Advances in Lattice QCD Studies of Light Meson & Light Baryon LCDAs

华俊 华南师范大学

On behalf of Lattice Parton Collaboration (LPC)

第五届中国格点量子色动力学研讨会

2025 10/10 @IMP

OUT LINE

Motivation About Meson & Baryon

- **Meson — ubiquitous messengers**

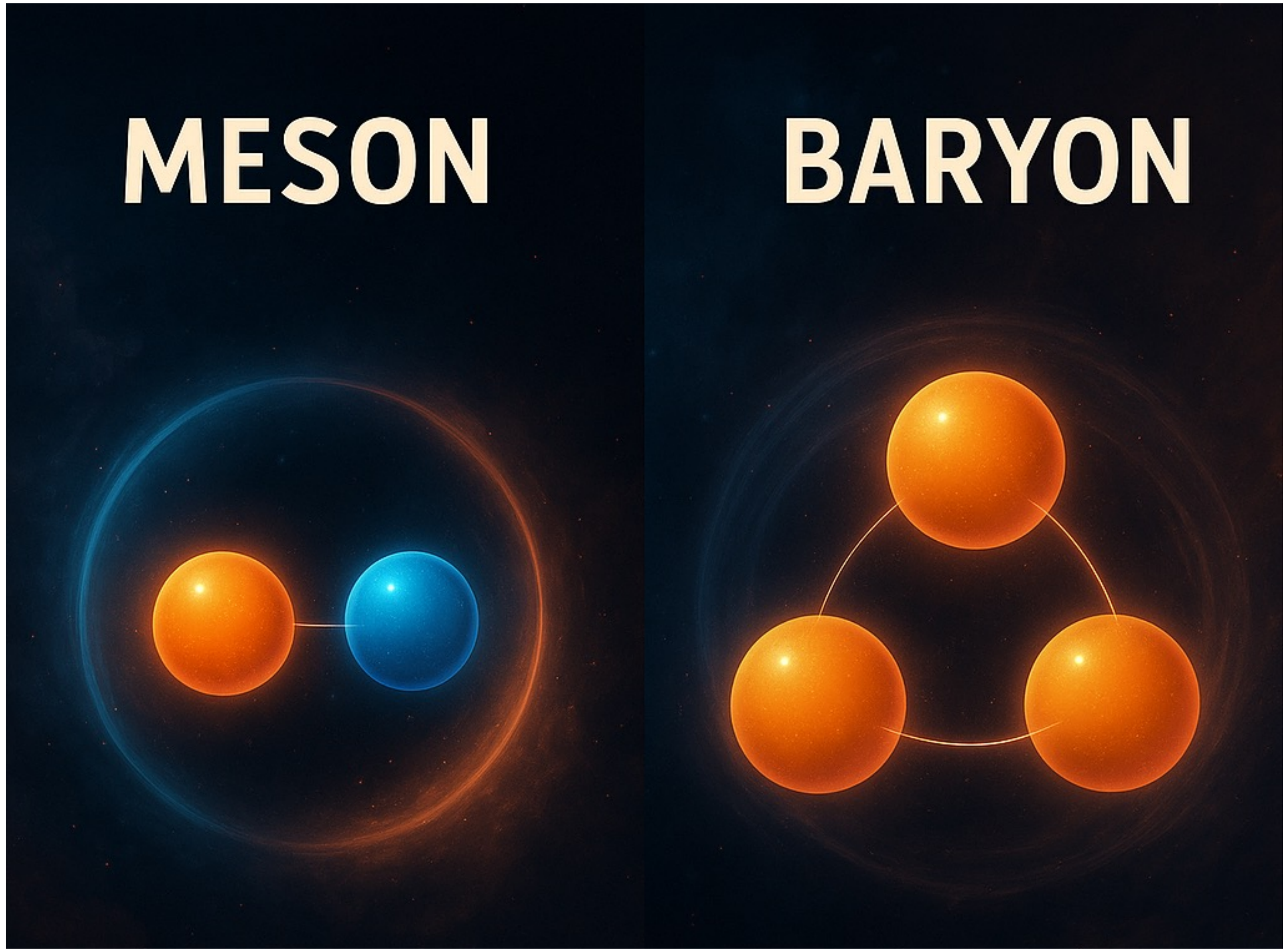
  Light Meson $SU(3)_{flavor}$ $8 \otimes 1$

  Heavy Meson $SU(4)_{flavor}$ $\cdots$

- **Baryon — cornerstone particles**

  Octet baryons

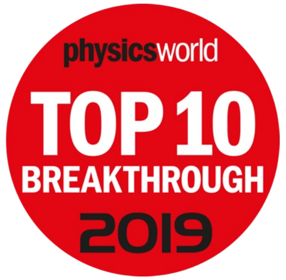
  Decuplet baryons



Motivation About Meson & Baryon

Meson and Baryon in Flavor Physics

- 1956, Parity violation in weak interaction
- 1964, Observation of CP violation in **Kaon meson**
- 1973, Kobayashi-Maskawa mechanism
- 2004, Observation of direct CPV in **B meson**
- 2019, Observation of direct CPV in **D meson**
- 2025, Observation of direct CPV in **Baryon $\Lambda_b^0 \to p$**



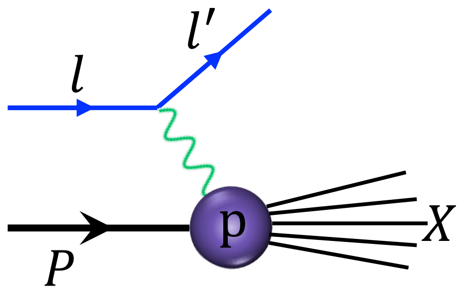
LHCb, Nature 643 8074 (2025)； Theoretical: *J.J.Han, et.al. PRL 134 221801(2025)*

Motivation About LCDA

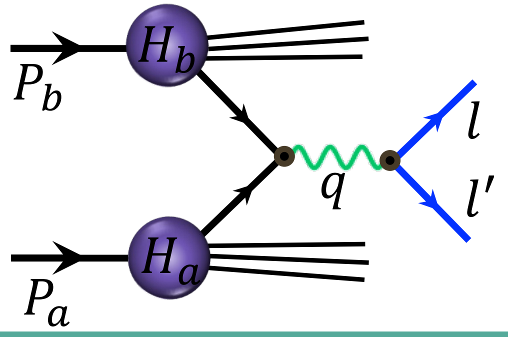
- One try to probe internal structure of nucleons → **Inclusive beam-target collision**

Defined PDFs

DIS



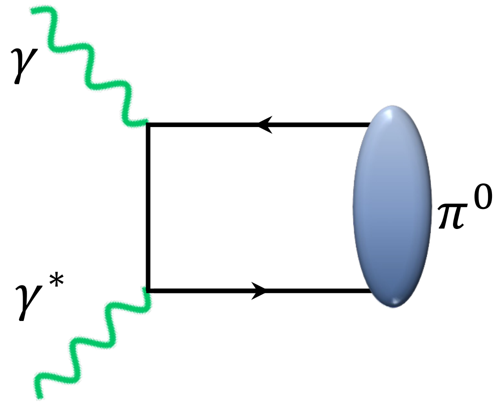
Drell-Yan



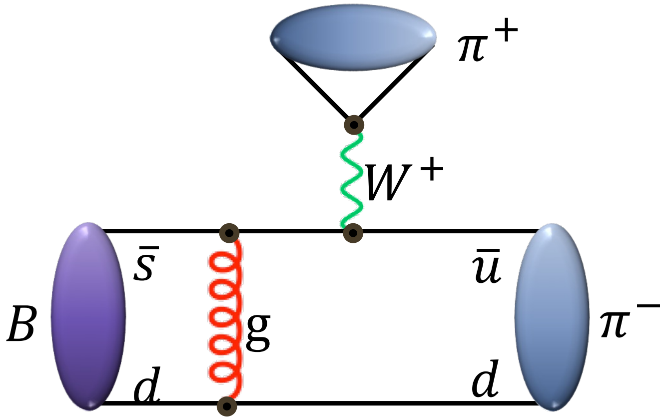
- One try to obtain richer QCD dynamical information → **Exclusive scattering**

Defined LCDAs

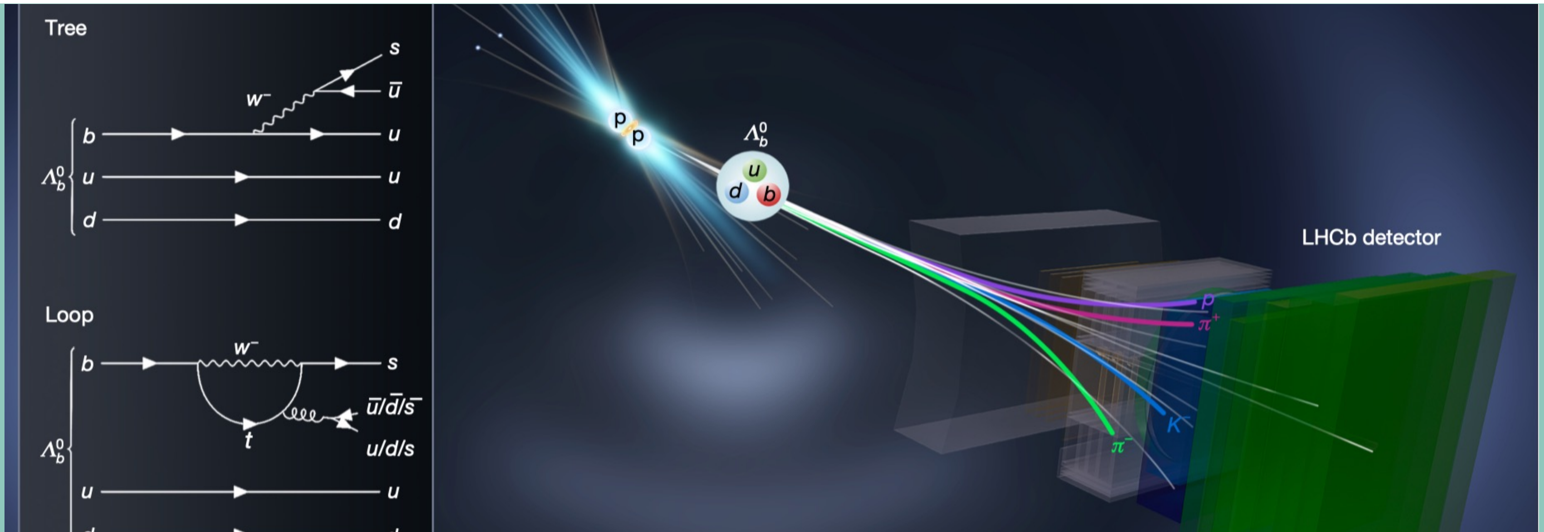
$\gamma\gamma^* \to \pi$



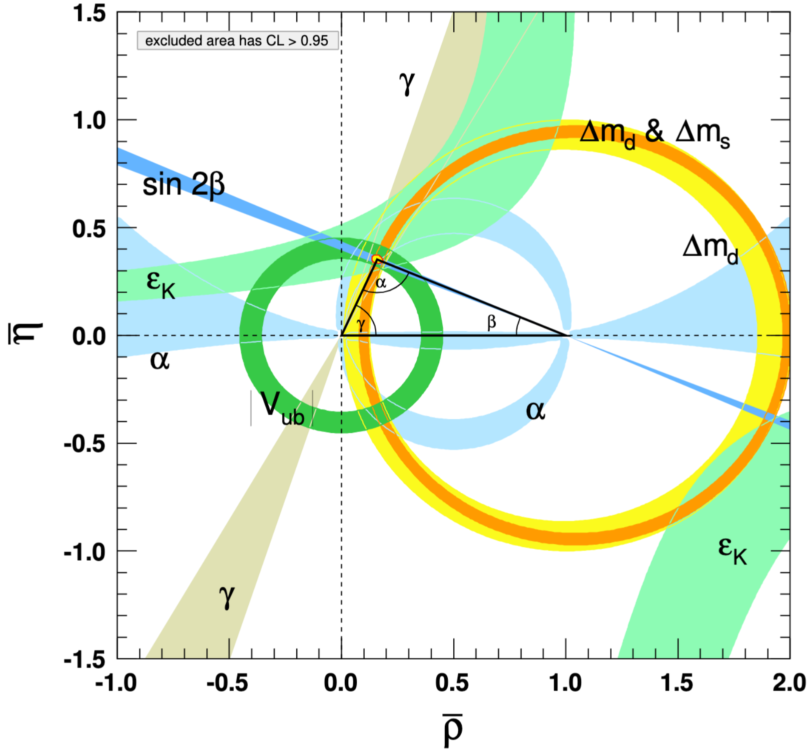
$B \to \pi^+\pi^-$



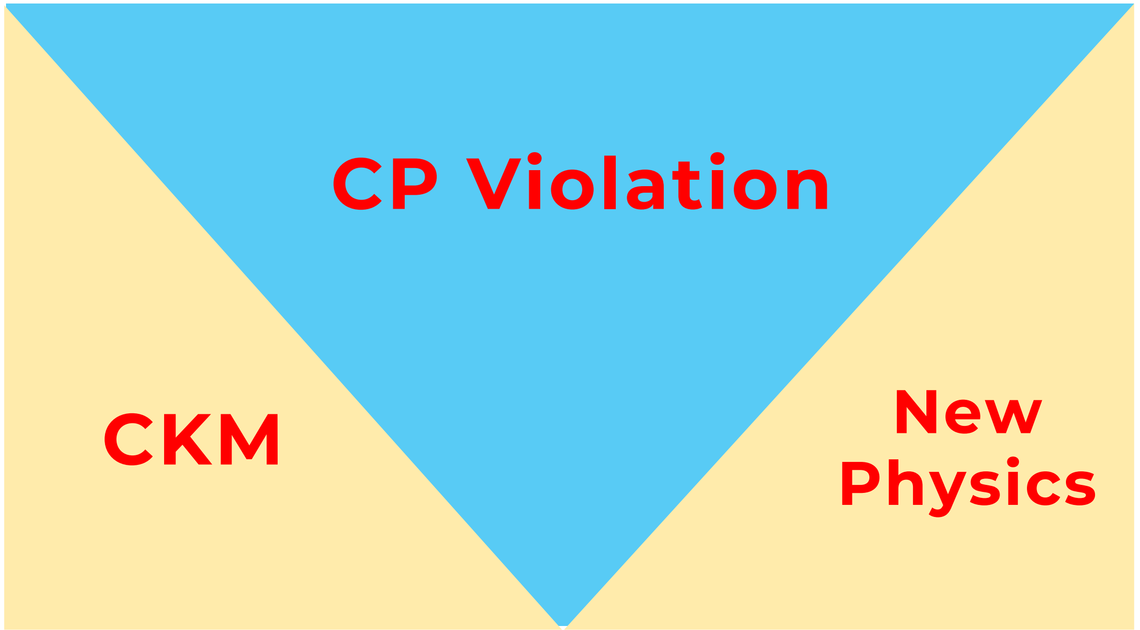
Motivation Exclusive Target



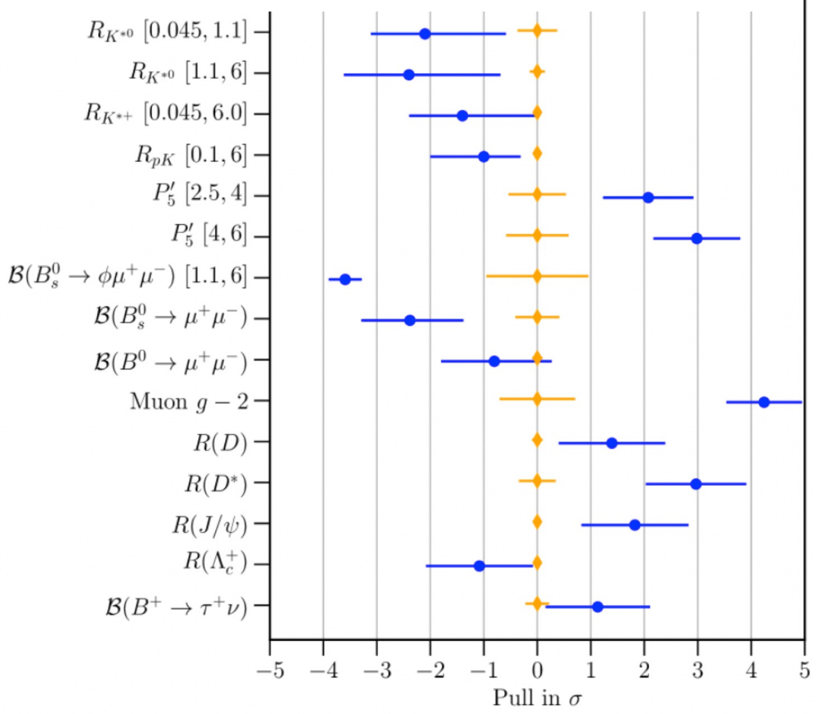
LHCb, Nature 643 8074 (2025)



PDG2024, PRD 110, 030001



Snowmass 2021, 2207.01851



Motivation About LCDA

Light Meson LCDAs: (1977 - now)

- Asymptotic form

  Chernyak, Zhitnitsky, 1977; Lepage, Brodsky, 1979; ……
- QCD Sum rules

  Chernyak, Zhitnitsky, 1982; Braun, Filyanov, 1989; ……
- Dyson-Schwinger Equation

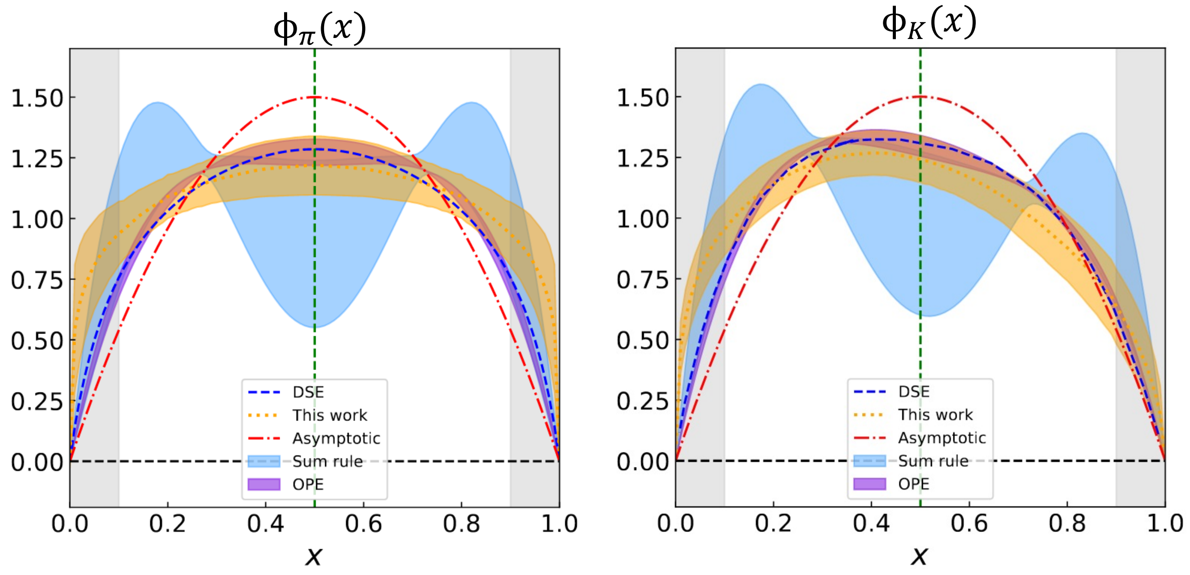
  Chang, Cloet, et.al, 2013; Gao, Chang, et.al, 2014; ……
- Global Fits

  Cheng, et.al, 2020; Hua, Li, Lu, Wang, Xing, 2021; ……
- Models

  Arriola, Broniowski, 2002; Zhong, Zhu, et.al, 2021; ……
- **Lattice QCD with OPE**

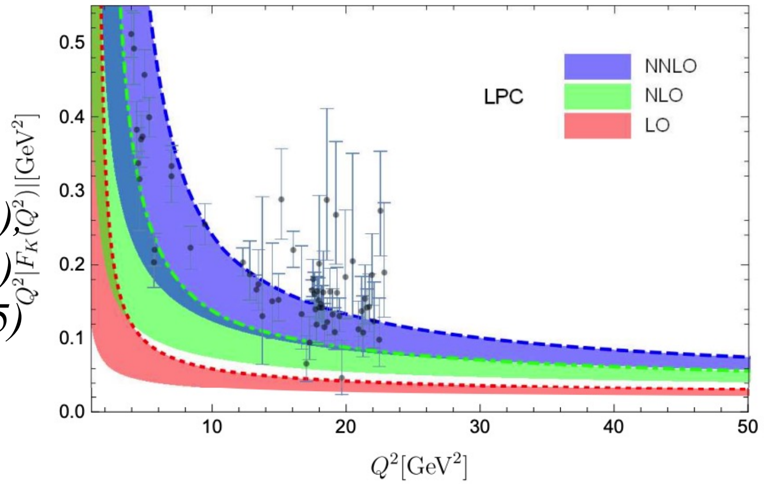
  Braun, et al., 2016; RQCD collaboration, 2019, 2020; ……
- **Lattice QCD with LaMET**

  LP3, 2019; LPC, 2021, 2022; ……



Application:

L.B.Chen et.al
PRL 132 201901(2024)
PLB 870 139886(2025)
PRL 135 131903 (2025)



Next Stage for LCDA ?

02

Advances of Framework

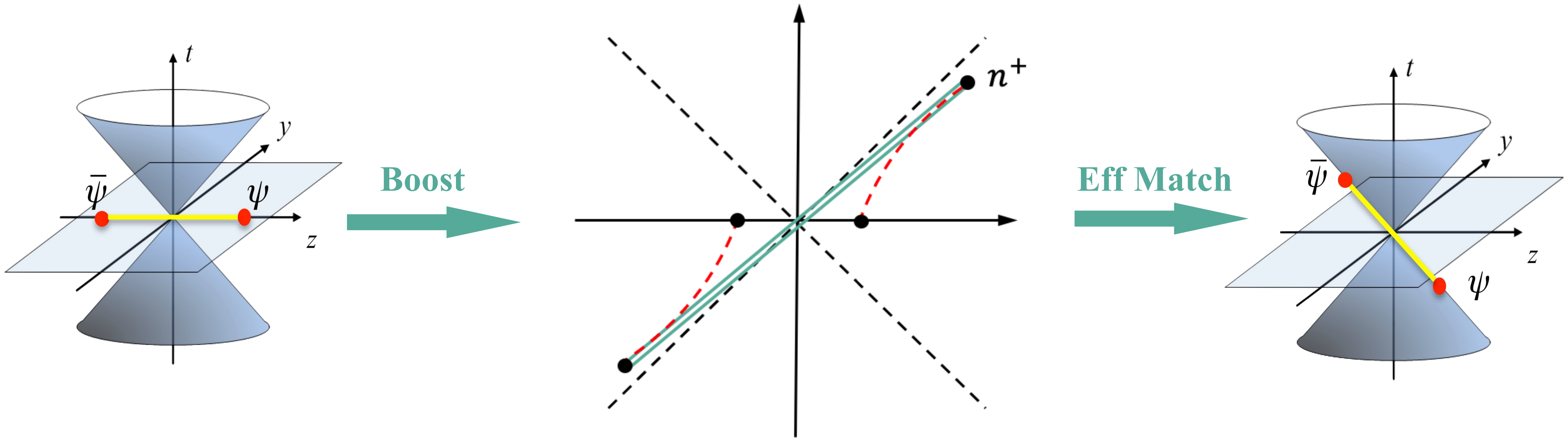
Renormalon resummation(LRR); Renormalization group resummation(RGR);

Two loop matching

Advances of Framework

Large momentum effective theory (LaMET): the light-cone non-local operator correlated to Euclidean non-local operator with a large momentum boost

Ji, PRL 110 262002 (2013)



LaMET factorization:

$$\phi(y,\mu_R)=\int dx C(x,y,\mu_R,\mu)\tilde{\phi}(x,\mu)+\mathcal{O}\left(\frac{1}{(yP^z)^2},\frac{1}{[(1-y)P^z]^2}\right)$$

What's update Version ?

(Focus on LCDA related)

Advances of Framework

Advances: (2021-)

- Hybrid & Self renormalization

  LPC NPB 964 115311, 969 115443(2021)
- Renormalon resummation (LRR)

  NPB 993 116282(2023)
- RGR & Threshold resummation

  NPB 993, JHEP 03 045(2025)
- Two loop matching

  arXiv:2504.09367
- Power correction estimation

  PRD 110 094038(2025), 112 016013(2025)

Linear divergence

Multi-a, extract linear-div

Match with perturbative $\overline{MS}$

Renormalon in perturbative results

Self renorm & Renormalon resum

JH et.al. NPB 993 116282(2023), XDJ et.al. JHEP 03 045(2025)

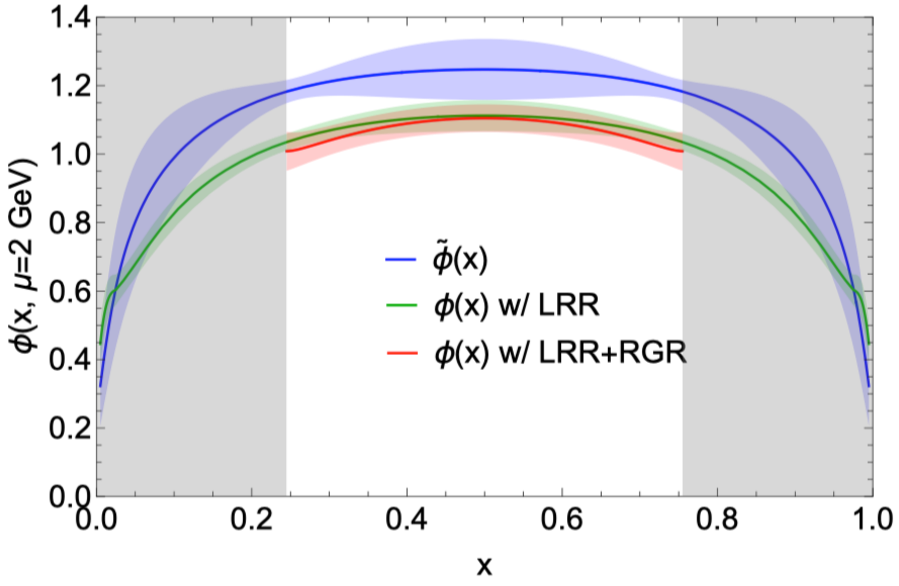
Linear divergence → Multi-a, extract linear-div → Match with perturbative $\overline{MS}$

$$Z(z,a)_R = \exp\left[\frac{kz}{a\ln\left[a\Lambda_{\mathrm{QCD}}\right]} + m_0 z + f(z)a + \frac{3C_F}{b_0}\ln\left[\frac{\ln\left[1/(a\Lambda_{\mathrm{QCD}})\right]}{\ln\left[\mu/\Lambda_{\mathrm{QCD}}\right]}\right] + \ln\left[1+\frac{d}{\ln\left(a\Lambda_{\mathrm{QCD}}\right)}\right]\right]$$

- Extract from multi lattice spacings of zero momentum matrix
- Extract from matching with perturbative $\overline{MS}$ quasi

Renormalon ambiguity occurs

LRR+RGR

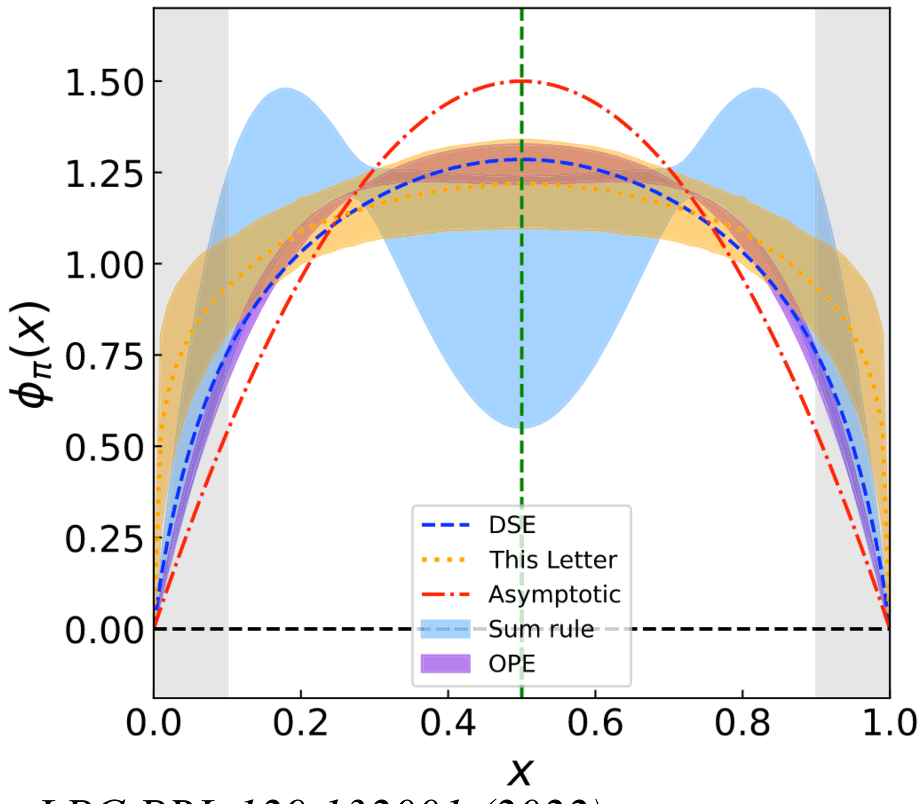


A LRR guarantee a more comprehensive self renormalization

RGR & Threshold resummation

LPC NPB 969 115443(2021), JH et.al. NPB 993 116282(2023)

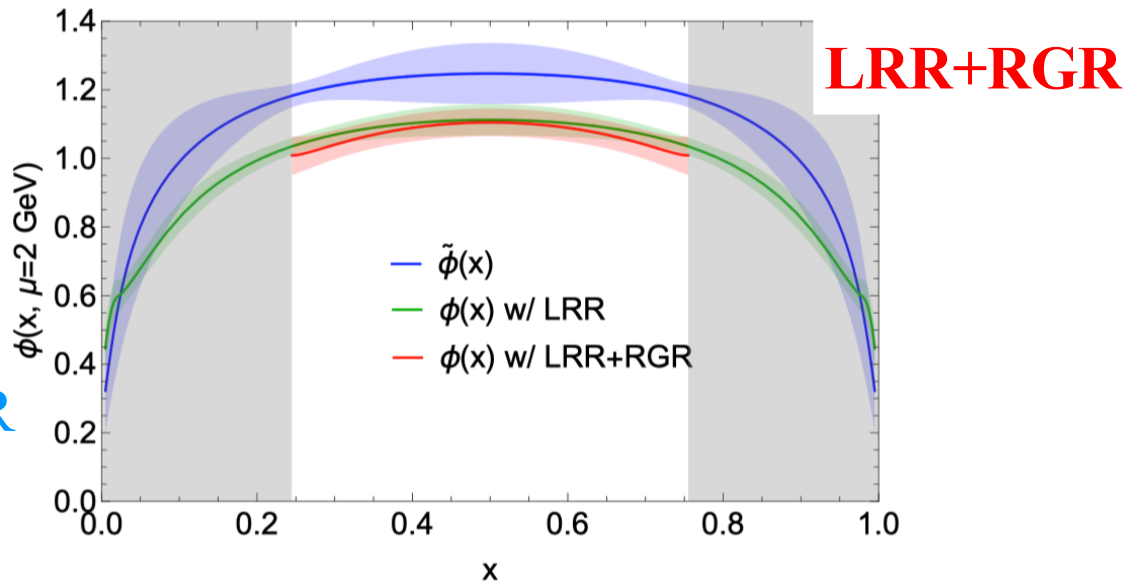
$$\phi(y,\mu_R)=\int dx C(x,y,\mu_R,\mu)\tilde{\phi}(x,\mu)+\mathcal{O}\left(\frac{1}{(yP^z)^2},\frac{1}{[(1-y)P^z]^2}\right)$$

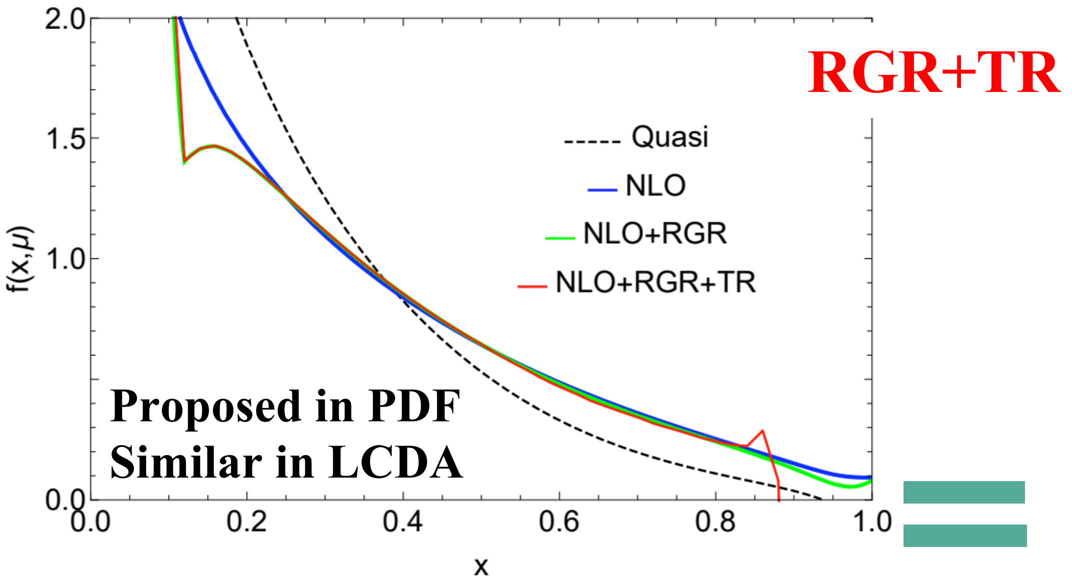


LPC PRL 129,132001 (2022)

- Scale of x: RGR
- Sum log in kernel: TR

Towards small x region control !





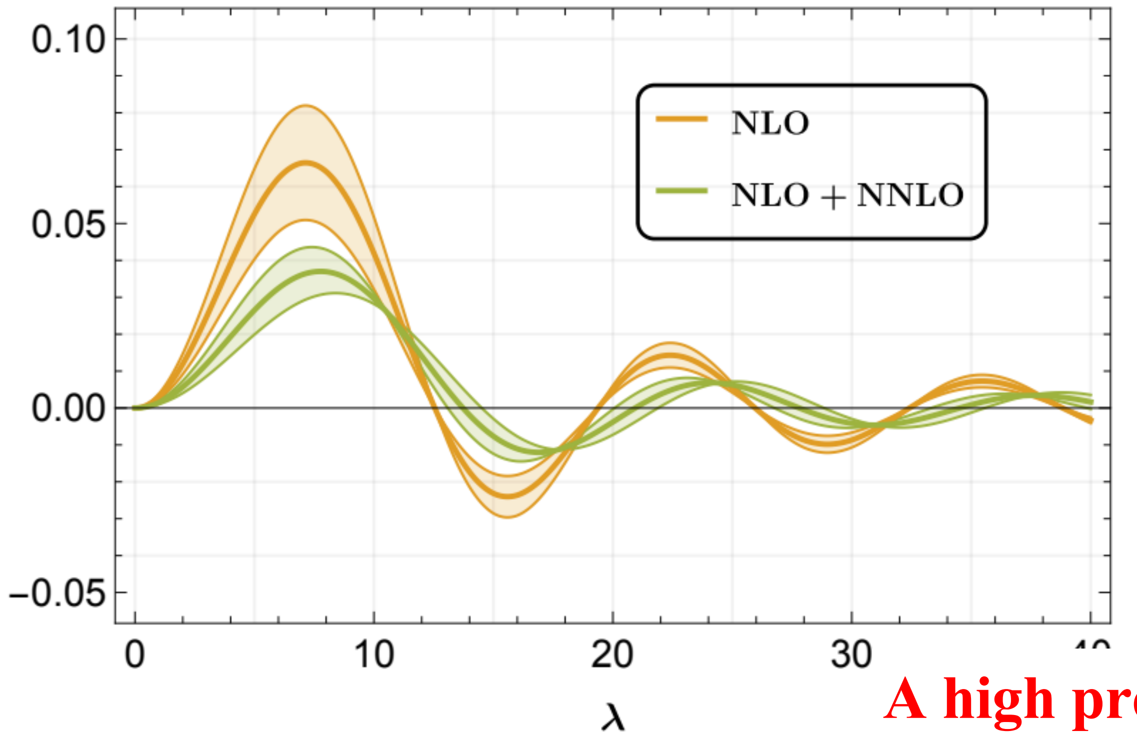
NNLO corrections:

YJ, FY & JHZ arXiv:2504.09367

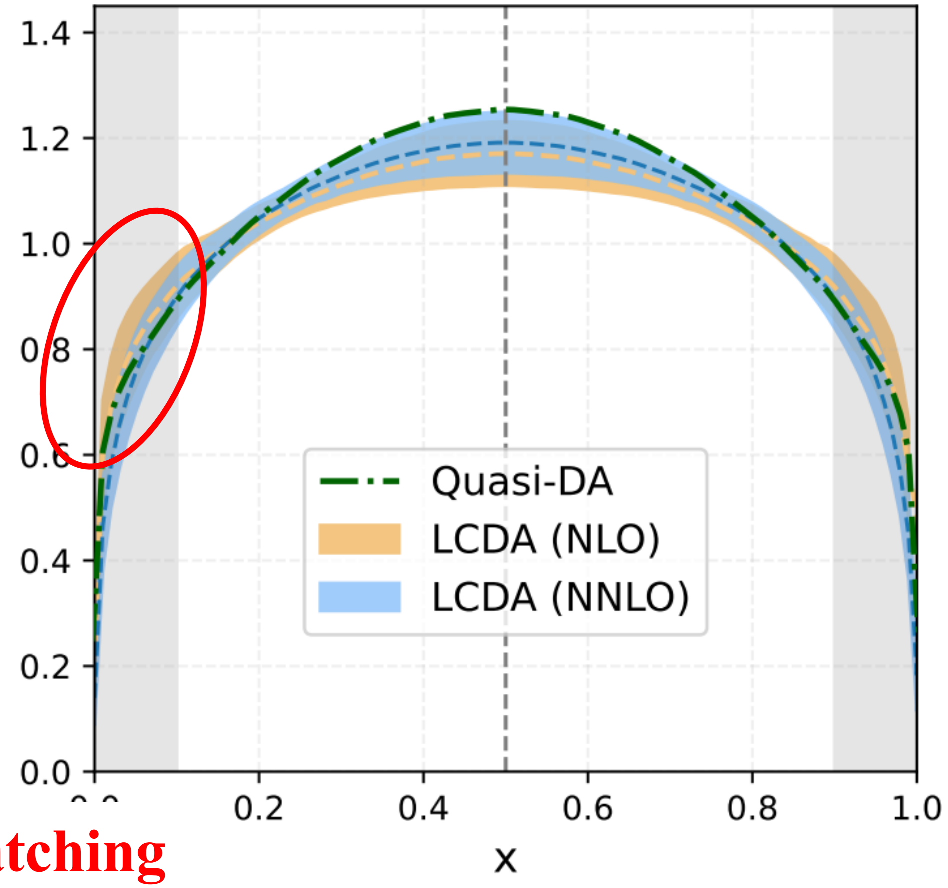
$$\phi_R(x, P^z) = \int dy C\left(x, y, \frac{\mu}{P^z}\right) \tilde{\phi}_R(y, \mu) + \mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(P^z)^2}\right)$$

Matching kernel

Coordinate space: $\mu = 2\ \mathrm{GeV}, \ z = 0.2\ \mathrm{fm}$



Momentum space:



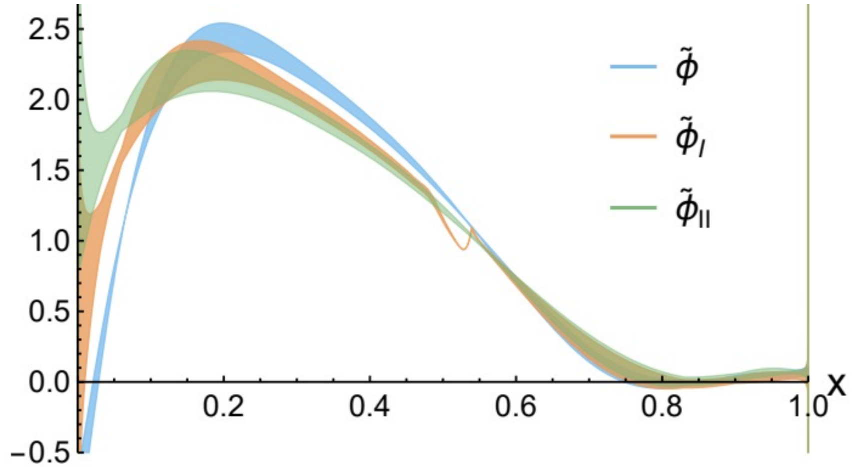
A high precision matching

Power corrections in LaMET:

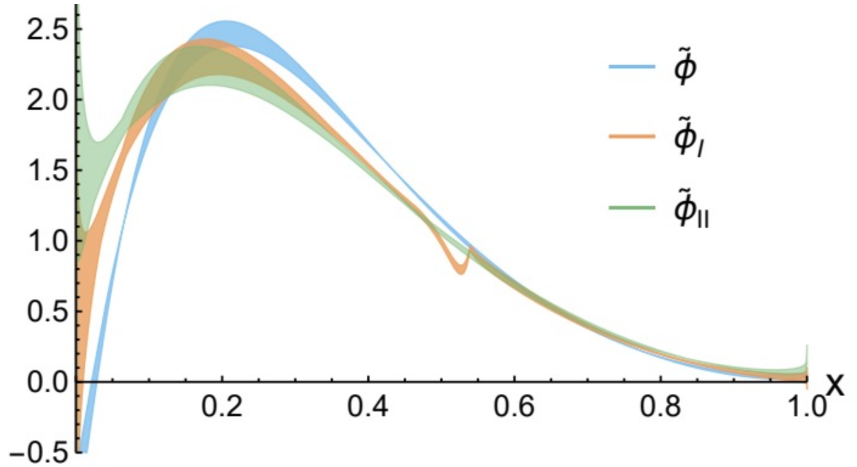
JLZ et.al. PRD 110 094038(2025), 112 016013(2025)

$$\phi_R\left(x, P^z\right) = \int dy C\left(x, y, \frac{\mu}{P^z}\right)\tilde{\phi}_R(y, \mu) + \mathcal{O}\left(\frac{m_H^2}{\left(P^z\right)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{\left(P^z\right)^2}\right)$$

- $\frac{m_H^2}{(P^z)^2}$: Match with OPE
- $\frac{\Lambda_{QCD}^2}{(P^z)^2}$: Estimated by renormalons



(e)



(f)

Towards precision calculation on updated framework. **How ?**

03

Advances of Precision Calculation

Simulation improvement; Precision check for meson LCDA;
Inverse Fourier Transform

Advances of Precision Calculation

Advances: (2021-)

- Kinematically-enhance

  PRD 112 L051502(2025)
- Precision check for meson LCDA

  … JHW et.al.
- Inverse Fourier transform

  arXiv:2505.14619, arXiv:2506.16689
- CLQCD ensembles & Pyquda

  CLQCD PRD 109(2024) , arXiv: 2411.08461
- … Gradient flow

  JHEP 06 210 (2024), arXiv:2507.18233

➢ **Precision**

Signal Improve

Kinematically-enhance:

RZ et.al. PRD 112 L051502(2025)

$$C_2^{\pi}(z,\vec{P},t)=\int d^3y e^{-i\vec{P}\cdot\vec{y}}\langle 0|\underbrace{\bar{\psi}_1(\vec{y},t)\Gamma_1 U(\vec{y},\vec{y}-z\hat{z})\psi_2(\vec{y}-z\hat{z},t)}_{Sink}\underbrace{\bar{\psi}_2(0,0)\Gamma_2\psi_1(0,0)}_{Src}|0\rangle$$

Decomposition in light-cone limit :

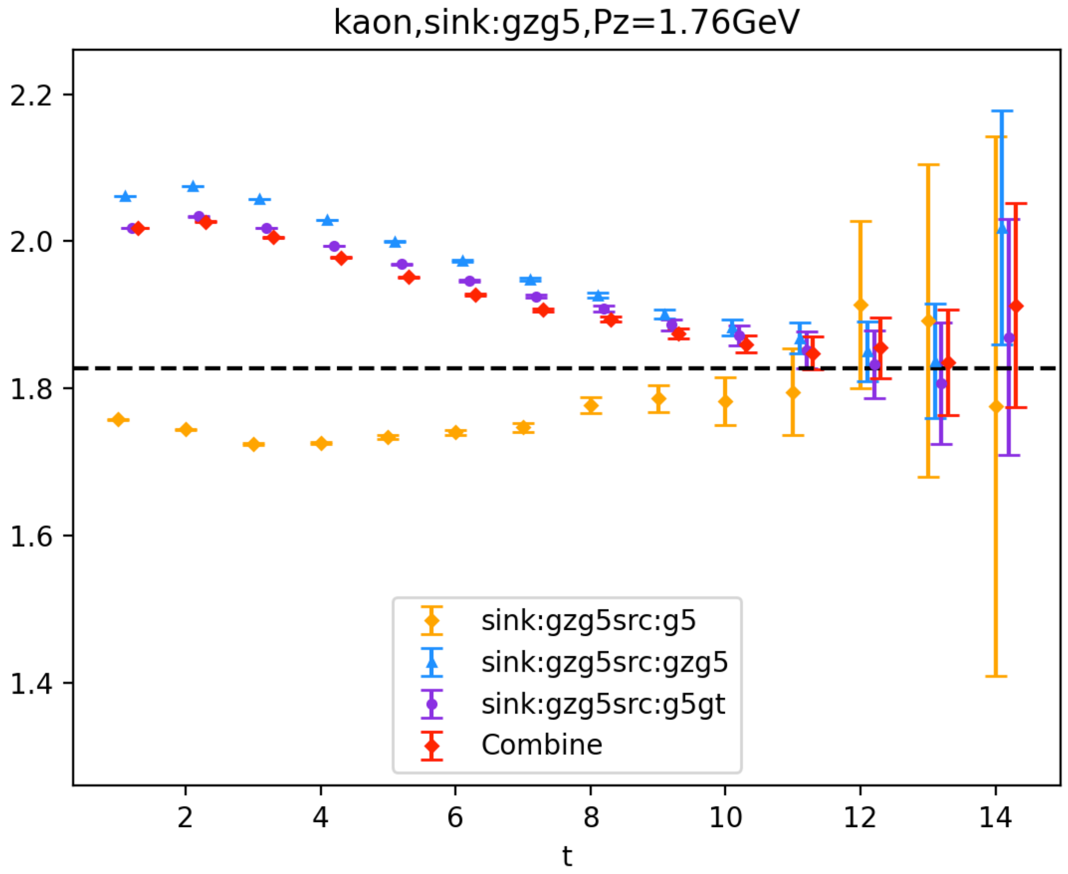
PLB 545,345 (2002), EPJC 33, 75(2004)

$$u_+^\dagger\gamma_5 d_+ = \sqrt{2}\bar{u}\gamma_+\gamma_5 d$$

$$(\bar{u}\gamma_5 d) = \left(u_+^\dagger\gamma_t\gamma_5 d_- + u_-^\dagger\gamma_t\gamma_5 d_+\right)/2$$

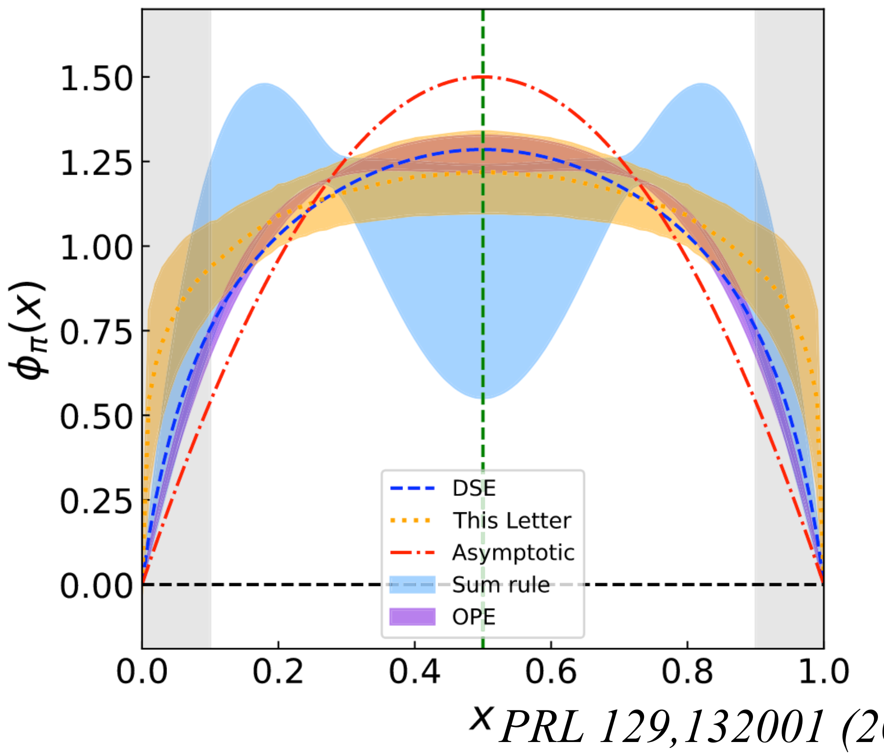
Can be further extended in baryon case.

See Muhua’s talk

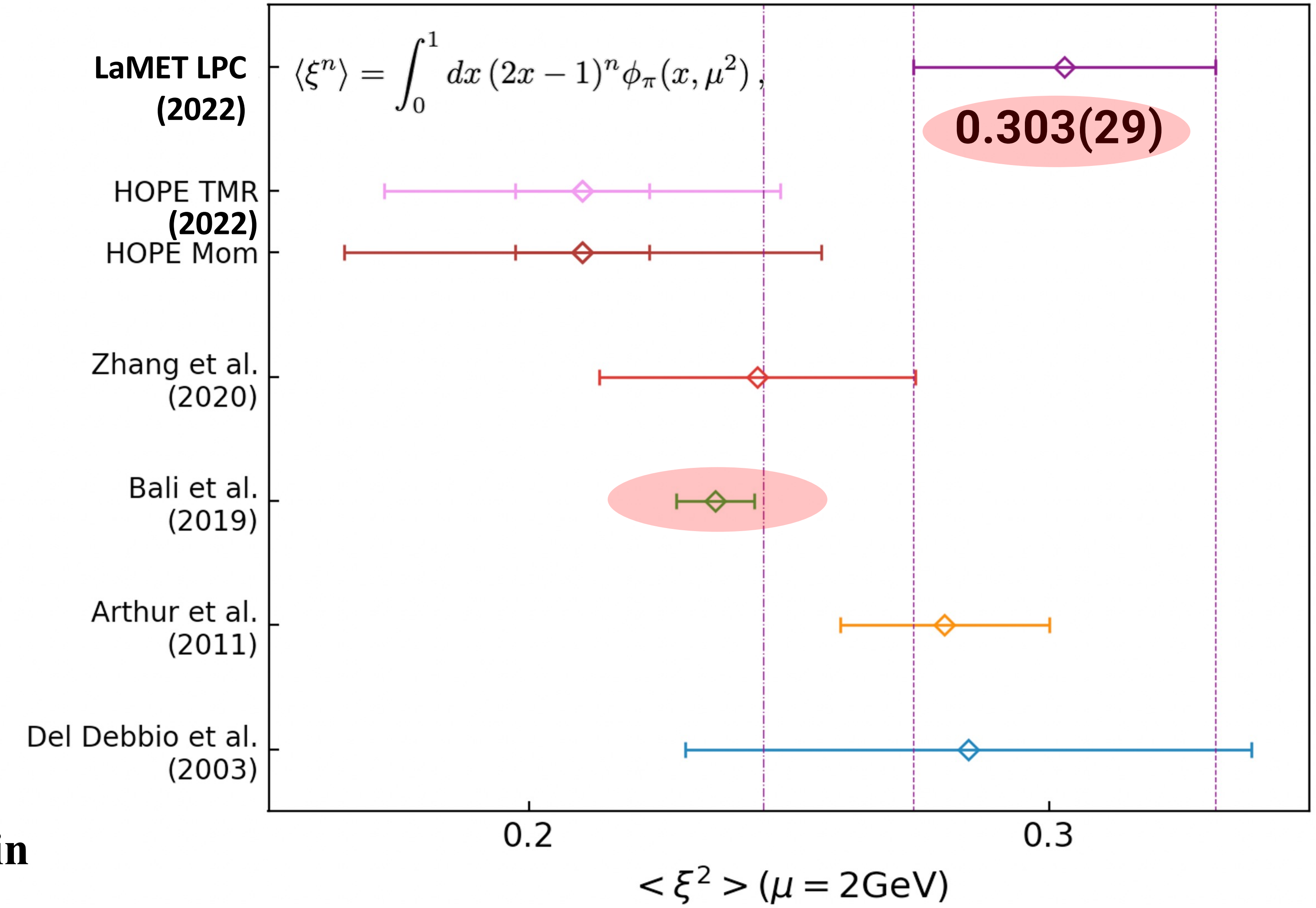


1 step towards precision calculation

Moments tension:



PRL 129,132001 (2022)



- **High precision under going in LaMET**
- **Moments tension between LaMET &OPE remain**

Precision Advances | Kinematically enhance | Precision meson LCDA | Inverse FT

- **OPE: Local moments**

- **Small z expansion: Non-local moments**

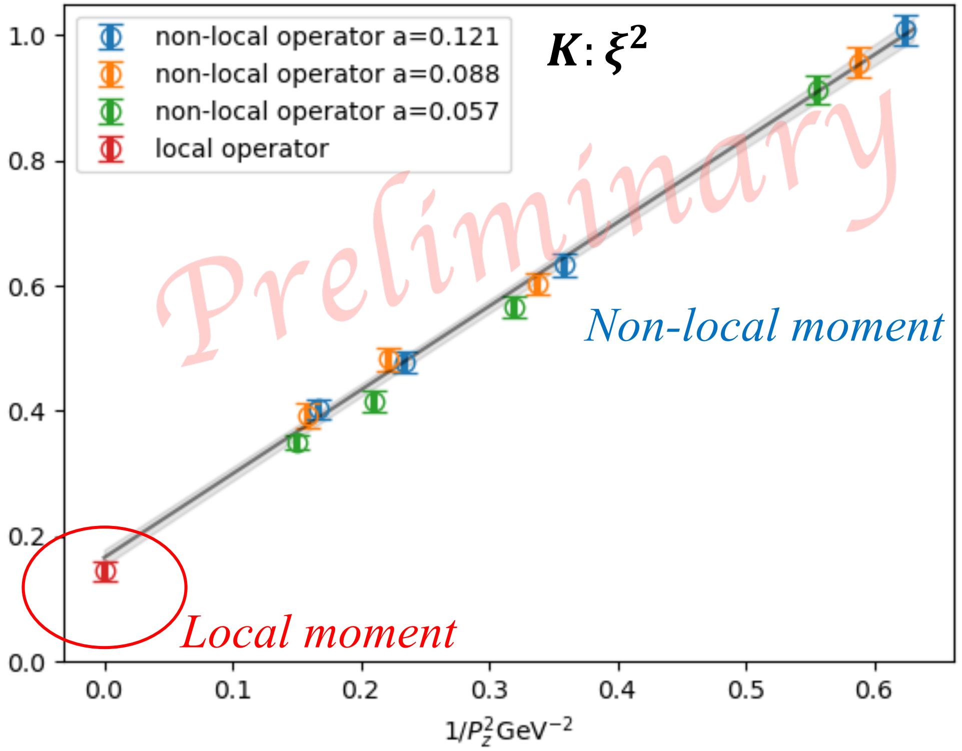
$$\xi \equiv 2x-1, \qquad \langle \xi^n \rangle_K(\mu) \equiv \int_0^1 dx\,(2x-1)^n\,\phi_K(x,\mu)$$

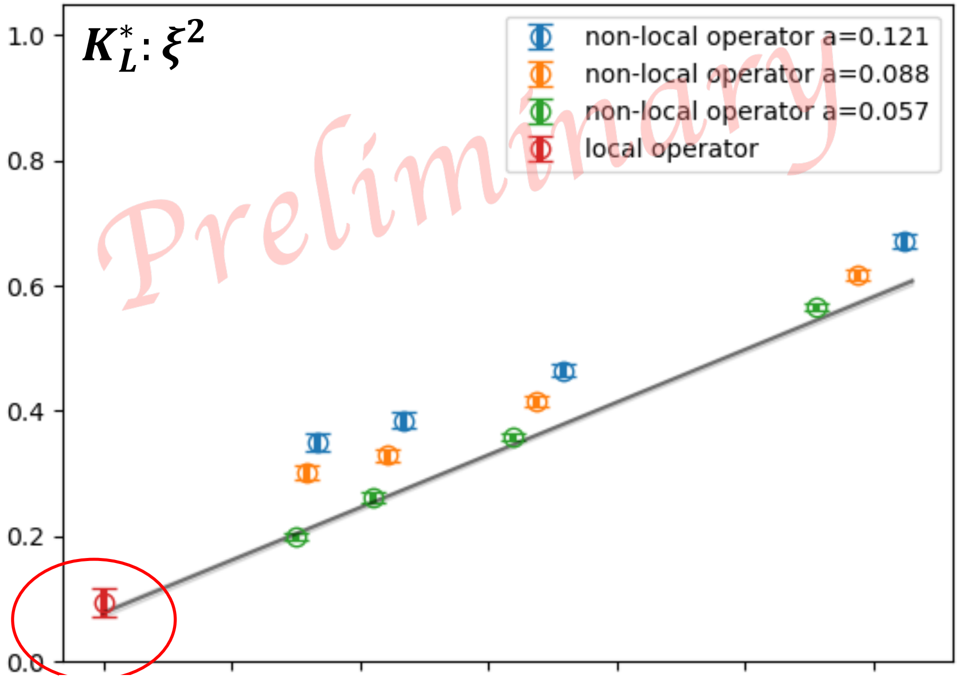
$$\langle 0|\bar{q}(0)\gamma^+\gamma_5(i\overleftrightarrow{D}^+)^n s(0)|K(P)\rangle = i f_K P^{+(n+1)}\langle \xi^n\rangle_K(\mu)$$

$$\tilde{\Phi}(z,P_z) \sim \int_0^1 dx\, e^{i(2x-1)P_z z}\phi_K(x)$$

$$\tilde{\Phi}(z,P_z) = \int_0^1 dx\left[1 + i(2x-1)P_z z - \frac{1}{2}(2x-1)^2 P_z^2 z^2 + \cdots\right]\phi_K(x)$$

Consistent moments from local and non-local





The first precision check in LaMET, then ?

Quasi from coordinate to momentum space: Limited discrete Fourier transform

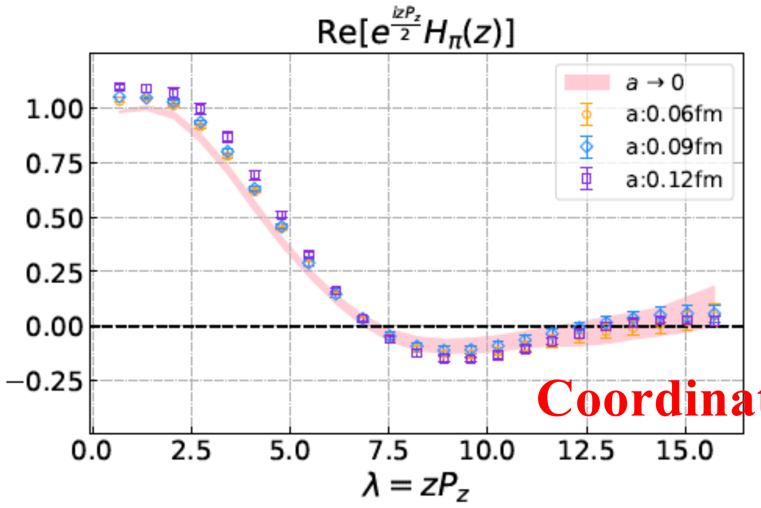
$$g(\lambda) = \int dx e^{-i\lambda x} f(x) \to f(x) = \frac{1}{2\pi} \sum_{\lambda_{\min}}^{\lambda_{max}} \Delta\lambda e^{i\lambda x} g(\lambda)$$

LPC NPB 964 115311(2021);—hybrid scheme Extrapolation

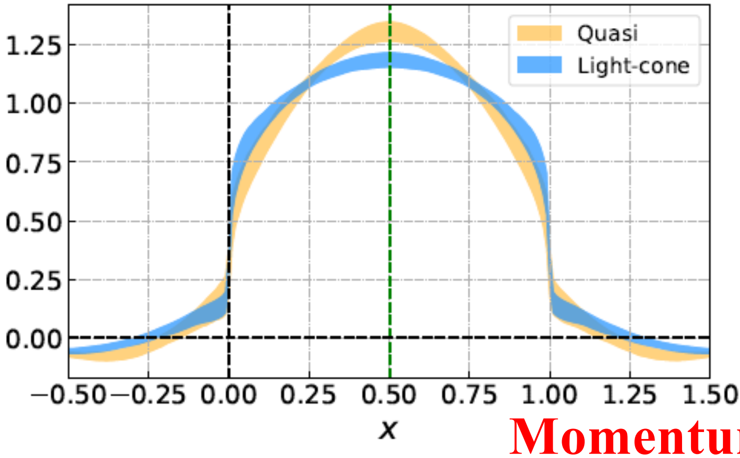
H.Dutrieux et.al, arXiv: 2504.17706; — inverse problem exist

J.W.Chen et.al. arXiv: 2505.14619; — can be circumvented

A.S.Xiong et.al. arXiv: 2506.16689; — can be addressed



Coordinate space:



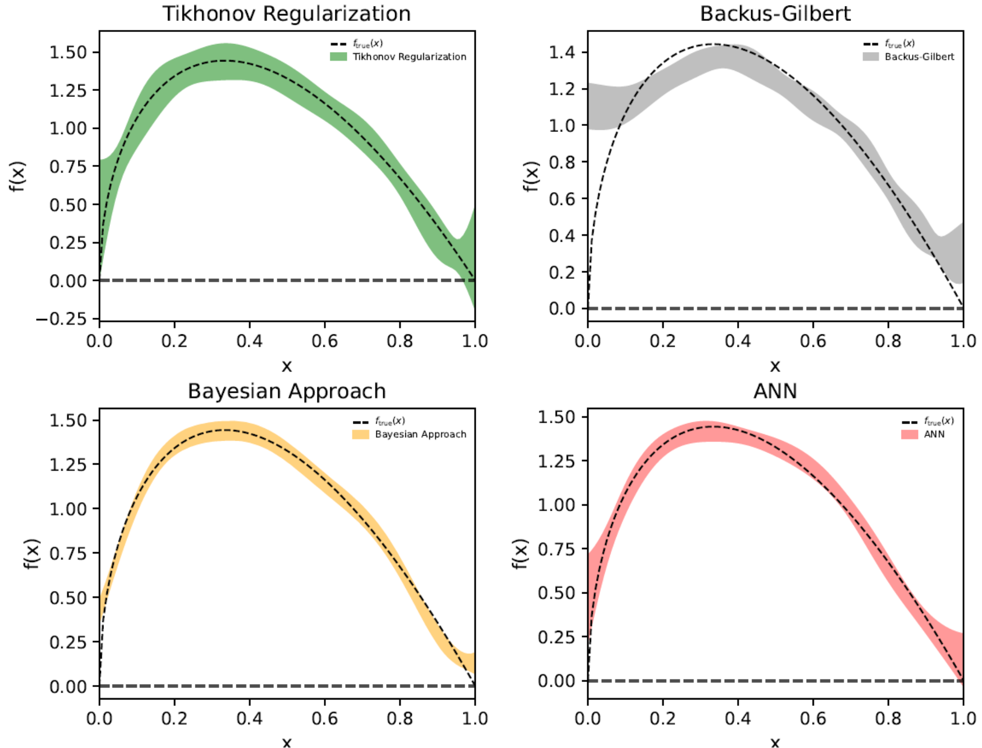
Momentum space:

Limited discrete Fourier transform: Can be circumvented

- Tikhonov regularization
- Backus-Gilbert
- Bayesian approach
- Artificial neurons network

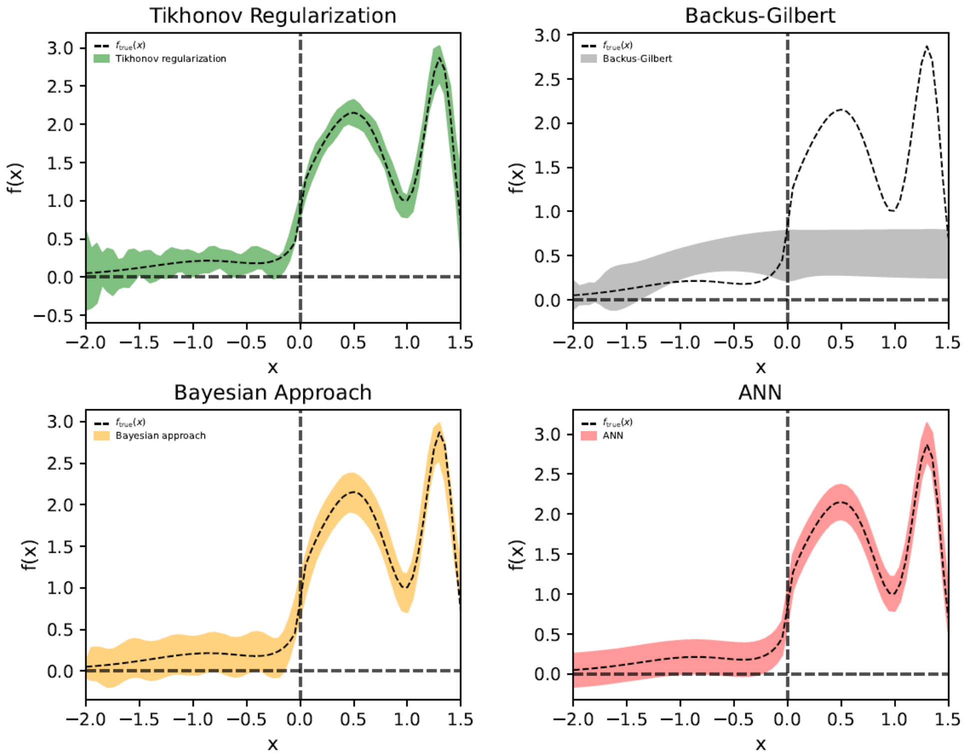
Regulator method

Approaches to the Inverse Fourier Transformation with Limited and Discrete Data 渝飞凌



Inverse method give

- a support plan for extrapolation
- a safer error estimate
- Maybe more …



04

Advances of Physical Target

From light meson to heavy meson & light baryon

B meson LCDA cannot be directly determined

× *OPE*

× *LaMET*

The most important input in heavy flavor physics

- Only models for heavy meson LCDAs

  Grozin, Neubert, 1997; Braun, Ivanov, Korchemsky, 2004; Beneke, Braun, Ji, Wei, 2018…
- Only models for heavy meson SF

  Korchemsky, Sterman, 1994; Bauer, Luke, Mannel, 2001; Neubert, 2005; Lee, Ligeti, Stewart, Tackmann, 2006; ……
- A new approach from Lattice ?

  XuJi et.al , PRD 2020, 2022, EPJC 2024

B meson LCDA cannot be directly determined

× *OPE*

× *LaMET*

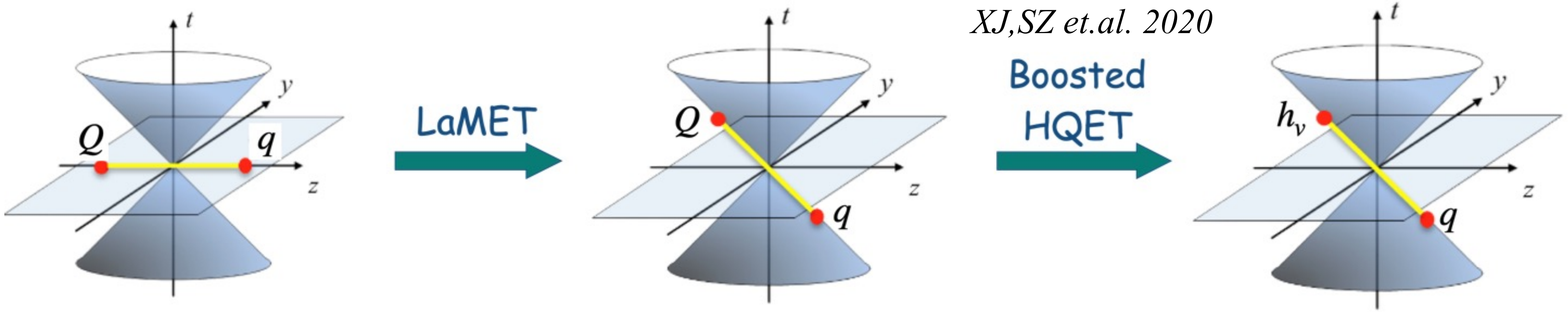
The most important input in heavy flavor physics

- **QCD LCDA**: quark distribution in **final-state** heavy mesons $B \to \boldsymbol{D}\ell\nu$:
- **HQET LCDA**: quark distribution in **initial-state** heavy mesons $\boldsymbol{B} \to D\ell\nu$

——Cusp divergence → Moments forbidden

Only solution：Two step matching

XJ,SZ et.al. 2020



LPC XYH, QAZ el.al. PRD 111, L111503, PRD 111, 034503, (2025) Targets Advances

Heavy meson LCDA

Light baryon LCDA

Light Baryon LCDAs: (1983 - now)

- Asymptotic form

  Chernyak, Zhitnitsky, 1983 ; ……
- QCD Sum rules

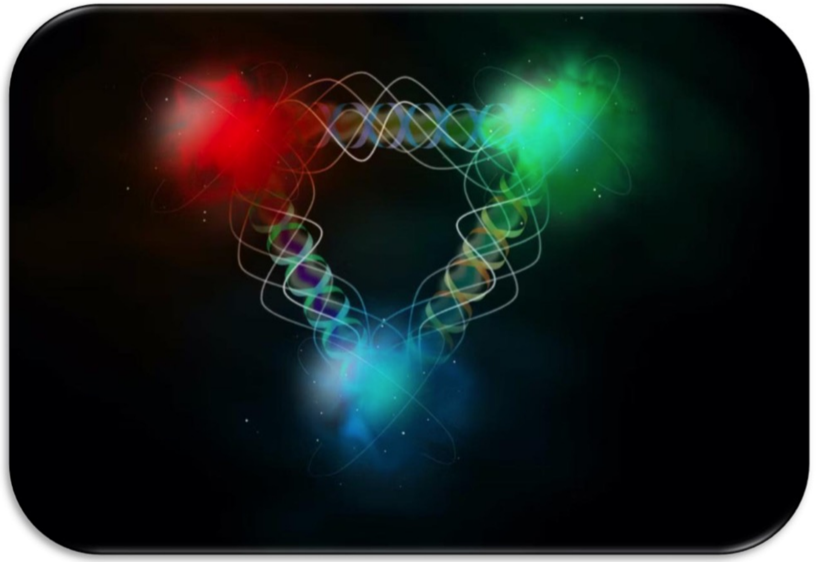
  King, Sachrajda,1987; Stefanis, Bergmann, 1993; Braun, et.al, 2000,2006; ……
- Models parametrization

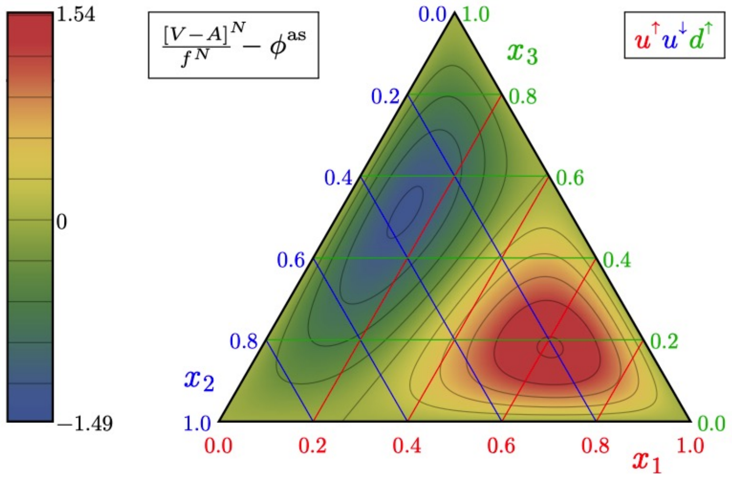
  Bell, Feldmann, Wang, Matthew 2013; ……
- Lattice QCD with OPE

  QCDSF, 2008, 2009; RQCD, 2016, 2019(2025) ……
- Lattice QCD with LaMET

  LPC, 2024,2025; ……

  PRD 111, 034510, arXiv:2508.08971





More is Different

Operator Product Expansion (OPE) VS Large Momentum Effective theory(LaMET)

Limited for only few moments:

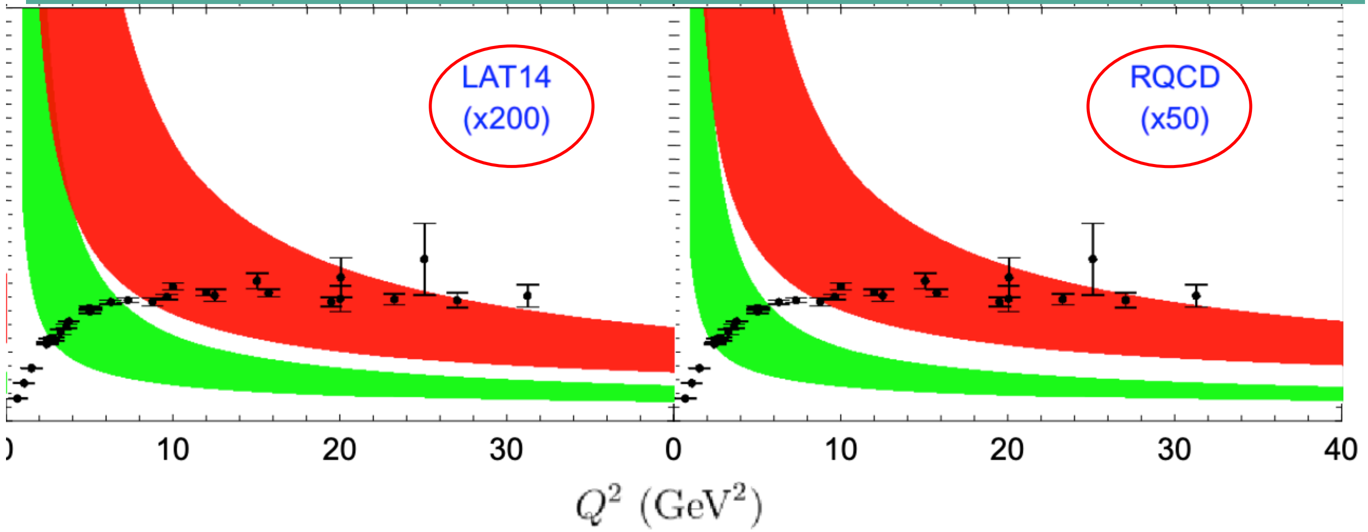
- First two moments for light mesons
- First moments for light baryons
- Inverse problem from moments to LCDA

High twist/moments are more important in Baryon

J.J.Han, et.al. EPJC 82 8 (2022), 686PRL 134 221801(2025)

L.B.Chen, et.al. PRL 135 131903 (2025)

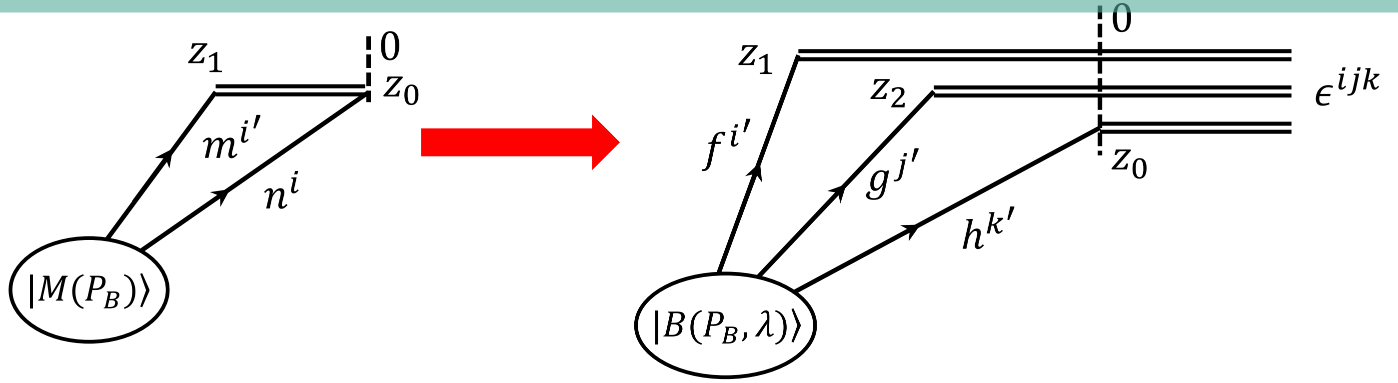
Nucleon Dirac form factors:

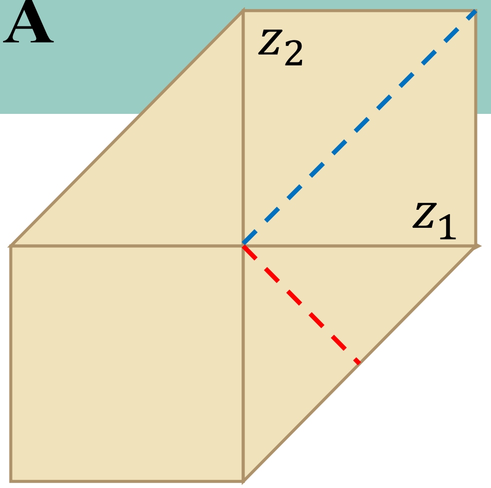


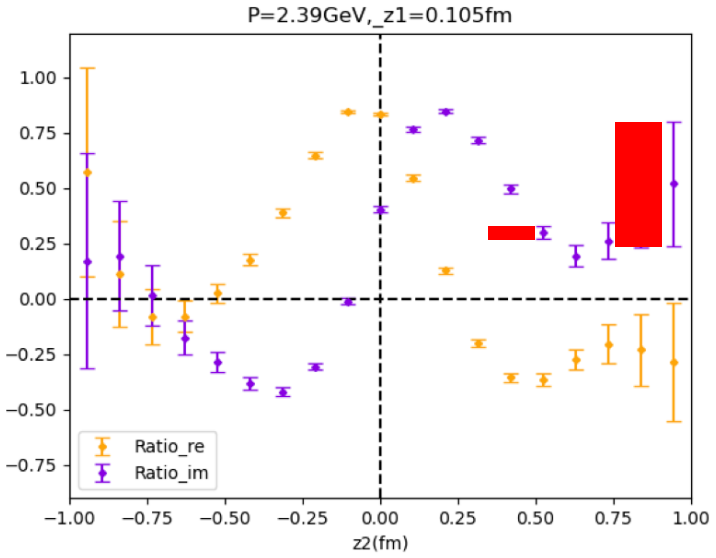
Soft contribution dominates in baryon decay , leading twist far from enough

Targets Advances | Heavy meson LCDA | Light baryon LCDA

Challenge from light meson LCDA to light baryon LCDA







- **From effective Wilson length → Signal to Noise Ratio**
  - Signal to Noise Ratio * 8 -> Statistics * 64
  - Totally 64*10*2
- **Dynamic enhance: SNR 200% improve (eff 400%) !**
- **Coding in Pyquda: Computing efficiency 800% !**

Thanks for X.Y.Jiang, arXiv: 2411.08461

x-dependent Light Baryon LCDAs from Lattice QCD 沐华 张 15:10 - 15:30 | Hybrid Renormalization Scheme for the Lattice Calculation of Baryon LCDAs 灏阳 白

Summary & Outlook

Framework advances:

- Renormalon resummation(LRR)
- Renormalization group resummation(RGR)
- Two loop matching

Simulation and analysis advances:

- Simulation improvement
- Inverse Fourier Transform
- Pyquda coding

LCDA
Precision Calculation
Seriously！

Physical targets advances:

- Consistent and precision meson LCDA
- Heavy meson LCDA
- Light baryon LCDA

Future potential:

- Gradient flow
- Lanczos
- ANN … …

Thanks for Your Attention！

Backup slides

We can also take start from the light-cone decomposition. We start from some convention on light-cone

$$\gamma^{\pm} = 1/\sqrt{2}\,(\gamma^0 \pm \gamma^3) \;\; \text{or} \;\; \text{on lattice} = 1/\sqrt{2}\,(\gamma^t \pm i\gamma^z) \tag{4}$$

$$P_{\pm} = (1/2)\,\gamma^{\mp}\,\gamma^{\pm}$$
$$\psi = \psi_+ + \psi_-, \quad \psi_{\pm} \equiv P_{\pm}\,\psi$$

For a quark bilinears constructed with only ψ_+ components describe the leading Fock states of mesons:

$$u_+^{\dagger}\,\gamma_5\,d_+ = \sqrt{2}\,\bar{u}\,\gamma_+\,\gamma_5\,d \tag{5}$$

Here, we have used

$$P_+\,\gamma_5\,P_+ = \sqrt{2}\,\gamma^+\,\gamma_5 \tag{6}$$

It will be trival for a leading twist contribution of a pseudoscalar meson should be $\gamma_+\gamma_5$ and $\gamma^t + i\gamma^z$ on lattice