Small Instantons and the Post-Inflationary QCD Axion in a Special Product GUT

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based on Hor, YN, Suzuki, Xu, arXiv:2504.02033 [hep-ph]

Strong CP Problem

QCD Lagrangian for strong interactions allows

$$\mathcal{L}_\theta = \theta \frac{g_s^2}{32\pi^2} G^{a\mu\nu} \widetilde{G}^a_{\mu\nu}$$

explicitly violating **CP** symmetry.

The physical strong CP phase : $\bar{\theta} \equiv \theta - \arg\det\left(M_u M_d\right)$

The current upper bound on the neutron electric dipole moment

➡ $|\bar{\theta}| < 10^{-11}$

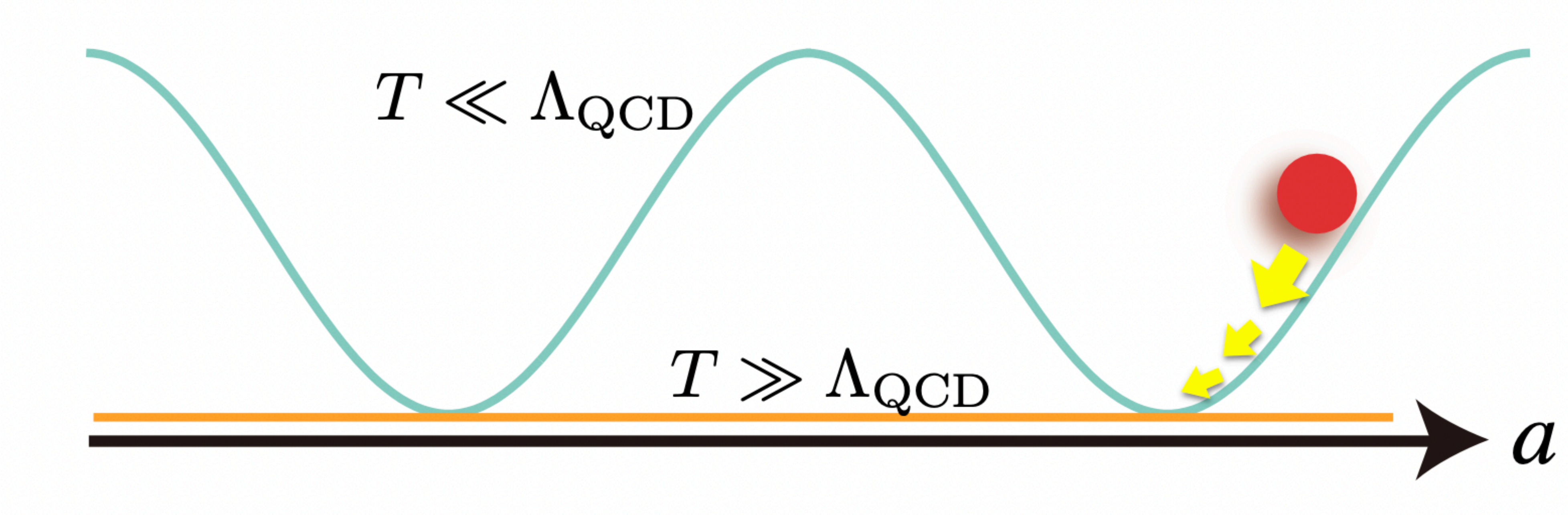
Why is $\bar{\theta}$ so small ??

Some shifts of $\bar{\theta}$ would not provide a visible change in our world.

Axion Solution

The most common explanation is **the Peccei-Quinn mechanism** that the strong CP phase is promoted to a dynamical variable.

$$\mathcal{L}_\theta = \left(\theta + \frac{a}{f_a}\right)\frac{g_s^2}{32\pi^2}G^{a\mu\nu}\widetilde{G}^a_{\mu\nu}$$



Fuminobu Takahashi slide

The **axion *a*** dynamically cancels the strong CP phase !

Axion Solution

- Axion is a pseudo-Nambu-Goldstone boson associated with spontaneous breaking of **a global U(1)$_{\text{PQ}}$ symmetry**.

- Non-perturbative QCD effects break the U(1)$_{\text{PQ}}$ explicitly and generate the axion potential :

$$V(a) \sim m_\pi^2 f_\pi^2 \cos\left(\theta + \frac{a}{f_a}\right)$$

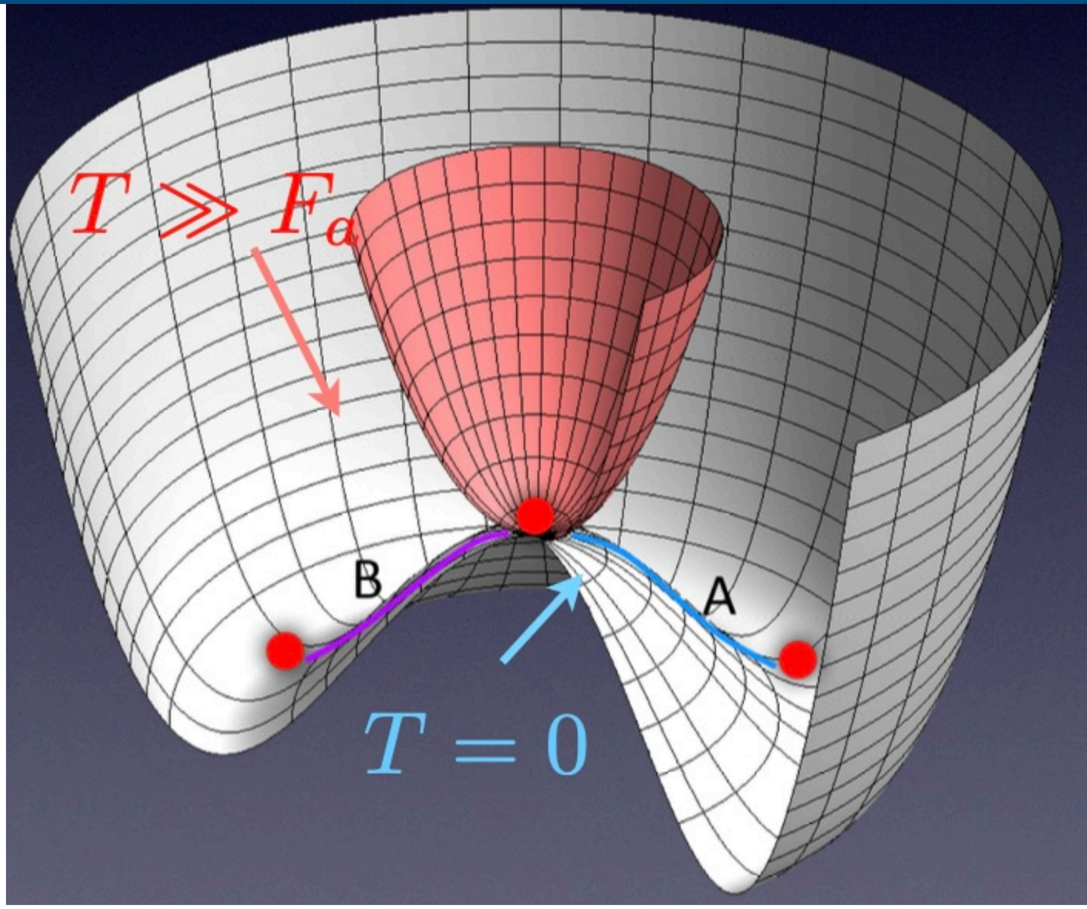
- Axion is a good **dark matter** candidate.

- Cosmological consequences :

★ U(1)$_{\text{PQ}}$ breaking **before** inflation ➡ Isocurvature perturbations

★ U(1)$_{\text{PQ}}$ breaking **after** inflation ➡ Formation of topological defects

Cosmic Strings



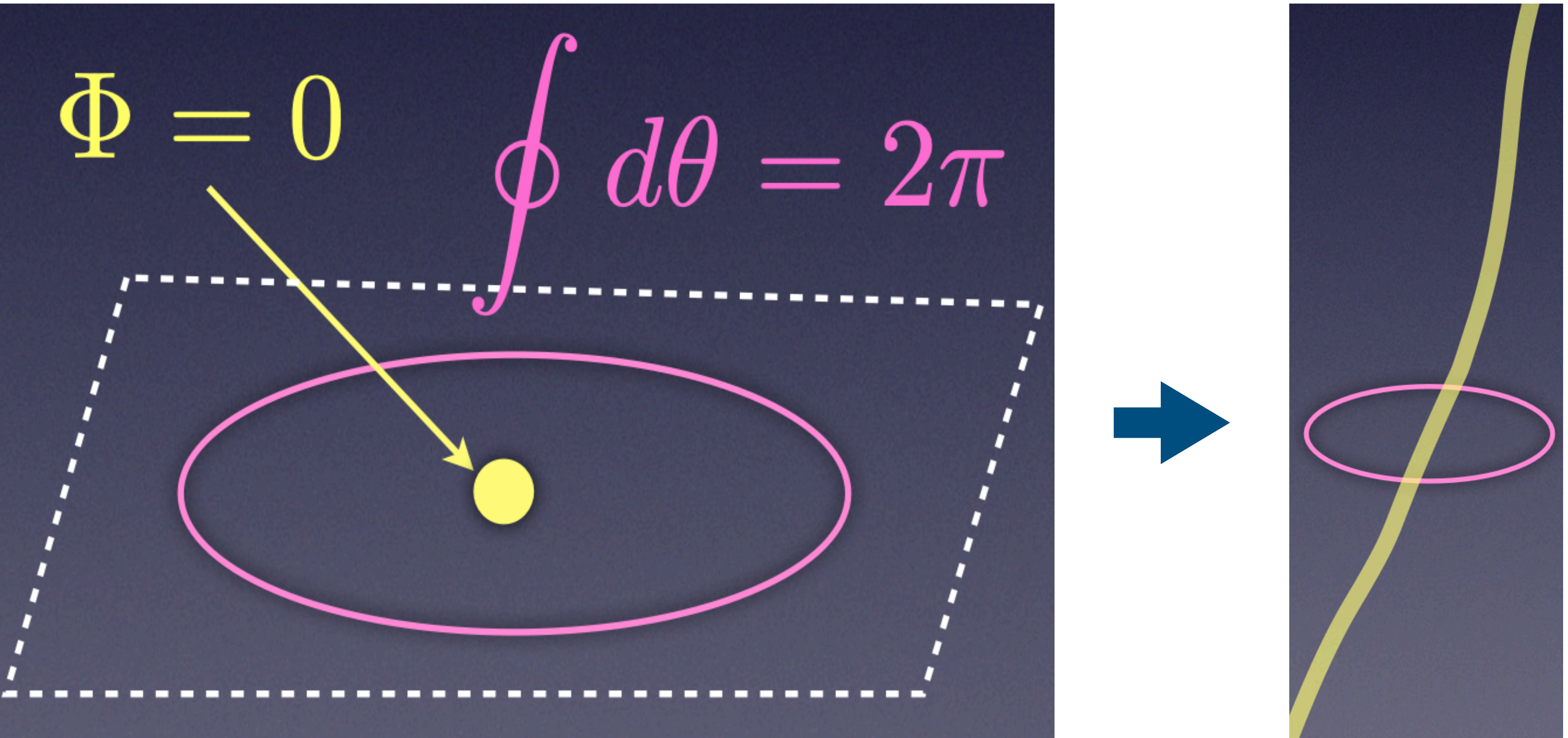
U(1)PQ breaking **after** inflation

Axion is a phase direction of PQ scalar :

$$\Phi_{\rm PQ} \sim v_{\rm PQ} e^{i\theta_a}$$

Axion field acquires spatial variations across the Universe.

Masahiro Kawasaki slide



Formation of **cosmic strings**

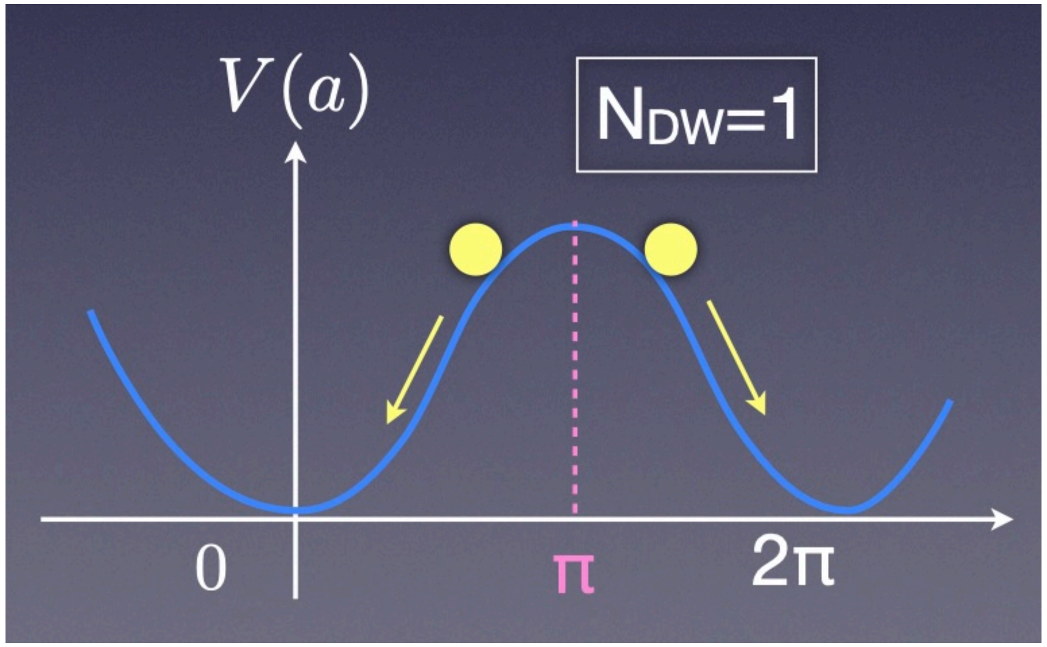
Domain Walls

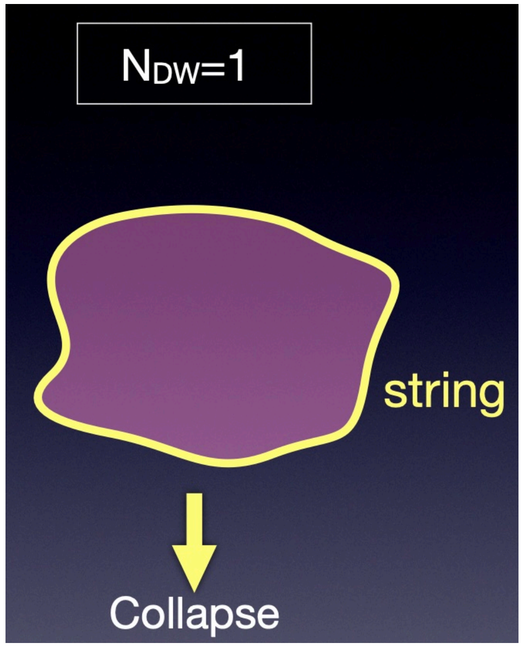
As the Universe expands and cools,
the axion gets a potential with $\underline{N_{\mathrm{DW}}}$ minima.

Domain wall number (associated with color anomaly)

$N_{\mathrm{DW}} = 1$

Domain walls form as disk-like structures attached to strings and eventually collapse due to their tension.





Masahiro Kawasaki slide

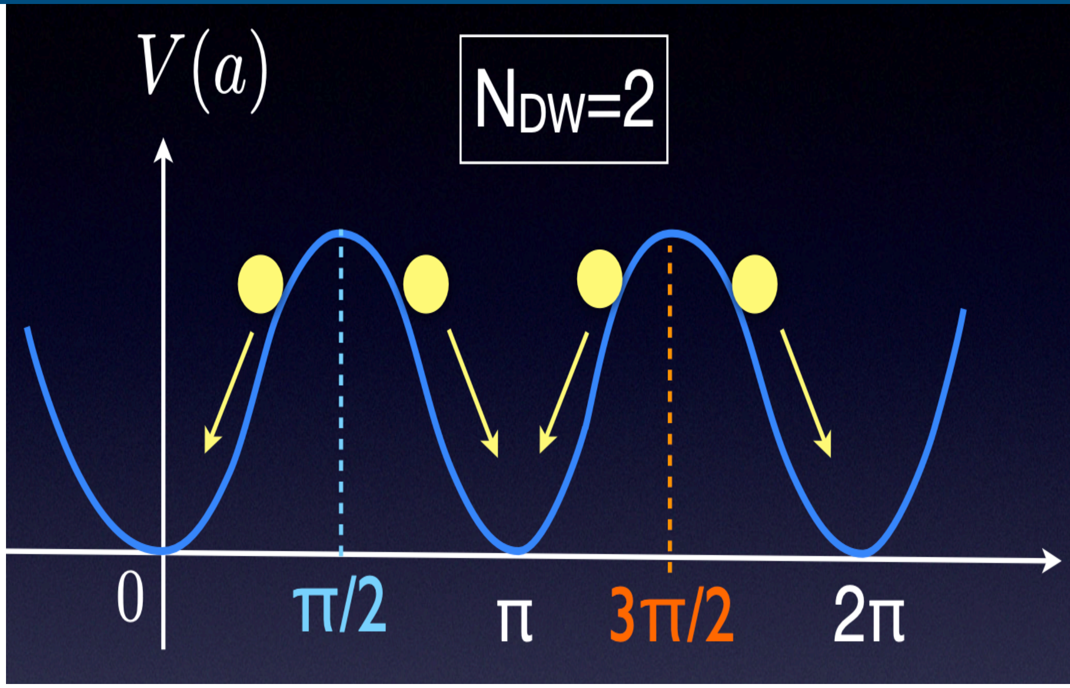
Decay of the string-domain wall network produces a lot of axions which dominate the axion DM abundance.

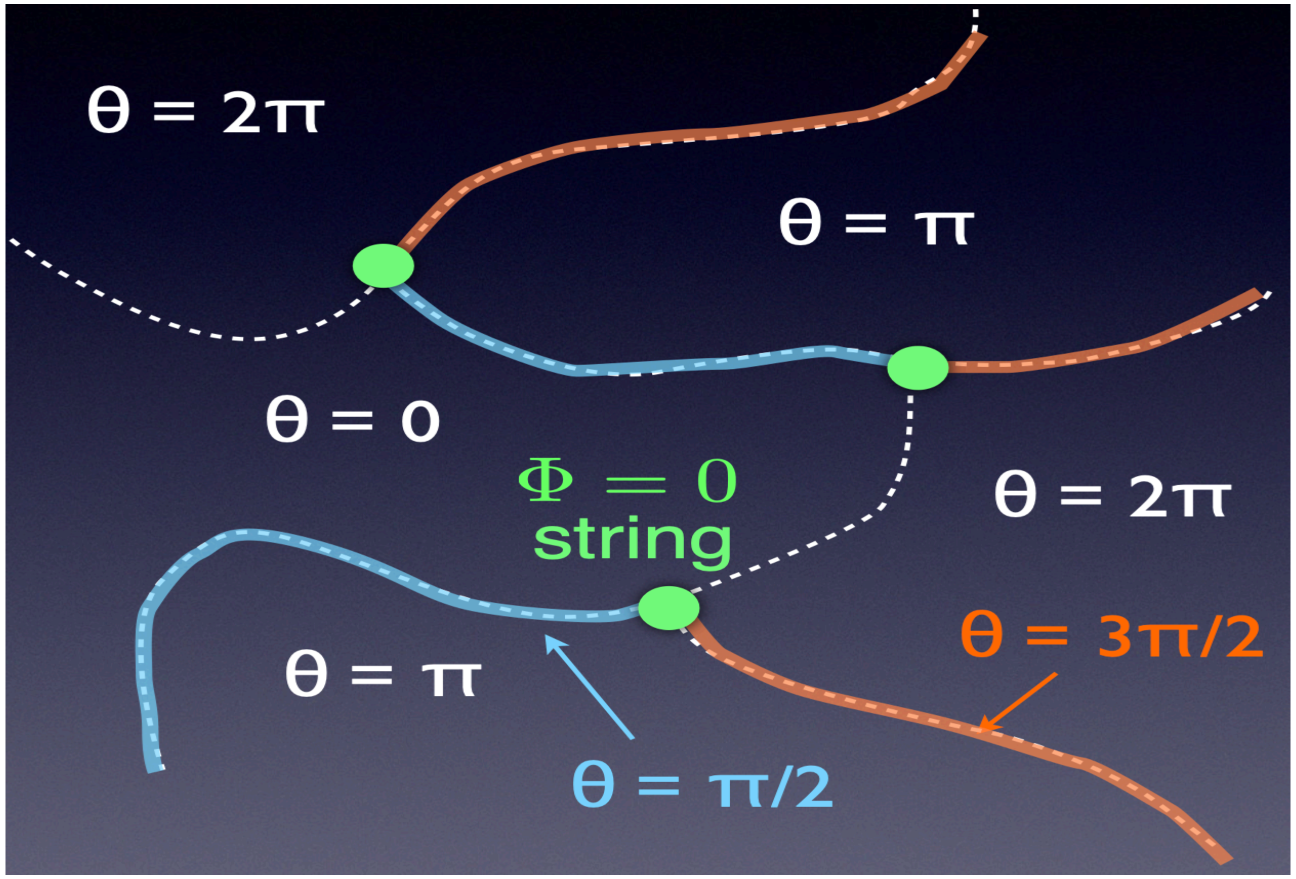
Domain Walls

$N_{DW} > 1$

Domain walls form a **stable** network that eventually dominates the Universe.

→ **Domain wall problem**





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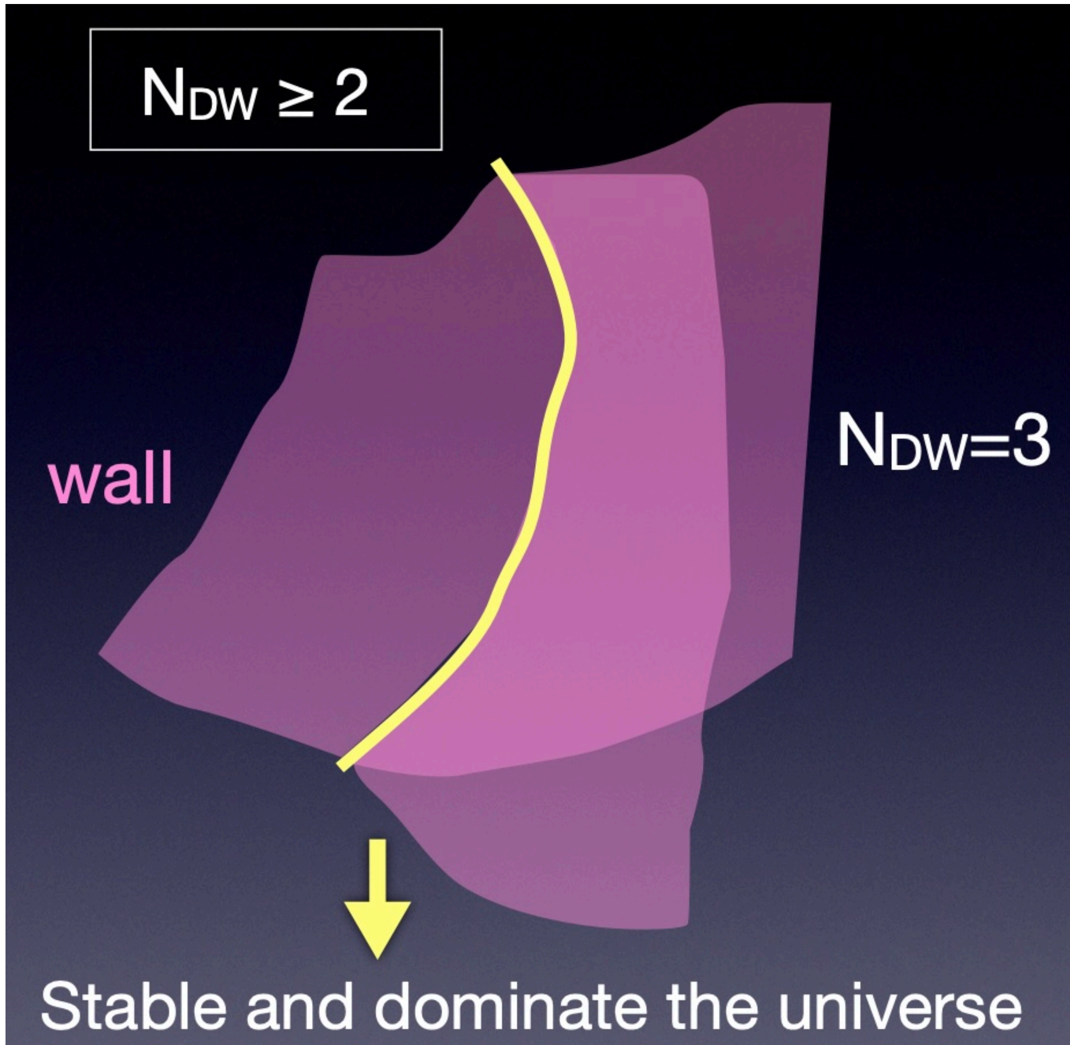
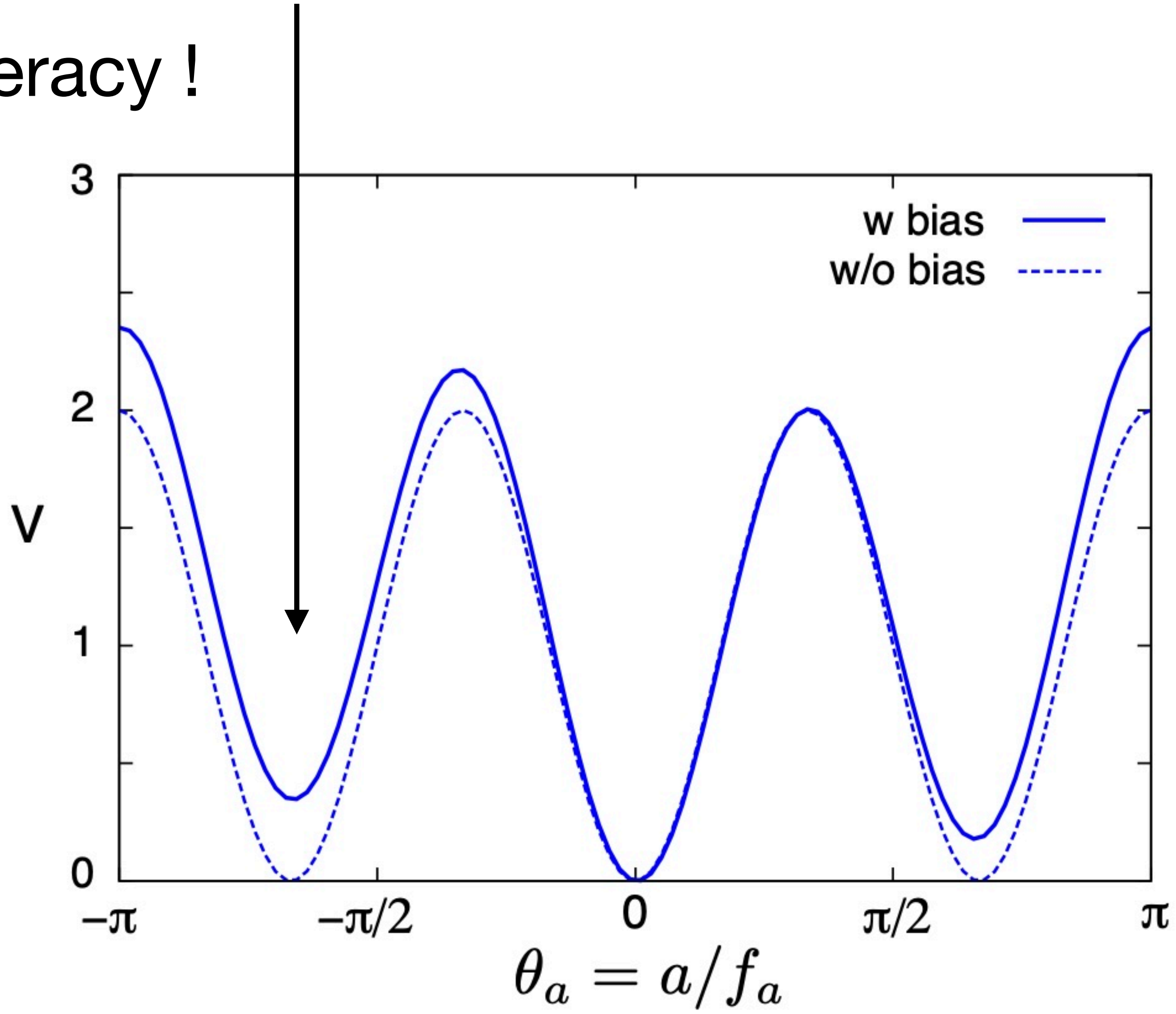


Bias Term

A potential solution to the domain wall problem is to introduce a small explicit breaking of the $U(1)_{PQ}$ symmetry (**Bias term**).

Bias term lifts the vacuum degeneracy !



→ **Domain walls collapse before they dominate the Universe.**

★ Axion DM abundance is changed.

However …

★ **Ad hoc introduction** of the bias term is unsatisfactory.

★ The minimum of the extra potential contribution needs to be **aligned with** that of the QCD potential to solve the strong CP problem.

Small Instanton

QCD θ-vacuum : superposition of *n*-vacua (energy degenerate but topologically distinct)

$$|\theta\rangle = \sum_{n=-\infty}^{\infty} e^{-in\theta}|n\rangle = \cdots + |0\rangle + e^{-i\theta}|1\rangle + \cdots$$

Instanton describes the tunneling effect between degenerate *n*-vacua

Instanton : localized object in Euclidean spacetime, satisfying Euclidean EOM with non-trivial topology and minimize Euclidean action

SU(2) BPST instanton solution with $Q = 1$:

$$\frac{g^2}{32\pi^2}\int d^4x\, F^{a\mu\nu}\widetilde{F}^a_{\mu\nu}\Big|_{\text{inst.}} = Q \quad (Q \in \mathbb{Z})$$

$$A^a_\mu(x)\Big|_{1-\text{inst.}} = 2\eta_{a\mu\nu}\frac{(x-x_0)_\nu}{(x-\underset{\text{Position}}{\underline{x_0}})^2 + \underset{\text{Instanton size}}{\underline{\rho^2}}}$$

Small Instanton

Instantons contribute to the axion potential $\propto \exp\left(-\frac{8\pi^2}{g^2(1/\rho)}\right)$

In QCD, large-size instantons dominate the axion potential due to asymptotic freedom.

A hidden gauge sector beyond QCD

⬇

Small instanton effects ➡ A possible origin of **the bias term** !

The SM gauge group is embedded into a larger UV gauge group.

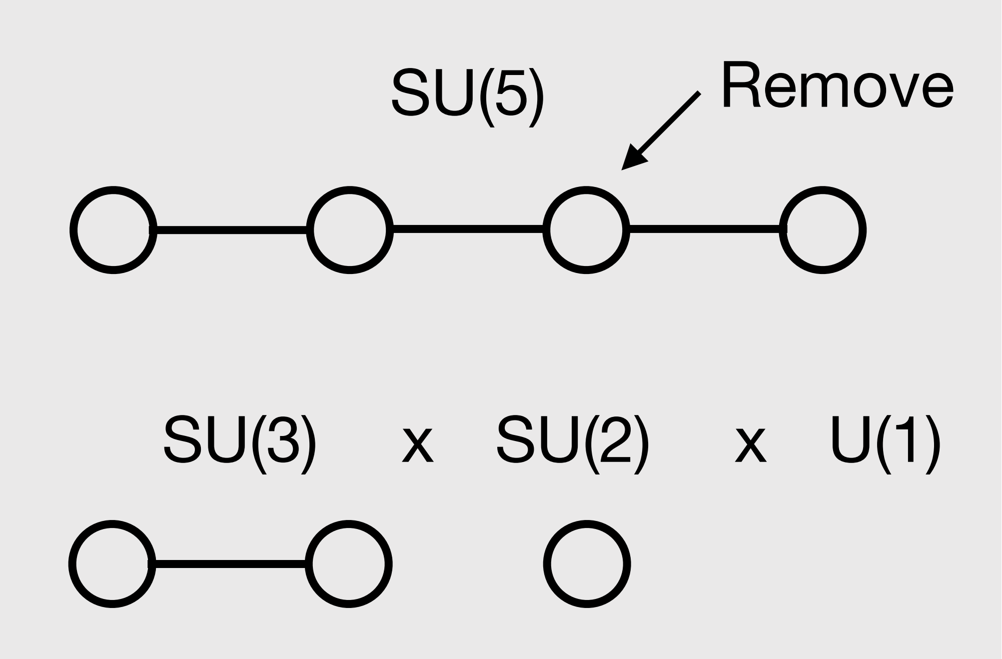
GUT is a natural candidate.

However, a naive embedding into SU(5) GUT does not work because the resulting small instanton effects **do not** lift the vacuum degeneracy.

Special Subalgebra

Simple Lie algebras possess not only **regular** subalgebras but also **special** subalgebras.

Regular subalgebras : systematically obtained by removing nodes from Dynkin diagrams.



Special subalgebras :

do not follow this scheme !

To identify the SM gauge group as such a special subgroup of a UV gauge group is essential to obtain **small instanton effects** resolving the vacuum degeneracy of the axion potential.

Special Product GUT

Our GUT model :

$$\underline{SU(10)} \times SU(5)_1 \to \underline{SU(5)_V} \supset SU(3)_C \times SU(2)_L \times U(1)_Y$$

$\supset SU(5)_2$: **Special embedding**

Diagonal subgroup

- All SM matter fields are charged under SU(5)$_1$.

- **A vector-like pair of PQ-charged fermions** transform as (anti-)fundamental reps. under SU(10), so that N_{DW} = 1.

- After GUT breaking, # of vector-like pairs of PQ-charged quarks is larger than one, due to special embedding.

  Domain wall problem ??

- The apparent vacuum degeneracy is lifted by **small instanton effects** on the axion potential that operates as a PQ-violating bias term.

Special Embedding

We focus on a gauge symmetry breaking : $SU(2N) \to SU(N)$

$T^m_{\rm UV}$ $(m = 1, ..., (2N)^2 - 1)$: SU(2N) generators

$T^a_{\rm IR}$ $(a = 1, ..., N^2 - 1)$: SU(N) generators

$$\boxed{T^a_{\rm IR} = \underbrace{\mathcal{O}^{am}}_{\text{Coefficients}} T^m_{\rm UV}}$$

r rep. of SU(N) is embedded into the fundamental rep. of SU(2N)

$$\mathrm{tr}(T^m_{\rm UV} T^n_{\rm UV}) = \frac{1}{2}\delta^{mn} \qquad \mathrm{tr}(T^a_{\rm IR} T^b_{\rm IR}) = \underbrace{T_{\rm IR}(\mathbf{r})}_{\text{Dynkin index}}\delta^{ab}$$

➡ $\mathcal{O}^{am}\mathcal{O}^{bn}\delta_{mn} = c\delta^{ab}$ $\boxed{c \equiv \frac{T_{\rm IR}(\mathbf{r})}{1/2}}$

Special embedding corresponds to c > 1.

Special Embedding

Consider **a Weyl fermion** that transforms as the fundamental rep. of SU(2N) but behaves as the **r** rep. of the SU(N) subgroup :

$$\mathcal{D}_\mu \psi = \partial_\mu \psi - i g_{\mathrm{UV}} A^m_{\mathrm{UV},\mu} (T^m_{\mathrm{UV}}) \psi$$

$$\supset \partial_\mu \psi - i g_{\mathrm{IR}} A^a_{\mathrm{IR},\mu} (T^a_{\mathrm{IR}}) \psi$$

A part of SU(2N) gauge field is expressed in terms of SU(N) gauge field :

$$A^l_{\mathrm{UV},\mu} = \frac{g_{\mathrm{IR}}}{g_{\mathrm{UV}}} \underline{A^a_{\mathrm{IR},\mu}} (\mathcal{O})^{al}$$

Canonically normalized kinetic term → $\boxed{g_{\mathrm{IR}} = g_{\mathrm{UV}} / \sqrt{c}}$

In our GUT model, **r** = **10** rep. of $SU(5) \subset SU(10)$ leading to **c = 3**.

Theta term : $\int \frac{g^2_{\mathrm{UV}}}{8\pi^2} \mathrm{tr}(F_{\mathrm{UV}} \wedge F_{\mathrm{UV}}) = \int \frac{c g^2_{\mathrm{IR}}}{8\pi^2} \mathrm{tr}(F_{\mathrm{IR}} \wedge F_{\mathrm{IR}})$

SU(10) x SU(5)

To achieve $SU(10) \times SU(5)_1 \to SU(5)_V$ we introduce a Higgs field :

Φ_a^{ij} : $(\mathbf{10},\ \overline{\mathbf{10}})$

$a\,(=1-10)$: SU(10) index

$i, j\,(=1-5)$: SU(5)$_1$ indices

VEV of Φ is described by the embedding of the $\overline{\mathbf{10}}$ rep. of SU(5)$_1$ into the anti-fundamental rep. of SU(10) :

$$\langle \Phi \rangle = v\mathbf{1}_{10\times 10}$$

A further breaking of $SU(5)_V \to SU(3)_C \times SU(2)_L \times U(1)_Y$ is induced by the VEV of an additional Higgs field in the **24** rep. of SU(5)$_1$.

Field	Spin	$SU(10)$	$SU(5)_1$	$U(1)_{\mathrm{PQ}}$	$U(1)_\eta$
Φ	0	$\mathbf{10}$	$\overline{\mathbf{10}}$	0	0
$\mathbf{24}_H^{(1)}$	0	$\mathbf{1}$	$\mathbf{24}$	0	0

SU(10) x SU(5)

Three generations of the SM quarks and leptons and the Higgs field are charged under SU(5)₁.

Field	Spin	$SU(10)$	$SU(5)_1$	$U(1)_{\rm PQ}$	$U(1)_\eta$
$\mathbf{10}_f^{(1)} (f = 1 - 3)$	$1/2$	$\mathbf{1}$	$\mathbf{10}$	0	0
$\bar{\mathbf{5}}_f^{(1)} (f = 1 - 3)$	$1/2$	$\mathbf{1}$	$\bar{\mathbf{5}}$	0	0
$\mathbf{5}_H^{(1)}$	0	$\mathbf{1}$	$\mathbf{5}$	0	0

The PQ mechanism is implemented by a PQ breaking field and **a vector-like pair of PQ-charged fermions** (**KSVZ fermions**) that transform as (anti-)fundamental reps. under SU(10).

Field	Spin	$SU(10)$	$SU(5)_1$	$U(1)_{\rm PQ}$	$U(1)_\eta$
ψ	$1/2$	$\mathbf{10}$	$\mathbf{1}$	$+1$	-1
$\bar{\psi}$	$1/2$	$\overline{\mathbf{10}}$	$\mathbf{1}$	0	$+1$
$\Phi_{\rm PQ}$	0	$\mathbf{1}$	$\mathbf{1}$	-1	0

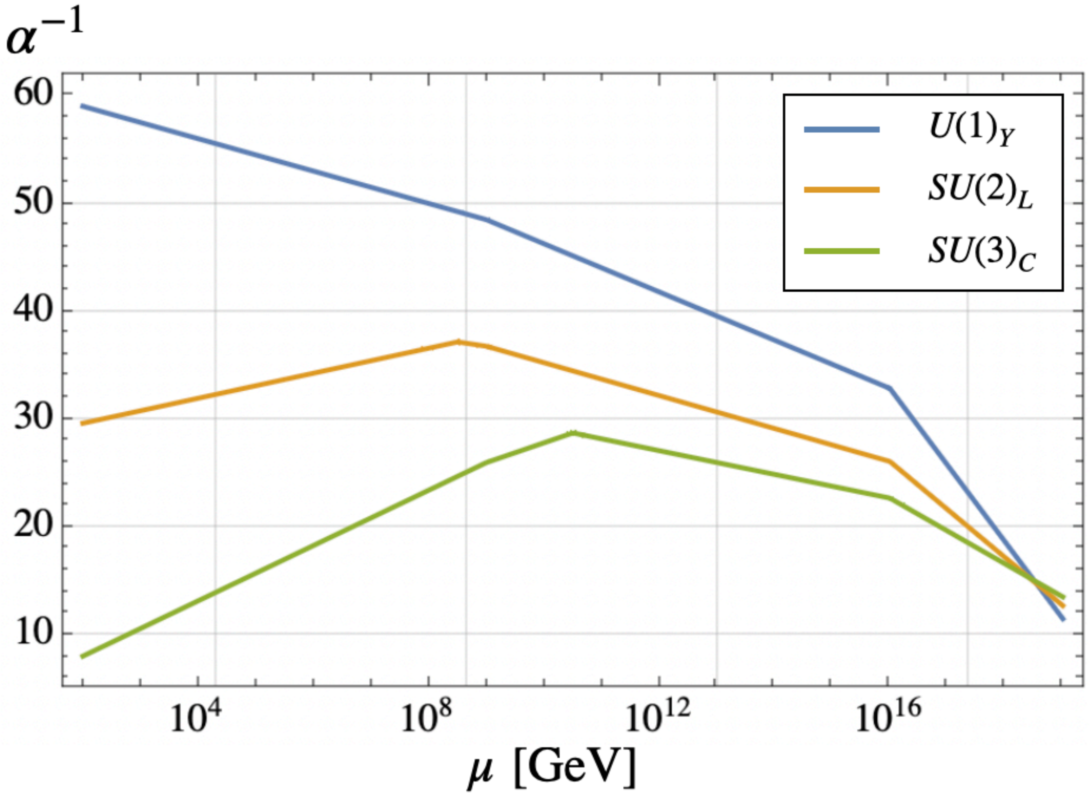
→ **$N_{\rm DW} = 1$**

SU(10) x SU(5)

In non-SUSY models, a gauge coupling unification is not automatic.

To achieve unification, we introduce **extra light fermions**.

Embedded into GUT multiplets



One-loop RGE

Field	Spin	$SU(10)$	$SU(5)_1$	$U(1)_{\rm PQ}$	$U(1)_\eta$
$\Psi_{99,f'}$ $(f'=1-4)$	$1/2$	$\mathbf{99}$	$\mathbf{1}$	0	0
$\Psi^{(1)}_{75,f'}$ $(f'=1-4)$	$1/2$	$\mathbf{1}$	$\mathbf{75}$	0	0

Under $SU(5) \subset SU(10)$ $\quad \mathbf{99} = \mathbf{75} \oplus \underline{\mathbf{24}}$

All components except the **SU(2)L triplet and SU(3)c octet** acquire large masses.

Spontaneous CPV

The minimum of the axion potential generated by small instanton effects needs to be aligned with that of non-perturbative QCD effects.

Spontaneous CP violation

We introduce complex scalar fields with $\arg(\langle\eta_\alpha\rangle) = \mathcal{O}(1)$

Field	Spin	$SU(10)$	$SU(5)_1$	$U(1)_{\rm PQ}$	$U(1)_\eta$
$\eta_\alpha (a = 1, 2)$	0	$\mathbf{1}$	$\mathbf{1}$	0	-1

Forbid dangerous terms at the classical level.

To reproduce the CKM phase, η couple to the mixing term between the KSVZ fermion sector and the SM sector :

$$\mathcal{L} \sim \Phi_{\rm PQ}\psi\bar{\psi} + \sum_{f=1-3,\alpha=1,2} a^u_{\alpha f}\eta_\alpha \bar{\psi}^a (\Phi)^{ij}_a \mathbf{10}^{(1)}_{ij,f}$$

$$+ y^u_{ff'}\mathbf{10}^{(1)}_f \mathbf{10}^{(1)}_{f'}\mathbf{5}_H + y^d_{ff'}\mathbf{10}^{(1)}_f \bar{\mathbf{5}}^{(1)}_{f'}\mathbf{5}^\dagger_H$$

All coefficients are real.

Spontaneous CPV

The setup is similar to the **Nelson-Barr mechanism**.

Up-type quark mass matrix :

$$\mathcal{L} \sim (\underset{\substack{\uparrow \\ \mathbf{10}_f^{(1)}}}{\underline{q_{u\,f}}}\ \ \underset{\substack{\uparrow \\ \bar{\psi}}}{\underline{U}}\ \ \underset{\substack{\uparrow \\ \psi}}{\underline{Q_u}})\mathcal{M}_u \begin{pmatrix} \bar{u}_{f'} \\ \bar{U} \\ \bar{Q}_u \end{pmatrix} \qquad \mathcal{M}_u = \begin{pmatrix} (m_u)_{f\,f'} & 0 & A^* \\ A^\dagger & v_{\mathrm{PQ}} & 0 \\ 0 & 0 & v_{\mathrm{PQ}} \end{pmatrix}$$

$$A^* = \sum_\alpha a^u_{\alpha f}\eta_\alpha \qquad (m_u)_{f\,f'} \equiv y^u_{f\,f'} v_{\mathrm{SM}}$$

O(1) CKM phase is properly generated when $(a^u\langle\eta\rangle)_f \gtrsim v_{\mathrm{PQ}}$

Since <u>determinant is real</u>, the physical θ-parameters of SU(10), SU(5)$_1$ or SU(3)$_C$ <u>vanish at the tree-level</u>.

Radiative corrections can still generate nonzero corrections.

Axion Potential

- QCD effects

Theta terms @ UV :

$$\int \frac{\theta_{10} g_{10}^2}{8\pi^2} \mathrm{tr}(F_{10} \wedge F_{10}) + \frac{\theta_5 g_5^2}{8\pi^2} \mathrm{tr}(F_5 \wedge F_5)$$

After the breaking of $SU(10) \times SU(5)_1 \to SU(5)_V$

Theta term @ IR : $\int \frac{\theta_c g^2}{8\pi^2} \mathrm{tr}(F \wedge \underline{F})$

$SU(5)_V$ gauge field

$$\frac{1}{g^2} = \frac{\boxed{3}}{g_{10}^2} + \frac{1}{g_5^2} \qquad \theta_c = \underline{3}\theta_{10} + \theta_5$$

Special embedding

Axion : $\Phi_{\mathrm{PQ}} \sim v_{\mathrm{PQ}} e^{i\underline{\theta_a}}$

Axion Potential

- **QCD effects**

A vector-like pair of SU(2)L doublet KSVZ (anti-)quarks and a pair of SU(2)L singlet (anti-)quarks appear after GUT breaking.

Axion coupling to gluons :

$$\int (3\theta_a + \theta_c) \frac{g^2}{8\pi^2} \mathrm{tr}(G \wedge G)$$

Pion mass and decay constant

$$\Rightarrow \quad V_{\mathrm{QCD}} \approx -\frac{m_u m_d}{(m_u + m_d)^2} \underline{m_\pi^2 f_\pi^2} \cos\left(3\theta_a + \underline{\bar{\theta}_c}\right)$$

Physical phase

A contribution from radiative corrections with spontaneous CPV : $\bar{\theta}_c \ll 10^{-2}$

Axion potential seems to correspond to the case with **NDW = 3.**

Axion Potential

- **Small instanton effects**

The instanton effects can be captured by **a local fermion operator**.

★ One flavor of KSVZ fermions in the (anti-)fundamental reps. of SU(10)

★ Four flavors of Weyl fermions in the **99** rep. of SU(10)

We assume an approximate chiral symmetry :

$$\Psi_{99} \to \Psi_{99} e^{i\beta}, \quad \Psi_{75}^{(1)} \to \Psi_{75}^{(1)} e^{-i\beta}$$

➡ Mass terms are suppressed by a small parameter κ ≪ 1 :

$$\mathcal{L} \sim \kappa^2 M (\Psi_{99})^a_b (\Psi_{99})^b_a + \kappa^{\dagger 2} \underline{M} (\Psi_{75}^{(1)})^{kl}_{ij} (\Psi_{75}^{(1)})^{ij}_{kl}$$

GUT scale $M \sim M_{\mathrm{Pl}}$

24 multiplet within **99** acquires a mass of $\mathcal{O}(\kappa^2 M)$

Axion Potential

- **Small instanton effects**

Instanton NDA

Csaki, D’Agnolo, Kuflik, Ruhdorfer (2024)

$$V_{\rm bias} \approx C_{10}\left(\frac{2\pi}{\alpha_{\rm UV}(M)}\right)^{2\times 10}(\Phi_{\rm PQ}+\Phi^*_{\rm PQ})$$

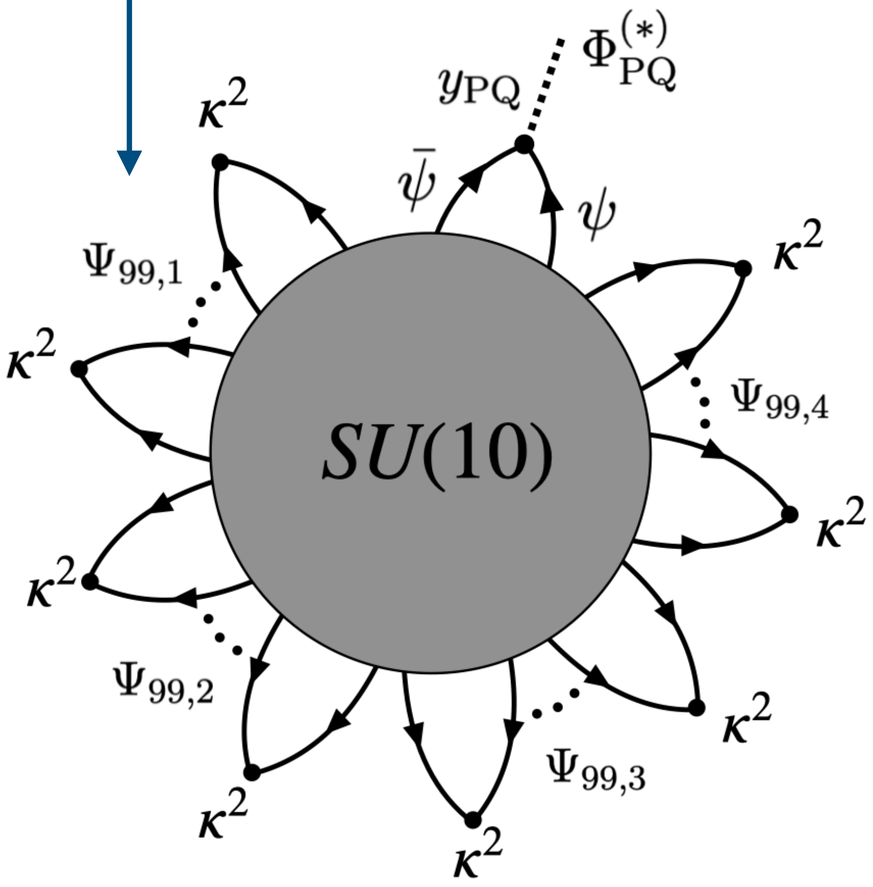
$$\times \int \frac{d\rho}{\rho^5}(\Lambda_{SU(10)}\rho)^{b_0}\,\underline{e^{-2\pi^2\rho^2M^2}}\,y_{\rm PQ}(\kappa^2 M\rho)^{10N_F}\rho$$

$$\approx (\kappa^2)^{10N_F} C_{10}\left(\frac{2\pi}{\alpha_{\rm UV}(M)}\right)^{2\times 10} e^{-2\pi^2}$$

$$\times \frac{\Phi_{\rm PQ}}{M} M^4 e^{-2\pi/\alpha_{\rm UV}(M)} + c.c.$$

Suppression originating from SU(10) breaking

Each flavor of Ψ_{99} has 2T(Adj) = 20 legs closed by 10 mass vertices.



’t Hooft vertex

$$\Lambda^{b_0}_{SU(10)} = M^{b_0} e^{-\frac{8\pi^2}{g^2_{\rm UV}(M)}}$$

b_0 : one-loop beta function coefficient

C_{10} : SU(10) instanton density

$y_{\rm PQ}$: Yukawa coupling of $\Phi_{\rm PQ}$ and KSVZ fermions

Axion Potential

- **Small instanton effects**

We use the following estimate in our analysis :

$$V_{\rm bias} = 3\times 10^2 (\kappa^2)^{40} \frac{e^{-2\pi/\alpha_{\rm UV}}}{\alpha_{\rm UV}^{20}} e^{-2\pi^2} M^3 \Phi_{\rm PQ} + c.c.$$

$$= 6\times 10^2 \epsilon \frac{e^{-2\pi/\alpha_{\rm UV}}}{\alpha_{\rm UV}^{20}} e^{-2\pi^2} M^3 v_{\rm PQ} \cos(\theta_a) \qquad \epsilon \equiv (\kappa^2)^{40}$$

This axion potential corresponds to the case with **$N_{\rm DW} = 1$**.

cf. QCD effects

$$V_{\rm QCD} \approx -\frac{m_u m_d}{(m_u+m_d)^2} m_\pi^2 f_\pi^2 \cos\left(3\theta_a + \bar{\theta}_c\right)$$

Axion potential from small instanton effects provides a bias term !

Post-Inflation Axion

PQ symmetry is spontaneously broken after reheating.

We focus on the scenario where the axion field starts to oscillate due to the axion mass originated from **the bias term** :

$$m_{\rm bias}^2 \equiv \frac{\partial^2 V_{\rm bias}/\partial\theta_a^2}{v_{\rm PQ}^2}$$

Temperature when the oscillation starts :

$$m_a(T_1) \approx m_{\rm bias} = 3H(T_1) \qquad H(T_1)^2 \approx \frac{\pi^2}{90 M_{\rm Pl}^2} g_* T_1^4$$

$$T_1 > 0.98\,{\rm GeV}\left(\frac{v_{\rm PQ}/3}{10^{12}\,{\rm GeV}}\right)^{-0.19} \equiv \underline{T_{1,{\rm QCD}}}$$

Temperature when the axion would start to oscillate with non-perturbative QCD effects if there was no bias term

Domain walls decay in a similar way as the standard $N_{\rm DW} = 1$ case.

Post-Inflation Axion

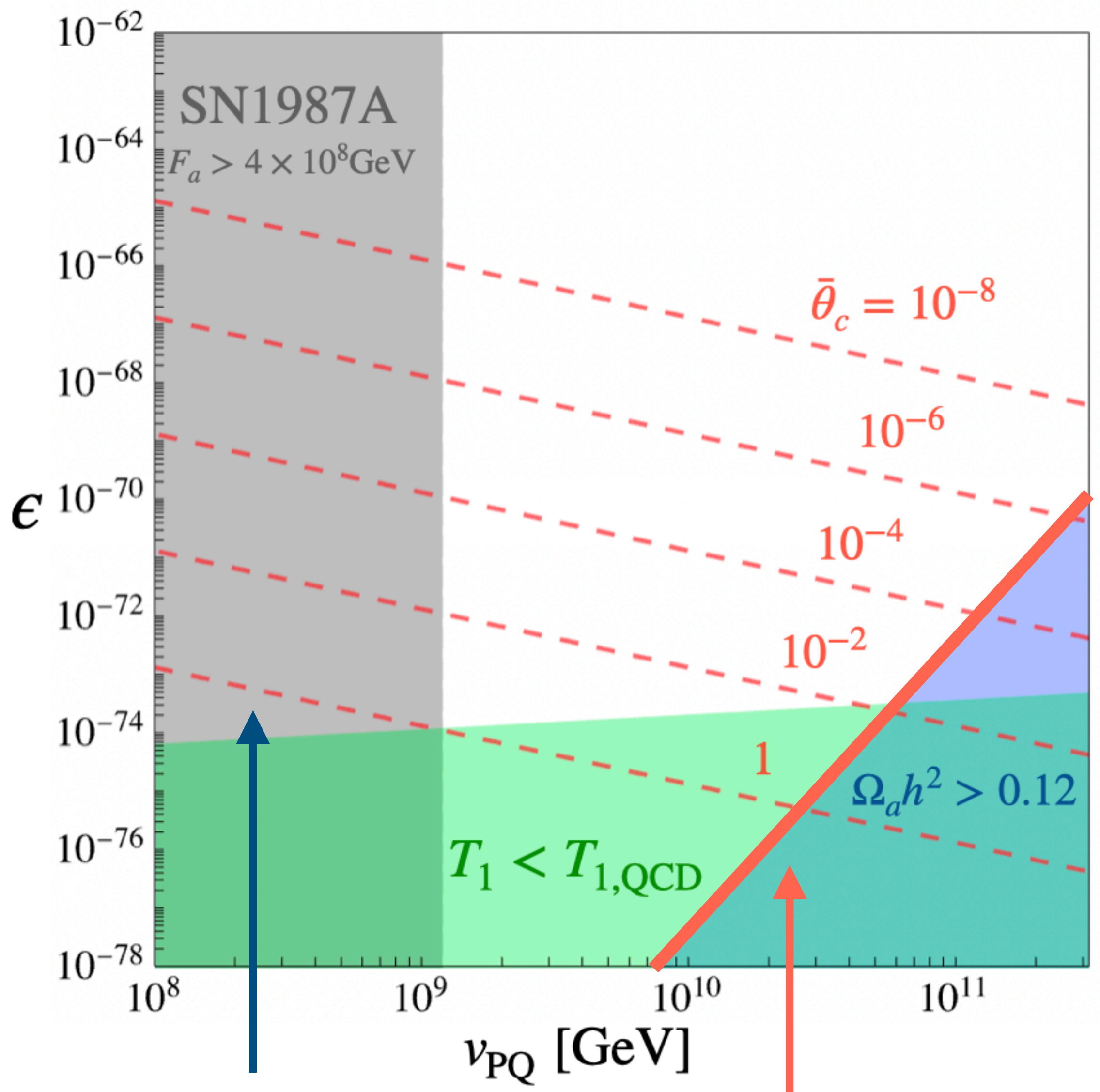
Axion abundance :

$$\Omega_a h^2 \approx 2 \times 10^{-12} \frac{v_{\rm PQ}}{T_1}$$

A large bias term shifts the axion potential minimum :

$$\Delta\bar{\theta} \equiv \frac{m_{\rm bias}^2 v_{\rm PQ}^2}{m_a^2 F_a^2} \bar{\theta}_c$$

Neutron EDM ➡ $\Delta\bar{\theta} \lesssim 10^{-10}$



$$\Delta\bar{\theta} = 10^{-10}$$

Correct axion DM abundance

A regime explored by ongoing and future experiments ! → $v_{\rm PQ} \gtrsim 6 \times 10^{10}\,{\rm GeV}$

Summary

- We have presented **a special product GUT** model equipped with a viable post-inflationary QCD axion.
- The model includes a vector-like pair of KSVZ fermions with NDW = 1.
- After GUT breaking, # of vector-like pairs of PQ-charged quarks is larger than one, which seems to encounter the domain wall problem.
- **Small instanton effects** on the axion potential operate as a PQ-violating bias term and allow the decay of domain walls.
- We have achieved a domain-wall-free UV completion for an IR model where NDW appears larger than one.
- The model gives a prediction for a dark matter axion window different from the ordinary NDW = 1 case.

Thank you.