Are axion solutions to the CP problem fine-tuned?

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1. The CP problem

2. Fine tuning

3. Axion solutions

4. Are axion solutions to the CP problem fine-tuned?

Section 1

The CP problem

QCD parameters

- QCD can be written schematically as (see e.g., Dine (2000))

$$\mathcal{L} \sim \frac{1}{\alpha_S} GG + \bar{q} i \not{D} q - \bar{q} M q$$

- Described by a coupling, α_S, and the quark mass matrix
- What about the CP violating

$$\tilde{G}G?$$

- That's a **total derivative**; can be written as $\tilde{G}G = \partial_\mu K^\mu$

Wait, there's another parameter

- We can ignore total derivatives, right?
- Wrong, it cannot be neglected because of **finite-action instantons**
- Thus, must consider

$$\boxed{\mathcal{L} \sim \frac{1}{\alpha_S} GG + \bar{q} i \not{D} q - \bar{q} M q + \theta \, \tilde{G} G}$$

- A new parameter, θ? Can it be shifted away?
- After chiral rotations and shifts in CP-violating phases, phyically invariant parameter remains:

$$\boxed{\bar{\theta} = \theta + \arg \det M}$$

Electric dipole moments

- Electric dipole moments (EDMs) violate CP;

$$\boxed{H = -\boldsymbol{d} \cdot \boldsymbol{E}}$$

- For the neutron, the relevant low-energy operator is $\bar{\boldsymbol{n}} \gamma_5 \sigma_{\mu\nu} \boldsymbol{n}\, \boldsymbol{F}^{\mu\nu}$
- The contribution from $\bar{\theta}$ to the neutron EDM

$$\boxed{|d| = 3.6 \times 10^{16}\, \bar{\theta}\, e\,\mathrm{cm}}$$

- Contributions from other CP violating phase are about $10^{-32}\, e\,\mathrm{cm}$
- Constraints on neutron EDM (Abel et al., 2020) imply that

$$\boxed{\bar{\theta} \lesssim 10^{-10}}$$

Why is the QCD angle so small?

- We almost forgot about $\bar{\theta}$. Turns out it must be tiny anyway. **Weird**
- The effective angle was a sum of two contributions

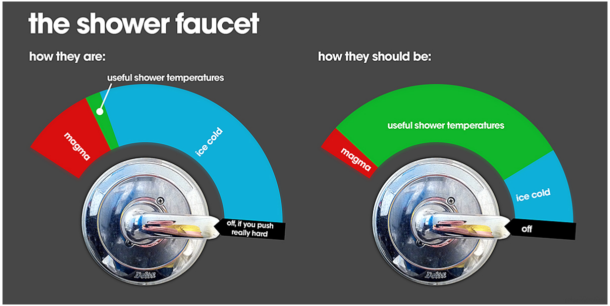
$$\boxed{\bar{\theta} = \theta + \arg\det M}$$

- Why do they cancel precisely or why are they both so small?
- Maybe $\bar{\theta} = 0$? CP broken in nature; nothing special about $\bar{\theta} = 0$
- Life would be much the same if $\bar{\theta} \gg 10^{-10}$. **No obvious anthropics** (see e.g., Dine et al. (2018)) though maybe $\bar{\theta} > 0.1$ spoils nucleosynthesis (Lee et al., 2020)
- Set it and forget it? $\bar{\theta}$ only RG stable in simple models (Hook, 2019)
- Does this **fine-tuning** matter?

Section 2

Fine tuning

Fine-tuning in everyday life

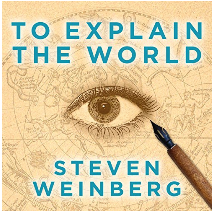


We know that showers that require **fine-tuning** are bad showers!

Fine-tuning in physics

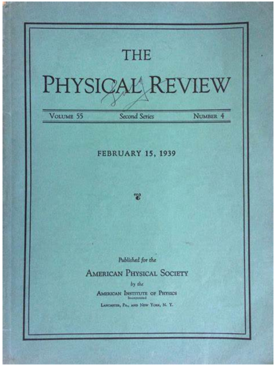
In high-energy physics, a theory is considered **fine-tuned or unnatural** if small variations in its parameters result in dramatic changes in its predictions. For reviews, see Nelson (1985); Giudice (2008); Craig (2023)

Fine-tuning in a scientific theory is like a cry of distress from nature, complaining that something needs to be better explained ***(Weinberg, 2015)***



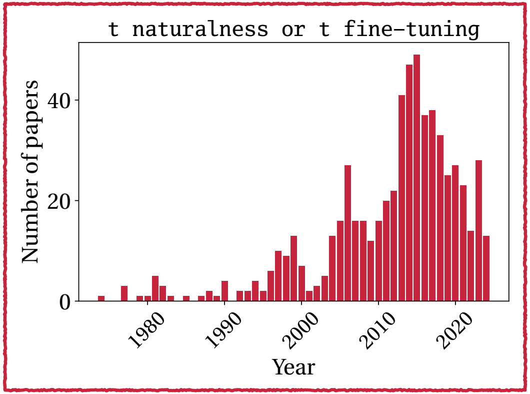
History of fine-tuning

- **1934** — Weisskopf's calculation of electron self-energy (Weisskopf, 1934)
- **1938** — Dirac's large numbers hypothesis (Dirac, 1938)
- **1973** — Wilson understanding of effective field theory (Wilson and Kogut, 1974)
- **1974** — Gaillard and Lee predict charm quark mass (Gaillard and Lee, 1974)
- **1988** — Weinberg makes anthropic argument (Weinberg, 1989)



Popularity of fine-tuning — data from INSPIRE

- **1974** — first hit by Georgi and Pais (1974)
- **1979** — ’t Hooft (1980)
- **1987** — Barbieri-Giudice measure (Barbieri and Giudice, 1988)
- **2000** — fine-tuning at LEP (Kane and King, 1999)
- **2006** — pre-LHC forecasts
- **2010 onward** — LHC-era



Foundations of fine-tuning

By **Bayes' theorem**, for a model M and experimental data D

$$P(M \mid D) = \frac{P(D \mid M)\, P(M)}{P(D)}$$

- $P(M)$ — Prior belief in the model
- $P(M \mid D)$ — Posterior belief after seeing data
- $P(D \mid M)$ — Evidence, which can be written as an integral over a model's parameters for prior $p(\Theta \mid M)$

$$P(D \mid M) = \int P(D \mid M, \Theta)\, p(\Theta \mid M)\, \mathrm{d}\Theta$$

Occam's Razor

Entities should not be multiplied beyond necessity

- Celebrated in physical sciences by e.g., Aristotle, Galileo and Newton (Sober, 2015; McFadden, 2023)
- Criticized in social sciences (Gelman, 2009) and biology (Crick, 1989)

> *Occam's razor is a useful tool in the physical sciences, but it can be dangerous in biology. Physicists may create models that are too neat, too powerful, and too clean.* ***(Crick, 1989)***

Foundations for Occam's Razor

> *Ockham says that we should prefer the simpler [hypothesis] ... intuition assents at once. But this only set the stage for centuries of discussion over precisely what is meant by simplicity ... It is interesting to see the mechanism by which Bayes's theorem usually justifies but in some cases modifies this intuition* ***(Jaynes, 1979)***

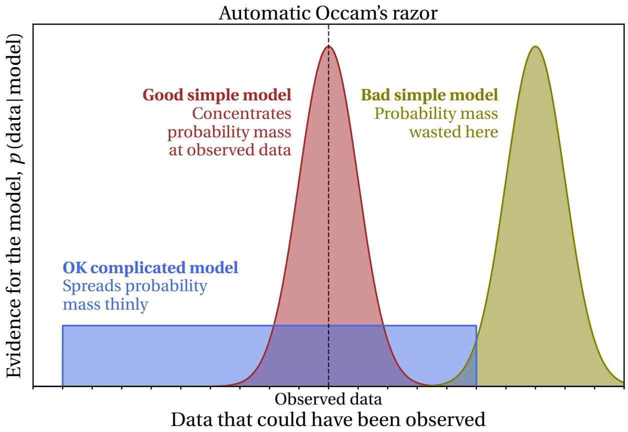
> *The Bayes factor is seen to function as a fully automatic Occam's Razor — cutting back to the simpler model whenever there is nothing to be lost by so doing* ***(Smith and Spiegelhalter, 1980)***

Foundations for Occam's Razor

> *Ockham's razor, far from being merely an ad hoc principle, can under many practical situations in science be justified as a consequence of Bayesian inference* ***(Jefferys and Berger, 1992)***

> *One might think that one has to build a prior over models which explicitly favours simpler models. But as we will see, Occam's Razor is in fact embodied in the application of Bayesian theory* ***(Rasmussen and Ghahramani, 2000)***

Illustration of the Automatic Razor (MacKay, 1991, 2003)



Section 3

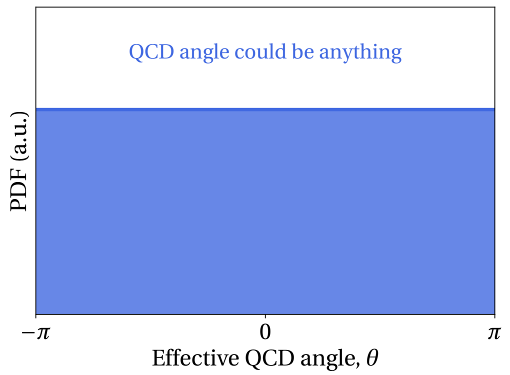
Axion solutions

The CP problem once more

- We have that

$$\bar{\theta} = \theta + \arg\det M$$

- Assuming flat distributions for the angles θ and $\arg\det M$, we predict that QCD angle could be anything
- Wouldn't it be nice to build a model that predicts $\bar{\theta} = 0$?



Solutions to the strong CP problem

There are a few ways forward. For reviews see e.g., Peccei (2008); Hook (2019); Strumia (2025)

- Make the problem trivial by rotation — e.g., if $m_u = 0$, $\bar{\theta}$ can be rotated away ('t Hooft, 1976), but simple $m_u = 0$ approach ruled out by lattice QCD (Davies et al., 2022)
- Fix it in the UV — impose a CP symmetry and break it spontaneously in a controlled way e.g., Nelson (1984); Barr (1984)
- Relax $\bar{\theta} \to 0$ by **dynamics**

$\Rightarrow$ axion solutions

The QCD axion

- Relaxing $\bar{\theta} \to 0$ natural because of Vafa and Witten (1984) theorem tells us that minimum energy state at $\theta = 0$
- However, θ is a parameter, so it cannot relax to that minimum
- We need to **promote** it to a **dynamical** field to relax it — the axion and the Peccei-Quinn mechanism (Peccei and Quinn, 1977; Weinberg, 1978; Wilczek, 1978)

The axion

- Add a pseudo-scalar field, a
- By effective-field theory rules, write down

$$\boxed{\left(\theta + \frac{a}{f}\right) \tilde{G}G}$$

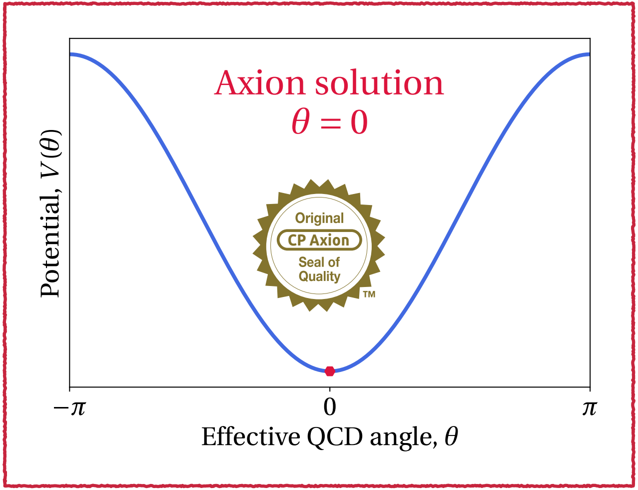
- **Anomalous** global symmetry!

$$\boxed{\theta \to \theta - \alpha \text{ and } a \to a + \alpha f}$$

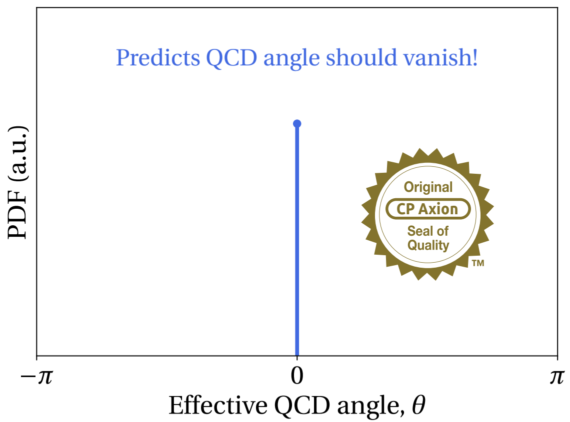
The PQ mechanism

- By anomalous symmetry, anywhere we had $\bar{\theta}$, we now write $a/f + \bar{\theta}$
- Contributions to EDM $\propto a/f + \bar{\theta}$
- By Vafa and Witten (1984) theorem, the ground state now at $a/f + \bar{\theta} = 0$
- Thus dynamical axion field **relaxes** effective QCD angle to zero

Original CP Axion



Axion



Predicts QCD angle should vanish!
Original
CP Axion
Seal of
Quality
TM
PDF (a.u.)
$-\pi$
0
π
Effective QCD angle, θ

Favored by automatic razor

- The QCD axion predicts $\bar{\theta} = 0$
- The SM predicts $\bar{\theta}$ could be anything
- We observe $\bar{\theta} \lesssim 10^{-10}$
- **QCD axion favored by about 10^{10}**

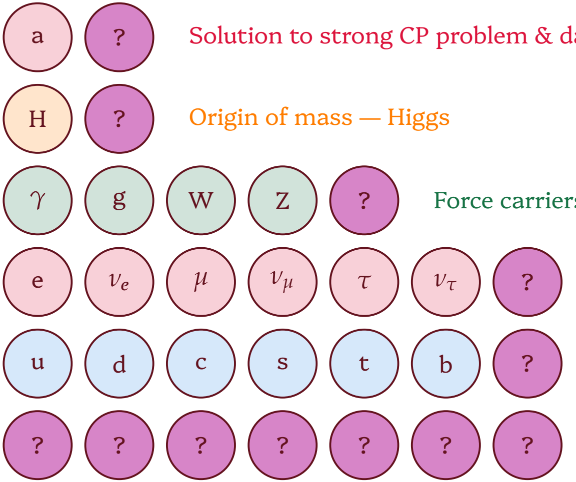
Section 4

Are axion solutions to the CP problem fine-tuned?

Realistic theory

- This was only an EFT
- We can embed in a UV theory by making axion pseudo-Goldstone boson from a complex scalar with a global $U(1)$ symmetry
- What would the automatic razor say here?

Beyond the Standard Model



Solution to strong CP problem & dark matter — axion

Origin of mass — Higgs

Force carriers

Matter — leptons

Matter — quarks

New Physics

Quantum gravity

Quantum gravity breaks all global symmetries

- *Maybe*. Folk theorem but no one actually knows
- What happens to global charge in a black hole? See e.g. Kallosh et al. (1995)
- This means that there are gravitational corrections to the axion potential that break the shift symmetry (Ghigna et al., 1992; Kamionkowski and March-Russell, 1992; Barr and Seckel, 1992)

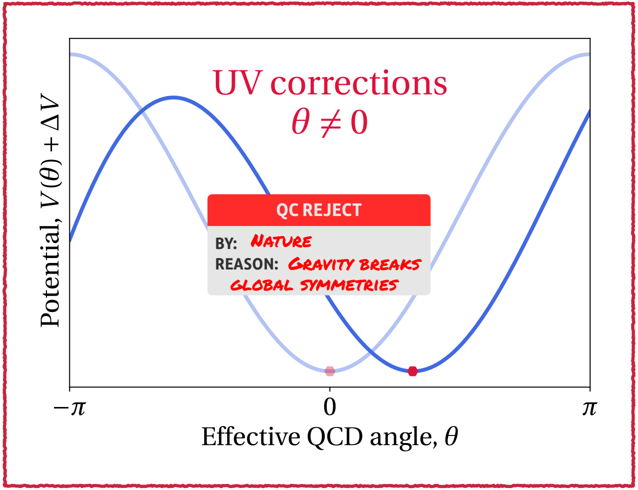
Quality problem

- This is the quality problem — the anomalous shift symmetry in the EFT and PQ-symmetry in a UV theory aren't protected from gravity
- In particular, in an EFT approach we can write higher-order Planck-suppressed operators that spoil the axion potential
- The axion no longer relaxes to $a/f + \bar{\theta} \to 0$

What happened to Vafa and Witten (1984) theorem?

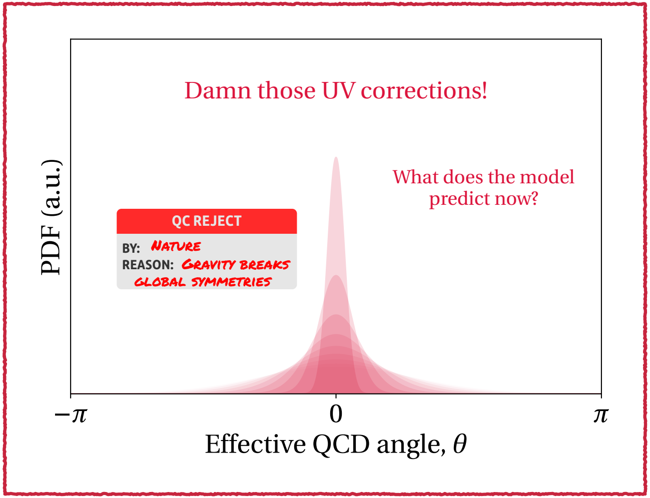
- We must add appropriate dynamics to the Vafa and Witten (1984) result
- Gravity breaks the anomalous shift symmetry
- Axion no longer appears only as $a/f + \bar{\theta}$
- **Axion cannot relax $a/f + \bar{\theta} \to 0$**

Potential with gravity turned on



Predictions with gravity turned on

- This is just a cartoon; what does it really look like?
- Depends on assumptions about Wilson coefficients of Planck-suppressed operators
- Can we compute it?



Is θ a parameter after all?

This is a subtle matter (Dvali, 2006; Kaplan et al., 2025)

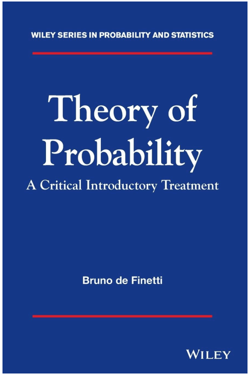
> *[T]he θ term of QCD* ***is not a parameter of the theory*** *that can be set to zero by imposition of symmetries on the Hamiltonian. Rather it reflects a choice of vacuum state or eigenstate of the QCD vacuum ...*
> *In classical physics, the Lagrangian and Hamiltonian are dual descriptions of the same physical system. Parameters of the Lagrangian are also parameters of the Hamiltonian. In quantum mechanics this is not the case* ***(Kaplan et al., 2025)***

Types of Uncertainty (O'Hagan, 2004)

- **Epistemic Uncertainty:** Due to lack of knowledge
  - Can be reduced by gathering more information
  - From Greek *episteme* (knowledge)
- **Aleatory Uncertainty:** Due to randomness and chance
  - Unpredictable, no matter how much information is available
  - From Latin *alea* (a dice game)

Roman dice — probability interpreted as fate & divine intervention from gods

De Finetti’s View on Probability (de Finetti, 2017)



PROBABILITY DOES NOT EXIST

- Probability, if regarded as something with objective existence, is a misleading misconception
- Comparison to superstitions like phlogiston, ether, and absolute space and time
- Objective probabilities are an attempt to materialize our uncertainty
- de Finetti wrote an appendix on quantum theory, but possibly wasn’t aware of Bell’s inequality

Does it matter?

- In $\bar{\theta} = \theta + \arg\det M$, the first term random (from quantum) and the second term unknown
- Unknown and random are **treated identically** in Bayesian formalism. Analysis of fine-tuning unchanged
- If θ isn't a parameter, though, imposing symmetries to forbid it make no sense
- Dynamic solutions, e.g. axion, are unaffected

Conclusions

- Strong CP problem justified in Bayesian formalism
- Extent to which quality problem spoils axion solutions in Bayesian formalism unclear at present — *please don't scoop us*
- Two interpretations of QCD angle θ
  - **Unknown parameter** — we don't know it, our uncertainty is epistemic
  - **Random** — outcome of quantum mechanical measurement, uncertainty is aleatoric
- Doesn't make any difference for understanding quality problem
- Strengthens case for dynamic solutions such as axion

References I

Abel C. and others , (2020). *Measurement of the Permanent Electric Dipole Moment of the Neutron*, *Phys. Rev. Lett.* **124** 081803 [2001.11966].

Barbieri R. and Giudice G. F., (1988). *Upper Bounds on Supersymmetric Particle Masses*, *Nucl. Phys. B* **306** 63.

Barr S. M., (1984). *Solving the Strong CP Problem Without the Peccei-Quinn Symmetry*, *Phys. Rev. Lett.* **53** 329.

Barr S. M. and Seckel D., (1992). *Planck scale corrections to axion models*, *Phys. Rev. D* **46** 539.

Craig N., (2023). *Naturalness: past, present, and future*, *Eur. Phys. J. C* **83** 825 [2205.05708].

Crick F., (1989). *What mad pursuit: A personal view of scientific discovery*. Basic Books.

References II

Davies D., Dine M. and Lehmann B. V., (2022). *Light Quarks at Large N*, [2201.05719].

de Finetti B., (2017). *Theory of probability: A critical introductory treatment*. Wiley, 2nd edition, DOI.

Dine M., (2000). *TASI lectures on the strong CP problem*, in *Theoretical Advanced Study Institute in Elementary Particle Physics (TASI 2000): Flavor Physics for the Millennium*, pp. 349–369, [hep-ph/0011376].

Dine M., Stephenson Haskins L., Ubaldi L. and Xu D., (2018). *Some Remarks on Anthropic Approaches to the Strong CP Problem*, *JHEP* **05** 171 [1801.03466].

Dirac P. A. M., (1938). *New basis for cosmology*, *Proc. Roy. Soc. Lond. A* **165** 199.

References III

Dvali G., (2006). *A Vacuum accumulation solution to the strong CP problem*, *Phys. Rev. D* **74** 025019 [hep-th/0510053].

Gaillard M. K. and Lee B. W., (1974). *Rare Decay Modes of the K-Mesons in Gauge Theories*, *Phys. Rev. D* **10** 897.

Gelman A., (2009). *Bayes, Jeffreys, Prior Distributions and the Philosophy of Statistics*, *Stat. Sci.* **24** [1001.2968].

Georgi H. and Pais A., (1974). *Calculability and Naturalness in Gauge Theories*, *Phys. Rev. D* **10** 539.

Ghigna S., Lusignoli M. and Roncadelli M., (1992). *Instability of the invisible axion*, *Phys. Lett. B* **283** 278.

References IV

Giudice G. F., (2008). *Naturally Speaking: The Naturalness Criterion and Physics at the LHC*, in Kane G. and Pierce A., editors, *Perspectives on LHC Physics*, pp. 155–178. World Scientific, [0801.2562], DOI.

Hook A., (2019). *TASI Lectures on the Strong CP Problem and Axions*, *PoS* **TASI2018** 004 [1812.02669].

Jaynes E. T., (1979). Review of *Inference, Method, and Decision: Towards a Bayesian Philosophy of Science* by R.D. Rosenkrantz, *J. Am. Stat. Assoc.* **74** 740.

Jefferys W. H. and Berger J. O., (1992). *The application of robust Bayesian analysis to hypothesis testing and Occam's razor*, *J. Ital. Stat. Soc.* **1** 17.

References V

Kallosh R., Linde A. D., Linde D. A. and Susskind L., (1995). *Gravity and global symmetries*, *Phys. Rev. D* **52** 912 [hep-th/9502069].

Kamionkowski M. and March-Russell J., (1992). *Planck scale physics and the Peccei-Quinn mechanism*, *Phys. Lett. B* **282** 137 [hep-th/9202003].

Kane G. L. and King S. F., (1999). *Naturalness implications of LEP results*, *Phys. Lett. B* **451** 113 [hep-ph/9810374].

Kaplan D. E., Melia T. and Rajendran S., (2025). *What can solve the Strong CP problem?*, [2505.08358].

Lee D., Meißner U.-G., Olive K. A., Shifman M. and Vonk T., (2020). *θ -dependence of light nuclei and nucleosynthesis*, *Phys. Rev. Res.* **2** 033392 [2006.12321].

References VI

MacKay D. J. C., (1991). *Bayesian Model Comparison and Backprop Nets*, in Moody J., Hanson S. and Lippmann R., editors, *Advances in Neural Information Processing Systems*, volume 4. Morgan-Kaufmann.

MacKay D. J. C., (2003). *Information theory, inference and learning algorithms*. Cambridge University Press, URL.

McFadden J., (2023). *Razor sharp: The role of Occam's razor in science*, *Ann. New York Acad. Sci.* **1530** 8–17.

Nelson A. E., (1984). *Naturally Weak CP Violation*, *Phys. Lett. B* **136** 387.

Nelson P., (1985). *Naturalness in Theoretical Physics: Internal constraints on theories, especially the requirement of naturalness, play a pivotal role in physics*, *Am. Sci.* **73** 60.

References VII

O'Hagan T., (2004). *Dicing with the Unknown*, *Significance* **1** 132–133.

Peccei R. D., (2008). *The Strong CP problem and axions*, *Lect. Notes Phys.* **741** 3 [hep-ph/0607268].

Peccei R. D. and Quinn H. R., (1977). *CP Conservation in the Presence of Instantons*, *Phys. Rev. Lett.* **38** 1440.

Rasmussen C. and Ghahramani Z., (2000). *Occam's Razor*, in Leen T., Dietterich T. and Tresp V., editors, *Advances in Neural Information Processing Systems*, volume 13. MIT Press.

Smith A. F. M. and Spiegelhalter D. J., (1980). *Bayes Factors and Choice Criteria for Linear Models*, *J. R. Stat. Soc. B* **42** 213.

References VIII

Sober E., (2015). *Ockham's razors: a user's manual*. Cambridge University Press.

Strumia A., (2025). *Solving the strong CP problem*, [2501.16427].

't Hooft G., (1976). *Symmetry Breaking Through Bell-Jackiw Anomalies*, *Phys. Rev. Lett.* **37** 8.

't Hooft G., (1980). *Naturalness, chiral symmetry, and spontaneous chiral symmetry breaking*, *NATO Sci. Ser. B* **59** 135.

Vafa C. and Witten E., (1984). *Parity Conservation in QCD*, *Phys. Rev. Lett.* **53** 535.

Weinberg S., (2015). *To Explain the World: The Discovery of Modern Science*. Penguin Books Limited.

Weinberg S., (1978). *A New Light Boson?*, *Phys. Rev. Lett.* **40** 223.

References IX

Weinberg S., (1989). *The Cosmological Constant Problem*, *Rev. Mod. Phys.* **61** 1.

Weisskopf V., (1934). *On the self-energy of the electron*, *Z. Phys.* **89** 27.

Wilczek F., (1978). *Problem of Strong P and T Invariance in the Presence of Instantons*, *Phys. Rev. Lett.* **40** 279.

Wilson K. G. and Kogut J. B., (1974). *The Renormalization group and the epsilon expansion*, *Phys. Rept.* **12** 75.