# Contraining DVCS Compton Form Factors Using Lattice QCD calculations

Yuan-Yuan Huang

Oct. 2025

Collaborators: Xu Cao, Taifu Feng, K. Kumericki, Yu Lu

中国科学院近代物理研究所
Institute of Modern Physics, Chinese Academy of Sciences

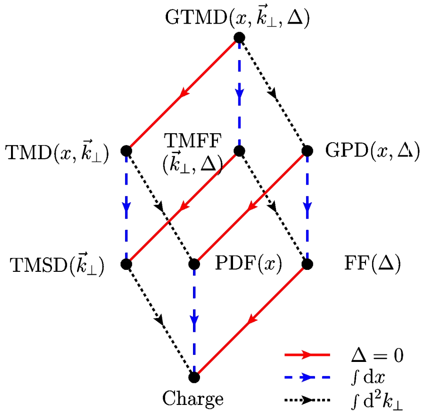
河北大学
HEBEI UNIVERSITY

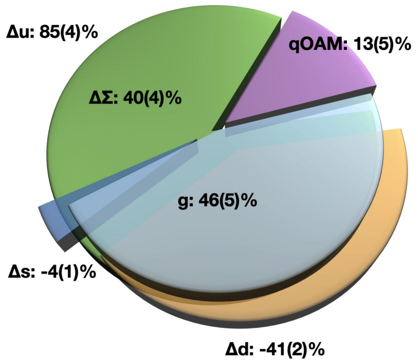
# Contents

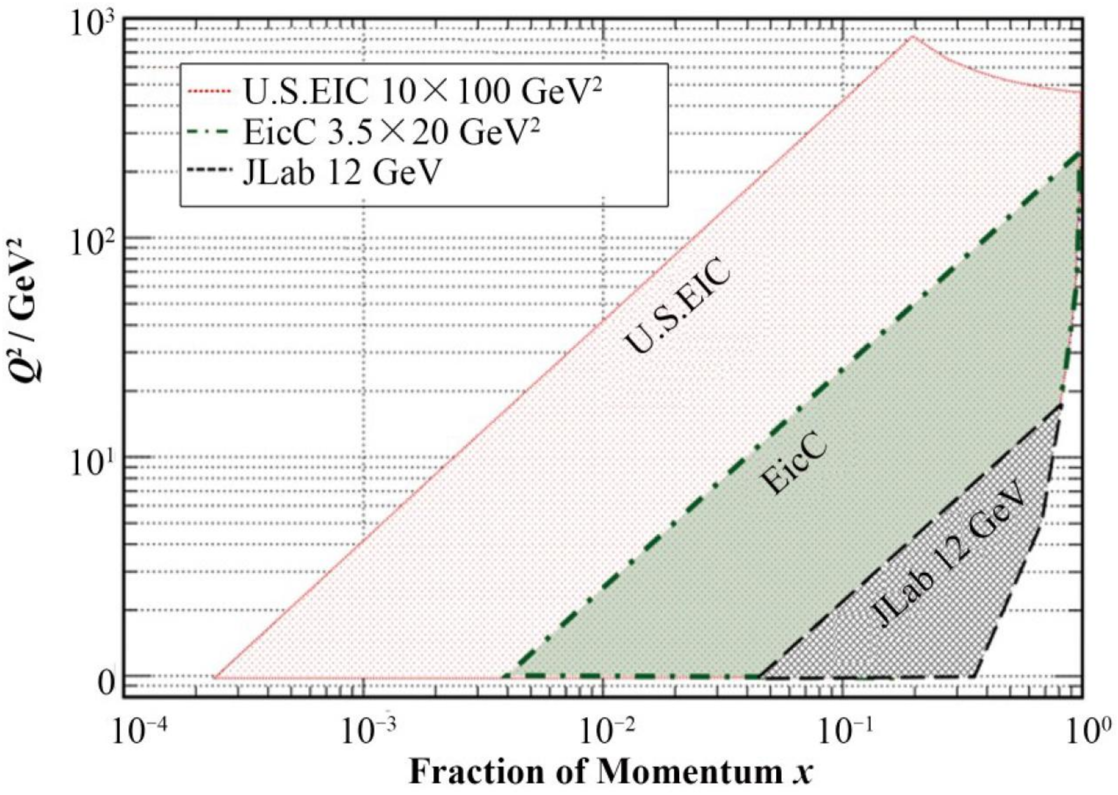
# Introduction

- **GPD: General Parton Distributions (trans. spatial position b┴ & longi. Momentum):**
- How does proton's spin influence the spatial distribution of partons?
- …… probed by the exclusive process

- **From 1D to 3D picture of hadron & nuclei**
- **Origin of the Proton/Meson spin**




- **中国科学: 物理学力学天文学, 50: 112005 (2020)**
- **核技术, 43(2): 020001 (2020) ;Front. Phys. 16, 64701 (2021)**

# Introduction

- **GPD(General Parton Distributions):**
- The key theoretical tool for describing the three-dimensional structure of hadrons.

- **CFF(Compton Form Factor):**
- The bridge connecting GPD theory with experimental observables.CFF is the convolution integral of GPDs and QCD Wilson coefficients

$$\mathcal{F}^q(\xi,t) = \mathcal{C}(x,\xi;Q^2)\otimes F^q(x,\xi,t)$$
$$= e_q^2\int_{-1}^{+1} dx\left[\frac{1}{\xi-x-i\epsilon}-\frac{1}{\xi+x-i\epsilon}\right]F^q(x,\xi,t)$$

$$\widetilde{\mathcal{F}}^q(\xi,t) = \mathcal{C}(x,\xi;Q^2)\otimes \widetilde{F}^q(x,\xi,t)$$
$$= e_q^2\int_{-1}^{+1} dx\left[\frac{1}{\xi-x-i0}+\frac{1}{\xi+x-i0}\right]\widetilde{F}^q(x,\xi,t)$$

$$\Re e\mathcal{F}(\xi,t) = e_q^2\,P.V.\int_{-1}^{+1} dx\left[\frac{1}{\xi-x}-\frac{1}{\xi+x}\right]F^q(x,\xi,t)$$

$$\Im m\mathcal{F}(\xi,t) = \pi e_q^2\Big(F^q(\xi,\xi,t)-F^q(-\xi,\xi,t)\Big)$$

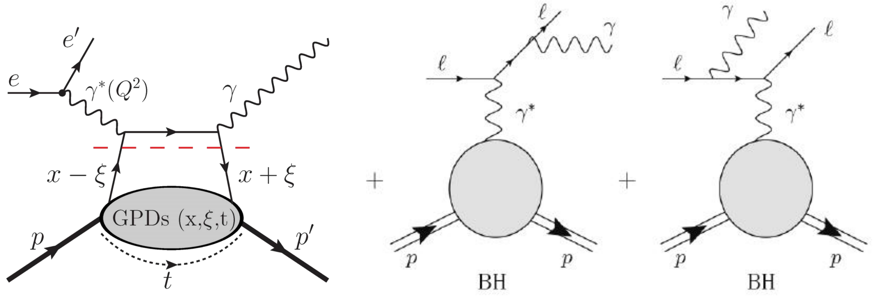
$$\Re e\widetilde{\mathcal{F}}(\xi,t) = e_q^2\,P.V.\int_{-1}^{+1} dx\left[\frac{1}{\xi-x}+\frac{1}{\xi+x}\right]\widetilde{F}^q(x,\xi,t)$$

$$\Im m\widetilde{\mathcal{F}}(\xi,t) = \pi e_q^2\Big(\widetilde{F}^q(\xi,\xi,t)+\widetilde{F}^q(-\xi,\xi,t)\Big)$$

arXiv:2207.10766 [hep-ph].

# Introduction

- **DVCS(Deeply Virtual Compton Scattering):**
- By measuring CFFs via DVCS, GPDs are indirectly decoded. The Bethe-Heitler process is a background process in DVCS measurements.




$$|T|^2 = |T_{\rm BH} + T_{\rm DVCS}|^2 = |T_{\rm BH}|^2 + |T_{\rm DVCS}|^2 + \mathcal{I}$$

$$\mathcal{I} = T^*_{BH}T_{DVCS} + T^*_{DVCS}T_{BH}$$

- **CFFs measured by the asymmetries:**

$$A_{LU,I}^{\sin\phi} \propto \mathrm{Im}\left[F_1\mathcal{H} + \xi(F_1+F_2)\widetilde{\mathcal{H}} - \frac{t}{4m^2}F_2\mathcal{E}\right],$$

$$A_{UL,I}^{\sin\phi} \propto \mathrm{Im}\left[\xi(F_1+F_2)(\mathcal{H}+\frac{\xi}{1+\xi}\mathcal{E}) + F_1\widetilde{\mathcal{H}} - \xi(\frac{\xi}{1+\xi}F_1 + \frac{t}{4M^2}F_2)\widetilde{\mathcal{E}}\right]$$

$$A_{LL,I}^{\cos\phi} \propto \mathrm{Re}\left[\xi(F_1+F_2)(\mathcal{H}+\frac{\xi}{1+\xi}\mathcal{E}) + F_1\widetilde{\mathcal{H}} - \xi(\frac{\xi}{1+\xi}F_1 + \frac{t}{4M^2}F_2)\widetilde{\mathcal{E}}\right]$$

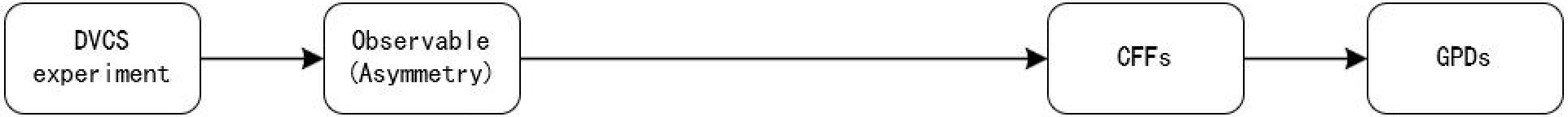
$$A_{UT,I}^{\sin(\phi-\phi_s)\cos\phi} \propto \mathrm{Im}\left[-\frac{t}{4M^2}(F_2\mathcal{H} - F_1\mathcal{E}) + \xi^2(F_1 + \frac{t}{4M^2}F_2)(\mathcal{H}+\mathcal{E}) - \xi^2(F_1+F_2)(\widetilde{\mathcal{H}} + \frac{t}{4M^2}\widetilde{\mathcal{E}})\right],$$

$$A_{UT,I}^{\cos(\phi-\phi_s)\sin\phi} \propto \mathrm{Im}\left(F_2\widetilde{\mathcal{H}} - F_1\xi\widetilde{\mathcal{E}}\right),$$

$$A_{LT,I}^{\sin(\phi-\phi_s)\sin\phi} \propto \mathrm{Re}\left(F_2\mathcal{H} - F_1\mathcal{E}\right),$$

$$A_{LT,I}^{\cos(\phi-\phi_s)\cos\phi} \propto \mathrm{Re}\left(F_2\widetilde{\mathcal{H}} - F_1\xi\widetilde{\mathcal{E}}\right).$$

# Introduction



# Introduction

- All order dispersion relation with arbitrary subtraction: **Eur.Phys.J.C 85 (2025) 1, 105 e-Print: 2410.13518**

$$\sum_{j\ \mathrm{even}}^{n} h_j \xi^{-j} = \Re\mathcal{H}(\xi) - \frac{1}{\pi}\int_0^1 \Im\mathcal{H}(x)\left(\frac{x}{\xi}\right)^n \left[\frac{1}{\xi - x} - (-1)^n \frac{1}{\xi + x}\right] \mathrm{d}x$$

- for j=0：
$$\mathrm{Re}\mathcal{H}(\xi, t) = \Delta(t) + \frac{1}{\pi}\mathrm{P.V.}\int_0^1 dx \left(\frac{1}{\xi - x} - \frac{1}{\xi + x}\right)\mathrm{Im}\mathcal{H}(x, t)$$
e0 = - $\Delta(t)$ for CFF E

- for j>0:
$$h_{2j} = \frac{2}{\pi}\int_0^1 \Im\mathcal{H}(\xi')\,(\xi')^{2j-1}\,\mathrm{d}\xi'$$
and e_{2j} for CFF E

These higher order subtraction constants are related to generalized form factors

$$\int_{-1}^{1} \mathrm{d}x\, x^{m-p_a} H^a(x,\xi,t) = \sum_{j=0}^{\lfloor \frac{m}{2} \rfloor} A^a_{m;2j}(t)(2\xi)^{2j} + \mathrm{mod}(2,m)(2\xi)^{m+1} C^a_m(t),$$

$$\int_{-1}^{1} \mathrm{d}x\, x^{m-p_a} E^a(x,\xi,t) = \sum_{j=0}^{\lfloor \frac{m}{2} \rfloor} B^a_{m;2j}(t)(2\xi)^{2j} - \mathrm{mod}(2,m)(2\xi)^{m+1} C^a_m(t),$$

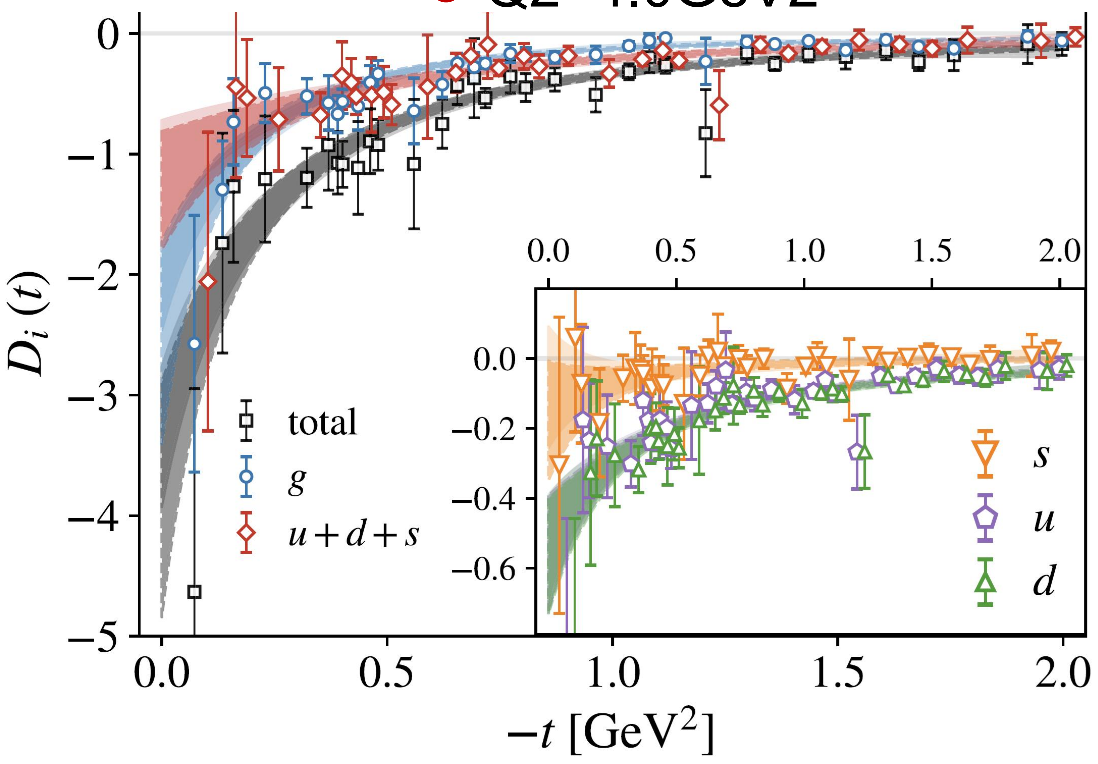
calculable in LQCD, e.g. PRD.77.094502 &108.014507 with bigger pion mass

# Introduction

- PHYSICAL REVIEW LETTERS 132, 251904 (2024)

$m_\pi \approx 170$ MeV

- Q2=4.0GeV2




$$\Delta^q(t) = 2\int_{-1}^{1} \frac{D^q(z,t)}{1-z}\mathrm{d}z = 4\sum_{k=0}^{+\infty} d^q_{2k+1}$$

$$\bar{D}^q_1(t) = \int_{-1}^{1} zD^q(z,t)\mathrm{d}z = \frac{4}{5}d^q_1$$

$$h^q_0(t) \simeq 4d^q_1 = 5\bar{D}^q_1(t)$$

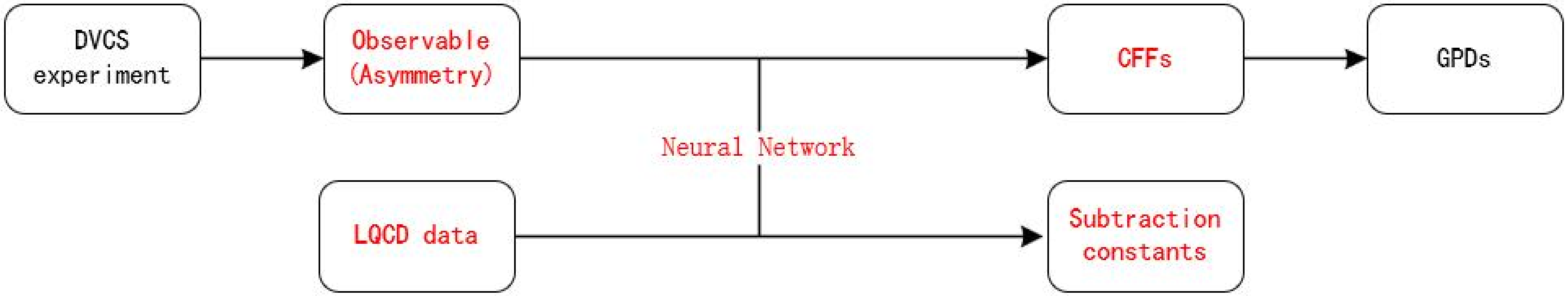
Flavor unseparated subtract const.:

$$\Delta(t) = \sum_q e_q^2 \Delta^q(t)$$

$$\mathrm{Re}\mathcal{H}(\xi,t) = \Delta(t) + \frac{1}{\pi}\mathrm{P.V.}\int_0^1 dx\left(\frac{1}{\xi-x} - \frac{1}{\xi+x}\right)\mathrm{Im}\mathcal{H}(x,t)$$

$$\langle N(\mathbf{p}',s')|\hat{T}^{\mu\nu}|N(\mathbf{p},s)\rangle = \frac{1}{m}\bar{u}(\mathbf{p}',s')\left[P^\mu P^\nu A(t) + iP^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho J(t) + \frac{1}{4}(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)D(t)\right]u(\mathbf{p},s),$$

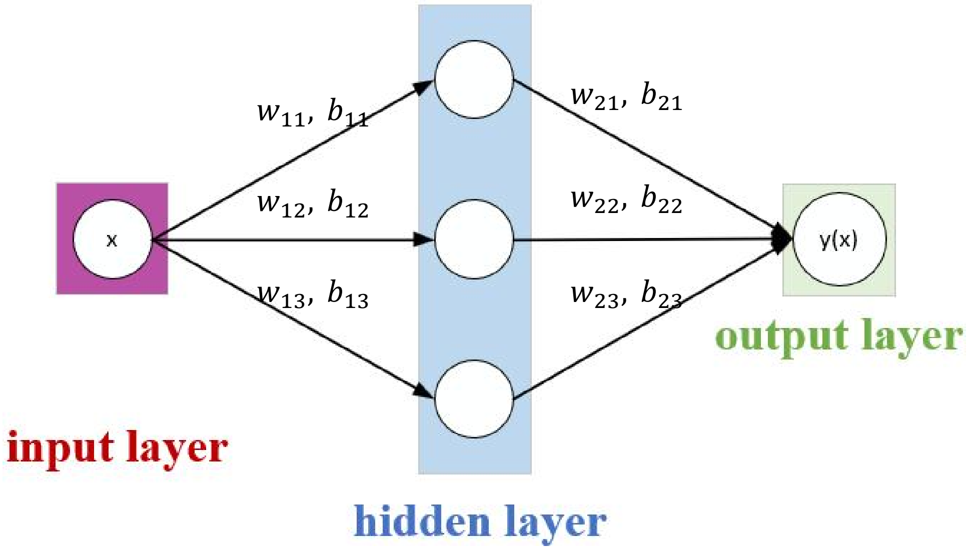
# Introduction

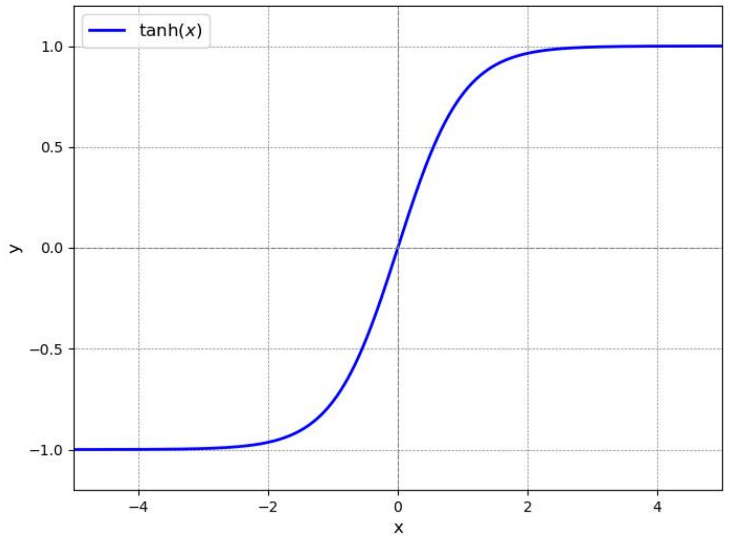


# Neural network

- **Perceptron and Activation function**

- The simplest neural network



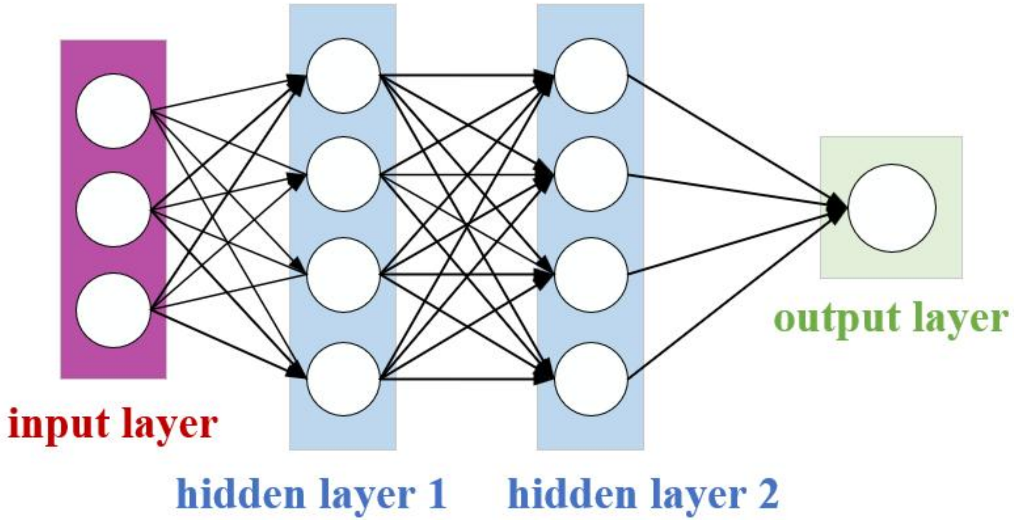



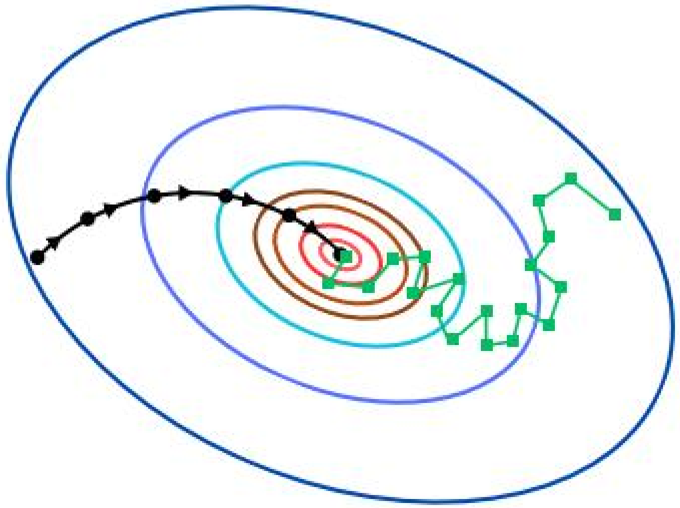

$$y(x) = \sum w_{2i} tanh(w_{1i}x + b_{1i}) + b_{2i}$$

# Neural network

- **Multilayer Perceptron and loss function**

- The simplest neural network



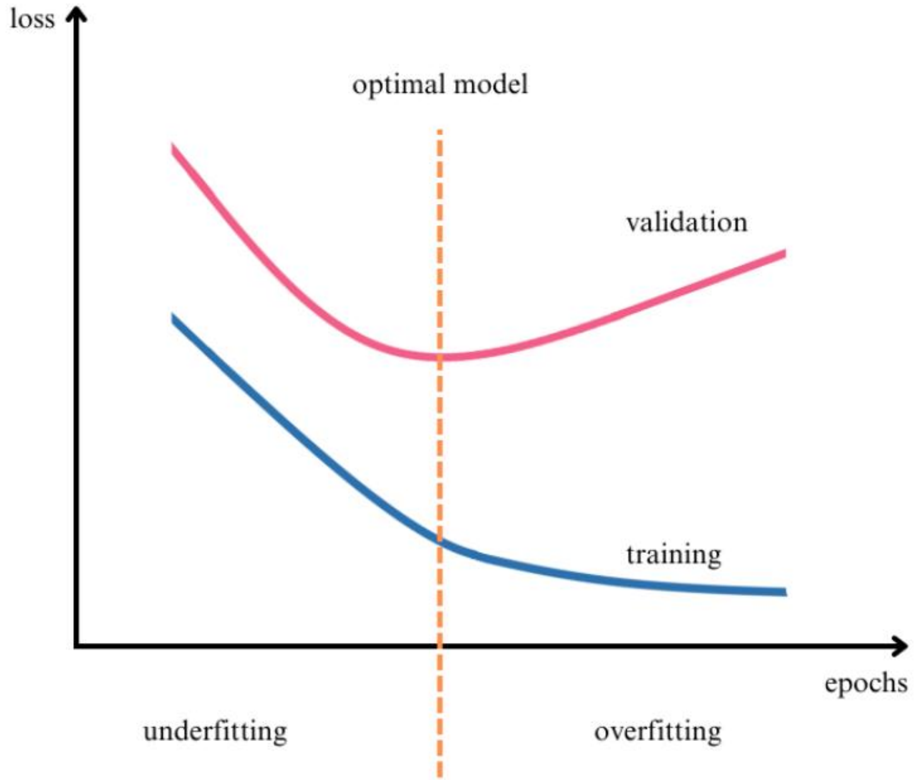
$$y(x) = (W_i((W_{i-1}(......(W_1(x) + B_1) + ......) + B_{i-1}) + B_i)$$

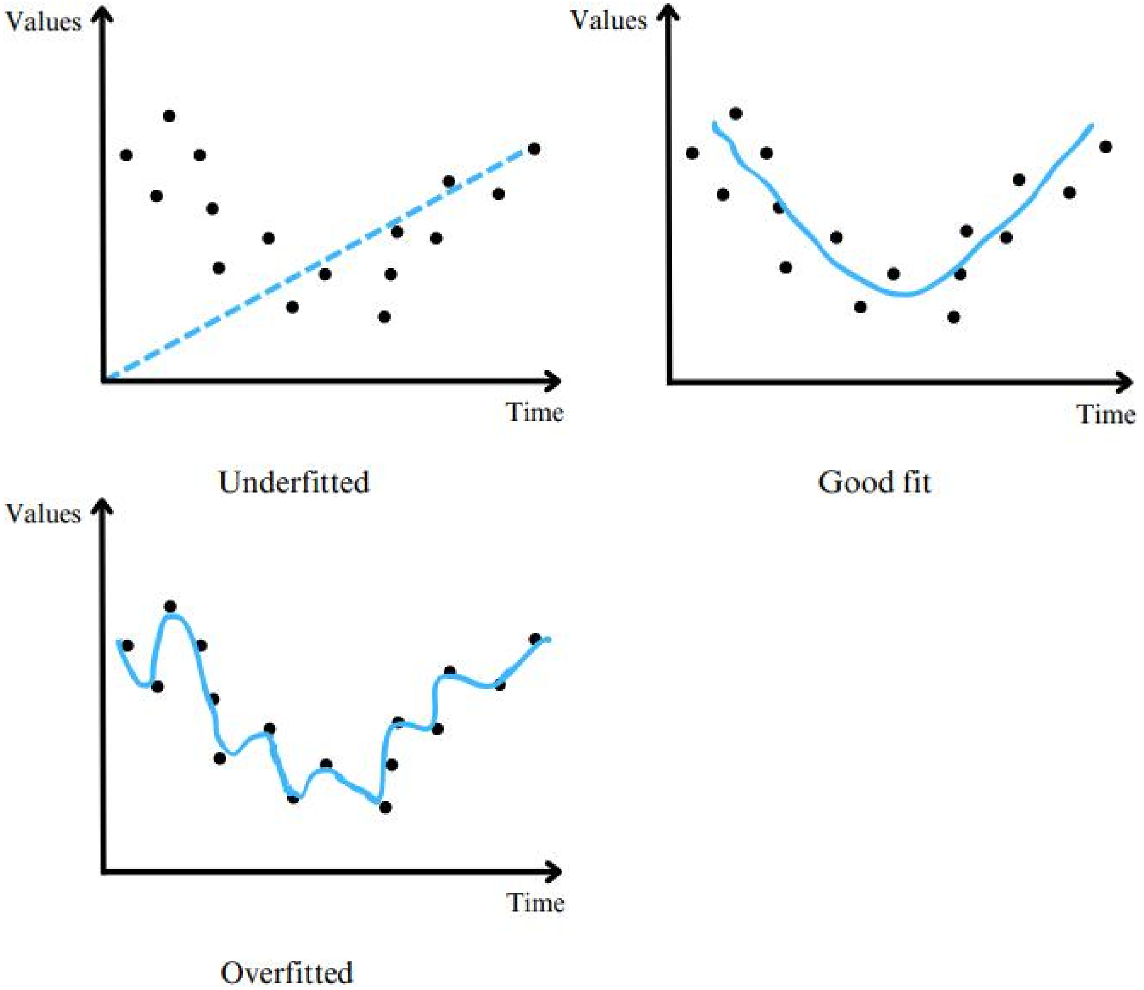
$$Loss = (y(x) - y_{obs})^2$$

$$w' = w - \frac{\nabla Loss}{\nabla w}$$

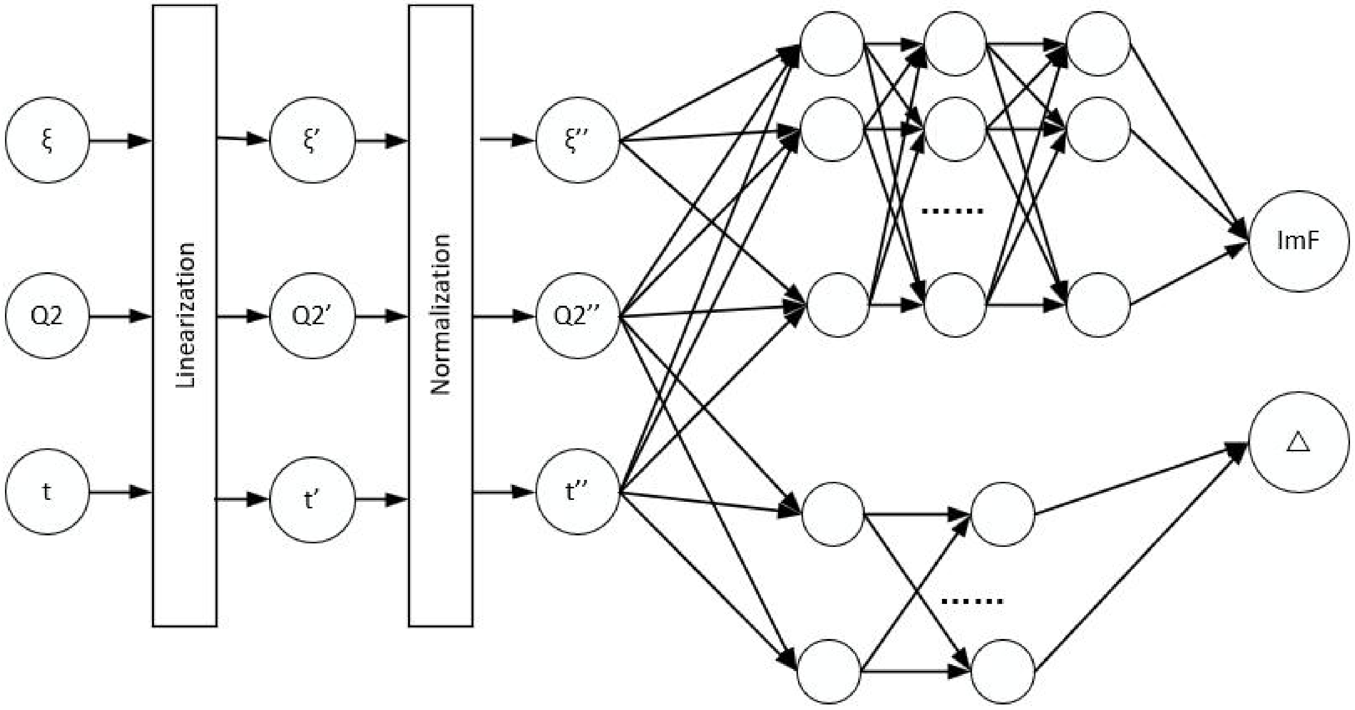
# Neural network

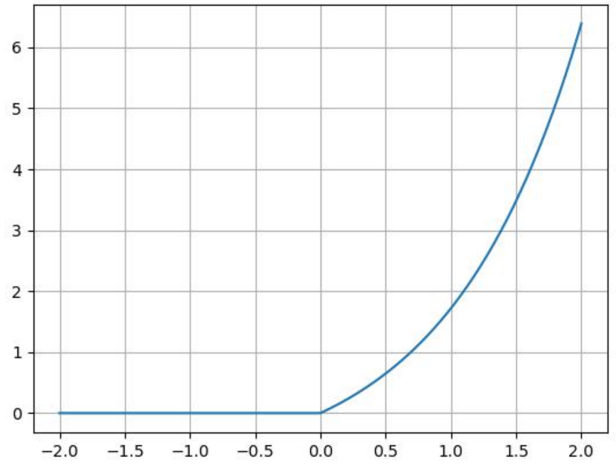
- **Early stopping**



# Neural network



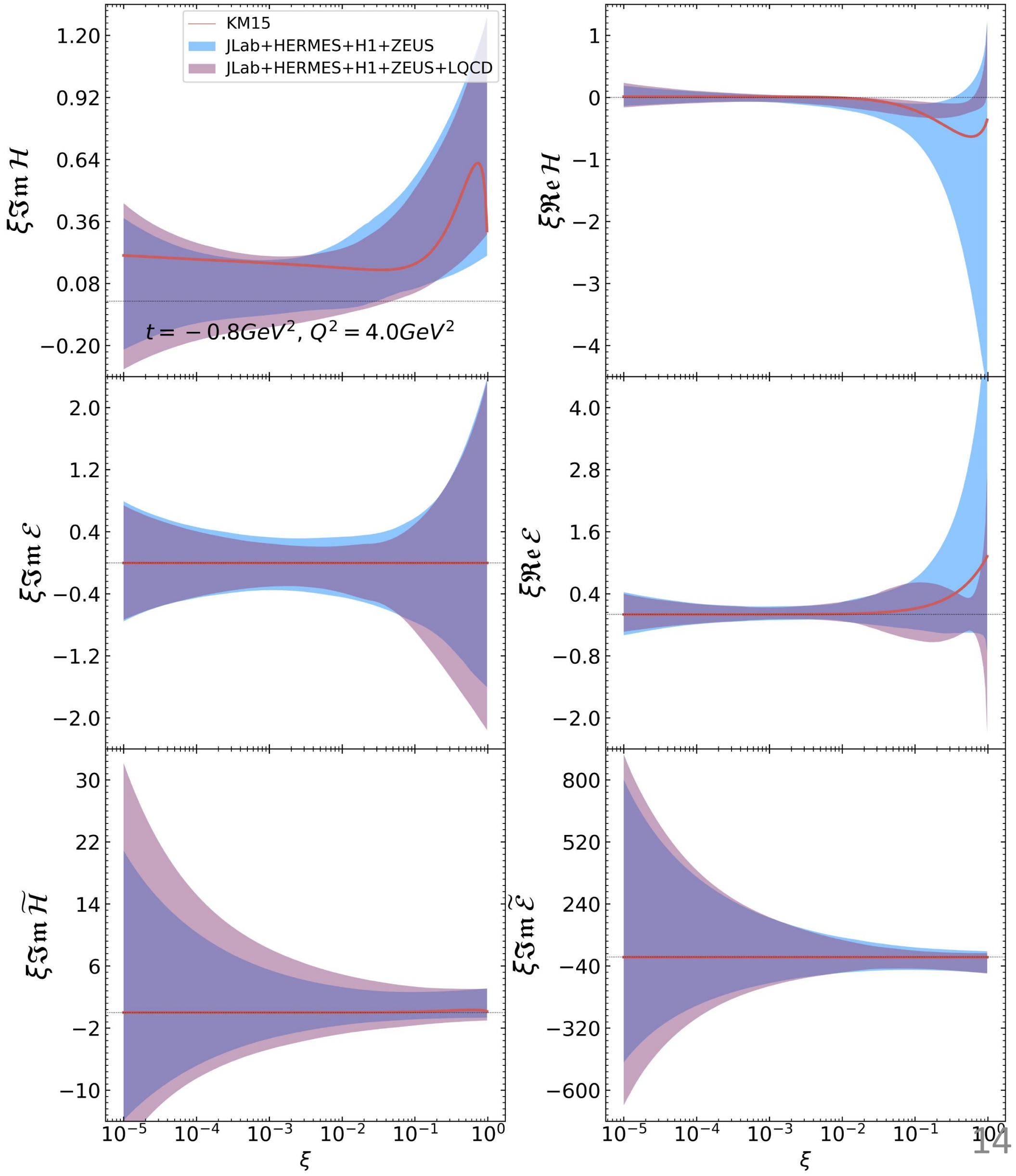

$$f(x) = \begin{cases} e^x - 1, x > 0 \\ 0, x < 0 \end{cases}$$

$$\mathcal{L}_\theta^{\chi^2}(\boldsymbol{x}, \sigma) = \mathcal{L}_\theta^{\chi^2}(\boldsymbol{x}, \sigma_{\text{DVCS}}) + \lambda \mathcal{L}_\theta^{\chi^2}(\boldsymbol{x}, \sigma_{\text{LQCD}})$$

# Add LQCD data of the Gravitation Form Factors

- 
- **The blue part fit DVCS data without LQCD data**
  Consistent with the result of
  PRL 125, 232005 (2020)

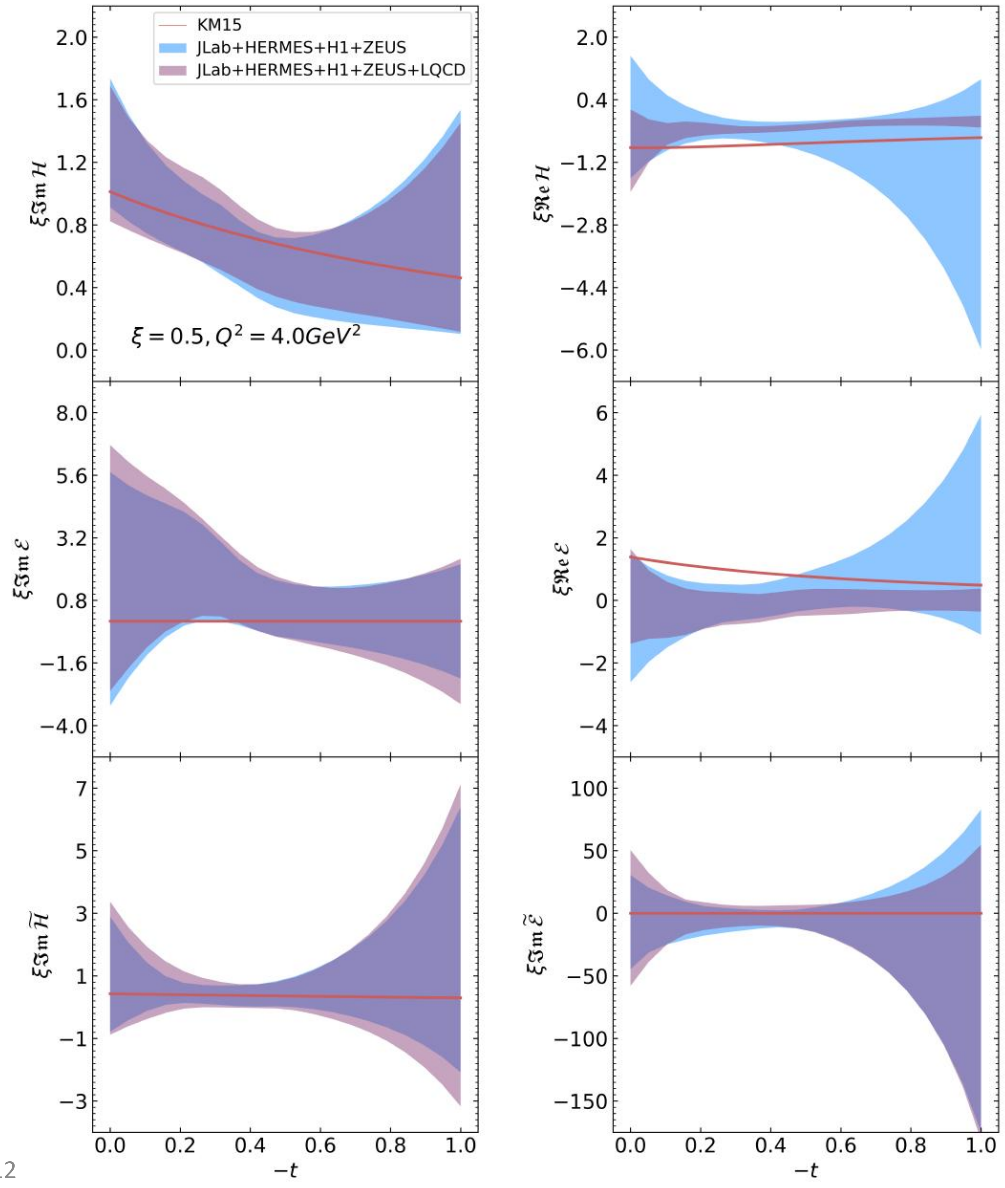
**-t = 0.8GeV2, Q2=4.0GeV2**

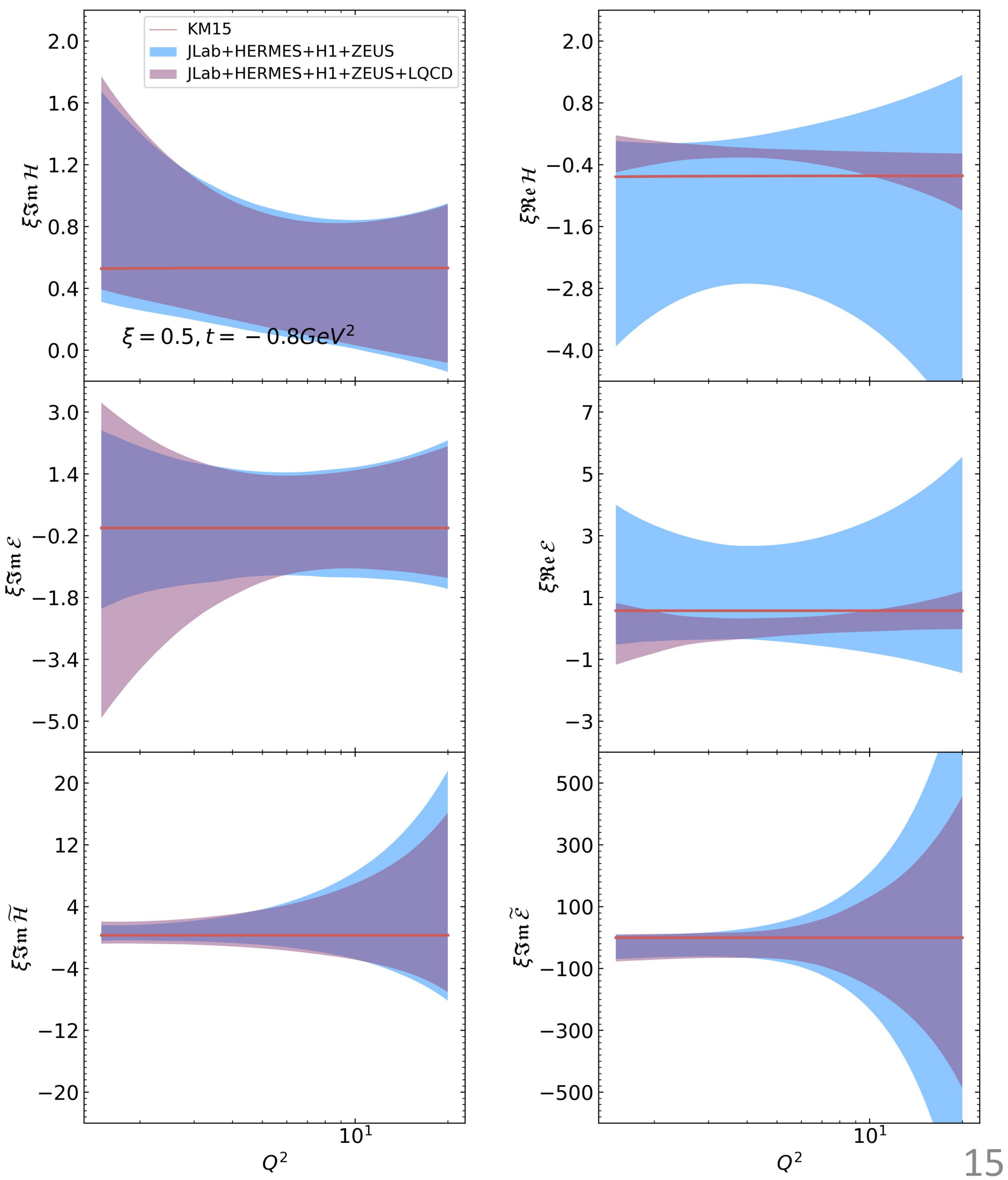



- **The red part fit DVCS data with LQCD data**
  Effectively constrains the real parts of H and E

# Add LQCD data of the Gravitation Form Factors

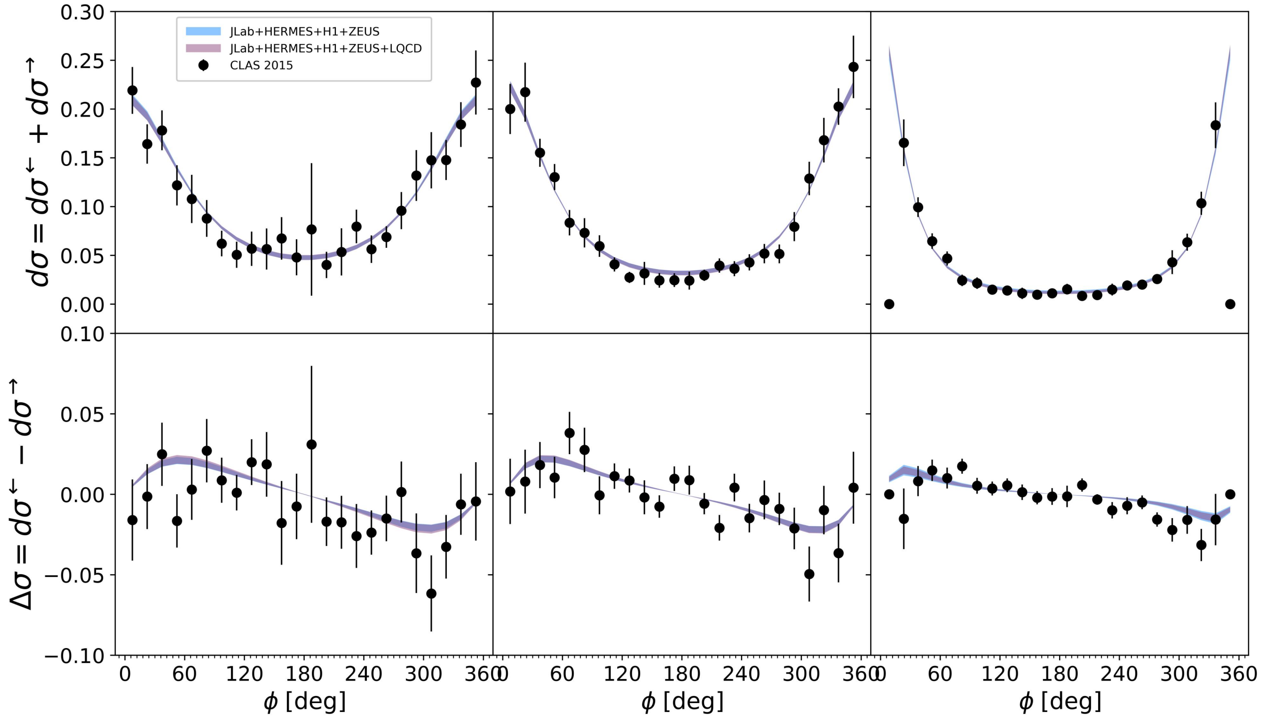
- **xi = 0.5, Q2=4.0GeV2** **xi=0.5, -t=0.8GeV2**



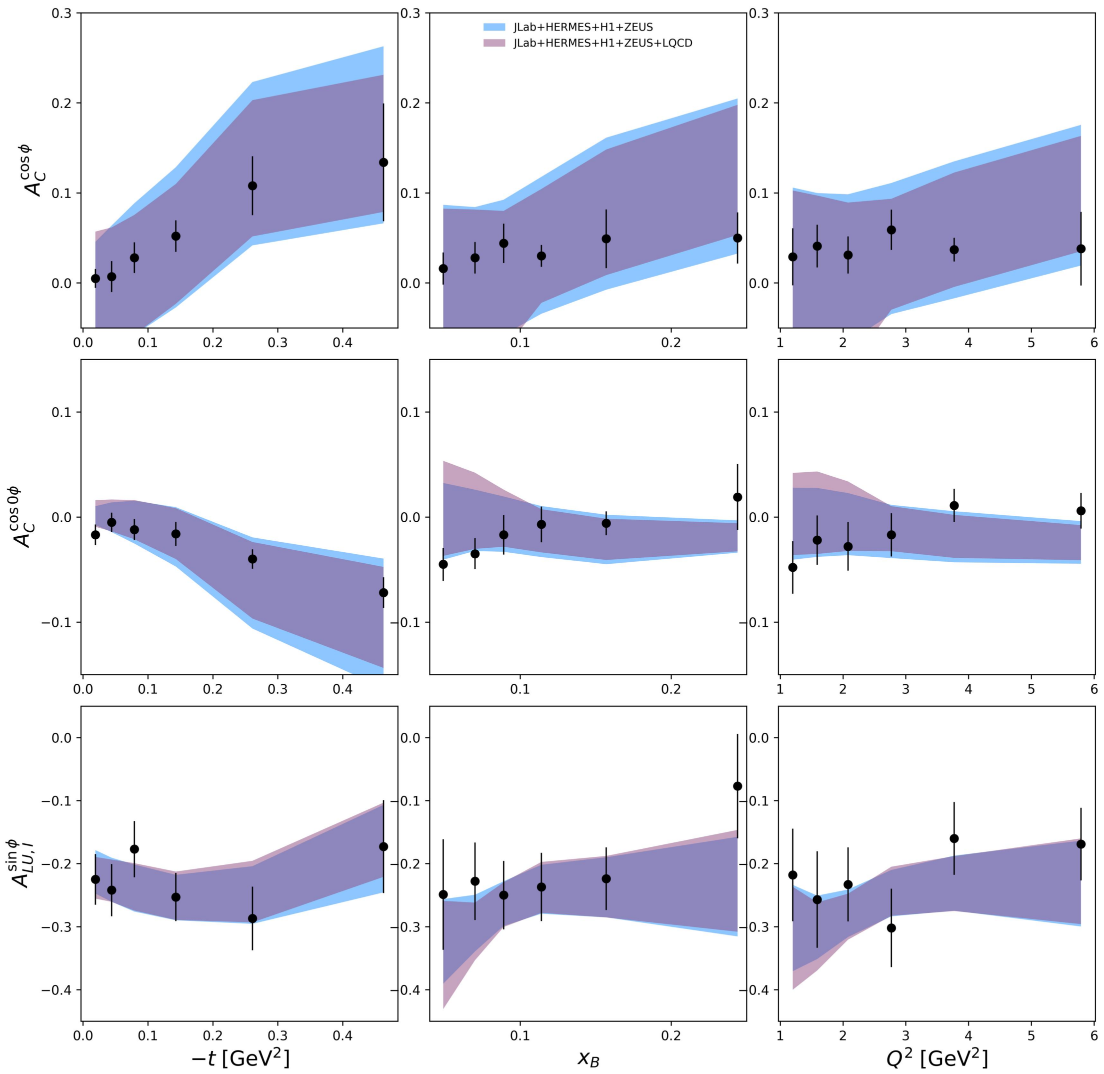



# Add LQCD data of the Gravitation Form Factors

- **CLAS2015**



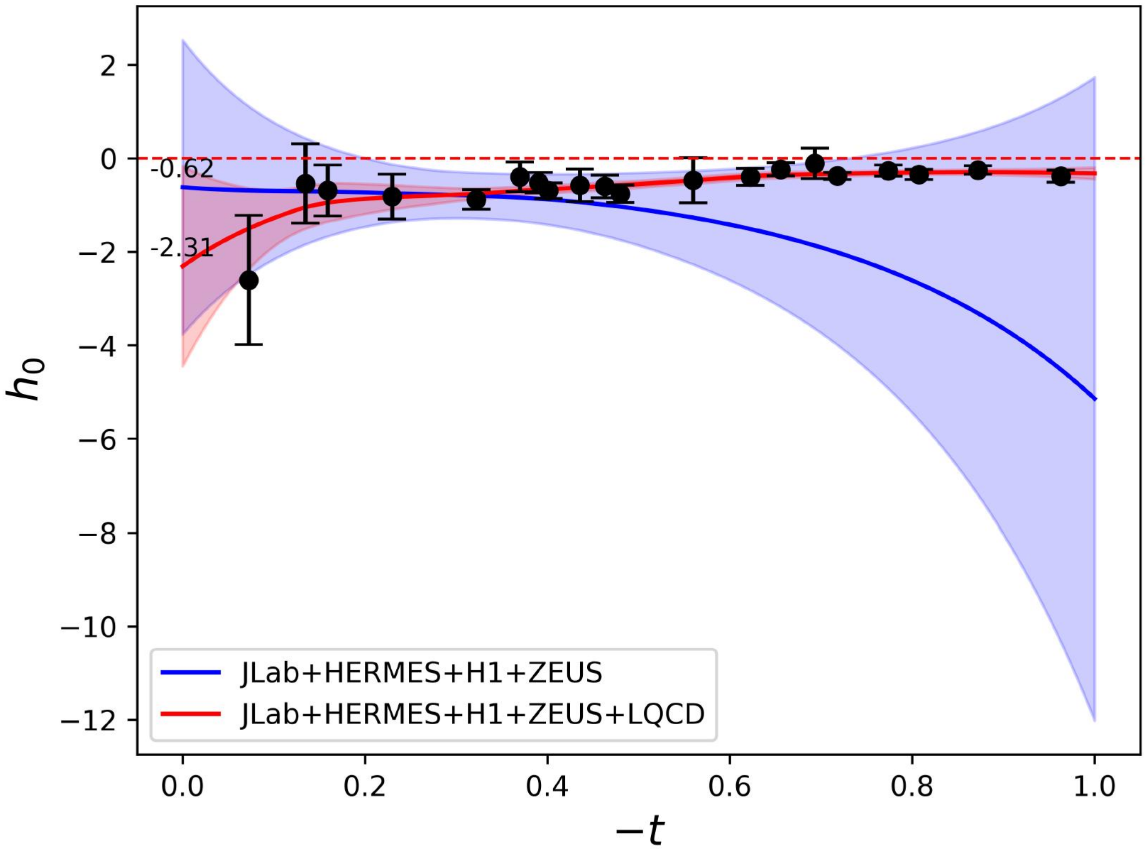

**HERMES2012**

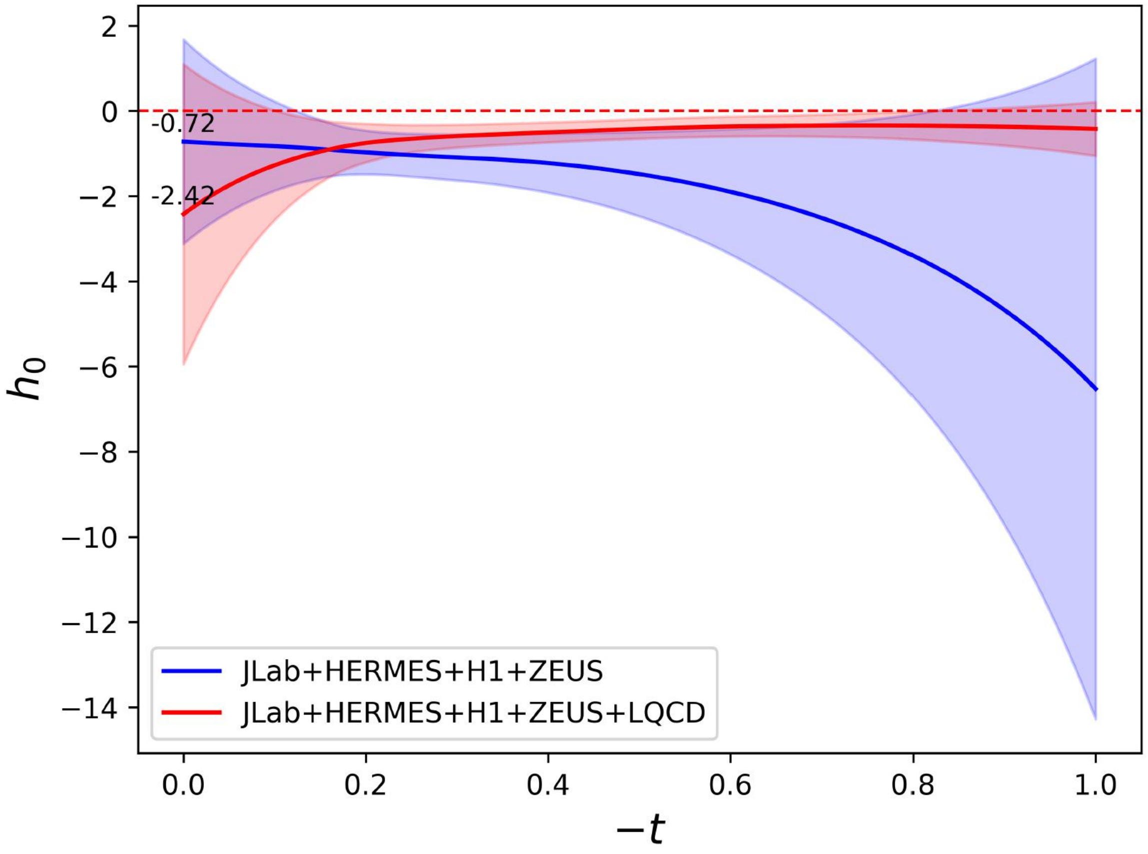



| $\chi^2_{DVCS}$ | $\chi^2_{DVCS+LQCD}$ |
|---|---|
| 0.87 | 1.27 |

# Add LQCD data of the Gravitation Form Factors

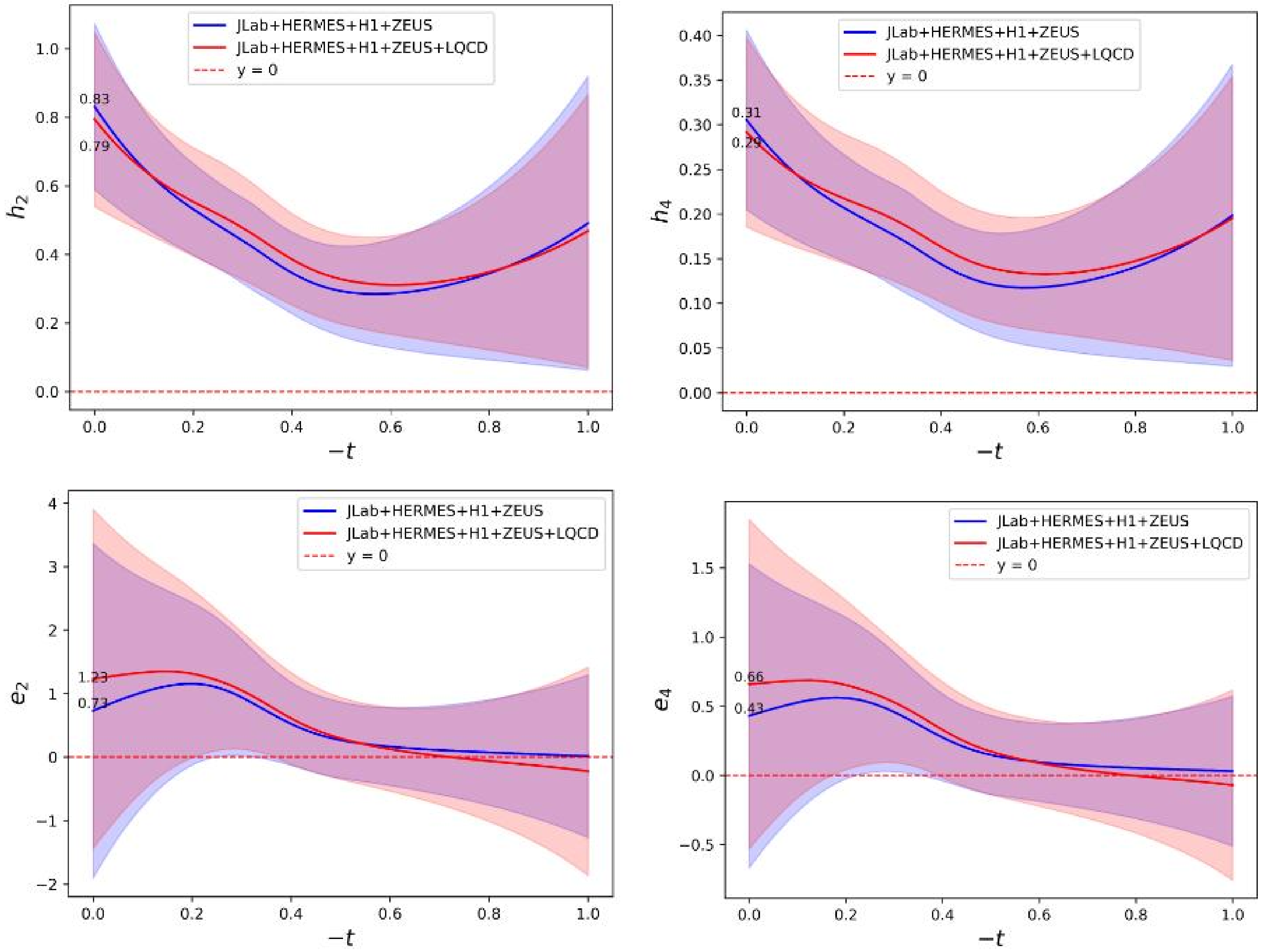
- for j=0： $\mathrm{Re}\mathcal{H}(\xi, t) = \Delta(t) + \frac{1}{\pi}\mathrm{P.V.}\int_0^1 dx \left(\frac{1}{\xi - x} - \frac{1}{\xi + x}\right)\mathrm{Im}\mathcal{H}(x, t)$ e0 = - $\Delta(t)$ for CFF E
- **Q2 = 4.0 GeV2** **Q2 = 2.0 GeV2**



# Add LQCD data of the Gravitation Form Factors

- for j>0: $h_{2j} = \frac{2}{\pi}\int_0^1 \Im\mathcal{H}(\xi')\,(\xi')^{2j-1}\,\mathrm{d}\xi'$ and e_{2j} for CFF E
- **Q2 = 4.0 GeV2**



# Summary

- **1. For CFF**

  The inclusion of LQCD data has shown a significant constraint on both ReH and ReE.

- **2. For observables**

  The chi-square values of the observables exhibit strong consistency before and after incorporating LQCD data.

- **3. For subtraction constants**

  The inclusion of LQCD data provides a good constraint on LO and demonstrates good consistency in higher-order subtraction constants.

Thanks