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Chiral Extrapolation of the nucleon mass at two-loop order

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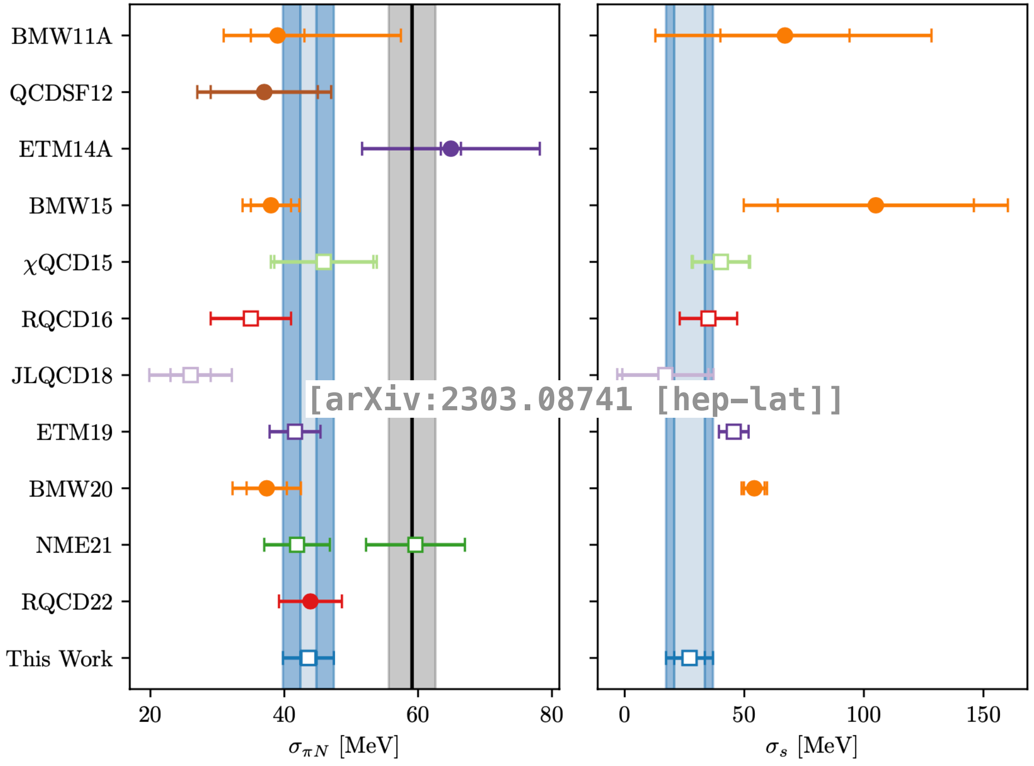
Outline

- Introduction
- Leading two-loop result of the nucleon mass
- Chiral extrapolation of lattice QCD data
- Summary and outlook

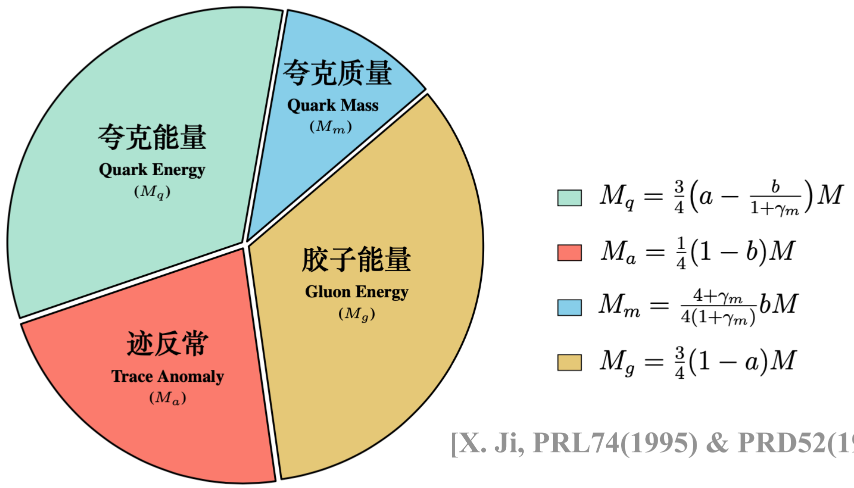
Nucleon mass physics

- High precision determination of the nucleon mass
  - Sigma term: tension between lattice QCD and phenomenological results
  - WIMP dark matter: scalar coupling of the nucleon to dark matter
  - The proton mass decomposition
  - …



Feynman—Hellmann Theorem

$$\sigma_{\pi N} = \frac{\hat{m}}{2m_N}\langle N\,|\,\bar{u}u + \bar{d}d\,|\,N\rangle = M_\pi^2 \frac{\partial m_N}{\partial M_\pi^2}$$

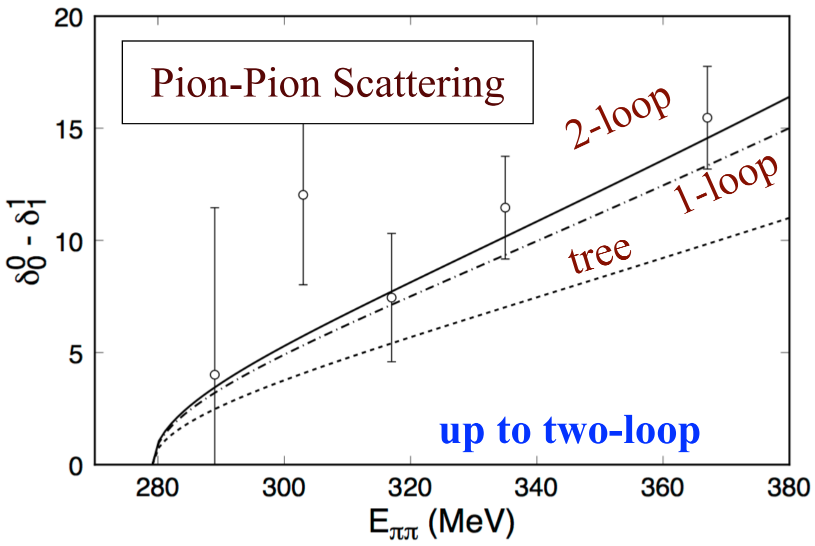


[X. Ji, PRL74(1995) & PRD52(1995)]

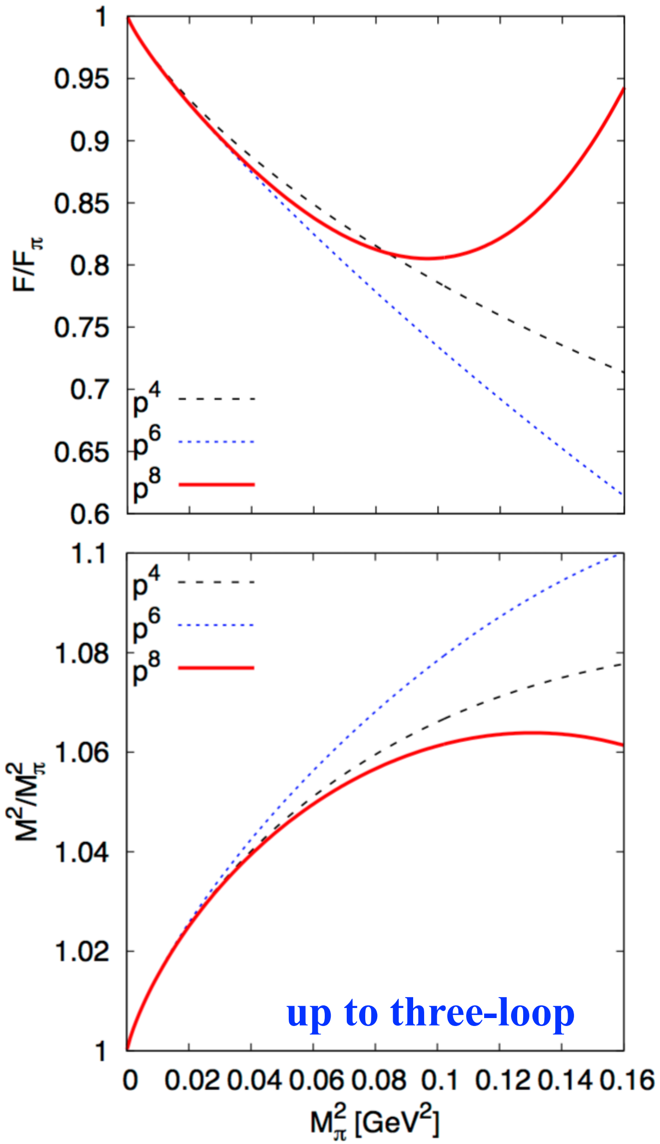
Status of ChPT beyond one-loop

- **Pure meson sector:** a great triumph has been achieved.
  - ➡ Two-loop calculations become standard and good convergence can be seen.
    - Pion mass
    - Pion decay constant
    - Electromagnetic form factor
    - $\pi\pi$ scattering
    - πK scattering
    - $K_{\ell 4}$
    - …

[Bijnens, et al., PLB (1996)]



[Bijnens and Truedsson, JHEP (2017)]



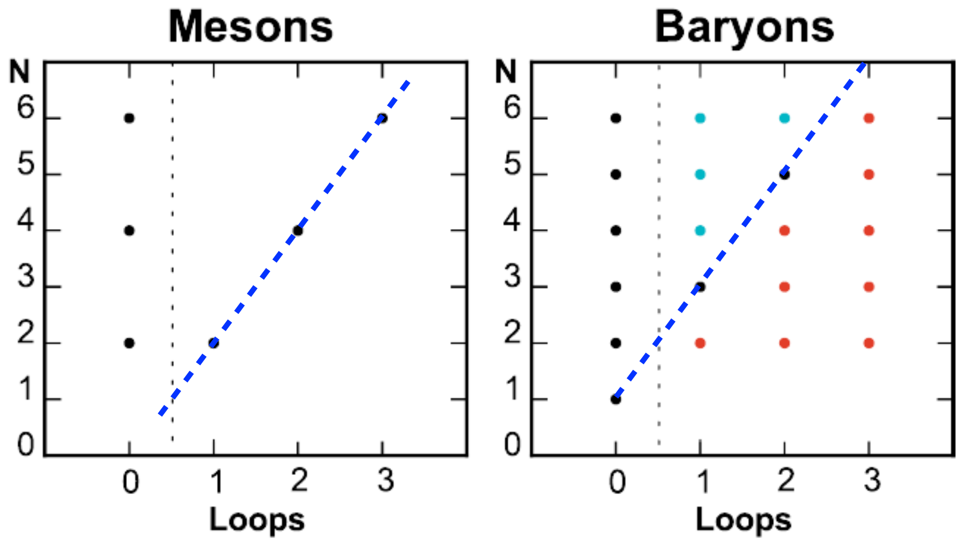
Status of ChPT beyond one-loop

- **Baryon sector:** multi-loop calculation is much more complex than pure meson sector
  - Dirac algebra & axial coupling between baryons and Goldstone bosons

    $\longrightarrow$ *More diagrams and master integrals*
  - Unequal mass

    $\longrightarrow$ *Hard to compute multi-loop integrals*
  - The introduction of non-zero baryon mass in the chiral limit

    $\longrightarrow$ *the notable Power Counting Breaking (PCB) problem!*

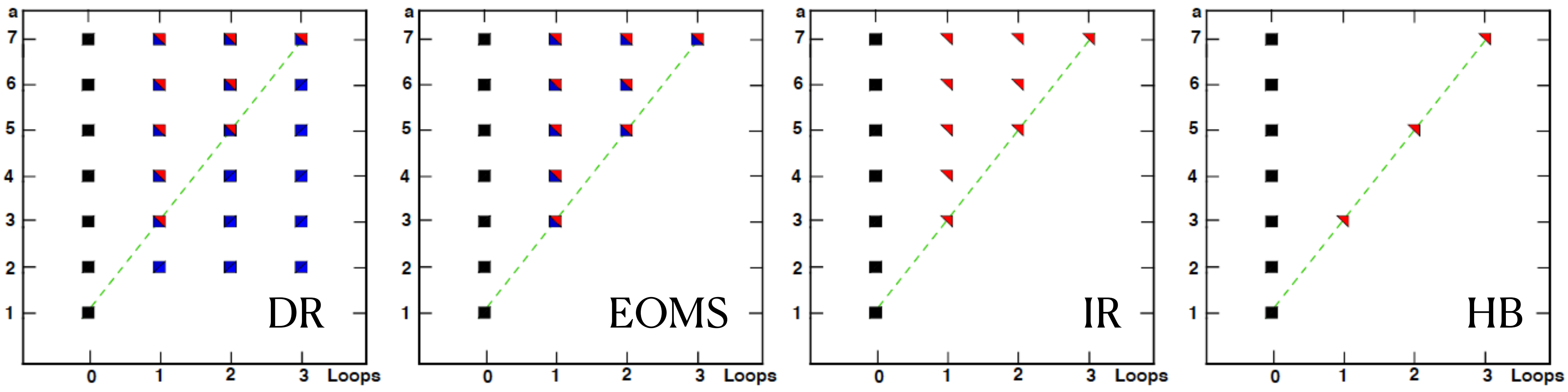


Naive power counting rule !!!

Red dots denote PCB terms!!!

Solutions to the PCB problem

- **Various approaches**
  - Heavy Baryon (HB) formalism [Jenkins and Manohar,PLB255'91]
  - Infrared Regularisation (IR) prescription [T.Becher and H.Leutwyler,Eur.Phys.J.C9'99]
  - Extended-On-Mass-Shell (EOMS) scheme [T.Fuchs,J.Gegelia,G.Japaridze and S.Scherer,PRD68'03]



philosophy: HB & IR: throwing away EOMS: reorganisation

Status of the nucleon mass at two loop

- **Previous works on the nucleon mass in ChPT**
  - 1999: HB formalism up to $O(p^5)$ [McGovern & Birse, PLB446(1999)]
  - 2007/2008: IR prescription up to $O(p^6)$ [truncated]

    [Schindeler, Djukanovic, Gegelia & Scherer, PLB649(2007), NPA803(2008)]
  - 2024: EOMS up to $O(p^6)$ [truncated]

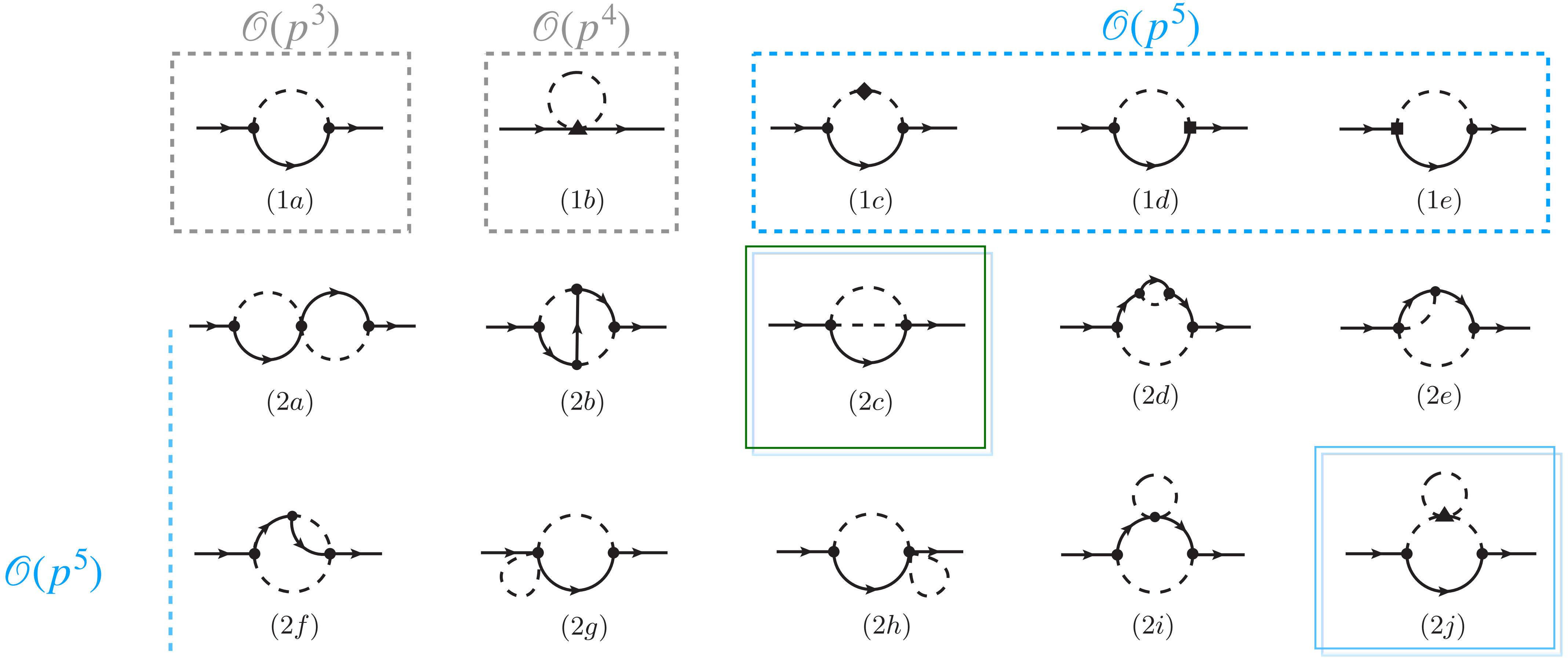
    [Conrad, Gasparyan & Epelbaum, PoS CD2021, 074 (2024), EPJ Web Conf. 303, 02001(2024)]

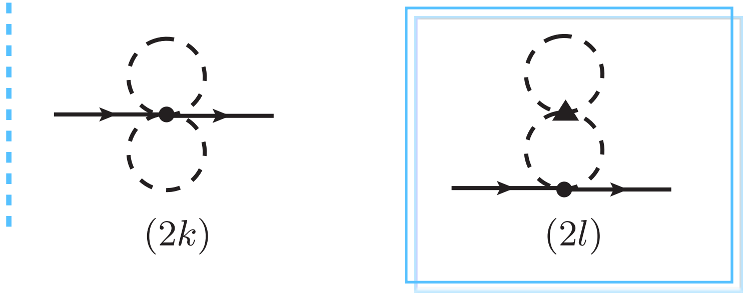
    [Chen, Hu, Mo & Jia, arXiv:2406.040124 [hep-ph]]

$$\boxed{m_N = m + k_1 M^2 + k_2 M^3 + k_3 M^4 \ln\frac{M}{\mu} + k_4 M^4 + k_5 M^5 \ln\frac{M}{\mu} + k_6 M^5 + k_7 M^6 \ln^2\frac{M}{\mu} + k_8 M^6 \ln\frac{M}{\mu} + k_9 M^6}$$

- **Our work: full EOMS chiral result up to $\mathcal{O}(p^5)$**
  - All the necessary analytical two-loop structure
  - Avoid too many unknown LECs

The nucleon mass at two-loop order





- “t channel”: incorporating $\pi\pi$ rescattering effect
- “s channel”: incorporating $\pi\pi N$ contribution

[Liang, Chen, Guo, Guo, and **DLY**, JHEP(2025)]

Two-loop nucleon mass in BChPT

- **Chiral expression up to $\mathcal{O}(p^5)$**
  - Tree + one loop + two loop $\quad m_N = -4c_1M^2 - 2e_mM^4 + \Delta m_N^{(1)} + \Delta m_N^{(2)}$
  - One-loop: $\quad \Delta m_N^{(1)} = \boxed{m_N^{(1a)}} + m_N^{(1b)} + m_N^{(1c)} + m_N^{(1d)} + m_N^{(1e)}\,,$
  - Two-loop: $\quad \Delta m_N^{(2)} = \boxed{m_N^{(2a)}} + m_N^{(2b)} + \cdots + m_N^{(2l)} + m_N^{(2')} + m_N^{\text{sub.}}\,.$

$$m_N^{(1a)} = \frac{3g^2 m\,\kappa}{2iF^2}\left[J_{01} + M^2 J_{11}\right]$$

$$m_N^{(2a)} = \frac{3g^2\,\kappa^2}{16mF^4}\Big[8m^2M^4 I_{11011} + 8m^2M^2(I_{01011} + I_{10011}) + 2m^2\big[4I_{00011} + I_{11(-1)01} + I_{11(-1)10} - I_{110(-1)1} - I_{1101(-1)}\big] + 2(M^2 - 2m^2)I_{11000} - I_{110(-1)0} - I_{1100(-1)}\Big],$$

✓ $J_{\nu_1\nu_2}$ and $I_{\nu_1\nu_2\nu_3\nu_4\nu_5}$ are one- and two-loop Feynman integrals.

Feynman integrals

Calculation of Feynman Integrals

In our work

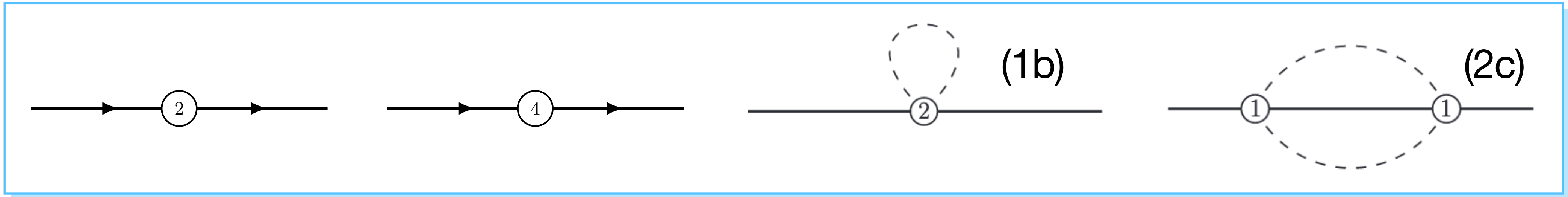
- Numerical ways: AMFLOW, AmpRed, FIESTA, pySecdec
  Liu,Ma,2022 Chen,2024
- Analytic ways: Differential Equations with UT-Basis, Mellin Barnes···

Analytic calculation of the high order corrections have many benefits. We can obtain the numerical effect of high order corrections very fast, expand them at any kinematic point, and integrate them to obtain more interesting results.

Slide by L. B. Chen

Renormalization of non-renormalizable EFT

- **Non-local UV divergences**
  - Pedagogic example with $g_A = 0$



- One-loop: $$m_N^{(1b)} = -\frac{3(d(c_3-2c_1)+c_2)M^4}{F^2 d}\left[-\frac{1}{\Lambda\epsilon}+\frac{1}{\Lambda}\log\frac{M^2}{\mu^2}+\mathcal{O}(\epsilon)\right]$$
- Two-loop: $$m_N^{(2c)} = \frac{3\left(6M^4 m+8M^2m^3-3m^5\right)}{32F^4\Lambda^2\epsilon^2}+\frac{132M^4m-16M^2m^3-9m^5}{128F^4\Lambda^2\epsilon}$$
$$+\frac{3m^3\left(3m^2-8M^2\right)}{16F^4\Lambda^2\epsilon}\log\frac{m^2}{\mu^2}-\frac{9M^4m}{8F^4\Lambda^2\epsilon}\log\frac{M^2}{\mu^2}+\text{finite}+\mathcal{O}(\epsilon)$$

✓ Local UV div. tree-loop counter term

✓ How to deal with non-local UV div.?

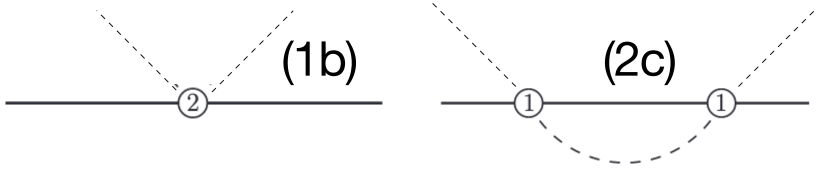
Renormalization of non-renormalizable EFT

- **UV renormalization at two-loop level**
  - Non-local UV: split the bare LECs in one-loop diagram

$$m_N^{(1b)} = -\frac{3(d(c_3-2c_1)+c_2)M^4}{F^2 d}\left[-\frac{1}{\Lambda\epsilon} + \boxed{\frac{1}{\Lambda}\log\frac{M^2}{\mu^2}} + \mathcal{O}(\epsilon)\right]$$

$$X = \boxed{-\frac{\beta_X^{(1)}}{\epsilon\Lambda}} + X^r + \mathcal{O}(\epsilon)\ , \quad X \in \{c_1, c_2, c_3\}$$

Pion-nucleon scattering up to $\mathcal{O}(\epsilon)$



$$\beta_{c_1}^{(1)} = 0\ , \quad \beta_{c_2}^{(1)} = \frac{m}{2F^2}\ , \quad \beta_{c_3}^{(1)} = \frac{m}{4F^2}$$

  - Local UV: split the bare LECs in the tree diagrams

$$m_N = \underbrace{m + 4c_1 M^2 - e_m M^4}_{\text{tree}} + m_N^{(1b)} + m_N^{(2c)}$$

$$X = \frac{\beta_X^{(22)}}{\epsilon^2\Lambda^2} - \frac{\beta_X^{(21)}}{\epsilon\Lambda^2} - \frac{\beta_X^{(11)}}{\epsilon\Lambda} + X^r\ , \quad X \in \{m, c_1, e_m\}$$

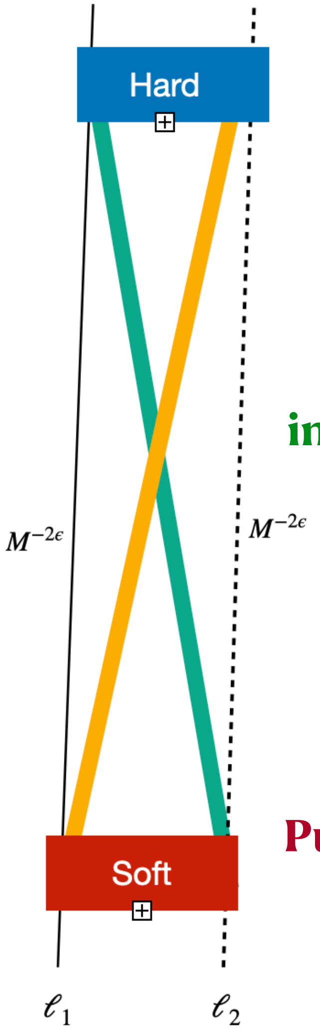
Two-loop UV renormalised parameters!

Chiral expansion of two-loop integrals

- **Dimensional counting analysis**
  - i.e. Equivalent to strategy of regions

[Gegelia, Japaridze and Turashvili, Theor. Math. Phys. 101(1994)]

[Beneke and Smirnov, NPB522(1998)]



Pure infrared regular part

[Local PCB term]

infrared regular-singular part

[Non-local PCB term]

Pure infrared singular part

[PCR-conserving term (IR)]

$$I_{\nu_1\nu_2\nu_3\nu_4\nu_5} = \int\!\!\int \frac{\mathrm{d}^d\ell_1}{(i\pi^{d/2})}\frac{\mathrm{d}^d\ell_2}{(i\pi^{d/2})}\frac{1}{\mathscr{D}_1^{\nu_1}\mathscr{D}_2^{\nu_2}\mathscr{D}_3^{\nu_3}\mathscr{D}_4^{\nu_4}\mathscr{D}_5^{\nu_5}}$$

$$\ell_i \longrightarrow M^{\alpha_i}\,\tilde{\ell}_i$$

$$I_{\nu_1\nu_2\nu_3\nu_4\nu_5} = \sum_{\alpha_1,\alpha_2} M^{\varphi(\alpha_1,\alpha_2)} \times I^{(\alpha_1,\alpha_2)}_{\nu_1\nu_2\nu_3\nu_4\nu_5}$$

$$(\alpha_1,\alpha_2) \in \{(0,0),(0,1),(1,0),(1,1)\}$$

$$I_{\nu_1\nu_2\nu_3\nu_4\nu_5} = I^{(0,0)}_{\dots} + M^{-2\epsilon}I^{(0,1)}_{\dots} + M^{-2\epsilon}I^{(1,0)}_{\dots} + M^{-4\epsilon}I^{(1,1)}_{\dots}$$

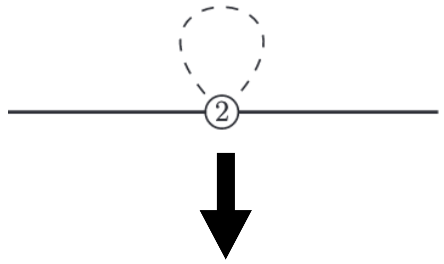
PCB renormalisation

- **EOMS scheme at two-loop:** absorption of PCB terms by LECs

Two-loop

non-local PCB term

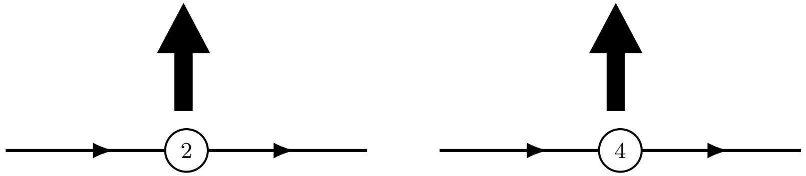
One-loop



$$X^r = \widetilde{X} + \delta_X^{(1)}\ , \quad X \in \{c_1, c_2, c_3\}$$

2-loop local PCB term

$$X^r = \widetilde{X} + \delta_X^{(21)} + \delta_X^{(22)}\ , \quad X \in \{m, c_1, e_m\}$$



Tree

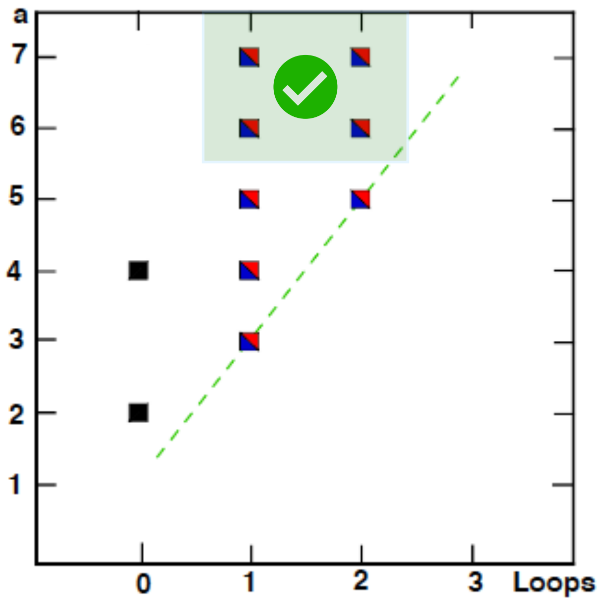
- $\delta_X^{(21)}$: cancel 1-loop local PCB term
- $\delta_X^{(22)}$: cancel 2-loop local PCB term

Final two-loop representation

- **The renormalised nucleon mass up to $\mathcal{O}(p^5)$**

$$m_N = \widetilde{m} - \underbrace{4\tilde{c}_1 M^2}_{\mathcal{O}(p^2)} + \underbrace{\bar{m}_N^{(1a)}}_{\mathcal{O}(p^3)} - \underbrace{2\tilde{e}_m M^4 + \bar{m}_N^{(1b)}}_{\mathcal{O}(p^4)} + \underbrace{\bar{m}_N^{(1c)} + 2\bar{m}_N^{(1d)} + \bar{m}_N^{2-\mathrm{loop}} + \bar{m}_N^{\mathrm{sub-diag.}}}_{\mathcal{O}(p^5)}$$

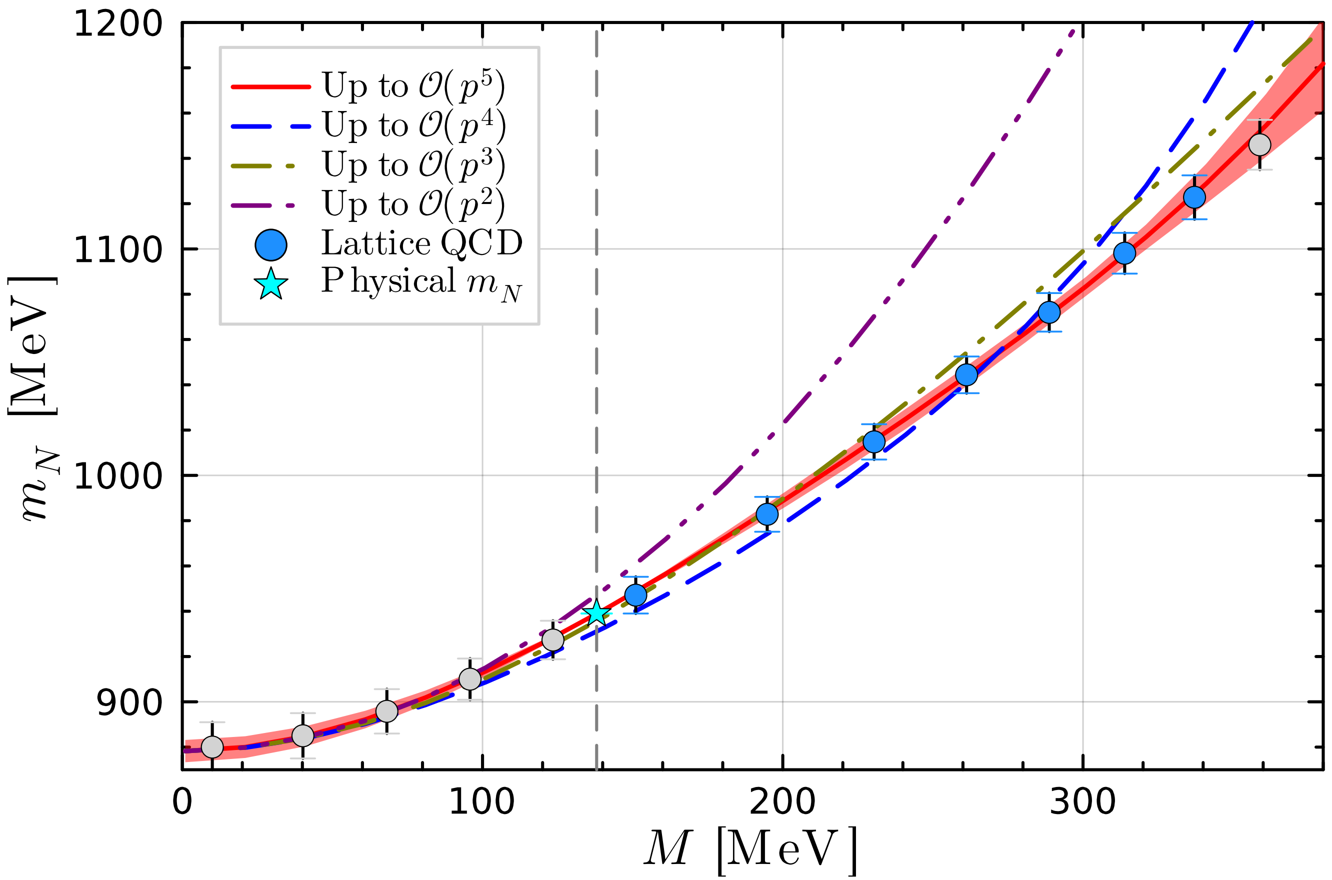
$$m_N^{\text{sub-diag.}} \sim \sum_X \left\{ \left(-\frac{\beta_X^{(1)}}{\epsilon\Lambda} \right) \times [\text{linear term in } \epsilon \text{ from one loops}] \right\}$$



- **Merits**
  - Faithful structure: respect original analytic property
  - Controllable accuracy: possess correct power counting rule
  - Renormalisation scale independent

Chiral extrapolation

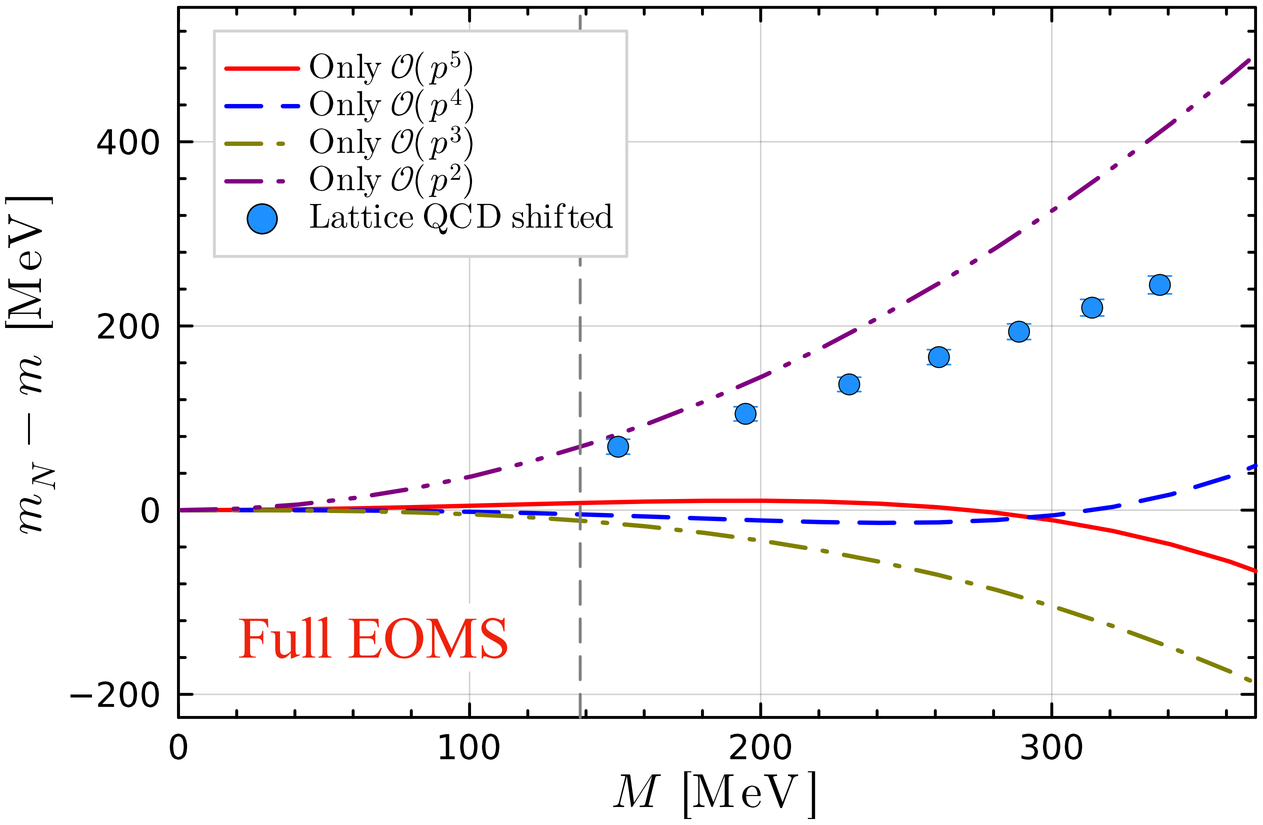
- Full EOMS



  - Original analyticity guarantees the successfulness of chiral extrapolation
  - Relativistic renormalisation scheme possesses good convergency

Convergency property

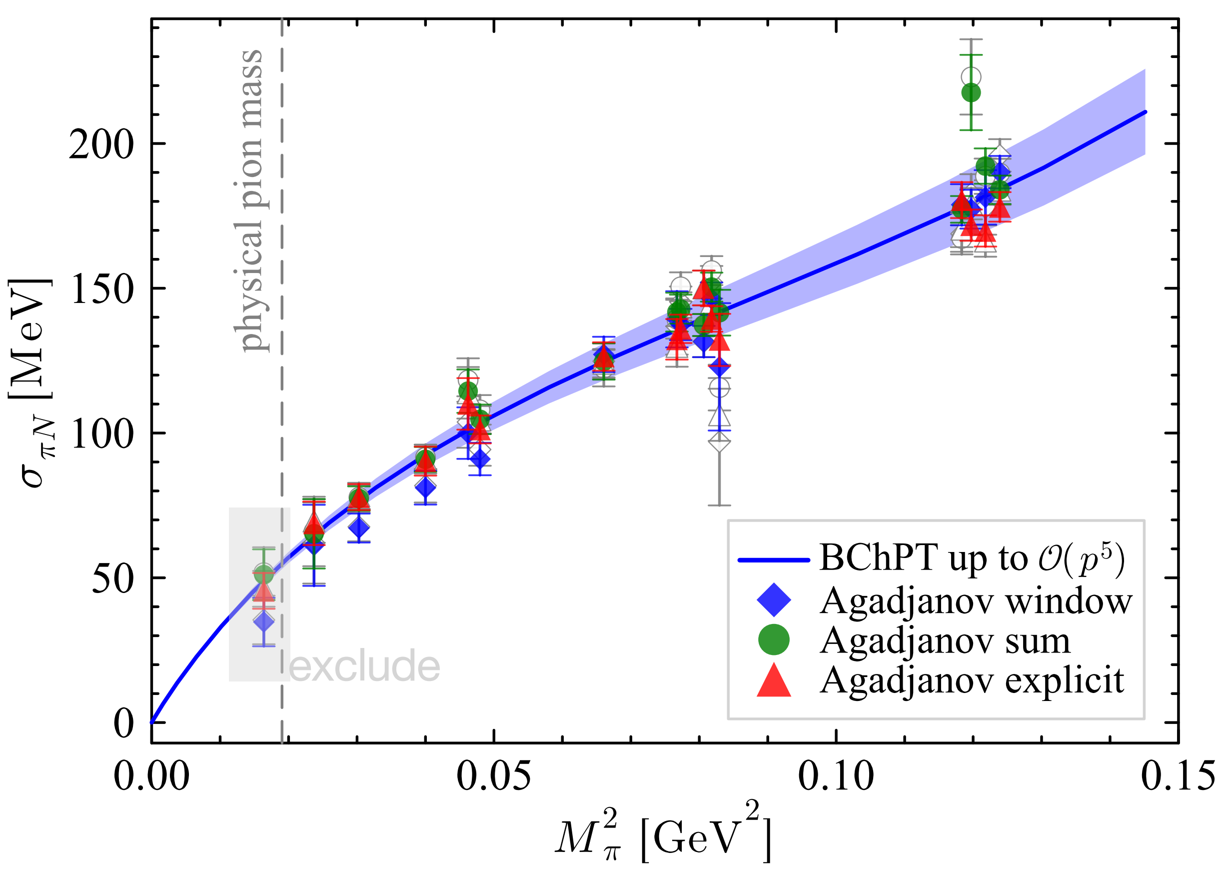
- Assess the convergency of the chiral expansion

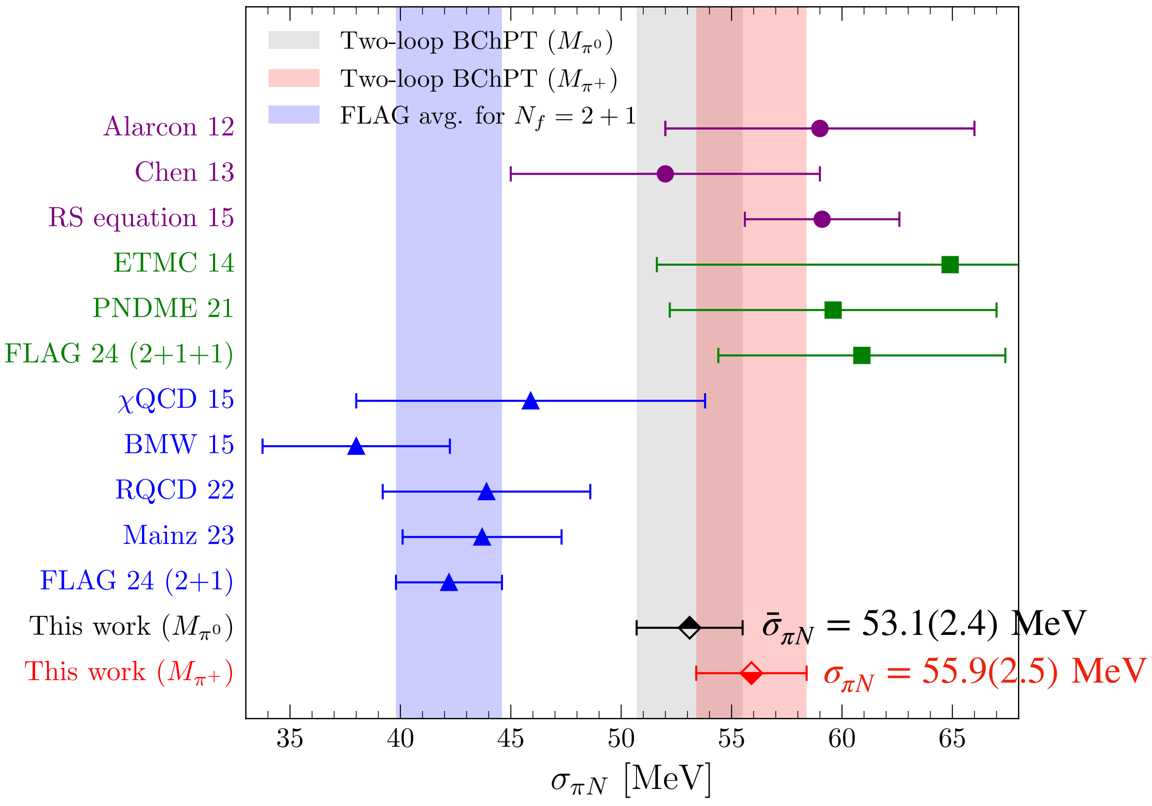


  - Two-loop contribution is small, approximately 10 MeV, for pion mass < 300 MeV;

Sigma term

- Sigma term via Feynman-Hellman theorem [Liang, Chen, Guo, Guo, and **DLY**, arXiv:2508.11435 [hep-ph]]





Resolving the tension between lattice QCD and phenomenology!

Alternative two-loop representation

- **EOMS truncated at $\mathcal{O}(p^5)$**

$$m_N = m + k_1 M^2 + k_2 M^3 + k_3 M^4 \ln\frac{M}{\mu} + k_4 M^4 + k_5 M^5 \ln\frac{M}{\mu} + k_6 M^5$$

$$k_1^{\text{EOMS}} = -4c_1 \,,$$

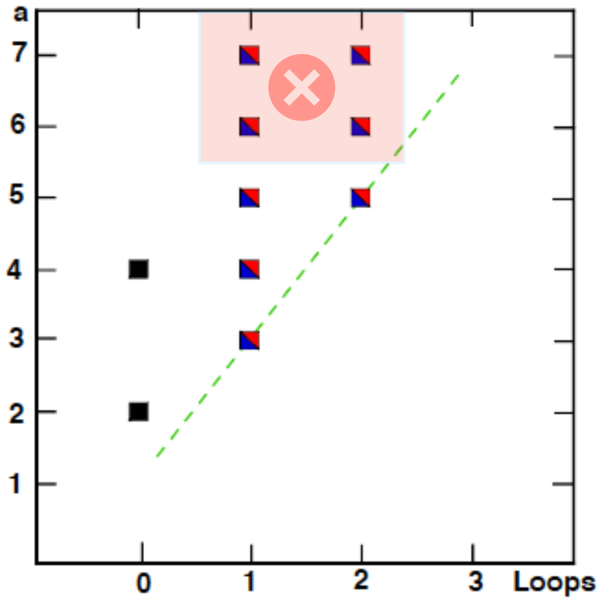
$$k_2^{\text{EOMS}} = -\frac{3g^2\pi}{2F^2\Lambda} \,,$$

$$k_3^{\text{EOMS}} = -\frac{3g^2}{2F^2 m\Lambda} + \frac{3(8c_1 - c_2 - 4c_3)}{2F^2\Lambda} \,,$$

$$k_4^{\text{EOMS}} = -2e_m + \frac{3g^2}{2F^2 m\Lambda} + \frac{3c_2}{8F^2\Lambda} \,,$$

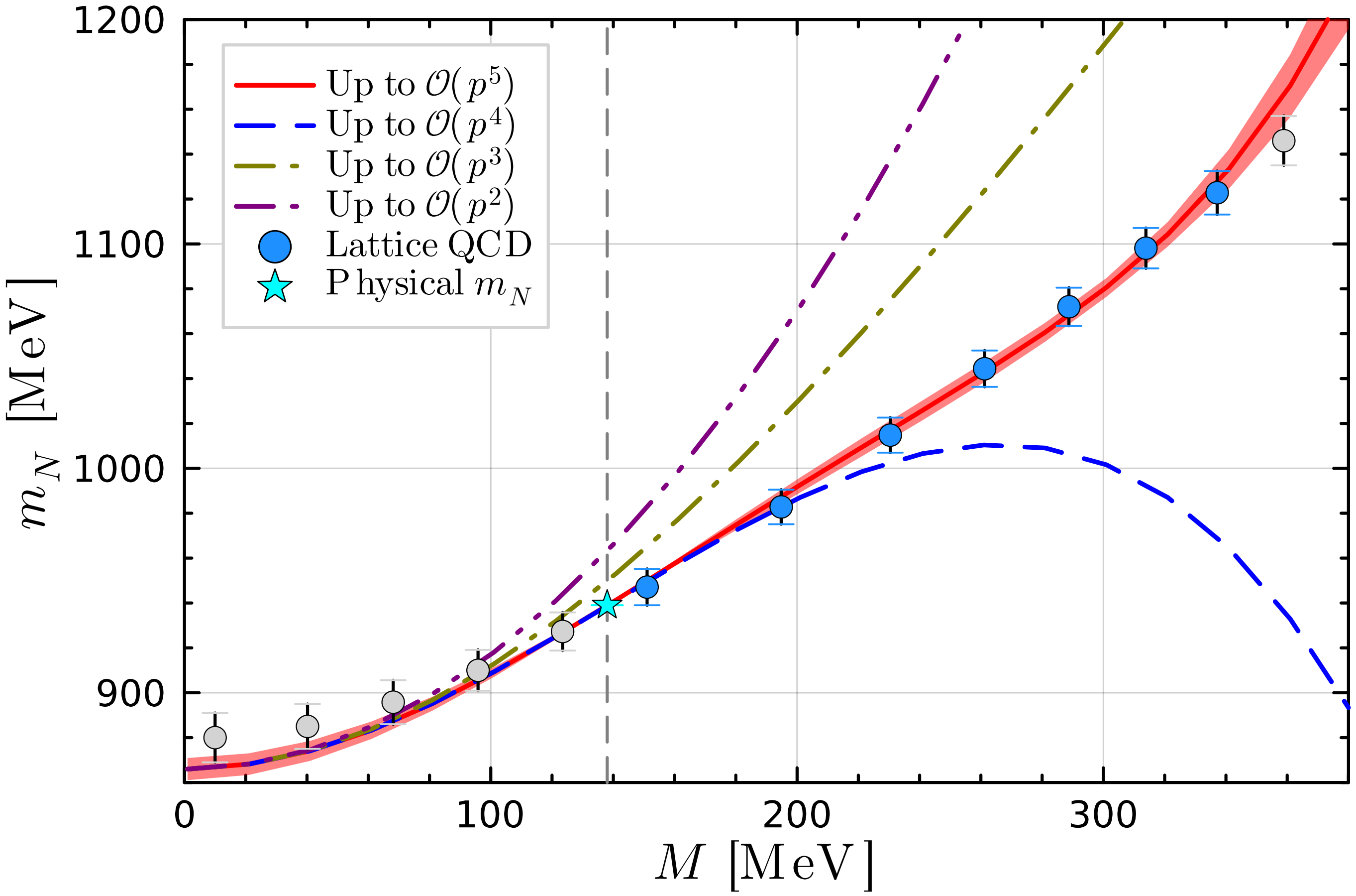
$$k_5^{\text{EOMS}} = \frac{3g^2(16g^2-3)\pi}{4F^4\Lambda^2} \,,$$

$$k_6^{\text{EOMS}} = \frac{19\pi g^4}{4F^4\Lambda^2} + \frac{6\pi g(d_{18}-2d_{16})}{F^2\Lambda} + g^2\left[\frac{6\pi}{F^4\Lambda^2} + \frac{3\pi\left(F^2 + 8m^2(2\ell_4 - 3\ell_3)\right)}{16F^4\Lambda m^2}\right] .$$



Chiral extrapolation

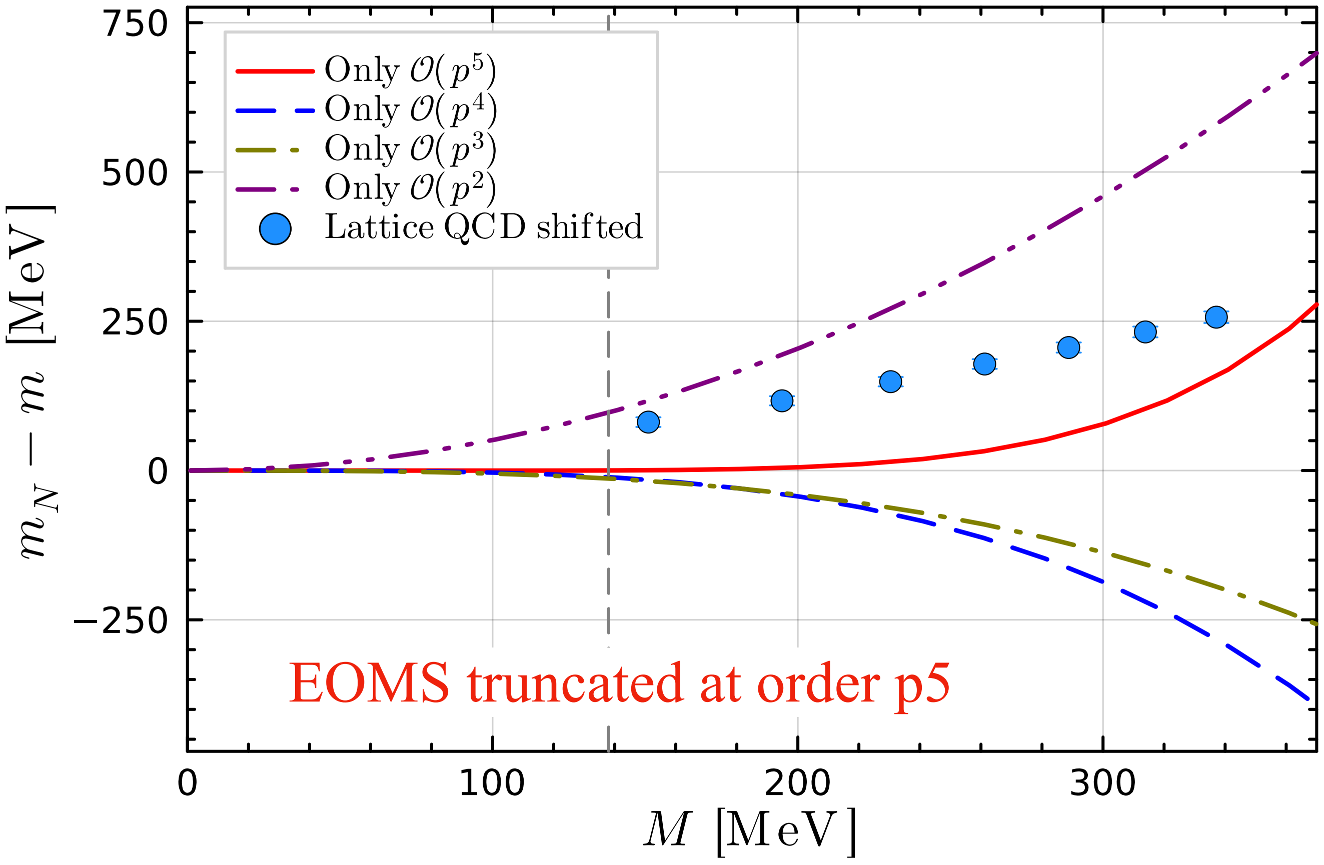
- EOMS truncated at order p5



  - Agree well in the fitting range, but deviate beyond
  - Weird behaviour of the pion mass dependence.

Convergency property

- Assess the convergency of the chiral expansion



  - Worse than full EOMS

Summary and outlook

- **Chiral representation of the nucleon mass in full-EOMS BChPT at two-loop order for the first time**
  - **Validity of EOMS at two-loop level:** the notable PCB issue addressed via dimensional counting analysis.
  - **Convergency:** the O(p5) contribution from the full EOMS is small around 10 MeV, implying that the chiral series in the full EOMS scheme converges very well.

$$\boxed{m_N = \{878.2 + \underbrace{68.8}_{\mathcal{O}(p^2)} + \underbrace{[-11.4]}_{\mathcal{O}(p^3)} + \underbrace{[-4.6]}_{\mathcal{O}(p^4)} + \underbrace{7.9}_{\mathcal{O}(p^5)}\}\ \text{MeV} = 938.9\ \text{MeV}}$$

  - **Comparison:** the EOMS-truncated and IR-truncated results change the analytic structure.

- **Outlook**
  - Provide reliable and high-order chiral expression for chiral extrapolation of lattice QCD data
  - Step into a promising era of two-loop BChPT, where the nucleon property can be explored with high precision!

Thanks!