Lattice study of heavy quark observables for QGP

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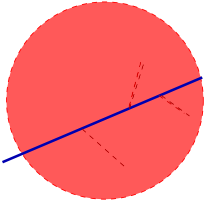
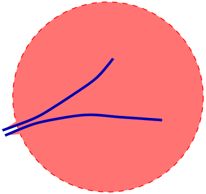
Heavy quark systems as probe of quark-gluon plasma

- In high energy heavy ion collisions in RHIC, LHC, ... deconfined medium (QGP) is formed, and lasts for $\mathcal{O}(10 \text{ fm})$.
- Heavy quarks generated early in the collision, $\tau \sim 1/2M$. Provide good probes of the medium.





- Heavy quark: Brownian motion through the QGP.
- Dissolution of quarkonia in deconfined medium.
- Such systems in the strongly coupled QGP can be studied using various color electric field correlators.

Heavy-light mesons in deconfined plasma

- For moderate momenta heavy quarks in a plasma, with $M \gg T$, a Langevin framework can be used.

$$\frac{dp_i}{dt} = \xi_i(t) - \eta_D(p) p_i, \qquad \langle \xi_i(t)\, \xi_j(t') \rangle = \kappa_{ij}\, \delta(t - t')$$

Svetitsky '88; Mustafa et al., '97; Moore & Teaney '05; Rapp & van Hees '05

- For low momenta, just one coefficient κ. Standard nonrelativistic relations:

$$\eta_D = \frac{\kappa}{2\,M\,T} \sim \tau_Q^R$$

- A field theoretic definition of κ can be given from the force-force correlator:

$$3\kappa = \frac{1}{\chi} \lim_{\omega \to 0} \int_{-\infty}^{\infty} dt\, e^{i\omega t} \int d^3x \left\langle \frac{1}{2} \left\{ F_i(t, x)\, ,\, F_i(0,0) \right\} \right\rangle$$

$$F^i = M \frac{dJ^i}{dt} = \phi^\dagger \left\{ -gE^i + \frac{\left[D^i, D^2 + c_b g \sigma \cdot B \right]}{2M} + \; .... \right\} \phi$$

Calculation of κ

- In leading order in $1/M$ one gets only the gE force.

$$\kappa = \frac{1}{3\chi} \lim_{\omega\to 0} \int_{-\infty}^{\infty} dt\, e^{i\omega t} \, \langle \mathrm{Tr}\, W(t,-\infty)^\dagger \, gE_i(t)\, W(t,0)\, gE_i(0)\, W(0,-\infty) \rangle$$

J. Casalderrey-Solana & D. Teaney, PRD 74 (2006) 085012;
S. Caron-Huot, M. Laine & G. Moore, JHEP 04 (2009) 53

- κ_E has been calculated to NLO in HTL PT.

$$\frac{\kappa_E}{T^3} = \frac{2\, g^4}{27\, \pi} \left[N_c \left(\ln \frac{2T}{m_D} + \xi \right) + \frac{N_f}{2} \left(\ln \frac{4T}{m_D} + \xi \right) + \frac{N_c\, m_D}{T} C + \mathcal{O}(g^2) \right]$$

- Series in g rather than in α. NLO corrections very large.

Caron-Huot & Moore, PRL 100 (2008) 052301

- κ_E can be connected to the EE Matsubara correlator. Can be calculated nonperturbatively using Lattice QCD.

Caron-Huot, Laine, Moore, JHEP (2009)
Banerjee, Datta, Gavai, Majumdar, PRD 85 (2012) 014510

$G_{EE}(\tau)$

- We studied $G_{EE}(\tau)$ for the gluonic plasma in the temperature range $T_c < T \lesssim 3.5\,T_c$.

  Banerjee, Datta, Gavai, Majumdar, 2012, NP A1038(2023)122721

- A discretization using $E_i = [D_i\,,\,D_0]$ turns out to be convenient for calculating the continuum correlator.

- Only a finite renormalization factor $Z_E(a)$, known to NLO.

  Christensen & Laine, PLB 755 (2016) 316

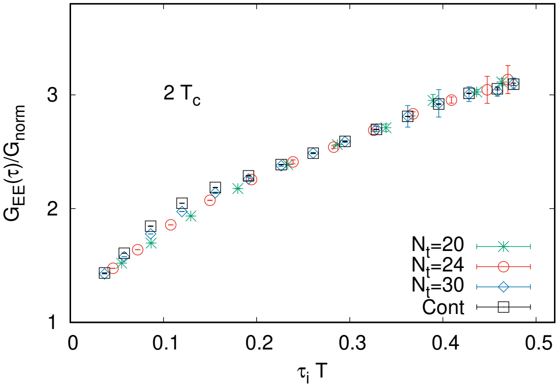
- For noise reduction multilevel technique was used.
- There have also been other calculations using Multilevel.

  Francis et al. PRD (2015); Brambilla, et al., PRD (2020)

- Besides, Gradient flow has also been used by some groups, for both noise reduction and operator renormalization.

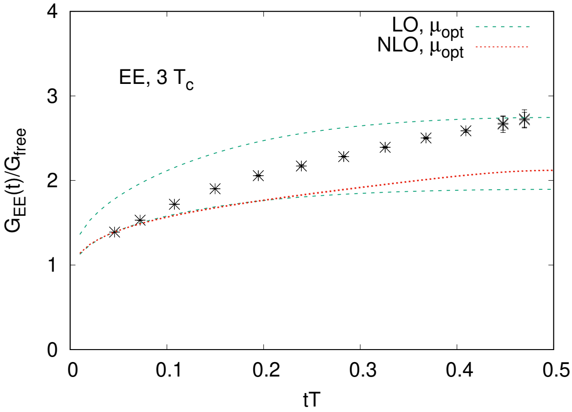
  Altenkort et al.,PRD103(2021)014511; Brambilla et al,PRD107(2023)054508

$G_{EE}(\tau)$ from lattice



- $\mu_{\rm opt}$ is the optimized scale for NLO.
- Here the correlators are normalized using the leading order behavior, $G_{\rm norm}(\tau) = \dfrac{G_{LO}(\tau)}{g^2 C_f}$

$G_{EE}(\tau)$ from lattice



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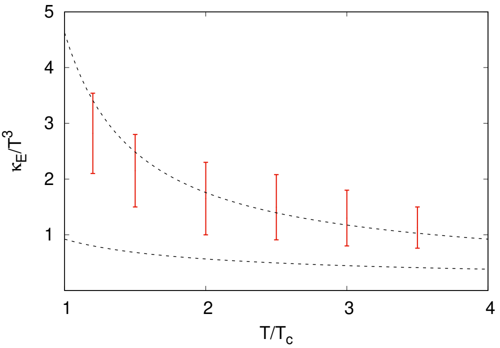
$\rho_{EE}(\omega)$ and κ_E

- $G_{EE}(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho_{EE}(\omega) \frac{\cosh \omega(\tau - 1/2T)}{\sinh \omega/2T}$
- κ_E can be extracted from the infrared behavior of $\rho_{EE}(\omega)$:
  $\rho_{IR} \underset{\omega\to 0}{\approx} \frac{\kappa_E \, \omega}{2T}$
- For $\omega \gg T$ we expect $\rho_{UV} \approx \frac{g^2(\mu) C_f \omega^3}{6\pi}$
- To extract $\rho_{EE}(\omega)$, we use these limiting behaviors.
  We use a linear regularization with these two limiting behaviors.
  We also use various model spectral functions with these limiting behaviors.
- Other methods of inversion, e.g., Brackus-Gilbert and Maximum entropy methods, have been used by other groups.

Altenkort et al. (2021)

κ_E from lattice



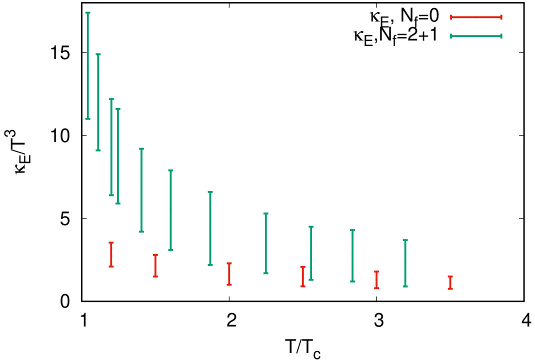
Banerjee, Datta, Gavai, Majumdar (2023)

- The error is dominated by uncertainty in the extraction of $\rho_{EE}(\omega)$.
- These values agree very well with the results of other groups, where available.
- Consistent with NLO pert. theory.

κ_E for QCD with $N_f = 2+1$

κ_E has also been calculated recently in a theory with 2+1 flavors of thermal quarks.

HotQCD: Bollweg et al., JHEP09(2025) 180; Altenkort et al., PRL 130(2023)231902



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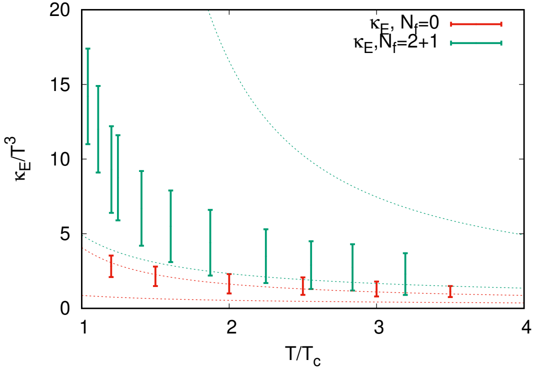

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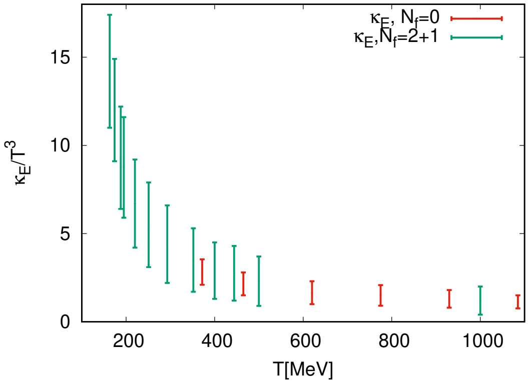
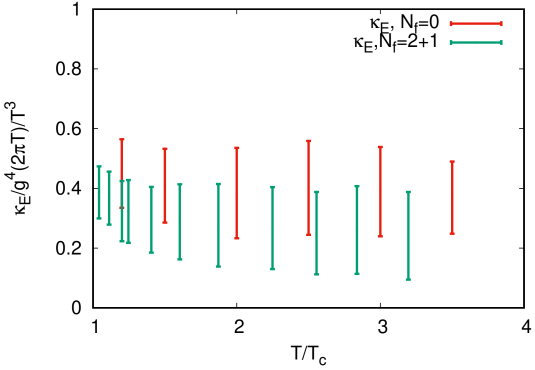


• Much of the N_f dependence can be attributed to the difference in T_c ($T_c^{N_f=0} \approx 310$ MeV).

Parametrization of κ_E

An interesting observation: (Altenkort et al., PRD 109 (2024) 114506) results for κ_E for T near T_c can be parametrized as $c\, g^4(2\pi T)\; T^3$, with similar c for both the gluonic and the 2+1 flavor theory.



- Not perturbative: in PT the coefficient c increases with N_f.
- Also in this temperature range g^5 term dominates in NLO PT.

$1/M$ correction

- At $\mathcal{O}\left(\frac{1}{M}\right)$ the magnetic field force needs to be included.
- Under certain assumptions, (Bouttefeux & Laine, JHEP 12 ('20) 150)

$$\kappa_Q \approx \kappa_E + \frac{2}{3}\langle v^2\rangle \kappa_B , \qquad \langle \gamma v^2\rangle = \frac{3T}{M_{\rm kin}}$$

- κ_B is similar to κ_E, for the $B - B$ correlator.
  The B operator has an anomalous dimension.
  We need to calulate

$$c_B(\mu)\, G_{BB}(\mu, T; \tau)$$

  where G_{BB} is the correlator renormalized at scale μ, and c_B is a Wilson coefficient. The μ dependence cancels in the product.
- This becomes equivalent to evaluating the $\overline{MS}$ correlator at scale $\mu_T \approx 19.2T$.

  M. Laine, JHEP 06 ('21) 139
- Renormalize the latice discretized operators, take continuum limit, and calculate $G_{BB}(\mu_T, \tau)$.

κ_B for gluonic plasma

- Results for κ_B for gluonic plasma in the temperature range 1-3 T_c.

Banerjee, Datta & Laine, JHEP 08 (2022) 128
Brambilla et al., PRD107(2023)074503, Altenkort et al., PRD 109(2024)114506

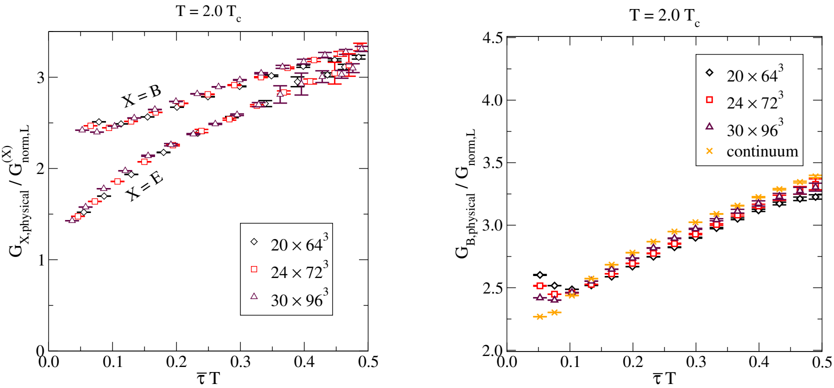
- Our calculation: clover discretization of B.
- The nonperturbative renormalization coefficients calculated by ALPHA are used.

Guazzini, Meyer, Sommer (2007)

$$\mathbf{G}_{\mathrm{BB}}(\tau, a) \xrightarrow[\substack{\mathbf{Z}_{\mathrm{SF}} \\ \text{(ALPHA)}}]{} \mathbf{G}^{\mathrm{SF}}_{\mathrm{BB}}(\tau,\mu_{\mathrm{SF}}) \xrightarrow[\mathbf{Z}_{\mathrm{SF},\,\overline{\mathrm{MS}}}]{a \to 0} \mathbf{G}^{\overline{\mathrm{MS}}}_{\mathrm{BB}}(\tau,\mu_{\mathrm{T}})$$

- Gradient flow has been used by the other groups.
  Also a $[D_i, D_j]$ regularization has been used.

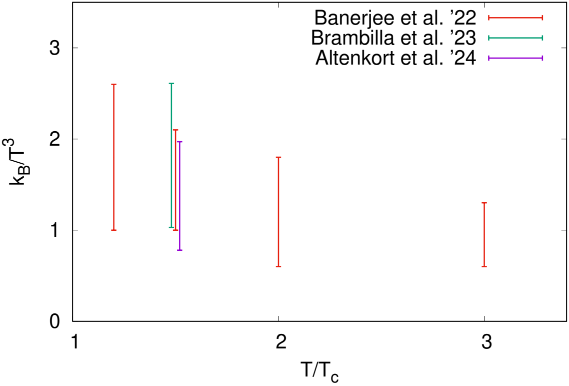
$G_{BB}(\tau)$



Banerjee, Datta & Laine, JHEP 08 (2022) 128

κ_B for gluonic plasma from BB correlator

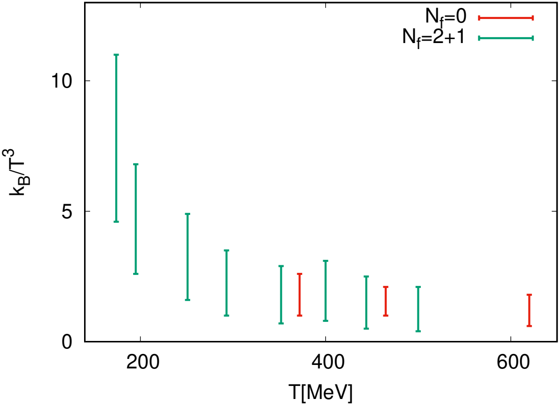
We can extract κ_B in a way analogous to that for κ_E.
The different calculations give consistent results.



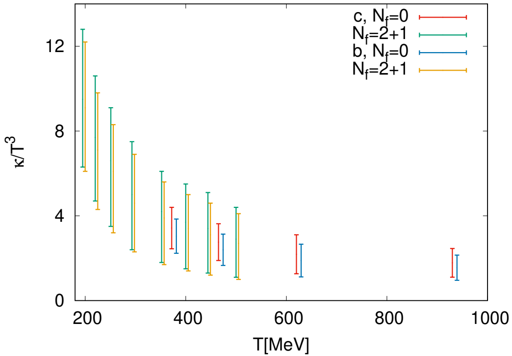
Effect of thermal quarks

The results for the theory with dynamical quarks have come out recently. The calculation uses gradient flow.

Bollweg et al., JHEP09(2025) 180



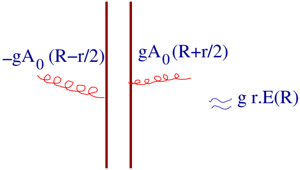
κ_c, κ_b

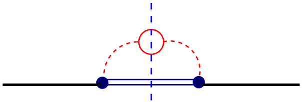


- Correction to the static limit is $\lesssim 10\%$ for bottom near T_c for the gluonic theory, rising to $\approx 15\%$ at 2 T_c. For the three flavor theory it is $< 10\%$ for $T < 1.6\,T_c$.
- Even for charm the $1/M$ correction is $\approx 25\%$ near T_c for the quenched and $< 20\%$ for the 3-flavor theory.

Quarkonia: color dipole in QGP

- For $r \ll 1/T$, e.g., for Υ, interaction of $\bar{Q}Q$ with medium: color dipole.





- Let us expand in r and further, write the $\bar{Q}Q$ in $|S\rangle$, $|O\rangle$ basis (pNRQCD).

$$\begin{aligned}
\mathcal{L}_{pNR} \; &= \; \mathrm{Tr} \quad \left[S^\dagger \left(i\partial_0 - V_s(r) \right) S \; + \; O^\dagger \left(iD_0 - V_o(r) \right) O \right] \\
&+ \; \mathrm{Tr} \quad \left[S^\dagger \vec{r}.g\vec{E} O \; + \; h.c. + \frac{1}{2} \, O^\dagger \{ \vec{r}.g\vec{E} \, , \, O \} \right] + \mathcal{O}\left(\frac{1}{M} \right)
\end{aligned}$$

Pineda & Soto (1998); Brambilla, et al., RMP 77 (2005) 1423

- Thermal correction depends on
$G(t) \; = \; \frac{1}{3} \sum_i \langle T \, E^{a,i}(t,\vec{0}) \, W^{ab}(t,0) \, E^{b,i}(0,\vec{0}) \rangle$.

Quarkonia in pNRQCD

- For $r \ll 1/T$ the quarkonia singlet-to-octet transition can be quantified by a transport coefficient κ.
- The Matsubara correlator $G(\tau)$ related to κ:

$$G(\tau) \;=\; \int_{-\infty}^{\infty} d\omega \; \rho(\omega) \; \frac{e^{\omega(1/2T \,-\, \tau)}}{2 \, \sinh \frac{\omega}{2T}}$$

$$\rho(\omega) \;=\; \rho_{\rm odd}(\omega) \;+\; \rho_{\rm even}(\omega)\,; \qquad \kappa \;=\; \lim_{\omega \to 0} \frac{T}{\omega} \; \rho_{\rm odd}(\omega)$$

B. Scheihing-Hitschfeld & X. Yao, PRD108 (2023) 054024

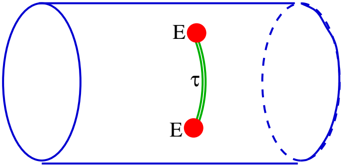
- Similarly, the octet-to-octet transition can be calculated from a transport coefficient $\kappa_{\rm oct}$ obtained from an EE correlator.

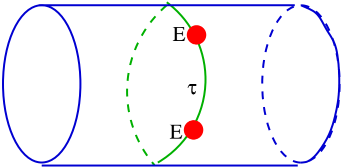
$G(\tau)$ and $G_{\text{fund}}(\tau)$

- In weak coupling, $\rho(\omega)$ has been calculated to NLO.

  B. Scheihing-Hitschfeld & X. Yao (2023)
  N. Brambilla, P. Panayiotou, S. Sappi & A. Vairo, JHEP 08 (2025) 219

- $G(\tau)$ looks similar in form to $G_{\text{fund}}(\tau)$ used to calculate the heavy quark momentum diffusion coefficient κ_E.

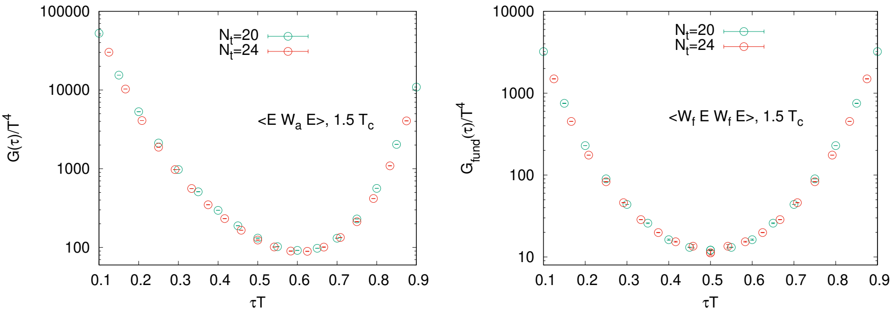




- $$\left.\begin{matrix} G(\tau)|_{LO} \\ G_{\text{fund}}(\tau)|_{LO} \end{matrix}\right\} = g^2 \begin{Bmatrix} N^2-1 \\ C_f \end{Bmatrix} G_{\text{norm}}(\tau)$$

- They differ in NLO.

Nonperturbative comparison

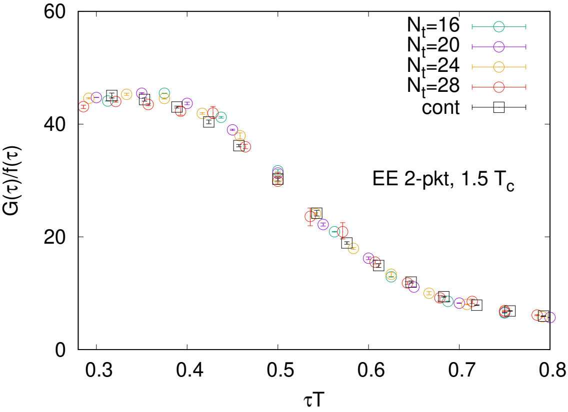


$$G(\tau; a) \;=\; Z_E^2(a)\, e^{\delta m(a)\,\tau}\; Z_{\rm bare}(\tau; a); \qquad a \;=\; \frac{1}{N_t\, T}$$

The two correlators are very different. In particular, $\langle E W_A E \rangle$ does not obey $\tau \to \beta - \tau$ symmetry.

N. Brambilla, S. Datta, M. Janer, V. Leino, J. Mayer-Steudte, P. Petreczky, A. Vairo, arXiv:2505.16603 (PRD)

$G(\tau)$: Continuum results



We have calculated the correlator both using multilevel and using gradient flow, to fix the renormalization.

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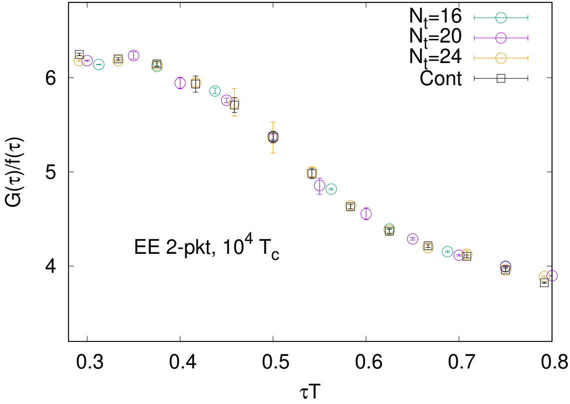

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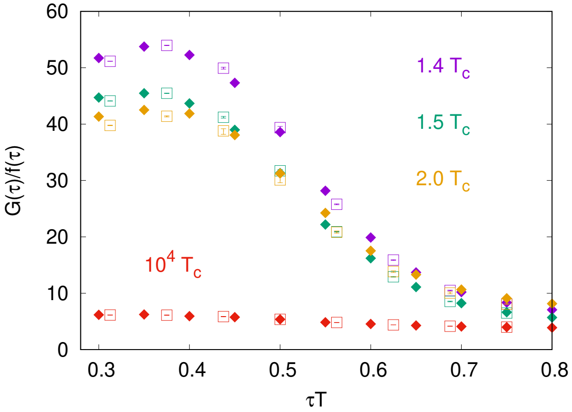
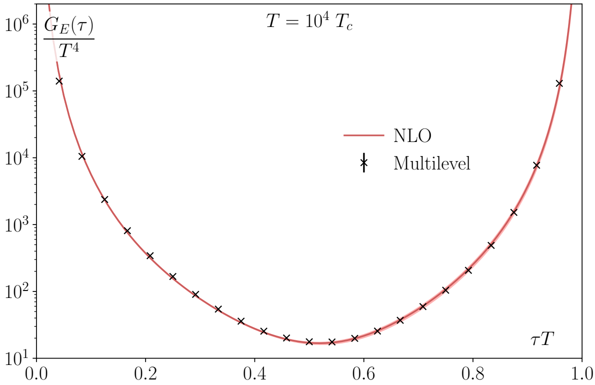
T dependence of $G(\tau)$



Datta, Banerjee, Brambilla, Janer, Leino, Mayer-Steudte, Petreczky, Singh, Vairo, JSPC 4 (2025) 100156 (2506.22594)

High temperature



At very high temperatures the nonperturbative correlator is close to the NLO correlator. But at temperatures a few T_c they are different.

Brambilla, Datta, Janer, Leino, Mayer-Steudte, Petreczky, Vairo, 2505.16603
Pert.th.: Brambilla, Panayiotou, Sappi, Vairo, 2505.16604

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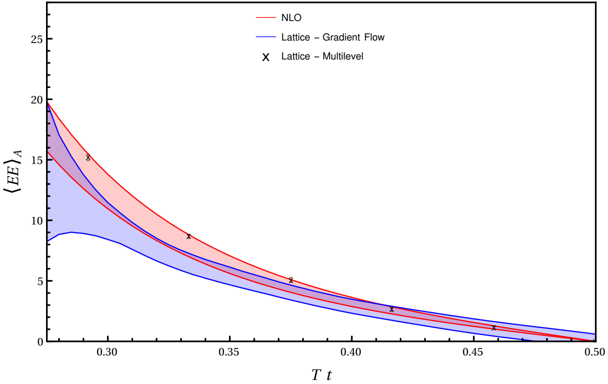

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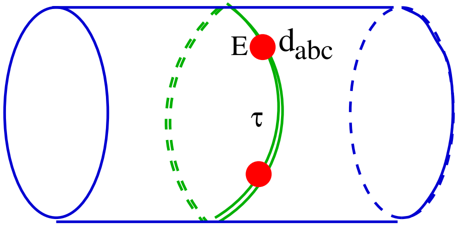
Pert.th.: Brambilla, Panayiotou, Sappi, Vairo, 2505.16604

Adjoint potential

The $O^\dagger \{r.gE\ ,\ O\}$ term in L_{pNR} leads to a correction to the octet potential, which can be obtained from the correlator $G_{\rm oct}(\tau)$.

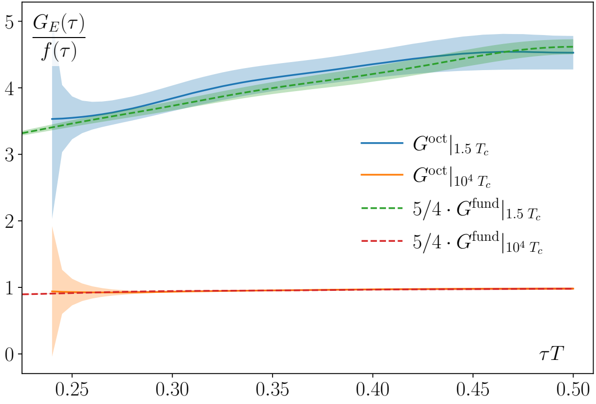


This has the same LO structure, modulo color factor:

$$G_{\rm oct}(\tau)(\tau) \;=\; g^2\, \frac{N^2-4}{N}\, G_{\rm norm}(\tau)$$

Octet-octet correlator

Structure of $G_{\text{oct}}(\tau)$, which calculates a correction of the thermal octet potential, is simple: it has a similar structure as $G_{\text{fund}}(\tau)$, and shows a color scaling, $\Rightarrow \kappa_{\text{oct}} \approx \frac{5}{4}\kappa_{\text{fund}}$



Brambilla, Datta, Janer, Leino, Mayer-Steudte, Petreczky, Vairo, 2505.16603

Summary

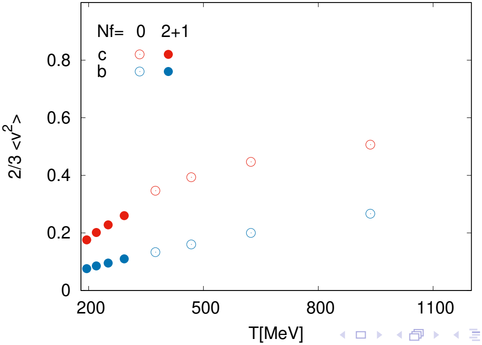
- Nonperturbative information about heavy quark systems in QGP can be obtained from a study of electric field corrlators.
- The heavy quark diffusion coefficient, in the leading order in $1/M$, can be obtained from $G_{EE}(\tau)$.
- Studied for both gluonic plasma and for full QCD in the temperature range of interest for URHIC experiments.
- The $\mathcal{O}\left(\frac{1}{M}\right)$ correction to the diffusion coefficient has also been calculated from a study of the BB correlators.
- The decay of quarkonia in QGP can also be calculated from thermal correlators of electric fields connected by adjoint Wilson lines.
- We show renormalized, continuum extrapolated results of it. The analysis of $\rho(\omega)$ is in progress.
- We also studied the correlators $G_{\rm oct}(\tau)$, relevant for the octet-octet transition in the plasma. $\kappa_{\rm oct} \approx \frac{5}{4}\kappa_E$.

EXTRA SLIDE: κ_Q to $\mathcal{O}\left(\frac{1}{M}\right)$

- To $\mathcal{O}\left(\frac{1}{M}\right)$ $\kappa_Q \approx \kappa_E + \frac{2}{3}\langle v^2\rangle\, \kappa_B$
- With $\kappa_B \lesssim \kappa_E$, $\langle v^2\rangle$ gives the size of $\mathcal{O}\left(\frac{1}{M}\right)$ correction.
- We obtained $\langle v^2\rangle$ from the constant part of $\frac{\langle J^i\, J^i\rangle}{\langle J^0\, J^0\rangle}$.

Burnier & Laine, JHEP 11 ('12) 086

- Calculated from the susceptibility in Altenkort et al.



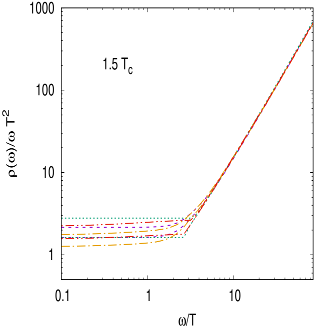
EXTRA SLIDE: $\rho(\omega)$ and κ_E

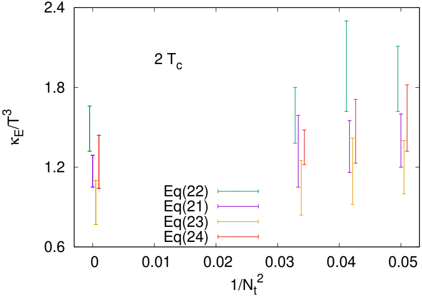
- $\rho^{IR}_{EE}(\omega) \equiv \frac{\kappa\,\omega}{2T}$, $\rho_{UV}(\omega) \equiv \frac{g^2(\mu) C_f \omega^3}{6\pi}$, $\mu = \max(\omega, \pi T)$
- Model $\rho(\omega)$ as
  $\rho_1(\omega) \equiv \max\left(\rho^{IR}_{EE}(\omega), \, c\rho_{UV}(\omega)\right)$
- Physically better motivated: Francis et al. '15
  $\rho_2(\omega) \equiv \sqrt{(\rho^{IR}_{EE}(\omega))^2 + (c\rho_{UV}(\omega))^2}$
- Also tried adding a Fourier mode (Francis et al. '15)

$$\rho_{1f,2f}(\omega) \equiv (1 + d \sin \pi y) \; \rho^{c=1}_{1,2}(\omega), \qquad y = \frac{\log\left(1 + \frac{\omega}{\pi T}\right)}{1 + \log\left(1 + \frac{\omega}{\pi T}\right)}$$

- At each temperature, calculate the correlator at three lattice spacings and take continuum limit.
- took a superset of the values obtained from different forms.
- Also looked at the fits for individual lattices.

EXTRA SLIDE: κ_E

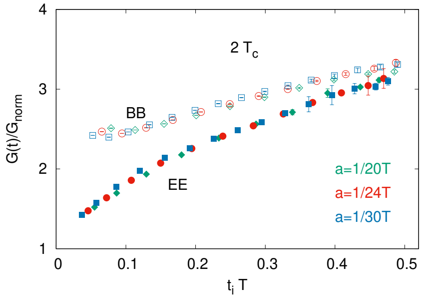


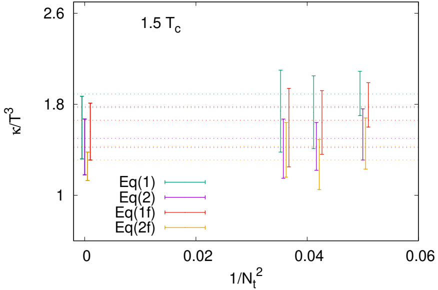


EXTRA SLIDE: Analysis of $G_{BB}(\tau)$

- $G_{BB}(\tau)$ has very different shape from $G_{EE}(\tau)$, in particular at short times.
- Analysis similar to the *EE* correlator, but subtle differences arise because of the anomalous dimension.

$$\rho_{EE}^{IR}(\omega) \equiv \frac{\kappa\,\omega}{2T}, \qquad \rho_{UV}(\omega) \equiv \frac{g^2(\mu) C_f \omega^3}{6\pi}, \qquad \mu = \max\left(\omega^{1-\frac{\gamma_0}{b_0}} (\pi T)^{\gamma_0/b_0}, \pi T\right)$$





EXTRA SLIDE: Quarkonia in QGP

- The evolution of quarkonia in QGP can be written down by studying the evolution of the quarkonium density matrix ρ obtained by integrating out the medium degrees of freedom.
- The evolution of ρ involves an effective Hamiltonian.

Y. Akamatsu, 2013-20

- ρ, H_{eff} can be split into singlet and octet subspaces,

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}, \; H_{\text{eff}} = \begin{pmatrix} H_s(T) & 0 \\ 0 & H_o(T) \end{pmatrix},$$

and $C_\pm, C_o$ control singlet $\longleftrightarrow$ octet and octet $\longleftrightarrow$ octet transitions, respectively.

Brambilla, Vairo, et al., 2017-2023

- $C_\pm \propto \sqrt{\kappa}\, r$, where the transport coefficient κ incorporates system information, and is related to $\tilde{G}(\omega \to 0)$.