

Bottom Meson Masses and Decay Constants From the Anisotropic Quark Action on CLQCD Ensembles

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1. Background
2. Anisotropic Quark Action on the Lattice
3. Correlation Functions Fitting
4. Renormalization
5. Extrapolation to Continuum
6. Comparison and Conclusion

- ◆ Leptonic decays of bottom mesons probe the quark-flavor-changing transitions. CKM matrix element can be determined with an experimental measurement of the decay rate.

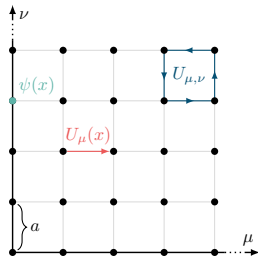
$$\Gamma(B^+ \rightarrow \ell^+ \nu_\ell) \propto f_B^2 |V_{ub}|^2 \quad \Gamma(B_s \rightarrow \ell^+ \ell^-) \propto f_{B_s}^2 |V_{tb}^* V_{ts}|^2 \quad \Gamma(\Upsilon \rightarrow e^+ e^-) \propto \alpha_{\text{QED}}^2 f_\Upsilon^2$$

- ◆ Heavy quark mass ($am_b > 1$) causes large discretization errors. Heavy-quark effective theory (HQET) or Nonrelativistic QCD (NRQCD) can be implemented on the lattice.
- ◆ Utilize finer lattice spacing $a \sim 0.03$ fm. *Bazavov et al., 2018, PRD MILC* *Hatton et al., 2021, PRD HPQCD ...*
- ◆ Relativistic anisotropic heavy quark action improves discretization errors.

- ◆ m_b, ζ, c_P : *Christ et al., 2007, PRD* *Aoki et al., 2012, PRD* *Christ et al., 2015, PRD ...*

- ◆ m_b, ν : *Chen, 2001, PRD* *Edwards et al., 2008, PRD* *Liu et al., 2010, PRD* *Leskovec et al., 2025, PRL ...*

Anisotropic Quark Action on the Lattice



isotropic clover quark action

$$S = a^4 \sum_x \bar{\psi}(x) \left[m_b + \gamma_\mu D_\mu - \frac{a}{2} D^2 - \frac{a}{2} c_{sw} \left(\sum_{\mu < \nu} \sigma_{\mu\nu} F_{\mu\nu} \right) \right] \psi(x)$$

covariant derivative

$$[U_\mu(x)\psi(x+\mu) - U_{-\mu}(x)\psi(x-\mu)] / 2a \Rightarrow D_\mu\psi(x)$$

field strength tensor

$$U_{\mu\nu}(x) \Rightarrow e^{ia^2 F_{\mu\nu}(x) + \mathcal{O}(a^3)} \quad U_{\mu\nu}(x) = U_\mu(x)U_\nu(x+\mu)U_\mu^\dagger(x+\nu)U_\nu^\dagger(x)$$

anisotropic quark action

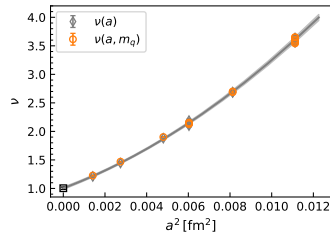
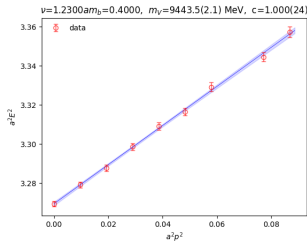
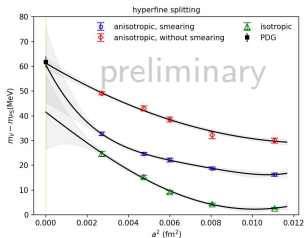
$$S = a^4 \sum_x \bar{\psi}(x) \left[m_q + \left(\gamma_0 D_0 - \frac{a}{2} D_0^2 \right) + \nu \sum_i \left(\gamma_i D_i - \frac{a}{2} D_i^2 \right) - \frac{a}{2} c_B \left(\sum_{i < j} \sigma_{ij} F_{ij} \right) - \frac{a}{2} c_E \left(\sum_i \sigma_{0i} F_{0i} \right) \right] \psi(x)$$

Chen, 2001, *Phys. Rev. D*
Liu et al., 2010, *Phys. Rev. D*

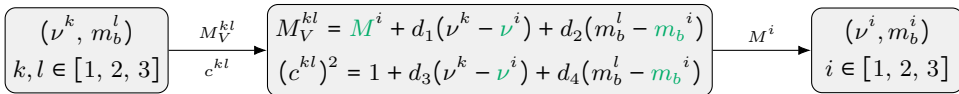
$$c_B = \frac{\nu}{u_0^3}, \quad c_E = \frac{1}{2}(1 + \nu) \frac{1}{u_0^3}, \quad u_0 = \left\langle \frac{\text{Re Tr} \sum_{x, \mu < \nu} U_{\mu\nu}(x)}{6N_c V} \right\rangle^{1/4}$$

Anisotropic Parameter Tuning

solve bottom quark propagators with the original gauge field (no stout smearing)



at each (m_b^k, ν^l) , fit the dispersion relation $E^{kl}(\mathbf{p})^2 = M_V^2(m_b^k, \nu^l) + (c^{kl})^2 \mathbf{p}^2$.



utilize the above formula to constrain $c \Rightarrow 1$ $M_V(m_b, \nu) \Rightarrow M_Y^{\text{phys}}$

$$\nu = \frac{\text{Sinh}(c_0 m_V a)}{c_0 m_V a} \times [1 + c_1(m_\pi^2 - (m_\pi^{\text{phys}})^2) + c_2(m_{\eta_s}^2 - (m_{\eta_s}^{\text{phys}})^2) + c_3 e^{-m_\pi L}]$$

$$M_V^i = l_{M_V}^0 m_b^i + l_{M_V}^1$$

$$X^i = l_X^0 m_b^i + l_X^1$$

Correlation Functions

Matrix element: decay constant from matrix element

$$\langle \Omega | \hat{A}_\mu(0) | P(p) \rangle = i f_P p_\mu ,$$

$$\langle \Omega | \hat{V}_\mu(0) | V(p, \epsilon) \rangle = f_V m_V \epsilon_\mu ,$$

$$\langle \Omega | \partial_\mu \hat{A}^\mu(x) | P(p) \rangle = (m_1^{\text{PC}} + m_2^{\text{PC}}) \langle \Omega | \hat{P}(x) | P(p) \rangle$$

$$\begin{aligned} C_2(t_f) &= \sum_{\mathbf{x}} \langle \Omega | \hat{O}(t_f, \mathbf{x}) \hat{O}^\dagger(0, 0) | \Omega \rangle \\ &= \sum_{n=0}^{\infty} |\langle \Omega | \hat{O}(t_f, \mathbf{x}) | n \rangle|^2 e^{-E_n t_f} \end{aligned}$$

Operator: local operators required for B , B_s , B_c

$$\hat{A}_{f_1 f_2}^\mu(x) = \bar{\psi}_{f_1}(x) \gamma^\mu \gamma^5 \psi_{f_2}(x) \quad \hat{V}_{f_1 f_2}^\mu(x) = \bar{\psi}_{f_1}(x) \gamma^\mu \psi_{f_2}(x) \quad \hat{P}_{f_1 f_2}(x) = \bar{\psi}_{f_1}(x) \gamma^5 \psi_{f_2}(x)$$

Renormalization: one field is of the light flavor and the other is of the heavy flavor

$$\hat{O}_{f_1 f_2}^R = Z_O^{f_1 f_2} \hat{O}_{f_1 f_2}^B \quad Z_P^{f_1 f_2} = Z_V^{f_1 f_2} \left. \frac{Z_P}{Z_V} \right|_{m_q \rightarrow 0} \quad Z_V^{lh} = \sqrt{Z_V^{ll} Z_V^{hh}}$$

2-pt correlator: wall-to-point(wp) and wall-to-wall(ww) two point correlation function

$$C_{2,wp}^{\Gamma_1 \Gamma_2}(t) = \frac{1}{L^3} \sum_{\mathbf{x}, \mathbf{y}, \mathbf{z}} \left\langle \bar{\psi}_1(\mathbf{x}, t_0 + t) \Gamma_1 \psi_2(\mathbf{x}, t_0 + t) [\bar{\psi}_1(\mathbf{y}, t_0) \Gamma_2 \psi_2(\mathbf{z}, t_0)]^\dagger \right\rangle$$

$$C_{2,ww}^{\Gamma_1 \Gamma_2}(t) = \frac{1}{L^3} \sum_{\mathbf{x}_1, \mathbf{x}_2, \mathbf{y}, \mathbf{z}} \left\langle \bar{\psi}_1(\mathbf{x}_1, t_0 + t) \Gamma_1 \psi_2(\mathbf{x}_2, t_0 + t) [\bar{\psi}_1(\mathbf{y}, t_0) \Gamma_2 \psi_2(\mathbf{z}, t_0)]^\dagger \right\rangle$$

Pseudoscalar Meson Correlation Function

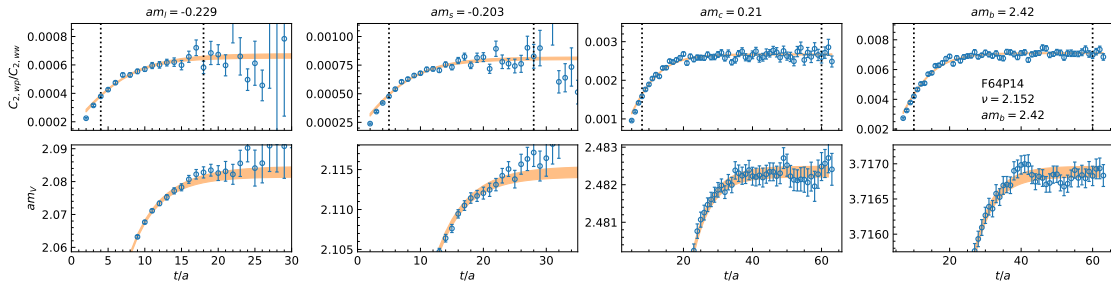
CLQCD ensembles used in the calculation

Symbol	$\hat{\beta}$	a (fm)	$\tilde{L}^3 \times \tilde{T}$	m_π (MeV)	m_{η_s} (MeV)	$m_\pi L$
C24P34	6.200	0.10524(05)(62)	$24^3 \times 64$	340.2(1.7)	748.61(75)	4.360(22)
C24P29			$24^3 \times 72$	292.3(1.0)	657.83(64)	3.746(13)
C32P29			$32^3 \times 64$	293.1(0.8)	658.80(43)	5.008(14)
C32P23			$32^3 \times 64$	227.9(1.2)	643.93(45)	3.894(20)
C48P23			$48^3 \times 96$	224.1(1.2)	644.08(62)	5.743(30)
C48P14			$48^3 \times 96$	136.4(1.7)	706.55(39)	3.495(44)
E28P35	6.308	0.08973(20)(53)	$28^3 \times 64$	351.4(1.4)	717.94(93)	4.462(17)
E32P29			$32^3 \times 64$	285.9(2.5)	699.6(4.4)	4.171(26)
E32P22			$32^3 \times 96$	215.5(2.6)	687.2(4.3)	3.145(33)
F32P30	6.410	0.07753(03)(45)	$32^3 \times 96$	300.4(1.2)	675.98(97)	3.780(15)
F48P30			$48^3 \times 96$	302.7(0.9)	674.76(58)	5.713(16)
F32P21			$32^3 \times 64$	210.3(2.3)	658.79(94)	2.646(28)
F48P21			$48^3 \times 96$	207.5(1.1)	661.94(64)	3.917(22)
F64P14			$64^3 \times 128$	135.6(1.5)	681.2(4.2)	3.413(31)
G36P29	6.498	0.06887(12)(41)	$36^3 \times 108$	297.2(0.9)	693.05(46)	3.731(11)
H48P32	6.720	0.05199(08)(31)	$48^3 \times 144$	316.6(1.0)	691.88(65)	4.000(12)
I64P30	7.020	0.03780(06)(23)	$64^3 \times 128$	310.6(2.5)	668.1(4.3)	3.808(20)

Vector Meson Correlation Function

$$C_{2,wp}^{V_i V_i}(t) = \frac{Z_{wp}}{2M_V} \left(e^{-M_V t} + e^{-M_V (T-t)} \right) \left(1 + R_{wp}^{V_i V_i}(t) \right) \quad C_{2,wp}^{V_i V_i}(t) / C_{2,ww}^{V_i V_i}(t) = \frac{f_V^2 M_V^2}{Z_{wp}} \frac{1 + R_{wp}^{V_i V_i}(t)}{1 + R_{ww}^{V_i V_i}(t)}$$

$$R_{wp(ww)}^{\Gamma_1 \Gamma_2}(t) = \sum_{n=1}^N r_{wp(ww),n}^{\Gamma_1 \Gamma_2} \left[e^{-(M+\delta E_n)t} + e^{-(M+\delta E_n)(T-t)} \right] / \left[e^{-Mt} + e^{-M(T-t)} \right]$$



The two-state fitting is needed to describe the correlator data. Signals are still good at $t \sim T/2$ so backward propagation is considered.

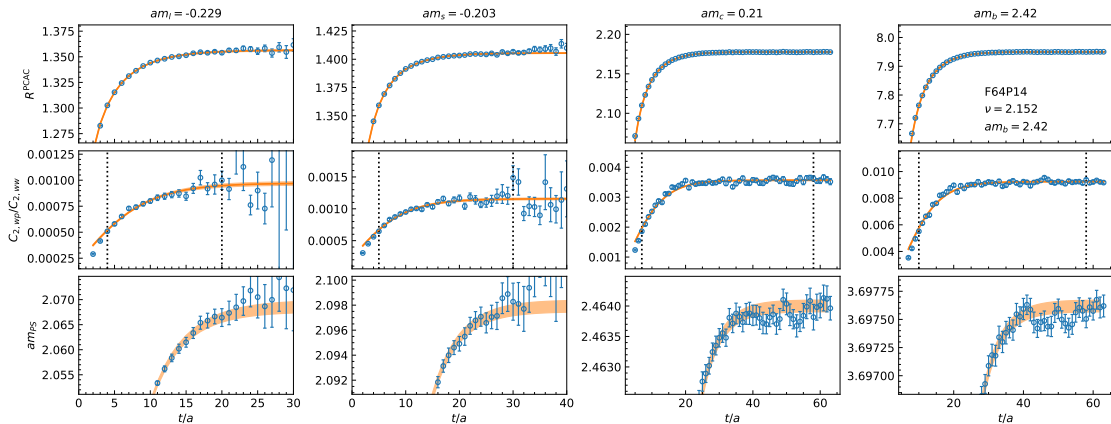
$$m_{\text{eff}} = \cosh^{-1} [C_2(t-1) + C_2(t+1)] / [2C_2(t)]$$

Pseudoscalar Meson Correlation Function

$$C_{2,wp}^{\text{PP}}(t) = \frac{Z_{wp}}{2M_{\text{PS}}} \left(e^{-M_{\text{PS}}t} + e^{-M_{\text{PS}}(T-t)} \right) (1 + R_{wp}^{\text{PP}}(t))$$

$$C_{2,wp}^{\text{PP}}(t)/C_{2,ww}^{\text{PP}}(t) = \frac{(f_{\text{P}} M_{\text{PS}}^2)^2}{(m_1^{\text{PC}} + m_2^{\text{PC}})^2} \frac{1}{Z_{wp}} \frac{1 + R_{wp}^{\text{PP}}(t)}{1 + R_{ww}^{\text{PP}}(t)}$$

$$\left[C_{2,wp}^{\text{A4P}}(t-1) - C_{2,wp}^{\text{A4P}}(t+1) \right] / 2C_{2,wp}^{\text{PP}}(t) = \frac{\text{Sinh}(M_{\text{PS}})}{M_{\text{PS}}} (m_1^{\text{PC}} + m_2^{\text{PC}}) \frac{1 + R_{wp}^{\text{A4P}}(t)}{1 + R_{wp}^{\text{PP}}(t)}$$

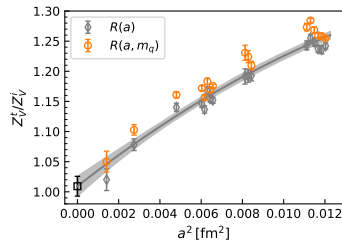
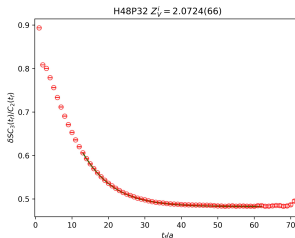
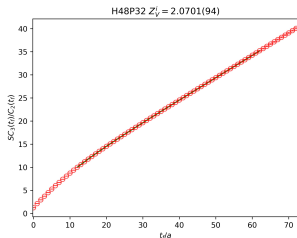


Renormalization

Z_V^t maybe different from Z_V^i

$$Z_V^i \langle P | V^i | P \rangle = 2P^i \quad Z_V^t \langle P | V^t | P \rangle = 2P^0$$

$$C_3^{PVP}(t_0, t, t_f) = \sum_{\mathbf{x}, \mathbf{y}} \langle P(\mathbf{x}, t_f) V(\mathbf{y}, t) P(\mathbf{0}, t_0) \rangle$$



$$R(t_f) = \frac{\sum_t C_3(t_f, t)}{C_2(t_f)} = \frac{c_0 t_f + (c_1 + c_2 t_f) e^{-\Delta E t_f} + c_3}{1 + d_1 e^{-\Delta E t_f}}$$

$$\delta R(t_f) = R(t_f + 1) - R(t_f) = c_0 + (c_1 + c_2 t_f) e^{-\Delta E t_f} + O(e^{-2\Delta E t_f})$$

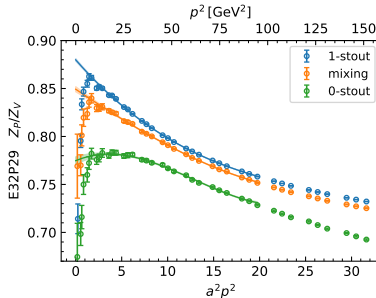
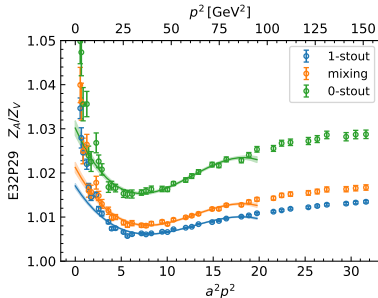
$$Z_V^\mu = \frac{p^\mu}{E} \frac{1}{c_0}$$

Numerical calculation shows that Z_V^t/Z_V^i goes to unity in the continuum limit

Renormalization

$$G_{\mathcal{O}_{f_1 f_2}}(p_1, p_2) = \sum_{x, y} e^{-i(p_1 \cdot x - p_2 \cdot y)} \langle \psi_{f_1}(x) [\bar{\psi}_{f_1}(x) \Gamma \psi_{f_2}(x)] \bar{\psi}_{f_2}(y) \rangle = S_{f_1}(p_1) \Lambda_{\mathcal{O}_{f_1 f_2}}(p_1, p_2) S_{f_2}(p_2)$$

$$S_f(p) = \sum_x e^{-ip \cdot x} \langle \psi_f(x) \bar{\psi}_f(0) \rangle \quad p_1^2 = p_2^2 = (p_1 - p_2)^2 = \mu^2$$



gauge field

with smearing $V_\mu(x)$

without smearing $U_\mu(x)$

1-stout: $S_{f_1}(V), S_{f_2}(V)$

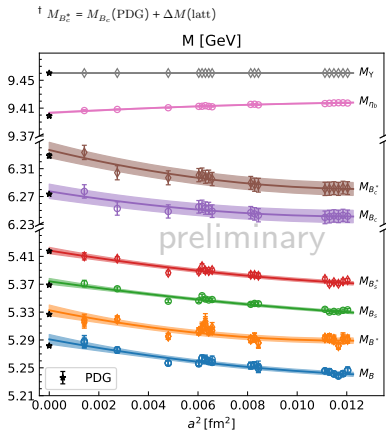
mixing: $S_{f_1}(U), S_{f_2}(V)$

0-stout: $S_{f_1}(U), S_{f_2}(U)$

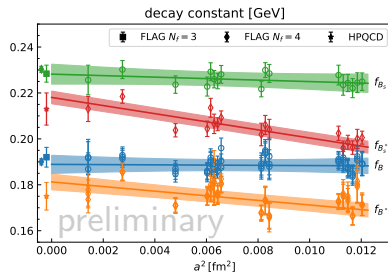
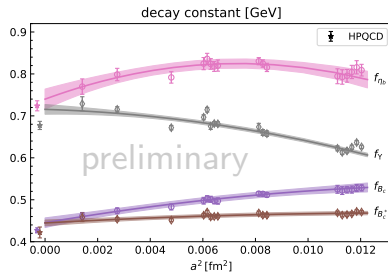
$$Z_P^{f_1 f_2} = Z_V^{f_1 f_2} \left. \frac{Z_P}{Z_V} \right|_{m_q \rightarrow 0} \quad \frac{1}{12q^2} \text{Tr}[q_\mu \Lambda_{V,B}^\mu(p_1, p_2) \not{q}] = \frac{Z_q^{\text{SMOM}}}{Z_V^{\text{SMOM}}} \quad \frac{1}{12i} \text{Tr}[\Lambda_{P,B}(p_1, p_2) \gamma_5] = \frac{Z_q^{\text{SMOM}}}{Z_P^{\text{SMOM}}}$$

$$Z^{\overline{\text{MS}}}(\mu_0) = R(\mu_0, \mu) C_{\text{SMOM}}^{\overline{\text{MS}}}(\mu) Z^{\text{SMOM}}(\mu)$$

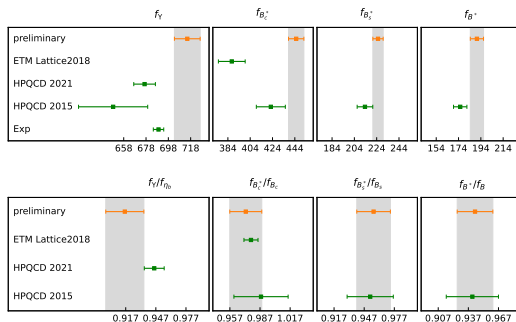
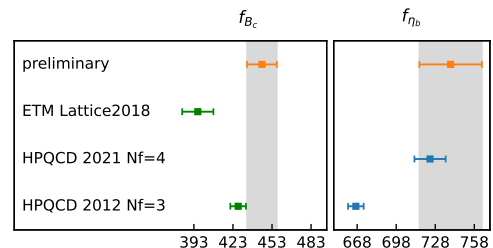
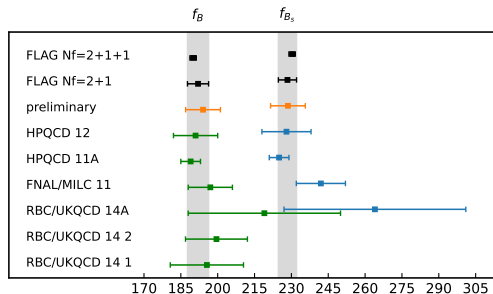
Extrapolation to the Continuum



$$X(m_\pi, m_{\eta_s}, a) = X(m_\pi^{\text{phys}}, m_{\eta_s}^{\text{phys}}, 0) + \sum_{n=1}^N c_n^X a^{2n} + d_1^X (m_\pi^2 - (m_\pi^{\text{phys}})^2) + d_2^X (m_{\eta_s}^2 - (m_{\eta_s}^{\text{phys}})^2)$$



Comparison



- ◇ f_B and f_{B_s} are consistent with FLAG averages
- ◇ lattice calculations for other bottom mesons decay constants are rare

- ◆ We present a systematic study of masses and decay constants of the mesons with bottom quark using the anisotropic fermion action on CLQCD ensembles at $a \in [0.04, 0.11]$ fm and $m_\pi \in [130, 360]$ MeV
- ◆ We propose a systematic framework to renormalize the quark bi-linear operator with the bottom quark field and verify it through the renormalized decay constants

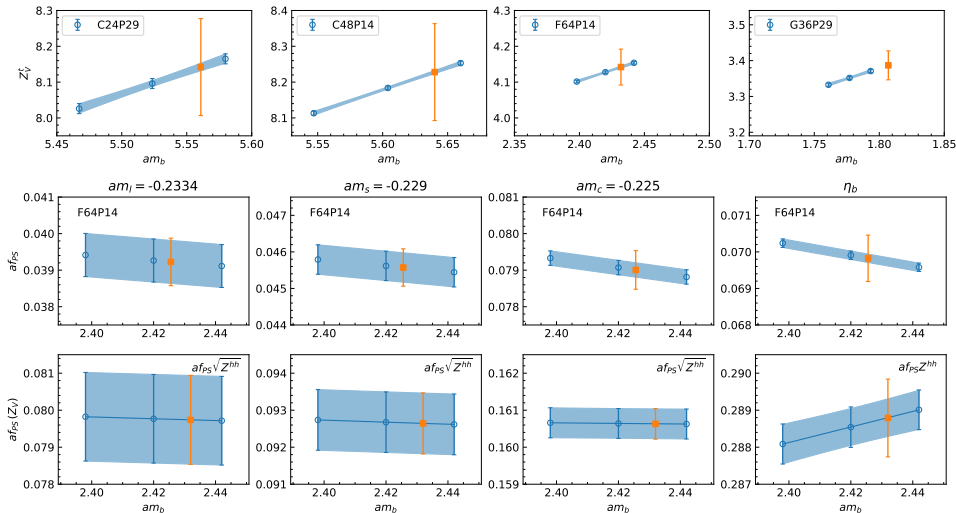
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Bottom Quark Mass Interpolation

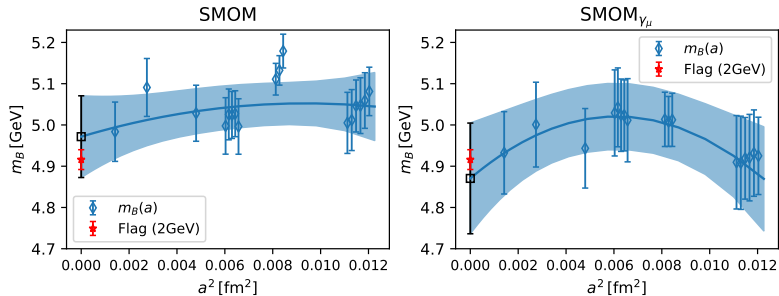
$$M_V^i = d_{MV}^0 m_b^i + d_{MV}^1 \quad X^i = d_X^0 m_b^i + d_X^1$$

Bare bottom quark values contain error from lattice spacing.

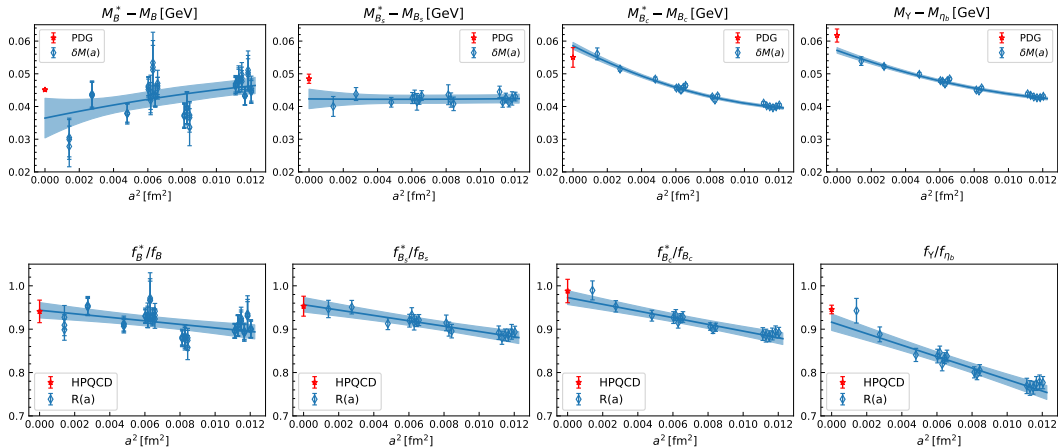
$af_{PS}^{B_l} \sqrt{Z^{hh}}$ show less m_b dependence

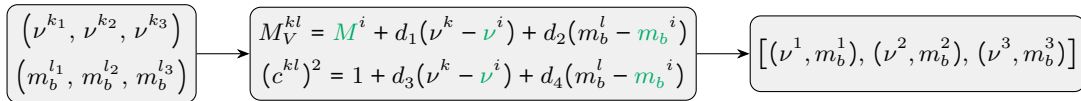


Bottom Quark Mass



Mass Splitting and Decay Constant Ratios





Reference I

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