Recent progress on the phase structure and EoS of dense two-color QCD

Etsuko Itou (YITP, Kyoto U./ RIKEN iTHEMS)

Recent Review:
EI “Lattice results for the equation of state in dense QCD-like theories”
arXiv: 2508.03090

International Workshop on Advances in Lattice QCD, 中国科学院近代物理研究所惠州研究部, 惠州, 中国, 2025/10/13

Two problems in finite-density QCD simulations

(1) **sign problem** $Z = \int \mathcal{D}U \boxed{[\det D]} e^{-S_g[U]}$

Fermion action must be real-positive

$\mu = 0 \quad D^\dagger = \gamma_5 D \gamma_5$ $\quad \det D$ real

$\mu \neq 0 \quad \Delta(-\mu)^\dagger = \gamma_5 \Delta(\mu) \gamma_5$ $\quad \det \Delta(\mu)$ complex

In two-color QCD $\det \Delta(\mu)$ is real,
since the fundamental reps. of SU(2) takes a pseudo-real reps.

(2) **numerical instability (onset problem)**

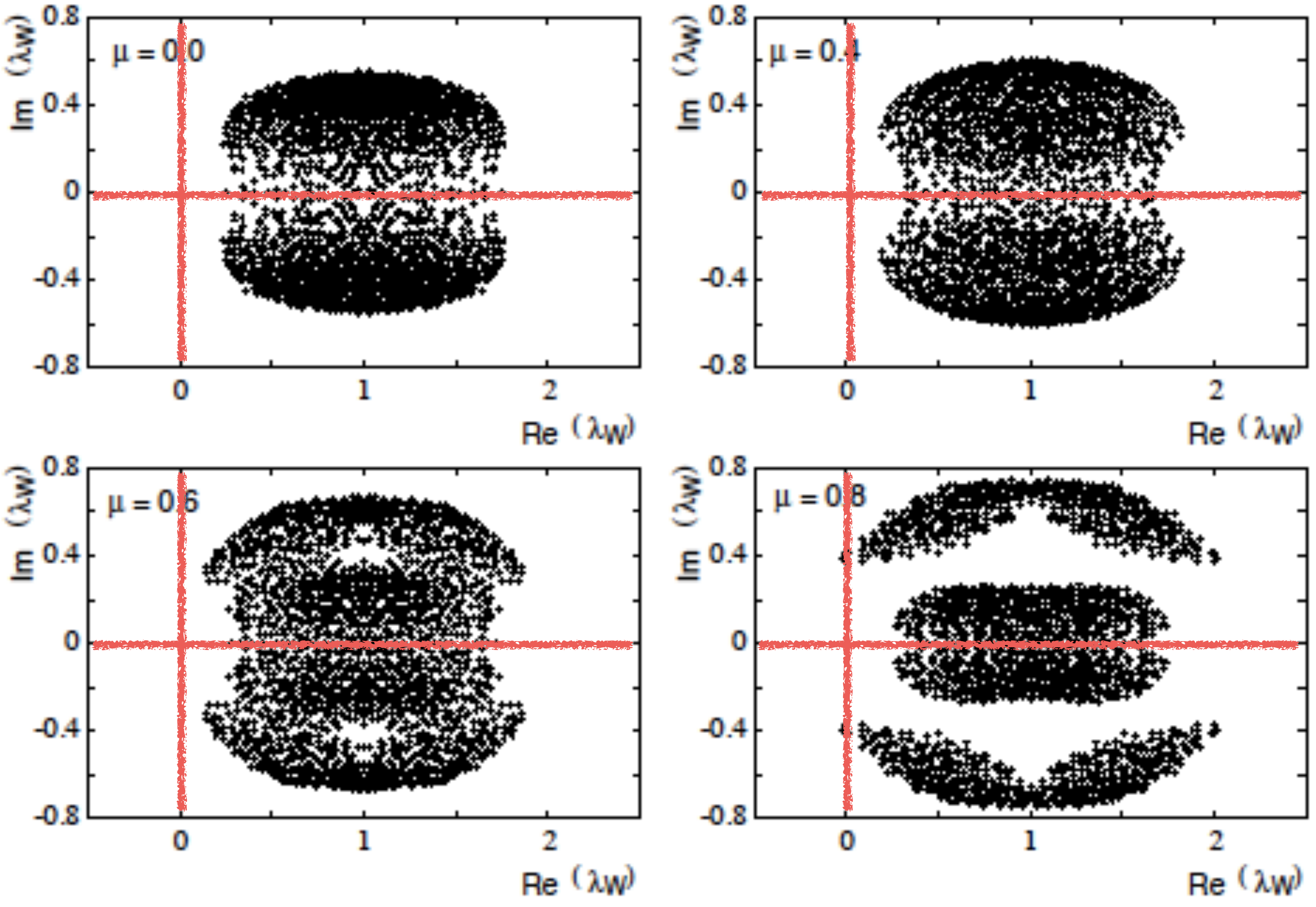
in the low-T and high-density regime:

$\mu / m_{PS} \geq 1/2$ in low-T $\quad m_{PS}$: pseudo-scalar (pion) mass at $\mu = 0$

Dynamical pair-creation and annihilation frequently occur,
then system becomes unstable

Eigenvalue of (Wilson) Dirac op.

Muroya et al. Nucl.Phys.Proc.Suppl. **94** (2001) 469



Beyond $a\mu = 0.8$ the simulation is broken-down

In fact, $am_{PS} \approx 1.5$ in their calculation.

So, $a\mu = 0.8$ is $\mu = m_{PS}/2$

Action with diquark source term

Fermion action in continuum limit

$$S_F^{cont.} = \underbrace{\int d^4x \bar{\psi}(x)(\gamma_\mu D_\mu + m)\psi(x)}_{\text{QCD}} + \underbrace{\mu \hat{N}}_{\text{Number op.}} - \underbrace{\frac{j}{2}(\bar{\psi}_1 K \bar{\psi}_2^T - \psi_2^T K \psi_1)}_{\text{diquark source}}$$

Related works on Nc=2 with even # flavor

Kogut et al. NPB642 (2002)18, **Alles et al.** NPB752 (2006)124,
Hands et al. NPB752 (2006) 124, PRD81 (2010) 091502,, EPJ. A47 (2011) 60, PRD87 (2013) 034507, **Kotov et al.** PRD94 (2016) 114510, JHEP 1803 (2018) 161

The QCD phase diagram appears in the j->0 limit

Theoretically, the term breaks U(1) baryon sym.

Fermion action on the lattice

$$\det[\mathcal{M}^\dagger \mathcal{M}]^{1/2} = \det[\Delta^\dagger(\mu)\Delta(\mu) + |\bar{J}|^2]^{1/2} \det[\Delta^\dagger(-\mu)\Delta(-\mu) + |J|^2]^{1/2}$$

j-source lifts the eigenvalue of Dirac op. up

Our strategy

(qualitatively) understand the QCD phase diagram

and EoS at low-T and high density

(1) sign problem

Avoid the sign problem (consider 2color 2flavor QCD)

(2) Numerical instability $\mu/m_{PS} \geq 1/2$ in low-T

Introduce the diquark source in the action

cf.) diquark -> π^- in 3-color QCD with isospin chemical

D. H. Rischke, D. T. Son and M. A. Stephanov, Phys. Rev. Lett.87(2001) 062001
D. T. Son and M. A. Stephanov, Phys. Atom. Nucl.64(2001) 83
B. B. Brandt, G. Endrodi and S. Schmalzbauer, Phys. Rev.D 97(2018) 05451

Our 2color QCD projects

- K.Iida, EI, T.-G. Lee: JHEP2001(2020)181
  Phase diagram by Lattice simulation (T=80MeV)
- T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253
  Phase diagram by 't Hooft anomaly matching
- K.Iida, EI, T.-G. Lee: PTEP2021(2021) 1, 013B0
  Scale setting of Lattice simulation
- K.Ishiguro, K.Iida, EI, PoS, Lattice 2021
  Flux tube and quark confinement by Lattice simulation
- K.Iida, EI, PTEP 2022 (2022) 11, 111B01
  Velocity of sound by Lattice simulation (T=80MeV)
- D. Suenaga, K.Murakami, EI, K.Iida, PRD 107, 054001 (2023) and
  Mass spectrum using effective model
- K.Murakami, D.Suenaga, K.Iida, EI, PoS, Lattice 2022
  Mass spectrum by Lattice simulation
- K.Murakami, K.Iida, EI, JHEP 02 (2024) 152
  Hadron potential w/ finite μ by Lattice simulation
- K.Iida, EI, K.Murakami, D. Suenaga, JHEP 10 (2024) 022
  Phase diagram and EoS by Lattice simulation (T=40MeV)

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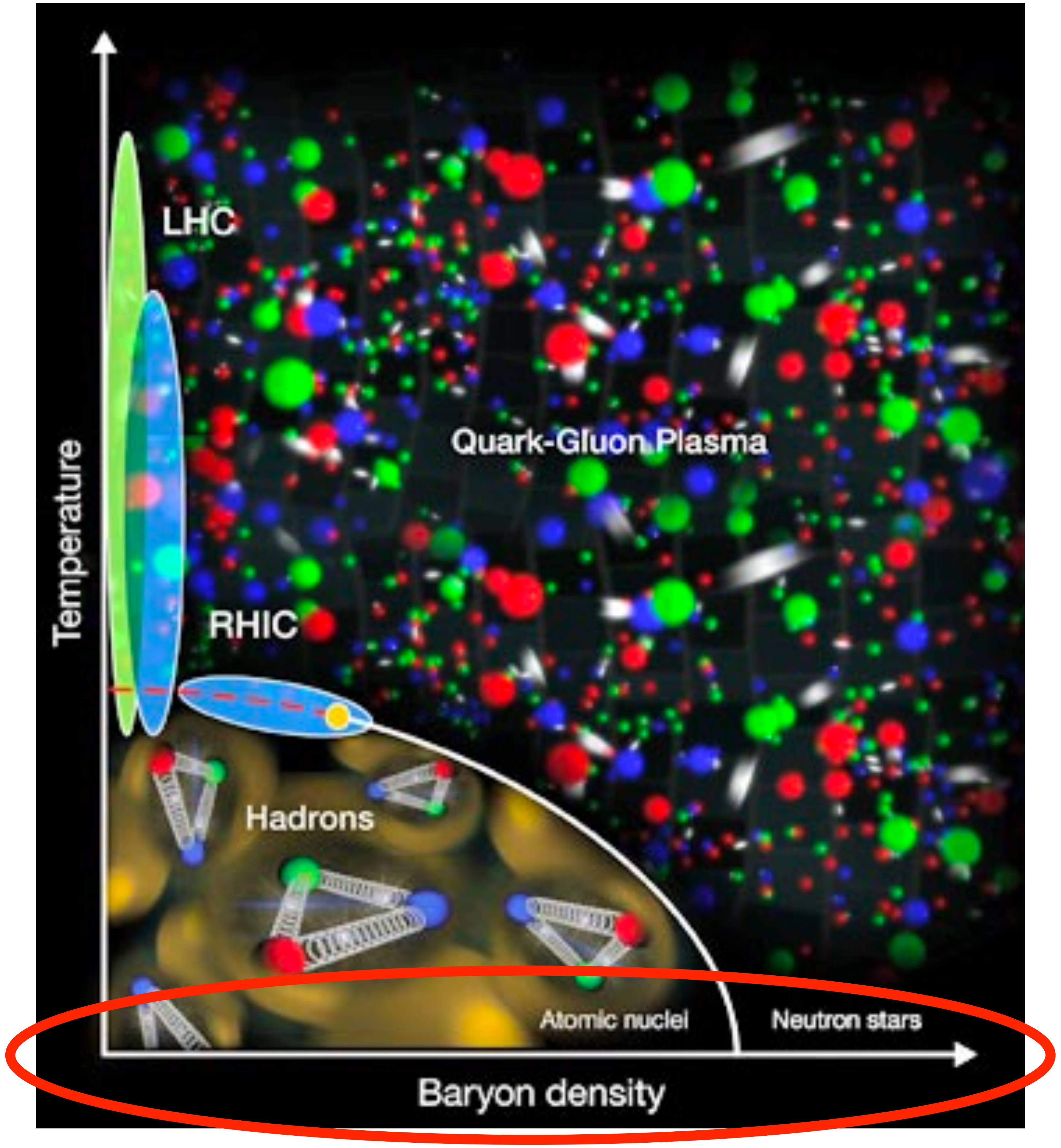
  Hadron potential w/ finite μ by Lattice simulation
- K.Iida, EI, K.Murakami, D. Suenaga, JHEP 10 (2024) 022
  Phase diagram and EoS by Lattice simulation (T=40MeV, 32^4 lattices)

Sound velocity in finite density regime?

EoS and sound velocity at low-T and high-μ



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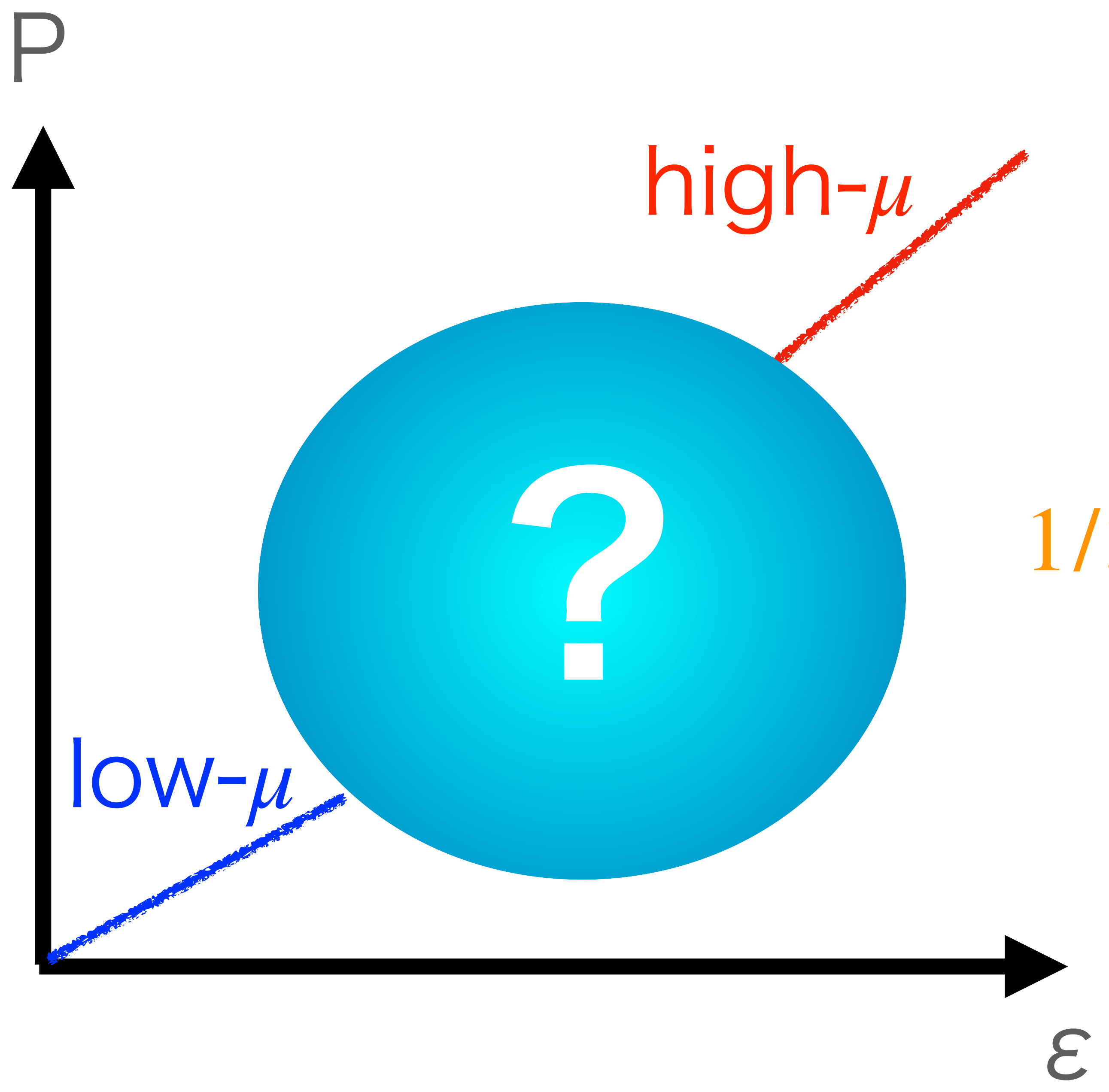
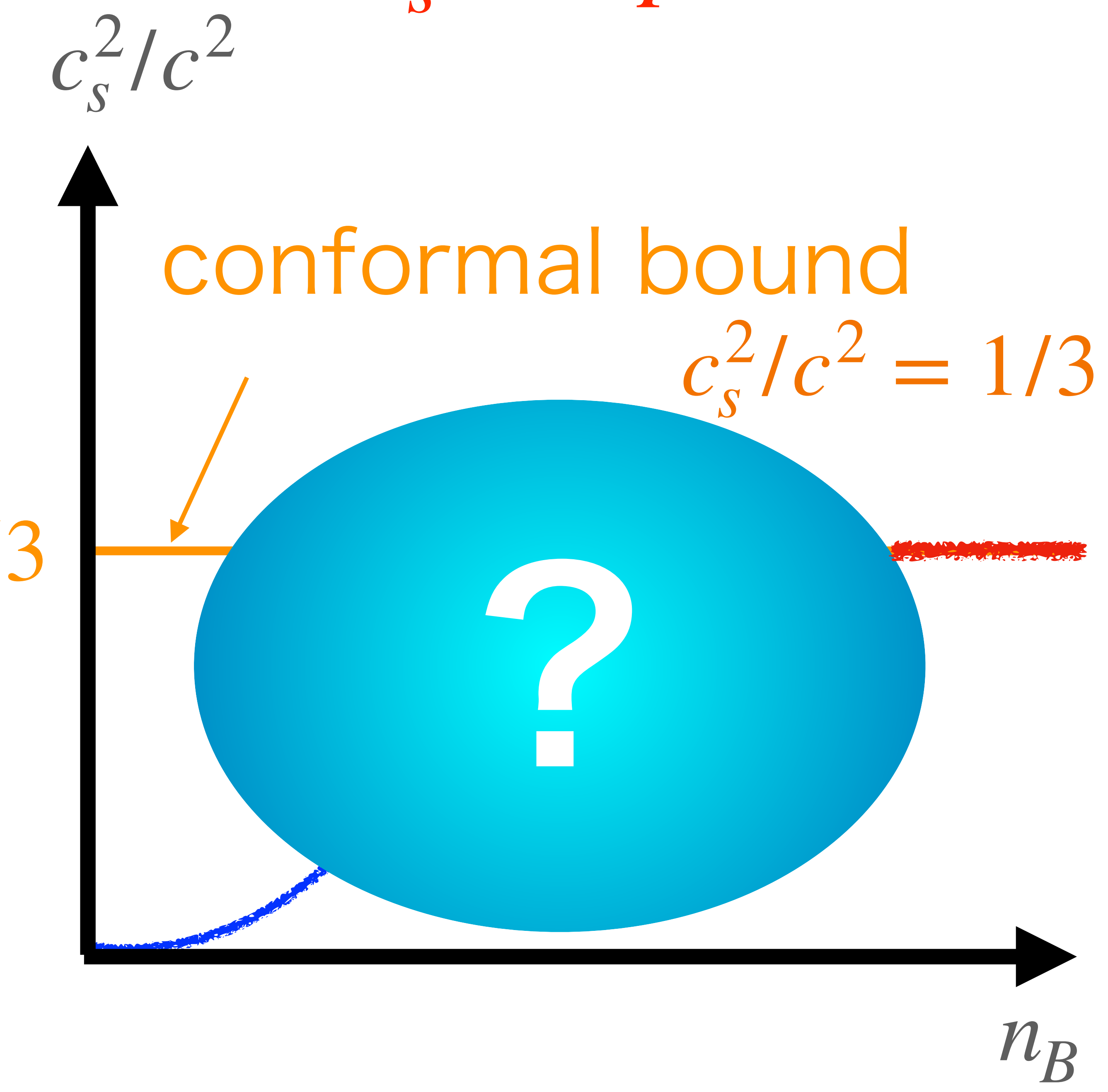
EoS

$p(\mu)$ vs $\varepsilon(\mu)$



Sound velocity

$c_s^2 = \partial p / \partial \epsilon$



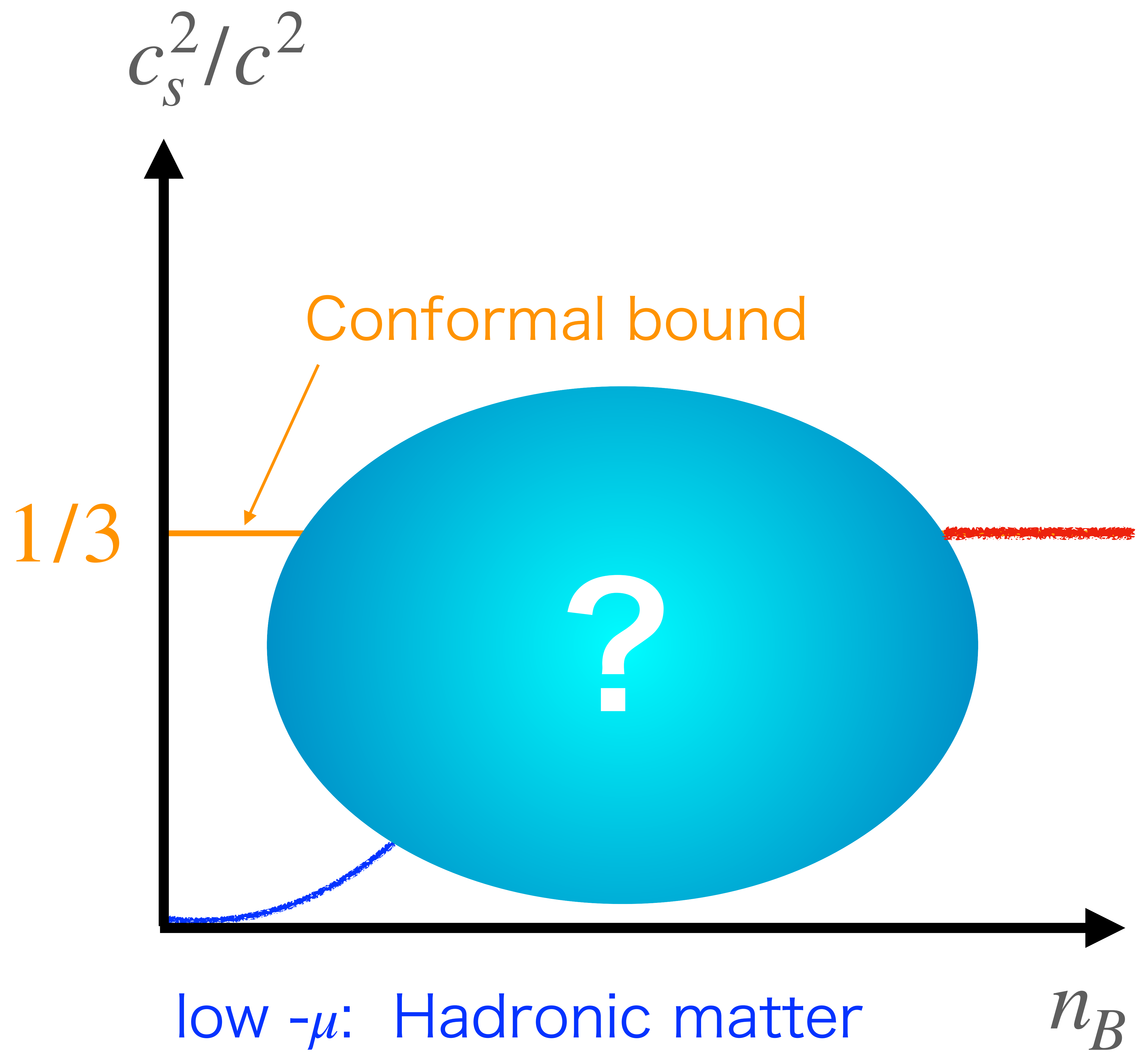
low -μ ($n_B \lesssim 2n_0$): Hadronic matter

high-μ ($5n_0 < n_B$): Quark matter

-> pQCD ($50n_0 < n_B$)

Prediction by phenomenology and effective models

Sound velocity has a peak?



low -μ: Hadronic matter

high-μ: Quark matter ~ pQCD

- Quark-hadron crossover picture consistent with observed neutron stars (M-R) suggests c_s^2 peaks at $n_B = 1 - 10n_0$

  Masuda,Hatsuda,Takatsuka (2013)
  Baym, Hatsuda, Kojo(2018)

- Quarkyonic matter model
  c_s^2 peaks at $n_B = 1 - 5n_0$

  McLerran and Reddy (2019)

- Microscopic interpretation on the origin of the peak = quark saturation
  (work for any # of color)

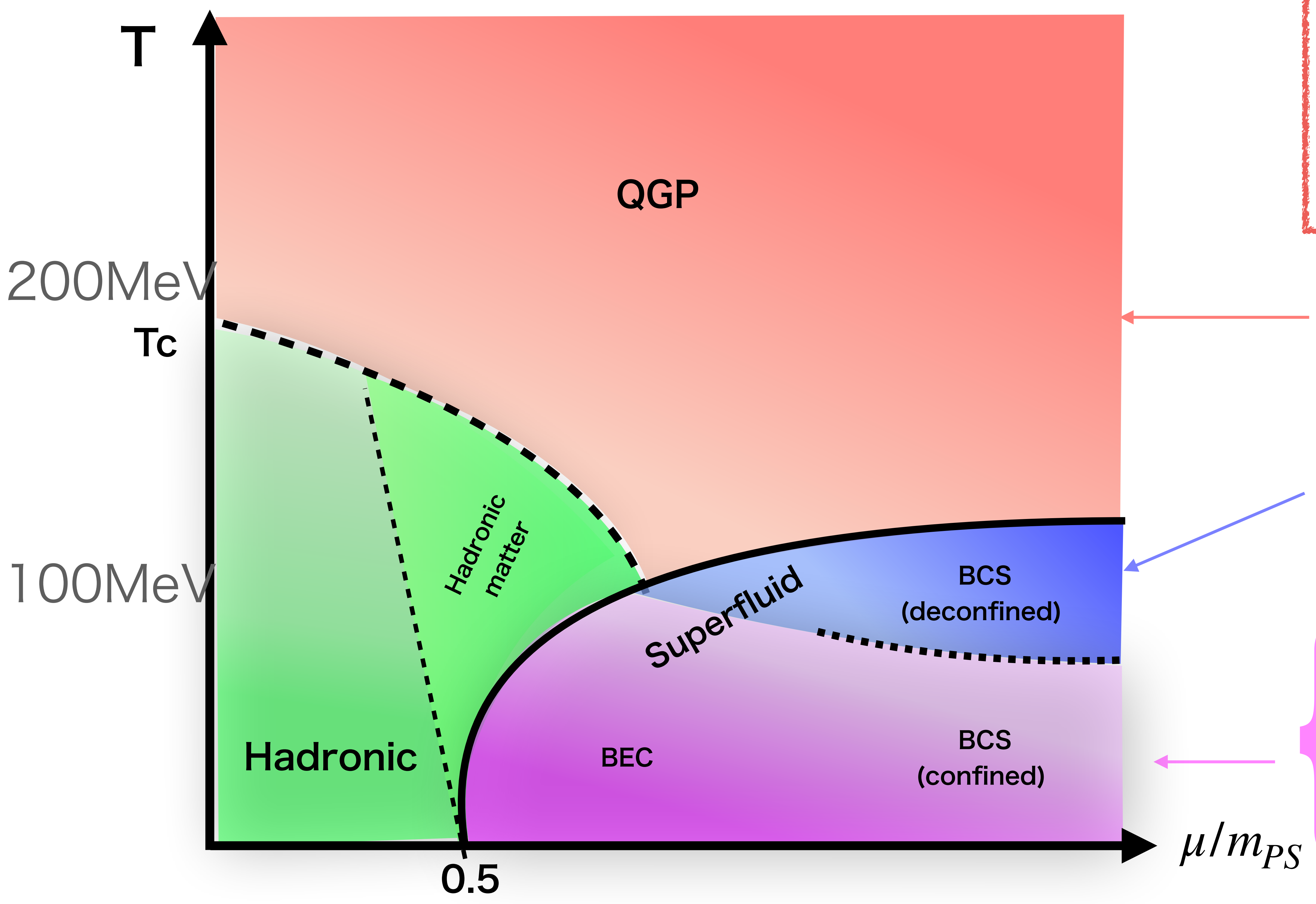
  Kojo (2021), Kojo and Suenaga (2022)

➡ Lattice study on 2color dense QCD
the sign problem is absent!!

2color QCD phase diagram

(1) K.Iida, EI, K.Murakami, D.Suenage arXiv: 2405.20566 [hep-lat]
(2) K.Iida, K.Ishiguro , EI, arXiv: 2111.13067
(3) K.Iida, EI, T.-G. Lee: PTEP2021(2021) 1, 013B0
(4) K.Iida, EI, T.-G. Lee: JHEP2001(2020)181
(5) T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253

Current status on 2color QCD phase diagram



At least 4 independent groups are studying the phase diagram

(1) S. Hands group : Wilson-Plaquette gauge + Wilson fermion
(2) Russian group : tree level improved Symanzik gauge + rooted staggered fermion
(3) Our group : Iwasaki gauge + Wilson fermion, Tc=200 MeV to fix the scale
(4) von Smekal group: Wilson/Improved gauge + rooted staggered fermion

T=158 MeV (**deconfined**, hadron -> QGP phase transition occurs)
T=130 MeV (**deconfined? QGP phase?** , 2019)

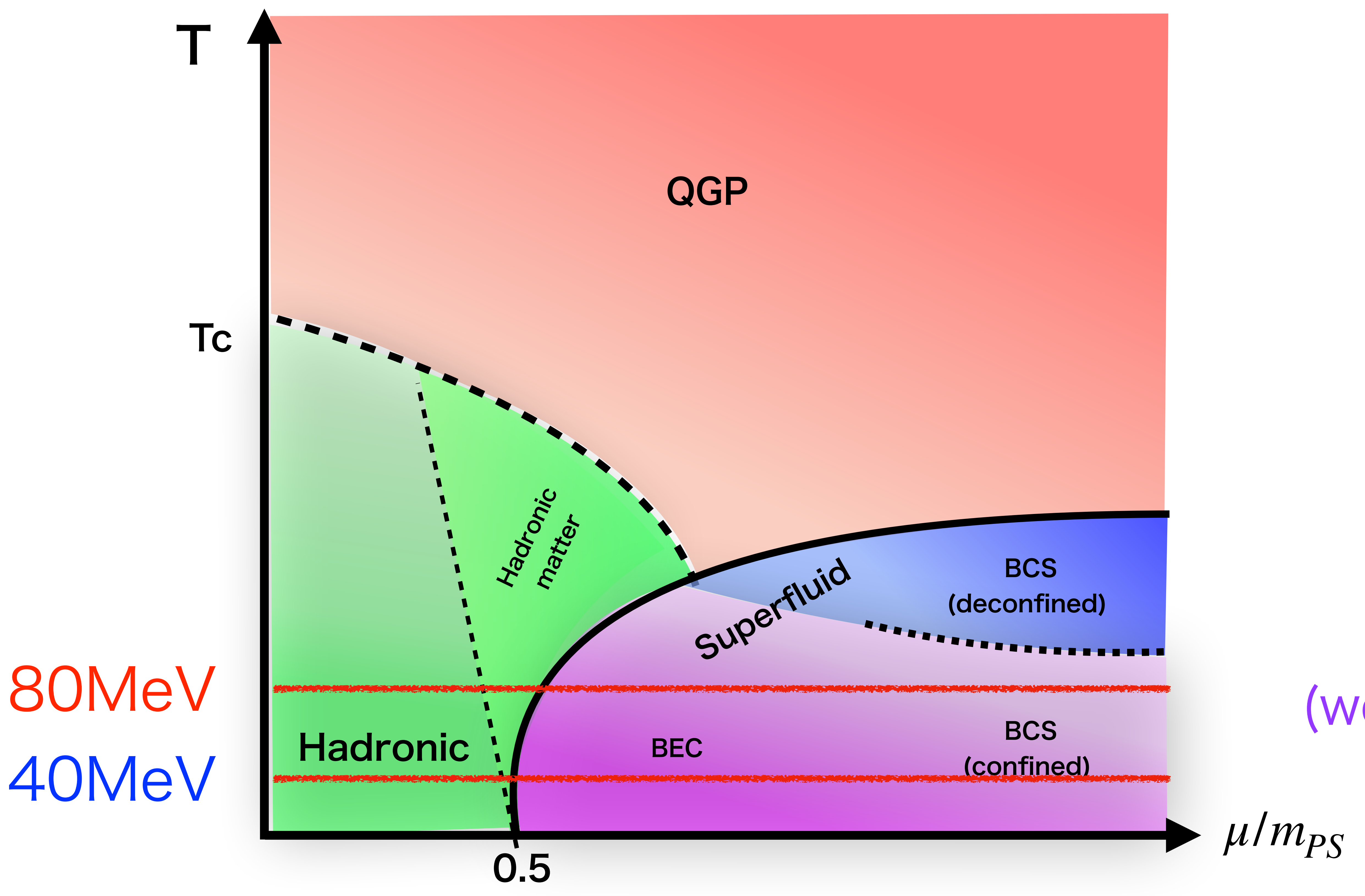
T=140 MeV (**deconfined** in high mu, <qq> is not zero, 2017, 2018, 2020)
T= 93 MeV (**deconfined** in high mu ?, also <qq> is not zero?, 2017)

T=87 MeV (**confined** in 2019)
T=79 MeV (**confined** even in high mu.2019)
T=55 MeV (**confined** in high mu, 2016)
T=47 MeV (**deconfined** coarse lattice in 2012, but **confined** in 2019)
T=45 MeV (**confined** in 2019)
T=40 MeV (**confined, 2024**)

- Even $T \approx 100\text{MeV}$ and $\mu/m_{PS} = 0.5$, superfluid phase emerges
- T_d (confine/deconfine)$\leq$ T_{SF} (superfluid/QGP) : constraint from 't Hooft anomaly matching

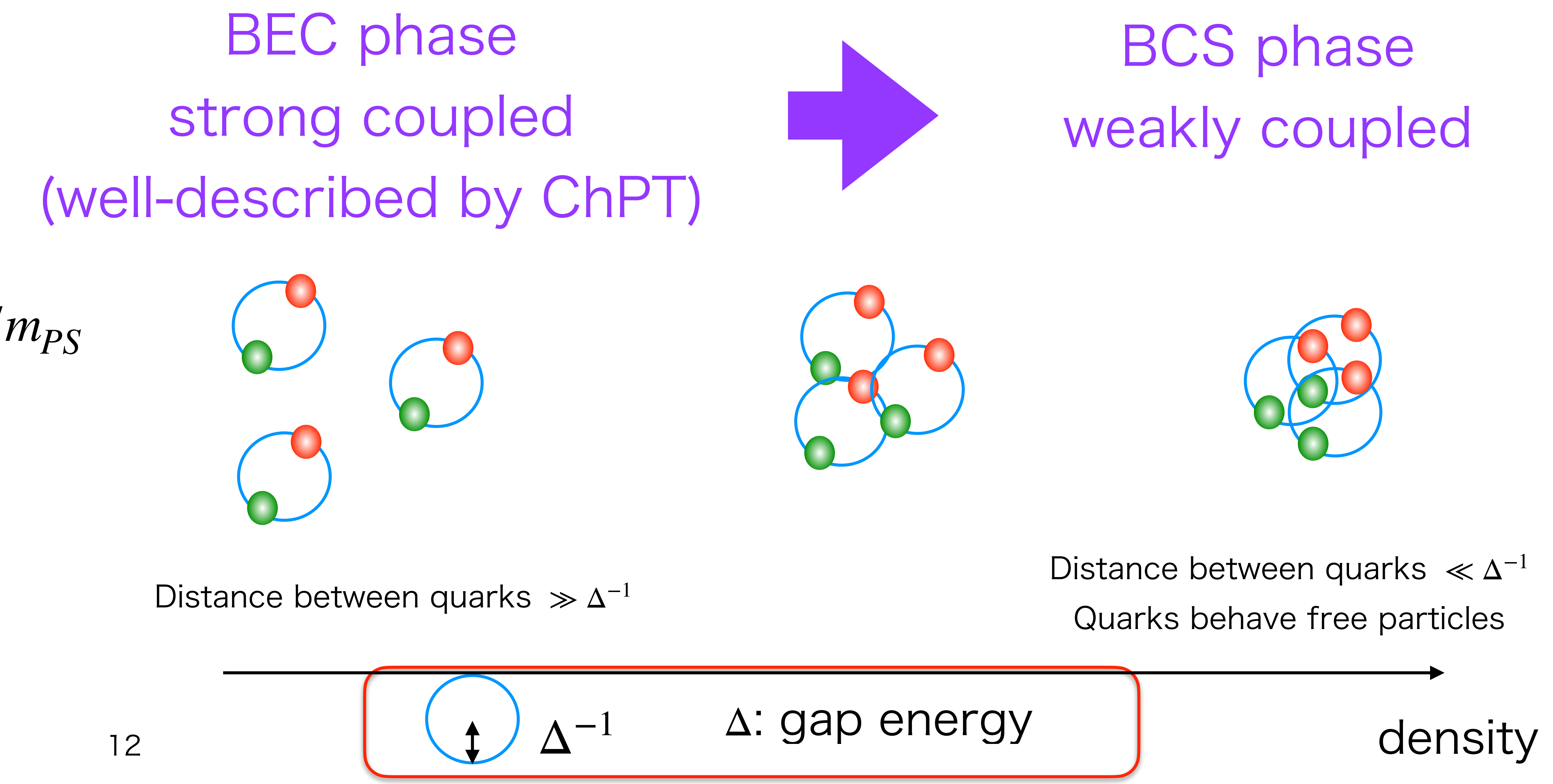
T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253

Three phases in low temperature regions



K.Iida, EI, T.-G. Lee: JHEP2001(2020)181
K.Iida, EI, K.Murakami, D.Suenage, JHEP 10 (2024) 022

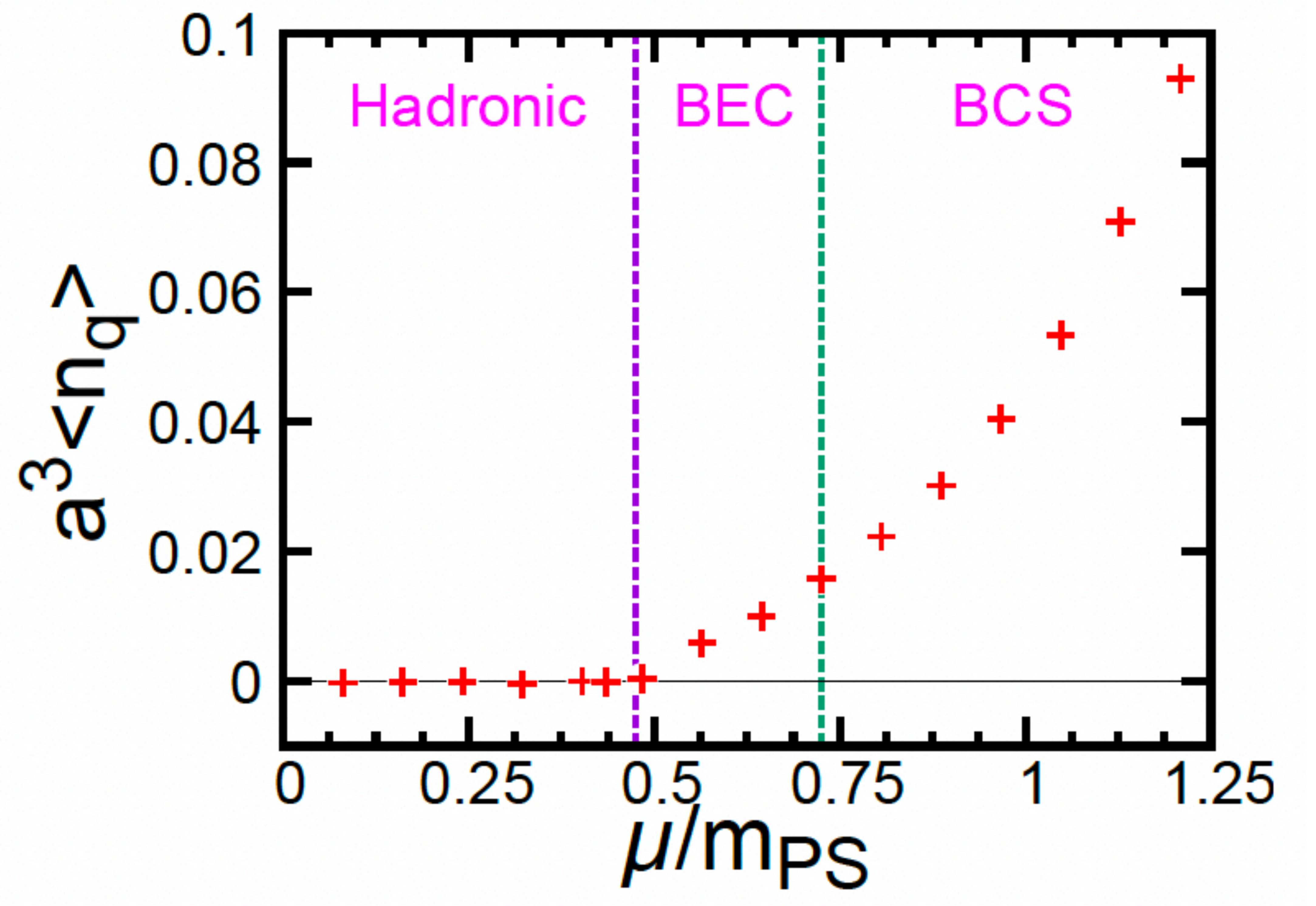
- We investigated T=80MeV and 40MeV
- Hadronic / Superfluidity phase transition around $\mu \approx m_{\mathrm{PS}}/2$
- BEC/BCS crossover in SF phase

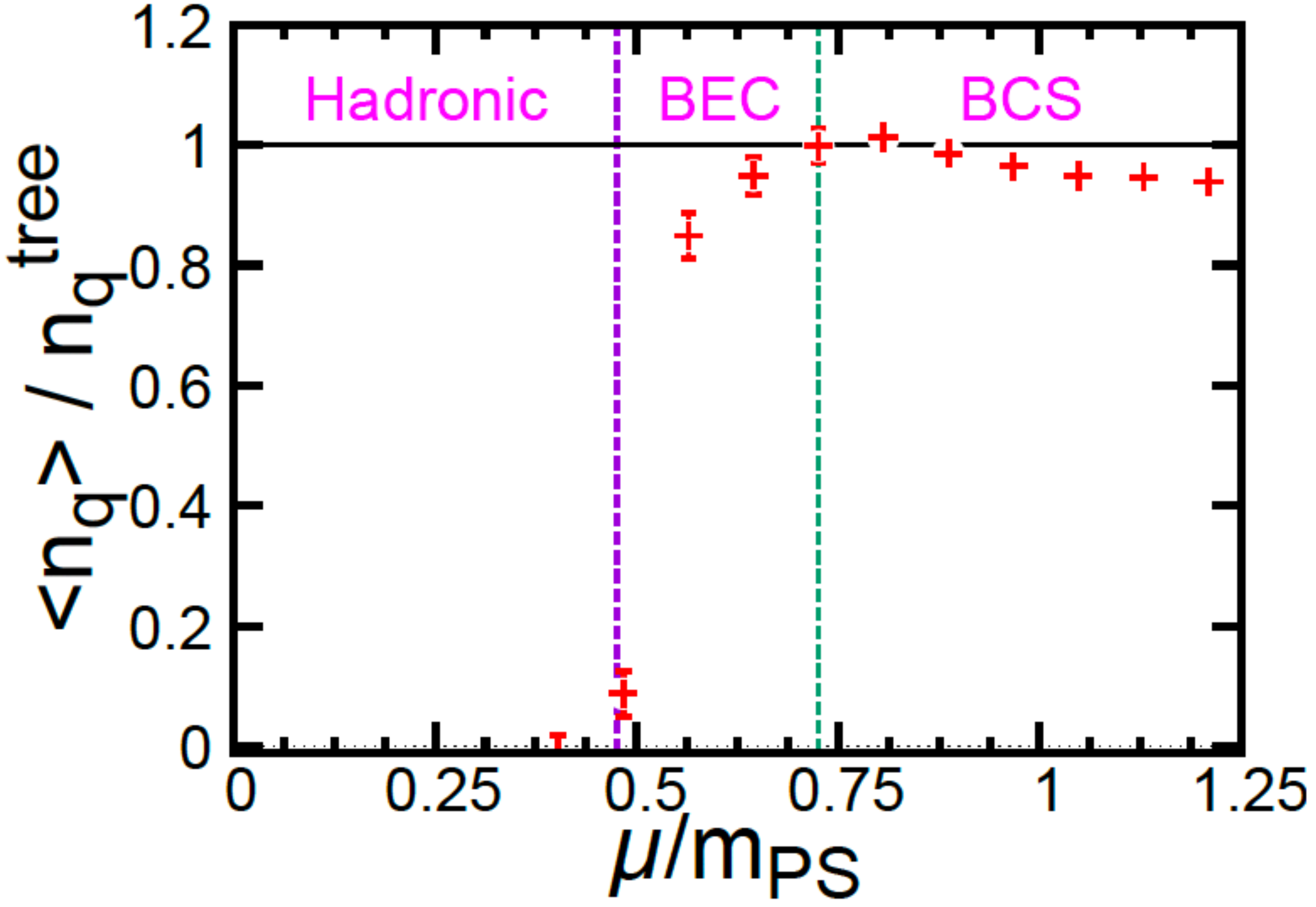


"definition" of BEC/BCS crossover

S. Hands, S. Kim and J.-I. Skullerud(2006)

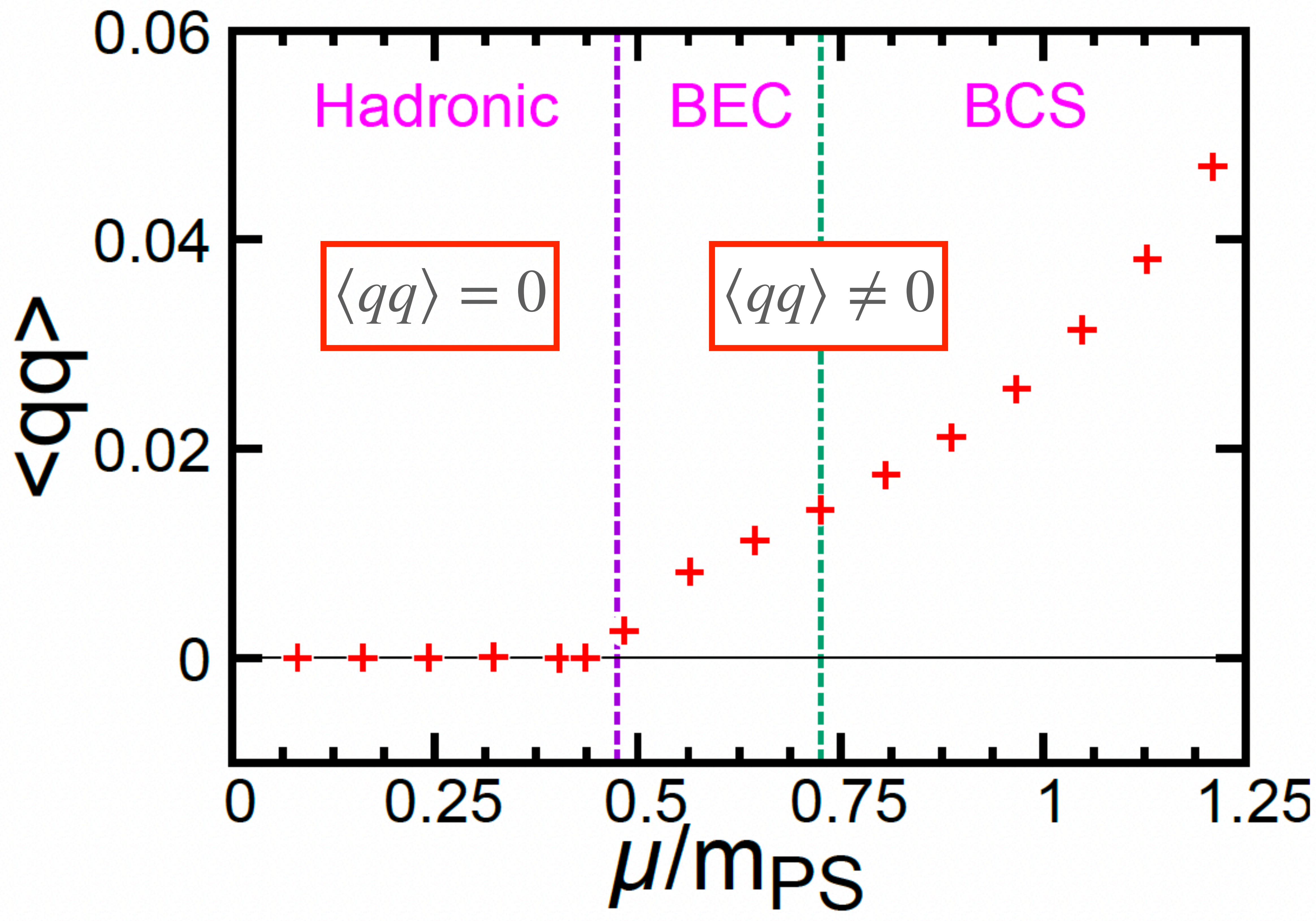
- If $\langle n_q \rangle \approx n_q^{\text{tree}}$, then the region is identified as BCS phase





Order parameter of Superfluidity : $\langle qq \rangle$

Diquark condensate



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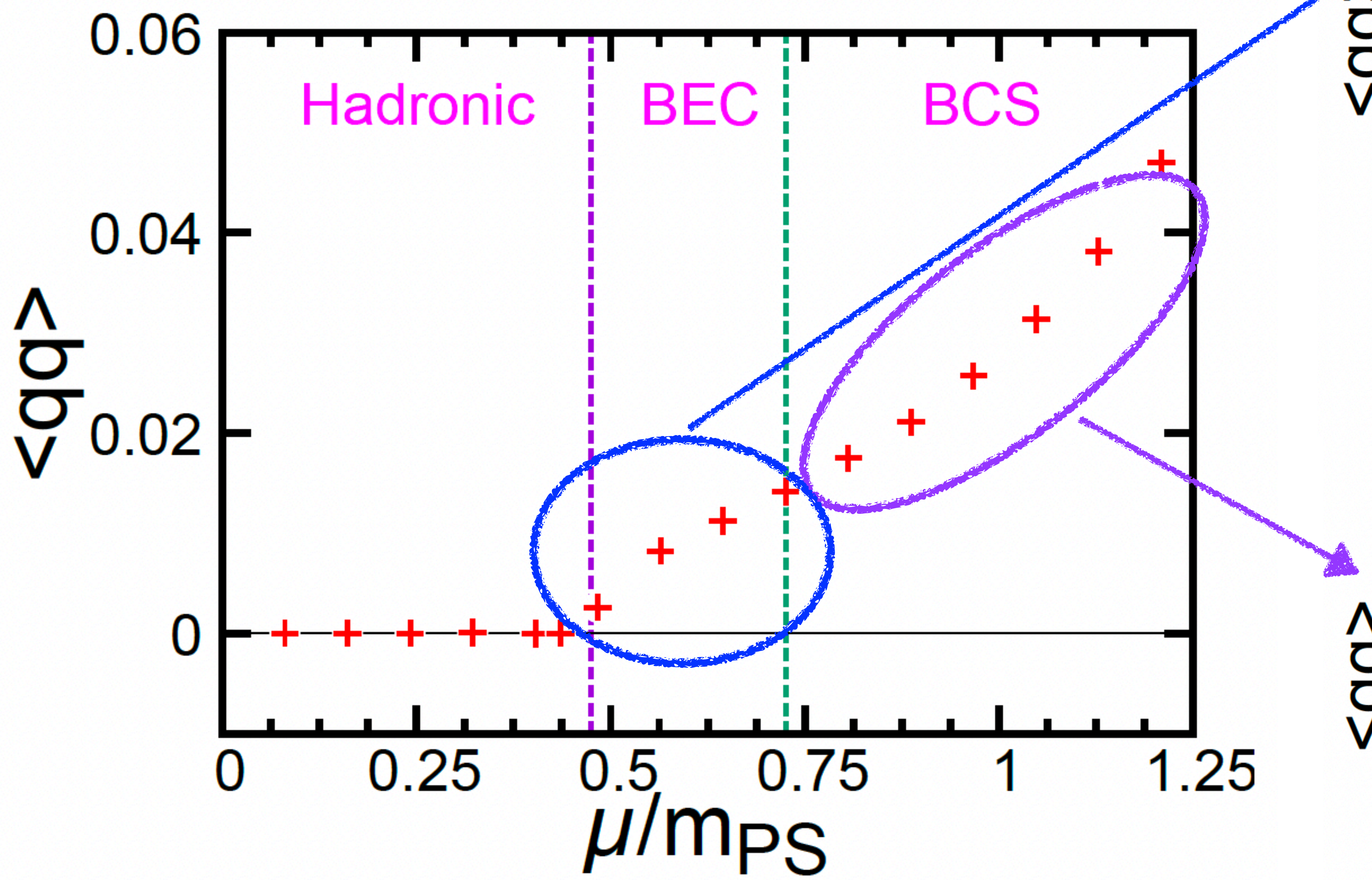
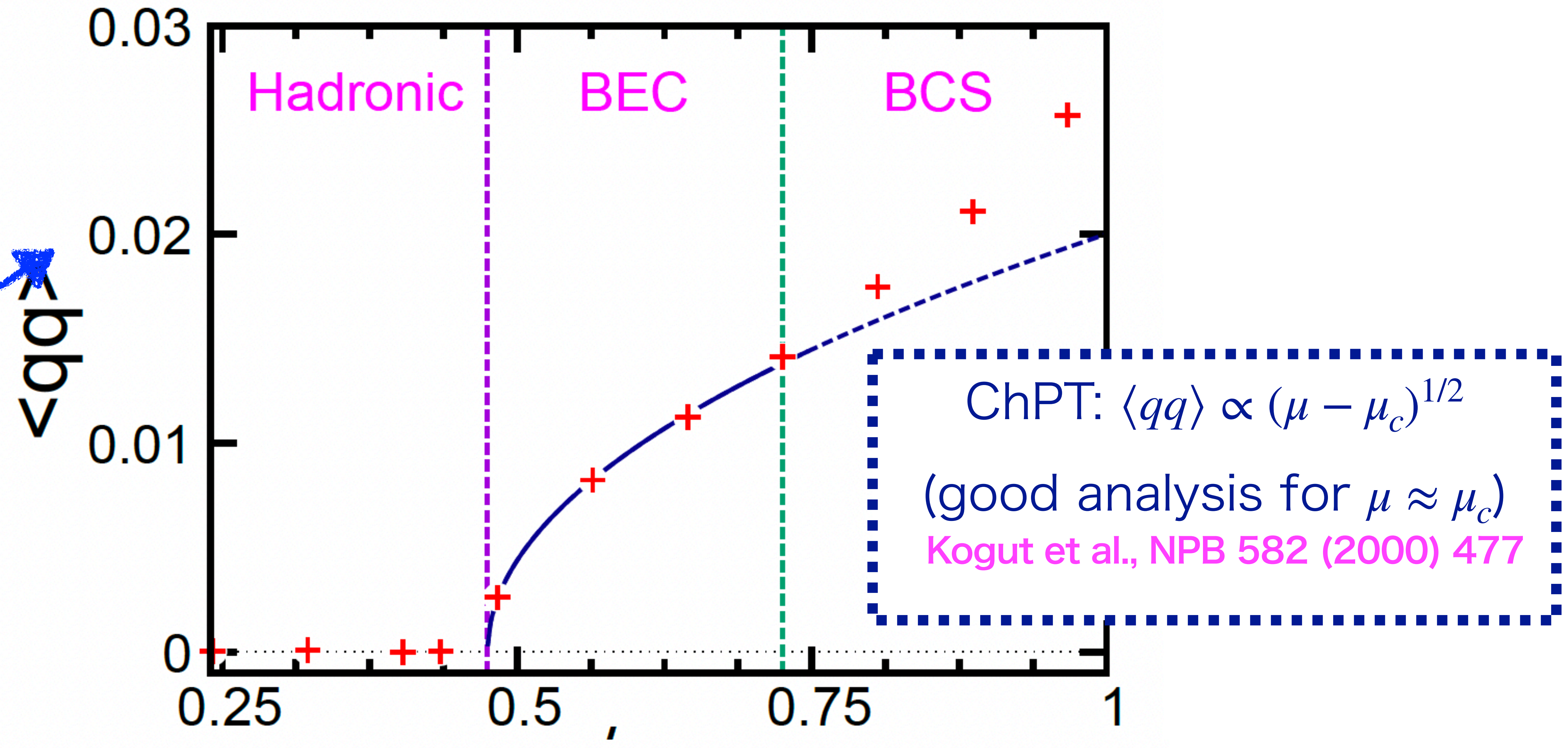
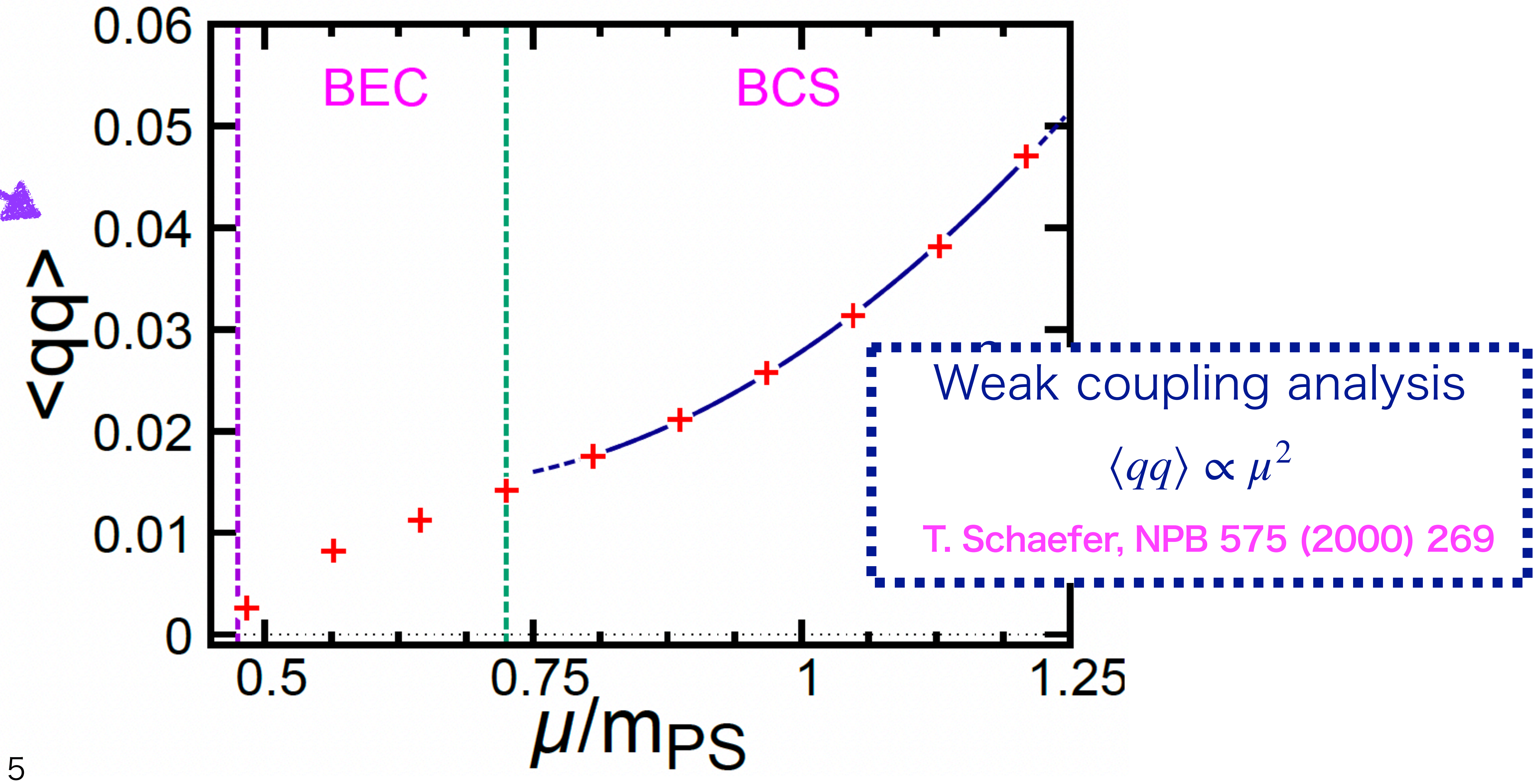


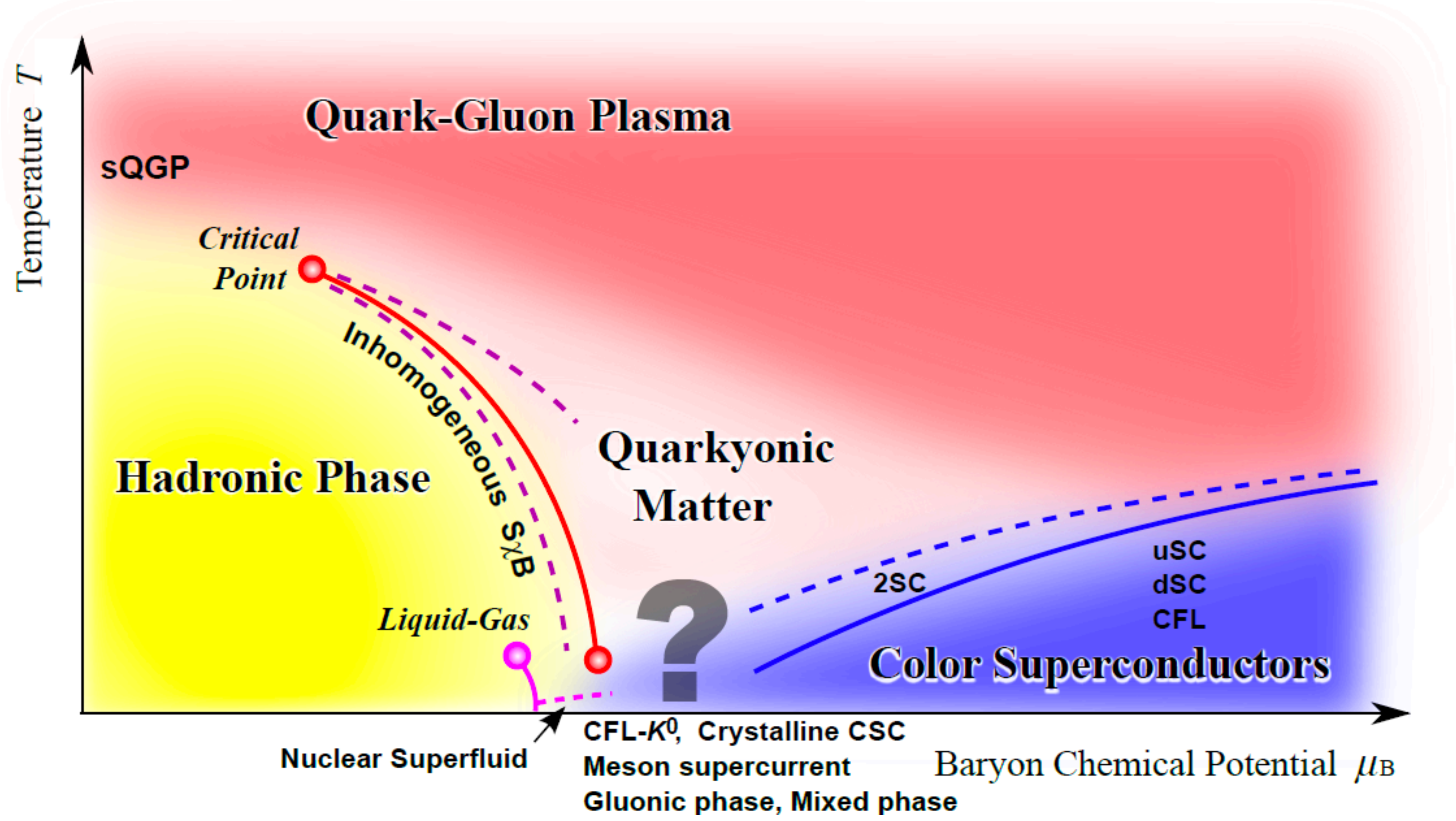




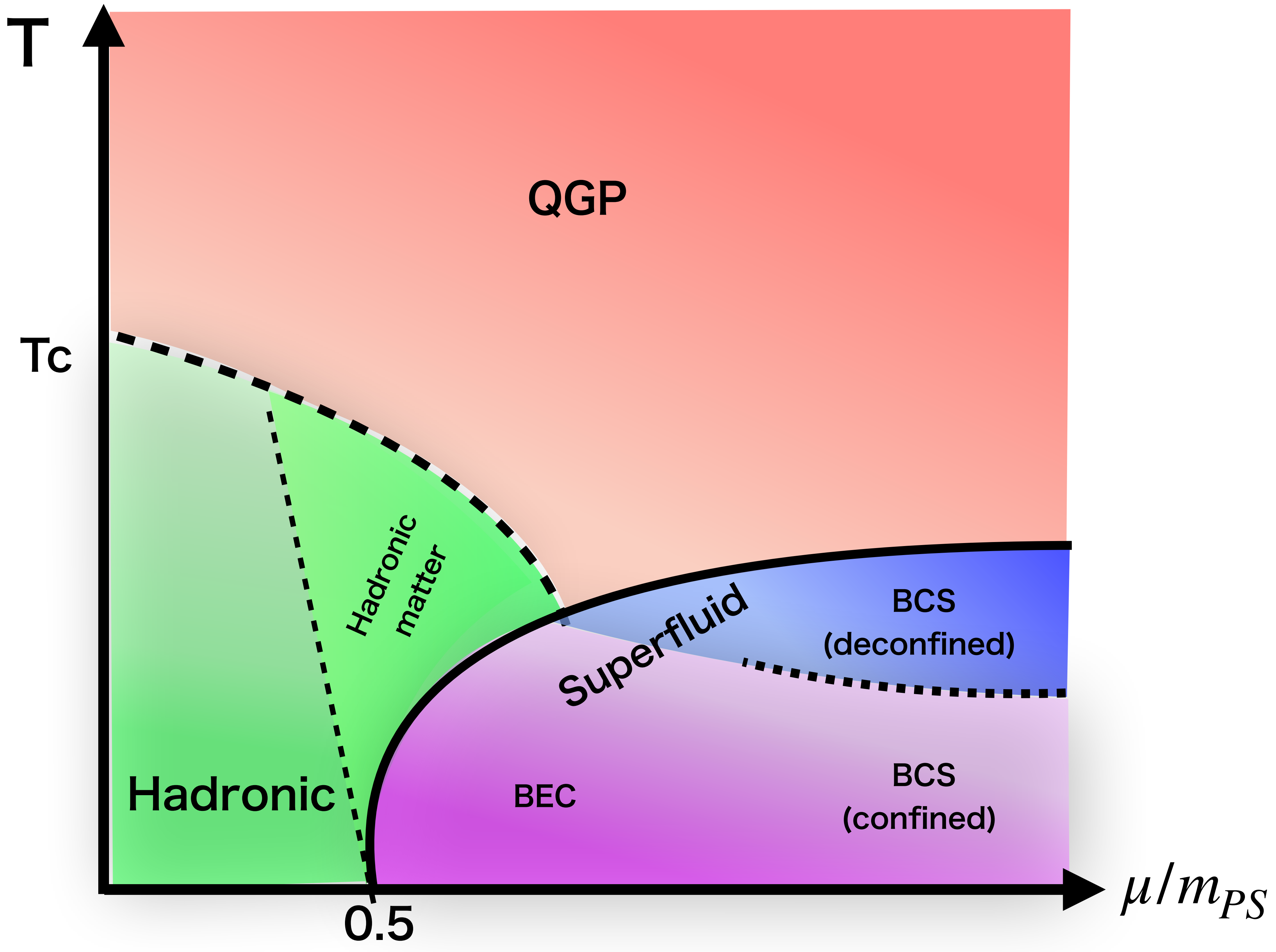
Phase diagram in T-μ plane

3 color QCD
expected phase diagram

Fukushima-Hatsuda (2010)



2 color QCD
numerically emerging phase diagram



Equation of state

K.Iida and EI, PTEP 2022 (2022) 11, 111B01
K.Iida, EI, K.Murakami, D.Suenaga, JHEP 10 (2024) 022

Equation of state

- trace anomaly: $\epsilon - 3p = \frac{1}{N_s^3}\left(a\frac{d\beta}{da}|_{LCP}\underline{\langle\frac{\partial S}{\partial \beta}\rangle_{sub.}} + a\frac{d\kappa}{da}|_{LCP}\langle\frac{\partial S}{\partial \kappa}\rangle_{sub.} + \underline{\cancel{a\frac{\partial j}{\partial a}\langle\frac{\partial S}{\partial j}\rangle}} \right)$

  No renormalization for μ $\langle\cdot\rangle_{sub.} = \langle\cdot\rangle_{\mu,T} - \langle\cdot\rangle_{\mu=0,T}$ Zero at $j \to 0$

- pressure: $p(\mu) = \int_{\mu_o}^{\mu} n_q(\mu')d\mu'$

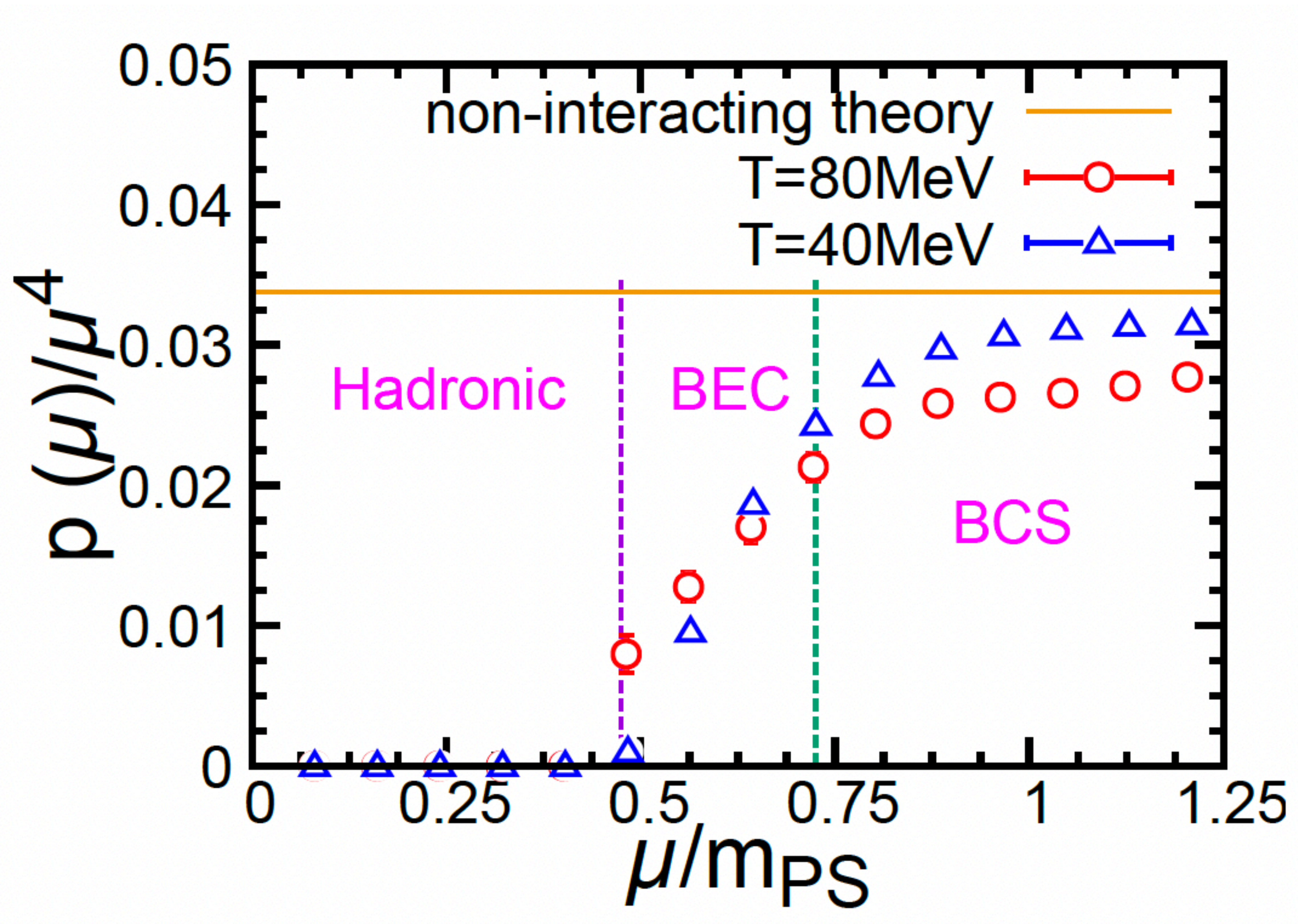
Early works for EoS in dense 2color QCD
Hands et al. (2006)
Hands et al. (2012), T~47MeV (coarse lattice)
Astrakhantsev et al. (2020), T~140MeV

Our work
Nonperturbatively calculate beta fn.

$a\frac{d\beta}{da} = -0.3521, \quad a\frac{d\kappa}{da} = 0.02817$

K.Iida, EI, T.-G. Lee: PTEP 2021 (2021) 1, 013B0

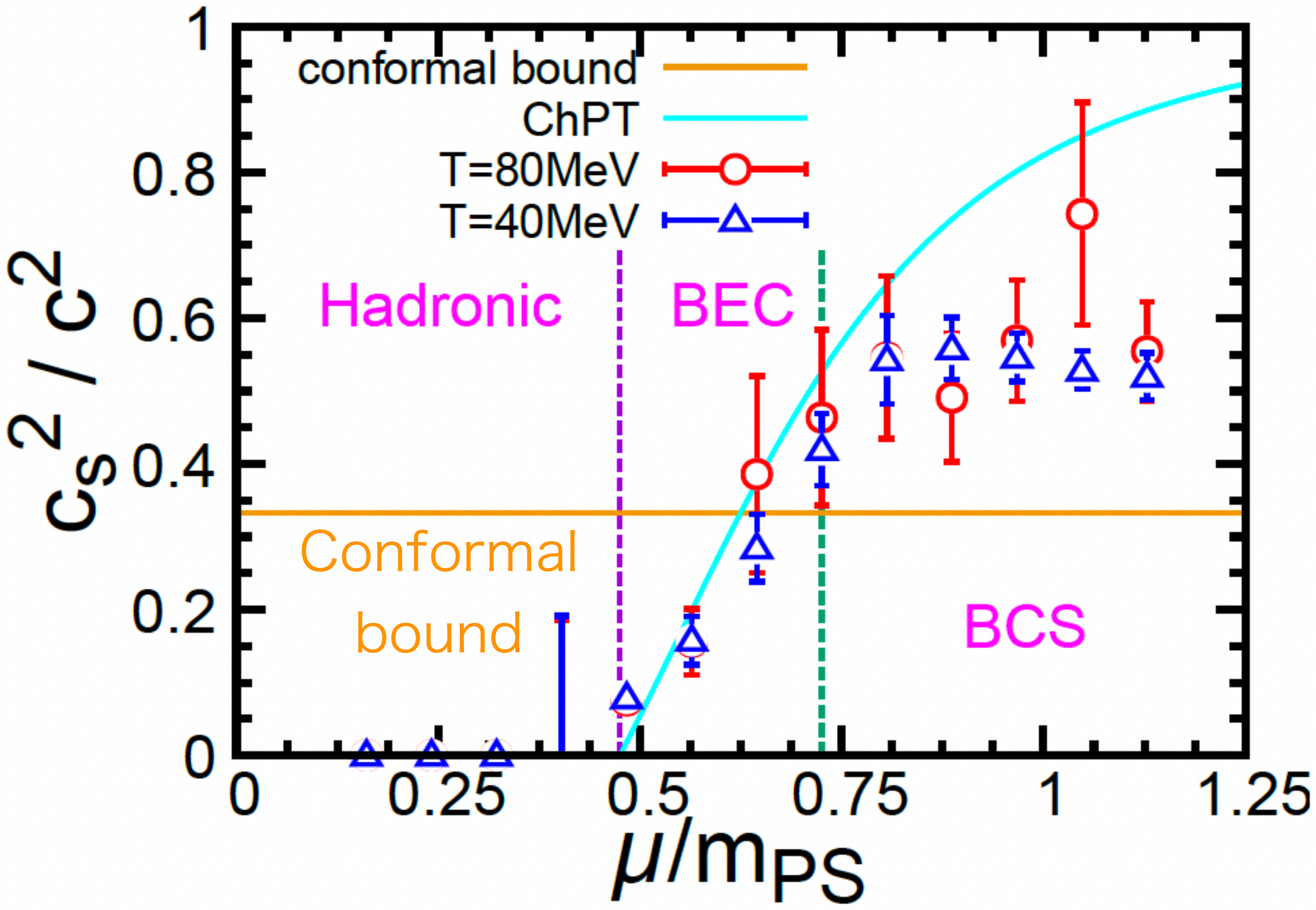
T dependence of EoS



- p increases more rapidly near the critical point at lower-T
- In high-μ, the data approaches the Stefan-Boltzmann limit (=non-interacting theory)

  $p_{SB}/\mu^4 = N_c N_f/(12\pi^2) \approx 0.03$
- Our largest data of p at T=40MeV reaches at 93% of p_{SB}

Square of sound velocity ($c_s^2/c^2 = \Delta p/\Delta e$)



- T-dependence of the sound velocity is negligible!
- In BEC phase, result is consistent with ChPT

  Chiral Perturbation Theory (ChPT)

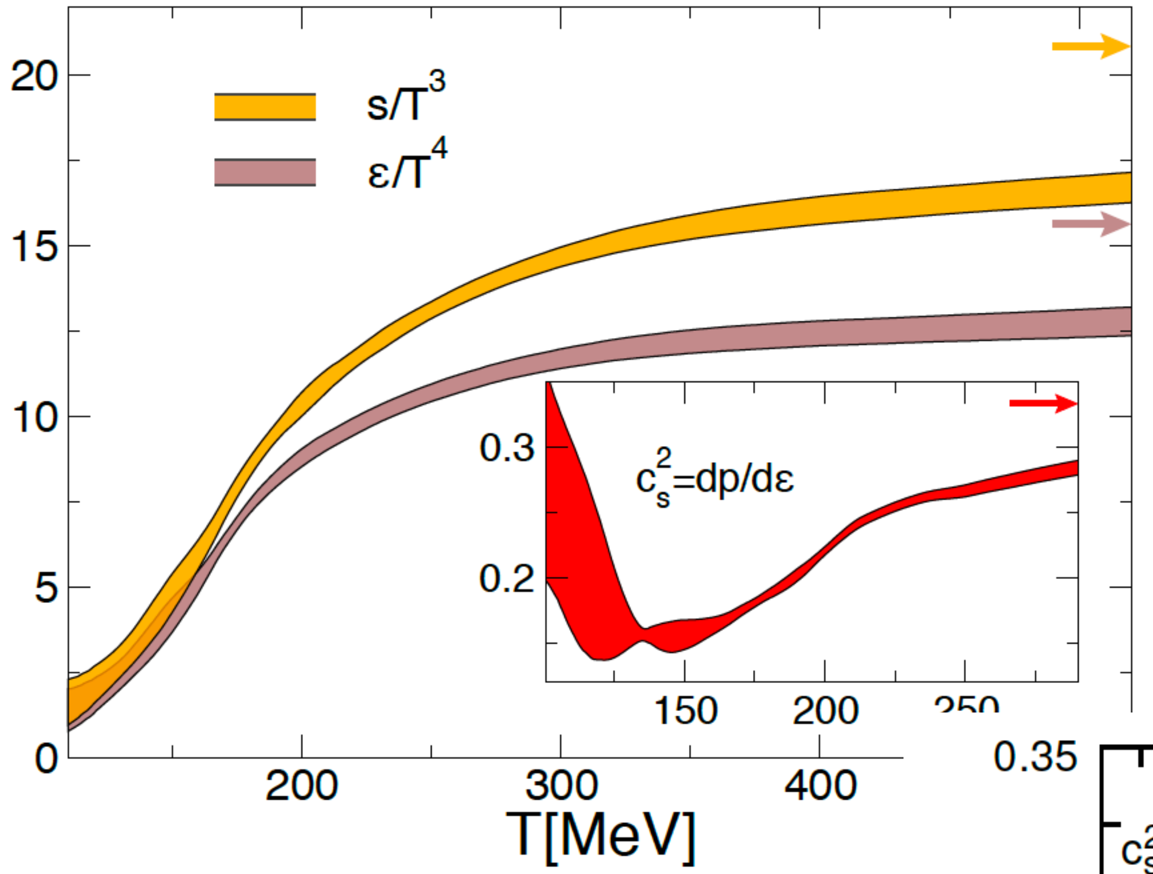
  $$c_s^2/c^2 = \frac{1-\mu_c^4/\mu^4}{1+3\mu_c^4/\mu^4}$$: no free parameter!

  Son and Stephanov (2001) : 3color QCD with isospin μ
  Hands, Kim, Skullerud (2006) : 2color QCD with real μ

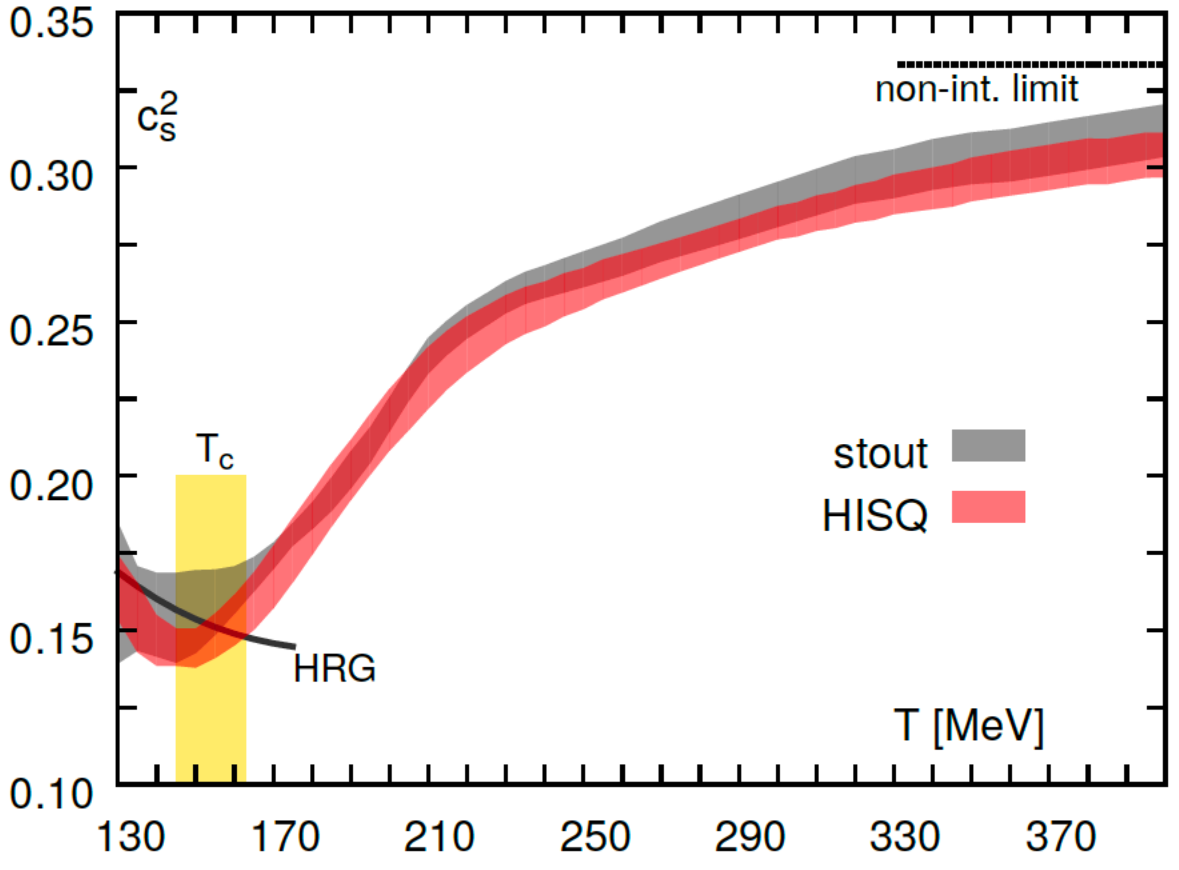
- c_s^2/c^2 exceeds the conformal bound

Sound velocity and phase transition

Finite Temperature transition (Nf=2+1 QCD)



Borsanyi et al. (2013)

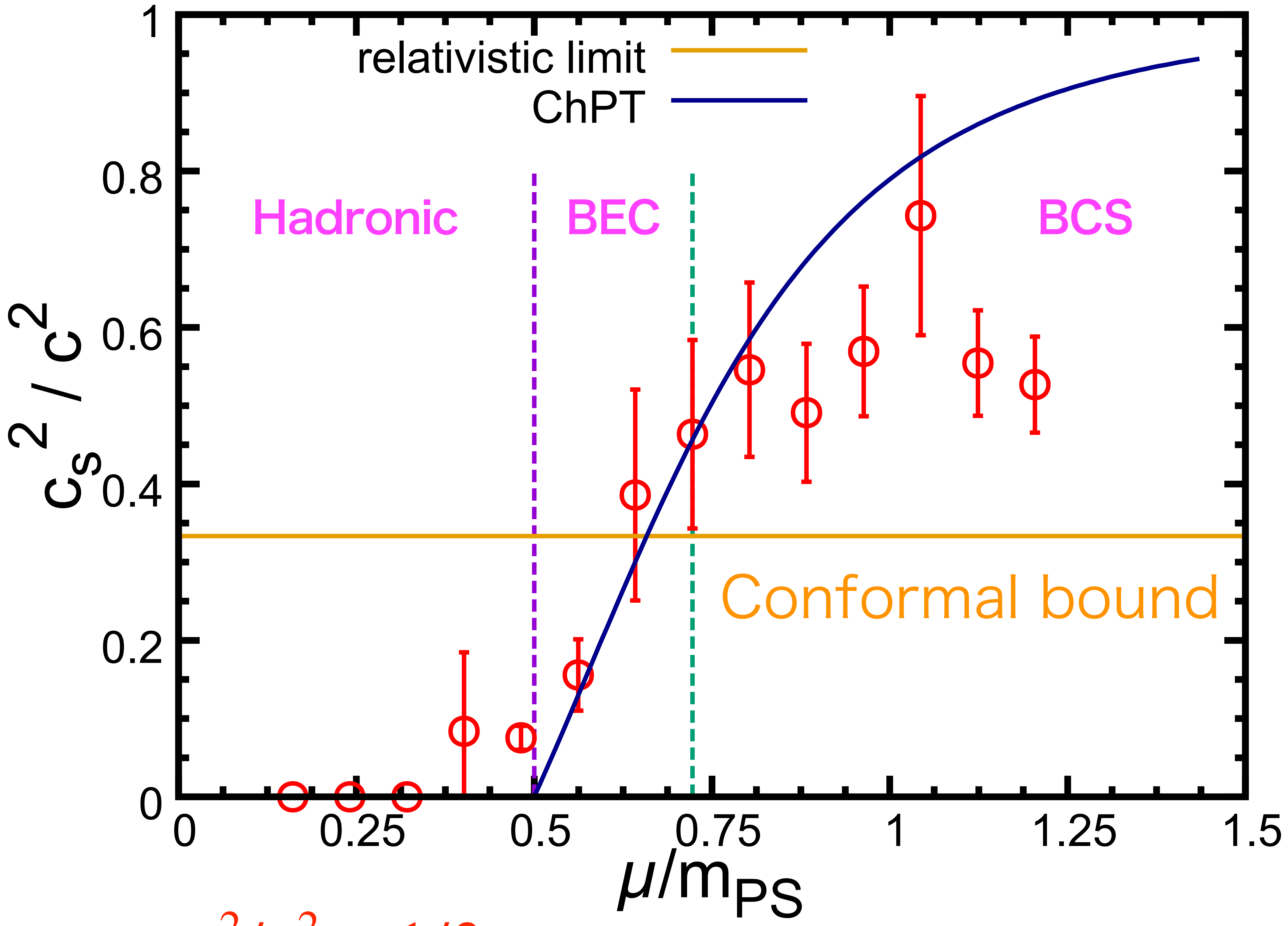


HotQCD (2014)

- Minimum around Tc
- Monotonically increases to $c_s^2/c^2 = 1/3$

Finite Density transition (Nf=2 2color QCD)

Iida and Ei arXiv: 2207.01253



- $c_s^2/c^2 > 1/3$
- previously unknown from any lattice calculations for QCD-like theories

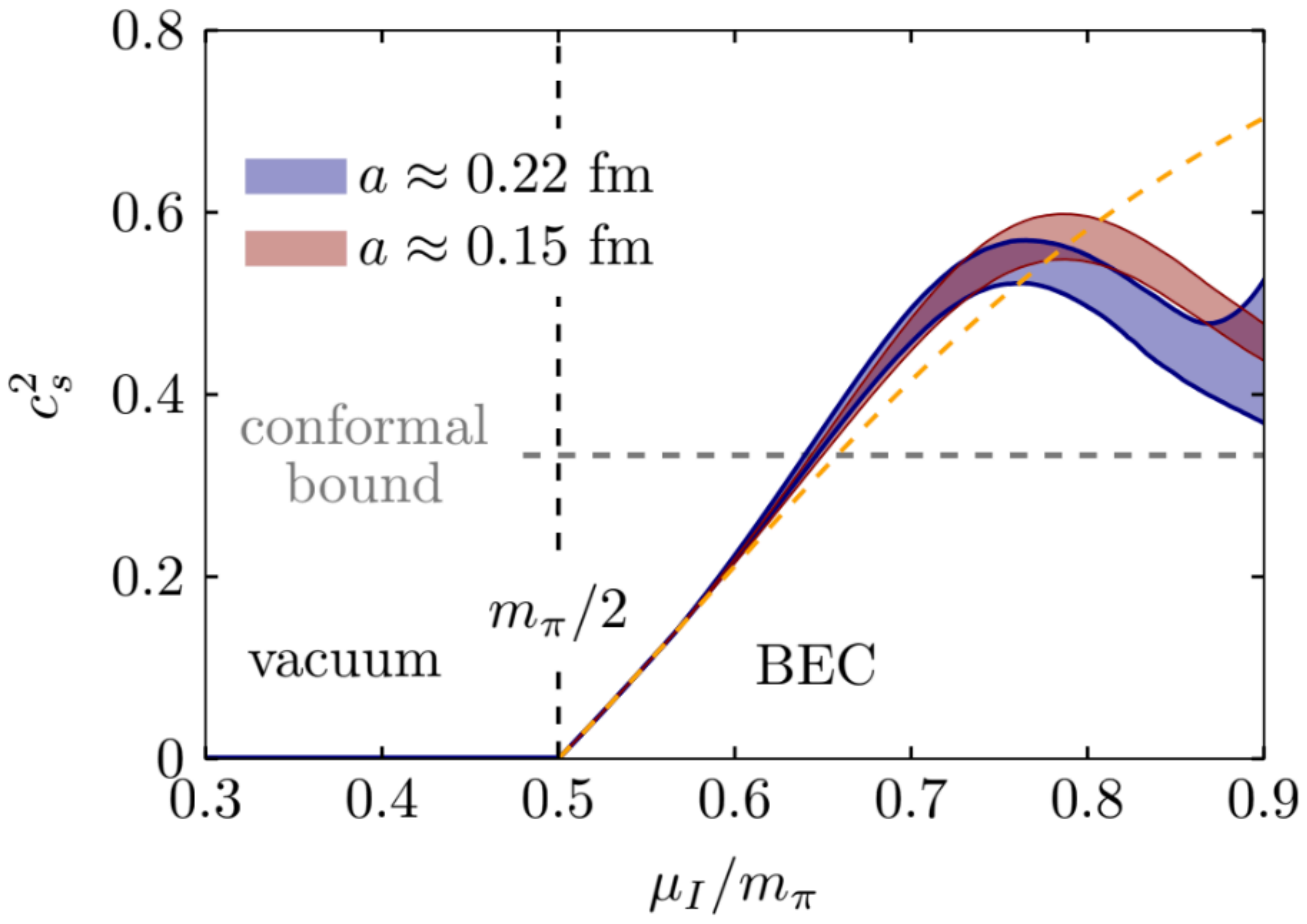
Lattice MC for 3 color QCD with isospin chemical potential

3 color QCD w/ Isospin-$\mu_I \approx$ 2color QCD w/ real μ

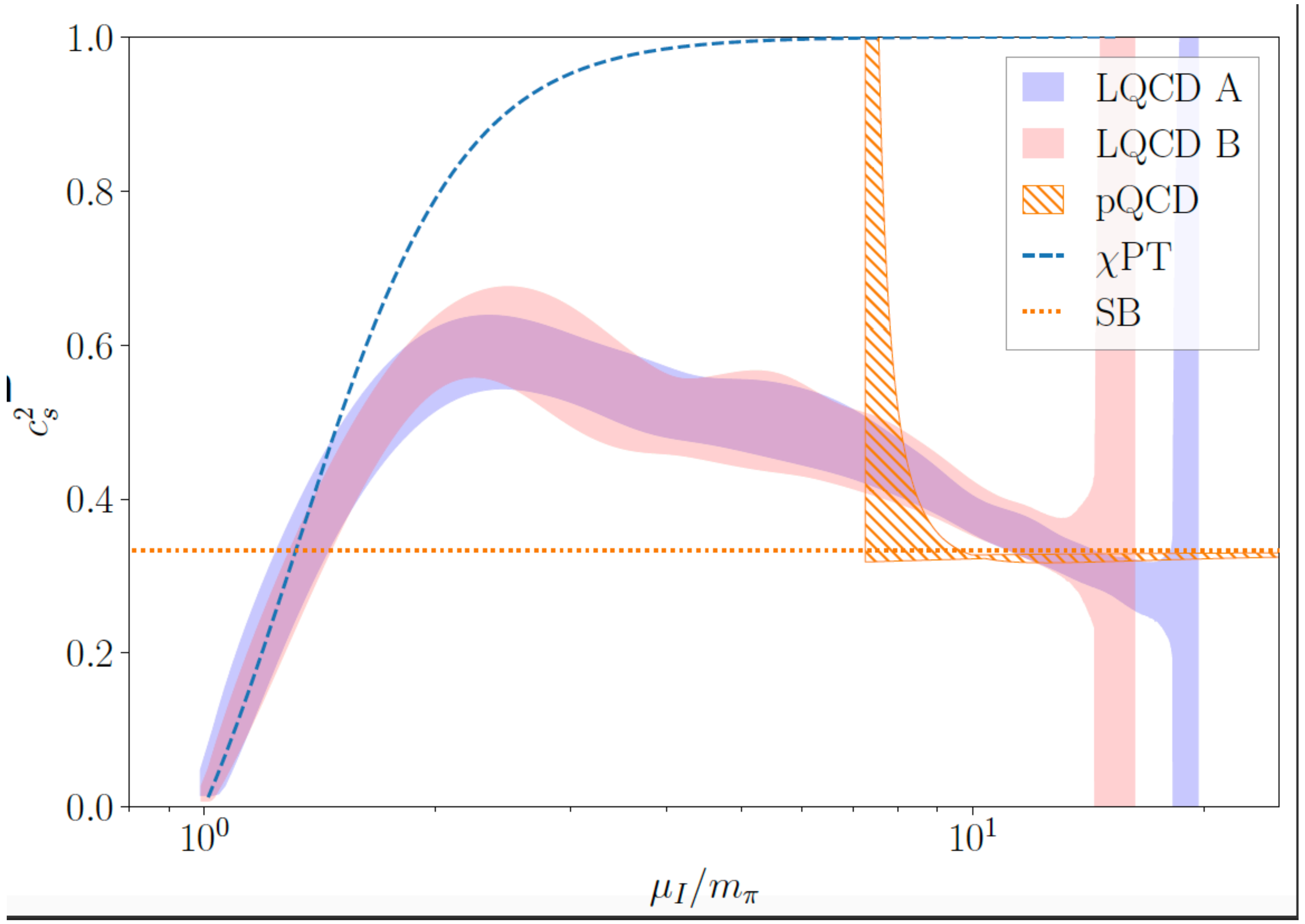
B. B. Brandt, F. Cuteri , G. Endrodi, arXiv: 2212.14016

R. Abbott et al. arXiv:2307.15014

Result with spline interpolation



New algorithm for n-point fn. calc.



Summary and future work

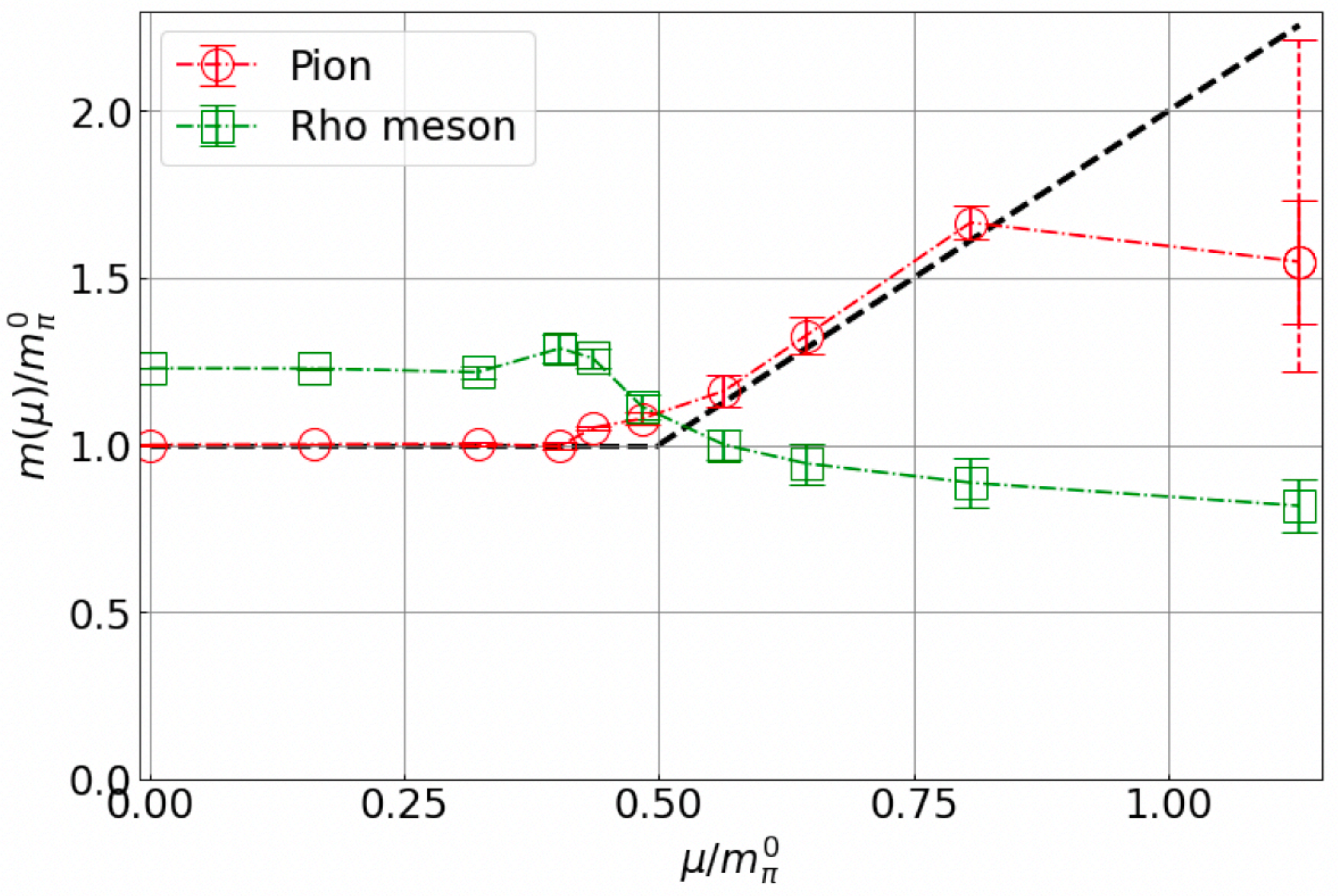
- Lattice numerical simulation for QCD-like theory w/o the sign problem has been ongoing
  2color finite-density QCD and 3color w/ isospin chemical potential are doable using exact algorithm
- Sound velocity exceeds the conformal bound in finite-μ QCD-like theory
  (All Lattice Monte Carlo results have satisfied $c_s^2/c^2 \leq 1/3$ for 40years!)
- Large sound velocity suggests stiffer-than-conventional picture of QCD matter
- Find a mechanism of a peak structure
  - quark saturation?(Kojo,Suenaga), negative trace anomaly?
  (McLerran, Fukushima, Fujimoto et al.), others?
  - Effective model analyses combined with the lattice results are also ongoing

cf.) D.Suenaga, M.Kawaguchi, Y.Fujimoto
Minato and Fukushima...

- Ongoing Lattice studies:
  => mass spectrum in superfluid phase
  => extended HAL QCD method in finite density
  => Find an explicit evidence of superfluidity (Fermi surface...)

Other interesting phenomena in superfluid phase

Mass spectrum



K.Murakami, D.Suenaga, K.Iida, EI,
PoS LATTICE2022 (2023) 154

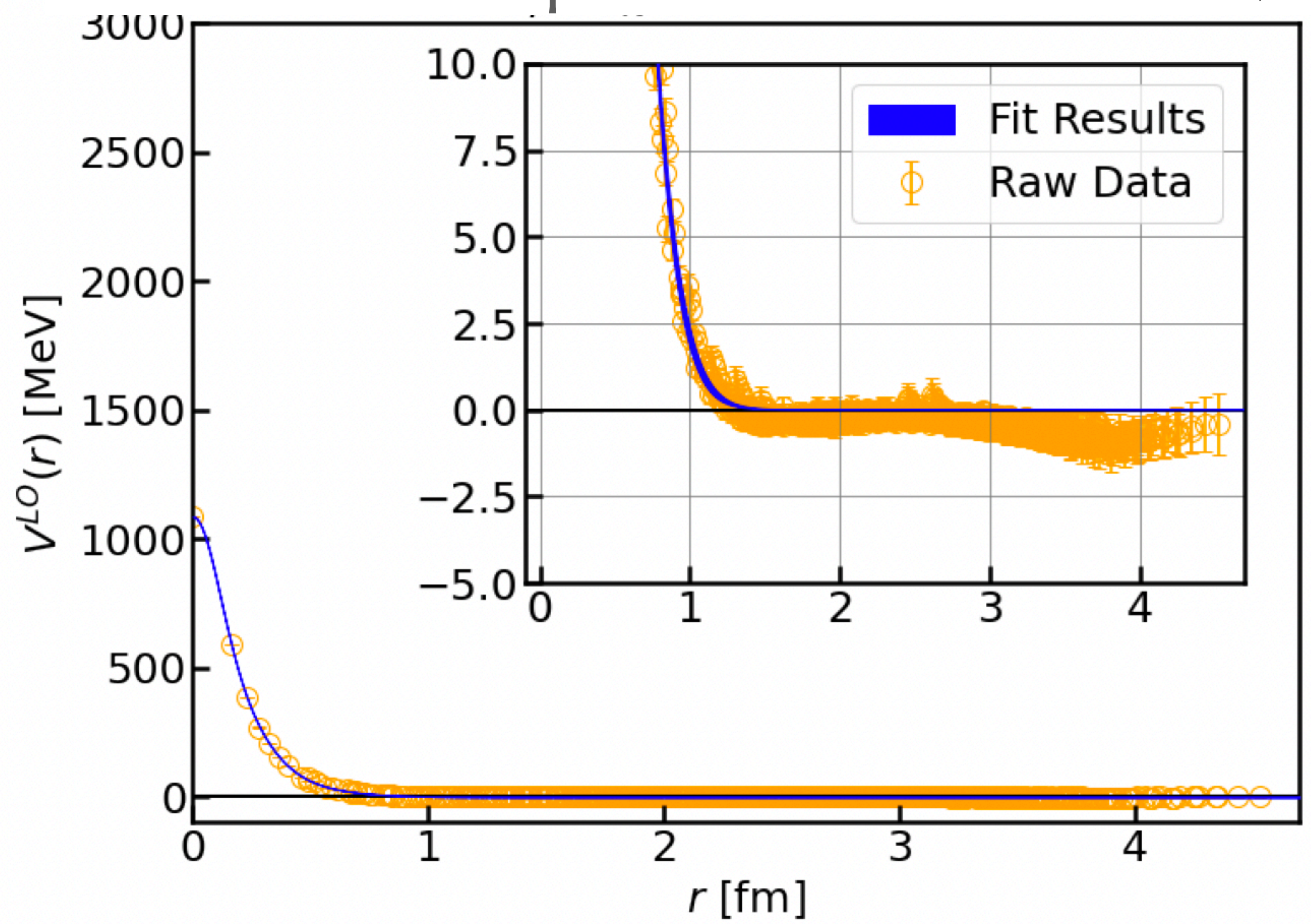
- It is observed that the order of hadron spectra are changed in superfluid phase
- rho meson becomes lighter than pion
- Such a changing is also predicted in 3color QCD Hatsuda-Lee(1992)

Hadron potential by HAL QCD method

- In hadronic phase, pion and diquark potential are equivalent because of extended flavor symmetry.

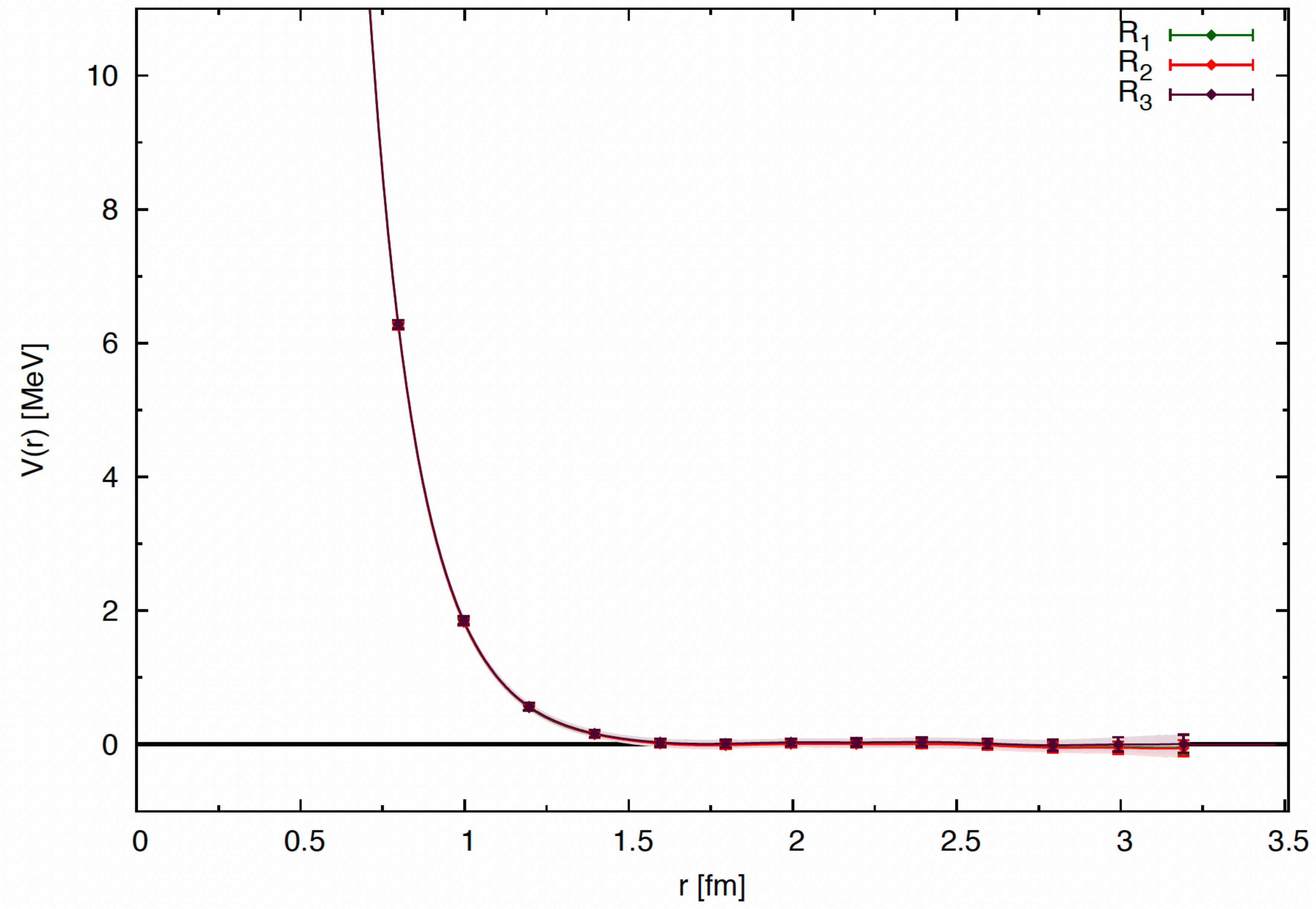
- Pion potential for 2color and 3color QCD are qualitatively same

Diquark-diquark potential
in hadronic phase of 2color QCD



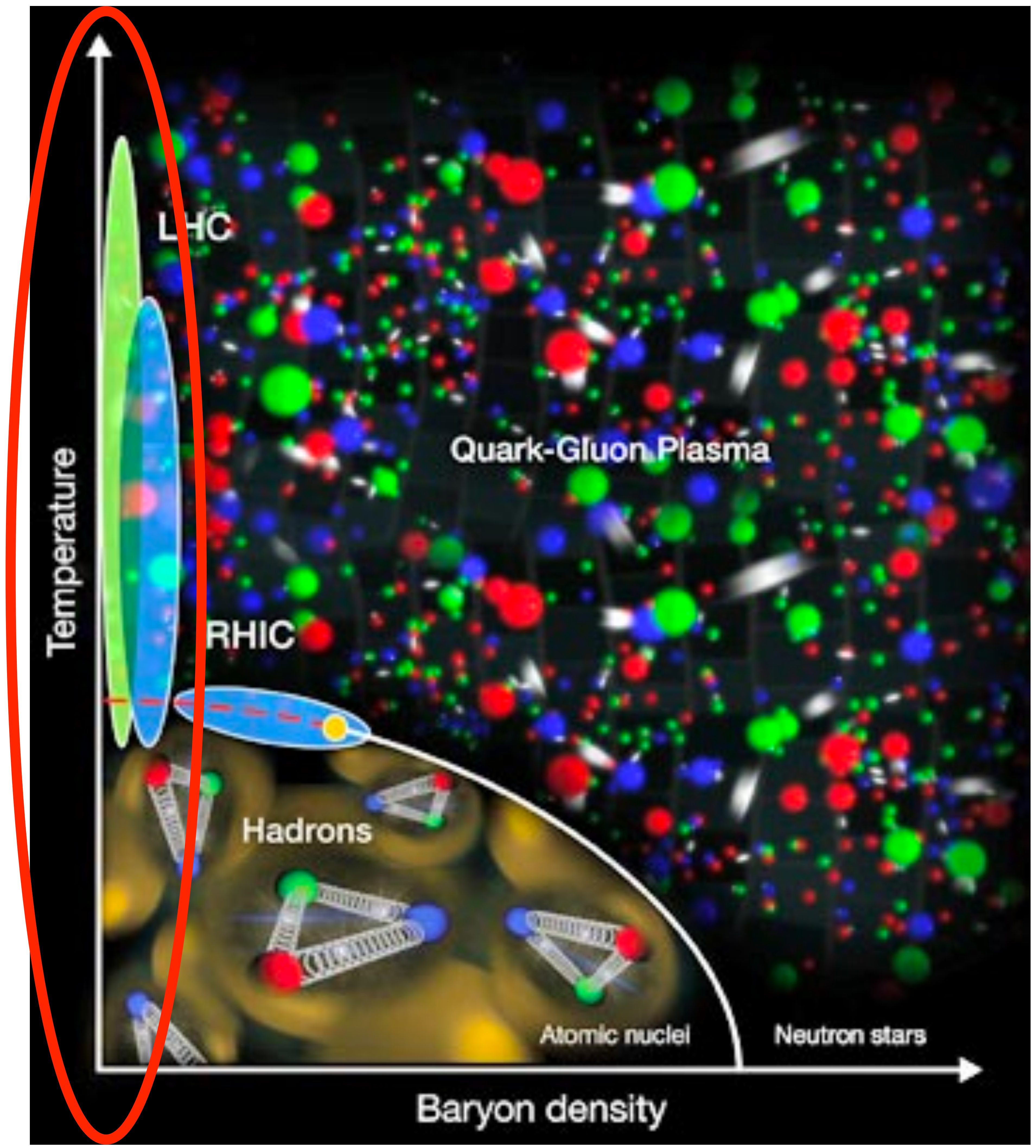
K.Murakami, K.Iida, EI, JHEP 11 (2023) 231

I=2 $\pi\pi$ potential of 3color QCD



T.Kurth et al.(HAL QCD coll.), JHEP12(2013)015 backup

EoS and sound velocity at zero μ



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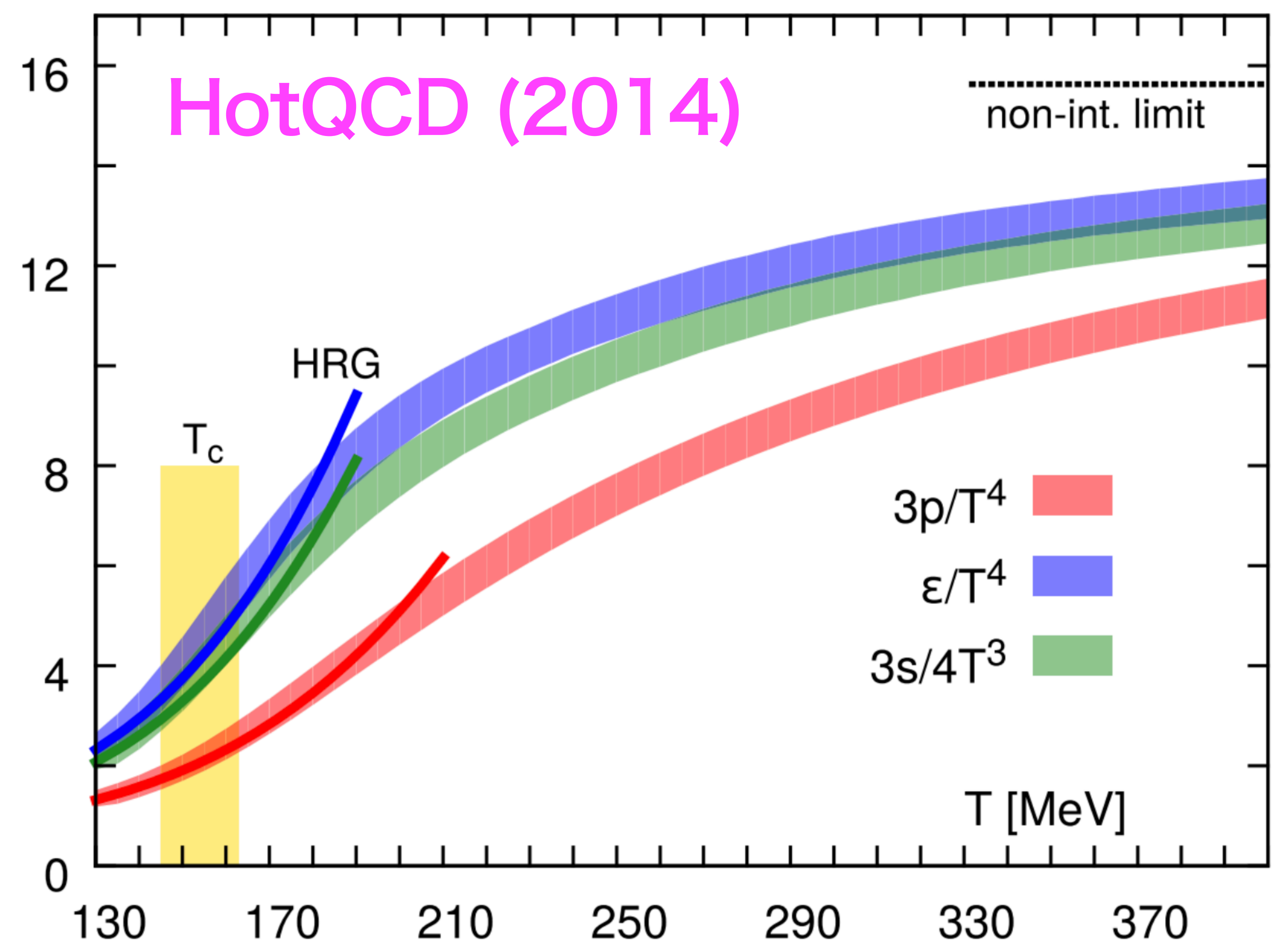
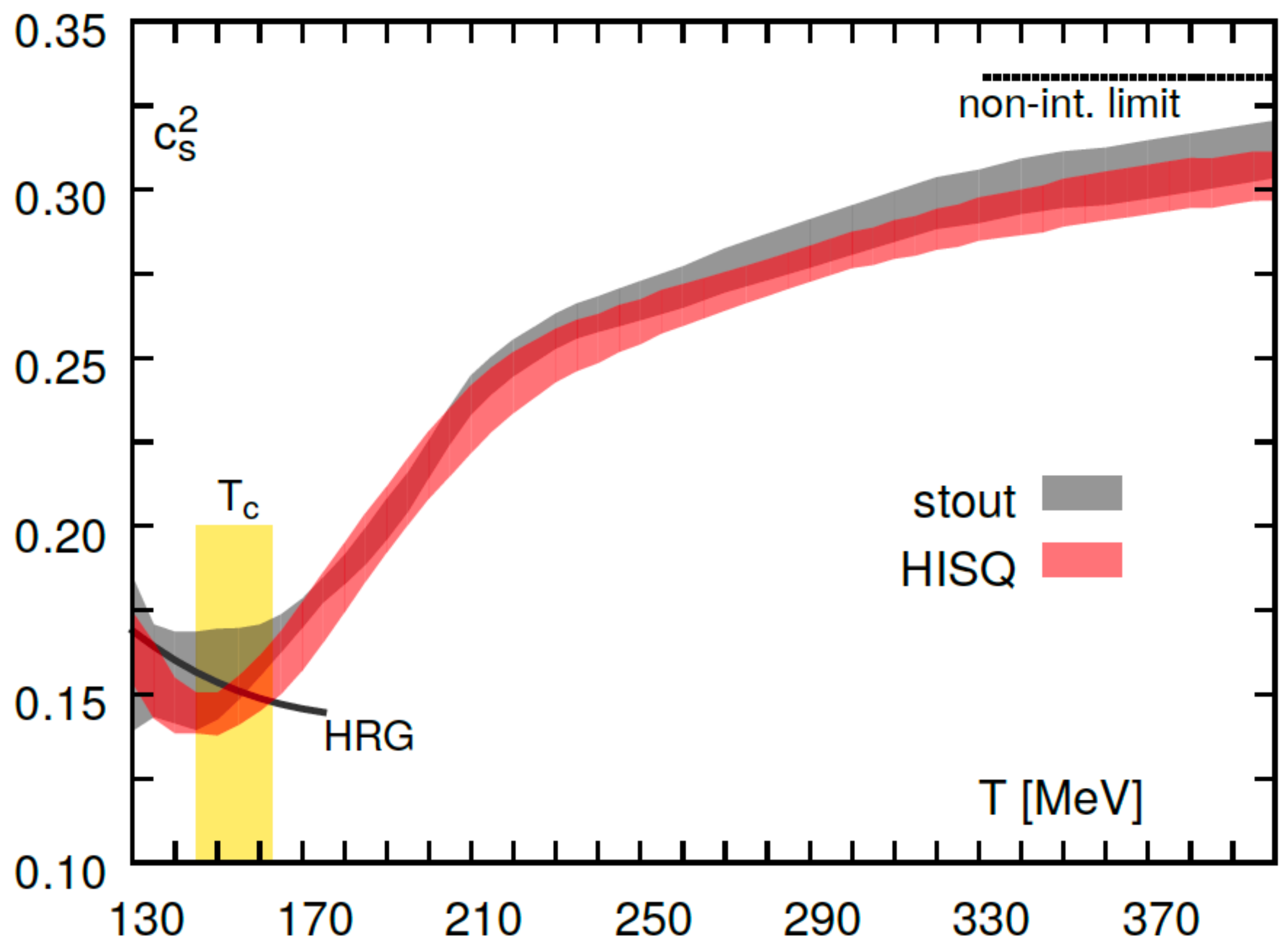
Finite Temperature transition (Nf=2+1 QCD)

EoS (p and ε)

Sound velocity $c_s^2/c^2 = \partial p/\partial \epsilon$

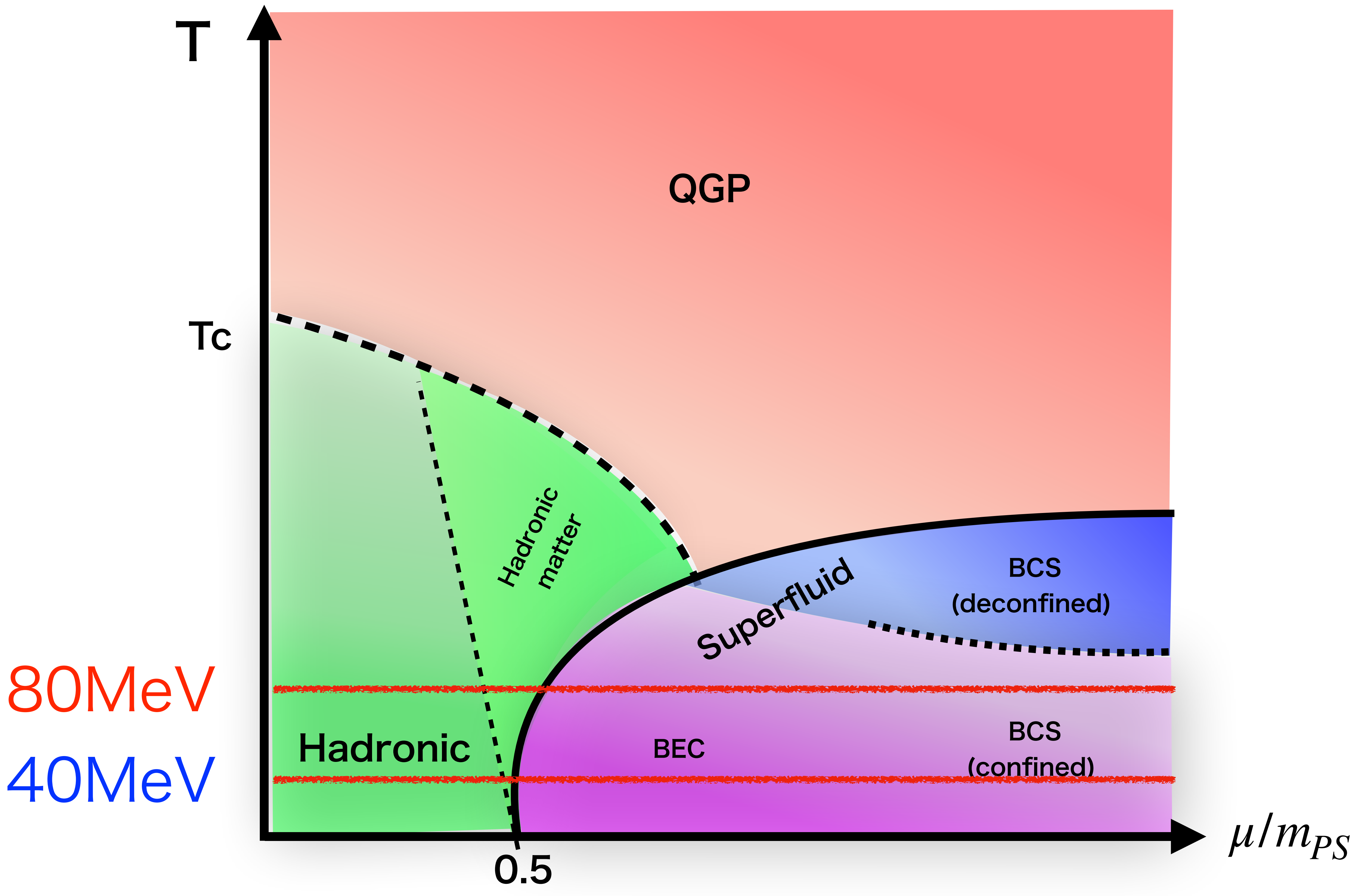




cf.) $c_s^2/c^2 = 1/3$: relativistic free theory where $e = 3p$

($c_s^2/c^2 \leq 1/3$: conformal bound)

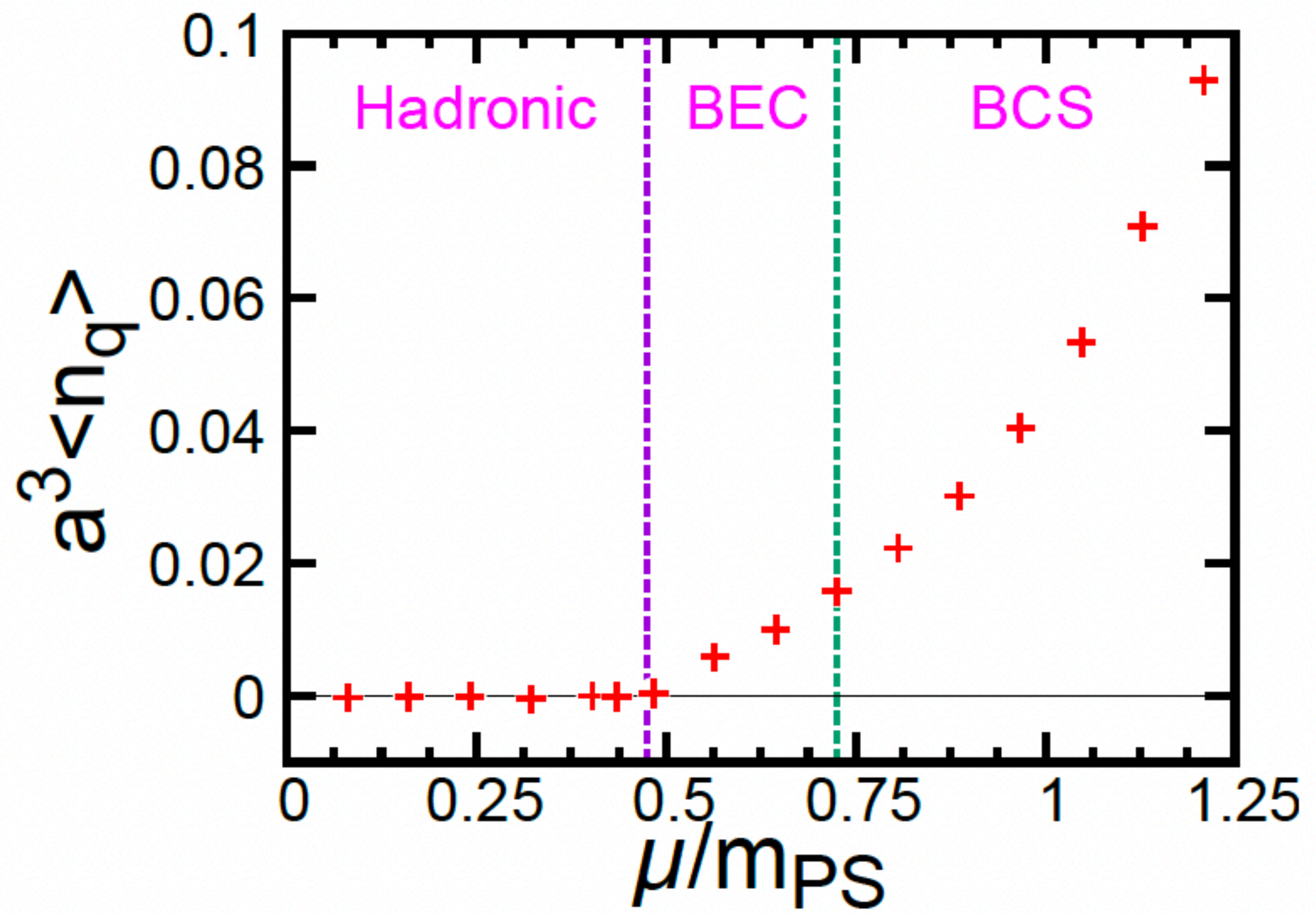
Current status on 2color QCD phase diagram



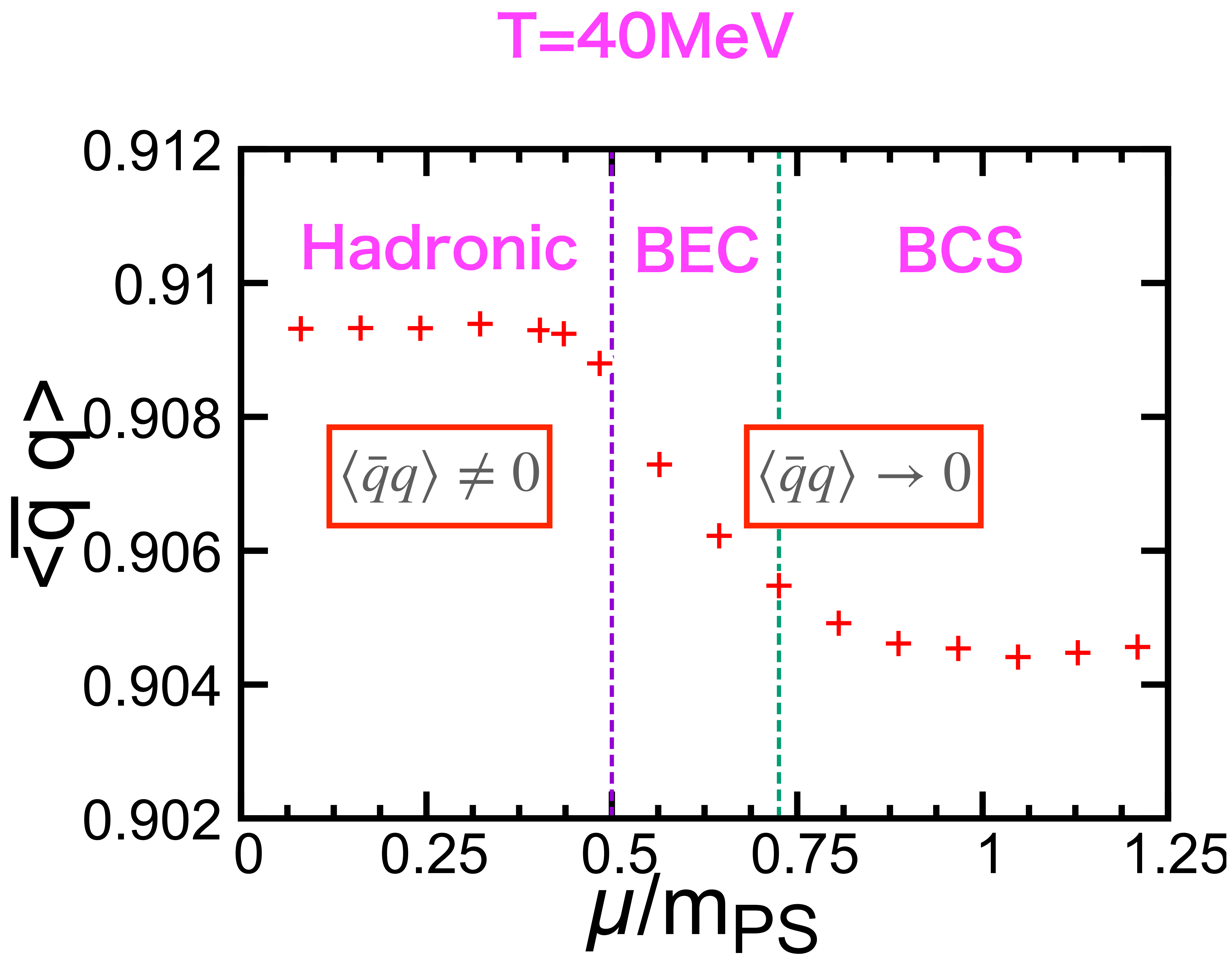
K.Iida, EI, T.-G. Lee: JHEP2001(2020)181
K.Iida, EI, K.Murakami, D.Suenage
arXiv: 2405.20566 [hep-lat]

- We investigated T=80MeV and 40MeV
- Hadronic / Superfluidity phase transition around $\mu = m_{PS}/2$

  Quark number density becomes non-zero because of pair-creation of lightest hadrons and the baryon symmetry is spontaneously broke



Chiral condensate : $\langle \bar{q}q \rangle$

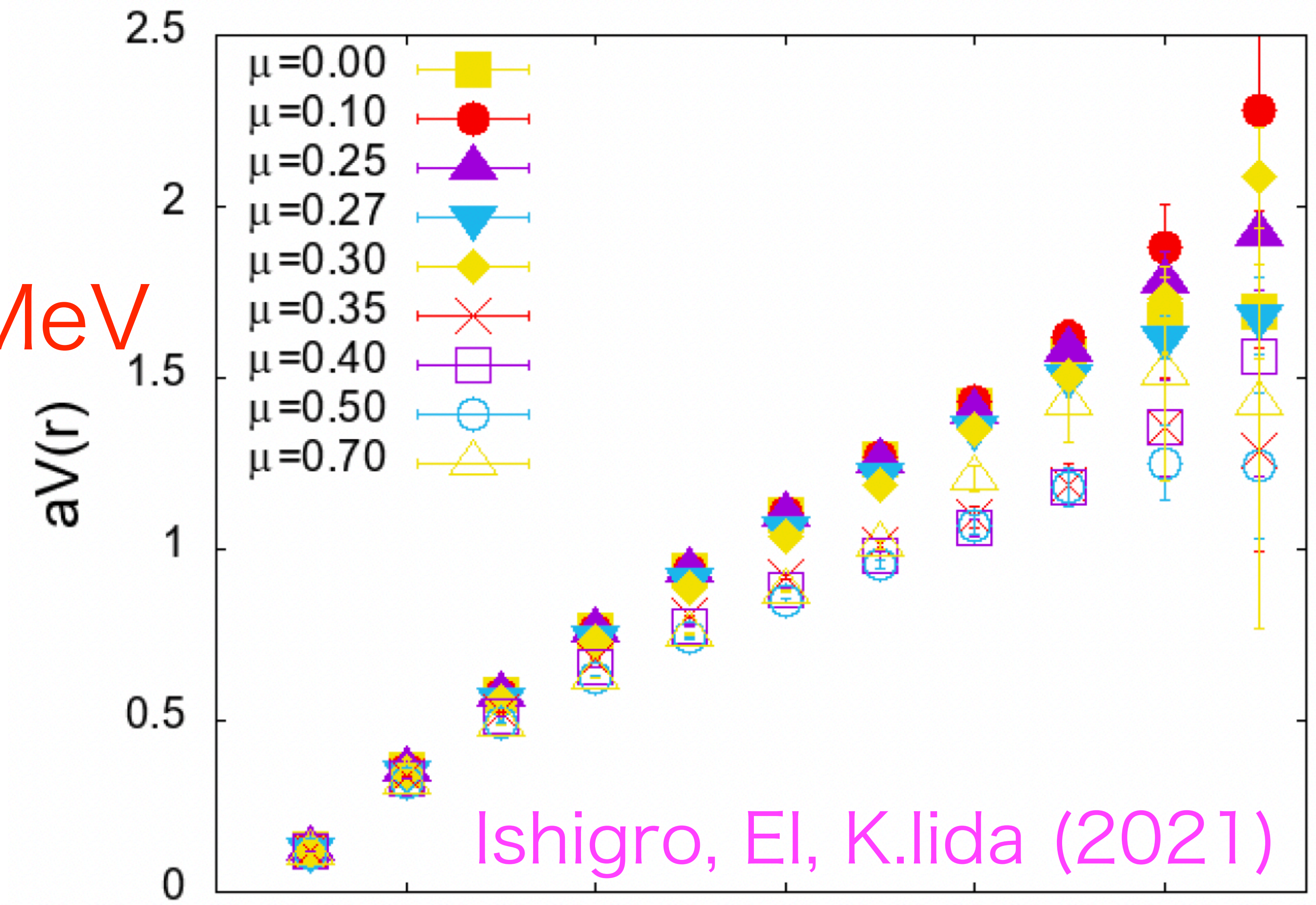


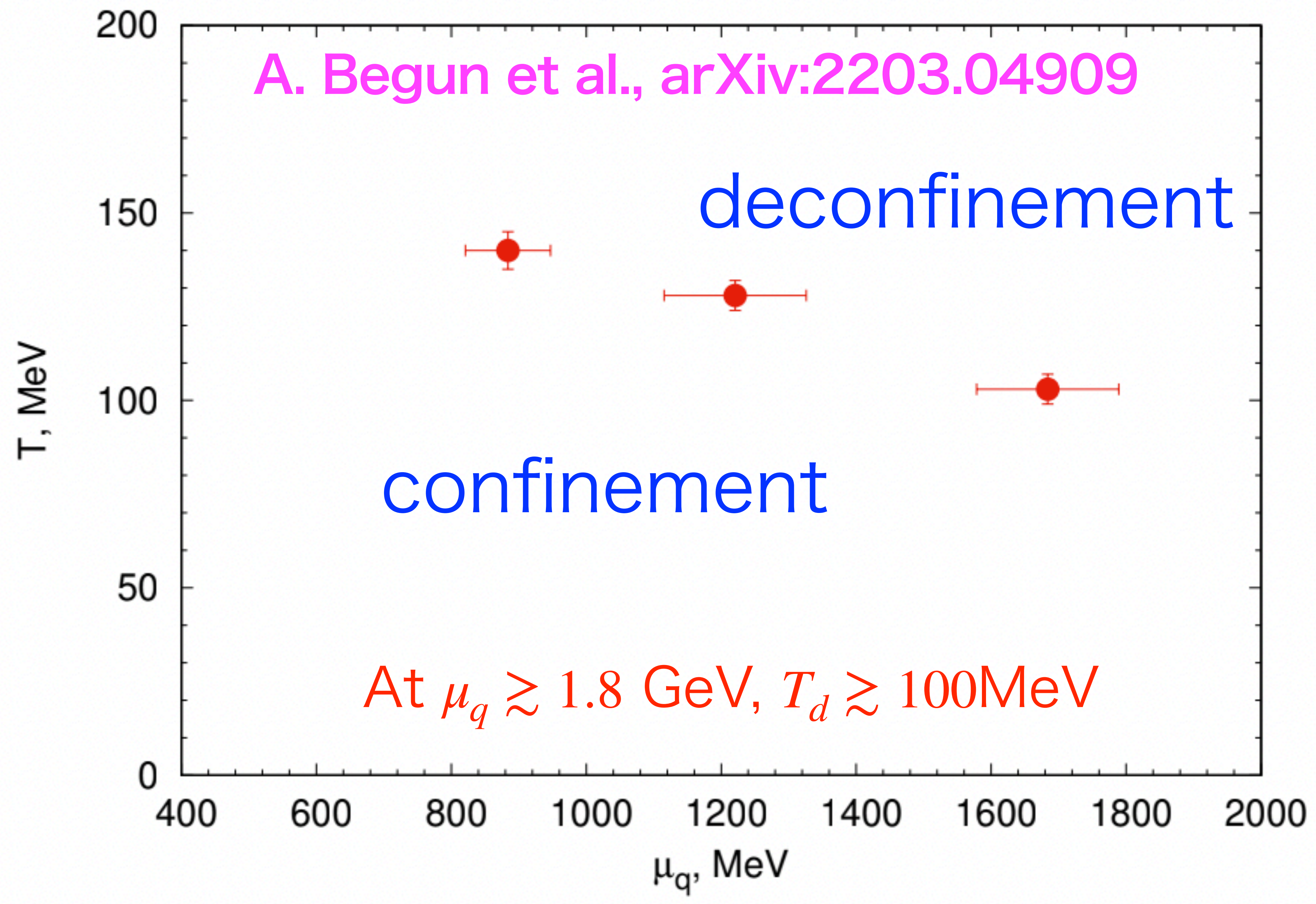
- Chiral symmetry is getting restored in Superfluid phase

- Our simulation uses the Wilson fermion, so does not show $\langle \bar{q}q \rangle = 0$ because of additive renormalization. But Russian group using the staggered fermion gives $\langle \bar{q}q \rangle = 0$

Confinement remains even in high density

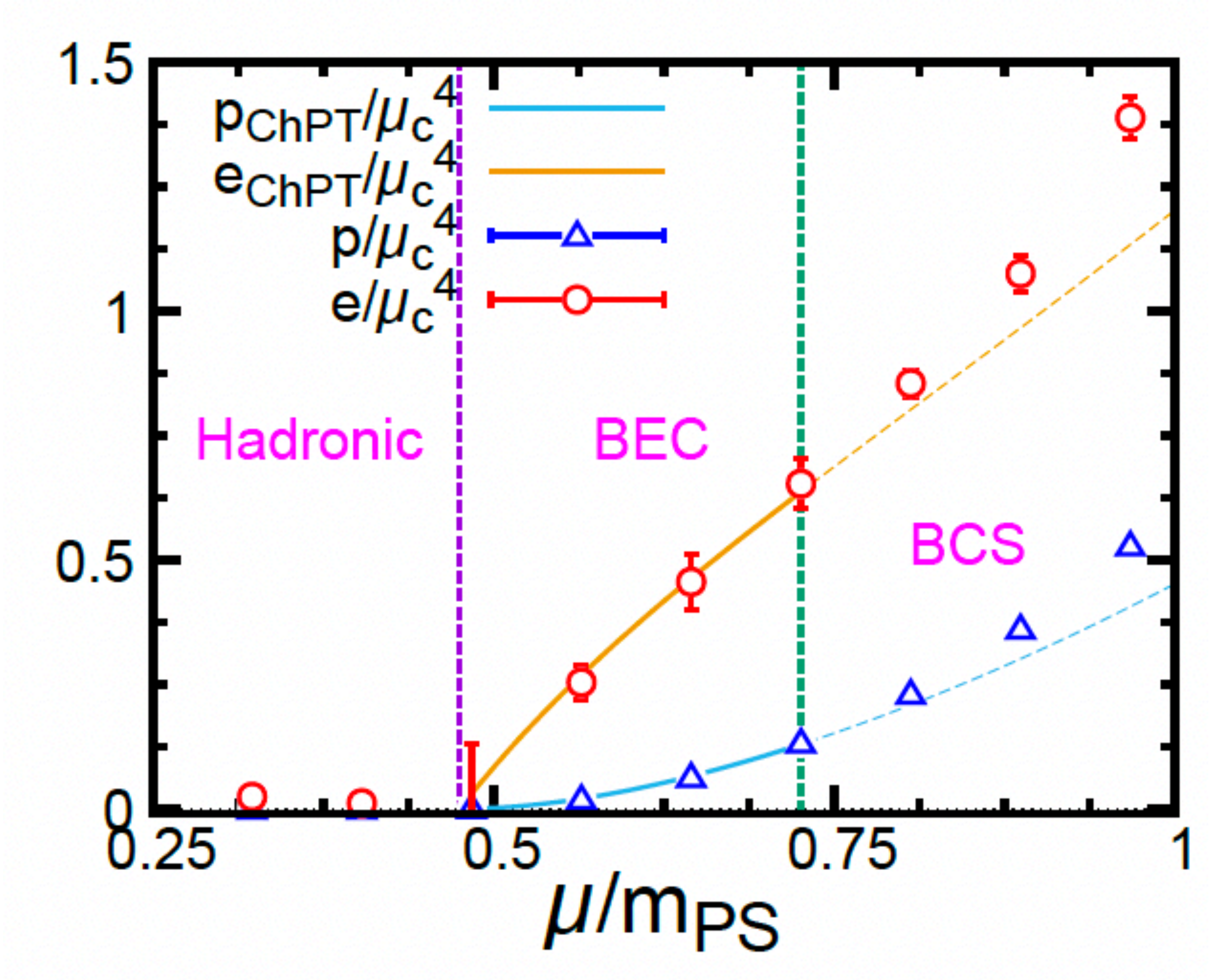
T=40MeV





- $q\bar{q}$ potential at T=40MeV also show a linear potential (=confinement)
- Other 2 groups also show: T~90-100MeV is the critical T for deconfinement
- In 2color QCD, the confinement occurs even at high-density. Hadronic superfluidity
- cf.) In real dense QCD, it is expected that quark dof would be relevant Condensate has color charge: $\langle (qq)^a \rangle$

EoS and consistency with ChPT result in BEC



- ChPT prediction (valid for near μ_c)

$$p_{\mathrm{ChPT}} = 4N_f F^2 \mu^2 \left(1 - \frac{\mu_c^2}{\mu^2}\right)^2$$

$$e_{\mathrm{ChPT}} = 4N_f F^2 \mu^2 \left(1 - \frac{\mu_c^2}{\mu^2}\right)\left(1 + 3\frac{\mu_c^2}{\mu^2}\right)$$

- We obtain the pion decay constant(F)
  from fit of p : F=51.1(5) MeV
  from fit of e : F=56.7(7) MeV
  cf.) F=60.8(1.6) by fitting of $\langle n_q \rangle$ at 140MeV
  (different mass, staggered fermion)

N. Astrakhantsev et al. (2020)

Counterexamples of conformal bound

N=4 SYM at finite density

PHYSICAL REVIEW D **94**, 106008 (2016)

Breaking the sound barrier in holography

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(Received 20 September 2016; published 15 November 2016)

It has been conjectured that the speed of sound in holographic models with UV fixed points has an upper bound set by the value of the quantity in conformal field theory. If true, this would set stringent constraints for the presence of strongly coupled quark matter in the cores of physical neutron stars, as the existence of two-solar-mass stars appears to demand a very stiff equation of state. In this article, we present a family of counterexamples to the speed of sound conjecture, consisting of strongly coupled theories at finite density. The theories we consider include $\mathcal{N} = 4$ super Yang-Mills at finite R-charge density and nonzero gaugino masses, while the holographic duals are Einstein-Maxwell theories with a minimally coupled scalar in a charged black hole geometry. We show that for a small breaking of conformal invariance, the speed of sound approaches the conformal value from above at large chemical potentials.

Evidence against a first-order phase transition in neutron star cores: impact of new data

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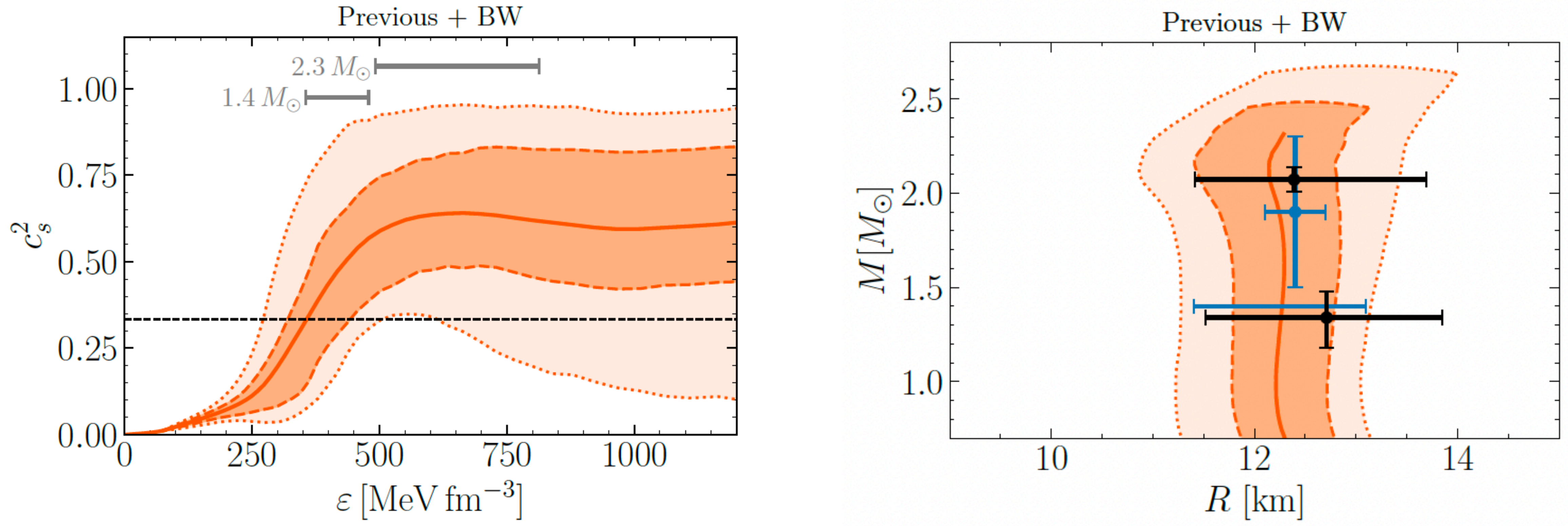
(Dated: June 13, 2023)

With the aim of exploring the evidence for or against phase transitions in cold and dense baryonic matter, the inference of the sound speed and equation-of-state for dense matter in neutron stars is extended in view of recent new observational data. The impact of the heavy (2.35 $M_\odot$) black widow pulsar PSR J0952-0607 and of the unusually light supernova remnant HESS J1731-347 is inspected. In addition a detailed re-analysis is performed of the low-density constraint based on chiral effective field theory and of the perturbative QCD constraint at asymptotically high densities, in order to clarify the influence of these constraints on the inference procedure. The trace anomaly measure, $\Delta = 1/3 - P/\varepsilon$, is also computed and discussed. A systematic Bayes factor assessment quantifies the evidence (or non-evidence) of a phase transition within the range of densities realised in the core of neutron stars. One of the consequences of including PSR J0952-0607 in the data base is a further stiffening of the equation-of-state, resulting for a typical 2.1 solar-mass neutron star in a reduced central density of less than five times the equilibrium density of normal nuclear matter. The evidence against the occurrence of a first-order phase transition in neutron star cores is further strengthened.

Bayesian analyses of recent observation data of neutron star

arXiv:2306.06218



Conformal bound (Holography bound)

Conformal bound: A conjecture proposed by A.Cherman et al., 2009

"maximal value of c_s^2/c^2 is 1/3 (non-interacting theory)

for a broad class of 4-dim. theories"

A bound on the speed of sound from holography

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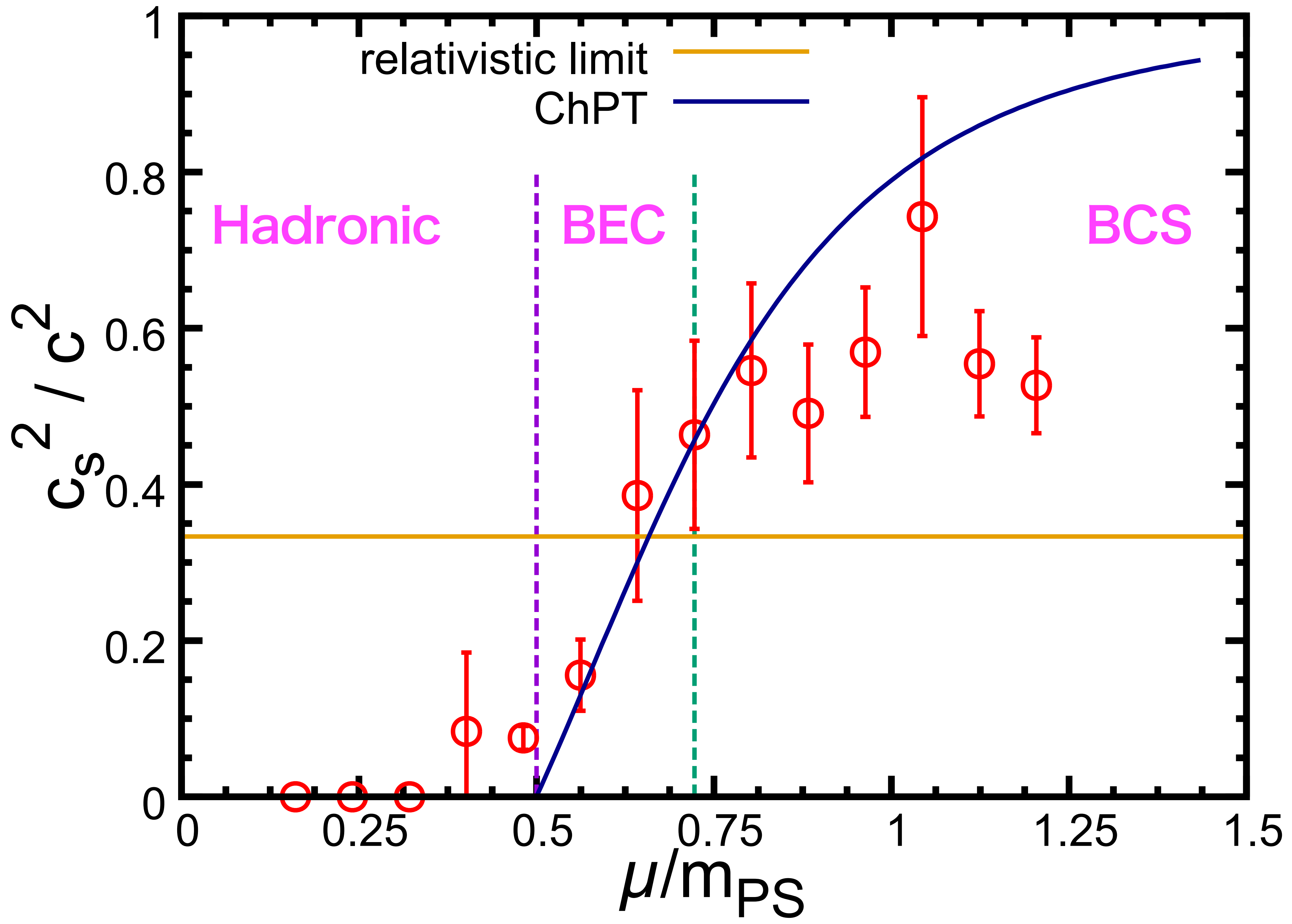
Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544

We show that the squared speed of sound v_s^2 is bounded from above at high temperatures by the conformal value of $1/3$ in a class of strongly coupled four-dimensional field theories, given some mild technical assumptions. This class consists of field theories that have gravity duals sourced by a single scalar field. There are no known examples to date of field theories with gravity duals for which v_s^2 exceeds $1/3$ in energetically favored configurations. We conjecture that $v_s^2 = 1/3$ represents an upper bound for a broad class of four-dimensional theories.

All Lattice Monte Carlo results have satisfied this bound for 40years!

Further high density?

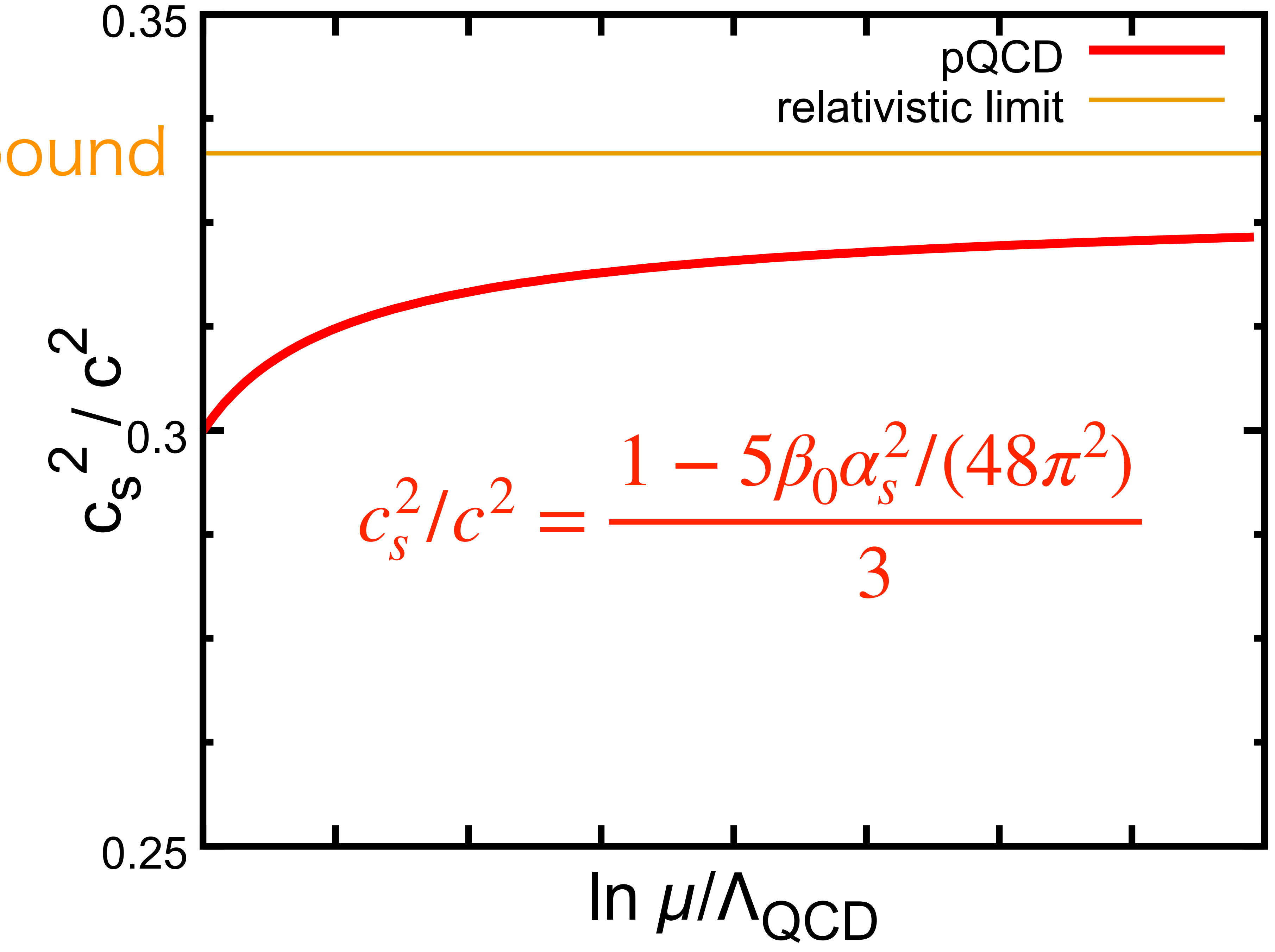
Kojo, Baym, Hatsuda (2021)



pQCD prediction

(Ultra high-density regime)

Conformal bound

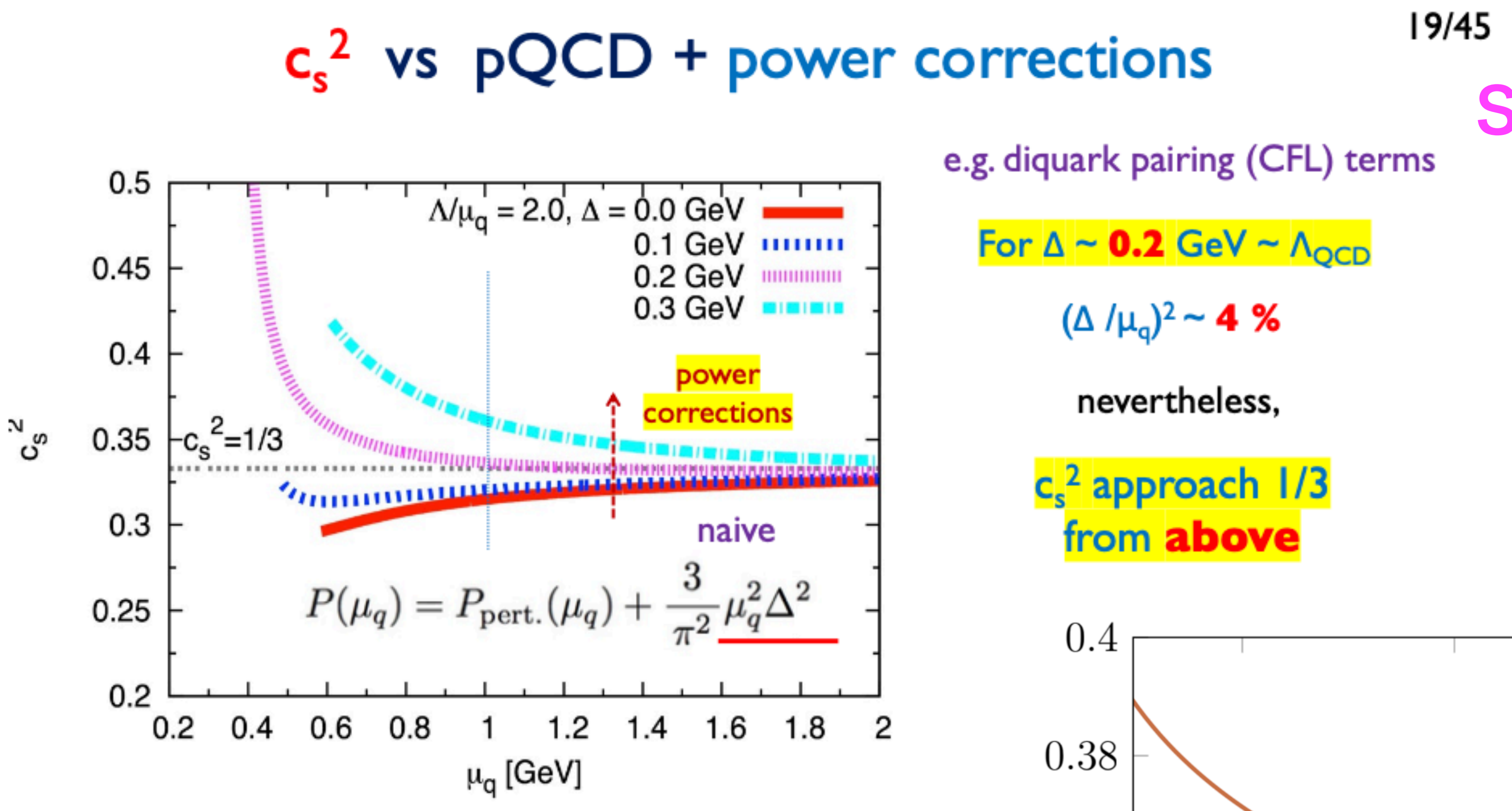


- Upper bound of chemical potential in lattice simulation comes from $a\mu \ll 1$
  (Here, we take $a\mu \leq 0.8$)
- To study high-density, the lighter mass / finer lattice spacing are needed

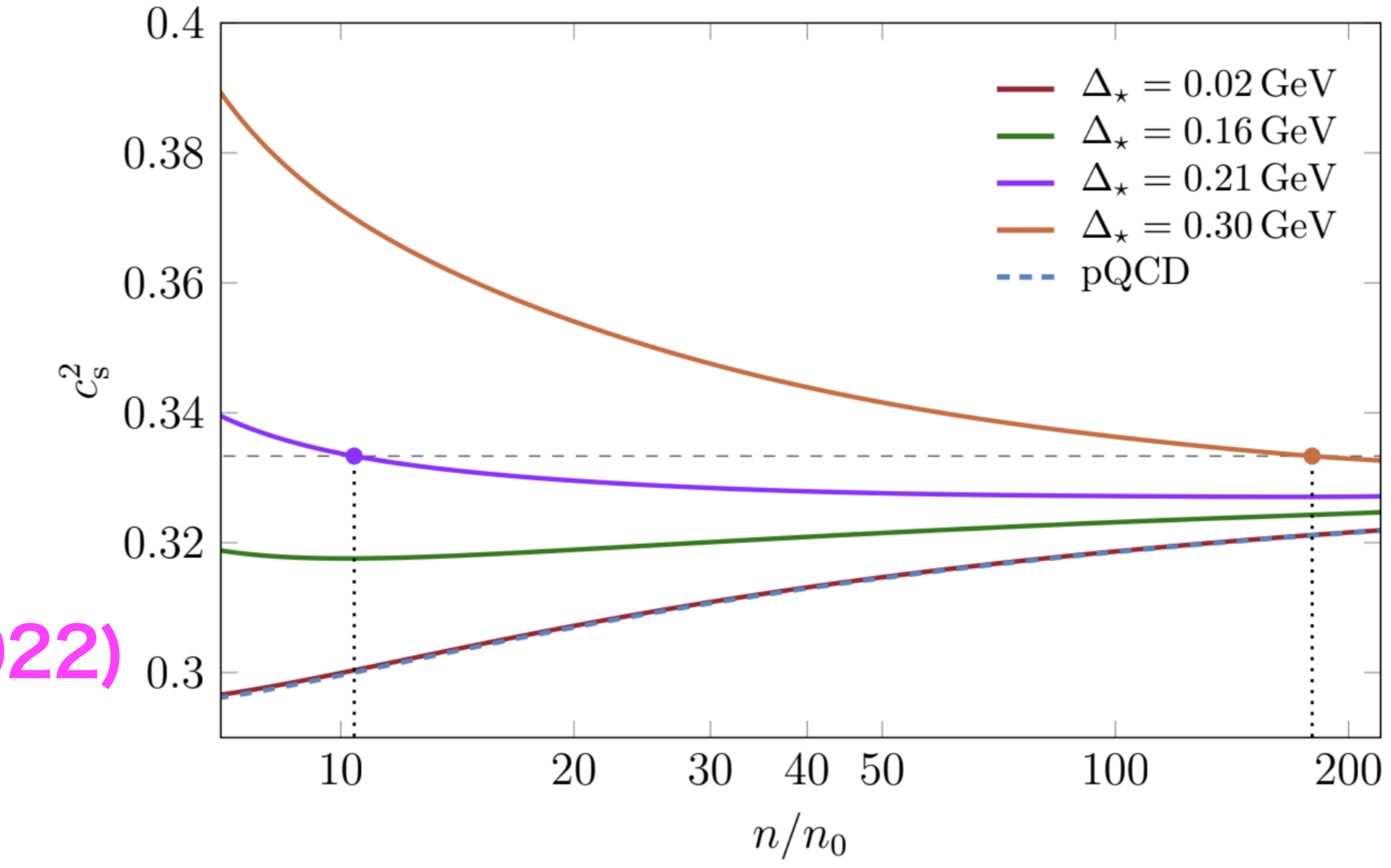
Further high density?

pQCD + power correction due to diquark gap

Slide by Kojo (2019)



fRG analysis
Braun, Geissel, Schallmo(2022)



Hard thermal loop resummation

Fujimoto and Fukushima(2021)