Study of the $D_s \to \phi \ell \nu_\ell$ semileptonic decay with (2+1)-flavor lattice QCD

Gaofeng Fan (樊高峰)[1,2]

In Collaboration with: Yu Meng[3], Chuan Liu[4], Zhaofeng Liu[1], Tinghong Shen[5], Ting-Xiao Wang[1], Ke-Long Zhang[6], Lei Zhang[2]

[1]Institute of High Energy Physics, [2]Nanjing University, [3]Zhengzhou University, [4]Peking University, [5]Wuhan University, [6]Central China Normal University

CLQCD

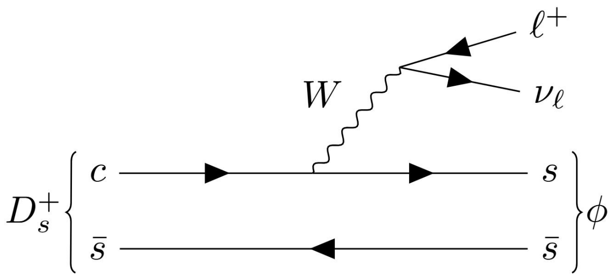
Based on arXiv:2510.xxxx

第五届中国格点量子色动力学研讨会

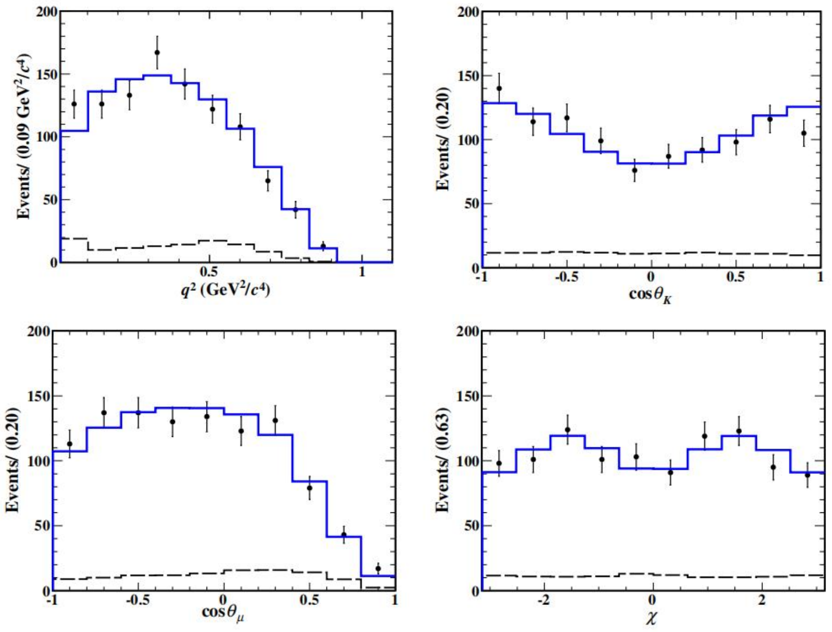
Outline

- Motivations
- Introduction to lattice QCD
- Lattice set up
- Methods
- Results
- Summary

Motivations



- Semi-leptonic decays offer an ideal place to deeply understand hadronic transitions in the nonperturbative region of QCD, and can help to explore the weak and strong interactions in charm sector
- Vector meson decay makes this transition difficult to model and bring additional polarization information
- Calculating branching fractions helps to test $\mu - e$ lepton flavor universality
- Combining with the experimental data, the CKM matrix element can be extracted, and it helps to test the unitarity of the CKM matrix and search for new physics beyond the SM



[BESIII, JHEP 12, 072 (2023)]

SM parameter

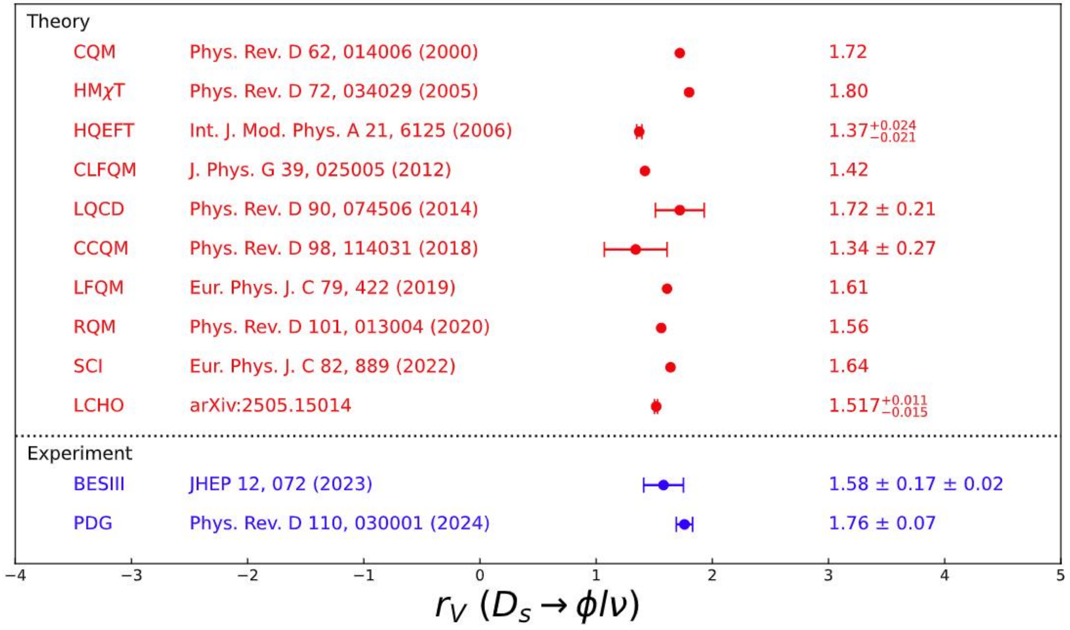
$$\boxed{\frac{d\Gamma(D_s \to \phi \ell \nu)}{dq^2}} = \frac{G_F^2 \boxed{|V_{cs}|^2} |\boldsymbol{p}_\phi| q^2}{96\pi^3 M_{D_s}^2} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) \boxed{\left(|H_+|^2 + |H_-|^2 + |H_0|^2\right)} + \frac{3m_\ell^2}{2q^2} \boxed{|H_t|^2}\right]$$

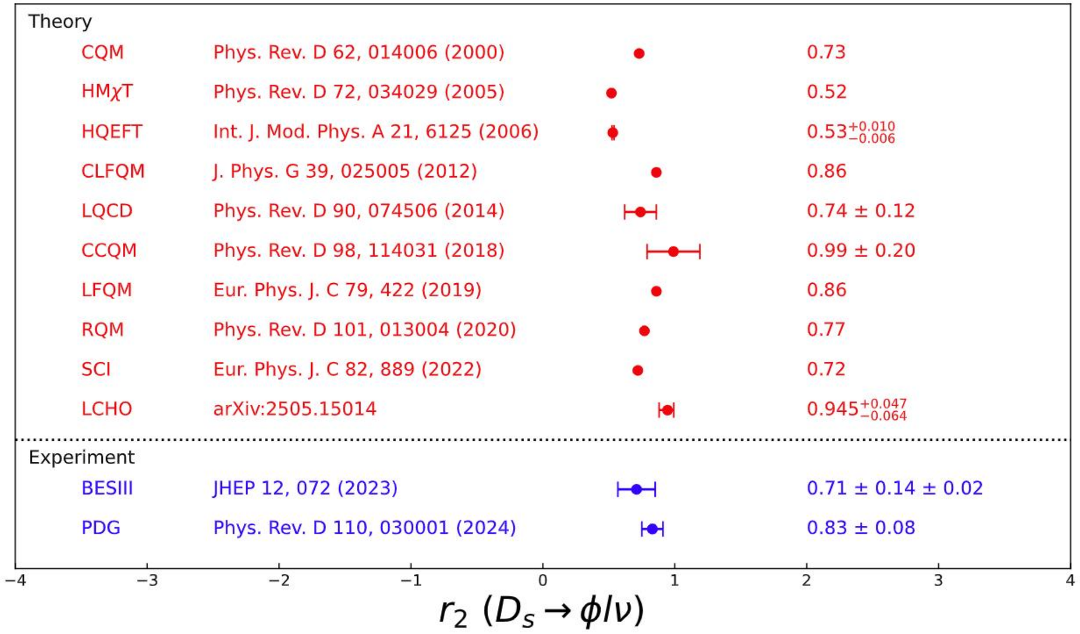
Experimental data

non-perturbative

Motivations

- Status of theoretical and experimental studies





A precise lattice calculation is important!

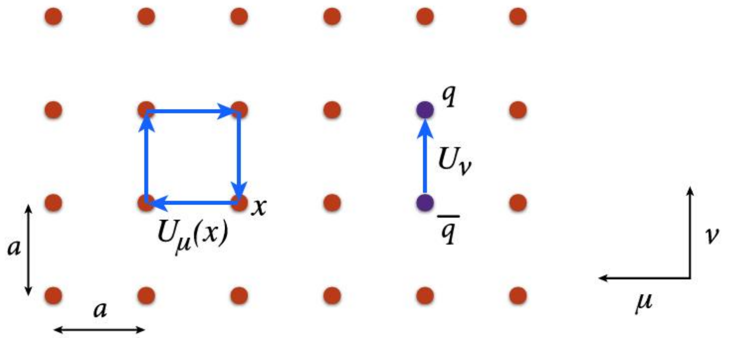
- Provide lattice QCD input to investigate the SU(3) symmetry (by combining with $D \to K^* l\nu$ calculation)

Introduction to lattice QCD

- Path integral in discrete Euclidean space

$$Z = \int [dU] \prod_f [dq_f][d\bar{q}_f] e^{-S_g[U] - \sum_f \bar{q}_f (D[U]+m_f) q_f}$$

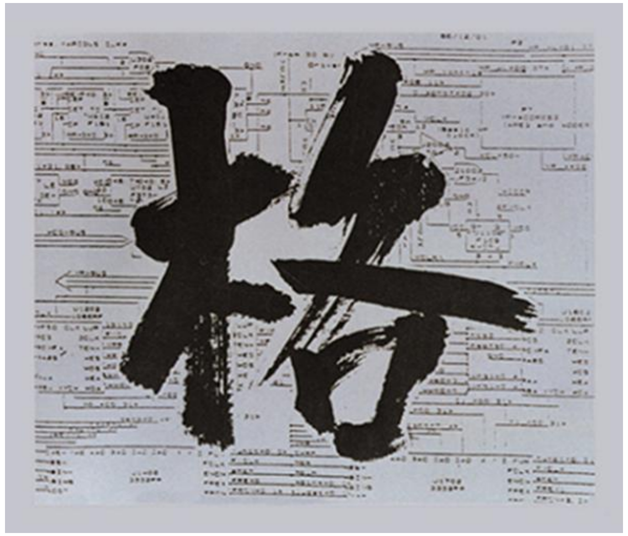
$$Z = \int [dU] e^{-S_g[U]} \prod_f \det(D[U] + m_f)$$



- Expectation values of gauge-invariant operators, also known as “correlation functions”

$$\langle \mathcal{O}(U,q,\bar{q}) \rangle = (1/Z) \int [dU] \prod_f [dq_f][d\bar{q}_f] \mathcal{O}(U,q,\bar{q}) e^{-S_g[U] - \sum_f \bar{q}_f (D[U]+m_f) q_f}$$

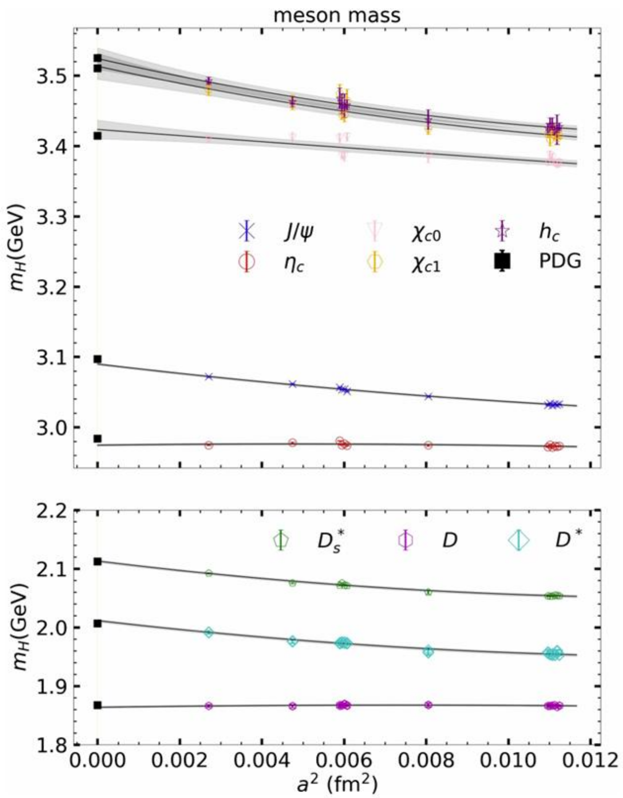
- Monte-Carlo method and data analysis



Lattice set up

- (2+1)-flavor Wilson-clover gauge ensembles [CLQCD, PRD 111, 054504 (2025)]
- Computer resources: “SongShan” supercomputer at Zhengzhou University
- Systematic calculation: four lattice spacing and four pion mass

	C24P29	C32P23	C32P29	F32P30	F48P21	G36P29	H48P32
a (fm)		0.10524(05)(62)		0.07753(03)(45)		0.06887(12)(41)	0.05199(08)(31)
am_l	−0.2770	−0.2790	−0.2770	−0.2295	−0.2320	−0.2150	−0.1850
am_s	−0.2400	−0.2400	−0.2400	−0.2050	−0.2050	−0.1926	−0.1700
am_s^{V}	−0.2356(1)	−0.2337(1)	−0.2358(1)	−0.2038(1)	−0.2025(1)	−0.1928(1)	−0.1701(1)
am_c^{V}	0.4159(07)	0.4190(07)	0.4150(06)	0.1974(05)	0.1997(04)	0.1433(12)	0.0551(07)
L (fm)	2.53	3.37	3.37	2.48	3.72	2.48	2.50
$L^3 \times T$	$24^3 \times 72$	$32^3 \times 64$	$32^3 \times 64$	$32^3 \times 96$	$48^3 \times 96$	$36^3 \times 108$	$48^3 \times 144$
N_{mea}	$450 \times 72 \times 2$	$333 \times 64 \times 3$	$397 \times 64 \times 2$	$360 \times 96 \times 2$	$241 \times 48 \times 4$	$300 \times 54 \times 2$	$300 \times 72 \times 2$
m_π (MeV/c^2)	292.3(1.0)	227.9(1.2)	293.1(0.8)	300.4(1.2)	207.5(1.1)	297.2(0.9)	316.6(1.0)
t	2 − 17	2 − 20	2 − 20	4 − 22	4 − 26	2 − 32	8 − 30
Z_V^s	0.85184(06)	0.85350(04)	0.85167(04)	0.86900(03)	0.86880(02)	0.87473(05)	0.88780(01)
Z_V^c	1.57353(18)	1.57644(12)	1.57163(14)	1.30566(07)	1.30673(04)	1.23990(13)	1.12882(11)
Z_A/Z_V	1.07244(70)	1.07375(40)	1.07648(63)	1.05549(54)	1.05434(88)	1.04500(22)	1.03802(28)



Methods (correlation function formulism)

- 2-point correlation function (2pt),

$$C^{(2)}(\vec{p}, t) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle \mathcal{O}_h(\vec{x}, t) \mathcal{O}_h^\dagger(0) \rangle$$

- 3-point correlation function (3pt) ,

$$\begin{aligned} C_{\mu\nu}(\vec{x}, t, t_s) &= \langle \mathcal{O}_{\phi_\nu}(t) J_\mu^W(0) \mathcal{O}_{D_s}^\dagger(-t_s) \rangle \\ &= \langle \bar{s}(t)\gamma_\nu s(t) \bar{s}(0)\gamma_\mu(1-\gamma_5)c(0)\bar{c}(-t_s)\gamma_5 s(-t_s) \rangle \\ &= \langle \mathrm{Tr}[\gamma_5\gamma_5 S_{-s}^\dagger(t, -t_s)\gamma_5\gamma_\nu S_s(t, 0)\gamma_\mu(1-\gamma_5)S_c(0, -t_s)] \rangle \end{aligned}$$

- Treat ϕ as stable: small width and $m_\phi - 2m_K < 0$ at heavier pion mass

$\phi(1020)$	$I^G(J^{PC}) = 0^-(1^{--})$
	Expand/Collapse All
$\phi(1020)$ MASS	1019.460 ± 0.016 MeV
$\phi(1020)$ WIDTH	4.249 ± 0.013 MeV (S = 1.1)

Methods (scalar function)

- The parameterization for $P \to V$ semileptonic matrix element

$$\langle \phi_\sigma(\vec{p}) | J_\mu^W(0) | D_s(p') \rangle = \frac{F_0(q^2)}{Mm} \epsilon_{\mu\sigma\alpha\beta} p'_\alpha p_\beta + F_1(q^2) \delta_{\mu\sigma} + \frac{F_2(q^2)}{Mm} p_\mu p'_\sigma + \frac{F_3(q^2)}{M^2} p'_\mu p'_\sigma$$

$$\langle \phi(\varepsilon, \vec{p}) | J_\mu^W(0) | D_s(p') \rangle = \varepsilon_\nu^* \varepsilon_{\mu\nu\alpha\beta} p'_\alpha p_\beta \frac{2V}{m+M} + (M+m) \varepsilon_\mu^* A_1 + \frac{\varepsilon^* \cdot q}{M+m} (p+p')_\mu A_2 - 2m \frac{\varepsilon^* \cdot q}{q^2} q_\mu (A_0 - A_3)$$

- Correlation functions $\longrightarrow$ Scalar functions $\longrightarrow$ Form factors

$\langle \phi_\sigma(\vec{p}) | J_\mu^W(0) | D_s(p') \rangle$ $\qquad$ $\tilde{\mathcal{I}}_j$ $\qquad$ V, A_0, A_1, A_2

- Relationship with the form factor

$$\begin{aligned} V &= \frac{(m+M)}{2mM} F_0, \\ A_1 &= \frac{F_1}{M+m}, \\ A_2 &= \frac{M+m}{2mM^2} (MF_2 + mF_3), \\ A_0 &= \frac{F_1}{2m} + \frac{m^2 - M^2 + q^2}{4m^2 M} F_2 + \frac{m^2 - M^2 - q^2}{4mM^2} F_3. \end{aligned}$$

- A_3 is not an independent form factor

$$A_3(q^2) = \frac{M+m}{2m} A_1(q^2) - \frac{M-m}{2m} A_2(q^2)$$

- $A_3(0) = A_0(0)$ is the kinematic constraint

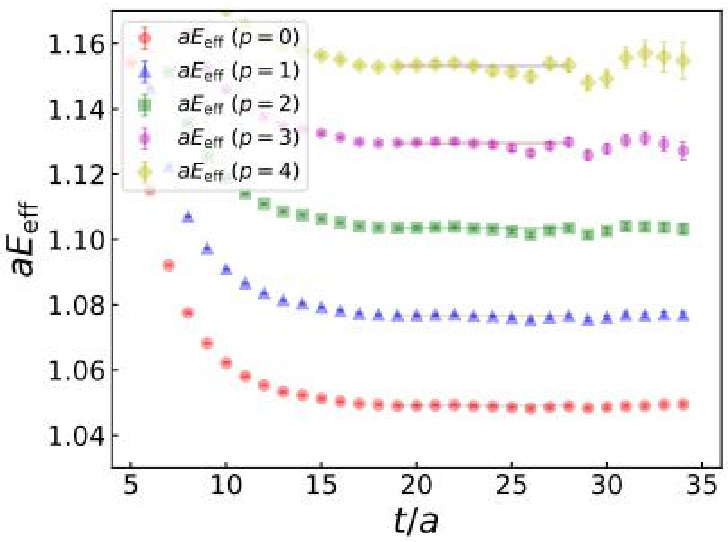
Methods (scalar function)

- A similar scalar function scheme has been used for high-precision calculation
  - $\Gamma(\eta_c \to 2\gamma) = 6.67(16)_{\mathrm{stat}}(6)_{\mathrm{syst}}\ \mathrm{keV}$ [Y. M et al, Science Bulletin 68, 1880 (2023)]
  - $\Gamma(D_s^* \to \gamma D_s) = 0.0549(54)\ \mathrm{keV}$ [Y. M et al, PRD 109, 074511 (2024)]
  - $\mathrm{Br}(J/\psi \to D e v_e) = 1.21(11) \times 10^{-11}$
    $\mathrm{Br}(J/\psi \to D \mu v_\mu) = 1.18(11) \times 10^{-11}$
    $\mathrm{Br}(J/\psi \to D_s e v_e) = 1.90(8) \times 10^{-10}$
    $\mathrm{Br}(J/\psi \to D_s \mu v_\mu) = 1.84(8) \times 10^{-10}$ [Y. M et al, PRD 110, 074510 (2024)]
  - $\mathrm{Br}(J/\psi \to \gamma \eta_c) = 2.49(11)_{\mathrm{lat}}(5)_{\mathrm{exp}}\%$ [Y. M et al, PRD 111, 014508 (2025)]

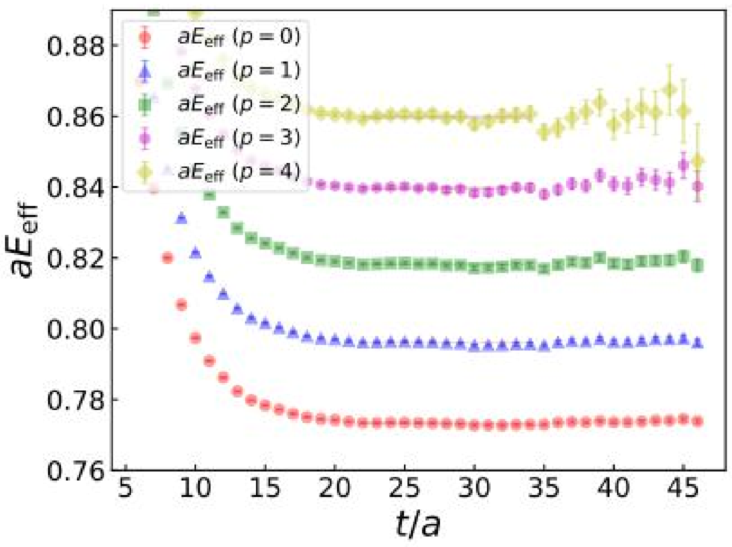
Results (2-point function fitting)

- Least χ^2 fitting considering covariance matrix between configurations and time
- There should be a plateau when meson ground states are dominant

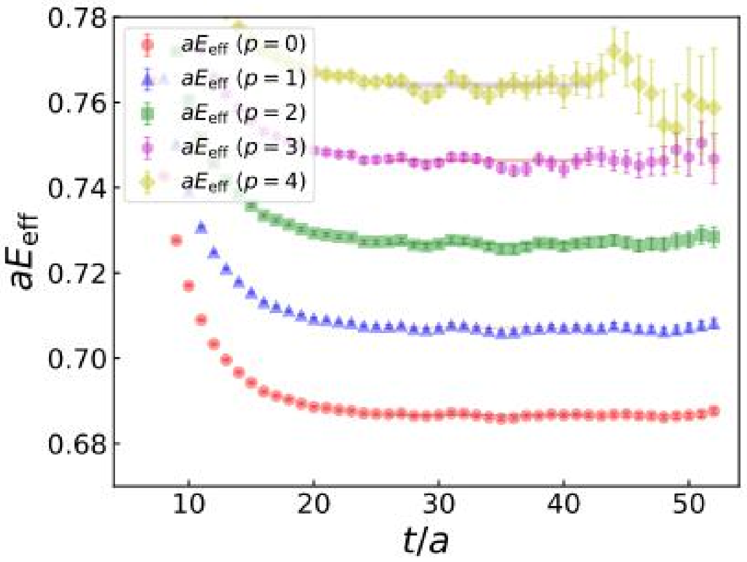
$$C^{(2)}(\vec{p}, t) = \frac{Z_h^2}{2E_h}\left(e^{-E_h t} + e^{-E_h(T-t)}\right)$$



(i) For C24P29 D_s meson.



(i) For F32P30 D_s meson.



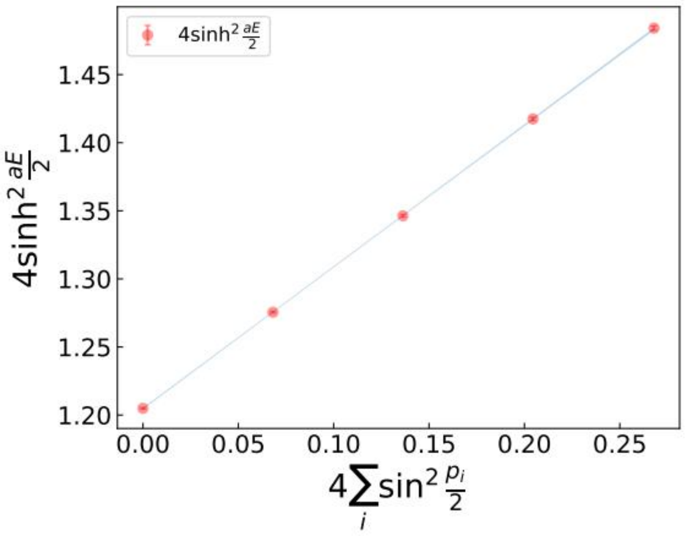
(i) For G36P29 D_s meson.

Results (dispersion relation)

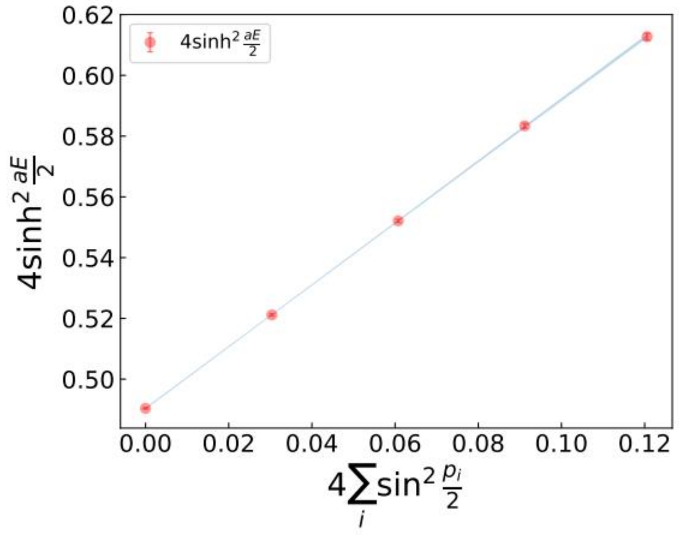
- We checked the dispersion relation of $\boldsymbol{D_s}$ meson at different momenta
- Use a discrete dispersion relation as the fitting function

$$4\sinh^2\frac{E_h}{2} = 4\sinh^2\frac{m_h}{2} + \mathcal{Z}^h_{\text{latt}}\cdot 4\sum_i \sin^2\frac{p_i}{2}$$

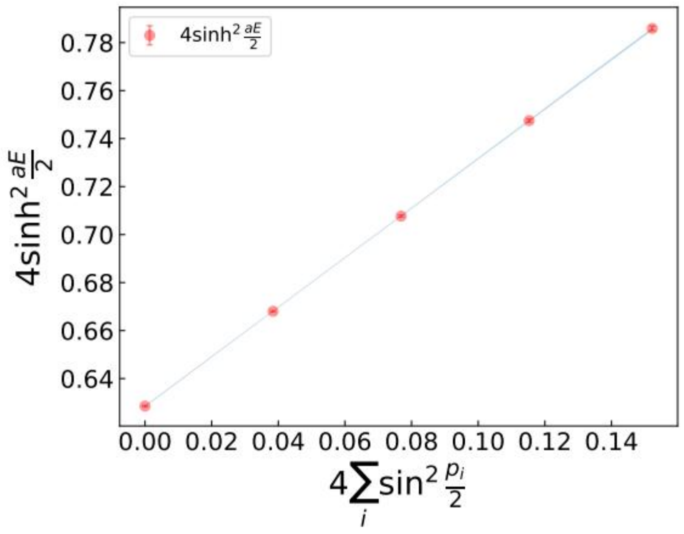
$\mathcal{Z}_{\text{latt}}$ is $1.0402(48), 1.0389(72), 1.0346(75), 1.0324(45), 1.0276(92), 1.0168(62), 1.0334(58)$



(ii) For C24P24 D_s meson.



(ii) For G36P29 D_s meson.

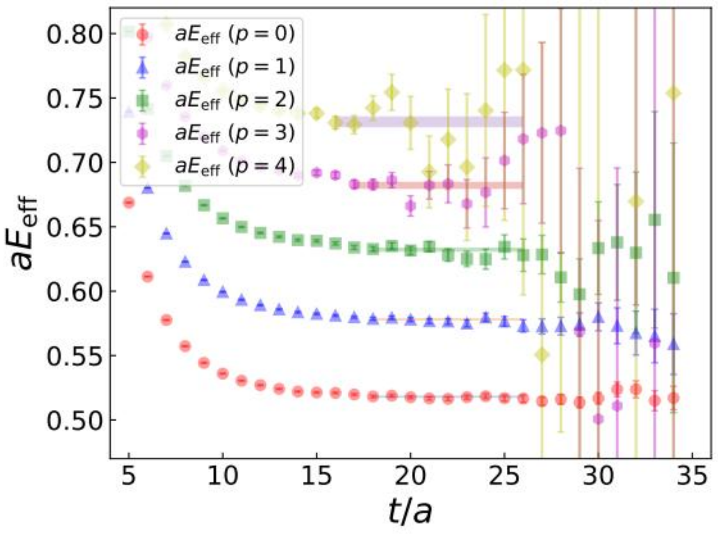


(ii) For F32P30 D_s meson.

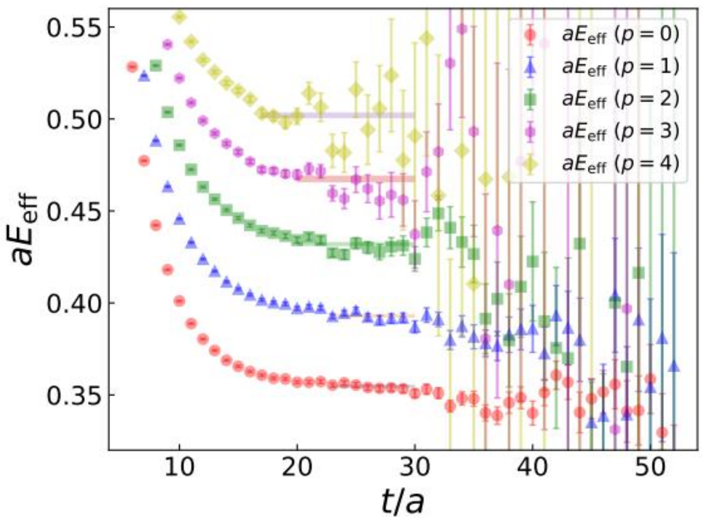
Results (2-point function fitting)

- Least χ^2 fitting considering covariance matrix between configurations and time
- There should be a plateau when meson ground states are dominant

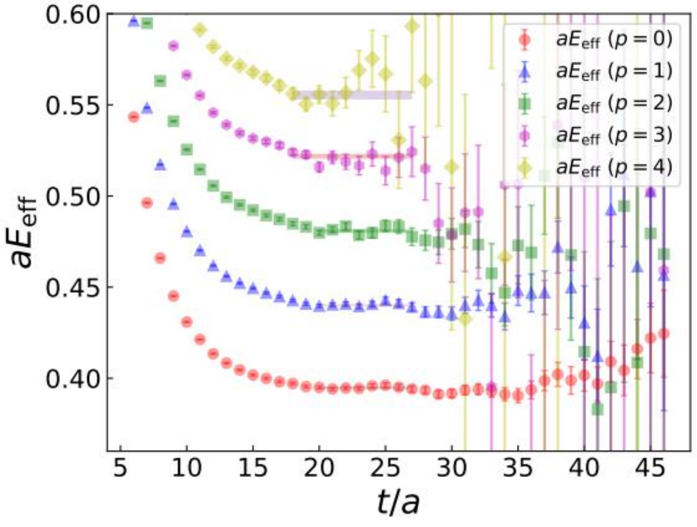
$$C^{(2)}(\vec{p}, t) = \left(-1 - \frac{|\vec{p}|^2}{3m_h^2}\right) \frac{Z_h^2}{2E_h} \left[e^{-E_h t} + e^{-E_h (T-t)}\right]$$



(iii) For C24P29 ϕ meson.



(iii) For G36P29 ϕ meson.



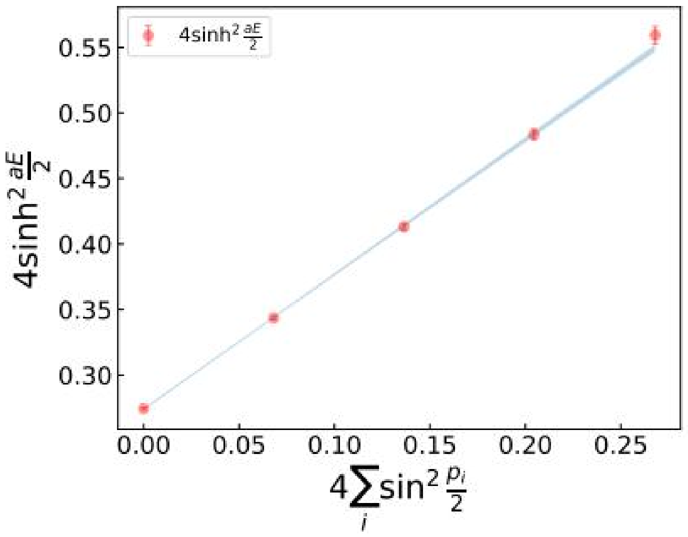
(iii) For F32P30 ϕ meson.

Results (dispersion relation)

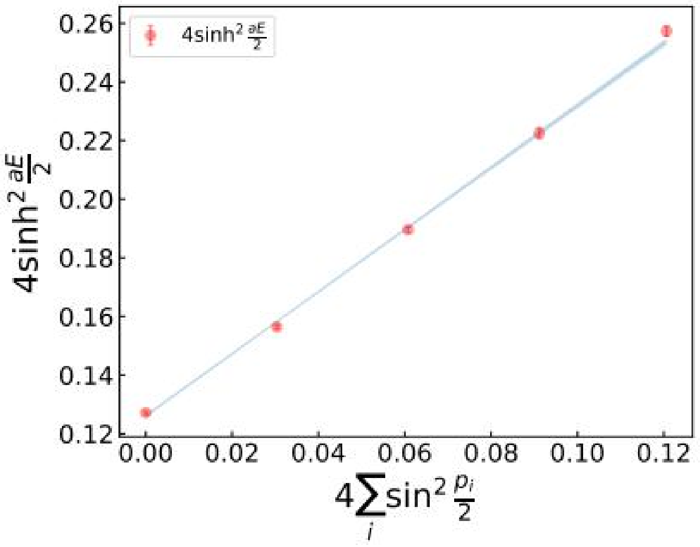
- We checked the dispersion relation of ϕ meson at different momenta
- Use a discrete dispersion relation as the fitting function

$$4\sinh^2\frac{E_h}{2} = 4\sinh^2\frac{m_h}{2} + \mathcal{Z}^h_{\mathrm{latt}}\cdot 4\sum_i \sin^2\frac{p_i}{2}$$

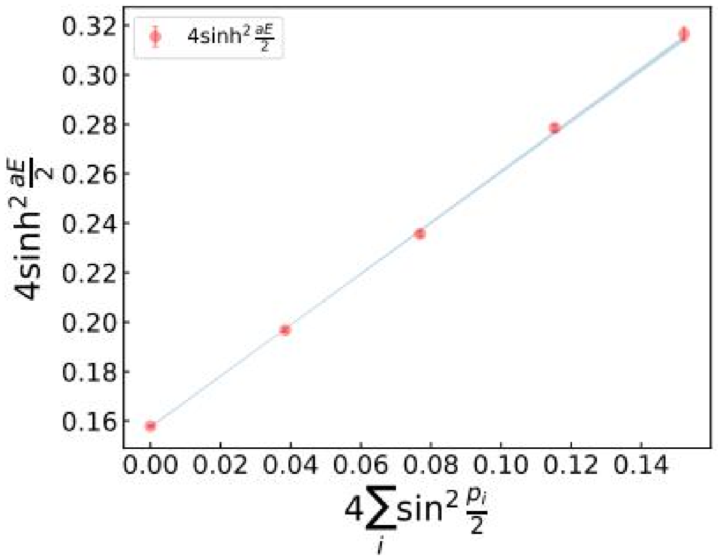
$\mathcal{Z}_{\mathrm{latt}}$ is 1.027(12), 1.042(13), 1.021(13), 1.0324(93), 1.061(15), 1.058(11), 1.057(13)



(iv) For C24P29 ϕ meson.



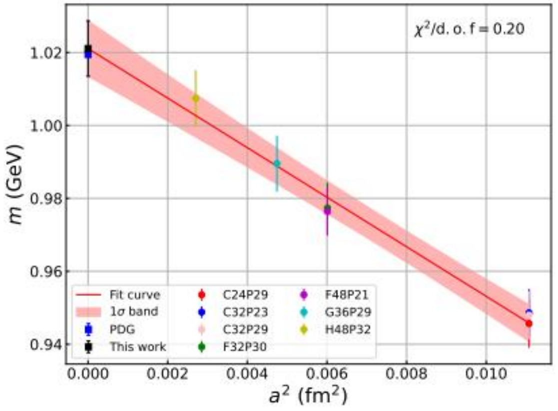
(iv) For G36P29 ϕ meson.



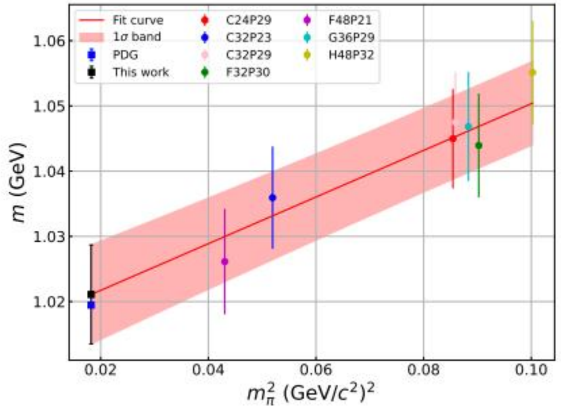
(iv) For F32P30 ϕ meson.

Results (mass and decay constant)

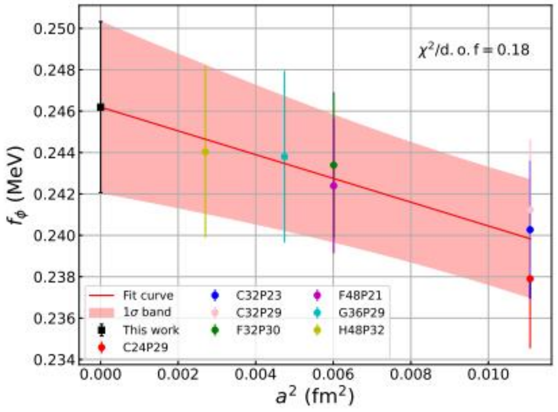
- We extrapolate masses and decay constants of $\boldsymbol{\phi}$ meson to get physical results



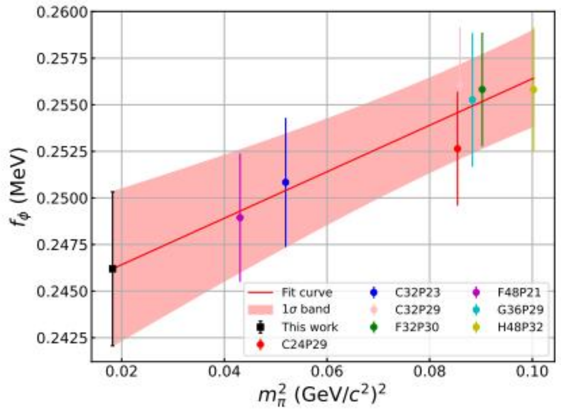
(i) ϕ mass at $m_\pi = 0.135$ GeV/c^2.



(ii) ϕ mass at $a = 0.0$ fm.



(iii) ϕ decay constant at $m_\pi = 0.135$ GeV/c^2.



(iv) ϕ decay constant at $a = 0.0$ fm.

$$m/f_\phi = c + da^2 + f\left(m_\pi^2 - m_{\pi,\text{phys}}^2\right)$$

$$m = 1.0211(76)\ \text{GeV}/c^2$$

$$f_\phi = 0.2462(41)\ \text{GeV}$$

Results (global fit)

- Extrapolate results to the physical pion mass and continuum limit using z-expansion

$$z\left(q^2, t_0\right) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

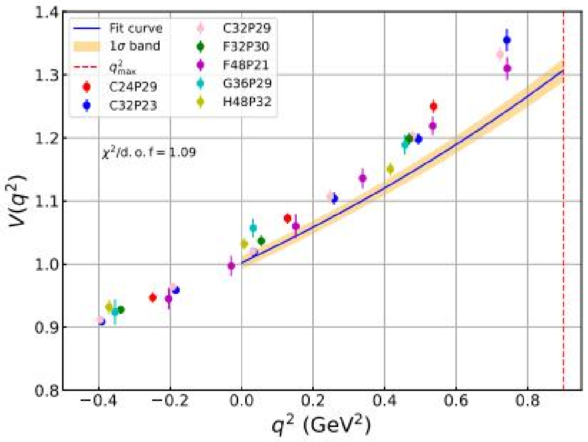
where $t_+ = \left(m_{D_s} + m_\phi\right)^2,\ t_0 = 0$

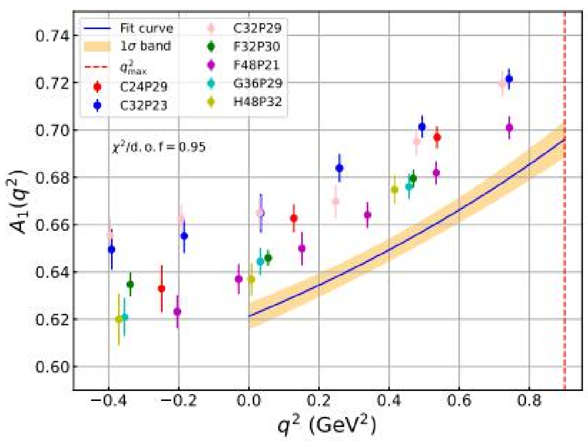
$$V\left(q^2, a, m_\pi\right) = \frac{1}{1 - q^2/m_{D_s^*}^2} \sum_{i=0}^{2} \left(c_i + d_i a^2\right)\left[1 + f_i\left(m_\pi^2 - m_{\pi,\mathrm{phys}}^2\right)\right] z^i$$

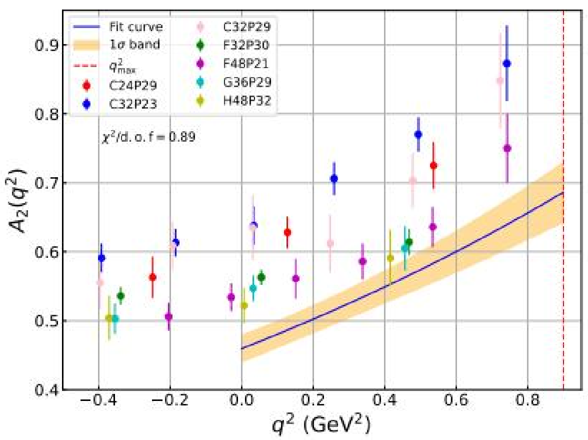
$$A_{0,1,2}\left(q^2, a, m_\pi\right) = \frac{1}{1 - q^2/m_{D_{s1}}^2} \sum_{i=0}^{2} \left(c_i + d_i a^2\right)\left[1 + f_i\left(m_\pi^2 - m_{\pi,\mathrm{phys}}^2\right)\right] z^i$$

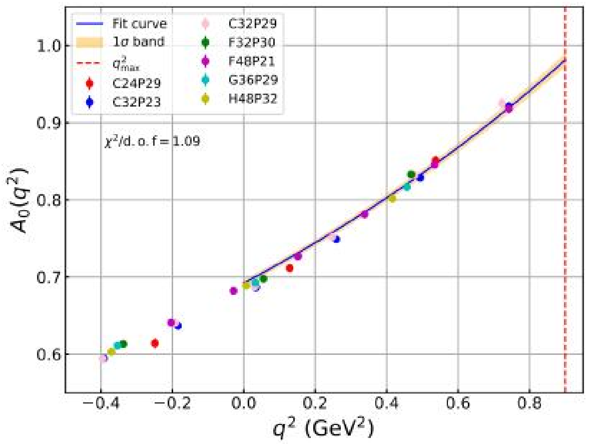
$$m_{\pi,\mathrm{phys}}^2 = 135.0\ \mathrm{MeV}/c^2,\ m_{D_s^*} = 2112.2\ \mathrm{MeV}/c^2,\ m_{D_{s1}} = 2459.5\ \mathrm{MeV}/c^2$$

- $A_3(0) - A_0(0) = 0.004(12)$, consistent with zero



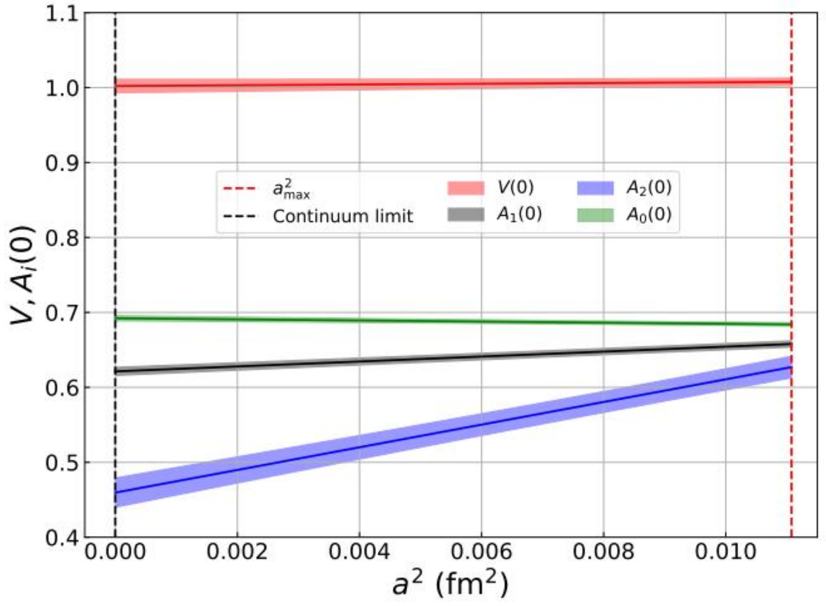




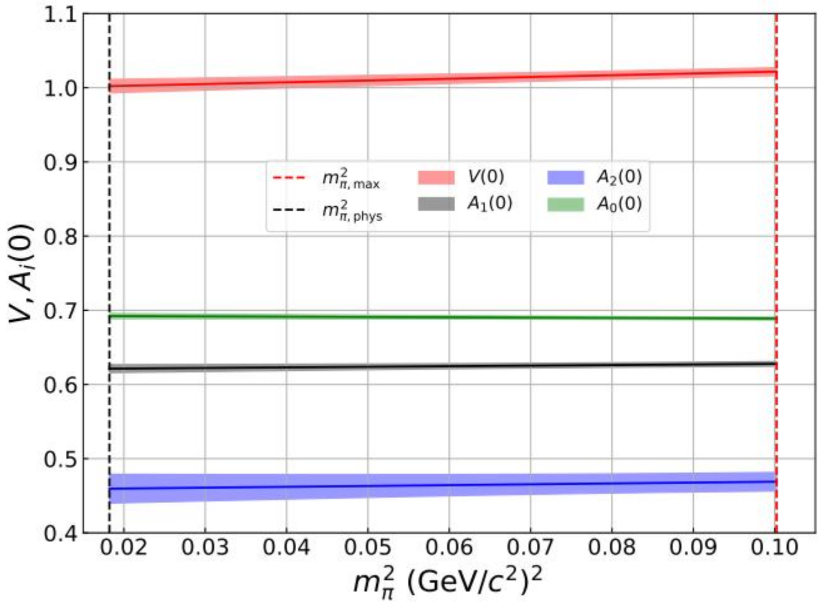


Results (a^2 and m_π^2 dependence)

- The form factors can be described well by the a^2-order term and the m_π^2-order term



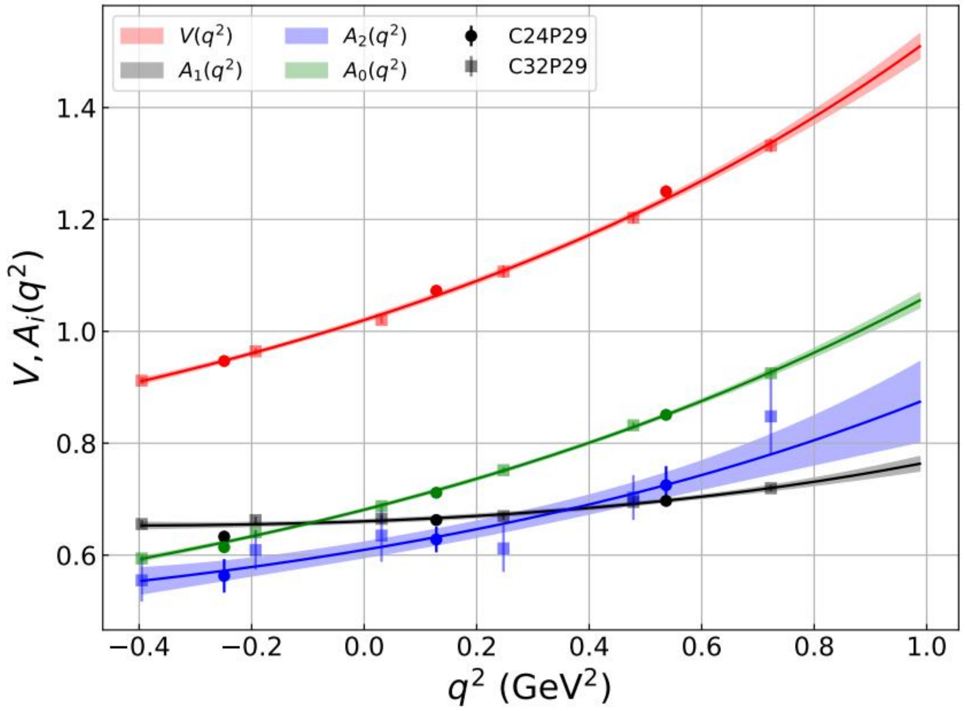
(i) Form factors at $m_\pi = 0.135$ GeV$/c^2$.



(ii) Form factors at $a = 0.0$ fm.

Results (finite-volume effect)

- The values of joint fitting are well consistent with the other two individual fittings within 1σ



C24P29, L=2.53 fm

C32P29, L=3.37 fm

	C24P29	χ^2/d.o.f	C32P29	χ^2/d.o.f	Combined	χ^2/d.o.f
$V(0)$	1.0271(66)	0.03	1.0167(43)	0.29	1.0202(36)	0.63
$A_1(0)$	0.6523(55)	0.01	0.6639(37)	0.24	0.6605(30)	0.82
$A_2(0)$	0.605(18)	0.01	0.616(20)	0.53	0.609(14)	0.38
$A_0(0)$	0.6759(45)	0.07	0.6835(29)	0.26	0.6811(24)	0.54

Results (parameterization schemes)

- Different parameterization schemes are consistent with each other within $1-2\sigma$

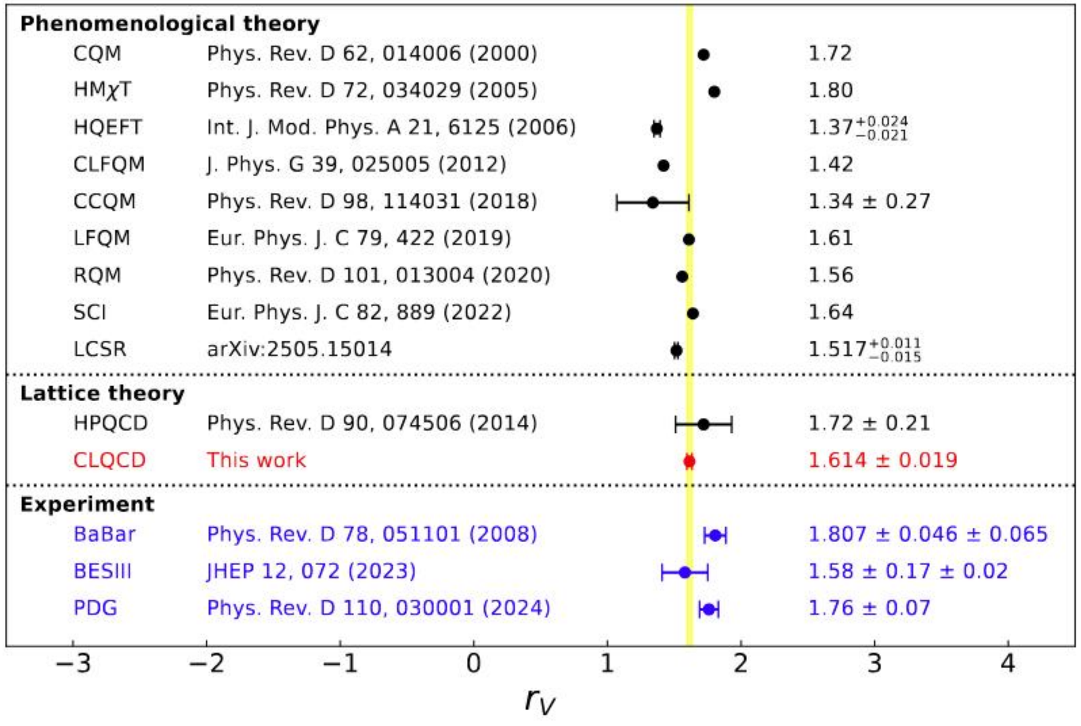
single pole
$$F\left(q^2, a, m_\pi\right)=\frac{1}{1-q^2/h^2}\left(c+da^2\right)\left[1+f\left(m_\pi^2-m_{\pi,\mathrm{phys}}^2\right)\right]$$

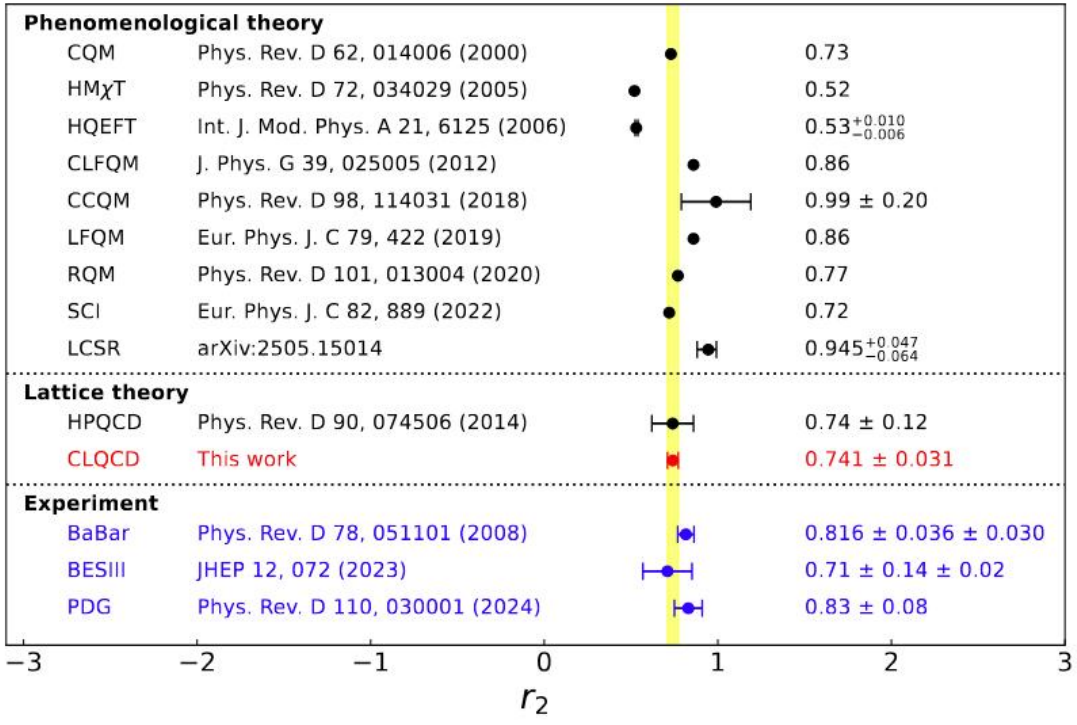
modified pole
$$F\left(q^2, a, m_\pi\right)=\frac{1}{\left(1-q^2/m_{\mathrm{pole}}^2\right)\left(1-hq^2/m_{\mathrm{pole}}^2\right)}\left(c+da^2\right)\left[1+f\left(m_\pi^2-m_{\pi,\mathrm{phys}}^2\right)\right]$$

	z-expansion	Single pole	Modified pole
$V(0)$	1.002(9)	1.002(9)	1.004(9)
$A_1(0)$	0.621(5)	0.624(4)	0.624(4)
$A_2(0)$	0.460(19)	0.470(19)	0.471(19)
$A_0(0)$	0.692(4)	0.688(3)	0.689(03)
$A_3(0)-A_0(0)$	0.004(12)	0.008(11)	0.006(11)
r_V	1.614(19)	1.606(18)	1.609(18)
r_2	0.741(31)	0.753(31)	0.755(31)
r_0	1.114(11)	1.1026(85)	1.1042(86)
$\mathcal{B}(D_s\to\phi\mu\nu_\mu)\times 10^2$	2.351(67)	2.367(54)	2.362(50)
$\mathcal{B}(D_s\to\phi e\nu_e)\times 10^2$	2.493(73)	2.511(59)	2.504(55)
$\mathcal{R}_{\mu/e}$	0.9432(13)	0.9427(12)	0.9431(12)

Results (r_V and r_2)

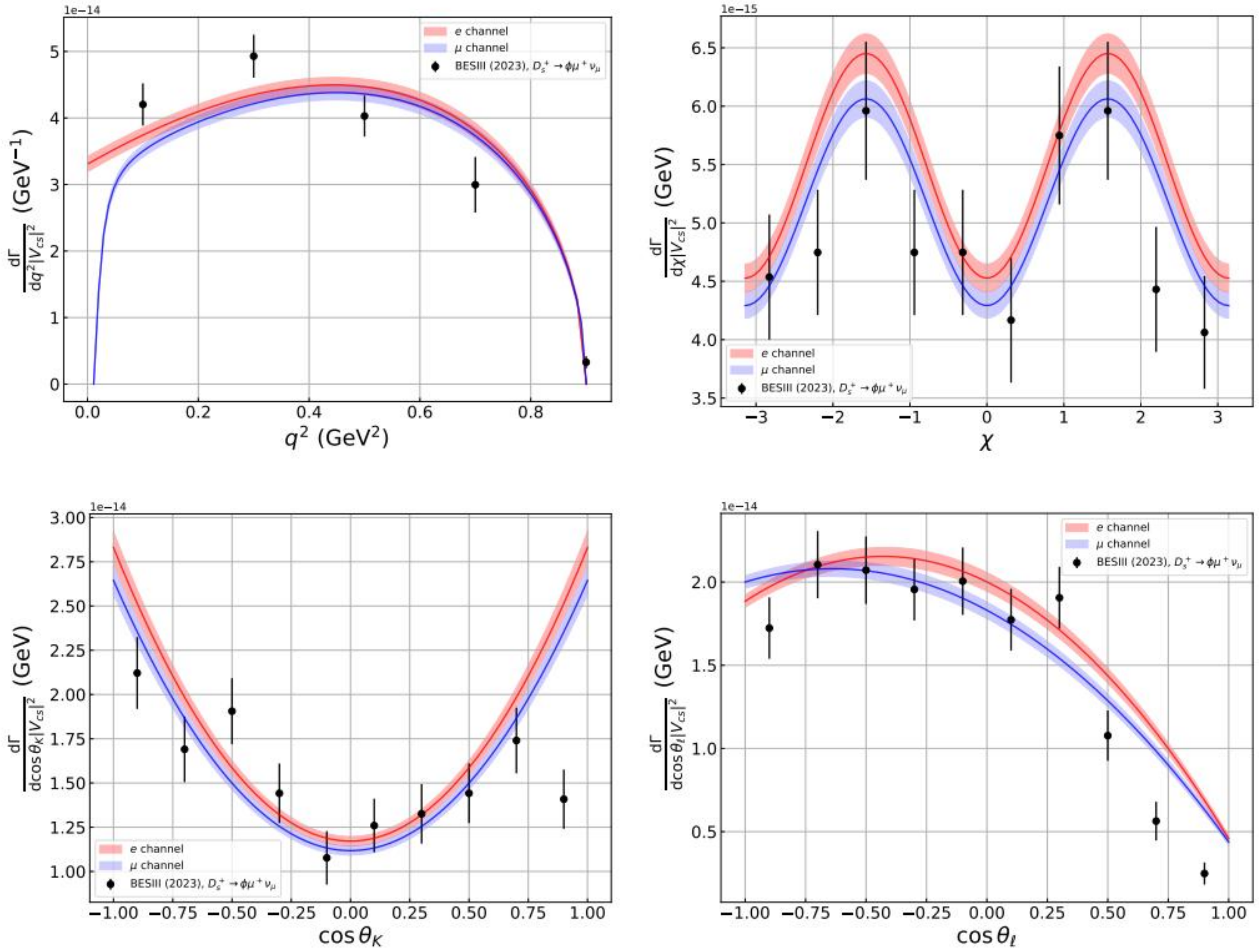
- Comparasions of the form factors $r_V \equiv V(0)/A_1(0)$ and $r_2 \equiv A_1(0)/A_1(0)$

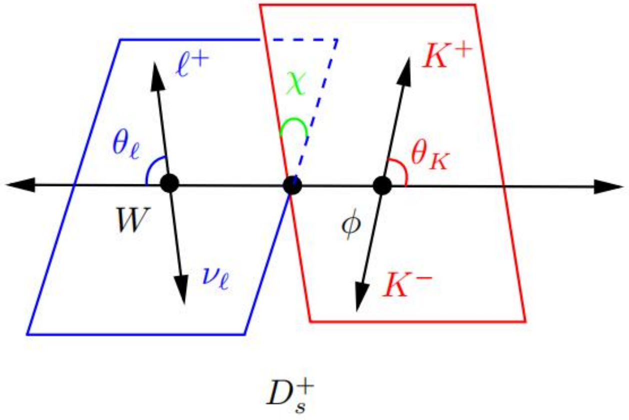




Results (differential decay width)

- Differential decay width [Front. Phys. (Beijing) 14 (2019) 64401]





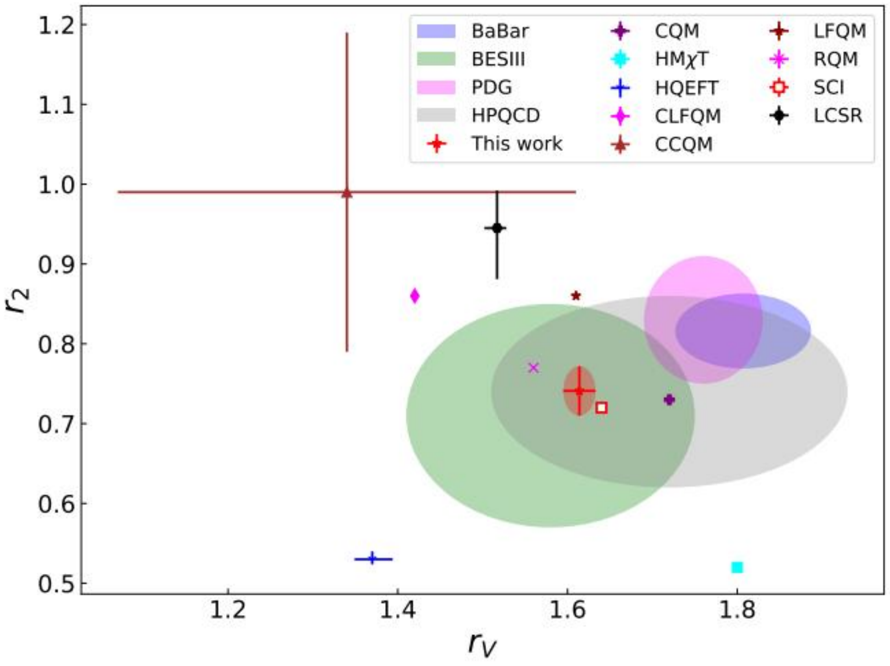
Results (summary)

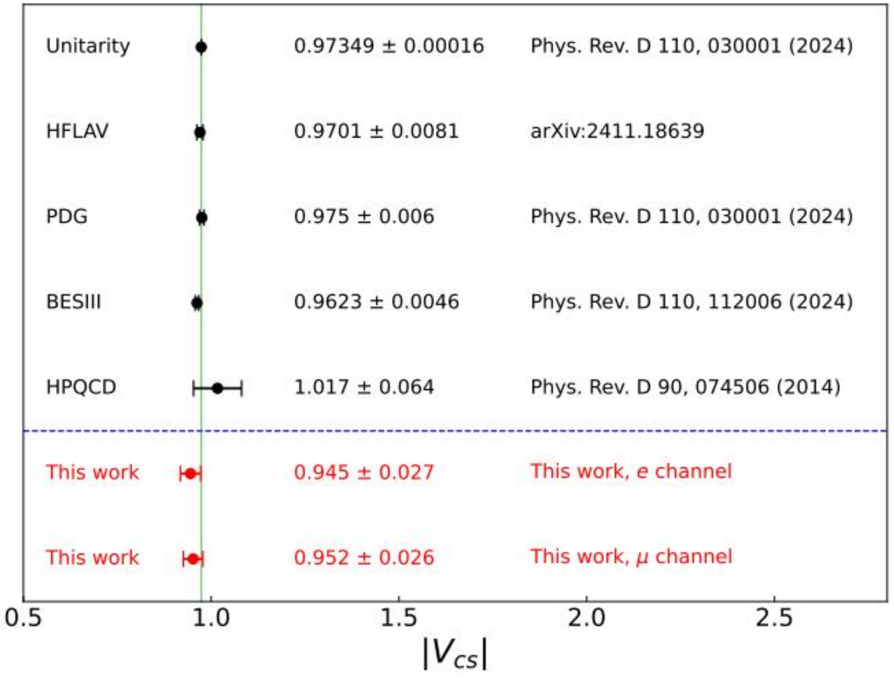
- Summary of preliminary results

$\mathcal{B}(D_s \to \phi\ell\nu_\ell) \times 10^2$	μ channel	e channel	$\mathcal{R}_{\mu/e}$
This work	2.351(67)	2.493(73)	0.9432(13)
BaBar	–	2.61(17)	–
CLEO	–	2.14(19)	–
BESIII (2018)	1.94(54)	2.26(46)	0.86(29)
BESIII (2023)	2.25(11)	–	–
PDG	2.24(11)	2.34(12)	0.957(68)

$$\frac{\mathrm{d}\Gamma(D_s \to \phi\ell\nu)}{\mathrm{d}q^2} = \frac{G_F^2 |V_{cs}|^2 |\boldsymbol{p}_\phi| q^2}{96\pi^3 M_{D_s}^2}\left(1 - \frac{m_\ell^2}{q^2}\right)^2 \left[\left(1 + \frac{m_\ell^2}{2q^2}\right)\left(|H_+|^2 + |H_-|^2 + |H_0|^2\right) + \frac{3m_\ell^2}{2q^2}|H_t|^2\right]$$

PDG $|V_{cs}|$ as input





Summary

- Mass and decay constant of the ϕ meson are calculated
- Form factors on seven lattice sets with different q^2 are calculated
- Extrapolate form factors to the physical pion mass and continuum limit
- Greatly improve the accuracy (1% - 4%) compared to HPQCD (12% - 16%)
- Branching fraction and $|V_{cs}|$ are calculated to the precision of 3%
- Preliminary work on $D \to K^* l\nu$ form factors are ongoing

Thank you for your attention!