PQ quality and scale hierarchy of extra-dimensional axions

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The 4-th International Conference on Axion Physics and Experiment (Axion 2025), Nanjing, July 28, 2025

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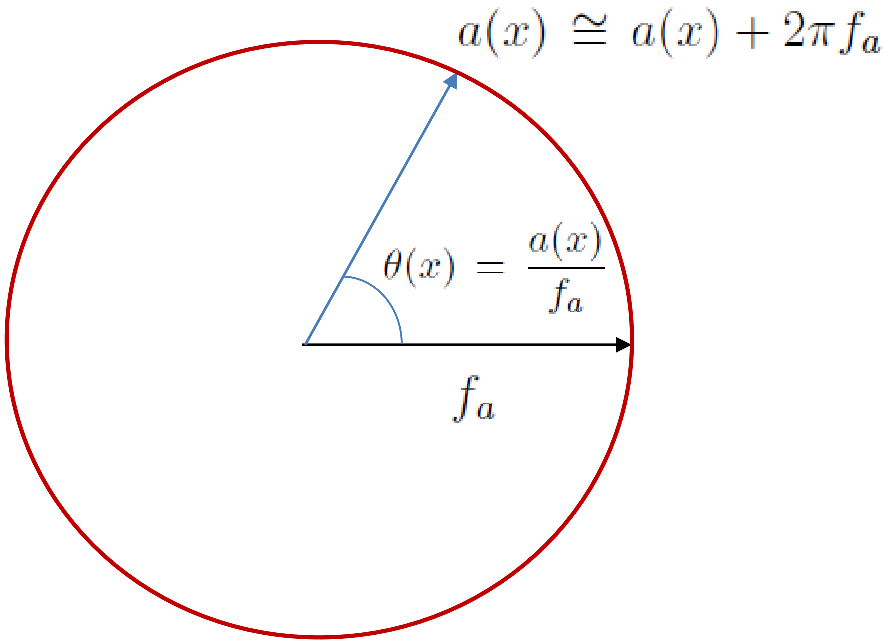
Axions are light pseudo-scalar bosons which have been postulated with many good motivations:

Strong CP problem, Dark matter, Axiogenesis, …

Also axions are ubiquitous in compactified string theory which is perhaps the best candidate for a theory incorporating both the SM of particle physics and quantum gravity .

Axions are described by a periodic field variable, so characterized by a mass scale f_a (=axion decay constant or axion scale) defining the radius of the circular field space:



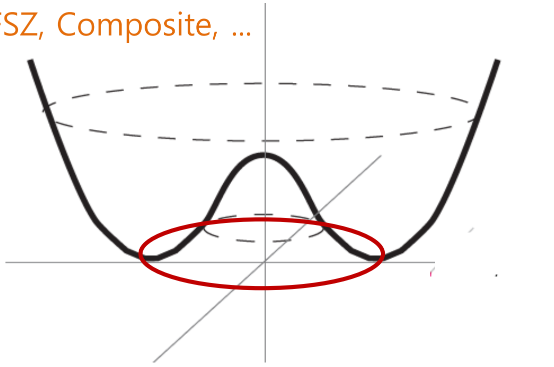
Angular nature of the axion field implies that all axion couplings involving a single axion field are inversely proportional to f_a .

Two types of axions

Field-theory axion (FTA) from the phase of complex scalar fields (either elementary or composite) Peccei & Quinn '77; Weinberg '78; Wilczek '78

KSVZ, DFSZ, Composite, ...

$$\sigma(x) = \rho(x) e^{ia(x)/f_a} \quad \left(\frac{f_a}{\sqrt{2}} = \langle \rho \rangle \right)$$

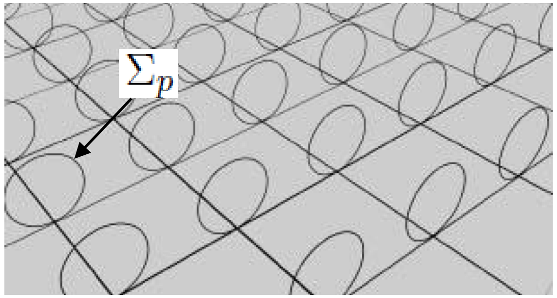


Extra-dimensional axion (EDA) from a p-form gauge field C_p in higher-dim theory compactified on internal space involving a closed p-dim surface Σ_p, e.g. string theory axions:

Witten '84, KC & Kim '85, Barr '85, ...

$$e^{i\int_{\Sigma_p} C_p} = e^{ia(x)/f_a}$$

$$\left(C_p = \frac{1}{p!} C_{[M_1 M_2 \ldots M_p]} \, dx^{M_1} \wedge dx^{M_2} \ldots \wedge dx^{M_p} \right)$$



Another key feature of axions is that their interactions are constrained by an approximate global U(1) Peccei-Quinn (PQ) symmetry which is non-linearly realized in low energy (<< f_a) limit, taking the form:

$$U(1)_{\rm PQ}: \quad a(x) \;\;\rightarrow\;\; a(x) + {\rm constant}$$

For QCD axions postulated to solve the strong CP problem, PQ-symmetry is required to be broken dominantly by the QCD anomaly, which is quite puzzling as quantum gravity generically breaks global symmetry.

(the PQ (axion) quality problem)

PQ-quality problem for QCD axion

* Strong CP problem

$$\mathcal{L}_{\rm QCD} \ni \frac{1}{32\pi^2}\bar{\theta} G^{\alpha}_{\mu\nu}\tilde{G}^{\alpha\mu\nu} \qquad \left(\bar{\theta} = \theta_{\rm QCD} + \arg\cdot{\rm Det}(M_u M_d)\right)$$

$$\Rightarrow \quad d_n \sim 10^{-16}\,\bar{\theta}\,e\cdot{\rm cm}$$

$$|d_n| < 10^{-26}\,e\cdot{\rm cm} \quad \Rightarrow \quad |\bar{\theta}| < 10^{-10}$$

* QCD axion solution

Peccei & Quinn '77; Weinberg '78; Wilczek '78

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD} + \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{1}{32\pi^2}\frac{a}{f_a} G^{a\mu\nu}\tilde{G}^a_{\mu\nu} + \Delta\mathcal{L}$$

$$\left(U(1)_{\rm PQ} : \; a(x) \to a(x) + \text{constant}\right)$$

PQ-breaking by QCD anomaly

Additional PQ-breaking

$$\Rightarrow \quad d_n \sim 10^{-16}\,\bar{\theta}_{\rm eff}\, e\cdot{\rm cm}$$

$$\left(\bar{\theta}_{\rm eff} = \langle \frac{a}{f_a}\rangle + \bar{\theta}\,\right)$$

$$V_{\rm axion} = V_{\rm QCD} + \Delta V$$

$$\left(V_{\rm QCD} \sim -m_\pi^2 f_\pi^2 \cos(\frac{a}{f_a} + \bar{\theta})\right)$$

To get $|\theta_{\rm eff}| < 10^{-10}$ w/o fine-tuning, all PQ-breakings other than the QCD anomaly should be negligible as

$$\Delta V < 10^{-10} m_\pi^2 f_\pi^2 \sim 10^{-14}\,{\rm GeV}^4$$

$$\Rightarrow \quad |\bar{\theta}_{\rm eff}| = |\langle \frac{a}{f_a}\rangle + \bar{\theta}| < 10^{-10}$$

($U(1)_{PQ}$ should be broken dominantly by the QCD anomaly.)

On the other hand, $U(1)_{PQ}$ can also be broken by quantum gravity effects:

blackhole evaporation, gravitational instantons, ...

As the quantum gravity mass scale >> 1 GeV, unless certain limitation on PQ-breaking is imposed, it is expected

$$\Delta V \gg 1\,\mathrm{GeV}^4$$

which would spoil the axion solution of the strong CP problem.

PQ-quality problem for the QCD axion:

What is the underlying feature of the QCD axion which would assure that PQ-breakings other than the QCD anomaly are all suppressed enough to have

$$\Delta V < 10^{-10} m_\pi^2 f_\pi^2 \sim 10^{-14}\,\mathrm{GeV}^4 \ ?$$

Questions on axion

* Can a qualified $U(1)_{PQ}$ secured from quantum gravity be achieved with a reasonable amount of model-building effort?

* Any connection between f_a and other mass scales such as M_{Pl}, M_{GUT}, M_{KK}, m_{SUSY}, ... ?

* Can $f_a \ll M_{\rm Pl}$ be achieved in a natural manner?

* Pre-inflationary or post-inflationary axion cosmology?

* Any testable prediction on the axion couplings to the SM particles?

Origins of f_a and qualified $U(1)_{PQ}$

In regard to the origin of f_a and qualified $U(1)_{PQ}$, EDA provides a simple and appealing answer.

KC '03, ...
Reece '24, Craig et al '24

- f_a is determined by the geometry of compactified extra-dim:

$f_a \ll M_{\rm Pl}$ can be easily achieved by the well-known extra-dimensional mechanism generating scale hierarchy, i.e. large or warped extra-dim.

- Qualified $U(1)_{PQ}$ originates from higher-dim gauge symmetry:

$$U(1)_C: \;\; C_p \;\rightarrow\; C_p + d\Lambda_{p-1}$$

((p-1)-form-valued gauge transformation)

$$U(1)_{\rm PQ}: \;\; C_p \;\rightarrow\; C_p + \alpha\omega_p \quad \left(\int_{\Sigma_p} \omega_p = 1\right)$$

(p-form global symmetry generated by the harmonic p-form ω_p associated with Σ_p)

$$\Rightarrow \qquad \frac{a(x)}{f_a} = \int_{\Sigma_p} C_p \;\rightarrow\; \frac{a(x)}{f_a} + \alpha$$

(4-dim axion is not a genuine local field, but an extended p-dim degree of freedom)

$$U(1)_C: \quad C_p \;\to\; C_p + d\Lambda_{p-1}$$

$$U(1)_{\rm PQ}: \quad C_p \;\to\; C_p + \alpha\omega_p \quad \left(\int_{\Sigma_p} \omega_p = 1\right)$$

$\alpha\omega_p = d\Lambda_{p-1}$ locally, but not globally

$\Rightarrow$ $U(1)_{\rm PQ} \simeq U(1)_C$ locally (but not globally) in the low energy limit when all $U(1)_C$-charged objects ((p-1)-brane) are integrated out.

PQ-breaking is severely constrained, e.g. $U(1)_{PQ}$ can be broken only in non-local manner through non-derivative couplings of C_p.

$$\Delta V \;\propto\; e^{-\mu_p {\rm Vol}(\Sigma_p)} \cos\left(a(x)/f_a\right) \qquad \left(\mu_p {\rm Vol}(\Sigma_p) \gg 1\right)$$

Yukawa suppression for nonlocal effect or nonperturbative suppression for instanton effect

Tension of $U(1)_C$-charged (p-1)-brane

PQ quality can be easily achieved:

$$\mu_p {\rm Vol}(\Sigma_p) \gtrsim \mathcal{O}(100) \qquad \Rightarrow \qquad \Delta V < 10^{-10} m_\pi^2 f_\pi^2 \sim 10^{-14}\,{\rm GeV}^4$$

Many essential features of EDA can be studied in the context of 5-dim field theory models.

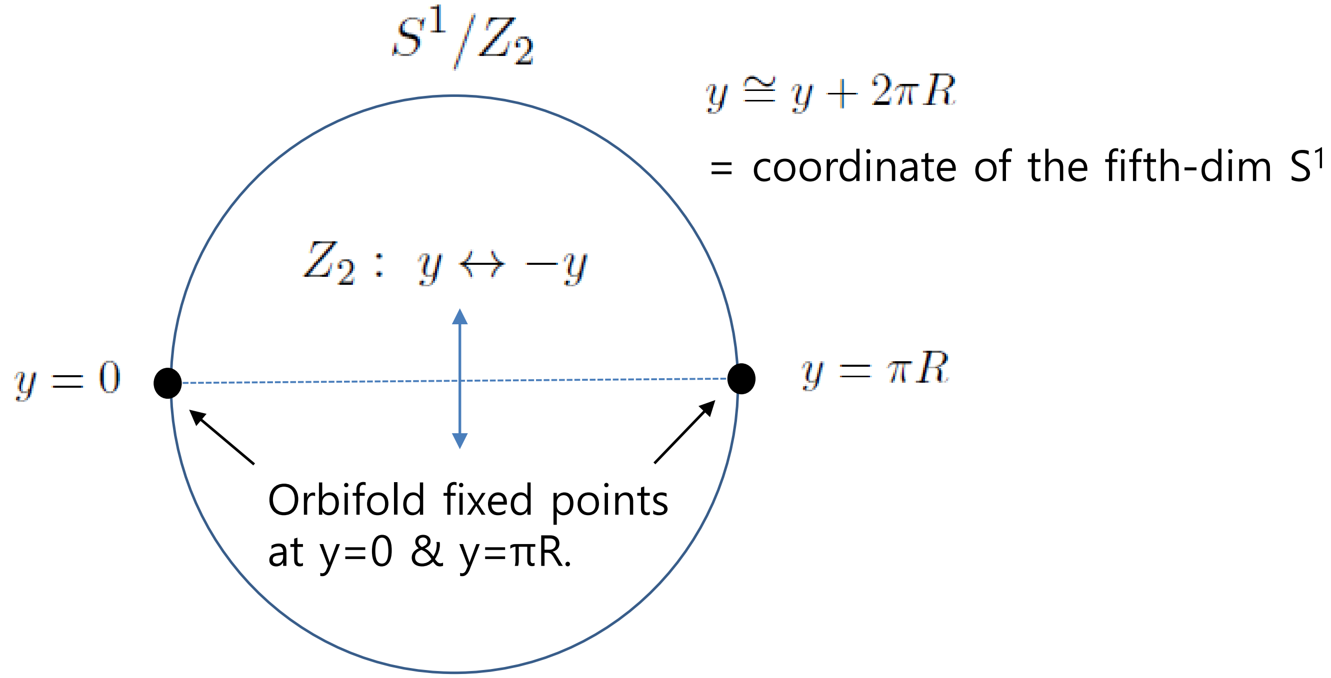
Particularly interesting example is an EDA from 1-form gauge field in warped 5-dim orbifold $M_4 \times S^1/Z_2$: KC '03,
KC, C.H. Lee & C.S. Shin, in preparation

- The axion potential induced by $U(1)_C$-charged (p-1)-brane, which we call an "irreducible axion potential" as it's existence is implied by the weak gravity conjecture, can be explicitly computed by familiar field-theoretic methods.

  (For p=1, $U(1)_C$-charged objects are point-like particles.)

- One can also study how the detailed feature of higher-dim theory such as warped geometry and orbifold boundary conditions affect the PQ-quality and f_a .

Field theory on 5-dim orbifold $M_4 \times S^1/Z_2$



* 5-dim 1-form gauge field giving 4-dim axion: $C_M = (C_\mu, C_5)$

$$C_\mu(x,y) = C_\mu(x, y+2\pi R) = -C_\mu(x,-y),$$

$$C_5(x,y) = C_5(x, y+2\pi R) = C_5(x,-y)$$

$$U(1)_C: \quad C_M \;\; \to C_M + \partial_M \Lambda$$

$$\big(\Lambda(y) = \Lambda(y+2\pi R) = -\Lambda(-y) \mod 2\pi\big)$$

$$U(1)_{\rm PQ}: \quad C_5 \;\; \to \;\; C_5 \; + \; \frac{\alpha}{2\pi R}$$

$$\frac{a(x)}{f_a} \; \equiv \; \theta_c(x) = \oint dy\, C_5(x,y) \cong \theta_c(x) + 2\pi \quad \Big(\Lambda(y) = \frac{ny}{R} \;\; (n \in \mathbf{Z})\Big)$$

$$\Rightarrow \qquad \theta_c(x) \quad \to \quad \theta_c(x) + \alpha$$

Imposing the orbifold boundary conditions does not affect the feature

$U(1)_{\rm PQ} \simeq U(1)_C$ locally (but not globally).

The weak gravity conjecture (= Consistency with quantum gravity) tells that there should exist $U(1)_C$-charged matter fields with

$$m \lesssim g_{5C} M_5^{3/2} \, .$$

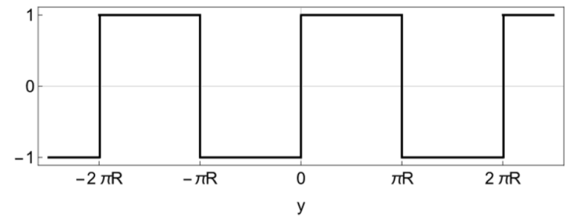
Arkani-Hamed et al '06

5-dim $U(1)_C$ gauge coupling and Planck mass

* $U(1)_C$-charged matter fields in 5-dim:

$$U(1)_C : \quad \Phi \to e^{iq_\Phi \Lambda} \Phi, \quad \tilde{\Phi} \to e^{iq_{\tilde{\Phi}} \epsilon(y) \Lambda} \tilde{\Phi}$$

$$\epsilon(y) = \pm 1$$



$$\Phi(y) = \Phi(y + 2\pi R) = \Gamma_\Phi \Phi^*(-y),$$

$$\tilde{\Phi}(y) = \tilde{\Phi}(y + 2\pi R) = \Gamma_{\tilde{\Phi}} \tilde{\Phi}(-y)$$

$$\left(\Phi = (\phi, \psi), \quad \tilde{\Phi} = (\tilde{\phi}, \tilde{\psi}), \quad \Gamma_\Phi^2 = \Gamma_{\tilde{\Phi}}^2 = 1 \right)$$

Z_2-odd gauge coupling

$$D_M \Phi = (\partial_M - iq_\Phi C_M)\Phi, \quad D_M \tilde{\Phi} = (\partial_M - iq_{\Phi} \epsilon(y) C_M)\tilde{\Phi}$$

* $U(1)_C$-neutral SM fields either in 5-dim bulk or at the 4-dim orbifold fixed points.

U(1)$_{PQ}$ can be broken only in non-local manner by non-derivative couplings of C_p .

* PQ-breaking by CS-term with integer-valued coefficients:

$$S_{CS} = \frac{1}{8\pi^2}\int C_p \wedge (n_1 G \wedge G + ...)$$

$$\Rightarrow \quad S_{\rm eff} = \frac{1}{32\pi^2}\int d^4x\, \theta_c(x)\, (n_1 G^{a\mu\nu}\tilde{G}^a_{\mu\nu} + ...)$$

PQ-breaking by QCD anomaly

Additional PQ-breaking with integer coefficients which can be set to zero if dangerous

* Irreducible PQ-breaking by the gauge interaction of U(1)$_C$-charged matter fields required by the weak gravity conjecture:

$$D_5\Phi = (\partial_y - iq_\Phi C_5)\Phi, \quad D_5\tilde{\Phi} = (\partial_y - iq_\Phi \epsilon(y) C_5)\tilde{\Phi} \qquad \left(C_5 = \frac{\theta_c(x)}{2\pi R}\right)$$

$\Rightarrow$ Irreducible axion potential:

$$\Delta V \propto e^{-S_{\rm ins}} \cos(q_\Phi \theta_c) \quad \left(S_{\rm ins} \sim \mu_p {\rm Vol}(\Sigma_p) \sim M_\Phi \pi R \gg 1\right)$$

Although the orbifold boundary conditions do not spoil "$U(1)_{PQ} \cong U(1)_C$ locally", they break $U(1)_C$ down to its Z_2-subgroup at the orbifold fixed-points, which can have an important consequence for the irreducible axion potential.

$$U(1)_C: \;\; C_M \;\; \to C_M + \partial_M \Lambda \quad \big(\Lambda(y) = \Lambda(y+2\pi R) = -\Lambda(-y) \mod 2\pi\big)$$

$$\Rightarrow \quad \Lambda(y=0,\pi R) = n\pi \;\; (n \in \mathbf{Z})$$

$U(1)_C$-breaking fixed-point interactions, e.g.

$$\mathcal{L}_{5D} \ni \;\; \delta(y)\Big(t_0\phi + \tilde{t}_0\tilde{\phi} + b_0\phi^2 + \tilde{b}_0\tilde{\phi}^2 + m_0\phi\tilde{\phi} + ...\Big)$$
$$+\delta(y-\pi R)\Big(t_\pi\phi + \tilde{t}_\pi\tilde{\phi} + b_\pi\phi^2 + \tilde{b}_\pi\tilde{\phi}^2 + m_\pi\phi\tilde{\phi} + ...\Big)$$

$$\big(t_{0,\pi} \neq \text{ for } q_\phi = \text{even}, \;\; m_{0,\pi} \neq 0 \text{ for } q_\phi + q_{\tilde{\phi}} = \text{even}, \;...\big)$$

Issues to be discussed

* Scale hierarchies among $M_{\rm Pl}$, f_a, M_{KK} generated by warped extra-dim

* Effects of "warping" and "$U(1)_C$-breaking fixed point interactions" on the PQ-quality, specifically on the irreducible axion potential

General linear-dilaton model for warped geometry

KC, Im, Shin '17
Im, Nilles, Olechowski '18

$$S_{\rm grav} = M_5^3 \int d^5x \sqrt{g} \left(\frac{R_5}{2} - \frac{1}{2}\partial_M S \partial^M S + 2\left(e^{-cS}k\right)^2 + \frac{6e^{-cS}k_1}{\sqrt{g_{55}}}\left(\delta(y) - \delta(y-\pi R)\right) \right)$$

$$\Rightarrow \quad ds^2 = e^{-2k_1|y|}(\eta_{\mu\nu}dx^\mu dx^\nu) + e^{-2k_2|y|}dy^2, \quad cS = -k_2|y|$$

$$\left(k_1 = -\frac{2k}{\sqrt{12-9c^2}},\ k_2 = 3c^2 k_1\right)$$

$$\frac{k_2}{k_1} = 3c^2 \quad \begin{cases} 0: \text{Randall-Sundrum} \\ 1: \text{Linear Dilaton} \\ \geq 6: \text{Heterotic M} \end{cases}$$

1 : Linear Dilaton (Antoniadis et al '11)

≥ 6 : Heterotic M (Horava & Witten '96)

$$M_{\rm Pl}^2 = \frac{2M_5^3}{2k_1 + k_2}\left(1 - e^{-(2k_1+k_2)\pi R}\right)$$

4-dim reduced Planck mass ~ 2×10^{18} GeV

Axion localization by warping and the resulting scale hierarchy

KC '03,
KC, C.H. Lee & C.S. Shin, in preparation

Wavefunction of 4-dim axion embedded in 5-dim gauge field:

$$C_{\mu 5} = \partial_\mu C_5 - \partial_y C_\mu \sim e^{(2k_1 - k_2)(y - \pi R)} \partial_\mu \theta_c$$

$$\Big(0 \leq y \leq \pi R, \ (2k_1 - k_2)\pi R \gg 1\Big)$$

$\theta_c(x) = \oint dy\, C_5(x,y)$ is localized near the IR fixed point $y = \pi R$, which results in an exponentially red-shifted axion scale:

$$\frac{f_a}{M_{\rm Pl}} \sim e^{-(k_1 - \frac{1}{2}k_2)\pi R}$$

* Scale hierarchy from warping for generic values of k_i $(i = 1, 2)$

$$\frac{M_{KK}}{M_{\rm Pl}} \sim 10^{-3} \frac{(k_1 - k_2)\pi R}{e^{(k_1-k_2)\pi R} - 1}$$

$$\frac{f_a}{M_{\rm Pl}} \sim 10^{-2} \sqrt{\frac{(2k_1 - k_2)\pi R}{e^{(2k_1-k_2)\pi R} - 1}}$$

Depending on k_2/k_1 and $k_1\pi R$, various different scale hierarchies between $M_{\rm Pl}$, f_a, M_{KK} can be realized.

$\big(k_2/k_1 = 0$: RS, $\quad k_2/k_1 = 1$: Linear dilaton, $\quad k_2/k_1 \geq 6$; Heterotic $M\big)$

* Irreducible axion potential

KC, C.H. Lee & C.S. Shin, in preparation

$$D_5\phi = \partial_y \phi - iqC_5\phi, \quad D_5\tilde{\phi} = \partial_y - i\tilde{q}\epsilon(y)C_5\tilde{\phi} \quad \left(C_5 = \frac{1}{2\pi R}\frac{a}{f_a} = \frac{\theta_c}{2\pi R}\right)$$

$$S = \int d^5x\sqrt{g}\left(D_M\phi D^M\phi^* + D_M\tilde{\phi}D^M\tilde{\phi}^* - M^2\phi\phi^* - \tilde{M}^2\tilde{\phi}\tilde{\phi}^* + \ldots\right)$$

$$+\frac{\delta(y)}{\sqrt{g_{55}}}\left(t_0\phi + \tilde{t}_0\tilde{\phi} + b_0\phi^2 + \tilde{b}_0\tilde{\phi}^2 + m_0\phi\tilde{\phi} + \ldots\right)$$

$$+\frac{\delta(y-\pi R)}{\sqrt{g_{55}}}\left(t_\pi\phi + \tilde{t}_\pi\tilde{\phi} + b_\pi\phi^2 + \tilde{b}_\pi\tilde{\phi}^2 + m_\pi\phi\tilde{\phi} + \ldots\ldots\right)$$

} $U(1)_C$ –breaking fixed-point interactions

The leading axion potential arises from

* the θ_c-dependent tree-level VEV of $\phi, \tilde{\phi}$ induced by $t_{0,\pi}, \tilde{t}_{0,\pi}$

* one-loop Casimir energy of the KK modes with θ_c-dependent KK masses $m_n(\theta_c)$:

Hosotani '83
Cheng et al '02

$$V_{1-\text{loop}} = \int \frac{d^4k}{(2\pi)^4}\sum_n \ln(k^2 + m_n(\theta_c)^2)$$

(These axion potentials can also be computed in the world-line formulation, for which the instanton interpretation is manifest.)

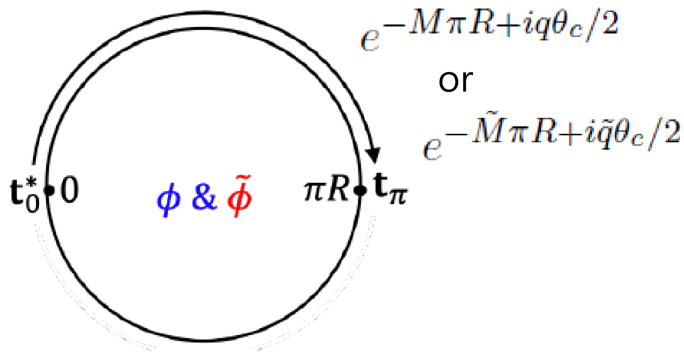
Axion potential induced by the U(1)$_C$-breaking fixed-point interactions in flat background geometry.

$$(k_1 = k_2 = 0)$$

* Tree-level axion potential from tadpole-terms ($t_{0,\pi}, \tilde{t}_{0,\pi}$) at fixed-points
(Worldline instantons stretched over the fundamental domain)

$$\delta(y)\big(t_0\phi + \tilde{t}_0\tilde{\phi}\big)$$

$$+\,\delta(y-\pi R)\big(t_\pi\phi + \tilde{t}_\pi\tilde{\phi}\big)$$



$$\Rightarrow \qquad \Delta V \simeq \frac{t_0 t_\pi}{M} e^{-M\pi R}\cos(q\theta_c/2) + \frac{\tilde{t}_0\tilde{t}_\pi}{\tilde{M}} e^{-\tilde{M}\pi R}\cos(\tilde{q}\theta_c/2)$$

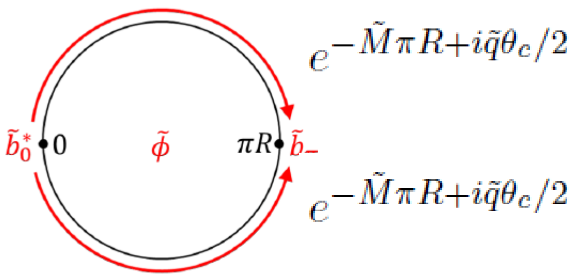
$$(q, \tilde{q} = \text{even integer})$$

* 1-loop axion potential induced by fixed-point interactions

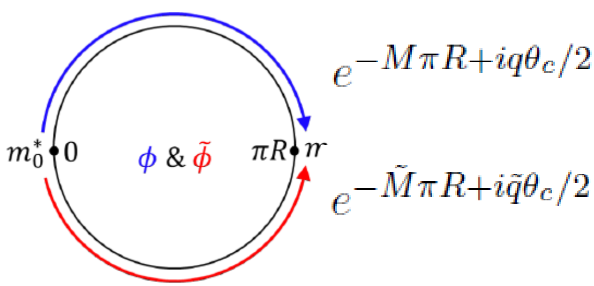
(Worldline instantons wrapped on the covering space S^1 with non-trivial transition at the fixed points)

$$\delta(y)\left(b_0\phi^2+\tilde{b}_0\tilde{\phi}^2+m_0\phi\tilde{\phi}\right)$$

$$+\ \delta(y-\pi R)\left(b_\pi\phi^2+\tilde{b}_\pi\tilde{\phi}^2+m_\pi\phi\tilde{\phi}\right)$$



$$\Rightarrow\quad \Delta V\simeq\frac{\tilde{b}_0\tilde{b}_\pi}{16\pi^2}\frac{1}{(2\pi R)^2}e^{-2\tilde{M}\pi R}\cos\left(\tilde{q}\theta_c\right)$$



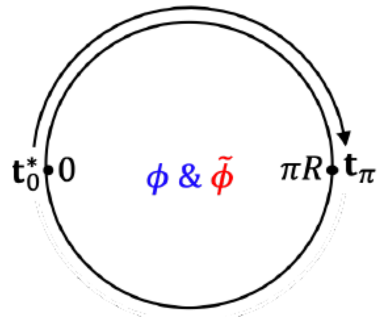
$$\Rightarrow\quad \Delta V\simeq\frac{m_0\tilde{m}_\pi}{64\pi^2}\frac{1}{(2\pi R)^2}e^{-(M+\tilde{M})\pi R}\cos\left((q+\tilde{q})\theta_c/2\right)$$

$$(q+\tilde{q}=\text{even integer})$$

$U(1)_C$-breaking fixed point interactions induce a variety of additional irreducible axion potentials, affecting the details of the PQ quality of the model.

However those additional axion potentials are all exponentially suppressed as expected, so there is no essential change of the PQ quality.

* Effect of warping on the PQ quality

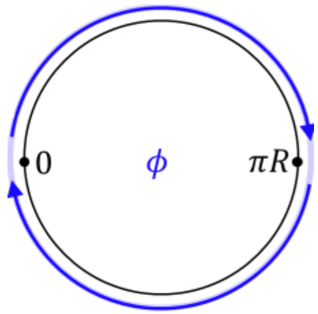


$$\Delta V \simeq \frac{t_0 t_\pi}{M} e^{-M\pi R} \cos(q\theta_c/2) \quad (k_1 = k_2 = 0)$$

$$\Rightarrow \quad \Delta V \simeq \frac{t_0 t_\pi e^{-2(k_1-k_2)\pi R}}{M} e^{-M_{\rm eff}\pi R} \cos\left(q\theta_c/2\right)$$

$$S_{\rm ins}(\theta_c): \; M\pi R \; \rightarrow \; M_{\rm eff}\pi R$$

$$\left(M_{\rm eff} = \sqrt{M^2 + (2k_1 - \frac{k_2}{2})^2} \right)$$



$$\Delta V \simeq \frac{1}{2}\frac{(2M\pi R)^2}{64\pi^2}\left(\frac{1}{\pi R}\right)^4 e^{-2M\pi R}\cos\theta_c \qquad (k_1 = k_2 = 0)$$

$$\Rightarrow \quad \Delta V \simeq \frac{1}{2}\frac{(2M_{\rm eff}\pi R)^2}{64\pi^2} M_{KK}^4 e^{-2M_{\rm eff}\pi R}\cos(\theta_c)$$

(for generic warped case with nonzero $k_{1,2}$)

	Flat		Warped
M_{KK} :	$\frac{1}{\pi R}$	$\rightarrow$	$\frac{k_1 - k_2}{e^{(k_1-k_2)\pi R} - 1}$
$S_{\rm ins}(\theta_c)$:	$2M\pi R$	$\rightarrow$	$2M_{\rm eff}\pi R$

$$\left(M_{\rm eff} = \sqrt{M^2 + (2k_1 - \frac{k_2}{2})^2}\right)$$

Warping improves the PQ-quality by

(i) red-shifting the characteristic mass scale of the axion potential,

(ii) enlarging the instanton action.

Conclusion

* In regard to the origin of qualified $U(1)_{PQ}$ and the axion scale, extra-dimensional axions (EDA) provide an appealing answer.

* Orbifolding the extra-dimension can cause a variety of additional PQ-breaking, changing some details of the PQ-quality, but it does not change the essential feature of EDA.

* Warped extra-dim can be useful for both the axion scale hierarchy and the PQ quality. It allows $f_a \ll M_{\rm Pl}$ in a natural manner, and also improves the PQ quality by i) red-shifting the KK scale and ii) enlarging the associated instanton action.