Charm (and bottom) tetraquarks from lattice QCD

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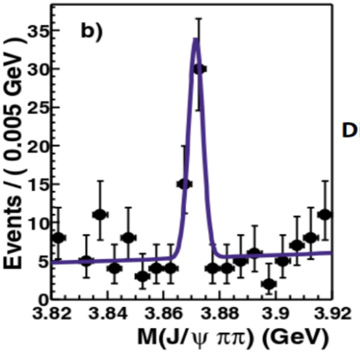
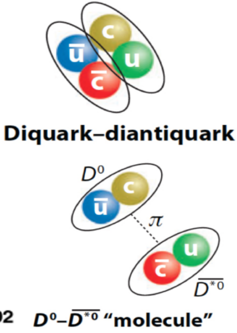
IMPCAS, Huizhou

X(3872): Belle 2003

- ✿ first observed in Belle 2003 (Belle PRL 2003)
- ✿ Quantum numbers, $J^{PC} = 1^{++}$ (LHCb, 2013)
- ✿ Appears within 1 MeV below $D^0\bar{D}^{*0}$ threshold.
- ✿ Preferred strong decay modes $J/\psi\ \omega$ and $J/\psi\ \rho$





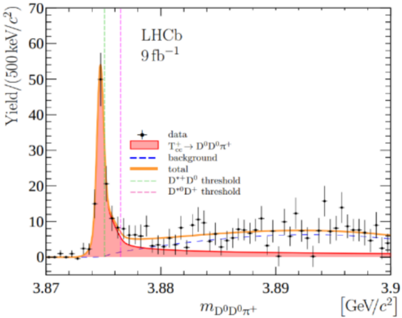
Belle hep-ex/0308029; CMS news

- ✿ Flavor info: $D^0 \equiv [c\bar{u}]$, $\bar{D}^{*0} \equiv [u\bar{c}]$, $J/\psi \equiv [c\bar{c}]$
- ✿ Flavor info: $\rho \equiv \pi \equiv [u\bar{u}] - [d\bar{d}]$, $\omega \equiv [u\bar{u}] + [d\bar{d}]$
- ✿ Isospin: A symmetry between u and d quarks $\equiv$ neutron-proton symmetry

Heisenberg 1932

- ✿ The isospin still uncertain
  - * nearly equal branching fraction to $J/\psi\ \omega$ and $J/\psi\ \rho$ decays.
  - * No charge partner candidates observed.

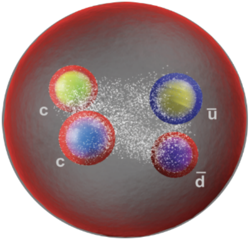
Doubly charm tetraquark: T_{cc}^{+}



LHCb: 2109.01038, 2109.01056

$$\delta m \equiv m_{T_{cc}^{+}} - (m_{D^{*+}} + m_{D^0})$$

$$\delta m_{\text{pole}} = -360 \pm 40^{+4}_{-0} \ \text{keV}/c^2\,,$$
$$\Gamma_{\text{pole}} = 48 \pm 2^{+0}_{-14}\,\text{keV}\,.$$

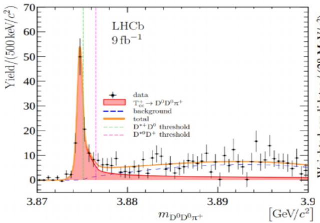


Courtesy: CERN Courier

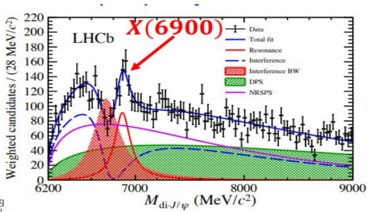
- The doubly charmed tetraquark T_{cc}^{+}, $I = 0$ and favours $J^P = 1^+$. Nature Phys., Nature Comm. 2022
  Striking similarities with the longest known heavy exotic, X(3872).
- No features observed in $D^0 D^+ \pi^+$: possibly not $I = 1$.
- Many more exotic tetraquark candidates discovered recently, T_{cs}, $T_{c\bar{s}}$, $X(6900)$.
  Prospects also for T_{bc} in the near future. See talk by Ivan Polyakov at Hadron 2023
- Doubly heavy tetraquarks: theory proposals date back to 1980s.

c.f. Ader&Richard PRD25(1982)2370

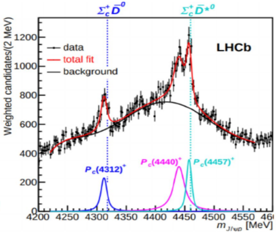
Beyond baryons and mesons in experiments



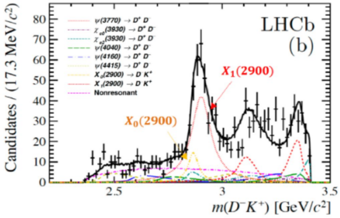
T_{cc} LHCb 2021



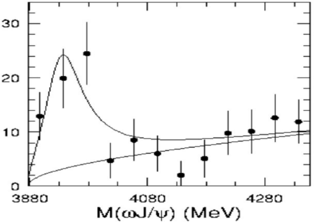
X(6900) LHCb 2020



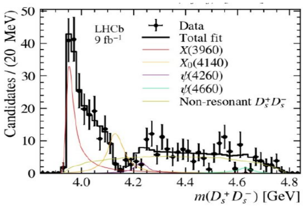
P_c LHCb 2019



X(2900) LHCb 2020

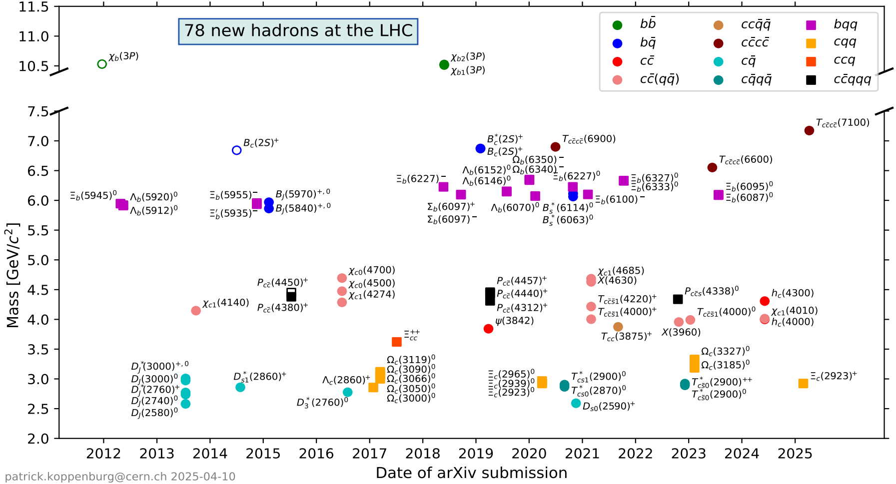


X(3915) Belle 2005



X(3960) LHCb 2021

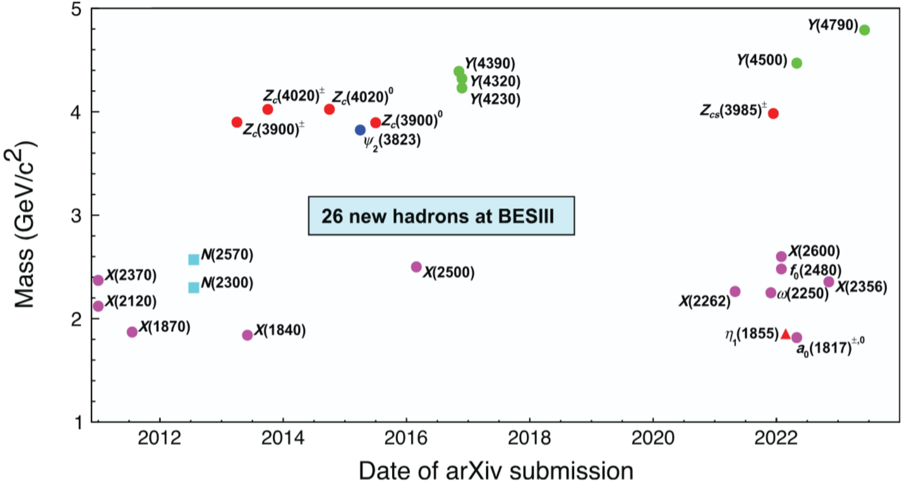
Summary of LHCb discoveries



https://www.nikhef.nl/~pkoppenb/particles.html

See a recent talk by Liming Zhang here

Summary of BESIII discoveries

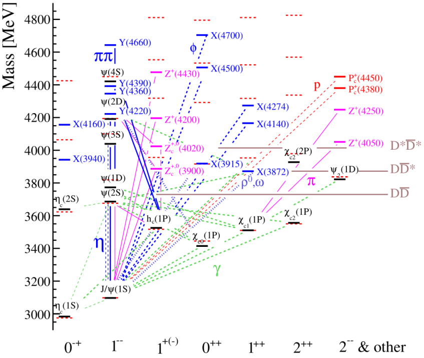


Liu and Mitchell Sci.Bull. 2310.09465

Prospects also in FTCF

Charmonium spectrum

- Rich experimental spectrum with several prospects.
  Exotics ...
- Comparison with simple minded model of hadrons.
  Mesons $\sim\ \bar{q}q$;   Baryons $\sim\ qqq$
- Model: Quantum mechanical system with a potential inspired from properties of strong interactions.
  Godfrey, (Isgur), ... 2015



Olsen *et al* 2017

A handful of models and several different predictions.

Need for hadron spectroscopy in QCD from first principles

Correlation functions

- Aim : to extract the physical states of QCD.
- Example case: mass of a pseudoscalar meson (pion)
  The simplest interpolating current: $\bar{\psi}\gamma_5\psi$
- Euclidean two point current-current correlation functions

$$
\begin{aligned}
C(t) &= \langle 0|[\bar{\psi}\gamma_5\psi](t)[\bar{\psi}\gamma_5\psi](0)|0\rangle \\
&= \langle 0|e^{Ht}[\bar{\psi}\gamma_5\psi](0)e^{-Ht}[\bar{\psi}\gamma_5\psi](0)|0\rangle \\
&= \sum_n e^{-E_n t}\langle 0|\bar{\psi}\gamma_5\psi(0)|n\rangle\langle n|\bar{\psi}\gamma_5\psi(0)|0\rangle \\
&= \sum_n |Z_n|^2 e^{-E_n t}
\end{aligned}
$$

Extraction of the mass spectrum

$C(t) = \sum_n |Z_n|^2 e^{-E_n t}$, which at large times, $C(t) \to |Z_0|^2 e^{-E_0 t}$

The operator can in principle couple with all the states that have its q. #s.
The strength of coupling Z_n determines the quality of signal.

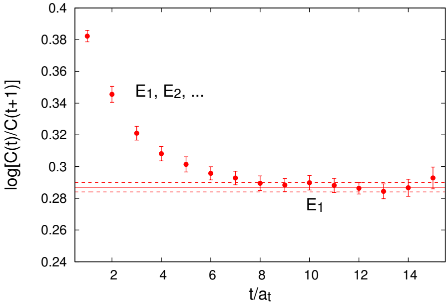
Effective mass defined as $m_{eff} = \frac{1}{dt} \log[\frac{C(t)}{C(t+dt)}]$
Mass extraction: Fit to $C(t)$ across multiple time slices.

Ground states: Single exponential fit forms

Excited states: Multi-exponential fit forms: Stability of fits!

Limited # time slices to extract excited state energies from multi-exponential fits.

Extraction of energy degenerate states is impossible this way.



Correlation matrices $C_{ji}(t)$ and GEVP

- Instead let us build a matrix of correlatiion functions:
  $$C_{ji}(t) = \langle 0|\Phi_j(t)\bar{\Phi}_i(0)|0\rangle = \sum_n \frac{Z_i^{n*} Z_j^n}{2E_n} e^{-E_n(t)}$$
  where $\Phi_j(t)$ and $\bar{\Phi}_i(0)$ are the desired interpolating operators. *c.f.* Detmold *et al.* 2403.00672
  $Z_j^n = \langle 0|\Phi_j|n\rangle$ are the operator-state overlaps.

- $C_{ji}(t)$ is Hermitian by construction. The eigensystem is automatically orthogonal.
  The eigenvalues representing the evolution of physical states.

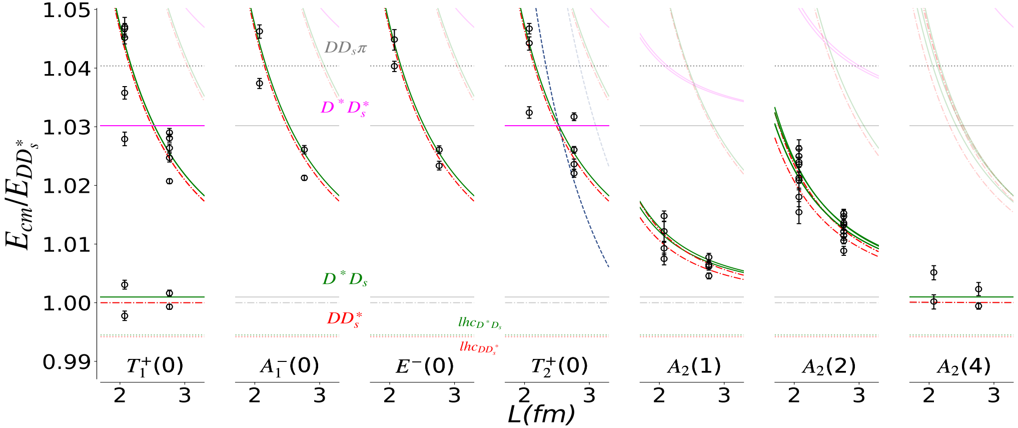
- Variational method: Application of Rayleigh-Ritz theorem.

- Solving the generalized eigenvalue problem for $C_{ji}(t)$. Michael NPB (1985)
  $$C_{ji}(t) v_j^{(n)}(t_0) = \lambda^{(n)}(t, t_0) C_{ji}(t_0) v_j^{(n)}(t_0)$$

- The m principal correlators given by eigenvalues behave as
  $$\lambda_n(t, t_0) \sim e^{-E_n(t-t_0)}(1 + \mathcal{O}(e^{-\partial E(t-t_0)})).$$

- Eigenvectors related to the operator state overlaps
  $$Z_j^n = \langle 0|\Phi_j|n\rangle \propto v_j^{(n)}(t_0)$$

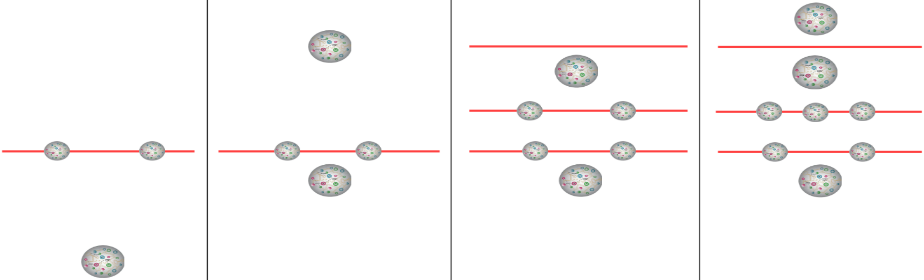
Excited state spectrum in DD_s-DD_s^*-D^*D_s-$D^*D_s^*$ system: An example



Shrimal, MP, *et al.*PRD 2504.16931

- CLS Ensembles : Spatial volumes: $L \sim 2$ fm and $L \sim 2.7$ fm; $a \sim 0.086$ fm
  $m_\pi \sim$ 280 MeV, $m_K \sim$ 467 MeV, ($m_c^?$): $m_D \sim$ 1762 MeV and 1927 MeV
- All bilocal meson-meson $[1_c \otimes 1_c \to 1_c]$ interpolators [All our results, unless specified]:
  $O^{M_1M_2}(\boldsymbol{P}) = \sum_k A_k M_1(\boldsymbol{p}_{1k}) M_2(\boldsymbol{p}_{2k}), \quad \boldsymbol{P} = \boldsymbol{p}_{1k} + \boldsymbol{p}_{2k}$

Complexity in Hadron spectroscopy



Straightforward

Deeply bound;
Strong decay stable;

$\pi, K, D, p, n, \Lambda, \Xi_{cc}, \ldots$

Exponential volume corrections
$[E_\infty - E_L \propto e^{-mL}]$

Relatively Easy

Shallow bound states;
Elastic resonances;
Only two body decays.

Deuteron,
$\Delta, \rho, D^*, D^*_{s0}, D_{s1}, \ldots$

Difficult

Inelastic resonances;
Multiple two-body final states;
Most hadrons.

Additionally three-body decays.

Quite complex

Inelastic resonances;
Multiple final state configs;
Most hadrons.

XYZTPs, glueballs, nuclei, ...

Power law volume corrections $\Delta E \propto \frac{a_0}{L^3} + O(\frac{1}{L^4})]$
Need a rigorous finite-volume amplitude analysis.

Finite volume spectrum and infinite volume physics

- On a finite volume Euclidean lattice : Discrete energy spectrum
  Cannot constrain infinite volume scattering amplitude away from threshold.

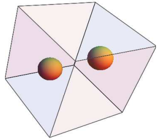
  Maiani-Testa PLB (1990)

- Non-interacting two-hadron levels are given by
  $$E(L) = \sqrt{m_1^2 + \boldsymbol{k}_1^2} + \sqrt{m_2^2 + \boldsymbol{k}_2^2} \text{ where } \boldsymbol{k}_{1,2} = \tfrac{2\pi}{L}(n_x, n_y, n_z).$$

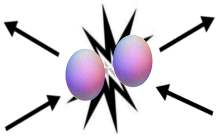
- Switching on the interaction: $\boldsymbol{k}_{1,2} \neq \frac{2\pi}{L}(n_x, n_y, n_z)$. *e.g.* in 1D $\boldsymbol{k}_{1,2} = \frac{2\pi}{L} n + \frac{2}{L}\delta(k)$.

- Lüscher's formalsim: **finite volume level shifts ⇔ infinite volume phase shifts.**

  Lüscher NPB (1991)



B



$\tilde{K}^{-1}$

- Generalizations of Lüscher's formalism: *c.f.* Briceño 1401.3312
  Quite complex problem: inelastic resonances ($R \to H_1 H_2, H_3 H_4$)
  Quantization condition is a determinant equation: $Det(B(L, k^2) - \tilde{K}^{-1}(k^2)) = 0$
  becomes an underconstrained problem with only few energy levels at hand.

Extensions and other methods

- Extensions within and beyond elastic scattering :
  - different inertial frames, boundary conditions
  - multiple scattering channels
  - particles with different identities   Briceño 1401.3312
  - 2-particle scattering in finite volume code: https://github.com/cjmorningstar10/TwoHadronsInBox
  - 3-particle scattering : Hansen, Sharpe, Lopez, Mai, Döring, Rusetsky, ...
- HALQCD method :
  - Determine the potential between scattering particles
  - Extract resonance information solving Schrödinger equation.

  Ishii *et al.* nucl-th/0611096; 1203.3642
- finite volume Hamiltonian EFT / Quantization condition in plane wave basis :
  - Constrain free parameters of the Hamiltonian based on lattice spectrum
  - Solve for EVP to extract resonance information.

  Hall *et al.* 1303.4157

  Meng & Epelbaum 2108.02709

  Mai & Döring 1709.08222
- Optical potential :

  Agadjanov *et al.* 1603.07205

  Hammer, Pang, Rusetsky, 1707.02176

Scattering amplitude parametrization

- Scattering amplitude: $S = 1 + i\frac{4k}{E_{cm}}t$
- For an elastic scattering, and assuming only S-wave,

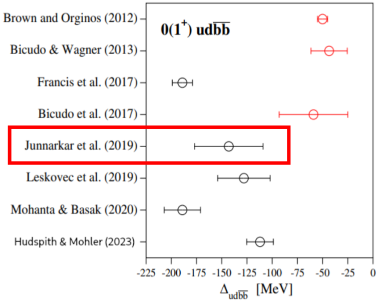
$$t^{-1} = \frac{2\tilde{K}^{-1}}{E_{cm}} - i\frac{2k}{E_{cm}}, \quad \text{with} \quad \tilde{K}^{-1} = k.cot\delta(k)$$

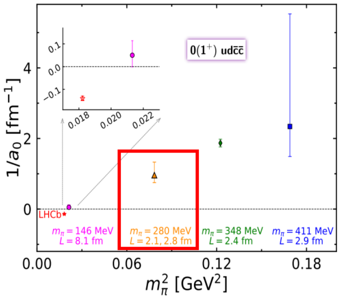
(virtual/bound) state constraint below threshold: $k.cot\delta(k) = (+/-)\sqrt{-k^2}$

- Lüscher's prescription: $k.cot\delta(k) = B(L, k^2)$: a known mathematical function. k^2 is determined from each extracted finite volume energy splittings.
- Parametrize $k.cot\delta(k)$ as different functions of k. Effective Range Expansion (ERE): $k.cot\delta(k) = a_0^{-1} + 0.5r_0k^2 + \beta_i k^{2i+4}$. The best fits determined to represent the energy dependence.
- For multichannel processes, $\tilde{K}^{-1}(k^2)$ and $B(L, k^2)$ become matrices, the Quantization conditions become a matrix determinant equation, each energy level gives a constraint, and each $\tilde{K}^{-1}$-matrix element* needs to be parametrized.

$$Det(\tilde{K}^{-1}(k^2) - B(L, k^2)) = 0$$

$\mathrm{T_{bb}}$ and $\mathrm{T_{cc}}$ using lattice QCD: $\mathrm{I(J^P) = 0(1^+)}$





- Deeper binding in doubly bottom tetraquarks $\mathcal{O}(100MeV)$. Fig: Hudspith&Mohler 2023

  Red box: ILGTI work on QQ tetraquarks: Junnarkar, Mathur, MP PRD 2019

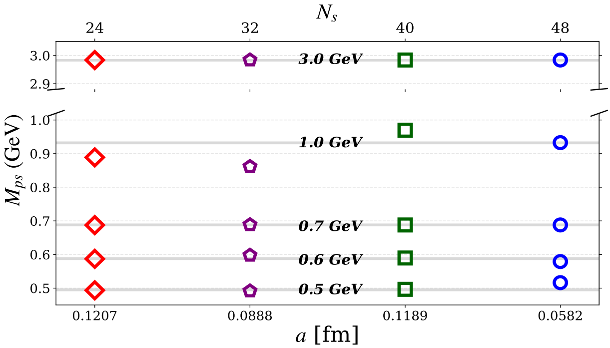
- Shallow bound state in doubly charm tetraquarks $\mathcal{O}(100keV)$. Fig: Lyu *et al.*PRL 2302.04505

  Red box: T_{cc} (RQCD) [PRL 2022] and its quark mass dependence [2402.14715].

  Chen *et al.* 2206.06185; Whyte, Wilson and Thomas 2405.15741

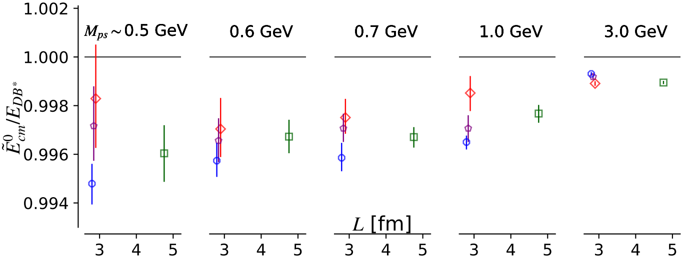
- No concrete conclusions in the bottom-charm tetraquark sector.
  A summary of different lattice investigations → see review by Pedro Bicudo, 2212.07793

Lattice setup for bottom hadron studies

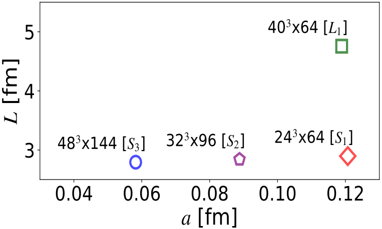


- MILC dynamical ensembles with $N_f = 2 + 1 + 1$ HISQ fields.
- Valence quark fields with masses ranging from light to charm: overlap action
- Bottom quark evolution using a NRQCD Hamiltonian.
  tuned using kinetic mass of 1S bottomonium spin averaged $\overline{M}^{\bar{b}b}$ Mathur *et al* Lattice 2016

The ground state spectrum: $\mathbf{I(J^P) = 0(1^+)}$

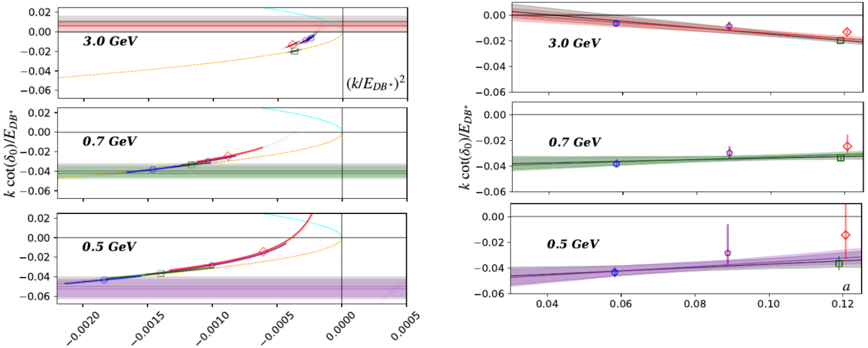


- Energy spectrum determined based on fits to $R^0(t)$.
  Automatic accounting for NRQCD additive correction.
- Energy reconstructed using $\tilde{E}^0 = \Delta E^0 + \overline{M}^{lat}_{B^*} + M^{lat}_D$
  where $\overline{M}^{lat}_{B^*} = M^{lat}_{B^*} - 0.5\overline{M}^{\bar{b}b,\ lat} + 0.5\overline{M}^{\bar{b}b,\ phys}$
- Consistent negative energy shifts.
  Decreasing magnitude with increasing $m_{u/d}$ or M_{ps}
- Non-trivial lattice spacing dependence.



MP *et al* PRL 2024

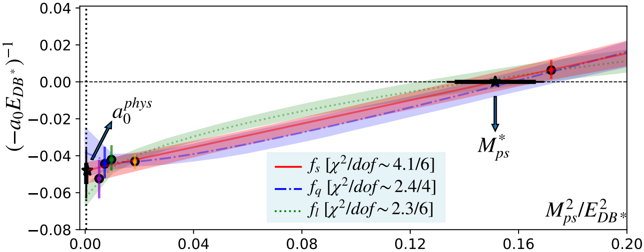
Finite volume analysis and continuum extrapolation: $\mathbf{I(J^P) = 0(1^+)}$



- Elastic DB^* scattering: finite volume analysis *à la* Lüscher. Only ground states used and only scattering length in an ERE. [$k cot\delta_0 \sim -1/a_0$] — Briceño PRD 2014
- A linear lattice spacing dependence assumed for the fitted amplitude.
- Determined DB^* scattering length in the continuum limit for all M_{ps}. Results indicate attractive interaction between D and B^* mesons at all M_{ps}.

MP *et al* PRL 2024

$\mathbf{M_{ps}}$ dependence of $\mathbf{DB^*}$ scattering length



- Light quark mass ($m_{u/d}$ or M_{ps}) dependence.

  $$f_l(M_{ps}) = \alpha_c + \alpha_l M_{ps}, \quad f_s(M_{ps}) = \beta_c + \beta_s M_{ps}^2, \quad \text{and} \quad f_q(M_{ps}) = \theta_c + \theta_l M_{ps} + \theta_s M_{ps}^2.$$

  indicates a real bound state at physical pion mass.

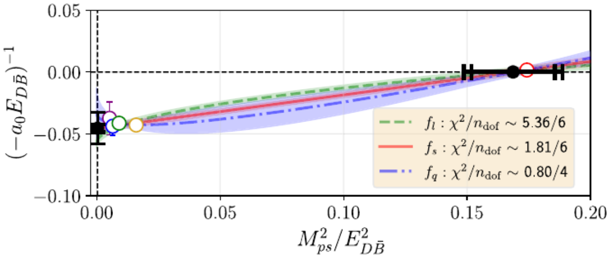
- DB^* scattering length and binding energy in the continuum limit

  $$a_0^{phys} = 0.57(^{+4}_{-5})(17)\ \text{fm} \quad \text{and} \quad \delta m_{T_{bc}} = -43(^{+6}_{-7})(^{+14}_{-24})\ \text{MeV}$$

- Results consistent with the nature of interaction with Alexandrou *et al.* PRL 2024, but conflicting numbers for binding energy estimates.

MP, Radhakrishnan, Mathur, PRL 2024

$\mathbf{M_{ps}}$ dependence of DB scattering length



- Light quark mass ($m_{u/d}$ or M_{ps}) dependence.

$$f_l(M_{ps}) = \alpha_c + \alpha_l M_{ps}, \quad f_s(M_{ps}) = \beta_c + \beta_s M_{ps}^2, \quad \text{and} \quad f_q(M_{ps}) = \theta_c + \theta_l M_{ps} + \theta_s M_{ps}^2.$$

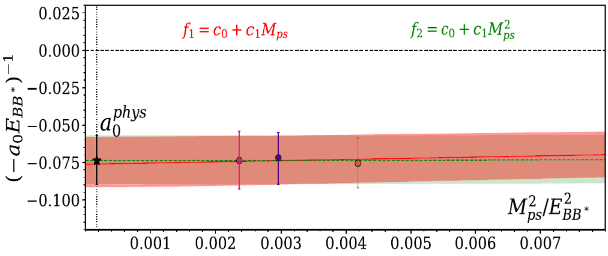
indicates a real bound state at physical pion mass.

- DB scattering length and binding energy in the continuum limit

$$a_0^{phys} = 0.61(^{+3}_{-4})(18)\ \text{fm} \quad \text{and} \quad \delta m_{T_{bc}} = -39(^{+4}_{-6})(^{+8}_{-18})\ \text{MeV}$$

Radhakrishnan, MP, Mathur, PRD 2024

$\mathbf{M_{ps}}$ dependence of $\mathbf{BB^*}$ scattering length



- Light quark mass ($m_{u/d}$ or M_{ps}) dependence.

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  indicates a real bound state at physical pion mass.

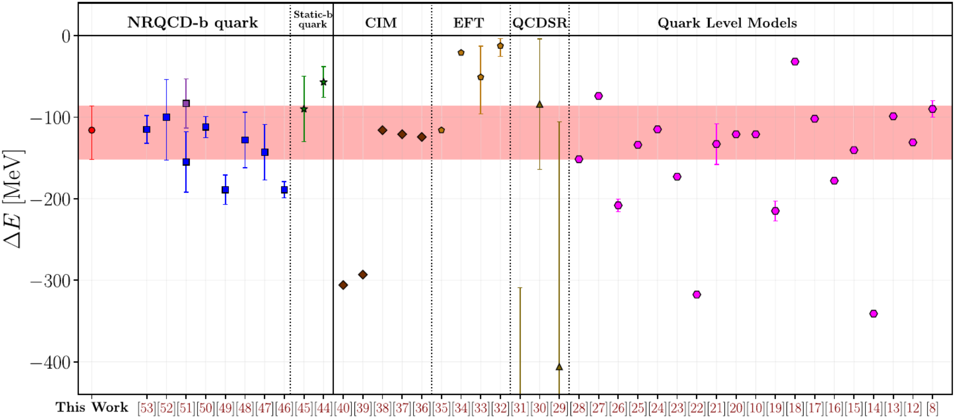
- BB^* scattering length and binding energy in the continuum limit

  $$a_0^{phys} = 0.25(^{+4}_{-3}) \text{ fm} \quad \text{and} \quad \delta m_{T_{bc}} = -116(^{+30}_{-36}) \text{ MeV}$$

- No signature of binding in KB^* and KB systems.

Tripathy, Mathur, MP PRD 2025; Junnarkar, Mathur, MP PRD 2018

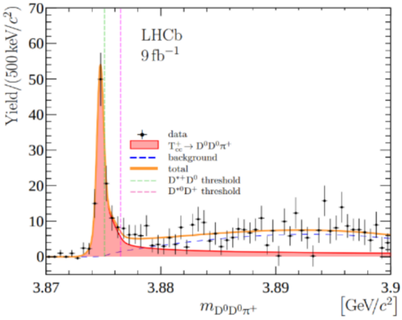
T_{bb} binding energy: Literature



- Consistent with the most lattice determinations.

Tripathy, Mathur, MP PRD 2025

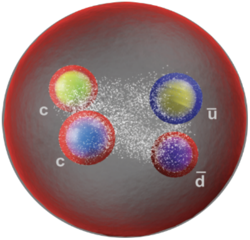
Doubly charm tetraquark: T_{cc}^+ [Revision]



LHCb: 2109.01038, 2109.01056

$$\delta m \equiv m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0})$$

$$\begin{aligned}\delta m_{\rm pole} &= -360 \pm 40\,^{+4}_{-0}\ \text{keV}/c^2\,,\\ \Gamma_{\rm pole} &= 48 \pm 2\,^{+0}_{-14}\ \text{keV}\,.\end{aligned}$$



Courtesy: CERN Courier

- The doubly charmed tetraquark T_{cc}^+, $I=0$ and favours $J^P=1^+$. Nature Phys., Nature Comm. 2022
  Striking similarities with the longest known heavy exotic, X(3872).
- No features observed in $D^0D^+\pi^+$: possibly not $I=1$.
- Many more exotic tetraquark candidates discovered recently, T_{cs}, $T_{c\bar{s}}$, $X(6900)$.
  Prospects also for T_{bc} in the near future. See talk by Ivan Polyakov at Hadron 2023
- Doubly heavy tetraquarks: theory proposals date back to 1980s.
  c.f. Ader&Richard PRD25(1982)2370
- Only two lattice calculations before discovery. No amplitude extraction.
  Cheung *et al.* 1709.01417; Junnarkar, Mathur, MP, 1810.12285

DD^* scattering amplitude and parametrization

- For the DD^* system [total spin equals 1], and assuming only $l < 2$, we have a 3×3 diagonal t matrix.

$$(t_l^{(J)})^{-1} = \frac{2(\tilde{K}_l^{(J)})^{-1}}{E_{cm} p^{2l}} - i\frac{2p}{E_{cm}}, \quad (\tilde{K}_l^{(J)})^{-1} = p^{2l+1} \cot \delta_l^{(J)}$$

- Using an effective range expansion near-threshold, we have

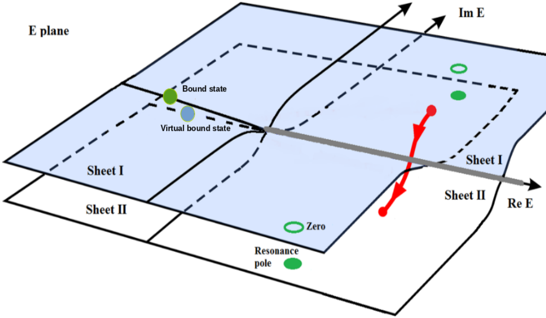
$$\tilde{K}^{-1} = \begin{array}{cc} \begin{bmatrix} \frac{1}{a_0^{(1)}} + \frac{r_0^{(1)} p^2}{2} & 0 & 0 \\ 0 & \frac{1}{a_1^{(0)}} + \frac{r_1^{(0)} p^2}{2} & 0 \\ 0 & 0 & \frac{1}{a_1^{(2)}} \end{bmatrix} & \begin{array}{ll} \text{J=1} & \text{l=0} \\ \text{J=0} & \text{l=1} \\ \text{J=2} & \text{l=1} \end{array} \end{array}$$

- Constraint on bound states:

$$p^{2l+1} cot(\delta_l) = -1^{\alpha} p^{2l} \sqrt{-p^2}$$

$\alpha = 1(2)$ for a real (virtual) bound state.

Virtual/bound states

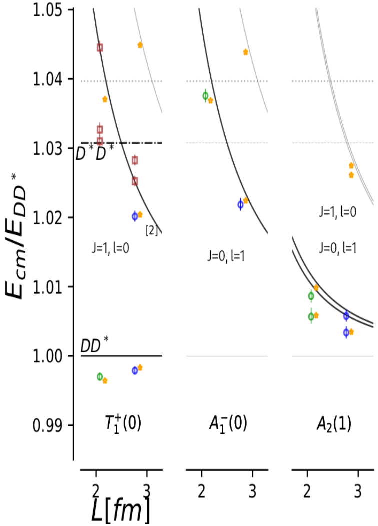


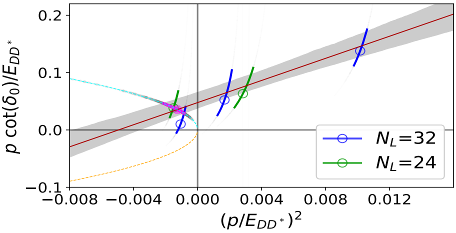
- $T \propto (p cot\delta_0 - ip)^{-1}$. Bound state is a pole in T with $p = i|p|$.
  Virtual bound state is a pole in T with $p = -i|p|$.

- An example for virtual bound state: spin-singlet dineutron.

- Only subthreshold real solutions allowed in the first Riemann sheet.
  Solutions in the second Riemann sheet: non-normalizable $[Im(p) = -|Im(p)|]$.

$I = 0$ DD^* scattering in $l = 0, 1$ @ $m_c^{(h)}$ with an ERE: T_{cc}^+





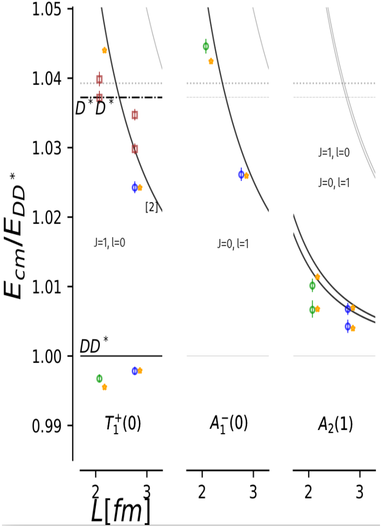
MP, Prelovsek PRL 2022

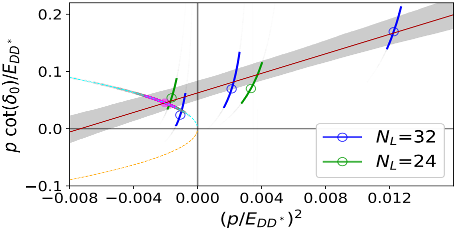
$m_\pi \sim 280$ MeV

- Fit quality:
  $$\chi^2/d.o.f. = 3.7/5.$$
- Fit parameters:
  $a_0^{(1)} = 1.04(0.29)$ fm & $r_0^{(1)} = 0.96(^{+0.18}_{-0.20})$ fm
  $a_1^{(0)} = 0.076(^{+0.008}_{-0.009})$ fm^3 & $r_1^{(0)} = 6.9(2.1)$ fm^{-1}
- Binding energy:
  $$\delta m_{T_{cc}} = -9.9(^{+3.6}_{-7.2}) \text{ MeV}.$$
- First evaluation of the DD^* amplitude in T_{cc} channel.

+/g refers to positive parity, -/u refers to negative parity.

$I = 0$ DD^* scattering in $l = 0, 1$ @ $m_c^{(l)}$ with an ERE: T_{cc}^+





MP, Prelovsek PRL 2022

$m_\pi \sim 280$ MeV

- Fit quality:
  $\chi^2/d.o.f. = 3.6/5.$
- Fit parameters:
  $a_0^{(1)} = 0.86(0.22)$ fm & $r_0^{(1)} = 0.92(^{+0.17}_{-0.19})$ fm
  $a_1^{(0)} = 0.117(^{+0.013}_{-0.014})$ fm^3 & $r_1^{(0)} = 8.6(^{+1.5}_{-1.1})$ fm^{-1}
- Binding energy:
  $\delta m_{T_{cc}} = -15.0(^{+4.6}_{-9.3})$ MeV.

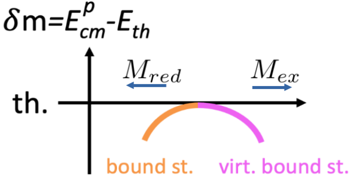
+/g refers to positive parity, -/u refers to negative parity.

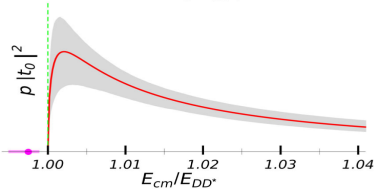
Our observations and inferences with ERE approach

- A shallow virtual bound state pole in s-wave related to T_{cc}.

	m_D [MeV]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(h)}$)	1927(1)	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(l)}$)	1762(1)	$-15.0(^{+4.6}_{-9.3})$	virtual bound st.
exp.	1864.85(5)	$-0.36(4)$	bound st.

- For $m_\pi > m_\pi^{phys}$, T_{cc} is expected to become a virtual bound state. At $m_\pi \sim$ 280 MeV, we indeed find a shallow virtual bound state.
- Observations in line with the expected behaviour of a near-threshold molecular bound state pole in simple Quantum Mechanical potentials.





MP, Prelovsek PRL 2022

- $M_{red}(\propto m_c)$ is the reduced mass of the DD^* system. E_{th} refers to $E_{DD^*} = M_D + M_{D^*}$.
- The mass of the particle exchanged during the interaction $M_{ex}(\propto m_{u/d})$.

Pion exchange interactions/left-hand cut: ERE and QC

- A two fold problem: (Unphysical pion masses used in lattice)

  $m_\pi > m_{D^*} - m_D \quad \Rightarrow \quad D^* \to D\pi$ is kinematicaly forbidden.

  $2 \to 2$ Generalized LQC: does not subthreshold lhc effects.

  Raposo&Hansen 2311.18793, Dawid *et al* 2303.04394, Hansen *et al* 2401.06609

  See recent talks by Hansen and Lopez

  ERE convergence fails at the nearest singularity.

  Left-hand cut in the DD^* system close below the DD^* threshold. Du *et al* 2303.09441

- Unphysical pion masses ($m_\pi > \Delta M = M_{D^*} - M_D$, stable D^* meson):

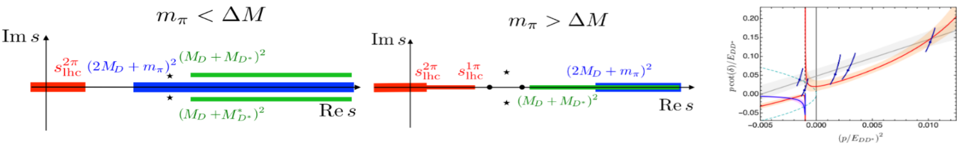


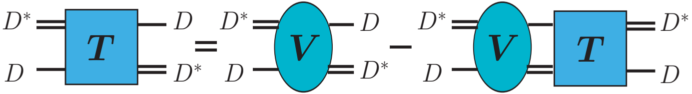
Figure taken from Du *et al* 2303.09441

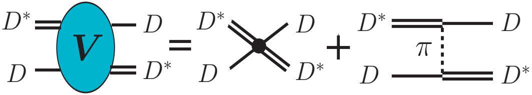
Long range pion exchange interactions: the origin of left-hand singularity and cut.

Fits with a potential that incorporates the one pion exchange:

Virtual bound states $\Rightarrow$ Virtual resonances

Solving Lippmann-Schwinger Equation for the DD^* amplitude





- The potential: a sum of short range and long range interactions

$$V(\boldsymbol{p}, \boldsymbol{p}') = V_{\rm CT}(p, p') + V_\pi^S(p, p') \quad \text{with} \quad V_{\rm CT}(p, p') = 2c_0 + 2c_2(p^2 + p'^2) + \mathcal{O}(p^4, p'^4)$$

- The scattering amplitude $\qquad T^{-1} \propto p \cot \delta_0 - ip$

- The pion decay constant f_π and $DD^*\pi$ coupling g_c
  at $m_\pi \sim 280$ MeV following the 1-loop χPT.

Du *et al* 2303.09441

One-pion exchange interaction/left-hand cut

- OPE from the lowest order NR Lagrangian

$$\mathcal{L} = \frac{g_c}{2f_\pi} \boldsymbol{D}^{*\dagger} \cdot \boldsymbol{\nabla}\pi^a \tau^a D + h.c. \ . \quad \Rightarrow \quad V_\pi(\boldsymbol{p}, \boldsymbol{p}') = 3\left(\frac{g_c}{2f_\pi}\right)^2 \frac{(\boldsymbol{\epsilon}\cdot\boldsymbol{q})(\boldsymbol{q}\cdot\boldsymbol{\epsilon}'^*)}{u - m_\pi^2}$$

Fleming *et al.* hep-ph/0703168, Hu&Mehen hep-ph/0511321

- Upon S-wave projection, we have

$$V_\pi^S(p,p) = \frac{g_c^2}{4f_\pi^2}\left[\frac{m_\pi^2 - q_0^2}{4p^2} \ln\left(1 + \frac{4p^2}{m_\pi^2 - q_0^2}\right) - 1\right]$$

Logarithmic function branch cut $\rightarrow$ infinite set of Riemann sheets

- With the finite branch point at

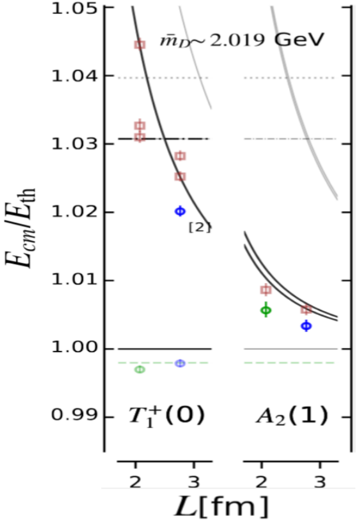
$$\boldsymbol{p^2_{\mathrm{lhc}} = \tfrac{1}{4}(q_0^2 - m_\pi^2) < 0} \text{ for all lattice setups.}$$

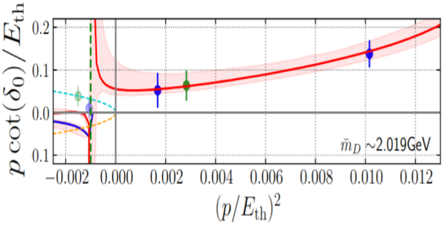
with $q_0 \simeq m_{D^*} - m_D$, where the $D^{(*)}$-meson recoil terms are ignored.

Du *et al.* 2303.09441

- Consequences:
  Complex phase shifts below the lhc.
  Modified near-threshold energy dependence.

$I = 0$ DD^* scattering in $l = 0$ @ $m_c^{(h)}$ with EFT: T_{cc}^+



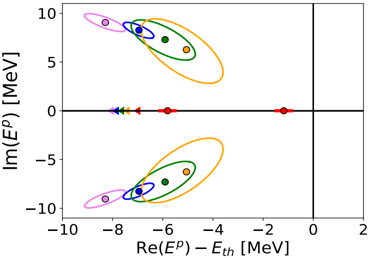


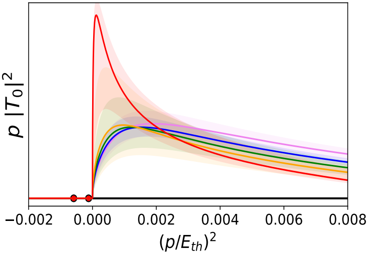
Collins, MP, *et al.* PRD 2024

- OPE pole at the *left-hand cut* branch point observed.
- Complex amplitudes along real axis along *left-hand cut*
- Hadronic poles observed as virtual resonance poles.
- Pure S-wave fits.
- E_{th} refers to $E_{DD^*} = M_D + M_{D^*}$

+/g refers to positive parity, -/u refers to negative parity.

Pole positions and scattering rate [EFT]

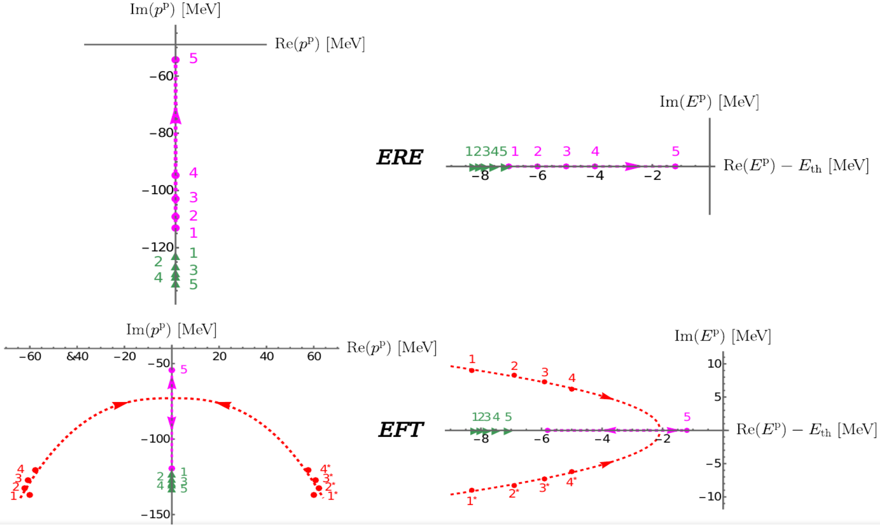




- Subthreshold resonance pole pair moving towards the real axis with increasing m_c.
- Collide on the real axis below threshold and turn back-to-back.
  At the heaviest m_c: virtual bound poles [in Red]
- With increasing m_c, subthreshold resonance poles evolves to become
  a pair of virtual bound poles.
- Enhancement in the DD^* scattering rate ($p|T_0|^2$).
  E_{th} refers to $E_{DD^*} = M_D + M_{D^*}$

Collins, MP, *et al.* PRD 2024

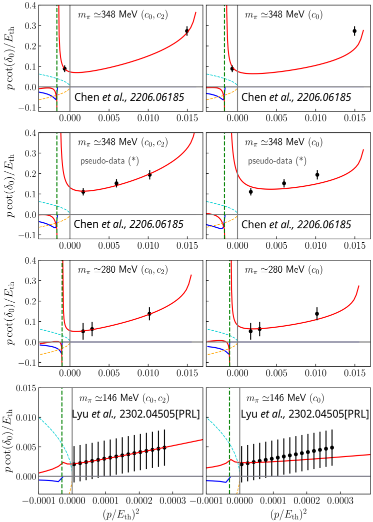
Pole trajectory of T_{cc}^+: ERE Vs EFT



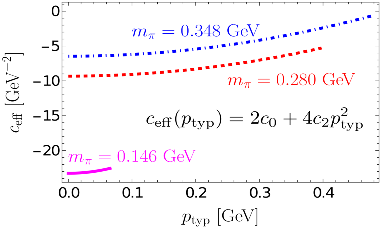
E_{th} refers to $E_{DD^*} = M_D + M_{D^*}$

Collins, MP, *et al.* PRD 2024

m_π dependence of the T_{cc} pole [EFT]



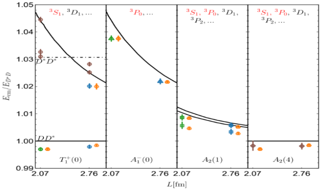
Collins, MP, *et al.* PRD 2024

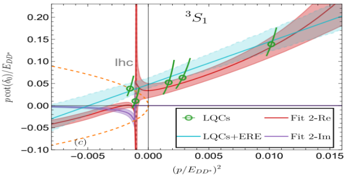


- Qualitative study of m_π dependence using $V_{\rm CT}(p, p') = 2c_0 + 2c_2(p^2 + p'^2)$
- Two parameter fit (c_0, c_2) [left] and a single parameter fit (c_0, with $c_2 = 0$) [right].
- Resonance poles at m_π ~348 and ~280 MeV. Shallow virtual bound poles at m_π =146 MeV.
- Stronger attraction for lighter m_π. [$c_{\rm eff}$] stronger binding in T_{cc} for lighter pions.
- m_π =146 MeV: HALQCD. Lyu *et al.* 2302.04505
  m_π =348 MeV: Chen *et al.* 2206.06185

Work around to LQC: A plane-wave approach and modified LQC

- An effective field theory incorporating OPE with a plane wave basis expansion [PlW].





Meng *et al.* 2312.01930

Virtual bound states ⇒ Virtual resonances [m_π ∼280 MeV]

- Modified 3-particle (Lüscher) Quantization Condition:

Hansen, Romero-Lopez, Sharpe, 2401.06609, Raposo, Hansen, 2311.18793

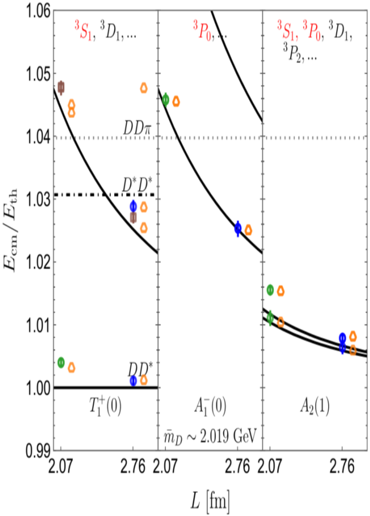
Dawid, Lopez, Sharpe 2409.17059, See a recent talk by Dawid here

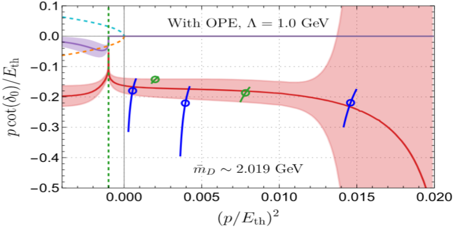
A rigorous procedure, but demands multiple lattice inputs.

- $D\pi$ finite volume spectrum up to the $D\pi\pi$ threshold.
- Isovector DD finite volume spectrum up to the $DD\pi$ threshold.
- Isoscalar $DD\pi$ finite volume spectrum up to the $DD\pi\pi$ threshold.

- Several followup investigations and proposals. *c.f.* Talks at Chiral Dynamics workshop: link

$I = 1$ DD^* scattering in $l = 0, 1$ @ $m_c^{(h)}$ with EFT+PlW





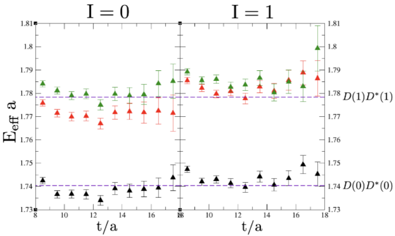
Meng, MP, *et al.* PRD 2025

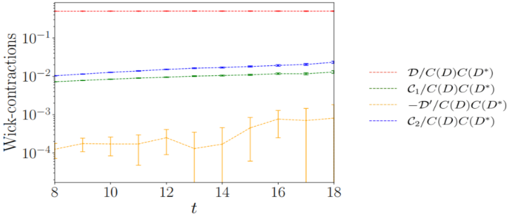
$m_\pi \sim 280$ MeV

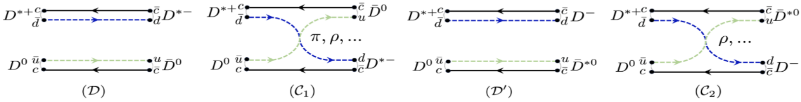
- Fit quality:
  $\chi^2/d.o.f. = 2.5/6.$
- Fit parameters:
  $a_0^{(1)} = -0.30(4)$ fm & $r_0^{(1)} = -0.89(55)$ fm
- No poles observed in the accessible region.
- First evaluation of the $I = 1$ DD^* amplitude.
  More recent investigation: Lachini *et al.* 2505.01363

+/g refers to positive parity, -/u refers to negative parity.

Isovector $\mathbf{T_{cc}}$: Weak repulsion







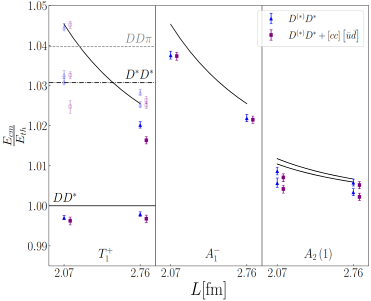
$$C_I^{DD^*}(t) = \mathcal{D} - \mathcal{C}_1 + (-)^{I+1}(\mathcal{D}' - \mathcal{C}_2)$$

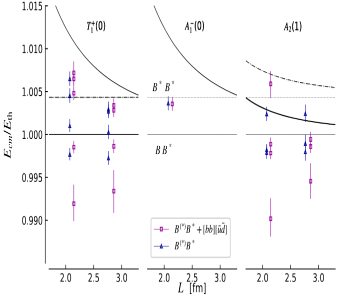
- $\mathcal{C}_2$ determining the isospin effects. Weakly repulsive interactions.

Ortiz-Pacheco, MP, *et al.*, 2312.13441

- Consistent with observations reported in Chen *et al.*, 2206.06185 [PLB]

Effect of local $[QQ]_{\bar{3}_c}[\bar{u}\bar{d}]_{3_c}$ operators: $I = 0$ DD^*/BB^* T_{QQ} Spectra

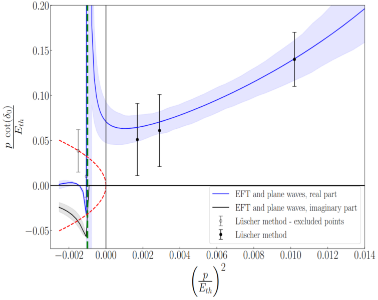


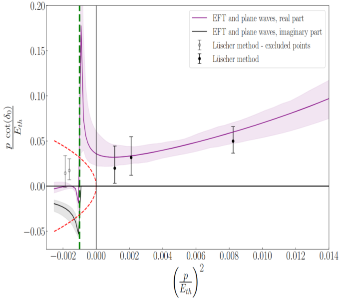


Prelovsek, MP, *et al.* PRD 2504.03473

- Bilocal meson-meson $[1_c \otimes 1_c \to 1_c]$ interpolators:
  $O^{M_1 M_2}(\boldsymbol{P}) = \sum_k A_k M_1(\boldsymbol{p}_{1k}) M_2(\boldsymbol{p}_{2k}), \quad \boldsymbol{P} = \boldsymbol{p}_{1k} + \boldsymbol{p}_{2k}$
- Local diquark-antidiquark $[\bar{3}_c \otimes 3_c \to 1_c]$ interpolators: $O^{4q}(\boldsymbol{P}) = [Q\Gamma_a Q]_{\bar{3}_c}[\bar{u}\Gamma_b \bar{d}]_{3_c}(\boldsymbol{P})$
- Left: DD^* Ground state energies nearly unaffected.
  First excitation in rest frame T_1^+ irrep gets significant downward shift.
- Right: BB^* spectra shows a significant change, already on the ground state energy estimates.
  Bottom quark approximated with a relativistic Wilson clover action.
  Caution: Uncontrolled discretization errors.

Effect of local $[cc]_{\bar{3}_c}[\bar{u}\bar{d}]_{3_c}$ operators: $I = 0$ DD^* Amplitude (T_{cc})

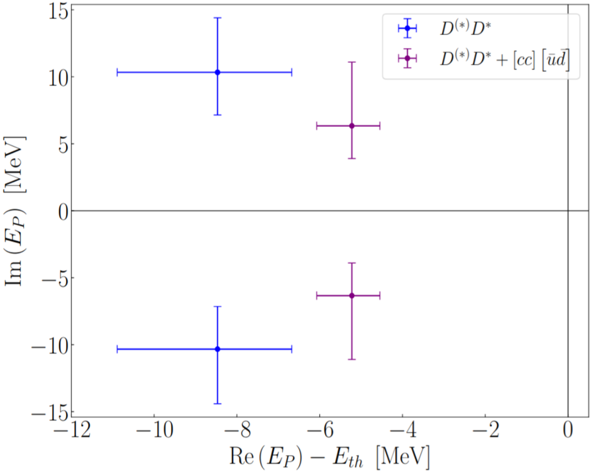




Prelovsek, MP, *et al.* PRD 2504.03473

- Amplitude extraction performed only for the charm sector.
- Left: Amplitude with purely $O^{M_1 M_2}(\boldsymbol{P})$.
- Right: Amplitude with full basis: $O^{M_1 M_2}(\boldsymbol{P})$ plus $O^{4q}(\boldsymbol{P})$.
- Potentially strong effects on the effective range and shape parameters. Enhanced binding.
- No amplitude extraction performed for bottom sector.
  Need for a multichannel analysis of the BB^*-B^*B^* system.

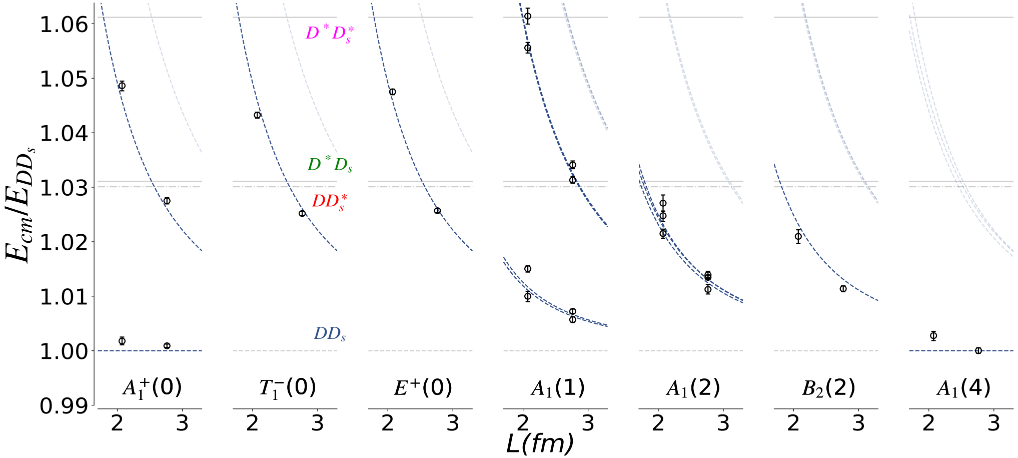
Effect of local $[cc]_{\bar{3}_c}[\bar{u}\bar{d}]_{3_c}$ operators: $I=0$ DD^* Poles (T_{cc})



Prelovsek, MP, *et al.* PRD 2504.03473

- Enhanced binding.

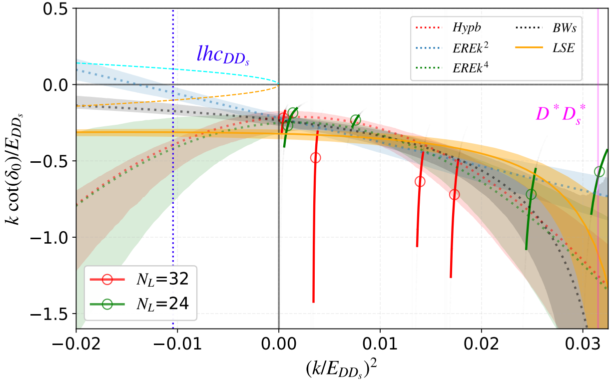
Elastic DD_s scattering: $I(J^P) = 1/2(0^+)\ T_{cc\bar{u}\bar{s}}$



Shrimal, Collins, Jana, MP, Prelovsek PRD 2504.16931

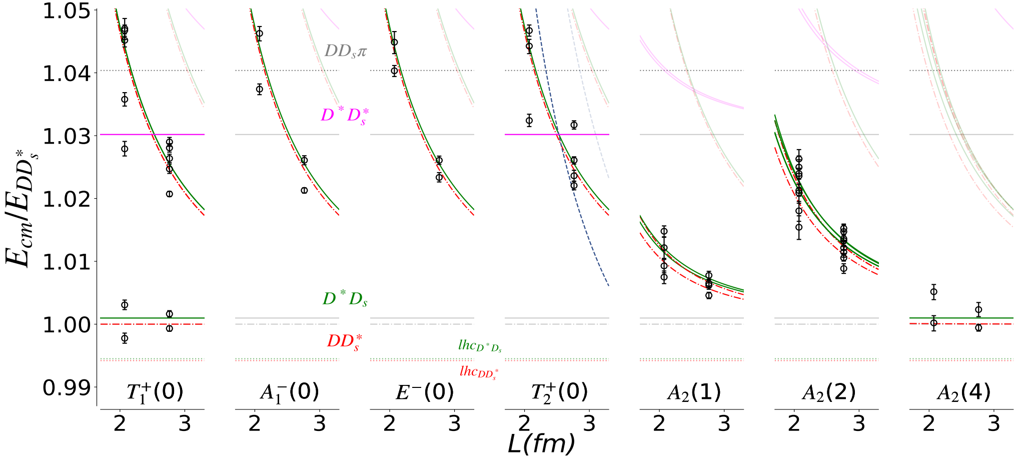
✿ CLS Ensembles : Spatial volumes: $L \sim 2$ fm and $L \sim 2.7$ fm; $a \sim 0.086$ fm
$m_\pi \sim$ 280 MeV, $m_K \sim$ 467 MeV, (m_c^h): $m_D \sim$ 1927 MeV

Elastic S-wave DD_s scattering amplitude $I(J^P) = 1/2(0^+)$ $T_{cc\bar{u}\bar{s}}$



- Only two previous lattice investigations. Both studies only in the rest frame. No amplitude extraction. Cheung *et al.* JHEP 2017; Junnarkar, Mathur, MP PRD 2018
- Energy shifts consistent with that observed in Cheung *et al.* JHEP 2017.
- Acausal poles with several parametrizations combined with Lüscher-based fits. Need to impose analyticity properties. Shrimal, Collins, Jana, MP, Prelovsek PRD 2504.16931
- No physically sensible poles identified in the accessible region.

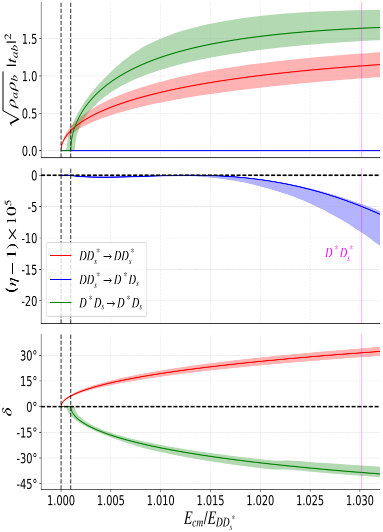
Inelastic DD_s-DD_s^*-D^*D_s-$D^*D_s^*$ spectra: $I(J^P) = 1/2(1^+)$ $T_{cc\bar{u}\bar{s}}$

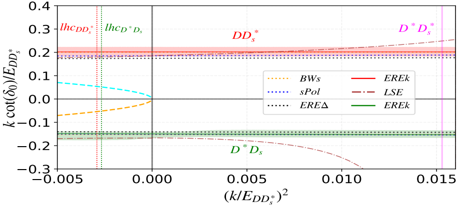


Shrimal, Collins, Jana, MP, Prelovsek PRD 2504.16931

✿ CLS Ensembles : Spatial volumes: $L \sim 2$ fm and $L \sim 2.7$ fm; $a \sim 0.086$ fm
$m_\pi \sim$ 280 MeV, $m_K \sim$ 467 MeV, (m_c^h): $m_D \sim$ 1927 MeV

Inelastic S-wave DD_s^*-D^*D_s amplitudes: $I(J^P) = 1/2(1^+)\ T_{cc\bar{u}\bar{s}}$



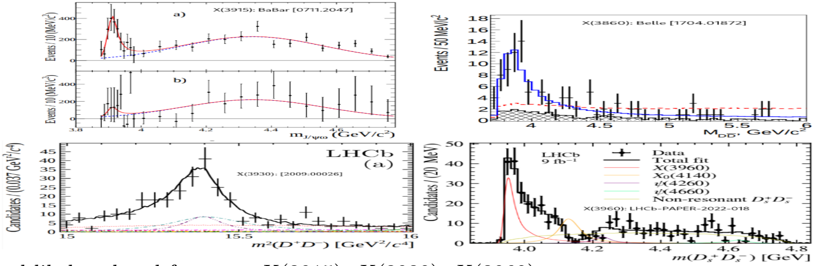


$$t_{ij} = \begin{cases} \dfrac{\eta\, e^{2i\delta_i} - 1}{2i\rho_i} & \text{if } i = j, \quad \rho_i = 2k_i/E_{cm}, \\ \dfrac{\sqrt{1-\eta^2}\, e^{i(\delta_i+\delta_j)}}{2\tilde{\rho}_i\tilde{\rho}_j} & \text{if } i \neq j, \quad \tilde{\rho}_i^{\,2} = \rho_i. \end{cases}$$

- Assuming a coupled system of DD_s^*-D^*D_s channels. Effects of DD_s and $D^*D_s^*$ channels ignored.
- Nontrivial pattern of energy shifts observed. Pattern consistent with that observed in Cheung *et al.* JHEP 2017
- Nearly decoupled system of DD_s^*-D^*D_s channels from the amplitudes and inelasticity.
- No pole features identified in the accessible region.

Shrimal, Collins, Jana, MP, Prelovsek PRD 2504.16931

Focus: Scalar charmonium-like states



- Several likely related features, $X(3915)$, $X(3930)$, $X(3960)$.
  Proximity to the $\bar{D}_s D_s$ threshold: Possible hidden strange content [$\mathbf{cs\bar{c}\bar{s}}$]
  $\Rightarrow$ narrow width from $\bar{D}D$

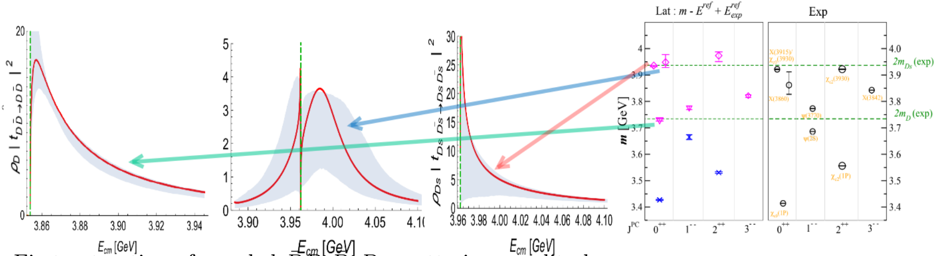
- Several phenomenological studies supporting this:
  Lebed Polosa 1602.08421, Chen *et al* 1706.09731, Bayar *et al* 2207.08490

- Another feature named as X(3860) observed by Belle. No evidence from LHCb.

- Yet unknown $\bar{D}D$ bound state, predicted by models.
  Gamermann *et al* 0612179, Hidalgo-Duque *et al* 1305.4487, Baru *et al* 1605.09649

- Such a $\bar{D}D$ bound state is supported by re-analysis of the exp. data.
  Danilkin *et al* 2111.15033, Ji *et al* 2212.00631.

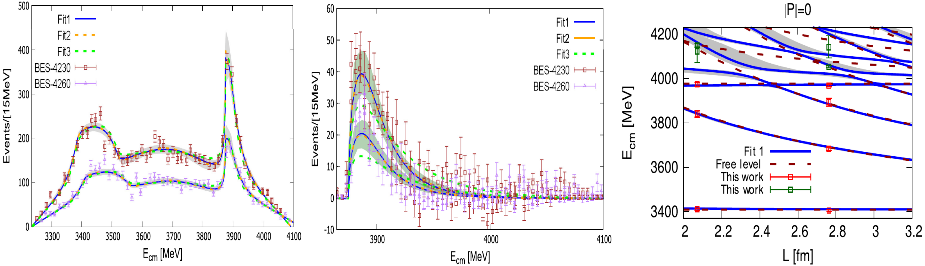
Charmonium-like resonances and bound states on the lattice



- ✿ First extraction of coupled $\bar{D}D$-$\bar{D}_sD_s$ scattering amplitude. $[\mathbf{\bar{c}c},\ \mathbf{\bar{c}c\bar{q}q};\ \mathbf{q} \rightarrow \mathbf{u}, \mathbf{d}, \mathbf{s}$, and $\mathbf{I = 0}]$.
- ✿ Lattice QCD ensembles : CLS Consortium
  $m_\pi \sim$ 280 MeV, $m_K \sim$ 467 MeV, $m_D \sim$ 1927 MeV, $a \sim 0.086$ fm
- ✿ In addition to conventional charmonium states, we observe candidates for three excited scalar charmonium states
  - $\Rightarrow$ a yet unobserved shallow $\bar{D}D$ bound state.
  - $\Rightarrow$ a $\bar{D}D$ resonance possibly related to X(3860).
  - $\Rightarrow$ a narrow resonance just below and with large coupling to $\bar{D}_sD_s$ threshold. possibly related to X(3960) / X(3930) / X(3915).
- ✿ Our (RQCD) recent publications on charmonium:

Collins, Mohler, MP, Piemonte, Prelovsek 2111.02934, 2011.02541, 1905.03506.

Isovector axialvector charmonium-like states from LFT and EFT



- First study of isovector charmonium-like states using multiple volume and inertial frames.
- Similated FV levels either consistent with noninteracting scenario or have mild -ve energy shifts. Most prominent in $D\bar{D}^*$ FV levels.
- Lattice QCD ensembles : CLS Consortium
  $m_\pi \sim$ 280 MeV, $m_K \sim$ 467 MeV, $m_D \sim$ 1927 MeV, $a \sim 0.086$ fm
- Highlights
  - $\Rightarrow$ Virtual poles below $D\bar{D}^*$ threshold in both charge parities.
  - $\Rightarrow$ Coupled $J/\psi\pi$-$D\bar{D}^*$ scattering as a function of m_π with an EFT framework.
    Results suggest EFT reasonably reconciles experimental lineshapes and FV energy levels
  - $\Rightarrow$ First study of s-wave $J/\psi\pi$ phase shifts using lattice.
- Our (RQCD) recent publications on charmonium:

Sadl, Collins, Guo, MP, Prelovsek, Yan 2406.09842.

Summary

- We have a handful of hadrons, with a large set of them still demanding an understanding based on first principles. The list is proliferating with several experimental efforts across the globe.

- Lattice QCD, being a suitable nonperturbative framework, has been used to study several of these hadrons.

- Made a 'very' brief outline of how hadron masses are extracted and how resonances can be studied in finite volume.

- Presented a selected examples of our own lattice investigations (doubly charm tetraquarks), particularly addressing near threshold poles.

- Many hadronic states remain unaddressed and several remaining challenges even before addressing lattice systematics. Formalisms accounting three body dynamics.
  New ideas to access highly excited states. ...

- Quark mass dependence as a probe to understand the nature of resonances.
  Heavy hadron sector serving as an excellent test bed.

- Lattice systematics: Need for huge computation resources.

Publications since 2018

Charmonium [RQCD]:

- MP *et al.* PRD 2019: *Identifying spin and parity of charmonia in flight with lattice QCD*
- Piemonte, MP *et al.* PRD 2019: *Charmonium resonances with* $J^{PC} = 1^{--}$ *and* 3^{--} *from* $\bar{D}D$ *scattering on the lattice*
- Prelovsek, MP *et al.* PRD 2021: *Charmonium-like resonances with* $J^{PC} = 0^{++}$, 2^{++} *in coupled* $\bar{D}D$-$\bar{D}_s D_s$ *scattering on the lattice*
- Sadl, MP, *et al.* PRD 2025: *Charmoniumlike channels* 1^+ *with isospin 1 from lattice and effective field theory*

Doubly charmed tetraquarks (covered in this talk) [RQCD]:

- MP, Prelovsek PRL 2022: *Signature of a doubly charm tetraquark pole in* DD^* *scattering on the lattice*
- Collins, MP *et al.* PRD 2024: *Toward the quark mass dependence of* T_{cc}^+ *from lattice QCD*
- Meng, MP *et al.* PRD 2025: *Doubly heavy tetraquark channel with isospin 1 from lattice QCD*
- Prelovsek, MP, *et al.* PRD 2025: *Doubly heavy tetraquarks from lattice QCD: incorporating diquark-antidiquark operators and the left-hand cut*
- Shrimal, MP, *et al.* PRD 2025: *Lattice study of* $cc\bar{u}\bar{s}$ *tetraquark channel in* $D^{(*)}D_s^{(*)}$ *scattering*

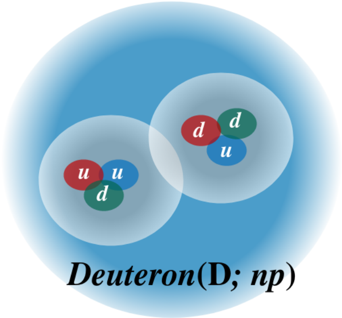
Publications since 2018

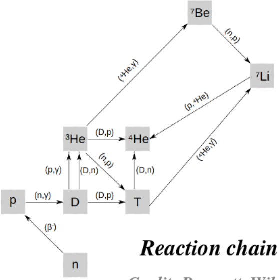
Bottom tetraquarks [with Indian colleagues]:

- Junnarkar, Mathur, MP PRD 2018: *Study of doubly heavy tetraquarks in Lattice QCD*
- MP, Radhakrishnan, Mathur *et al.* PRL 2024: *Bound isoscalar axial-vector $bc\bar{u}\bar{d}$ tetraquark T_{bc} from lattice QCD using two-meson and diquark-antidiquark variational basis*
- Radhakrishnan, Mathur, MP *et al.* PRD 2024: *Study of the isoscalar scalar $bc\bar{u}\bar{d}$ tetraquark T_{bc} with lattice QCD*
- Tripathy, Mathur, MP *et al.* PRD 2025: *$bb\bar{u}\bar{d}$ and $bs\bar{u}\bar{d}$ tetraquarks from lattice QCD using two-meson and diquark-antidiquark variational basis*

Thank you

Deuteron: the longest known dibaryon



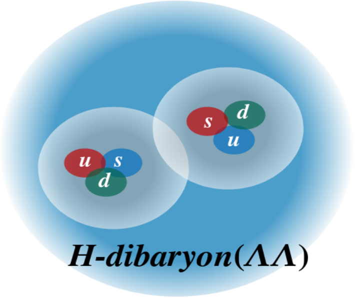


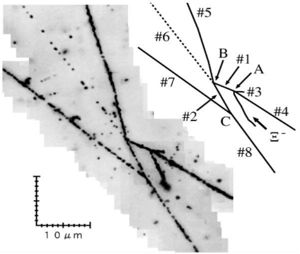
Reaction chain in BBN

Credit: Pamputt, Wikimedia Commons

- Nucleus of Deuterium discovered in 1932. Urey, Brickwedde & Murphy
- A very fine-tuned binding energy $\Delta E = M_D - M_p - M_n = 2.2$ MeV.
- Big bang Nucleosynthesis (BBN) has a deuteron bottleneck:
  Determines the abundances of light nuclei.
- How will the binding energy vary with quark masses?
  Could there be dineutrons or diprotons with heavier light quark masses.

The scalar dihyperon

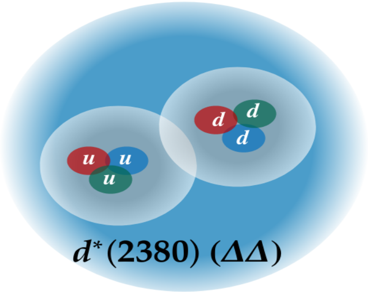


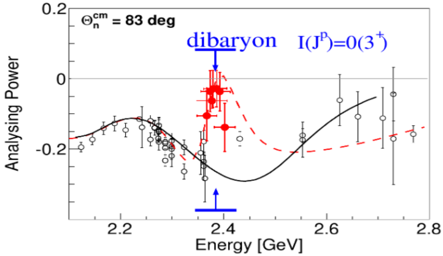


***NAGARA event*:**
Takahashi PRL 87, 212502 (2001)

- Bound *uuddss* flavor-singlet dihyperon with $J^P = 0^+$:
  Perhaps a stable Dihyperon — Jaffe PRL 38 195 (1977)
- NAGARA event: Strongest constraint on binding energy.
  $$B_H < B_{\Lambda\Lambda}^{Nagara} = 6.91 \pm 0.16 \text{ MeV}$$
  Takahashi *et al.*, PRL 87, 212502 (2001)
- ALICE @ LHC: constraints on $\Lambda\Lambda$ interactions from femtoscopic measurements
  ALICE 1905.07209 PLB

d* resonance





d* dibaryon: Clement 1610.05591, WASA@COSY/SAID 1402.6844 PRL

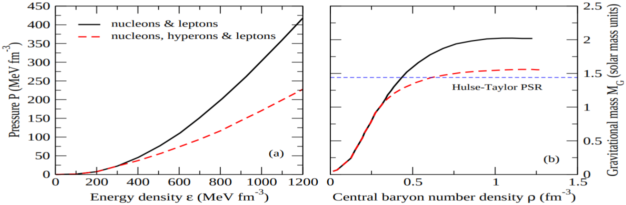
- Prediction for an isoscalar $\Delta\Delta$ configuration with $J^P = 3^+$.
  Assumed SU(6) symmetry. Dyson and Xuong PRL 13 815 (1964)
- Resonance feature at 2.38 GeV with $\Gamma \sim 70$ MeV and $I(J^P) = 0(3^+)$.
  Pole in the coupled ${}^3D_3 - {}^3G_3$ partial waves. Adlarson *et al.*, 1402.6844 PRL
- Whether isosymmetric partner of d^* with maximal isospin exists?
  Other possible nonstrange dibaryon candidates, if any.

Baryon-baryon interactions: Other prospects

- Hyperon formation $\Leftarrow$ Large nuclear densities in astrophysical objects

Bazavov *et al*, 1404.6511 PRL, 1404.4043 PLB

Chatterjee and Vidaña 1510.06306 EPJA, Vidaña *et al* 1706.09701 PLB



- A handful of experimental efforts using large nuclei reactions.
  Inputs on LECs to EFTs $\Rightarrow$ nuclear many body calculations.

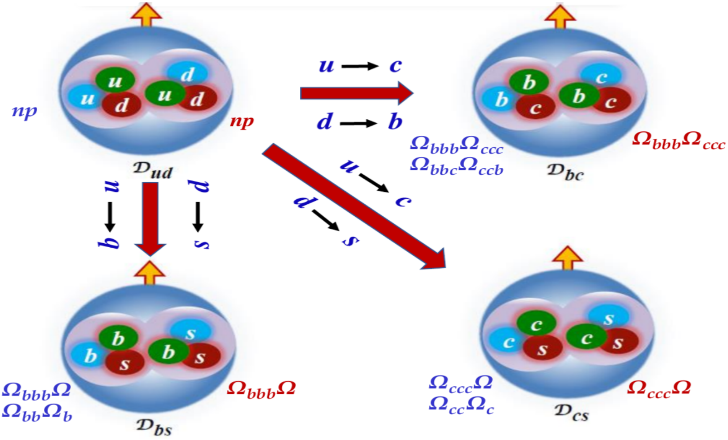
Epelbaum 2005, INT-NFPNP 2022, $0\nu\beta\beta$ PSWR 2022

- Heavy dibaryons: Relatively free of the light quark chiral dynamics.

- Heavy dibaryons: no near three or four particle thresholds.
  Simple model studies ($\Omega\Omega$ scattering): widely different inferences.

Richard *et al* 2005.06894 PRL, Liu *et al* 2107.04957 CPL, Huang *et al* 2011.00513 EPJC

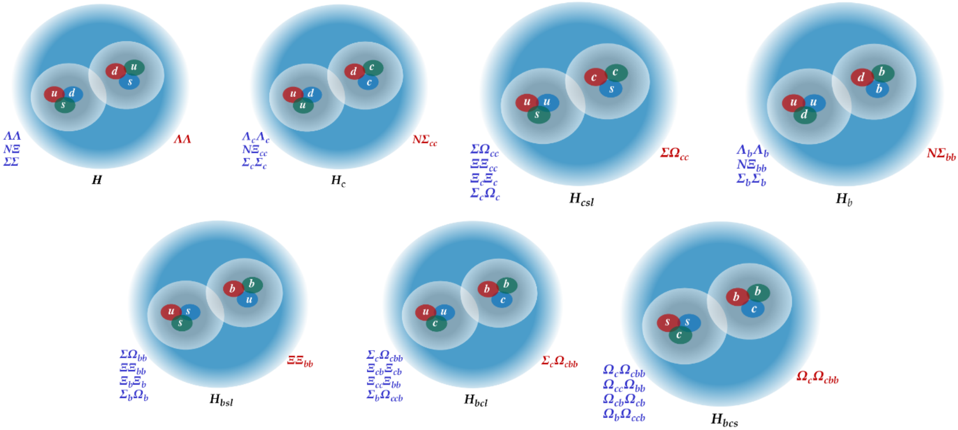
Deuteron-like Heavy dibaryons



Elastic thresholds in red text

Junnarkar and Mathur 1906.06054 PRL

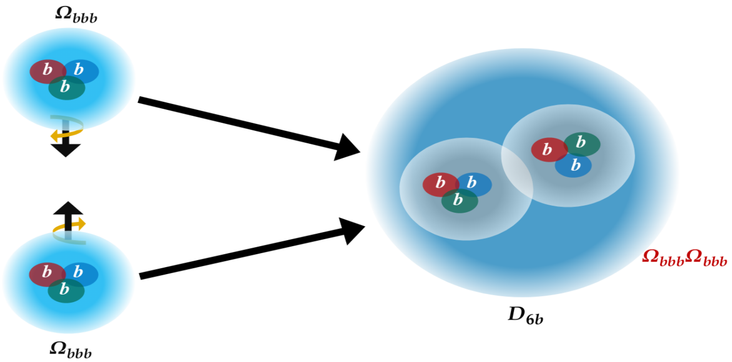
Triply flavored heavy dibaryons



Elastic thresholds in red text

Junnarkar and Mathur 2206.02942 PRD

Single flavored heavy dibaryons ($\mathcal{D}_{6q}$)



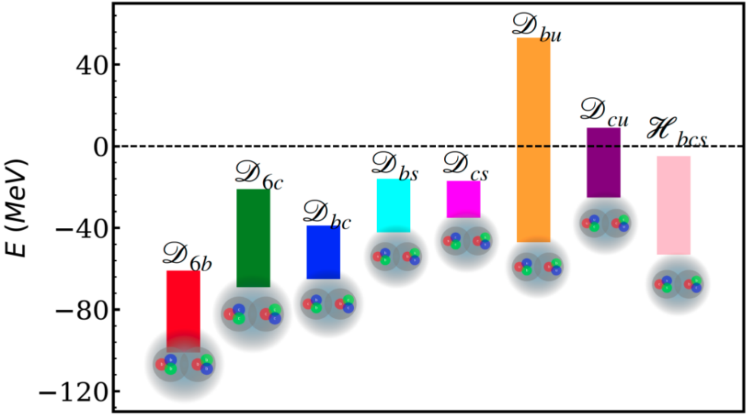
Heavy spin 0 single flavored partner of $d^*(2380)$?? Dyson and Xuong PRL 13 815 (1964)

Leading m_l dependence could arise from pair produced 2π exchanges.

Calculations at m_Q: Relatively cheap calculations with clean signals.

Mathur, MP and Chakraborty 2205.02862 PRL

Heavy dibaryons results summary

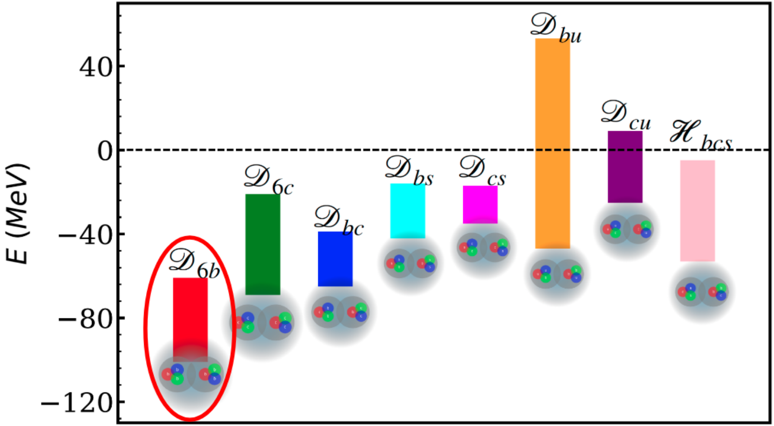


$\Delta E = E - M_{H_1} - M_{H_2}$

Junnarkar and Mathur 1906.06054 PRL ($\mathcal{D}_{bc}$, $\mathcal{D}_{bs}$, $\mathcal{D}_{cs}$, $\mathcal{D}_{bu}$, $\mathcal{D}_{cu}$),
Mathur, MP, Chakraborty 2205.02862 PRL ($\mathcal{D}_{6b}$),
Junnarkar and Mathur 2206.02942 PRD ($\mathcal{H}_{bcs}$),

Dhindsa, Mathur, MP 2206.02942 PRD ($\mathcal{D}_{6c}$, $\mathcal{D}_{6s}$)

Baryon-baryon interactions in heavy sector

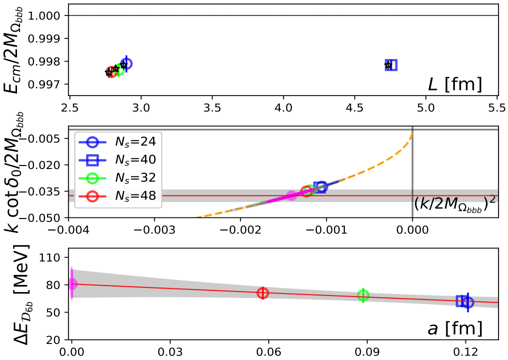


Mathur, MP, Chakraborty 2205.02862 PRL

Not limited to just a finite volume spectrum extraction.

Involved scattering analysis with a zero-range approximation.

Amplitude analysis and binding energy estimate



- Fits with "$-1/a_0^{[0]} - a/a_0^{[1]}$" is found to be the best with $\chi^2/d.o.f. = 0.7/2$

$$a_0^{[0]} = 0.18(^{+0.02}_{-0.02})\ \text{fm},$$

$$a_0^{[1]} = -0.18(^{+0.18}_{-0.11})\ \text{fm}^2$$

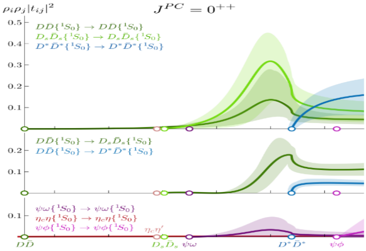
- Constraint $k.cot\delta(k) = -\sqrt{-k^2}$ gives us a bound state pole with

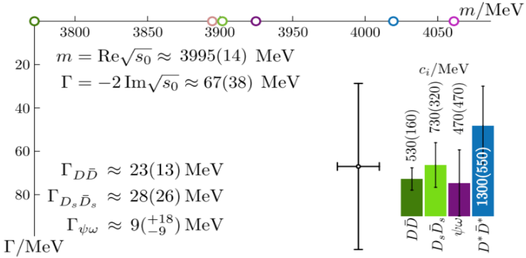
$$\Delta E^{cont}_{\mathcal{D}_{6b}} = -81(^{+14}_{-16})(14)\ \text{MeV}.$$

Using $M^{lphys}_{\Omega_{bbb}} = 14366(7)(9)$ MeV, we compute the mass of this bound state as

$$M^{phys}_{\mathcal{D}_{6b}} = 2M^{lphys}_{\Omega_{bbb}} + \Delta E^{cont}_{\mathcal{D}_{6b}} = 28651(^{+16}_{-17})(15)\ \text{MeV}$$

Recent lattice investigation by HSC





HSC 2309.14070, 2309.14071.

- Two-hadron channels considered: $\eta_c\eta$, $\eta_c\eta'$, $\bar{D}D$, $\bar{D}_sD_s$, $\psi\omega$, $\psi\phi$, $\bar{D}^*D^*$, $\chi_{c1}\eta$.

- Anisotropic lattice QCD ensembles : Hadron Spectrum Collaboration
  $m_\pi \sim$ 391 MeV, $m_K \sim$ 540 MeV, $m_D \sim$ 1852 MeV, $a_s \sim 0.12$ fm

- In addition to conventional charmonium states, only a single scalar resonance below 4 GeV
  $\Rightarrow$ with large coupling to all open charm channels.
  relation to X(3960) / X(3930) / X(3915) / $\chi_{c0}(3860)$ features ?

- Results in conflict with several other theoretical and experimental studies.
  Resolution: quark mass dependence ?