

Two-loop extraction of the pion-nucleon sigma term

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- Introduction
- The tension between lattice QCD and phenomenology
- Two-loop extraction of pion-nucleon sigma term
- Summary and outlook

Introduction

Why nucleon sigma term?

- Anatomy of the nucleon mass
 - Trace of the energy-momentum tensor

$$T_{\mu}^{\mu} = \left[\frac{\beta_{\text{QCD}}}{2g} G_{\mu\nu}^a G_a^{\mu\nu} + \sum_q \gamma_m m_q \bar{q}q \right] + \sum_q m_q \bar{q}q$$

- Explicit chiral symmetry breaking term (classical)
- γ_m anomalous dimension of the mass operator.
- Trace anomaly owing to quantum effects

- Nucleon mass budget

$$m_N = \langle N | \underbrace{\frac{\beta_{\text{QCD}}}{2g} G_{\mu\nu}^a G_a^{\mu\nu} + \gamma_m \sum_q m_q \bar{q}q}_{\text{Trace anomaly}} + \underbrace{(m_u \bar{u}u + m_d \bar{d}d + \cdots)}_{\text{sigma term}} | N \rangle$$

Why nucleon sigma term?

- Dark matter detection [Hill & Solon PLB(2012)]
 - Scalar coupling of the nucleon

$$\langle N | m_q \bar{q}q | N \rangle = m_N f_q$$

- Dark matter of scalar nature

$$\mathcal{L}_{\chi q} = C \frac{m_q}{\Lambda^3} \bar{\chi} \chi \bar{q} q \quad \Rightarrow \quad \sigma_{\chi N} = \frac{m_{\chi N}^2}{\pi \Lambda^6} \left| \sum_q C_q f_q \right|^2$$

- Condensate in nuclear matter [Kaiser & Weise PLB(2008)]

$$\frac{\langle \Psi | \bar{q}q | \Psi \rangle}{\langle 0 | \bar{q}q | 0 \rangle} = 1 - \frac{\rho}{F_\pi^2} \left\{ \frac{\sigma_{\pi N}}{M_\pi^2} \left[1 - \frac{3k_f^2}{10m_N^2} + \frac{9k_f^4}{56m_N^4} \right] + \dots \right\}$$

Basics of pion-nucleon sigma term

- **Definition**

- Scalar form factor of the nucleon $\sigma(t) = \frac{1}{2m_N} \langle N(p') | \hat{m}(\bar{u}u + \bar{d}d) | N(p) \rangle$, $t = (p' - p)^2$

- Nucleon sigma term (**Direct approach**) $\sigma_{\pi N} = \sigma(t = 0) = \frac{1}{2m_N} \langle N(p) | \hat{m}(\bar{u}u + \bar{d}d) | N(p) \rangle$

- Feynman-Hellman theorem $\frac{\partial m_N^2}{\partial \hat{m}} = \langle N(p) | \bar{u}u + \bar{d}d | N(p) \rangle$

- **Therefore, the nucleon sigma term via FH theorem. (**FH approach**)**

- Gell-Mann-Oakes-Renner relation: $M^2 = 2B_0\hat{m}$

$$\sigma_{\pi N} = \hat{m} \frac{\partial m_N}{\partial \hat{m}} = M^2 \frac{\partial m_N}{\partial M^2}$$

Nucleon mass as input

Status of sigma term from lattice QCD

- **FH approach**

- χ QCD 15 (2+1): $\sigma_{\pi N} = 45.9(7.4)(2.8)$ MeV [Yang, et al., [χ QCD], PRD(2016)]
- BMW 15 (2+1): $\sigma_{\pi N} = 38(8)(8)$ MeV [Durr, et al., PRL(2016)]
- RQCD 22 (2+1): $\sigma_{\pi N} = 43.9(4.7)$ MeV [Bali, et al., [RQCD], JHEP(2023)]
- ...

- **Direct approach**

- NME 21(2+1+1): $\sigma_{\pi N} = 59.6(7.4)$ MeV [Gupta, et al., PRL(2021)]
- Mainz 23 (2+1): $\sigma_{\pi N} = 43.7(3.6)$ MeV [Agadjanov, et al., PRL(2023)]
- ETMC 24 (2+1+1): $\sigma_{\pi N} = 41.9(8.1)$ MeV [Alexandrou, et al., PRD(2025)]
-
- In the direct approach, the $N\sigma$ or $N\pi\pi$ effects might be important [Barca, et al., arXiv:2508.09006 [hep-lat].

The tension between lattice QCD and phenomenology

Roy-Steiner equation determination

- Basic formula:
$$\sigma_{\pi N} = \underbrace{F_\pi^2 (d_{00}^+ + 2M_\pi^2 d_{01}^+)}_{\text{RS equation}} + \underbrace{(\Delta_D - \Delta_\sigma)}_{\text{Dis. Relation}} - \underbrace{\Delta_R}_{\text{B}\chi\text{PT}}$$

- Subthreshold parameters output of the RS equations

$$d_{00}^+ = -1.36(3)M_\pi^{-1} \quad [\text{KH} : -1.46(10)M_\pi^{-1}]$$

$$d_{01}^+ = 1.16(3)M_\pi^{-3} \quad [\text{KH} : 1.14(2) M_\pi^{-3}]$$

- $\Delta_D - \Delta_\sigma = (1.8 \pm 0.2) \text{ MeV}$ [Hoferichter, Ditsche, Kubis and Meissner, JHEP(2012)]
- $\Delta_R \lesssim 2 \text{ MeV}$ [Bernard, Kaiser and Meissner, PLB(1996)]
- Isospin breaking in the Cheng-Dashen theorem shifts $\sigma_{\pi N}$ by +3 MeV

$$\sigma_{\pi N} = (59.1 \pm 1.9_{\text{RS}} \pm 3.0_{\text{LET}}) \text{ MeV} = (59.1 \pm 3.5) \text{ MeV}$$

[NB: recover $\sigma_{\pi N} = 45 \text{ MeV}$ if KH80 scattering lengths are used] [Courtesy of Ulf-G. Meissner]

The long-standing tension

- Phenomenological (RS equation)

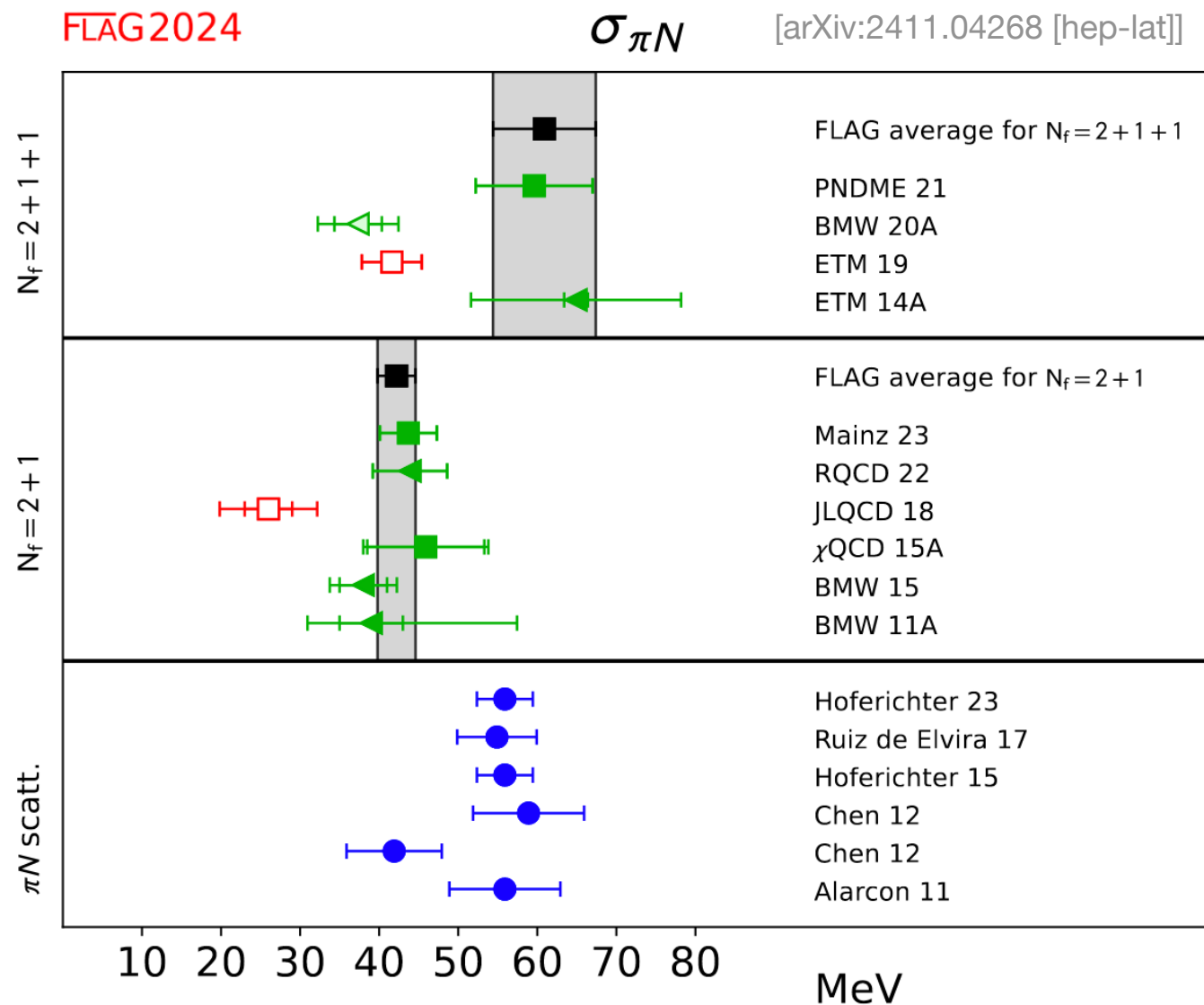
$$\sigma_{\pi N} = 59.1(3.5) \text{ MeV}$$

- FLAG Average $N_f = 2 + 1 + 1$

$$\sigma_{\pi N} = 60.9(6.5) \text{ MeV}$$

- FLAG Average $N_f = 2 + 1$

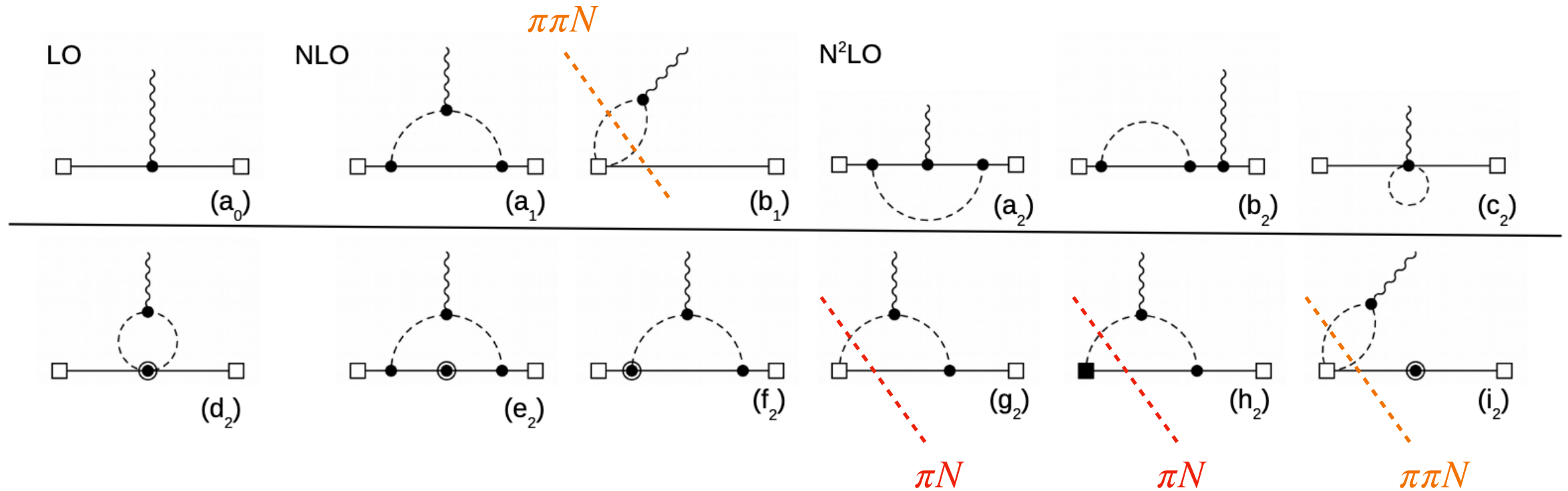
$$\sigma_{\pi N} = 42.2(2.4) \text{ MeV}$$



Tension between phenomenological and $N_f = 2 + 1$ lattice QCD determinations.

Excited-state contamination?

- ESC from an EFT point of view [Gupta, et al., PRL(2021)]



- Spin-1/2, positive parity: N , πN , $\pi\pi N$, ...

Excited-state contamination?

- **NME 21** [Gupta, et al., PRL(2021)]

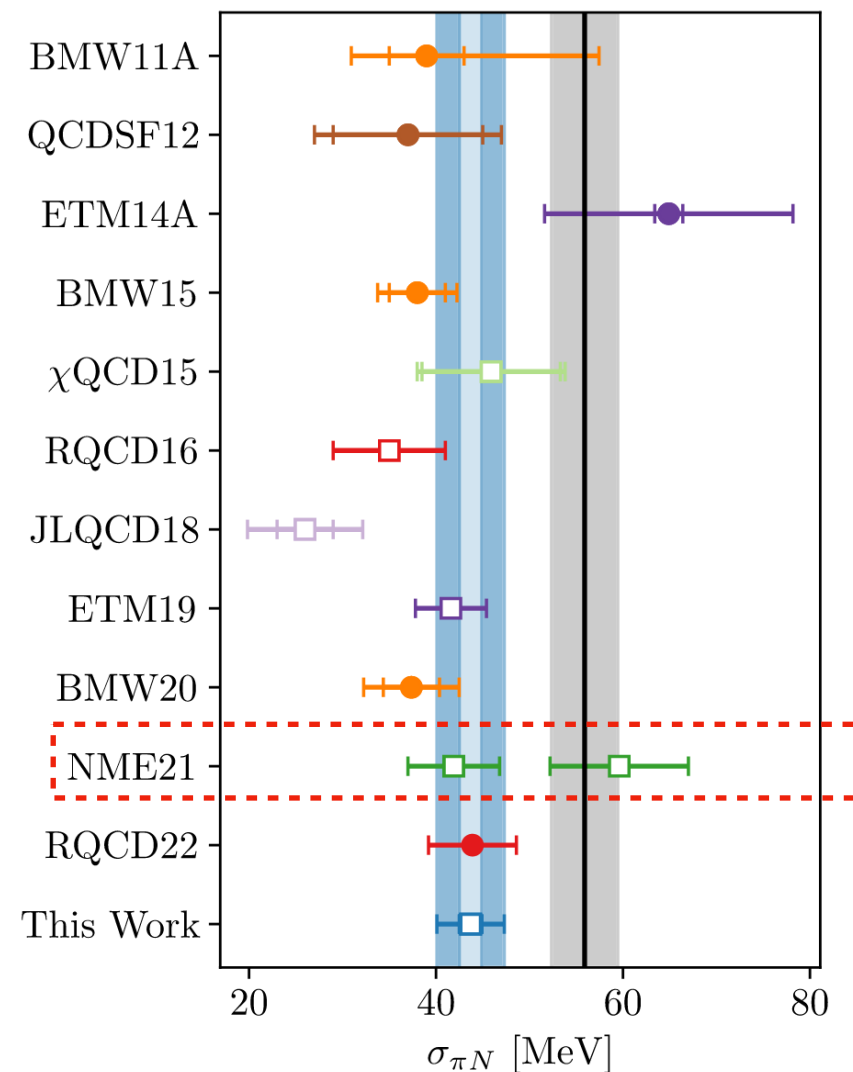
- πN & $\pi\pi N$ excited states (e.s.)

$$\sigma_{\pi N} = \begin{cases} 41.9(4.9) \text{ MeV} & \text{w/o e. s.} \\ 59.6(7.4) \text{ MeV} & \text{w e. s.} \end{cases}$$

- **Mainz 23** [Agadjanov, et al., PRL(2023)]

- An upward trend for $\sigma_{\pi N}$, when using priors similar to NME21, albeit not as pronounced.

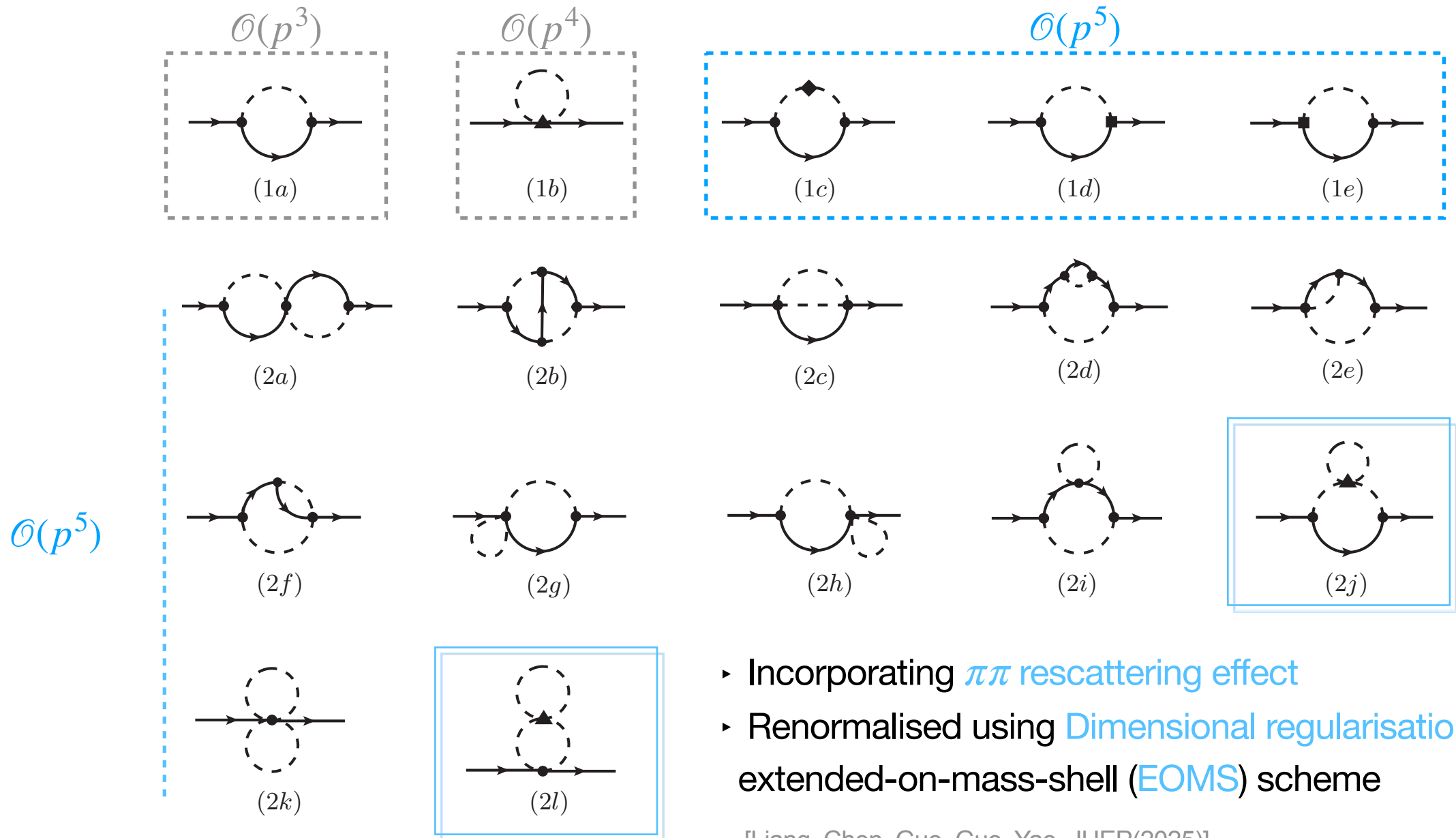
$$\sigma_{\pi N} = 43.7(3.6) \text{ MeV}$$



The subtraction of ESC does not play the sole role in alleviating the tension!

Two-loop extraction of pion-nucleon sigma term

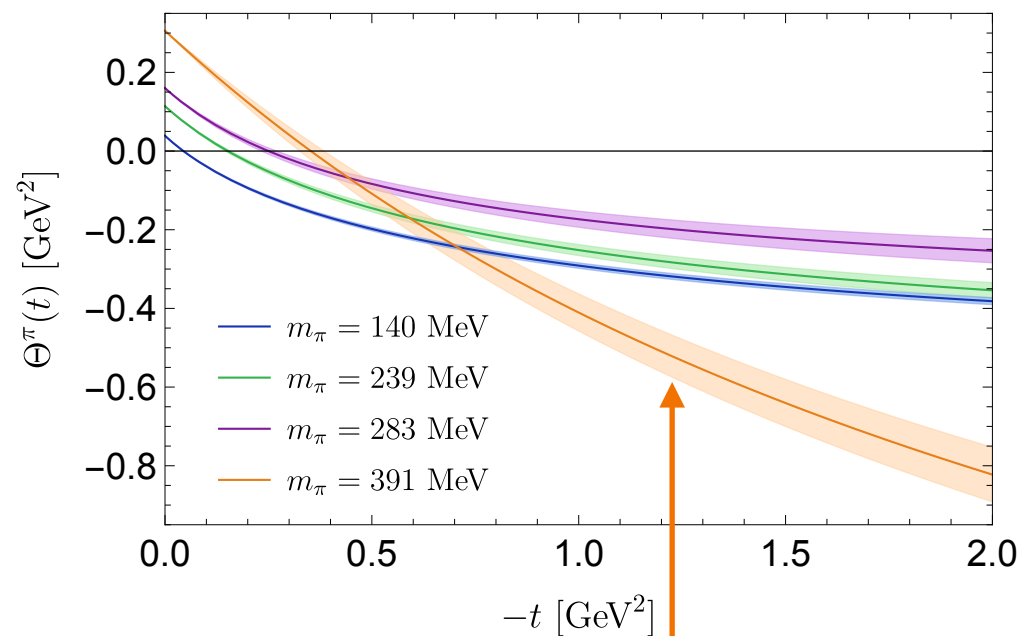
The nucleon mass at two-loop order



- Incorporating $\pi\pi$ rescattering effect
- Renormalised using Dimensional regularisation and extended-on-mass-shell (EOMS) scheme

$\pi\pi$ rescattering effect

- Using the $\pi\pi$ scattering phase shifts at unphysical pion masses (239 MeV, 283 MeV, 391 MeV) obtained from Roy equation analyses

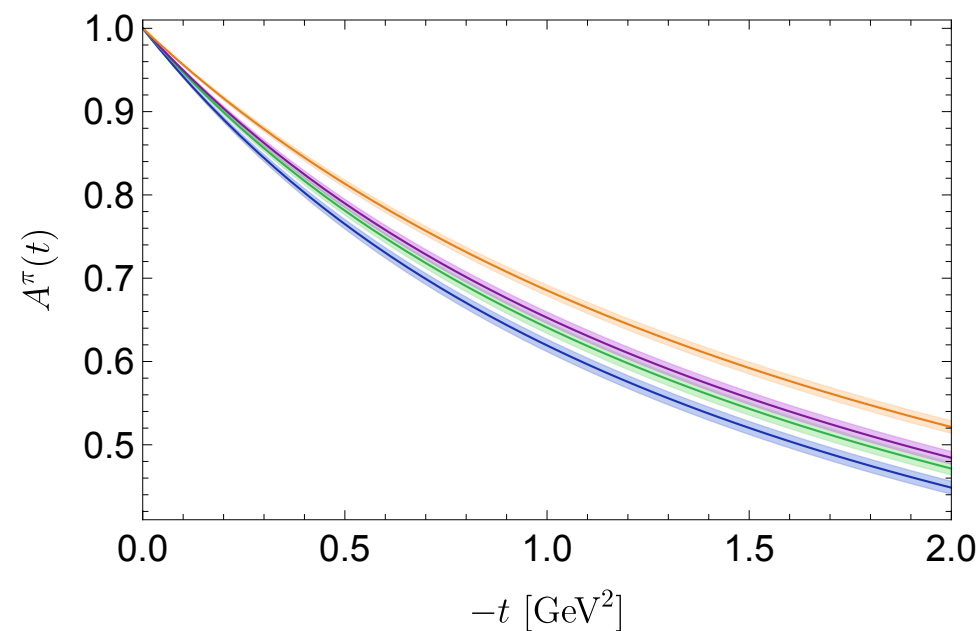


Fast change before $m_\pi = 391$ MeV:
 σ meson becomes a $\pi\pi$ bound state!

[R.A. Briceno et al. [HadSpec], PRL 118 (2017) 022002]

[Cao, Guo, Li, Yao, Nature Com.(2024)]

[Cao, Guo, Li, Wu, Yao, arXiv:2507.05375 [hep-ph]]



Isoscalar $\pi\pi$ contribution is significant
at unphysical pion masses!

Two-loop sigma term in $B\chi$ PT

- Chiral expression up to $\mathcal{O}(p^5)$

- Tree + one loop + two loop

$$\sigma_{\pi N} = -4c_1 M^2 - 4e_m M^4 + \Delta\sigma_{\pi N}^{(1)} + \Delta\sigma_{\pi N}^{(2)}$$

- One-loop: $\Delta\sigma_{\pi N}^{(1)} = \sigma_{\pi N}^{(1a)} + \sigma_{\pi N}^{(1b)} + \sigma_{\pi N}^{(1c)} + \sigma_{\pi N}^{(1d)} + \sigma_{\pi N}^{(1e)}$,

- Two-loop: $\Delta\sigma_{\pi N}^{(2)} = \sigma_{\pi N}^{(2a)} + \sigma_{\pi N}^{(2b)} + \dots + \sigma_{\pi N}^{(2l)} + \sigma_{\pi N}^{(2')} + \sigma_{\pi N}^{\text{sub.}}$.

$$\sigma_{\pi N}^{(1a)} = -\frac{3ig_A^2 M^2 m (J_{11} + M^2 J_{21})}{2F^2}$$

$$\sigma_{\pi N}^{(2a)} = \frac{3g_A^2 M^2 m}{2F^4} (M^4 I_{12011} + M^4 I_{21011} + M^2 I_{02011} + 2M^2 I_{11011} + M^2 I_{20011} + I_{10011} + I_{01011})$$

✓ $J_{\nu_1 \nu_2}$ and $I_{\nu_1 \nu_2 \nu_3 \nu_4 \nu_5}$ are one- and two-loop Feynman integrals.

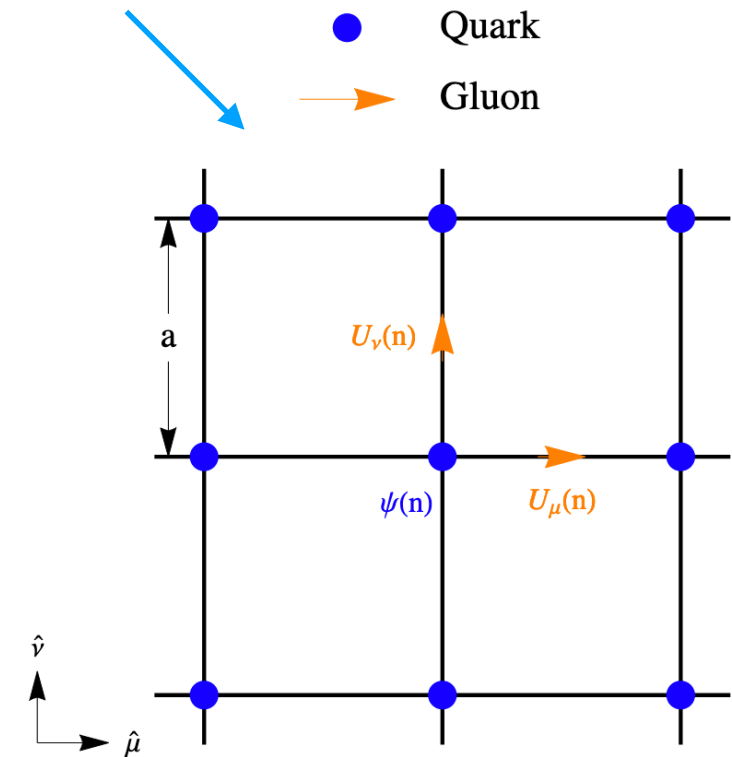
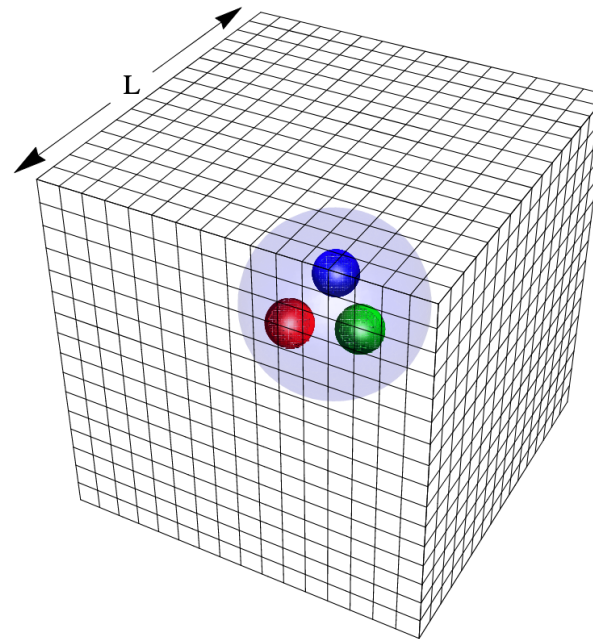
Sigma term on the lattice

- Lattice artifacts: finite-volume & lattice spacing corrections

Continuous Infinite volume

Discrete Finite volume

$$\sigma_{\pi N} \implies \sigma_{\pi N} + \Delta_L \sigma_{\pi N} + b_\pi \frac{a}{\sqrt{t_0}} M_\pi^2,$$



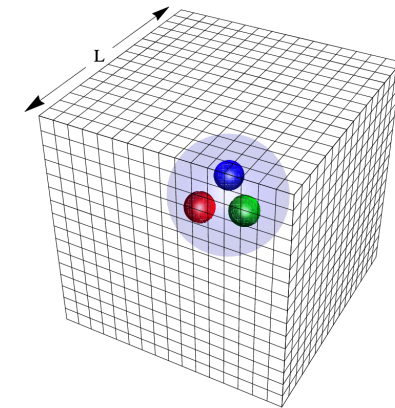
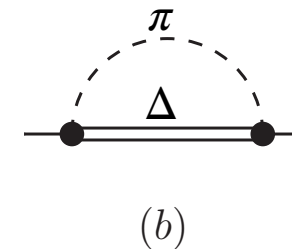
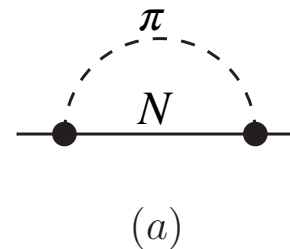
Sigma term on the lattice

- Our treatment of FV corrections

✓ ChPT at finite volume:

$$\Delta_L \sigma_{\pi N} = \Delta_L \sigma_{\pi N}^{(N)}(M_\pi; L) + \Delta_L \sigma_{\pi N}^{(\Delta)}(M_\pi; L)$$

[Liang, Yao, JHEP(2022)]



✓ Our full expression (N for example):

$$\Delta_L \sigma_{\pi N}^{(N)}(M_\pi; L) = \sum_{n_s} \vartheta(n_s) \int_0^1 dx_1 \frac{3g_A^2 m M_\pi^2}{32F^2 \pi^2} \left[2K_0(\sqrt{\mathcal{M}_N^2}) + \frac{L^2 M_\pi^2 n_s (x_1 - 1)}{\sqrt{\mathcal{M}_N^2}} K_1(\sqrt{\mathcal{M}_N^2}) \right]$$

Ensemble	H102	N101	H105	C101	S400	N451	D450	D452	N203	S201	N200	D200	E250	N302	J303	E300
BChPT w/o Δ	-4.72	-0.94	-6.17	-1.49	-8.22	-1.70	-0.68	-1.11	-2.86	-16.3	-3.90	-1.73	-0.52	-9.07	-3.80	-1.06
BChPT w Δ	-9.17	-1.62	-11.7	-2.52	-16.5	-3.03	-1.11	-1.75	-5.37	-33.3	-7.25	-2.89	-0.76	-18.3	-6.94	-1.69

✓ Mainz 23 uses the expression at infinite L and treat b_L free:

$$\Delta_L \sigma_{\pi N}^{(N)}(M_\pi; L) \xrightarrow{L \rightarrow \infty} b_L \left(\frac{M_\pi^3}{M_\pi L} - \frac{M_\pi^3}{2} \right) \exp(-M_\pi L), \quad b_L = 9g_A^2/(8\pi F^2).$$

Sigma term on the lattice

- **Direct approach: Mainz 23** [Agadjanov, et al., PRL(2023)]

- Effective scalar form factor (ESFF)

$$G_S^{\text{eff}}(t, t_s) \equiv \text{Re} \frac{C_3(t, t_s; \mathbf{0})}{C_2(t_s; \mathbf{0})}$$

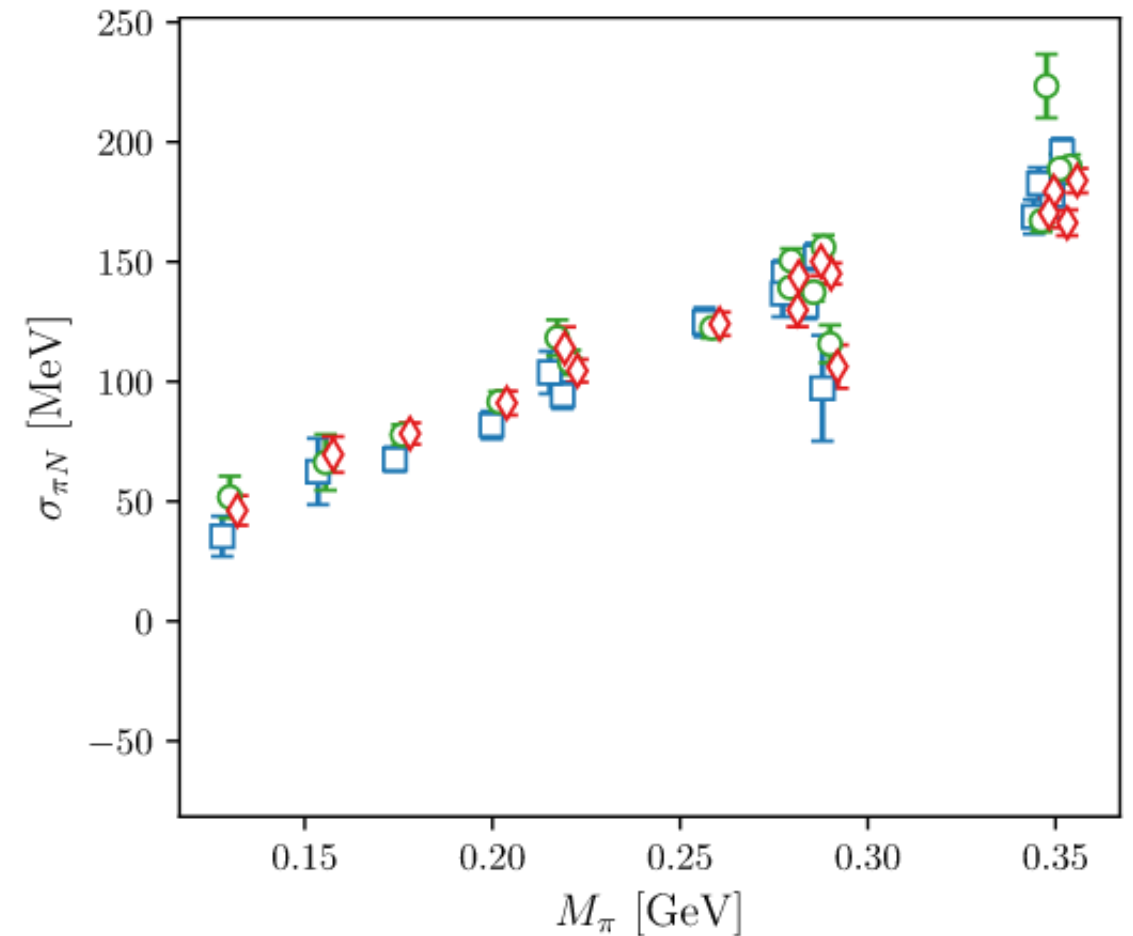
- Summation method: summed correlator (SC)

$$S(t_s) = a \sum_{t=a}^{t_s-a} G_S^{\text{eff}}(t, t_s) \xrightarrow{t_s \gg \Delta^{-1}} b_1 + (t_s - a)G_S$$

Δ is the energy gap between the g.s. and 1st e.s

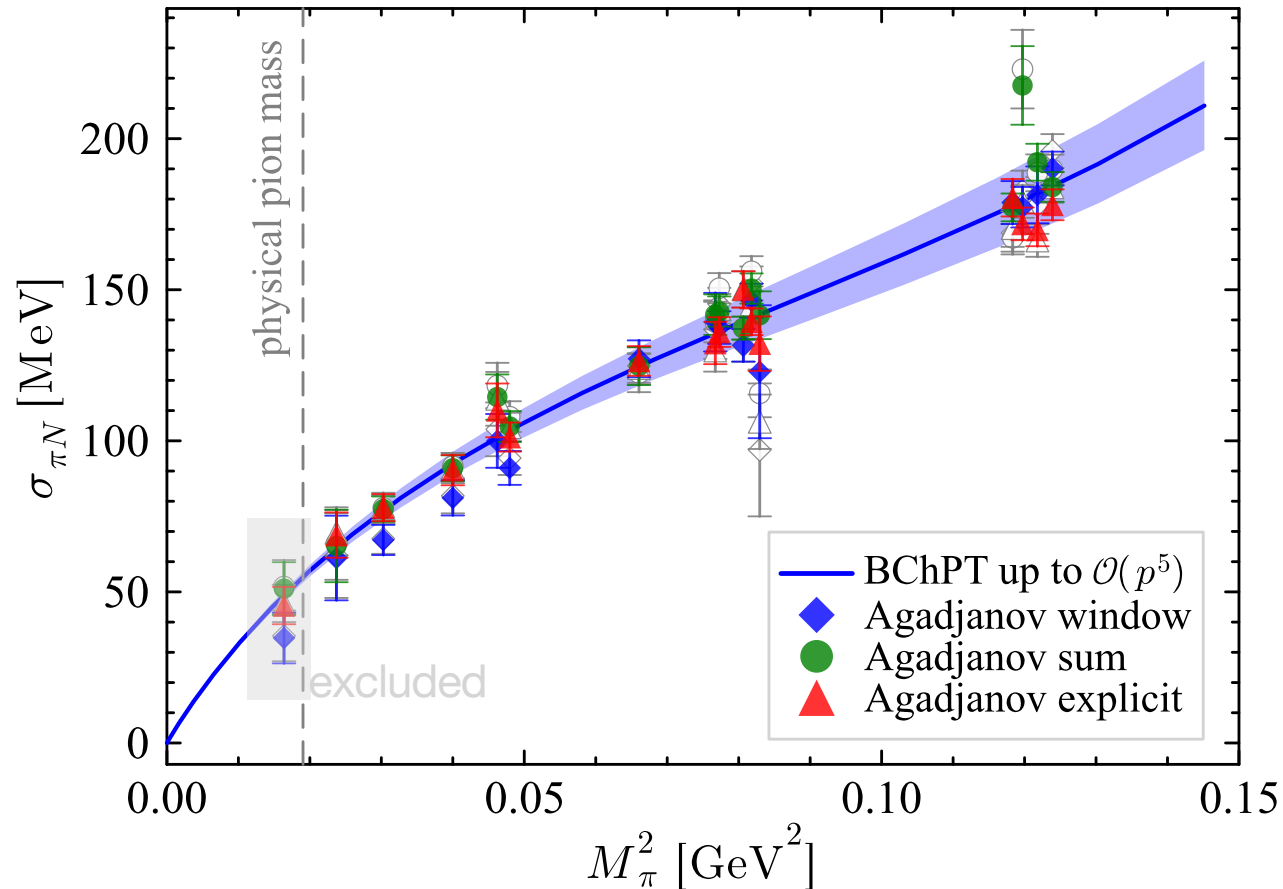
- **Excited-state analysis**

- ✓ “**w**indow”: window average of the SC
- ✓ “**g**reen”: explicit two-state fit to the SC
- ✓ “**E**xplicit”: explicit two-state fit to the ESFF



Chiral extrapolation

- Fit to Mainz 23



Fit procedure

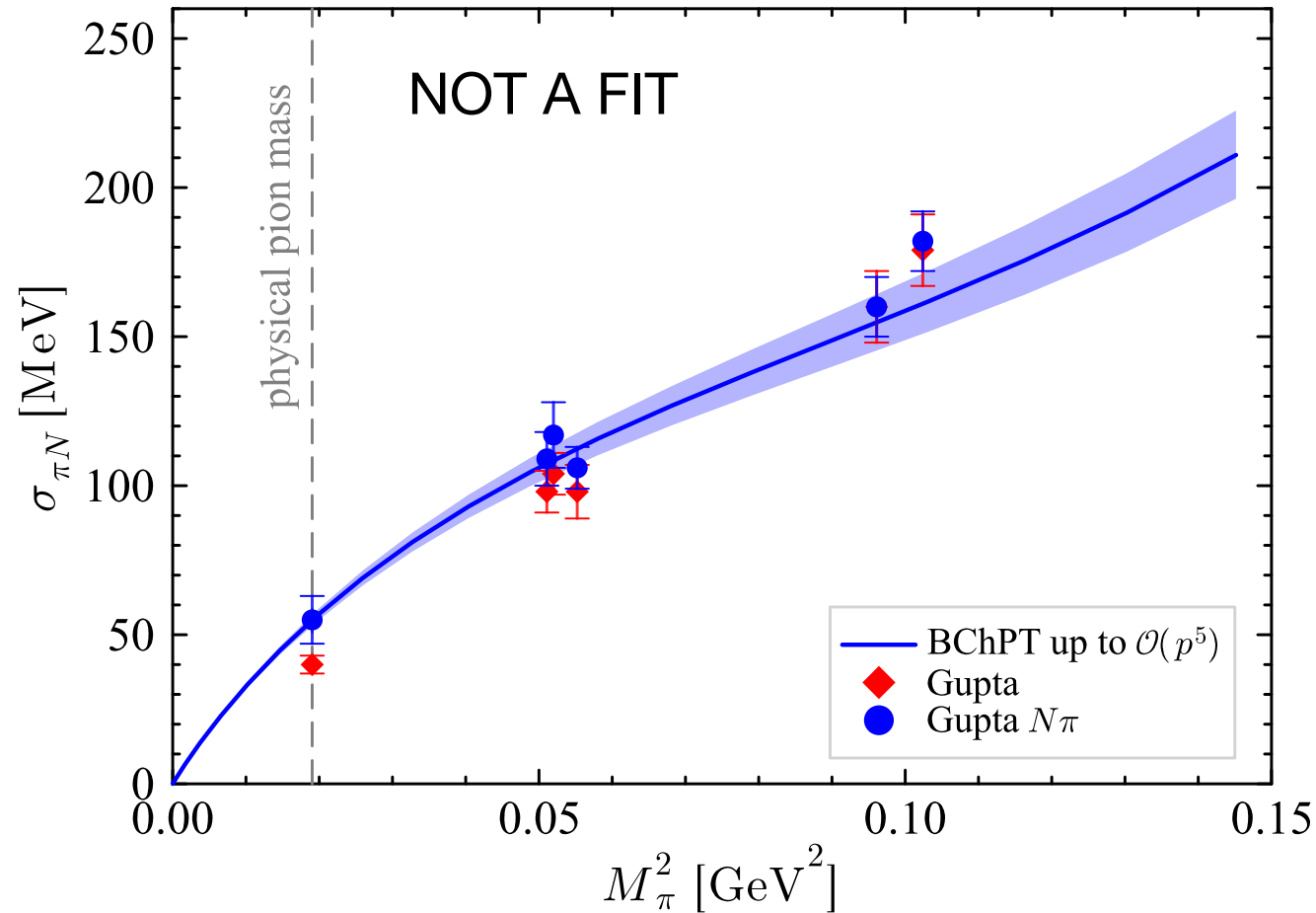
$$\chi^2 = \chi_{m_N}^2 + \omega \cdot (\chi_{\text{win}}^2 + \chi_{\text{sum}}^2 + \chi_{\text{exp}}^2)$$

- Constrained by physical m_N
- Three ESC approaches with equal weighting $\omega = 1/3$
- Subtracting Finite volume corrections and lattice spacing effects

LECs	Values	Correlation matrix		
		$\tilde{m}$	$\tilde{c}_1$	$\tilde{e}_m$
$\tilde{m}$ [MeV]	863.9 ± 2.1	1.000	0.944	-0.662
$\tilde{c}_1$ [GeV^{-1}]	-1.06 ± 0.02		1.000	-0.728
$\tilde{e}_m$ [GeV^{-3}]	-5.59 ± 0.21			1.000
$\chi^2/\text{d.o.f.}$	$18.24/(15 + 1 - 4) \simeq 1.52$			

Chiral extrapolation

- Comparison with NME 21 data

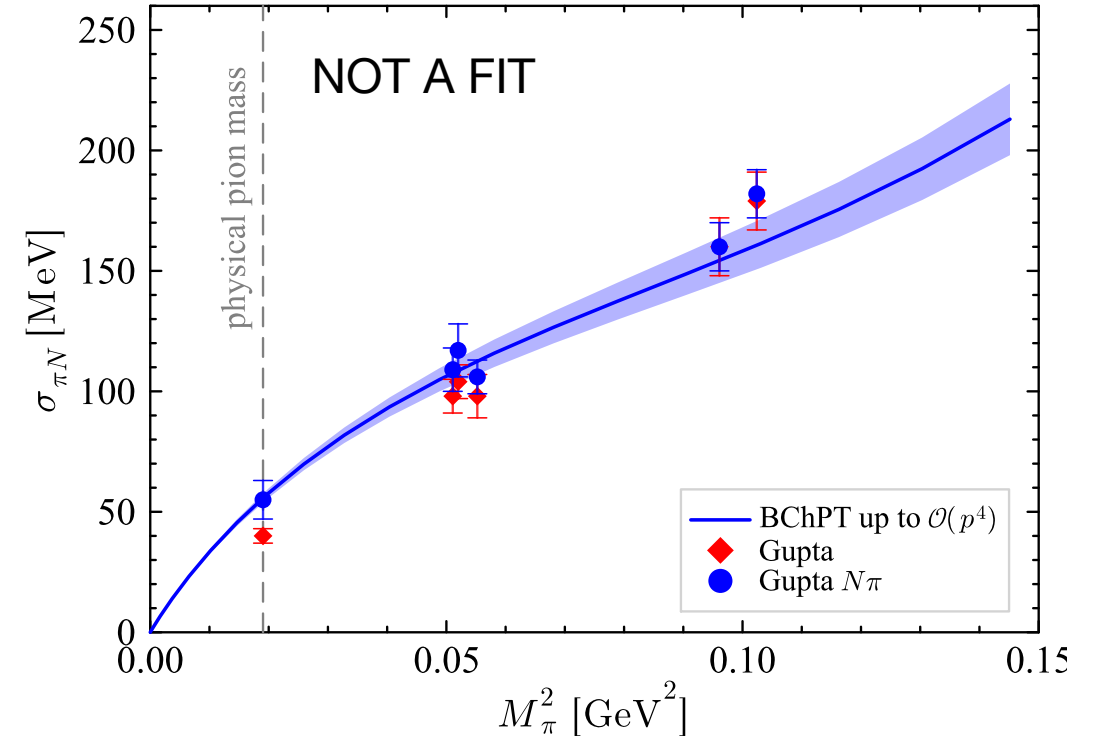
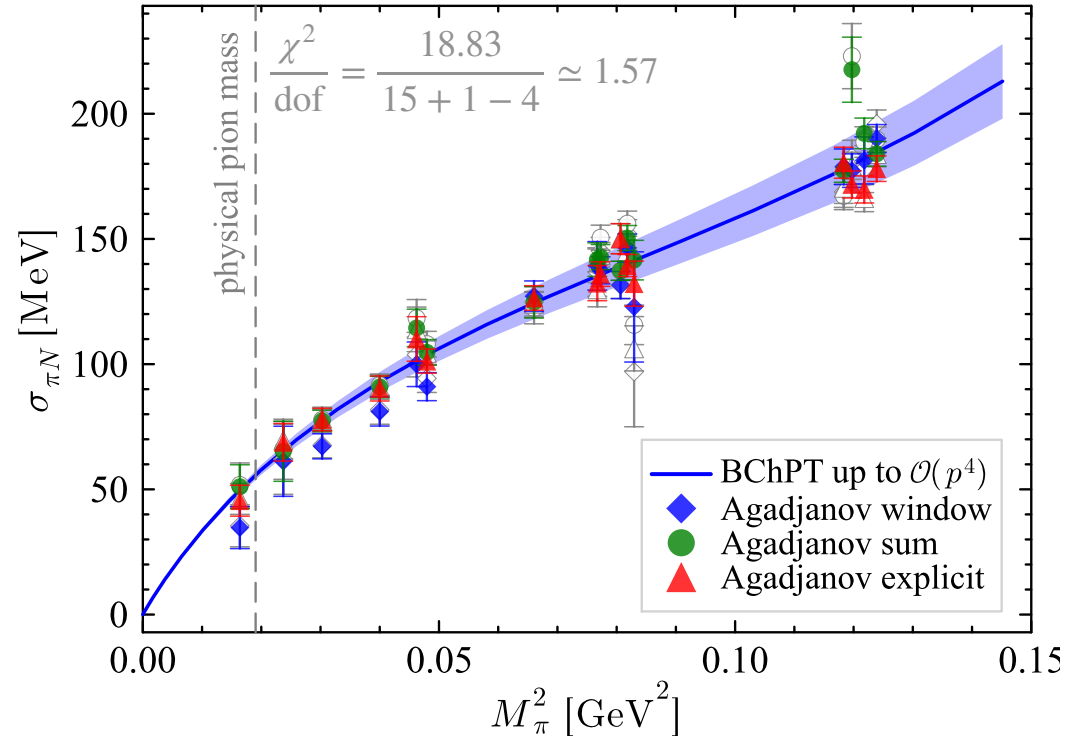


- Sigma term (M_{π^+}): $\sigma_{\pi N} = 55.9(1.9)$ MeV
- Sigma term (M_{π^0}): $\bar{\sigma}_{\pi N} = 53.1(1.8)$ MeV

In remarkable consistency with the dataset in which the ESC is properly accounted for.

Chiral extrapolation

- Comparative fit: up to $\mathcal{O}(p^4)$

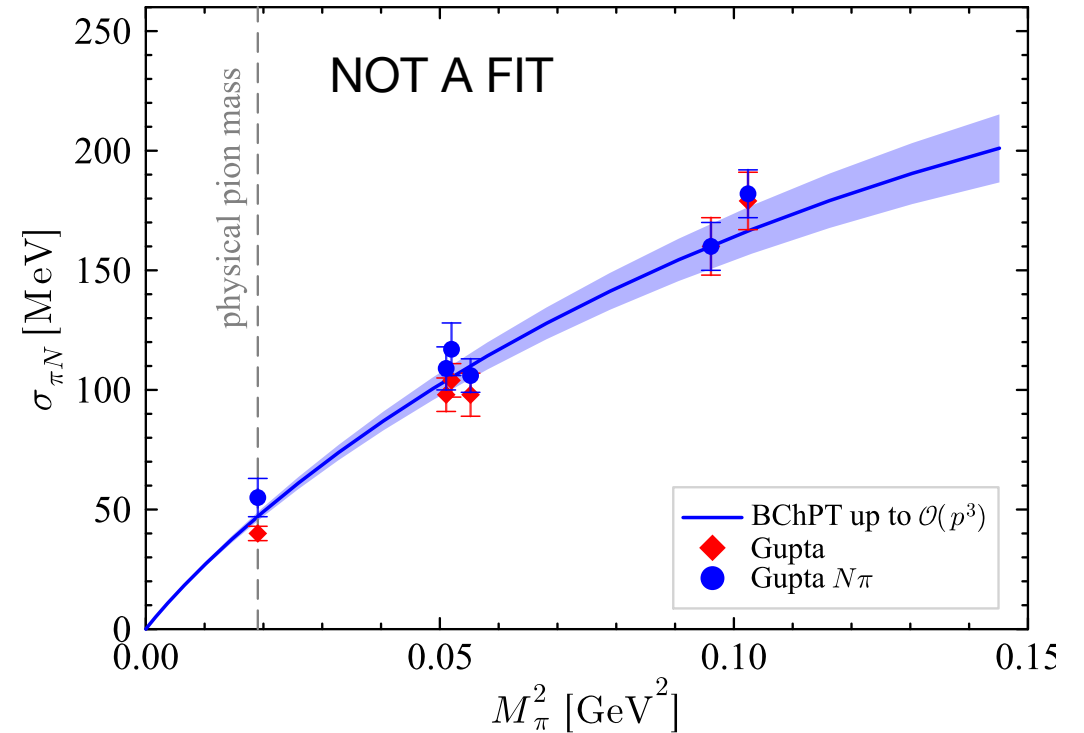
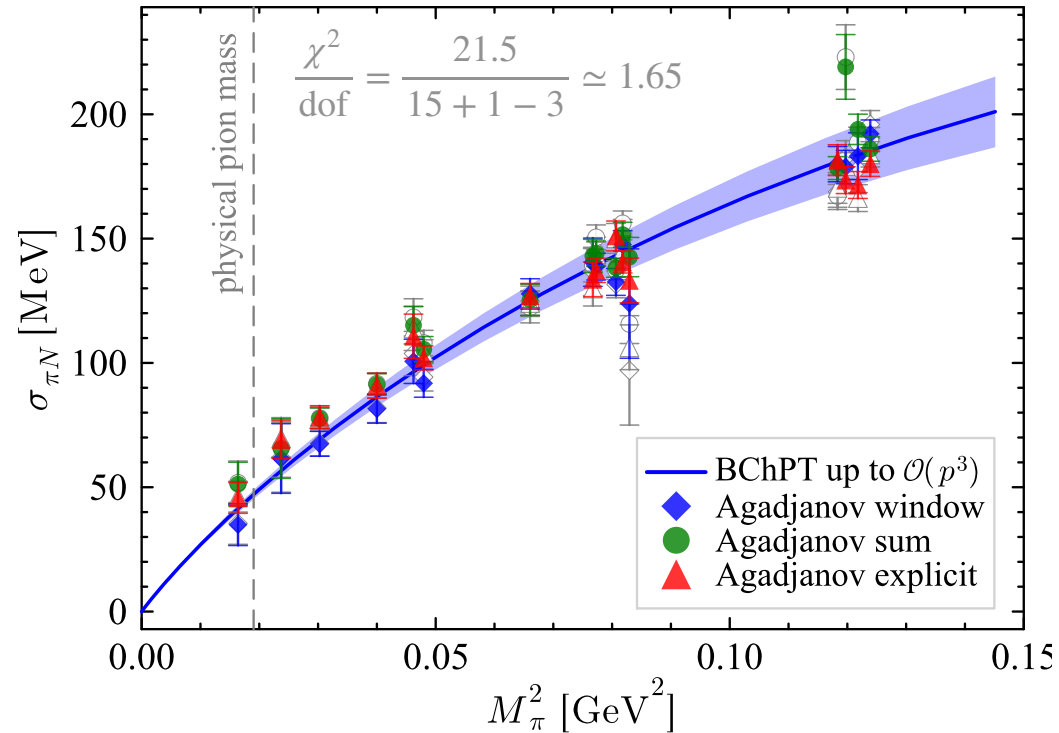


- Parameters: $\tilde{m} = 872.9(1.9)$ MeV, $\tilde{c}_1 = -1.09(2)$ GeV⁻¹, $\tilde{e}_m = -1.34(21)$ GeV⁻³
- Sigma term (M_{π^+}): $\sigma_{\pi N} = 56.7(1.9)$ MeV
- Sigma term (M_{π^0}): $\bar{\sigma}_{\pi N} = 53.9(1.8)$ MeV

The e_m term mimics part of the two-loop contribution through parameter adjustment

Chiral extrapolation

- Comparative fit: up to $\mathcal{O}(p^3)$



- Parameters: $\tilde{m} = 886.7(2)$ MeV, $\tilde{c}_1 = -0.84(2)$ GeV⁻¹
- Sigma term (M_{π^+}): $\sigma_{\pi N} = 48.1(1.9)$ MeV
- Sigma term (M_{π^0}): $\bar{\sigma}_{\pi N} = 45.4(1.8)$ MeV

The $\mathcal{O}(p^3)$ fit indeed yields small nucleon sigma term!

Two-loop extraction of $\sigma_{\pi N}$

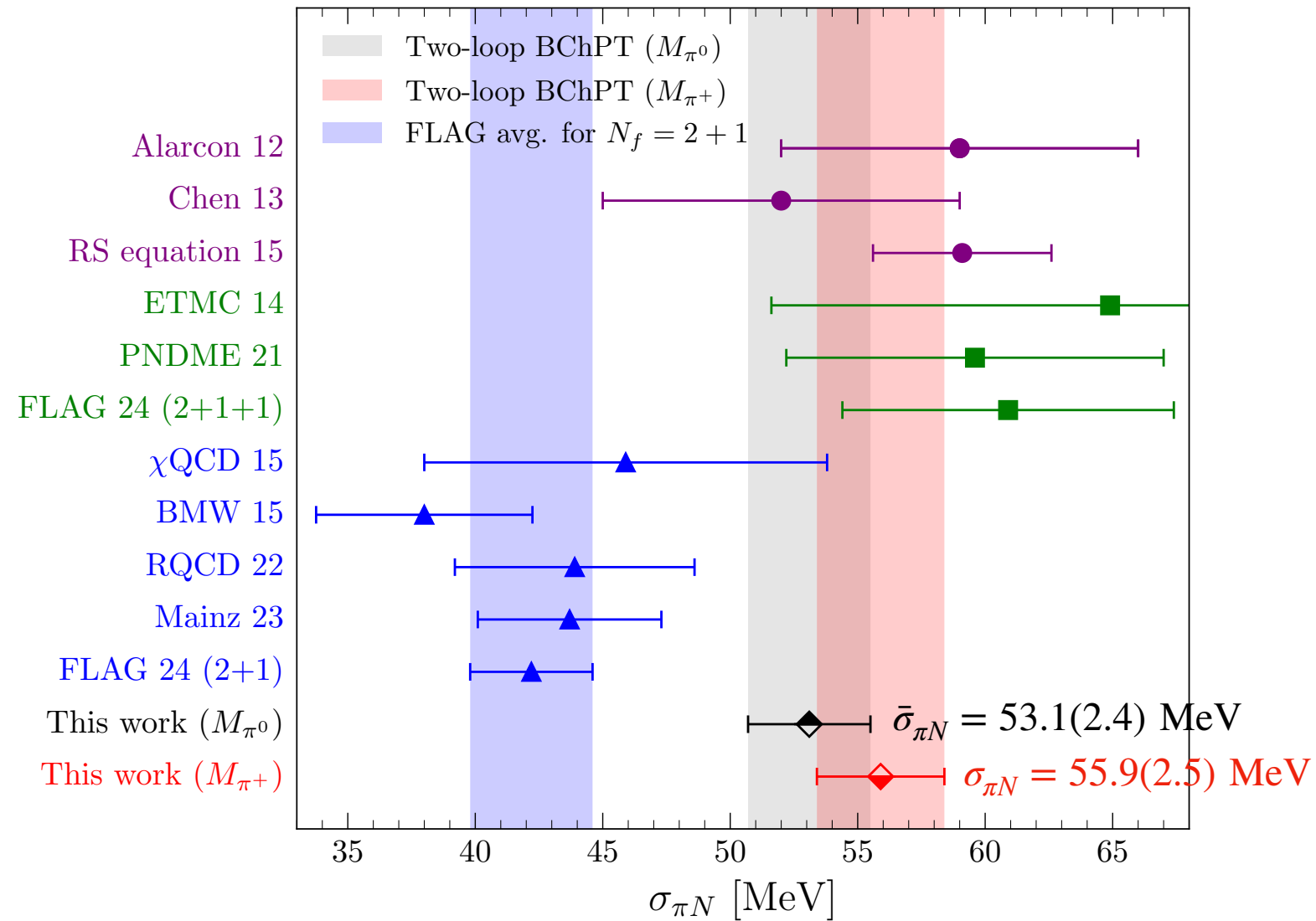
- Precise determination of $\sigma_{\pi N}$ in the physical world ($M_{\pi} = M_{\pi}^+$)

$$\sigma_{\pi N} = 55.9 \pm (1.9)_{\text{stat}} \pm (1.5)_{\text{sys}_1} \pm (0.6)_{\text{sys}_2} \text{ MeV} = 55.9(2.5) \text{ MeV}$$

- Error budget

- (i) **stat**: propagated from the 1σ errors of the fitted parameters ($\tilde{m}, \tilde{c}_1, \tilde{e}_m$);
- (ii) **sys₁**: arising from the errors in the one-loop renormalized LECs (c_2, c_3) involved in the $\mathcal{O}(p^4)$ chiral loop contribution;
- (iii) **sys₂**: due to truncation of the chiral expansion at $\mathcal{O}(p^5)$.

Comparison



Resolving the tension between lattice QCD and phenomenology!

Summary and outlook

Summary and outlook

- The two-loop representation of the sigma term is established in EOMS BChPT for the first time.
 - ▶ **Method 1:** from the nucleon mass via the Feynman-Hellmann theorem.
 - ▶ **Method 2:** a direct calculation of the forward isoscalar-scalar nucleon matrix element.
 - ▶ **Chiral extrapolation:** $N_f = 2 + 1$ lattice QCD data at unphysical quark masses.
 - ▶ **Our result of the sigma term:**

$$\sigma_{\pi N} = 55.9(2.5) \text{ MeV}$$

- The long-standing tension between lattice QCD and dispersive determinations is naturally resolved.
 - ▶ Owing to the incorporation of intermediate $\pi\pi$ rescattering effects that begin to contribute at two-loop order.
 - ▶ The general importance of $\pi\pi$ rescattering effects in observables coupled to isoscalar scalar currents.

Thanks!