_+)\eta_+\gamma_5 W_c(tn_+,0)h_v(0)|H_Q(v)\right\rangle$$



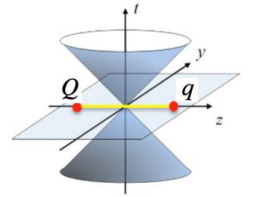
- **QCD LCDA**:

$$\phi(y,\mu) = \frac{1}{if_H}\int_{-\infty}^{+\infty}\frac{d\tau}{2\pi}e^{iyP_H\tau n_+}\langle 0|\bar{q}(\tau n_+)\acute{n}\gamma_5 W_c(\tau n_+,0)Q(0)|H(P_H)\rangle$$



- **Quasi-DA**:

$$\tilde{\phi}(x,P^z) = \int\frac{dz}{2\pi}e^{izP^z}\left\langle P^z|\bar{\psi}(z)\Gamma\mathcal{U}(z,0)\psi(0)|P^z\right\rangle$$



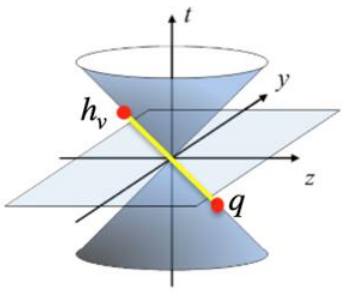
Sequential effective theory

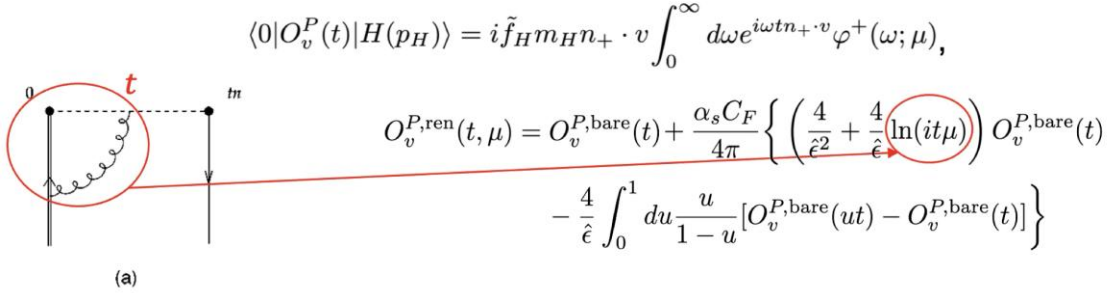
HQET LCDA:

$$\varphi^+(\omega,\mu) = \frac{1}{i\,\tilde{f}_{H_Q} m_{H_Q} n_+\cdot v}\int \frac{dt}{2\pi} e^{-i\omega t n_+\cdot v} \left\langle 0 \left| \bar{q}(t n_+) \not{n}_+ \gamma_5 W_c(t n_+, 0) h_v(0) \right| H_Q(v) \right\rangle$$

- **Direct calculation of HQET LCDAs faces the following issues:**

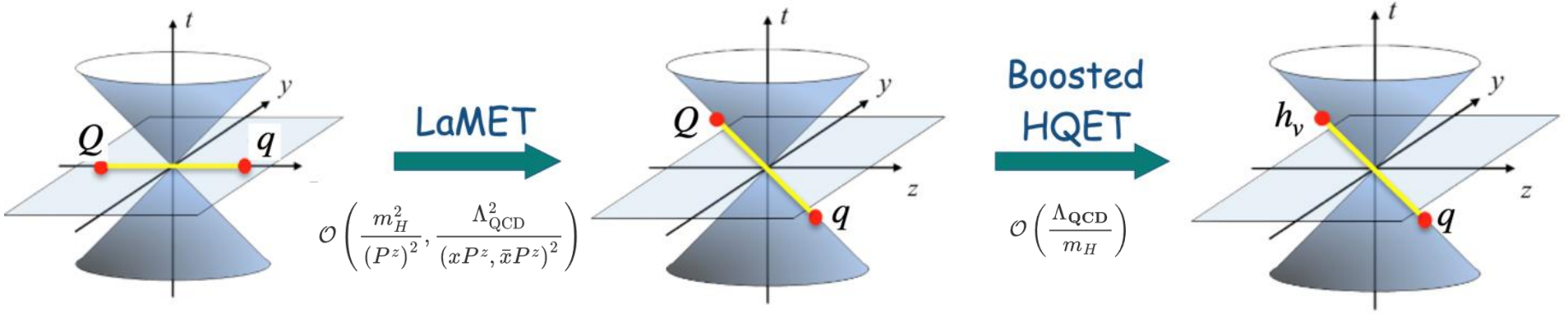
  - Light-like correlators
  - Cusp divergence





The HQET LCDA cannot be calculated directly on the lattice.

Sequential effective theory



Quasi-DA
Equal-time correlators

QCD LCDA
Light-like correlators
with **QCD fields**

HQET LCDA
Light-like correlators
with **HQET fields**

- **Scales: $\Lambda_{\rm QCD}$, m_Q, P^z**
- **LQCD calculable**

Liu, Wang, Xu, QAZ, Zhao, PRD 99, 094036 (2019)
Han, QAZ, et.al., PRD111, 034503, (2025)

LaMET:

- $\Lambda_{\rm QCD}, m_Q \ll P^z$
- **Integrate out P^z**

Hierarchy:
$\Lambda_{QCD} \ll m_Q \ll P^z$

bHQET:

- $\Lambda_{\rm QCD} \ll m_Q$
- **Integrate out m_Q**

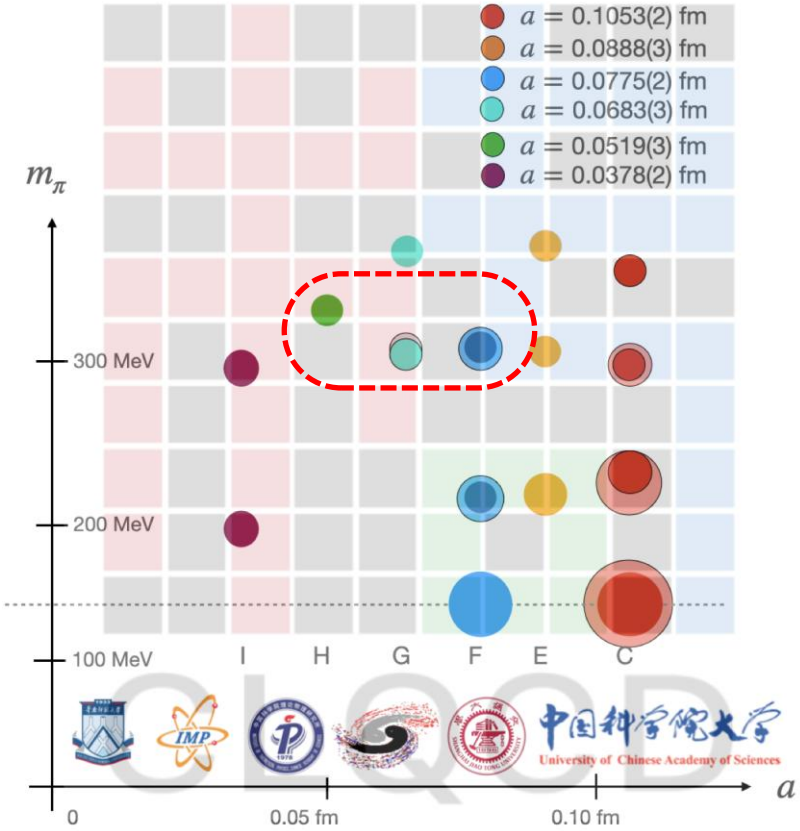
Ishaq, Jia, Xiong, Yang, PRL125, 132001 (2020)
Zhao, PRD 101, 071503 (2020)
Beneke, Finauri, Vos, Wei, JHEP 09, 066 (2023)

Lattice setup

- 2+1 flavor QCD ensembles generated by the **CLQCD collaboration.**
- Three lattice spacings $\boldsymbol{a}$={$\mathbf{0.078}$, $\mathbf{0.068}$, $\mathbf{0.052}$}fm, enables continuum extrapolation.
- Multiple momenta up to 3.5GeV to carry out the infinite momentum extrapolation.
- Different pion masses to ensure physical mass extrapolation.

Hu et.al., PRD109, 054507 (2024)
Du et.al., PRD111, 054504 (2025)

Ensemble	Volume	a (fm)	m_π(MeV)	N_{stats}	Pz
F32P21	32^3x64	0.0775	210		
F32P30	32^3x96	0.0775	303	900*27	0-2.5 GeV
G36P29	36^3x108	0.6826	295	600*27	0-2.5 GeV
H48P32	48^3x144	0.5187	317	550*27	0-3.5 GeV



Bare matrix elements

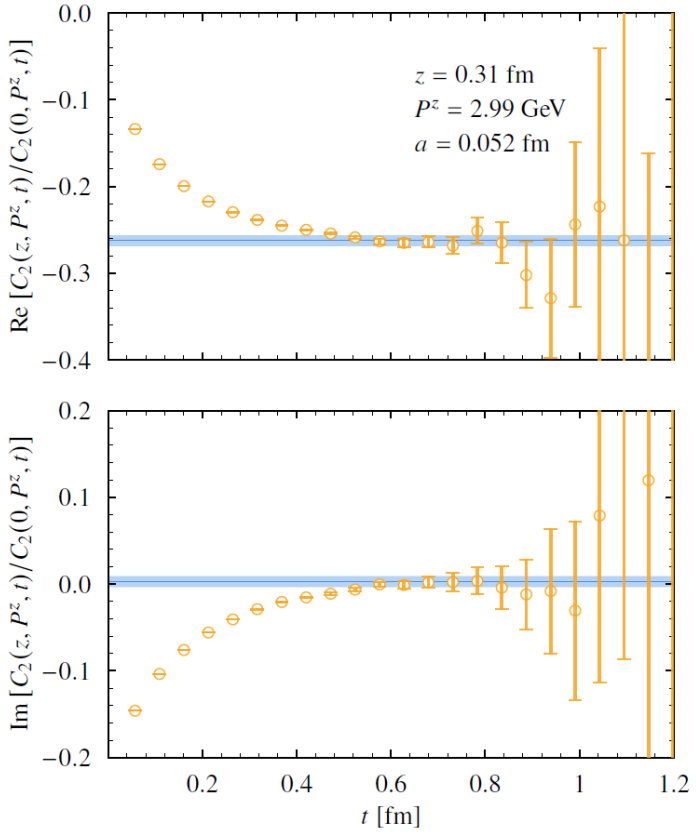
- Nonlocal 2-point correlation function:

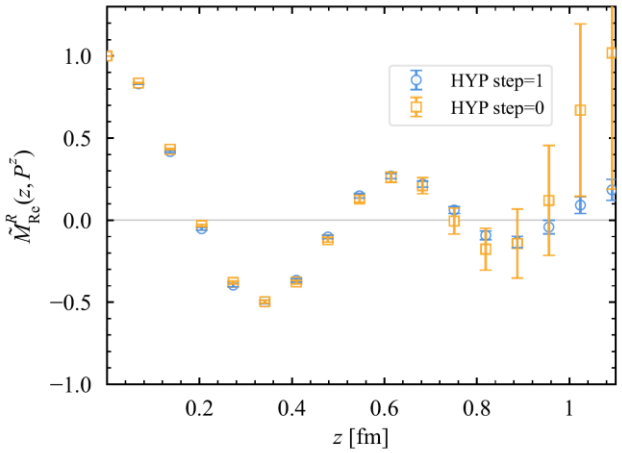
$$C_2(z,P^z,t) = \sum_{x^3} e^{-iP^z x^3} \left\langle \gamma_5 G_q^\dagger(x^3,t,0)\gamma_5 \Gamma_{sink} \times W_c(x^3,x^3+z)\Gamma_{source} G_Q(x^3+z,t,0)\right\rangle$$

- Bare matrix elements $\tilde{M}^B(z,P^z)$:

$$\frac{C_2(z,P^z,t)}{C_2(0,P^z,t)} = \tilde{M}^B(z,P^z)e^{-E_0 t} \times \left[\sum_{n=0}^{N_{\text{stat}}-1} A_n e^{-(E_n-E_0)t}\right].$$

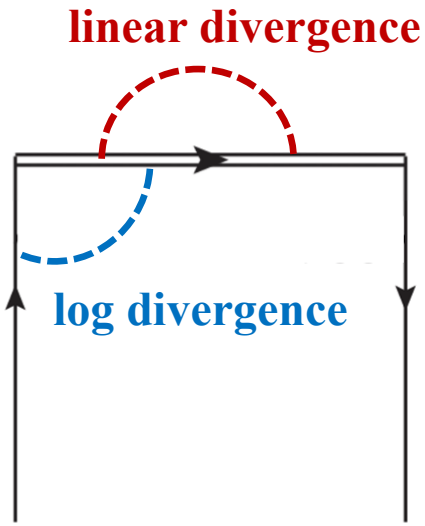
- Beside high statistics, **HYP smearing** is used to improve the signal.





Hybrid renormalization

- $\tilde{M}^B(z, P^z)$ contains both **linear** and **logarithmic** divergences, can be multiplicatively renormalized.



- **Hybrid renormalization scheme**:

$$\tilde{M}^R(z, P^z) = \frac{\tilde{M}^{(0)}(z, P^z, 1/a)}{\tilde{M}^{(0)}(z, P^z_{\text{ref}}, 1/a)}\theta(z_s - |z|) + \frac{\tilde{M}^{(0)}(z, P^z, 1/a)}{Z_R(z, 1/a)}\theta(|z| - z_s),$$

  - Short distance (perturbative region): **ratio scheme**

$$\langle 0|\bar{q}(z)\gamma^z\gamma_5 W_c(z,0)Q(0)|H(P^z)\rangle \propto P^z \quad \Rightarrow \quad P^z_{\text{ref}} = 0.5\text{GeV}$$

  - Long distance (nonperturbative region): **self renormalization scheme**

$$Z_R(z, 1/a) = \exp\left\{\frac{kz}{a\ln[a\Lambda_{\text{QCD}}]} + m_0 z + f(z)a^2 + \frac{3C_F}{b_0}\ln\left[\frac{\ln[1/(a\Lambda_{\text{QCD}})]}{\ln[\mu/\Lambda_{\text{QCD}}]}\right] + \ln\left[1 + \frac{d}{\ln[a\Lambda_{\text{QCD}}]}\right]\right\}$$

See Hao-Yang' talk

Hybrid renormalization

➢ **Self renormalization scheme**:

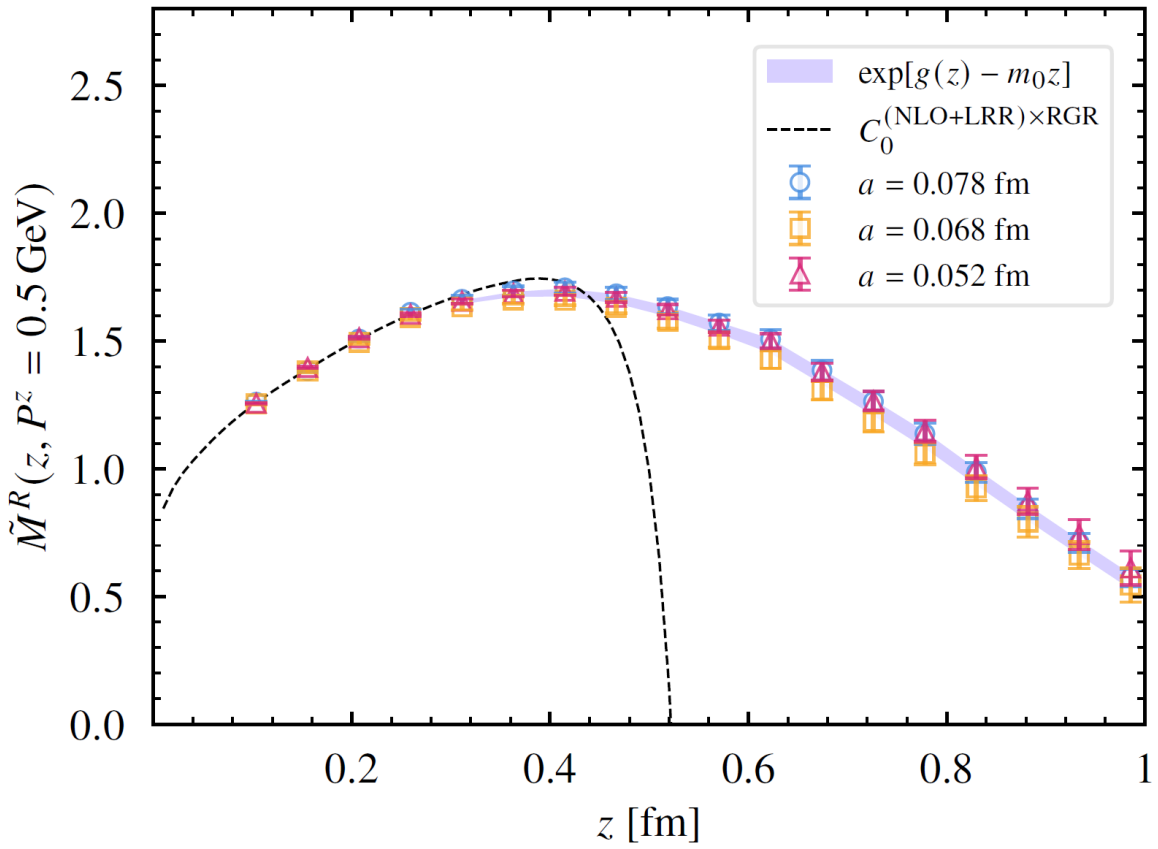
- **Short distance**:

  Lattice **agree well** with perturbation results (NLO, LRR+RGR-improvement)

  See Jun' talk

- **Long distance**:

$$Z_R(z,1/a) = \exp\left\{\frac{kz}{a\ln[a\Lambda_{\rm QCD}]} + m_0 z + f(z)a^2 + \frac{3C_F}{b_0}\ln\left[\frac{\ln[1/(a\Lambda_{\rm QCD})]}{\ln[\mu/\Lambda_{\rm QCD}]}\right] + \ln\left[1+\frac{d}{\ln[a\Lambda_{\rm QCD}]}\right]\right\}$$



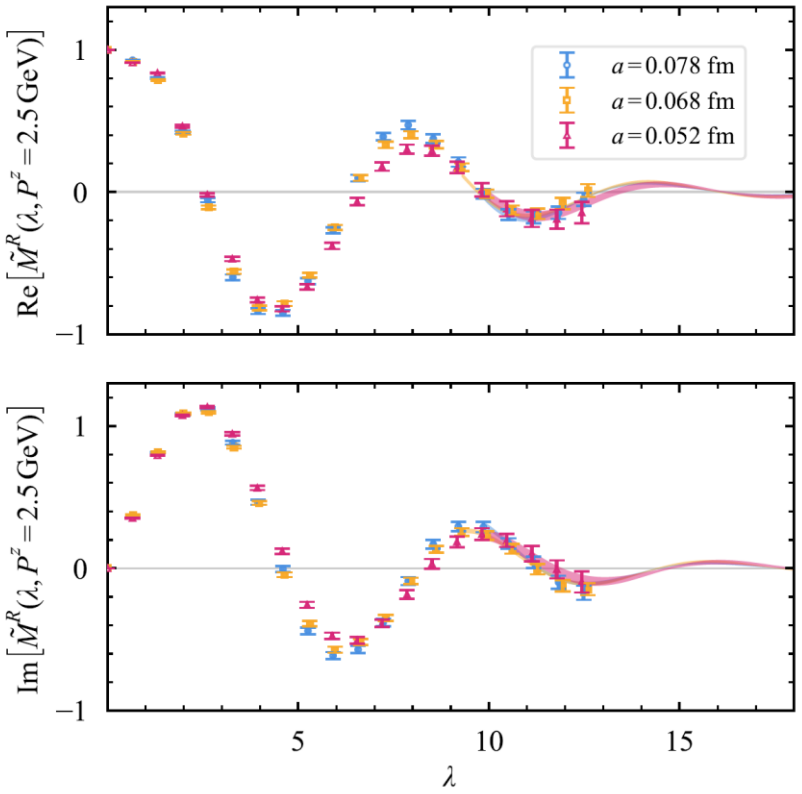
Fit results: $(\chi^2/d.o.f = 0.92)$

$k = 0.802(35)$, $\Lambda_{\rm QCD} = 0.192(75)$GeV,

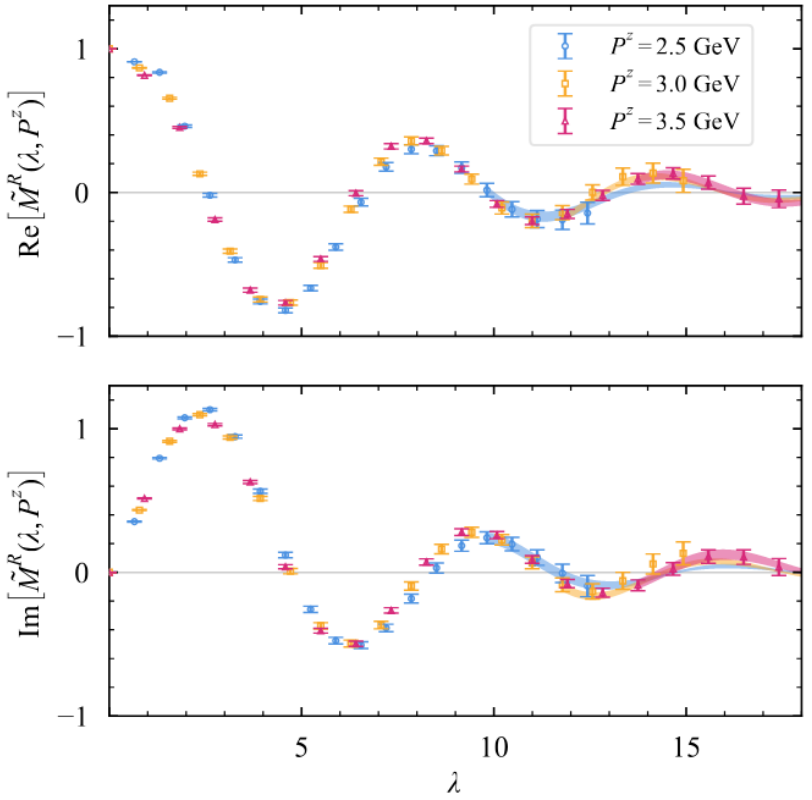
$m_0 = 0.241(98)$GeV, $d = -0.18(10)$

Renormalized matrix elements

- **a-dependence, $P^z = 2.5\text{GeV}$**



- **P^z-dependence, $a = 0.052\text{fm}$**



Quasi-DA: $\tilde{\phi}(x, P^z) = \int_{-\infty}^{+\infty} \frac{dz}{2\pi} e^{-ixP^z z} \tilde{M}^R(z, P^z).$

A brute-fore truncation at large λ will lead to strong oscillation after FT: $\tilde{h}_{\text{extra}}(\lambda) = \left[\frac{c_1}{(-i\lambda)^{n_1}} + e^{i\lambda}\frac{c_2}{(i\lambda)^{n_2}}\right] e^{-\lambda/\lambda_0}$

Matching in LaMET

Quasi-DA: $\tilde{\phi}(x, P^z) = \int_{-\infty}^{+\infty} \frac{dz}{2\pi} e^{-ixP^z z} \tilde{M}^R(z, P^z).$

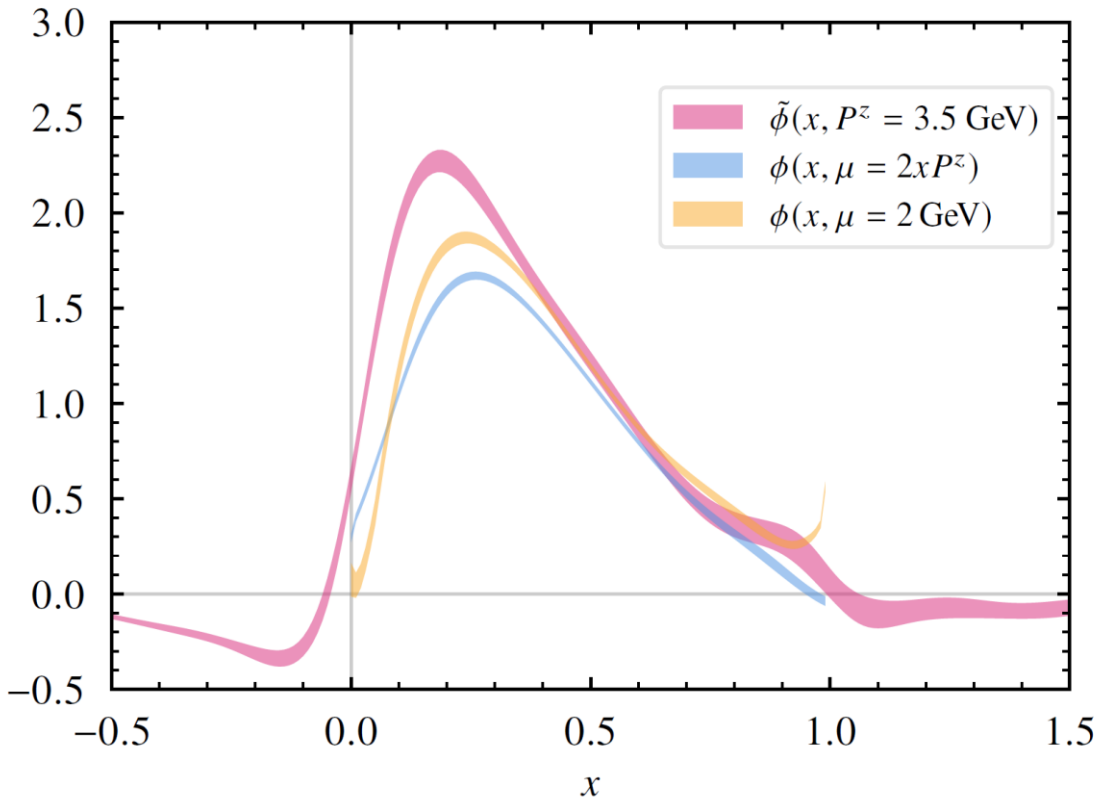
Matching from quasi-DA to QCD LCDA:

$$\phi(y,\mu) = \int_{-\infty}^{+\infty} \frac{dx}{|x|} \mathcal{C}\left(\frac{y}{x}, \frac{\mu}{xP^z}\right) \tilde{\phi}(x, P^z) + \mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(yP^z, (1-y)P^z)^2}\right).$$

Liu, Wang, Xu, QAZ, Zhao, PRD 99, 094036 (2019)
Han, QAZ, et.al., PRD111, 034503, (2025)

$$\frac{dC^{-1}(x, y, P^z, \mu)}{d\ln\mu^2} = \int_0^1 d\zeta V[\zeta, y, \alpha_s(\mu)] C^{-1}(x, \zeta, P^z, \mu)$$

$$\phi(y, 2yP^z) \overset{\rm ERBL}{\longrightarrow} \phi(y, 2{\rm GeV})$$

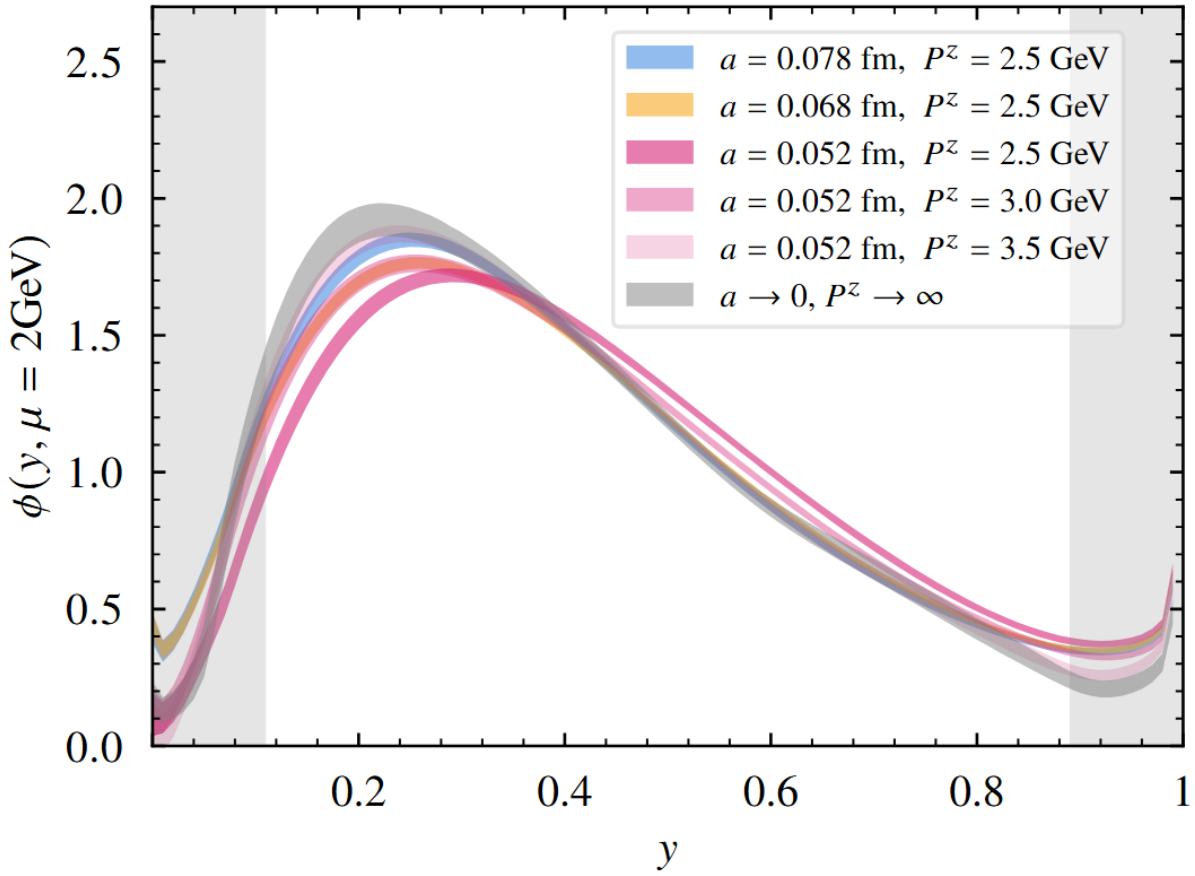


Quasi-DA and QCD LCDA

QCD LCDA of D meson

Continuum and infinite-momentum extrapolation:

$$\phi(y;a,P^z)=\phi(y;a\to 0,P^z\to\infty)\left[1+c_1(y)a^2+\mathcal{O}(a^4)\right]\left[1+\frac{c_2(y)}{(P^z)^2}+\mathcal{O}\left(\frac{1}{(P^z)^4}\right)\right]$$



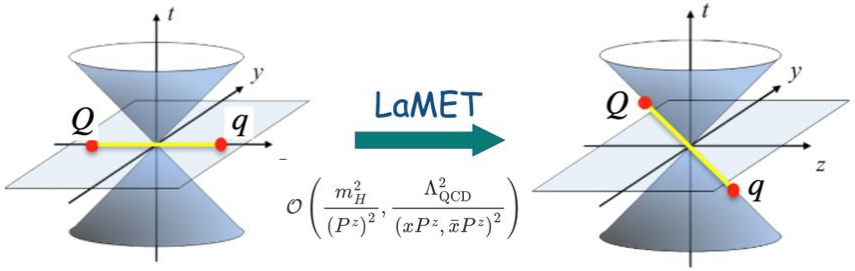
$$\phi(x,\mu;a,P^z,m_\pi)=\phi(x,\mu;m_\pi^{phys})\left[1+c_1(x)a^2\right]\left[1+\frac{c_2(x)}{(P^z)^2}\right]$$
$$\times\left[1+c_3(x)(m_\pi^2-(m_\pi^{phys})^2)\right]$$

in progress

Estimation of power corrections in LaMET matching

- Power corrections:

$\frac{\Lambda^2_{\rm QCD}}{(yP^z,(1-y)P^z)^2}$: Contribute to end-point region



$\frac{m_H^2}{(P^z)^2}$: All region, can be estimated by predictions without LaMET.

- Gegenbauer moments from LaMET
- Mellin moment from OPE

$$C_0^{(3/2)}(\xi) = \langle 1 \rangle,$$
$$C_1^{(3/2)}(\xi) = 3\langle \xi \rangle,$$
$$C_2^{(3/2)}(\xi) = \frac{3}{2}(5\langle \xi^2 \rangle - 1),$$

Power correction $\frac{m_H^2}{(P^z)^2}$ can be estimated from local moments

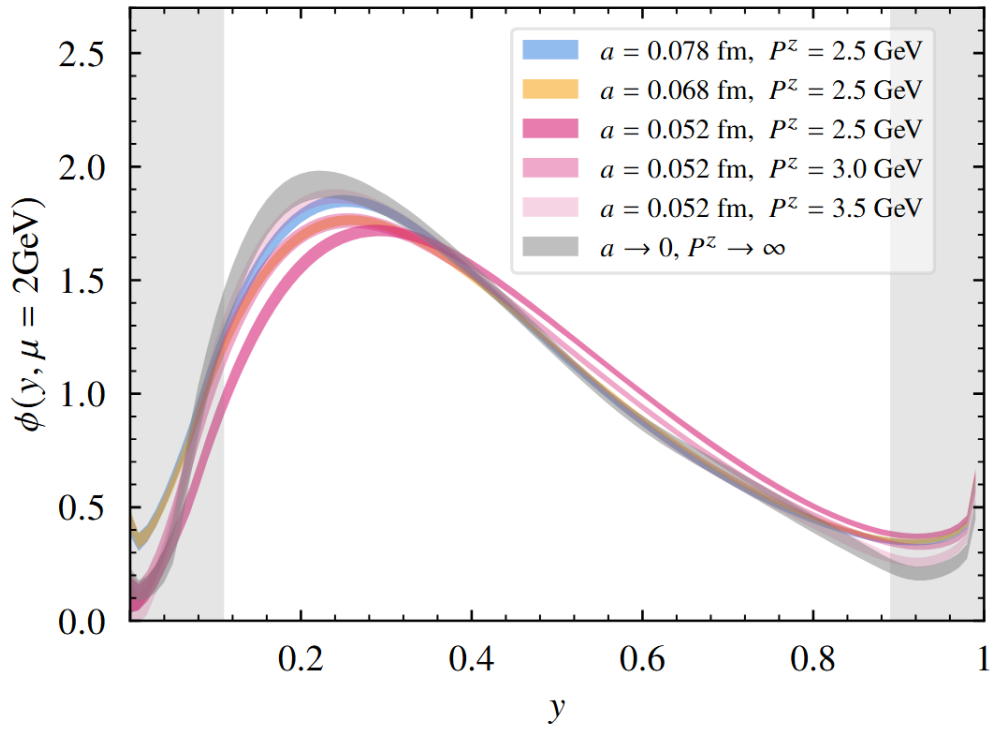
Estimation of power corrections in LaMET matching

- Gegenbauer moments from LaMET:

$$\phi(x,\mu^2) \;=\; 6x(1-x)\left[1+\sum_{n=1}^{\infty} a_n(\mu^2)\, C_n^{3/2}(2x-1)\right],$$

Fitting from data excluding the end-point region:

n	a_1	a_2	a_3	a_4
2	-0.397(18)	0.118(10)		
4	-0.412(20)	0.134(16)	-0.016(11)	0.005(8)
6	-0.391(22)	0.111(18)	0.026(20)	-0.019(15)
8	-0.376(24)	0.109(19)	0.041(22)	-0.034(18)



Estimation of power corrections in LaMET matching

- Moments from local operators:

$$\bar{q}(z_2 n)\not{n}\gamma_5 W_c(z_2 n, z_1 n)\, q'(z_1 n)$$
$$= \sum_{k,l=0}^{\infty} \frac{z_2^k z_1^l}{k!l!} n^\rho n^{\mu_1} \ldots n^{k+l} \mathcal{M}^{(k,l)}_{\rho\mu_1\ldots\mu_{k+l}} + \cdots,$$

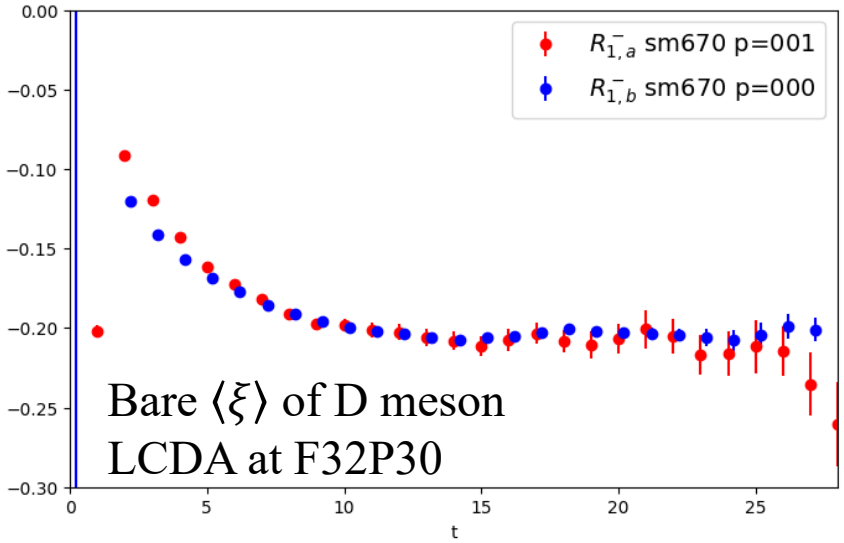
$$\mathcal{M}^{(k,l)}_{\rho\mu_1\ldots\mu_{k+l}} = \bar{q}(0)\overleftarrow{D}_{(\mu_1}\ldots\overleftarrow{D}_{\mu_k}\overrightarrow{D}_{\mu_{k+1}}\ldots\overrightarrow{D}_{\mu_{k+l}\gamma_\rho)}\gamma_5 q'(0).$$

- Definition of operator

$$P(x) = \bar{\psi}_1(x)\gamma_5\psi_2(x)$$
$$O^-_{\mu\nu}(x) = \bar{\psi}_1(x)\gamma_{(\mu}\Gamma[\overrightarrow{D}_{\nu)} - \overleftarrow{D}_{\nu)}]\psi_2(x)$$

- Mellin moment

$$R^-_{1,a} = \frac{i}{3}\sum_{j=1}^{3}\frac{1}{p_j}\frac{C_{O_{4j}P}(t)}{C_{A_4P}(t)} = \langle\xi\rangle \quad R^-_{1,b} = \frac{4E}{3E^2+\vec{p}^2}\frac{C_{O_{44}P}(t)}{C_{A_4P}(t)} = \langle\xi\rangle$$

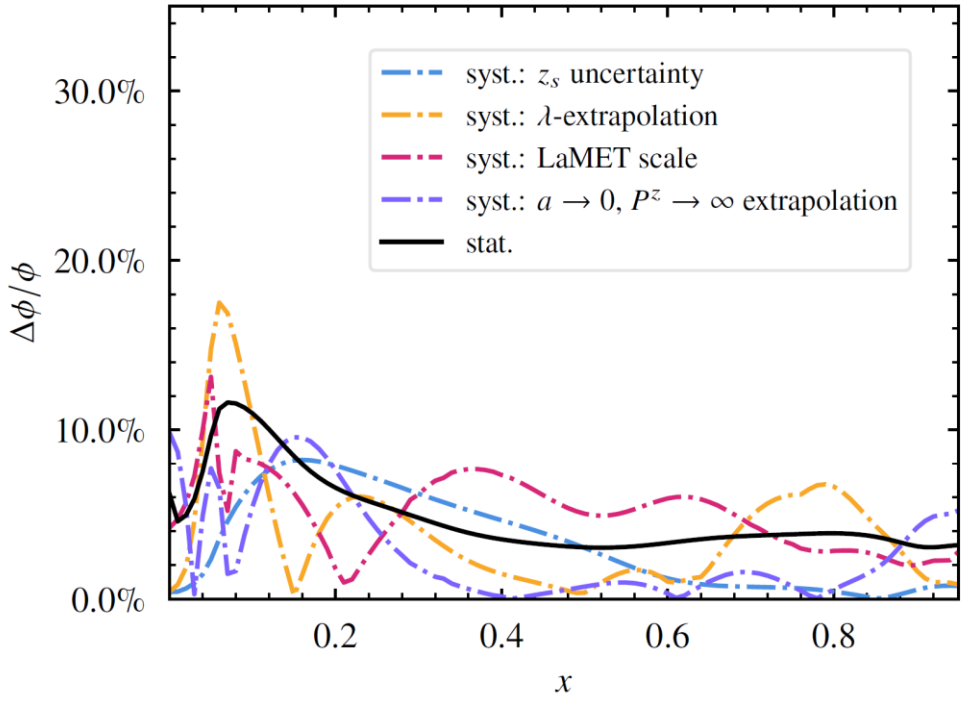


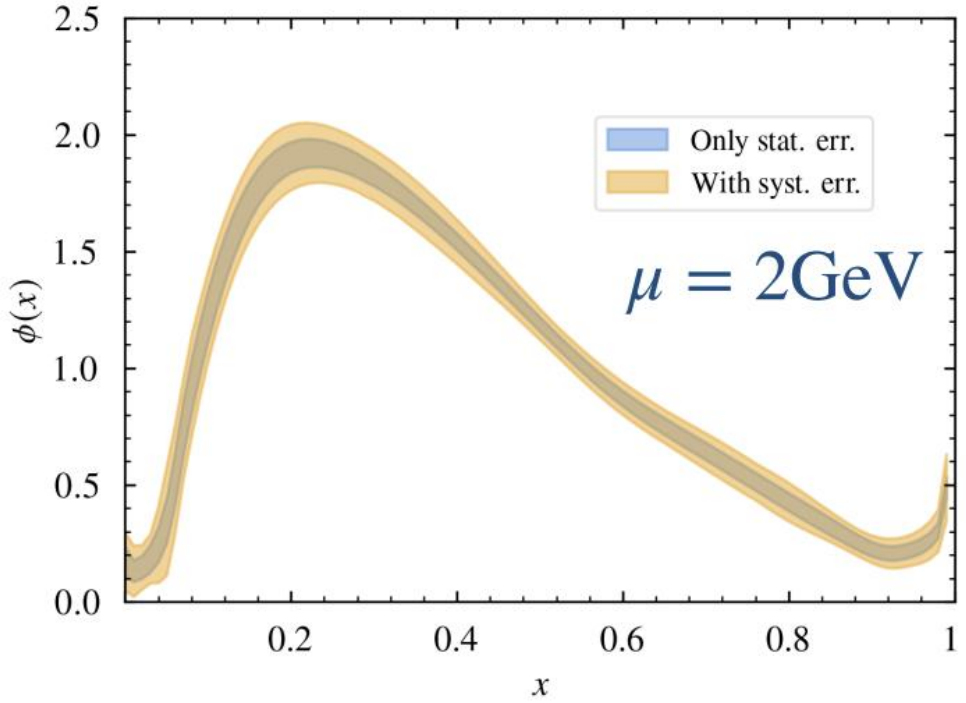
$$C_0^{(3/2)}(\xi) = \langle 1\rangle,$$
$$C_1^{(3/2)}(\xi) = 3\langle\xi\rangle,$$
$$C_2^{(3/2)}(\xi) = \frac{3}{2}(5\langle\xi^2\rangle - 1),$$

Power correction $\frac{m_H^2}{(P^z)^2}$ can be estimated from local moments

Analysis of systematic errors

- Sources of systematic error:
  - ✓ Hybrid renormalization — selection of z_s
  - ✓ Lambda extrapolation — extrapolation region
  - ✓ LaMET scale — $\mu = 2xP^z$
  - ✓ Extrapolation

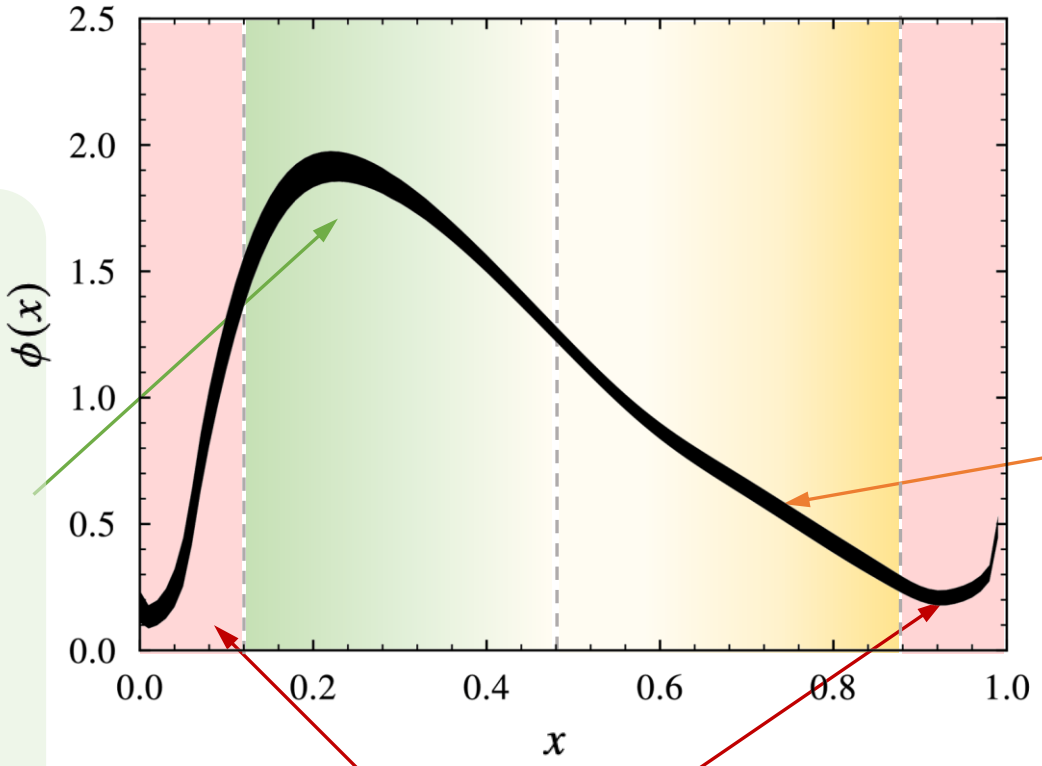




QCD LCDA of D meson

Peak region: x $\sim \frac{\Lambda_{\rm QCD}}{m_H}$

- Light quark carries small momentum fraction;
- Related to the HQET LCDA.



Tail region: x ~ 1

- Contain only hard-collinear physics;
- Suppressed in LCDA.

End-point region:

- LaMET matching kernel suffer large power corrections.
- Lattice QCD predictions fail

$$\mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(xP^z, \bar{x}P^z)^2}\right)$$

Ishaq, Jia, Xiong, Yang, PRL125(2020)132001
Beneke, Finauri, Vos, Wei, JHEP 09, 066 (2023)

Prediction for HQET LCDA

- Leading twist HQET LCDA:

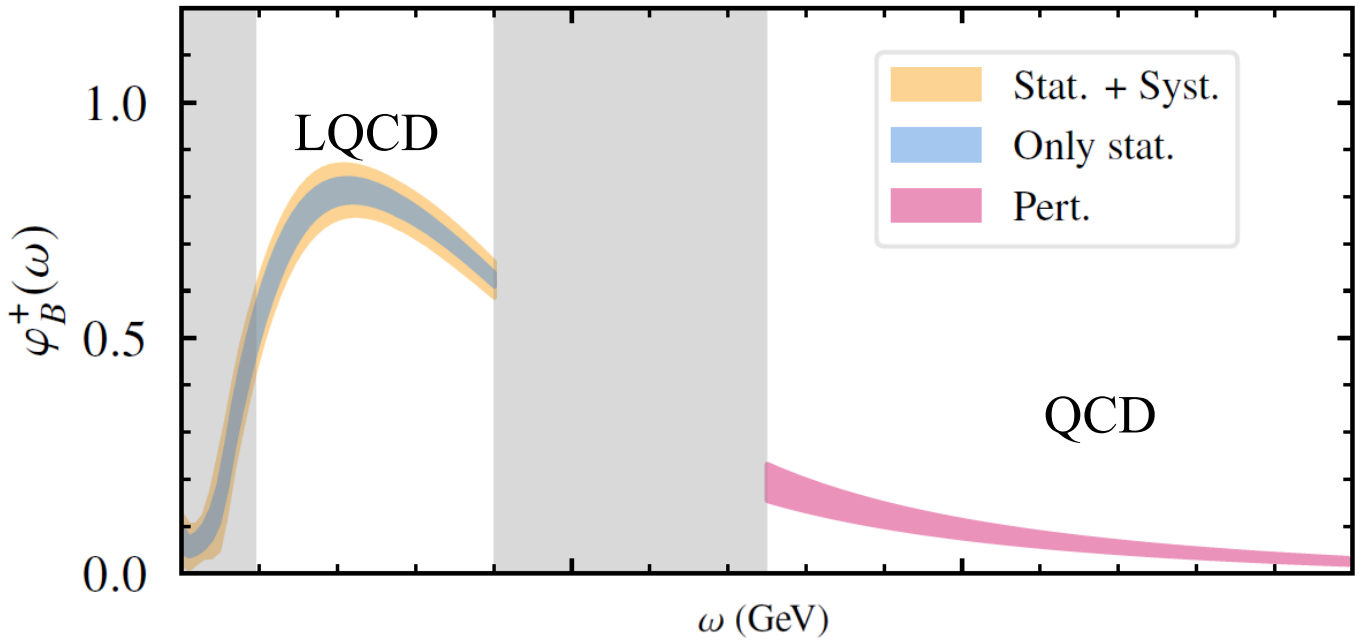
Lee, Neubert, et al. PRD 72 (2005) 094028

QCD LCDA, nonperturbative

$$\varphi^+(\omega,\mu)=\begin{cases}\frac{1}{m_H}\left[1-\frac{\alpha_s C_F}{4\pi}\left(\frac{1}{2}\ln^2\frac{\mu^2}{m_H^2}+\frac{1}{2}\ln\frac{\mu^2}{m_H^2}+\frac{3}{2}\ln\frac{\mu^2}{m_Q^2}+\frac{\pi^2}{12}+4\right)+\mathcal{O}\left(\alpha_s^2\right)\right]\phi(\frac{\omega}{m_H},\mu), & \omega\sim\Lambda_{\rm QCD}\\ \frac{\alpha_s C_F}{\pi\omega}\left[\left(\frac{1}{2}-\ln\frac{\omega}{\mu}\right)+\frac{4\bar{\Lambda}}{3\omega}\left(2-\ln\frac{\omega}{\mu}\right)\right]+\mathcal{O}\left(\alpha_s^2\right), & \omega\sim m_H\end{cases}$$

Beneke, Finauri, Vos, Wei, JHEP 09, 066 (2023)

Prediction for HQET LCDA from first-principle:

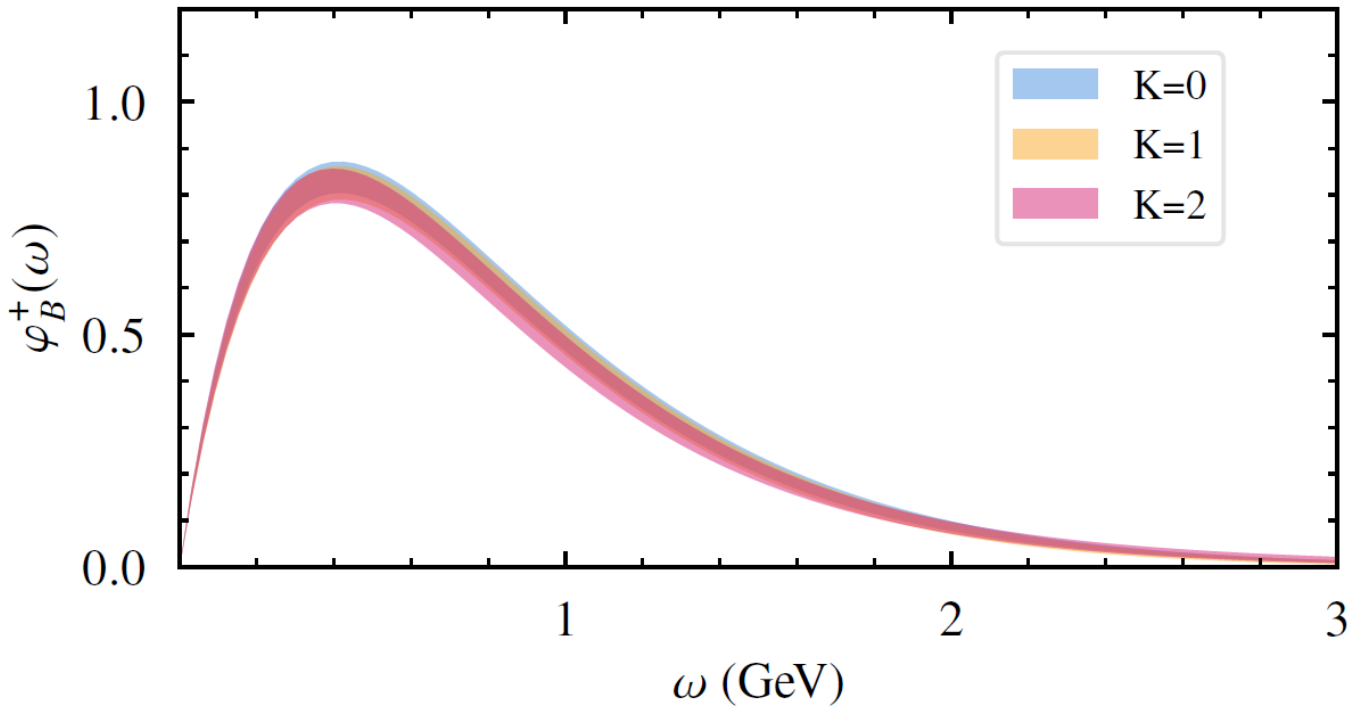


Prediction for HQET LCDA

- Rebuild the full distribution of the HQET LCDA from a **model-independent parametrization**:

Feldmann, Lughausen, Dyk, JHEP10, 162 (2020)

$$\varphi^+(\omega,\mu) = \frac{\omega e^{-\omega/\omega_0}}{\omega_0^2} \sum_{k=0}^{K} \frac{a_k(\mu)}{1+k} L_k^{(1)}(2\omega/\omega_0),$$



Inverse and inverse-logarithmic moments

- Predictions of inverse and inverse-logarithmic moments from the full distribution of HQET LCDA:

$$f_{B\to\pi}^{+}(0) = 0.122 \times \Big[1 \pm 0.07|_{S_0^\pi} \pm 0.11|_{\Lambda_q} \pm 0.02|_{\lambda_E^2/\lambda_H^2} {}^{+0.05}_{-0.06}|_{M^2} \pm 0.05|_{2\lambda_E^2+\lambda_H^2} {}^{+0.06}_{-0.010}|_{\mu_h} \pm 0.04|_{\mu} {}^{+1.36}_{-0.56}|_{\lambda_B} {}^{+0.25}_{-0.43}|_{\sigma_1,\sigma_2}\Big]$$

Cui, Huang, Shen, Wang, JHEP 03 (2023) 140

$$\lambda_B^{-1}(\mu) = \int_0^\infty \frac{d\omega}{\omega}\varphi^+(\omega,\mu),$$

$$\sigma_B^{(n)}(\mu) = \lambda_B(\mu) \int_0^\infty \frac{d\omega}{\omega} \ln\left(\frac{\mu}{\omega}\right)^{(n)} \varphi^+(\omega,\mu).$$

	λ_B (GeV)	$\sigma_B^{(1)}$
Our work	0.416 (20)	1.445 (33)
PRD98 (2018)	> 0.24	
JHEP (2020)	0.383 (153)	
PRD72 (2005)	0.48 (11)	1.6 (2)
PRD69 (2004)	0.46 (11)	1.4 (4)
PRD55 (1997)	0.35 (15)	
PRD101 (2020)	$0.343^{+0.064}_{-0.079}$	
PLB848 (2024)	0.338 (68)	

Summary and Outlook

- We have obtained precise QCD and HQET LCDAs for heavy mesons at leading power.
- Systematic uncertainties from non-perturbative renormalization, scale conversion, continuum and infinite-momentum extrapolation etc., are comprehensively addressed.
- The physical mass extrapolation is still in progress.
- Power correction in LaMET ($\propto m_H/P\^z$) can be estimated from local moments, this work is currently ongoing.

Thanks for your attention