Quantum Simulation of Particle Scattering

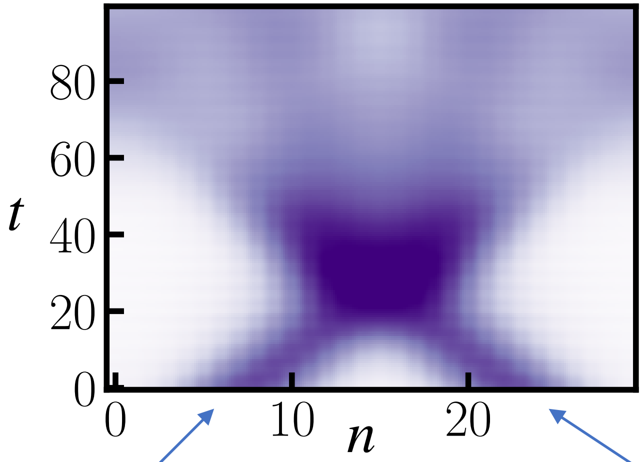
Yahui Chai

DESY. QUANTUM | Center for Quantum Technology and Applications

IBM Quantum

Outline

1. Quantum algorithm for meson scattering



- Simulation of dynamics

- Quantum subspace expansion (QSE) for stable particle construction

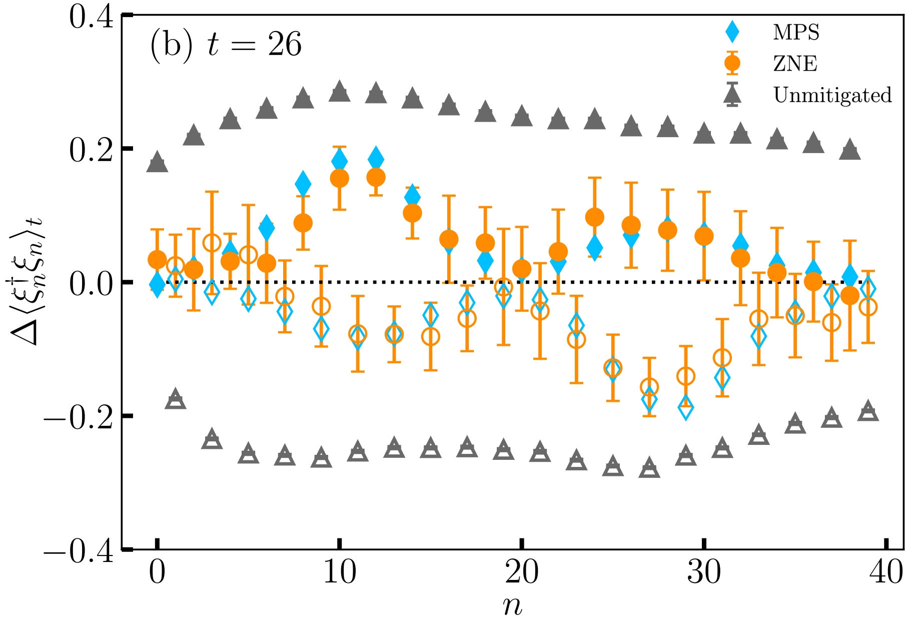
- Quantum circuit decomposition

https://arxiv.org/abs/2505.21240

2. Hardware run for fermion scattering

- Tensor network circuit compilation

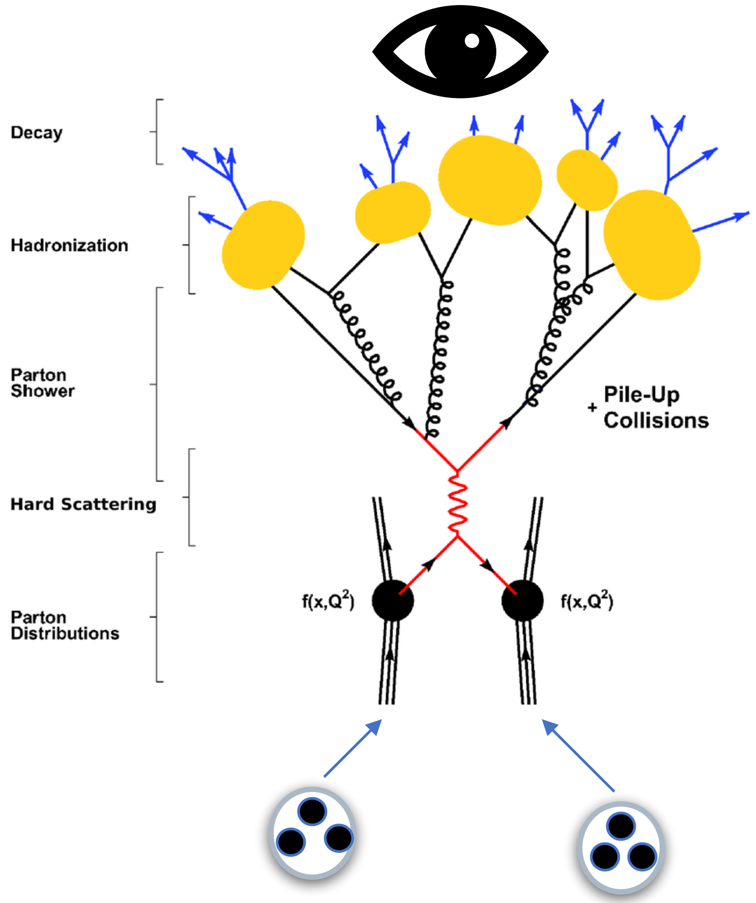
- Hardware run for 40 and 80 qubits



https://arxiv.org/abs/2507.17832

Motivation

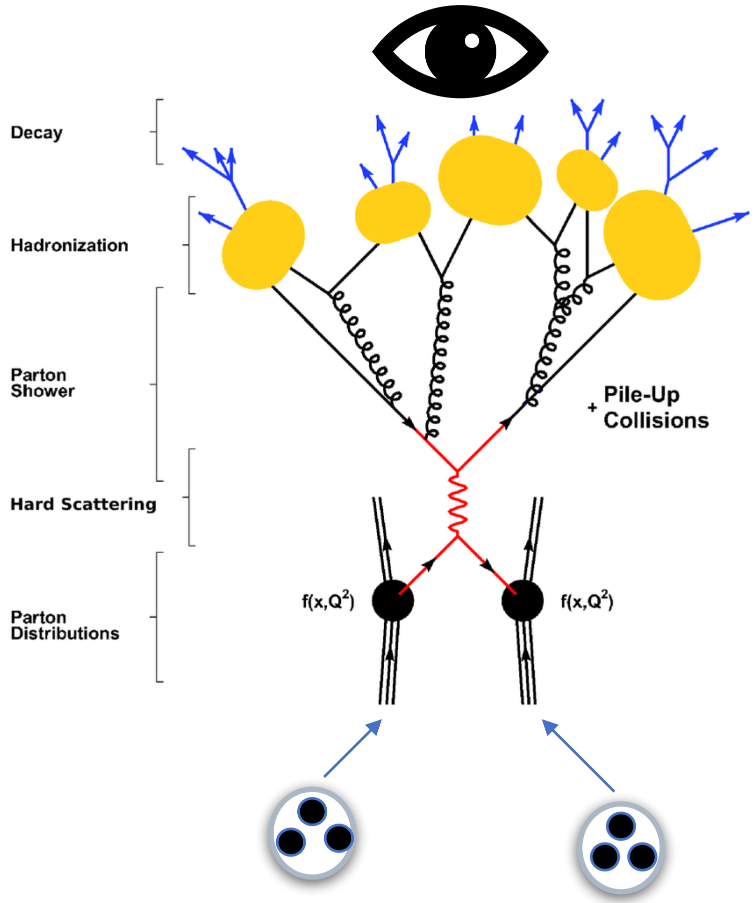
- Experiments on colliders: verify the theory, probe the particle structure.



Motivation

- Experiments on colliders: verify the theory, probe the particle structure.

- Real time dynamics in classical simulation
  - Conventional Monte Carlo: sign problem
  - Tensor Network : increasing entanglement with time-- increasing computation resource

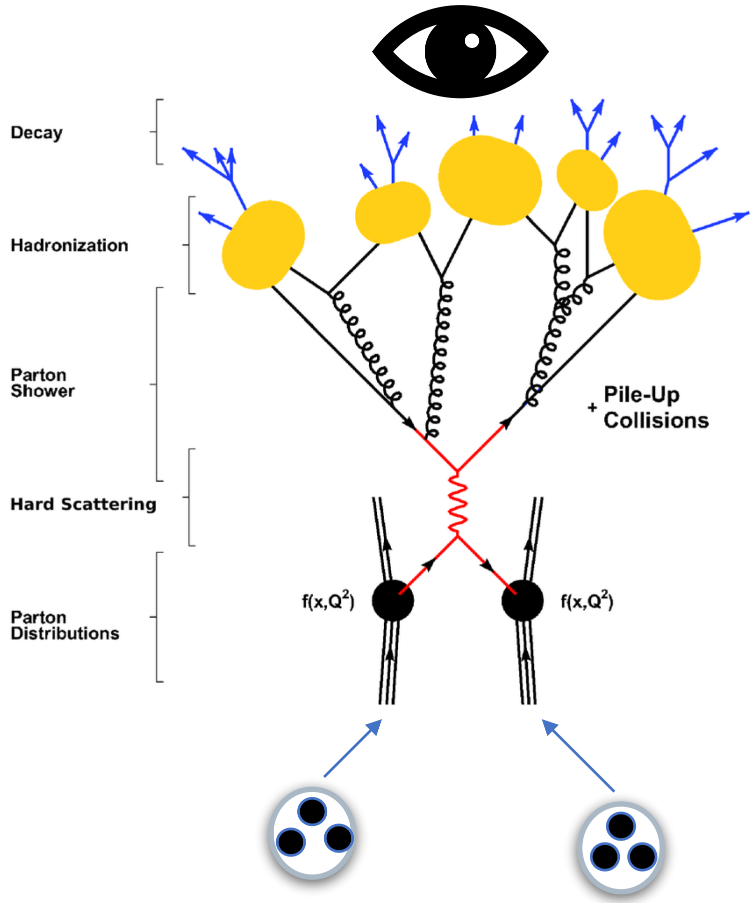


Motivation

- Experiments on colliders: verify the theory, probe the particle structure.

- Real time dynamics in classical simulation
    - Conventional Monte Carlo: sign problem
    - Tensor Network : increasing entanglement with time--increasing computation resource

- Quantum computers promise to efficiently simulate real-time dynamics



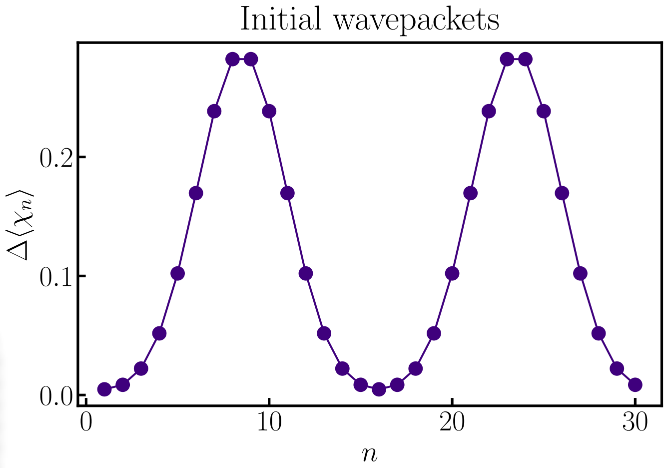
Scattering process on QC

1. Vacuum state $|\Omega\rangle$: ground state of Hamiltonian

Scattering process on QC

1. Vacuum state $|\Omega\rangle$: ground state of Hamiltonian

2. Initial state $|\psi(t=0)\rangle = \Phi_1^\dagger \Phi_2^\dagger |\Omega\rangle$: wave packets of particles.



Fermion wave packet: $\Phi^\dagger(\bar{n}, \bar{k}) = \sum_n \phi(n)\ \xi_n^\dagger = \sum_k \phi(k)\ \xi_k^\dagger$

Gaussian distribution

Meson wave packet: $\Phi^\dagger(\bar{n}, \bar{k}) = \sum_n \phi(n)\ b_n^\dagger = \sum_k \phi(k)\ b_k^\dagger$

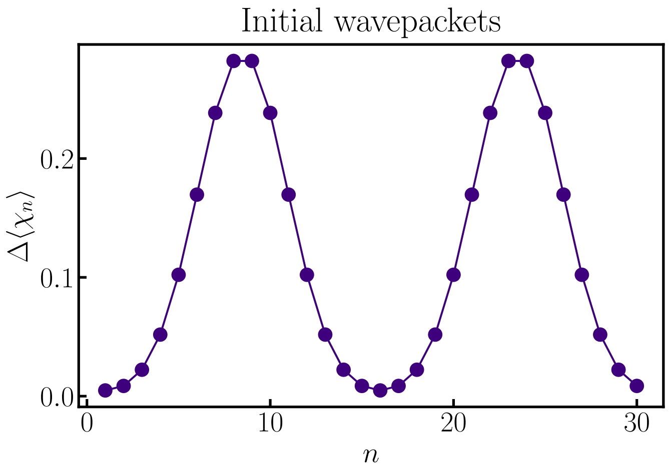
Stable particle operator construction

Efficient circuit decomposition for non-unitary operator

Scattering process on QC

1. Vacuum state $|\Omega\rangle$: ground state of Hamiltonian

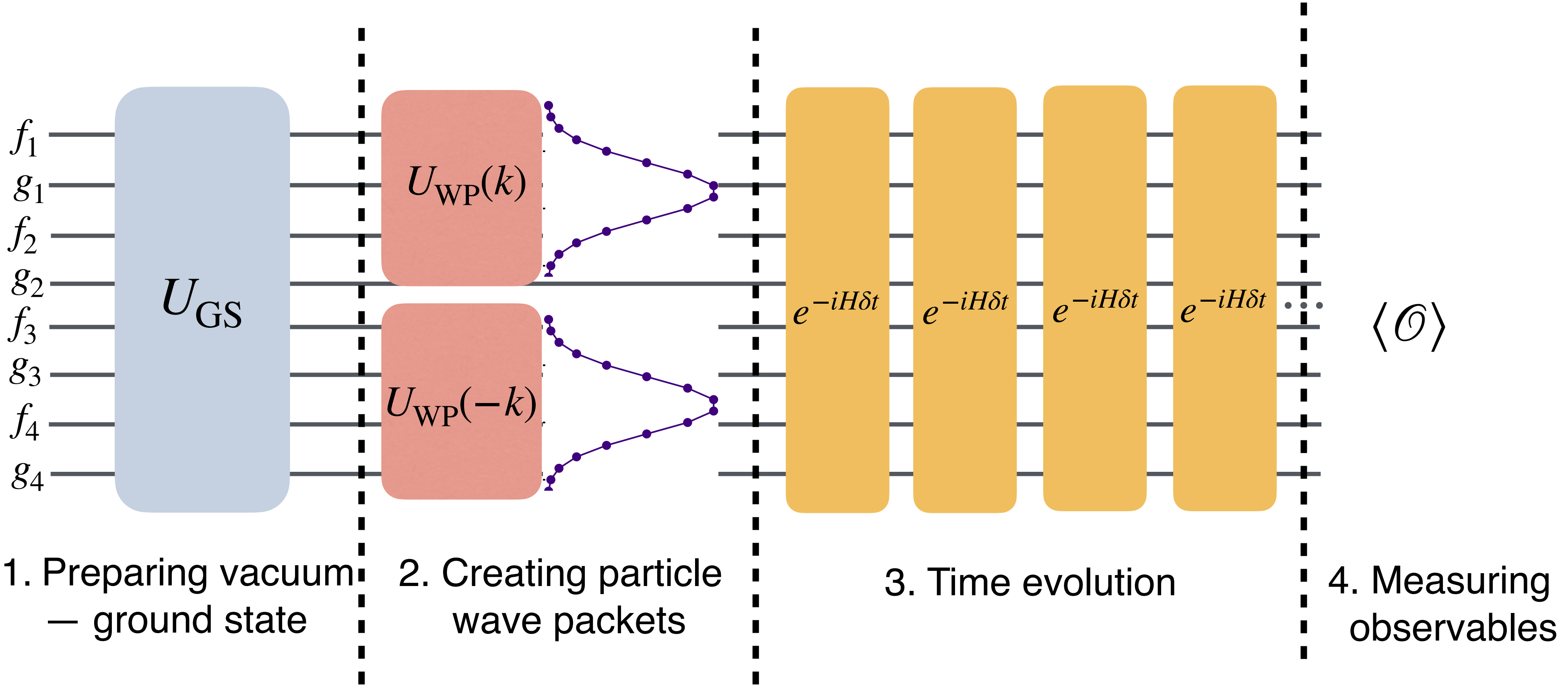
2. Initial state $|\psi(t=0)\rangle = \Phi_1^\dagger \Phi_2^\dagger |\Omega\rangle$: wave packets of particles.



3. Time evolution: $|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$

$$e^{-iHt} \approx \left(\prod_i e^{-iH_i\delta t}\right)^{t/\delta t} + \mathcal{O}(\delta t),$$

Scattering process on QC



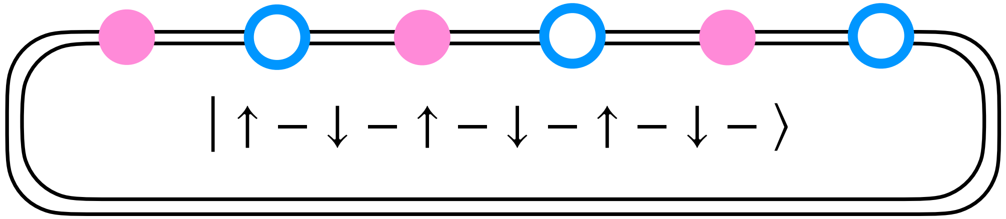
Meson scattering

- Stable meson creation operator construction
- Efficient circuit decomposition for non-unitary meson wave packet operator

Lattice $\mathbb{Z}_2$ gauge theory in 1+1 D

- Staggered fermion coupled with $\mathbb{Z}_2$ gauge field, periodic condition

$$H = \frac{1}{2a}\sum_{n=1}^{L}\left(\xi_n^\dagger \underset{\substack{\downarrow \\ Z}}{U_{n,n+1}} \xi_{n+1} + \text{h.c.}\right) + m\sum_{n=1}^{L}(-1)^n \xi_n^\dagger \xi_n + \varepsilon \sum_{n=1}^{L} \underset{\substack{\downarrow \\ X}}{E_{g,n}}$$

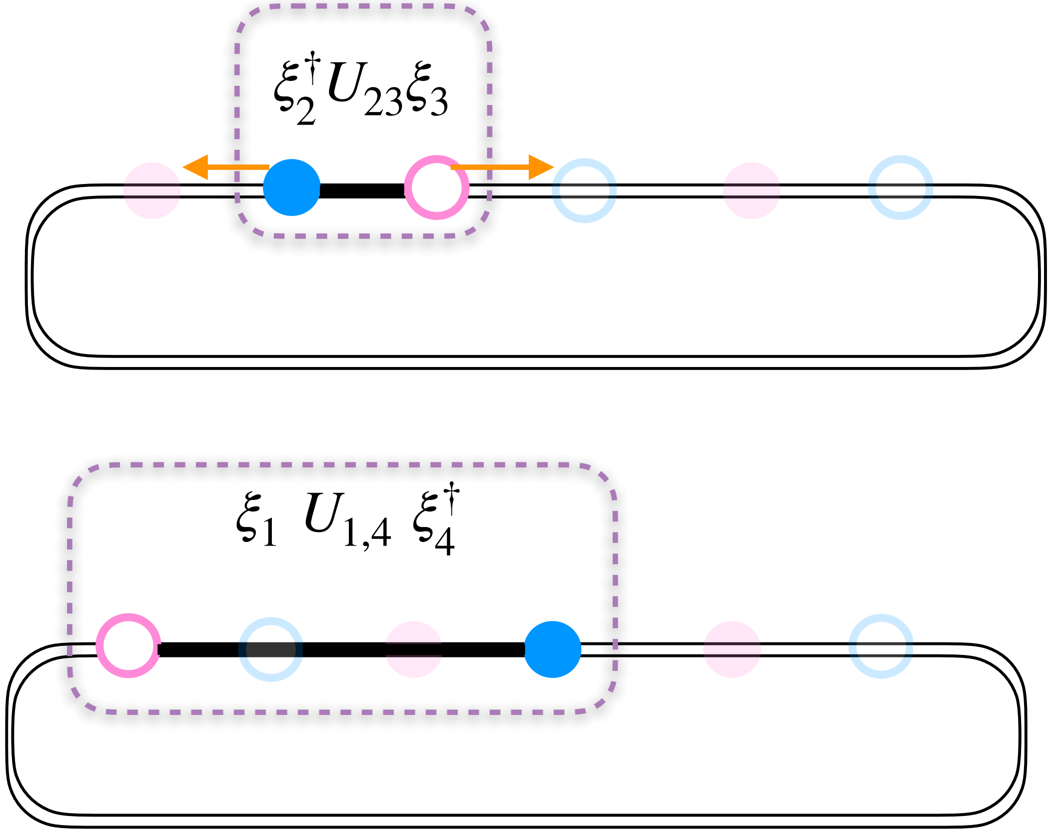


L sites, 2L qubits

- Quantum number: charge conjugation

Vector meson: c = -1
Scalar meson: c = +1

How to create a meson state?



Meson Creation Operator

$$b^{\dagger}_{k,c} = \sum_{I} a_I^{(k,c)} \boxed{M_I}$$

Operators, $M_I \equiv M_{(n,l)} \in \{\xi_n^{\dagger} U_{n,l} \xi_l \,|\, n, l = 1,2,\cdots L\}$

Meson Creation Operator

$$b^{\dagger}_{k,c} = \sum_{I} a_I^{(k,c)} \; M_I$$

$$|\,k, c\rangle_b = b^{\dagger}_{k,c}\,|\,\Omega\rangle = \sum_{I} a_I^{(k,c)} \; M_I\,|\,\Omega\rangle$$

$$H\,|\,k, c\rangle_b = E\,|\,k, c\rangle_b$$

$$C\,|\,k, c\rangle_b = ce^{ik}\,|\,k, c\rangle_b$$

Quantum subspace expansion

$$b_{k,c}^{\dagger} = \sum_{I} \boxed{a_I^{(k,c)}} M_I$$

Quantum subspace expansion

$$b^\dagger_{k,c} = \sum_I \boxed{a_I^{(k,c)}} M_I$$

$$\mathscr{H}_{I,J} = \langle \Omega | M_I^\dagger \; H \; M_J | \Omega \rangle$$

Quantum subspace expansion

$$b^\dagger_{k,c} = \sum_I \boxed{a_I^{(k,c)}} M_I$$

$$\mathscr{H}_{I,J} = \langle \Omega | M_I^\dagger \; H \; M_J | \Omega \rangle$$

$$\mathscr{C}_{I,J} = \langle \Omega | M_I^\dagger \; C \; M_J | \Omega \rangle$$

Quantum subspace expansion

$$b^{\dagger}_{k,c} = \sum_{I} \boxed{a_I^{(k,c)}} M_I$$

$$\mathscr{H}_{I,J} = \langle \Omega | M_I^{\dagger} \; H \; M_J | \Omega \rangle$$

$$\mathscr{C}_{I,J} = \langle \Omega | M_I^{\dagger} \; C \; M_J | \Omega \rangle$$

$$\mathscr{S}_{I,J} = \langle \Omega | M_I^{\dagger} \; M_J | \Omega \rangle$$

Quantum subspace expansion

$$b^{\dagger}_{k,c} = \sum_I a_I^{(k,c)} M_I$$

$$\mathcal{H}_{I,J} = \langle \Omega | M_I^{\dagger} \; H \; M_J | \Omega \rangle$$

$$\mathcal{C}_{I,J} = \langle \Omega | M_I^{\dagger} \; C \; M_J | \Omega \rangle$$

$$\mathcal{S}_{I,J} = \langle \Omega | M_I^{\dagger} \; M_J | \Omega \rangle$$

- QSE equation

$$\left(\mathcal{H} + \mathcal{C}\right) \vec{a}^{(k,c)} = \lambda \; \mathcal{S} \vec{a}^{(k,c)},$$

Quantum subspace expansion

$$b^{\dagger}_{k,c} = \sum_I \boxed{a_I^{(k,c)}} M_I$$

$$\mathscr{H}_{I,J} = \langle \Omega | M_I^{\dagger} \; H \; M_J | \Omega \rangle$$

$$\mathscr{C}_{I,J} = \langle \Omega | M_I^{\dagger} \; C \; M_J | \Omega \rangle$$

$$\mathscr{S}_{I,J} = \langle \Omega | M_I^{\dagger} \; M_J | \Omega \rangle$$

- QSE equation

$$\left(\mathscr{H} + \mathscr{C}\right) \vec{a}^{(k,c)} = \lambda \; \mathscr{S} \vec{a}^{(k,c)},$$

$$\lambda = E + c e^{-ika}$$

Quantum subspace expansion

$$b_{k,c}^\dagger = \sum_I \boxed{a_I^{(k,c)}} M_I$$

$$\mathscr{H}_{I,J} = \langle \Omega | M_I^\dagger \ H \ M_J | \Omega \rangle$$

$$\mathscr{C}_{I,J} = \langle \Omega | M_I^\dagger \ C \ M_J | \Omega \rangle$$

$$\mathscr{S}_{I,J} = \langle \Omega | M_I^\dagger \ M_J | \Omega \rangle$$

- QSE equation

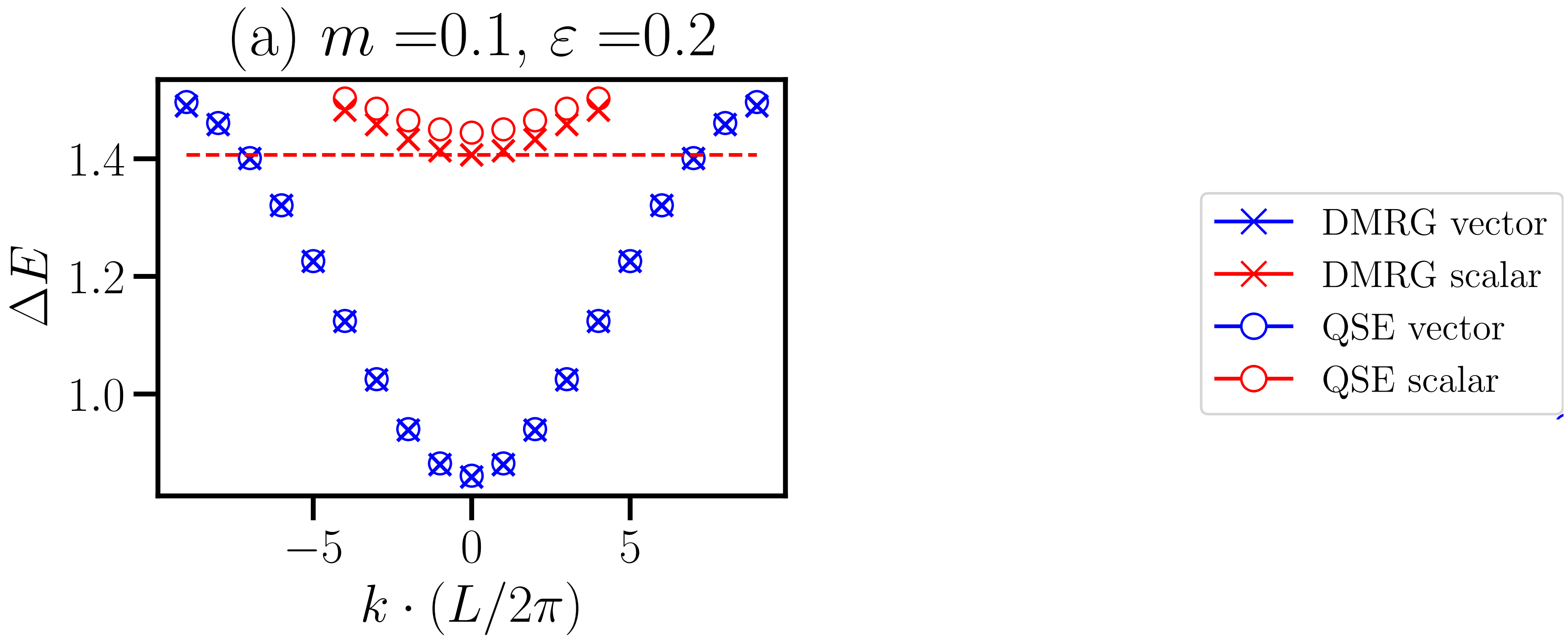
$$\left(\mathscr{H} + \mathscr{C} \right) \vec{a}^{(k,c)} = \lambda \ \mathscr{S} \vec{a}^{(k,c)},$$

$$\lambda = E + ce^{-ika}$$

$$\vec{a}^{k,c} = (a_{1,1}^{k,c}, a_{1,2}^{k,c}, \cdots)^T$$

Results from QSE for $L = 30$

- With coefficients from QSE: $|k, c\rangle_b = \sum_I a_I^{k,c} M_I |\Omega\rangle$, $\quad M_I \in \{\xi_n^\dagger W_{n,l} \xi_l \,|\, n, l = 1,2,\cdots L\}$

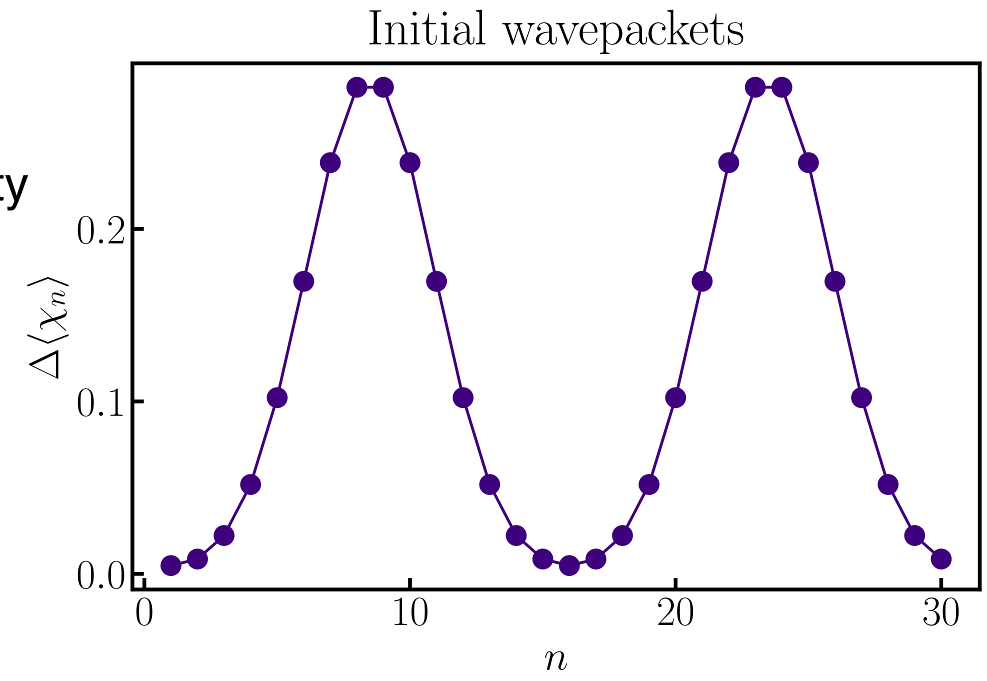


Wave packet of meson

$$B^\dagger_{\bar{k},\bar{x}} = \sum_{k\in\Lambda^*} \phi(k)_{\bar{k},\bar{x}}\ b^\dagger_{k,-1} \qquad \phi(k)_{\bar{k},\bar{x}} = \frac{1}{\sqrt{\mathcal{N}_\phi}} \exp(-ik\boxed{\bar{x}})\ \exp\left(-(k-\boxed{\bar{k}})^2/(4\boxed{\sigma_k^2})\right)$$

Staggered fermion density

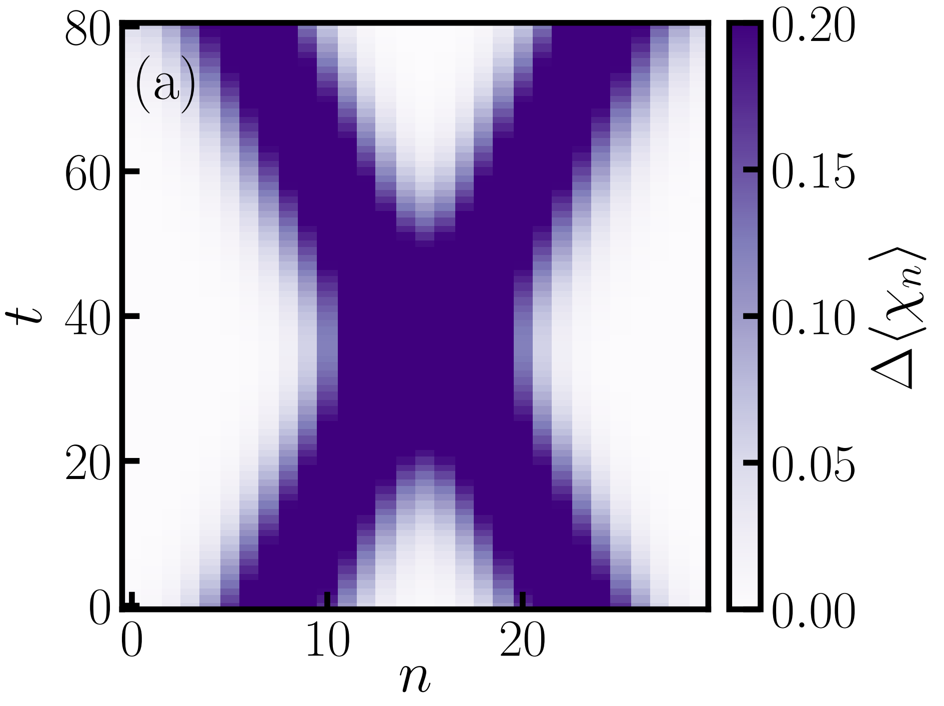
$$\langle \chi_n \rangle = \langle \xi_n^\dagger \xi_n \rangle$$



Scattering

$$B^\dagger_{\bar{k},\bar{x}} = \sum_{k\in\Lambda^*} \phi(k)_{\bar{k},\bar{x}}\; b^\dagger_{k,-1} \qquad \phi(k)_{\bar{k},\bar{x}} = \frac{1}{\sqrt{\mathcal{N}_\phi}} \exp(-ik\bar{x})\;\; \exp\left(-(k-\bar{k})^2/(4\sigma_k^2)\right)$$

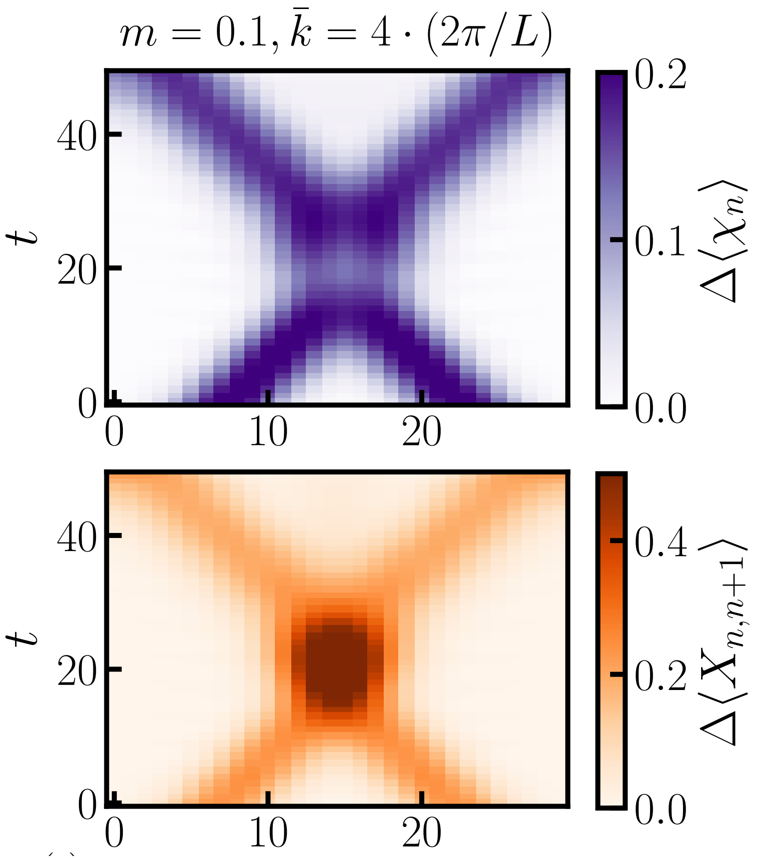
- Scattering:
  1. Initial state: $|\psi(t=0)\rangle = B^\dagger_{\bar{k},\bar{x}_1} B^\dagger_{-\bar{k},\bar{x}_2} |\Omega\rangle$
  2. Time evolution: $|\psi(t)\rangle = e^{-iHt}|\psi(t=0)\rangle$



Elastic Scattering

Inelastic Scattering

$m = 0.1,\ \varepsilon = 0.2$

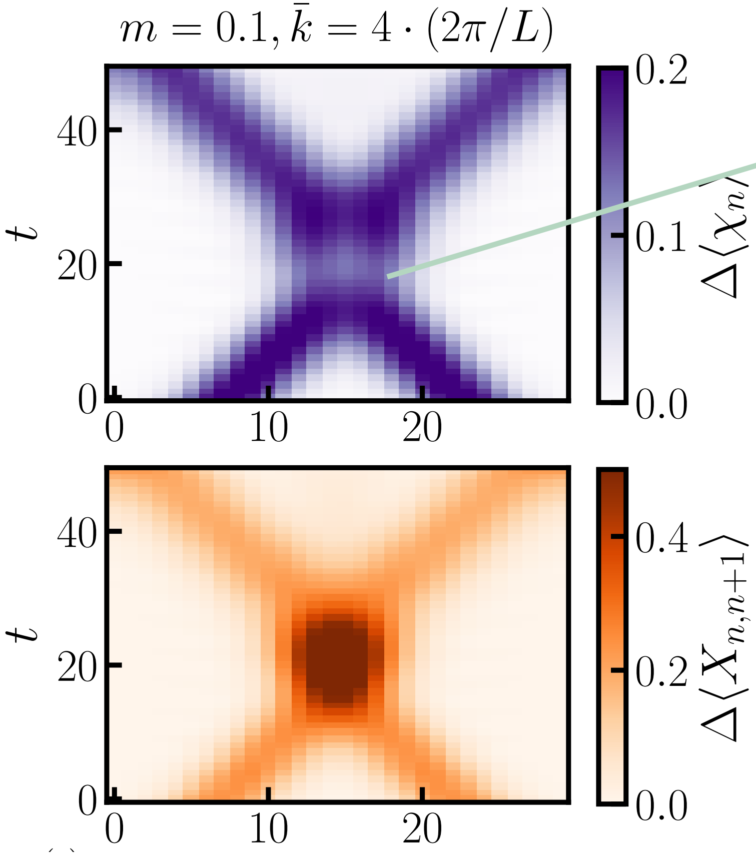


staggered fermion density

electric field density

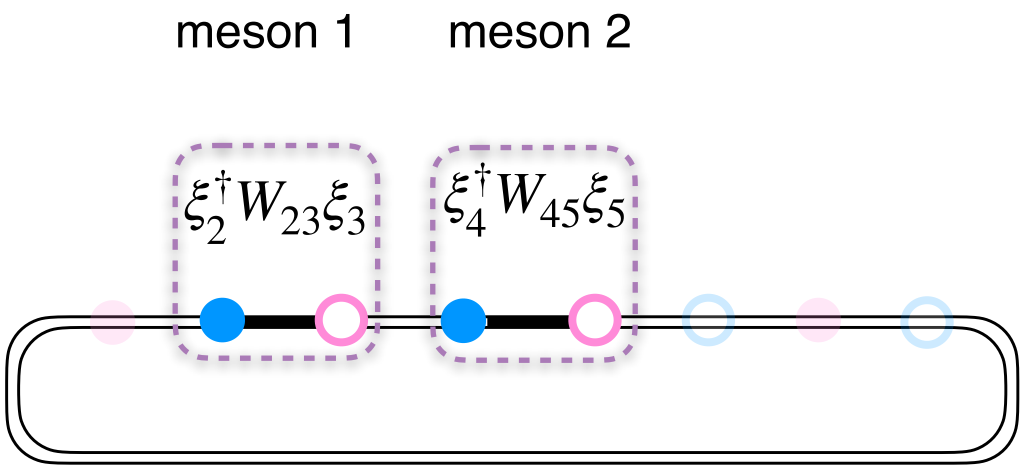
Inelastic Scattering

$m = 0.1,\ \varepsilon = 0.2$



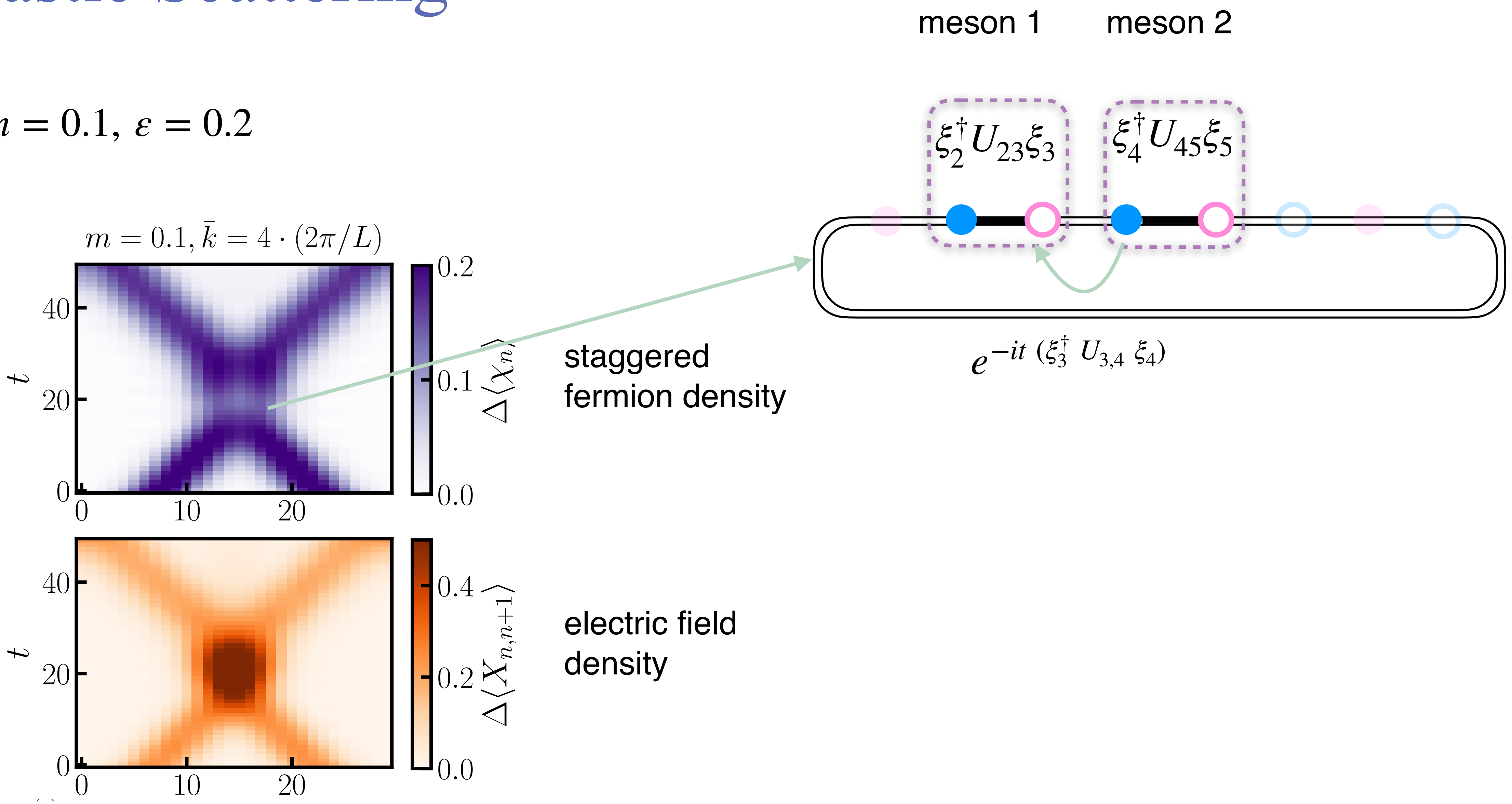
staggered fermion density

electric field density



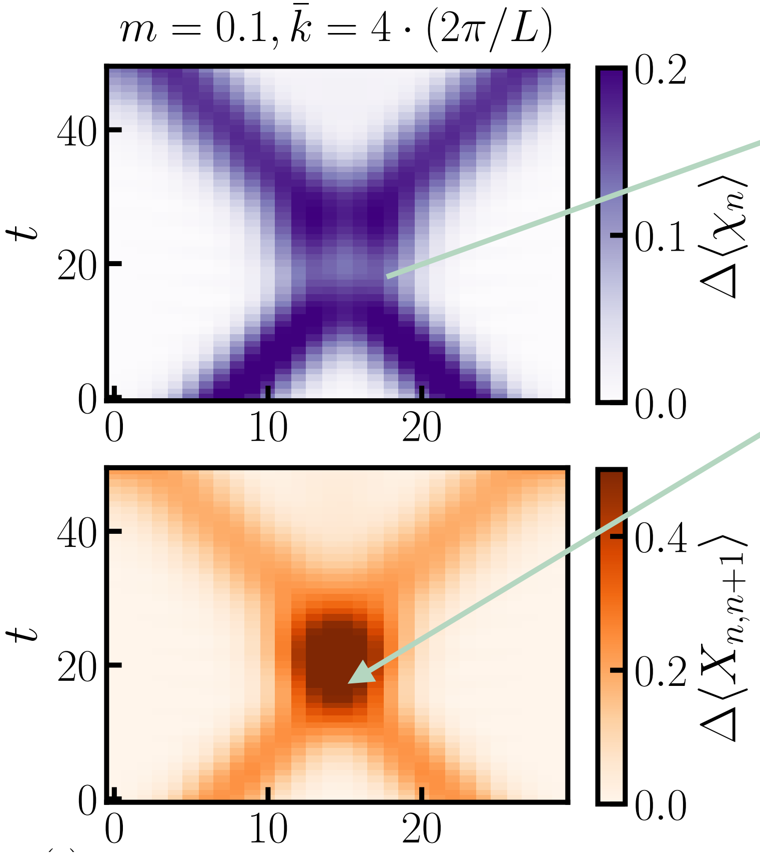
Inelastic Scattering

$m = 0.1,\ \varepsilon = 0.2$



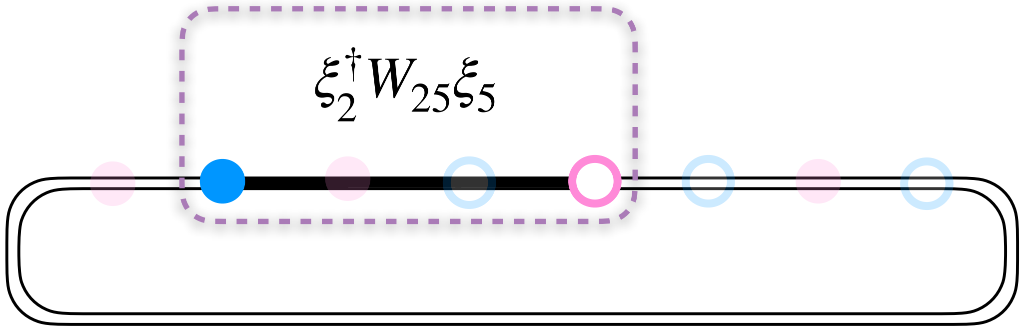
Inelastic Scattering

$m = 0.1,\ \varepsilon = 0.2$

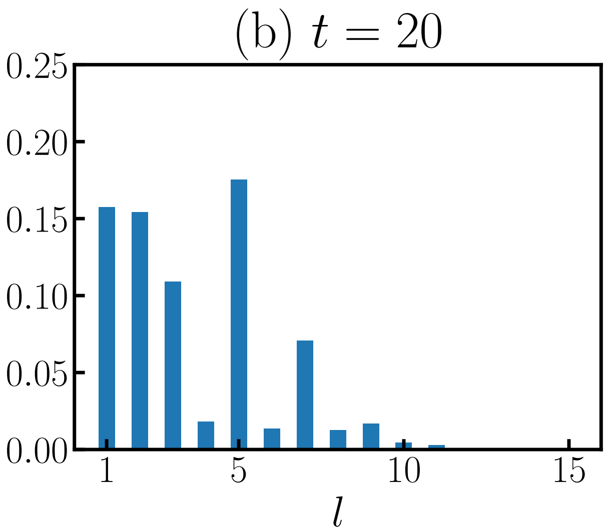


staggered fermion density

electric field density

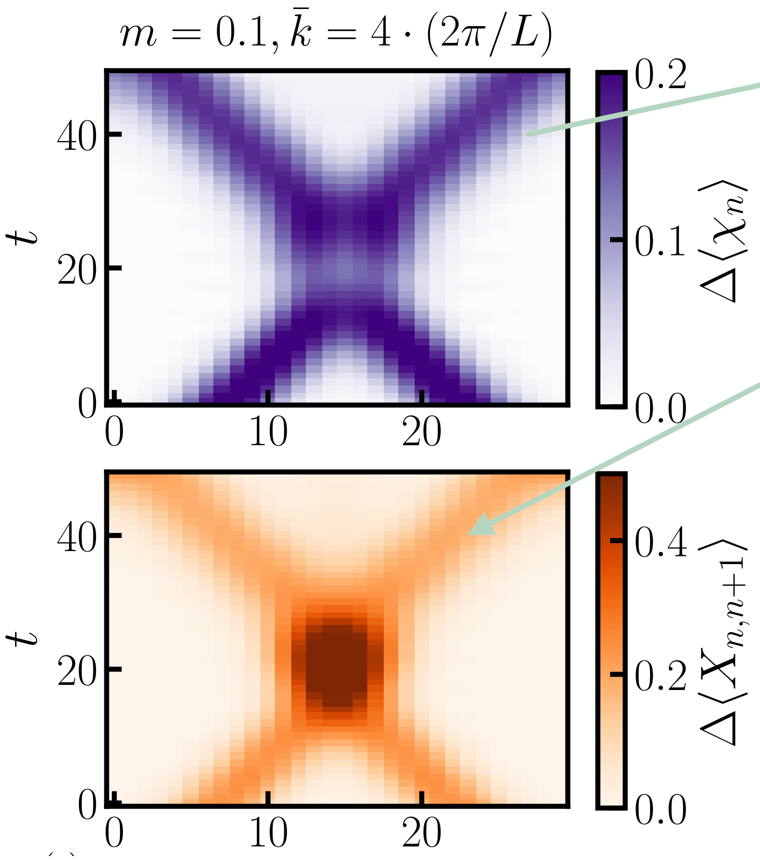


Probability of electric flux with length l



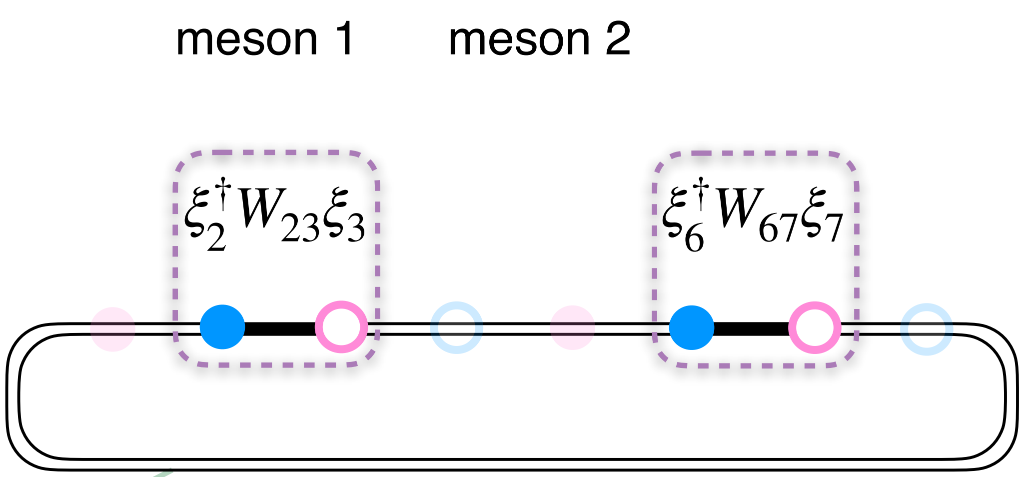
Inelastic Scattering

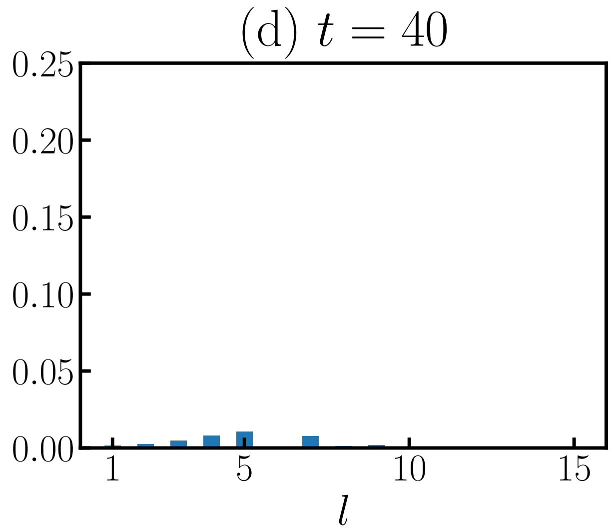
$m = 0.1,\ \varepsilon = 0.2$



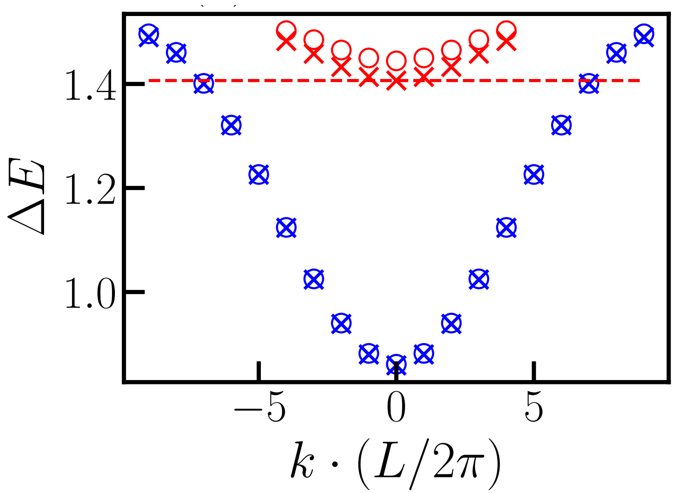
staggered
fermion density

electric field
density

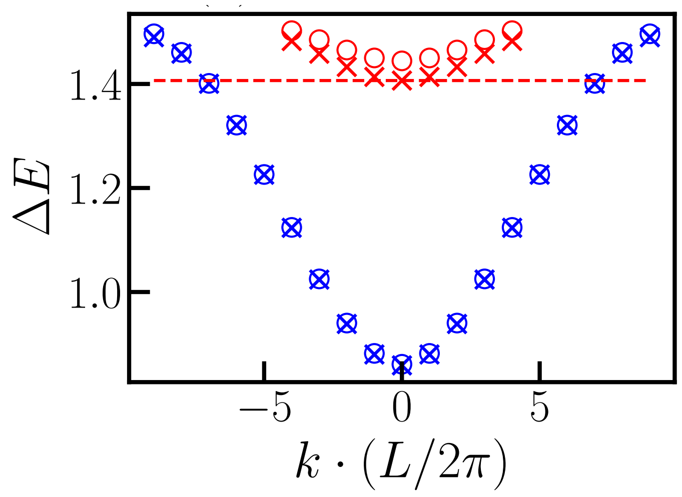


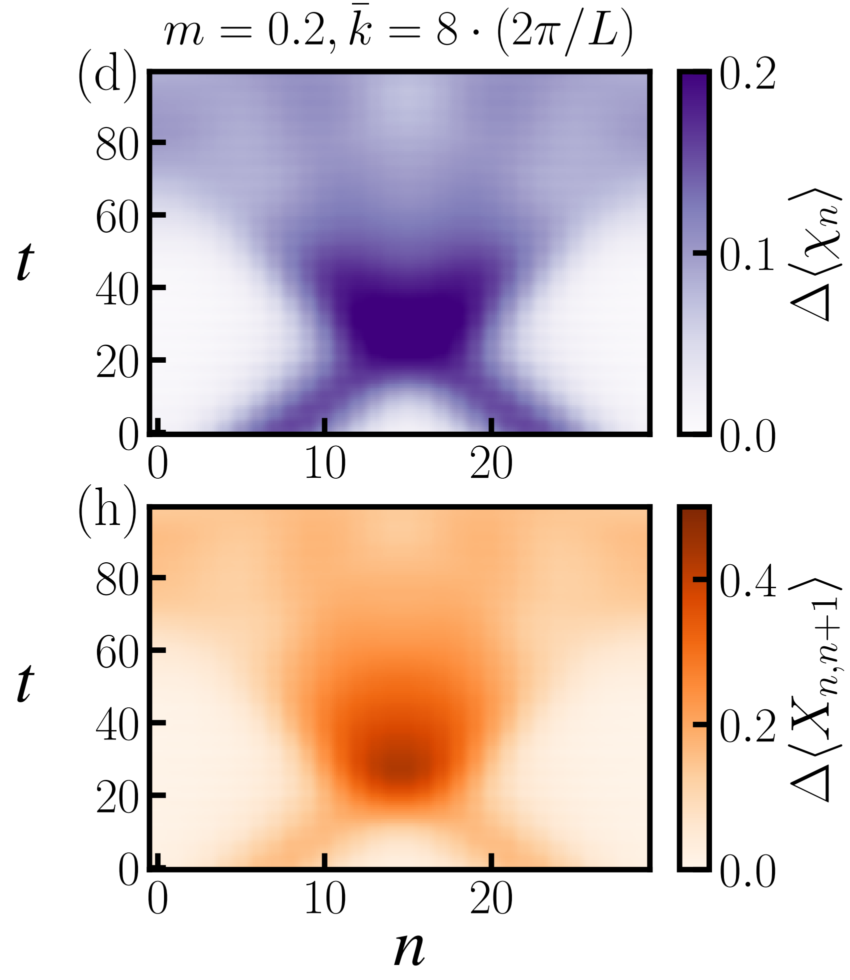


Inelastic Scattering



Inelastic Scattering

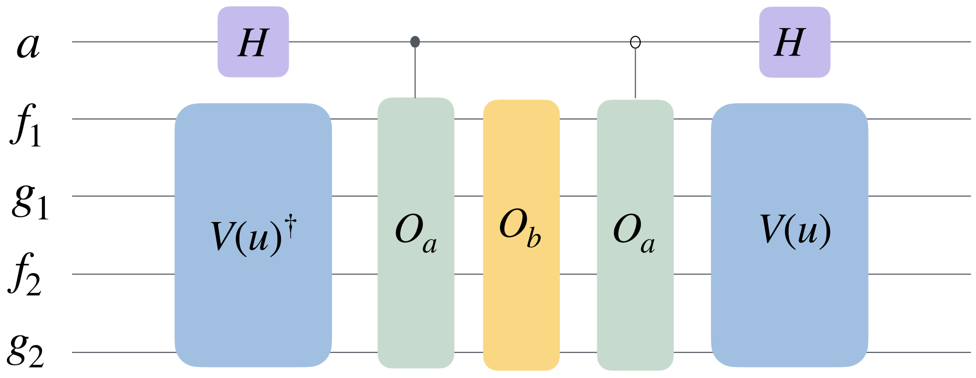




Quantum circuit for meson wave packet

- Non-unitary $$B^\dagger_{\bar{k},\bar{x}} = \sum_{k\in\Lambda^*} \phi(k)_{\bar{k},\bar{x}}\, b^\dagger_{k,-1},$$
$$= \sum_{k\in\Lambda^*} \sum_{nl} \phi(k)_{\bar{k},\bar{x}} a^{k,-1}_{nl}\ \xi^\dagger_n U_{n,l} \xi_l,$$

- Decomposition via Givens rotation

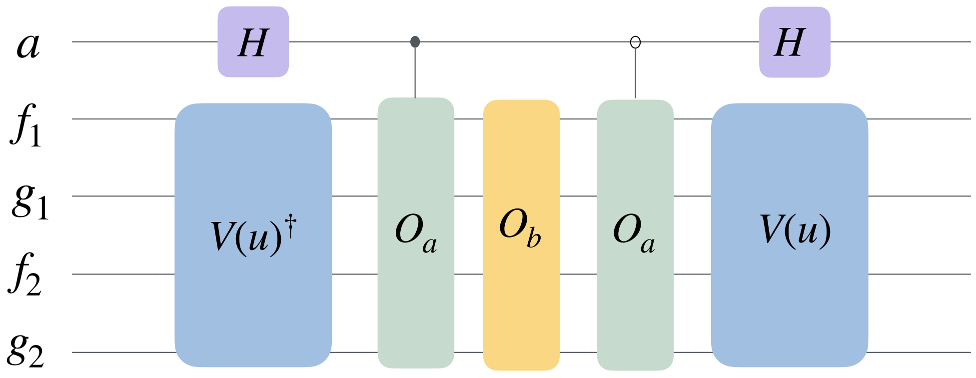


	CNOT number	CNOT depth
$V(u,\tilde{\xi})$ or $V(u,\tilde{\xi})^\dagger$	$2L(L-1)$	$4(2L-3)$
O_a	$4(L-1)$	$4(L-1)$
O_b	$12(L-1)$	$12(L-1)$
Total	$4L^2+16L-20$	$36L-44$

Quantum circuit for meson wave packet

- Non-unitary $$B^{\dagger}_{\bar{k},\bar{x}} = \sum_{k\in\Lambda^*} \phi(k)_{\bar{k},\bar{x}}\, b^{\dagger}_{k,-1},$$ $$= \sum_{k\in\Lambda^*} \sum_{nl} \phi(k)_{\bar{k},\bar{x}} a^{k,-1}_{nl}\, \xi^{\dagger}_n U_{n,l} \xi_l,$$

- Decomposition via Givens rotation



	CNOT number	CNOT depth
$V(u,\tilde{\xi})$ or $V(u,\tilde{\xi})^{\dagger}$	$2L(L-1)$	$4(2L-3)$
O_a	$4(L-1)$	$4(L-1)$
O_b	$12(L-1)$	$12(L-1)$
Total	$4L^2+16L-20$	$36L-44$

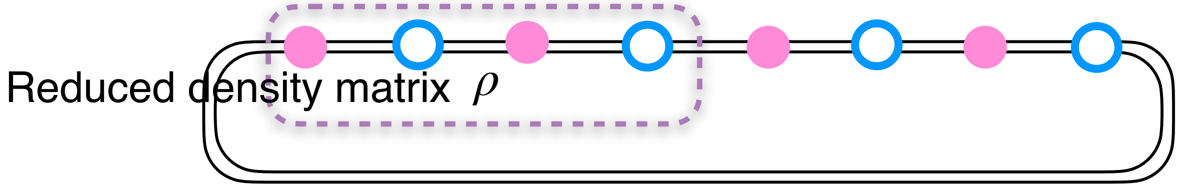
~1000

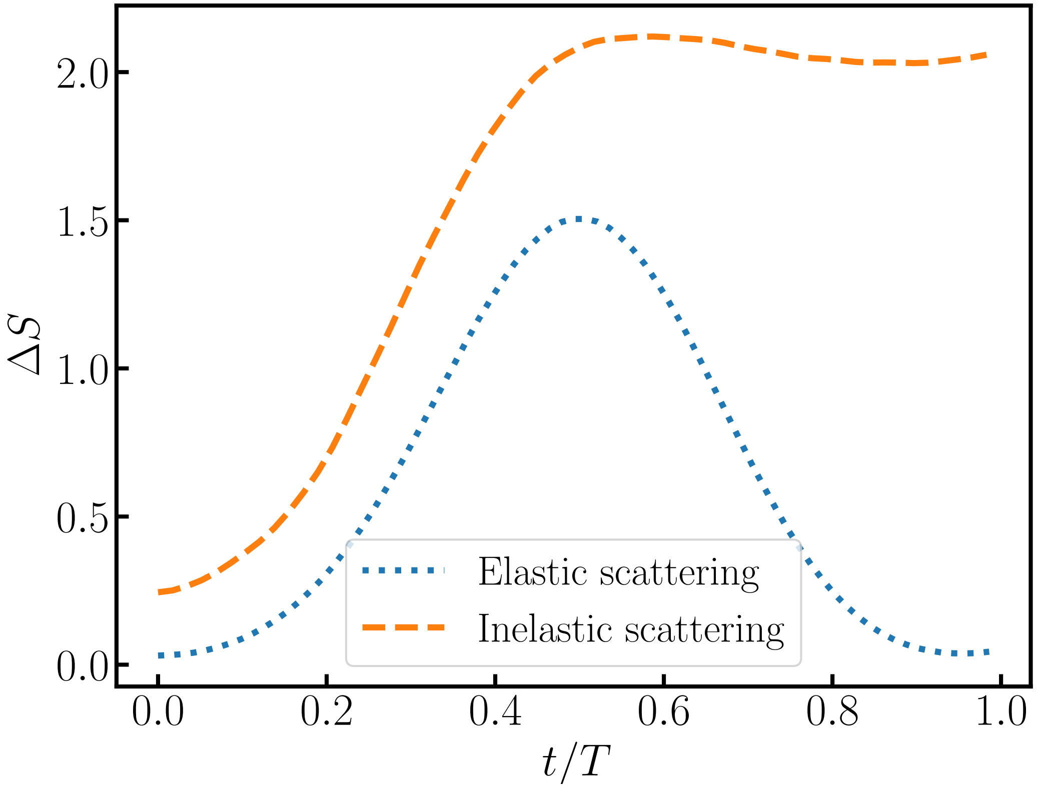
Hardware run?

Entanglement entropy

Larger entanglement entropy, harder for tensor networks

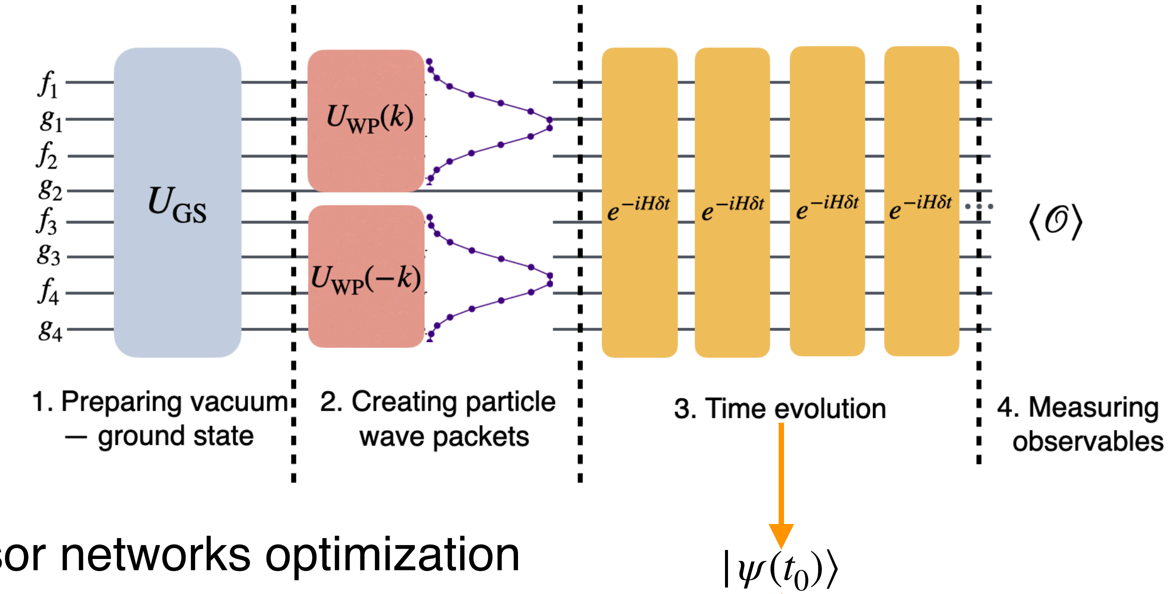
$$S(t) = -Tr[\rho(t)\log_2 \rho(t)]$$





Scattering process on QC

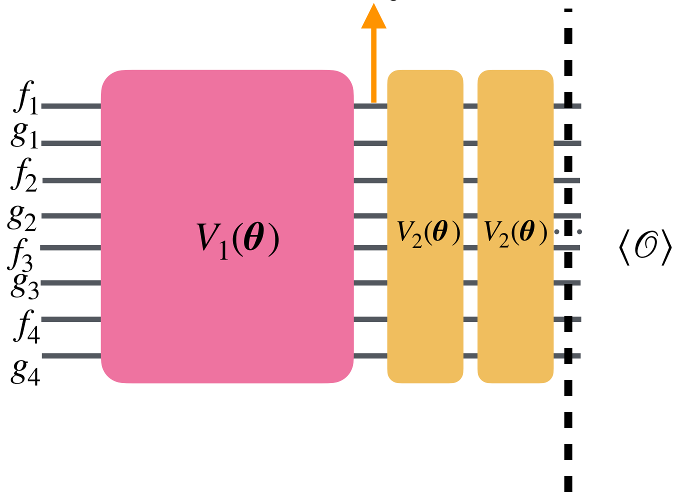
Conventional circuit



Circuit compression by tensor networks optimization

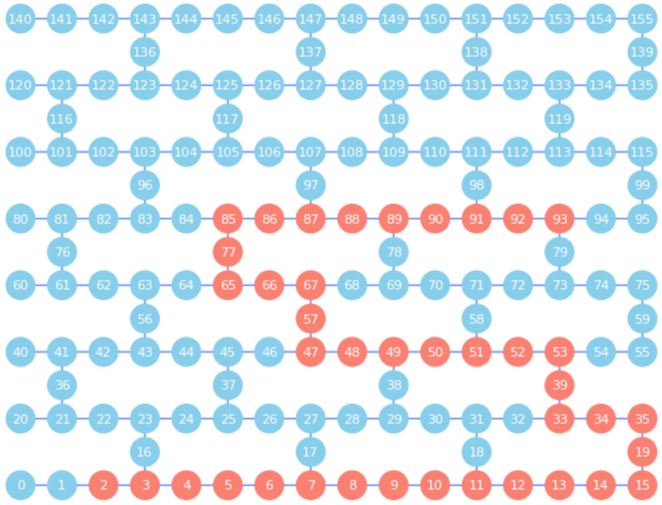
$$C_{\mathrm{State}}(\boldsymbol{\theta}) = \left| \langle \psi(t_0) \,|\, V_1(\boldsymbol{\theta}) \,|\, 0 \rangle \right|^2$$

$$C_{\mathrm{Uni}}(\boldsymbol{\theta}) = \frac{1}{2^{2N}} \left| \mathrm{Tr}(V_2(\boldsymbol{\theta})^\dagger e^{-iH\delta t}) \right|^2$$



Hardware run of fermion scattering on Thirring model 40 qubits

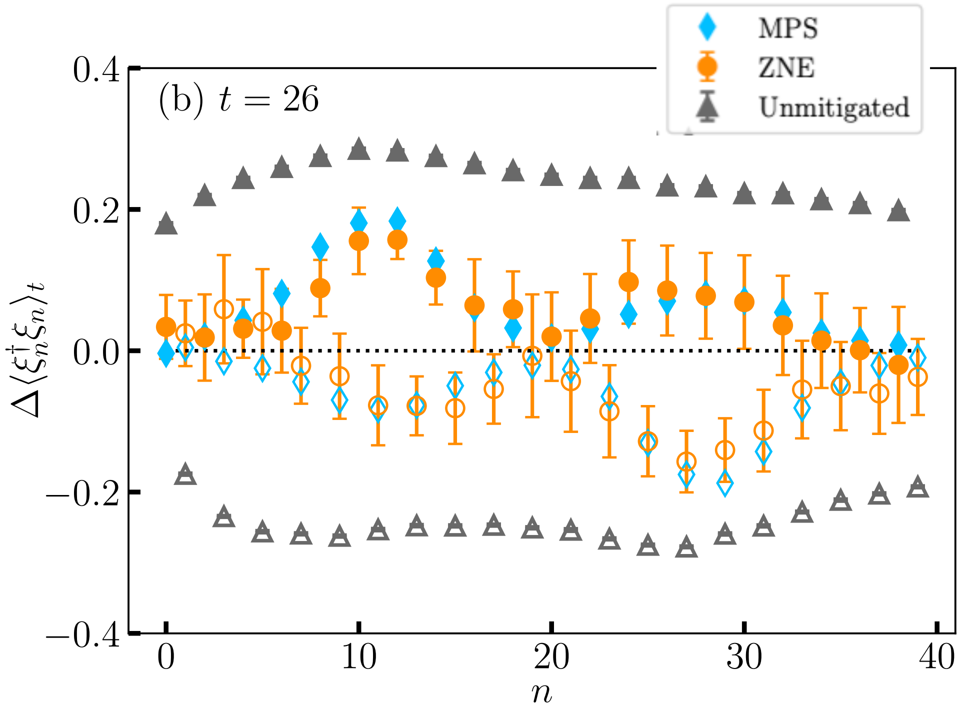
- IBM device ibm_fez
- Error mitigation: zero-noise extrapolation (ZNE)



$N = 40,\ T = 26,\ \Delta t = 2/3$		
	CNOT layers	CNOT gates
$\lvert\psi(t_0)\rangle$	36 (241)	702 (3371)
e^{-i2H}	12 (18)	234 (351)
In total	96 (331)	1872 (5126)

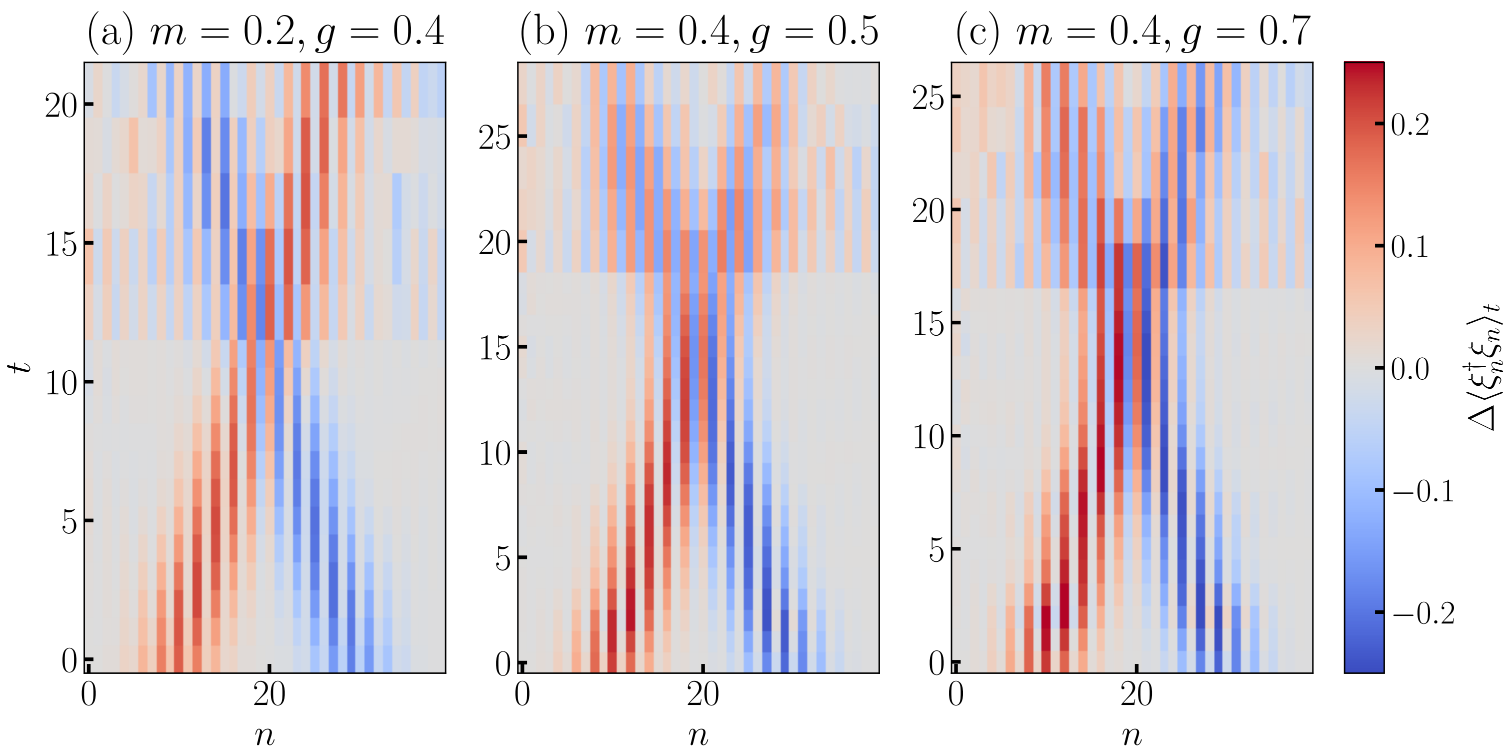
Our optimized circuit

Conventional approach



Hardware run for 40 qubits: full dynamics

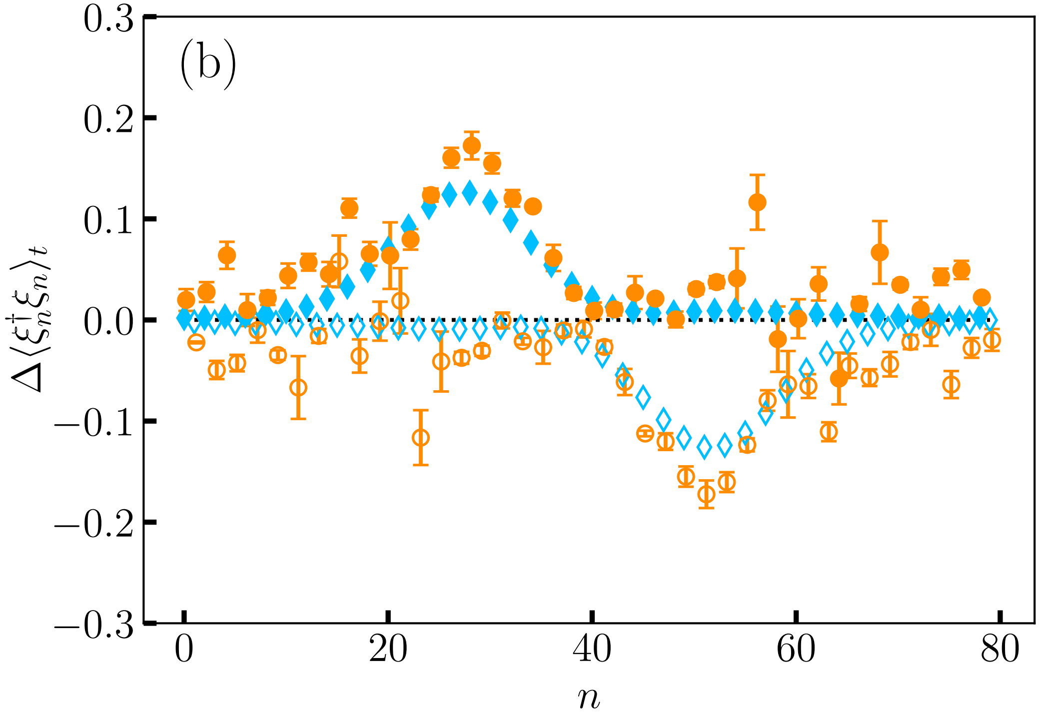
- IBM device ibm_fez
- Error mitigation: zero-noise extrapolation (ZNE)



Hardware run for 80 qubits: state preparation

- IBM device ibm_fez
- Error mitigation: zero-noise extrapolation (ZNE)

$N = 80,\ t_0 = 10,\ \Delta t = 2/3$		
	CNOT layers	CNOT gates
$\lvert\psi(t_0)\rangle$	25 (249)	948 (3987)



Summary

- Algorithm development for meson scattering:
    - QSE for eigenstates with specific momentum and charge conjugation
    - Circuit decomposition
    - Inelastic scattering: new particle production, string breaking, hadronization…
- Hardware run for fermion scattering, optimized circuit by TN
    - Full scattering for 40 qubits
    - State preparation for 80 qubits

Outlook

- Other Gauge theory, e.g., QED
- Higher dimension (2+1)D

Summary

- Algorithm development for meson scattering:
  - QSE for eigenstates with specific momentum and charge conjugation
  - Circuit decomposition
  - Inelastic scattering: new particle production, string breaking, hadronization…
- Hardware run for fermion scattering, optimized circuit by TN
  - Full scattering for 40 qubits
  - State preparation for 80 qubits

Outlook

- Other Gauge theory, e.g., QED
- Higher dimension (2+1)D

On the Market 🤩

yahui.chai@desy.de

Thank you!

Arianna Crippa
(CQTA)

Karl Jansen
(CQTA)

Stefan Kühn
(CQTA)

Vincent R.
Pascuzzi (IBM, NY)

Francesco
Tacchino (IBM
Zürich)

Ivano Tavernelli
(IBM Zürich)

Yibin Guo
(CQTA)

Joe Gibbs
University of Surrey

Zoe Holmes
EPFL

First fermion scattering paper

Y. Chai, A. Crippa, K. Jansen, S. Kühn, V. R. Pascuzzi, F. Tacchino, and I. Tavernelli, Quantum **9**, 1638 (2025).

Meson scattering paper

arXiv:2505.21240

Fermion scattering hardware run paper

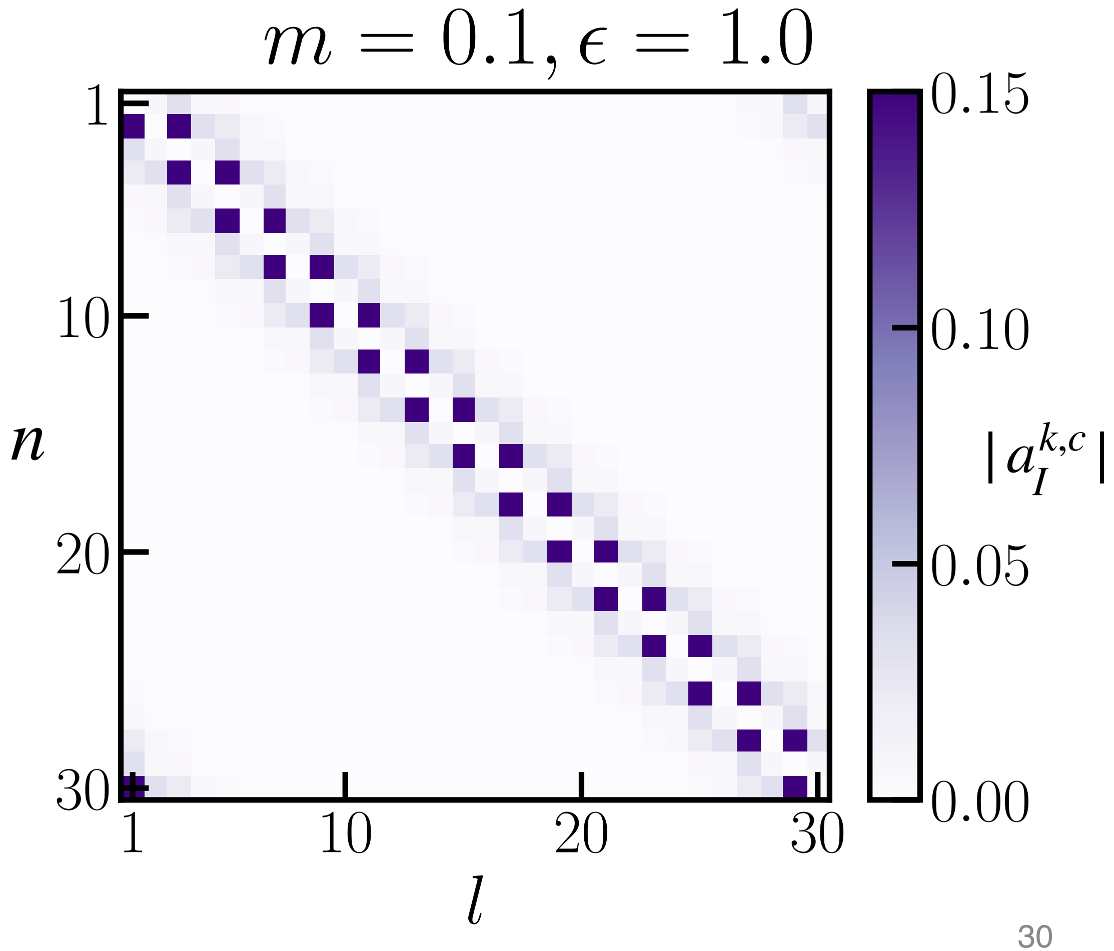
arXiv:2507.17832

DESY. QUANTUM | Center for Quantum Technology and Applications

IBM **Quantum**

Results from QSE

- With coefficients from QSE: $|k, c\rangle_b = \sum_I a_I^{k,c} M_I \, |\Omega\rangle$

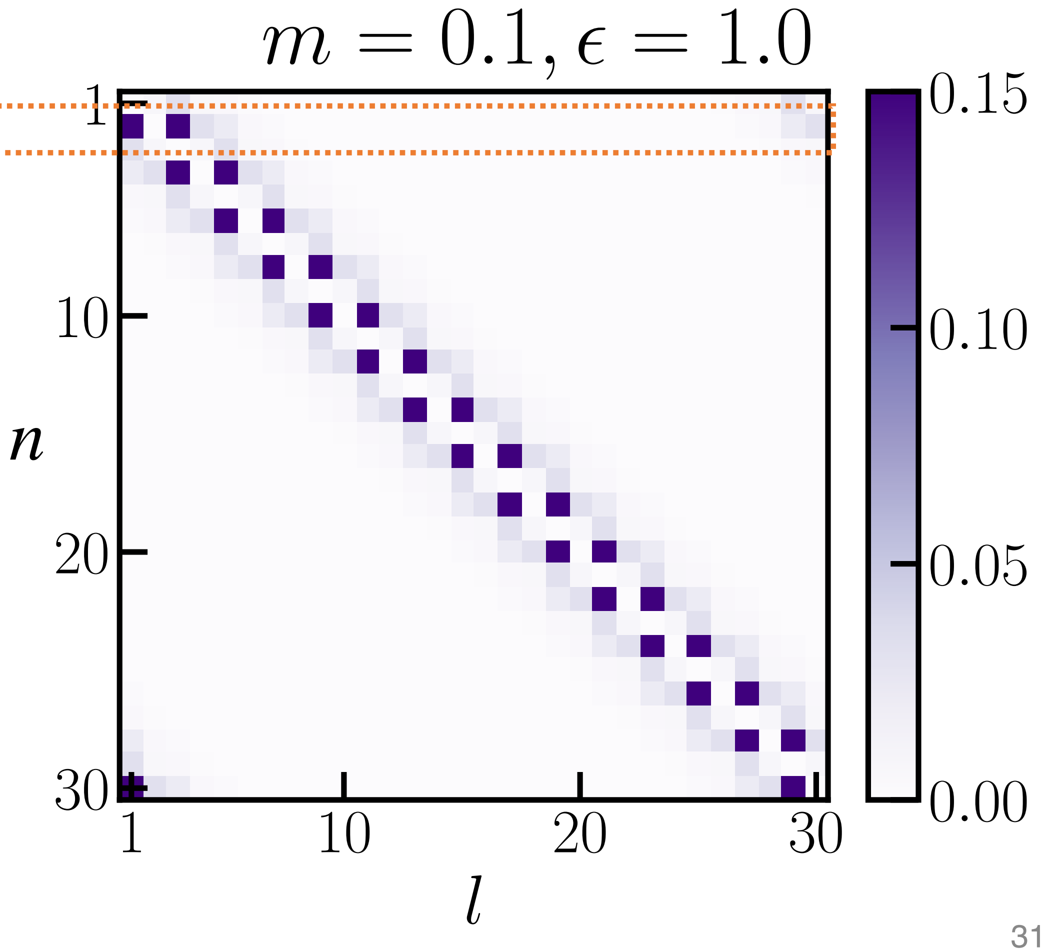


Results from QSE

- With coefficients from QSE: $|k,c\rangle_b = \sum_I a_I^{k,c} M_I \, |\Omega\rangle$

$\xi_2^\dagger W_{21} \xi_1$ $\xi_2^\dagger \xi_2$ $\xi_2^\dagger W_{23} \xi_3$ $\xi_2^\dagger W_{24} \xi_4$ $\xi_2^\dagger W_{25} \xi_5$

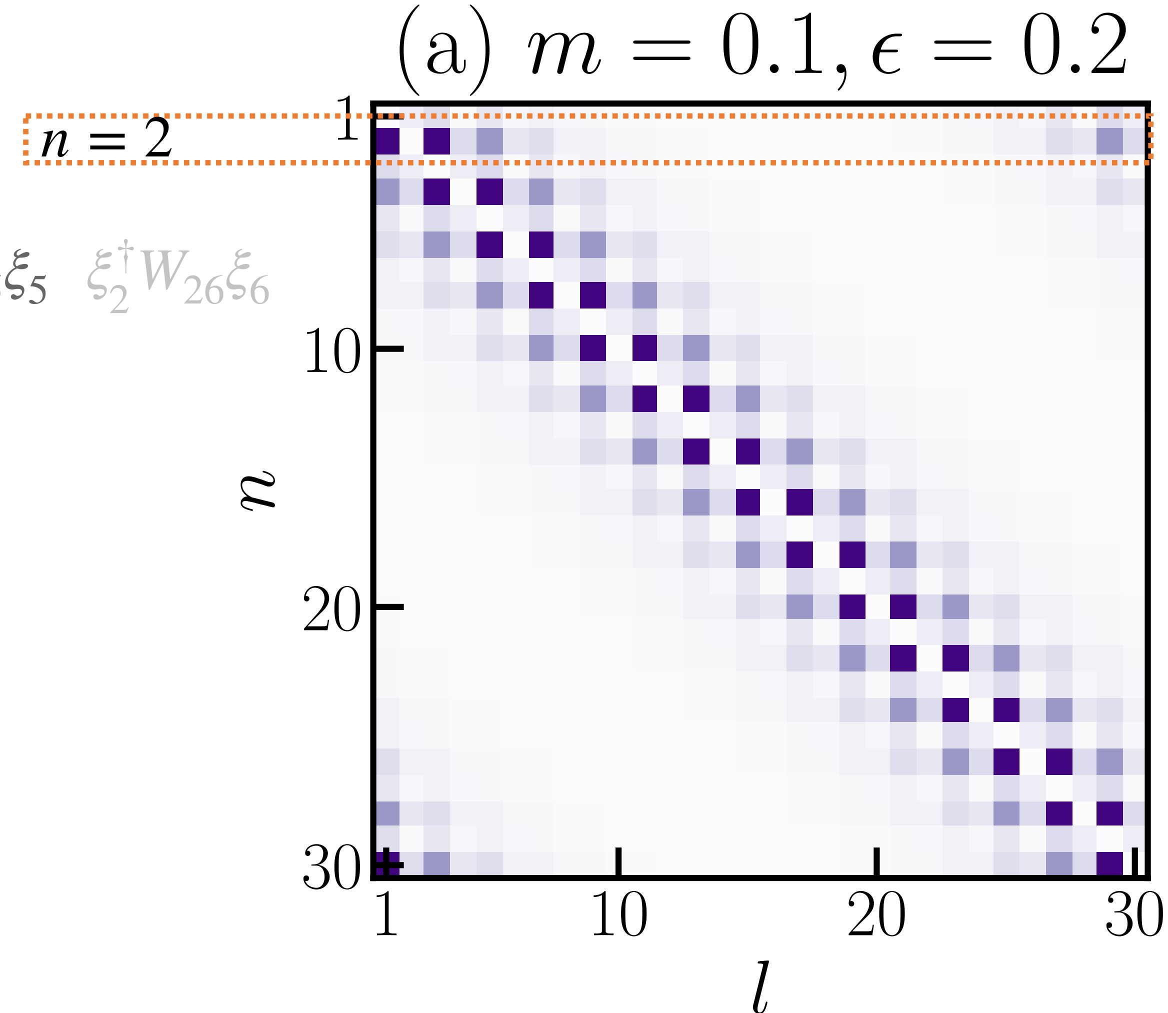
$n = 2$



Results from QSE

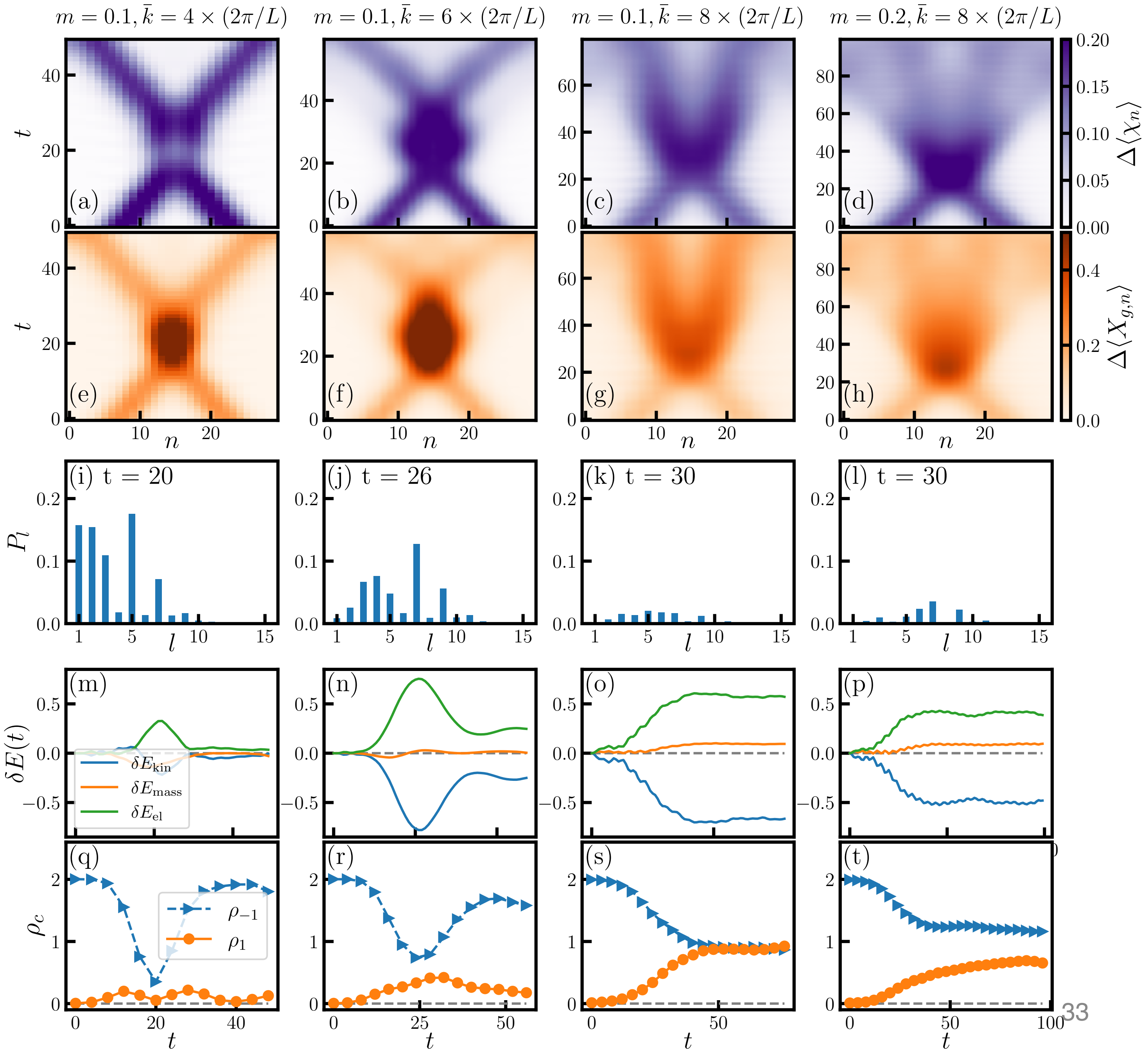
- With coefficients from QSE: $|k,c\rangle_b = \sum_I a_I^{k,c} M_I |\Omega\rangle$

$\xi_2^\dagger W_{21} \xi_1$ $\xi_2^\dagger \xi_2$ $\xi_2^\dagger W_{23} \xi_3$ $\xi_2^\dagger W_{24} \xi_4$ $\xi_2^\dagger W_{25} \xi_5$ $\xi_2^\dagger W_{26} \xi_6$



Scattering

$$P_l = \sum_{n=1}^{L-l} |\langle \psi(t) | \xi_n^\dagger W_{n,n+l} \xi_{n+l} | \Omega \rangle|^2 + \sum_{n=l+1}^{L} |\langle \psi(t) | \xi_n^\dagger W_{n,n-l} \xi_{n-l} | \Omega \rangle|^2$$



ZNE