核结构与反应少体问题2025

ab initio effective operator with continuum

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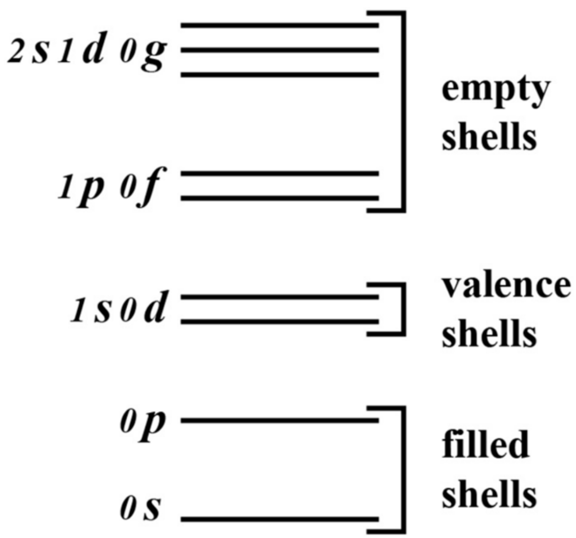
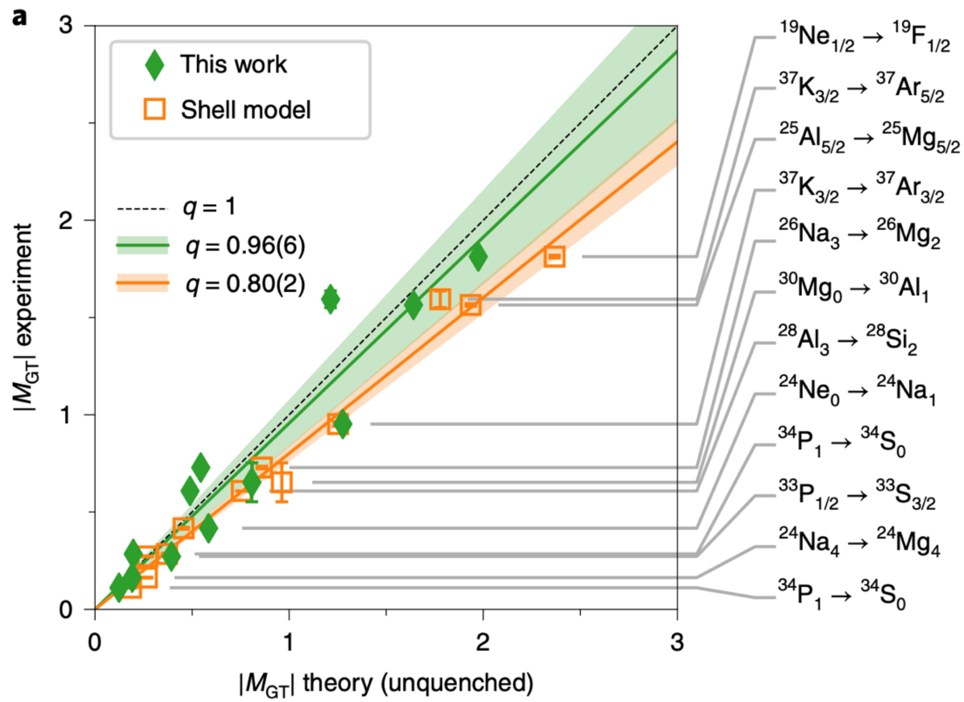
Outline

- Introduction
  - Effective operator in valence space
  - Open quantum systems
- Method
  - Many-body pertubation theory effective operator
- Results
  - Mirror symmetry breaking in dripline nuclei
  - EM transitions in ^{36}Ca and ^{38}Ca

Effective operator in valence space

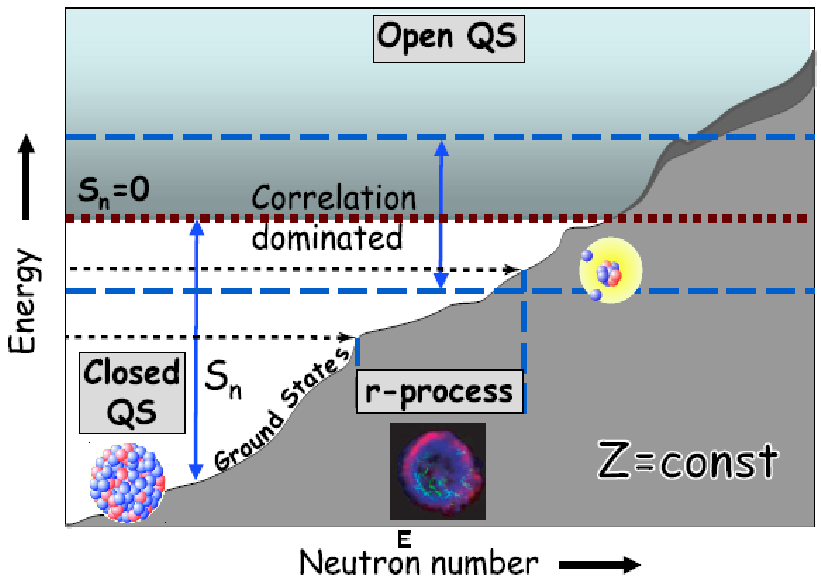




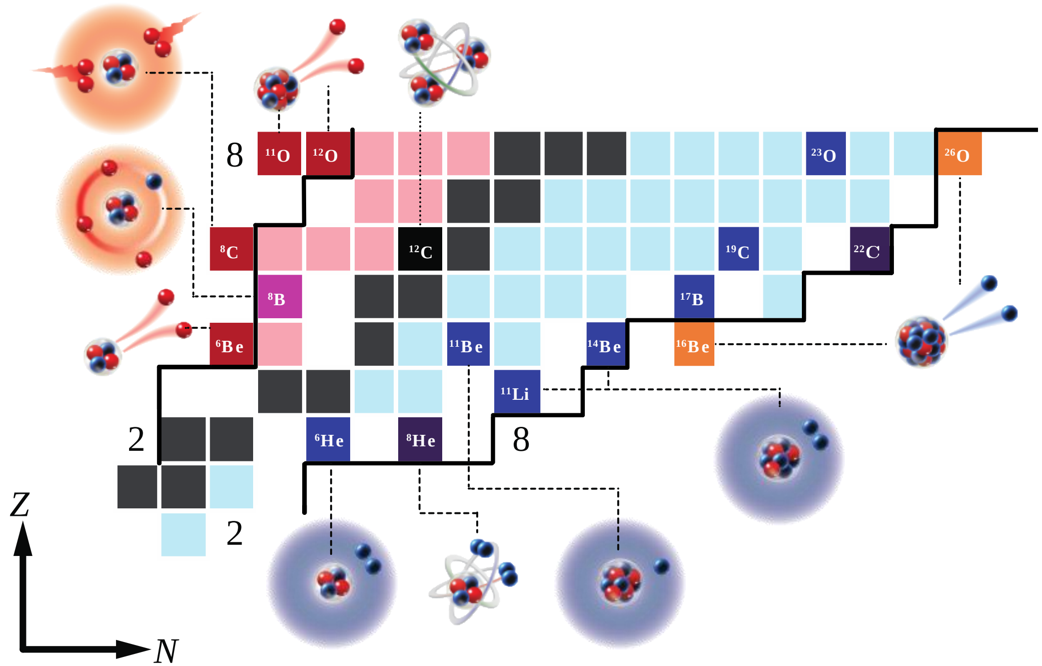
P. Gysbers *et al.* Nat. Phys. 15, 428 (2019)

- Valence space Hamiltonian and operator
  - phenomenological approach
  - appropriate many-body theory
- Observables in phenomenological approach
- Observables in ab initio many-body theory

Open Quantum Systems



N. Michel *et al.*, JPG: NPP 36, 013101 (2009)



From S. M. Wang’s talk

Resonance & continuum near threshold

- Halo & clustering
- Exotic decay
- Deformation
- New magic number
- …

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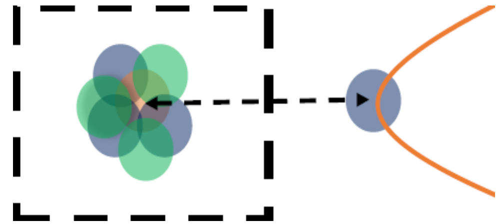
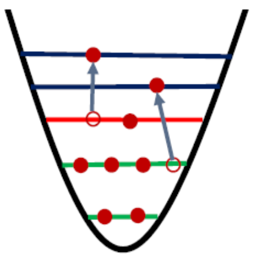
closed quantum system

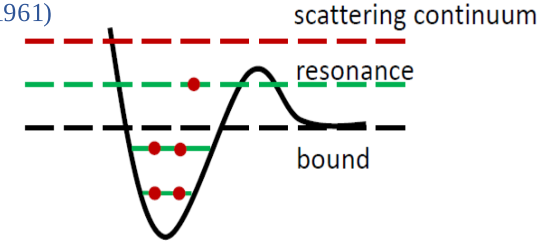
open quantum system



Hilbert Space → **Rigged Hilbert space**

Gel'fand I M *et al*, Generalized Functions vol 4 (1961)





bound → bound, resonant, continuum

HO basis

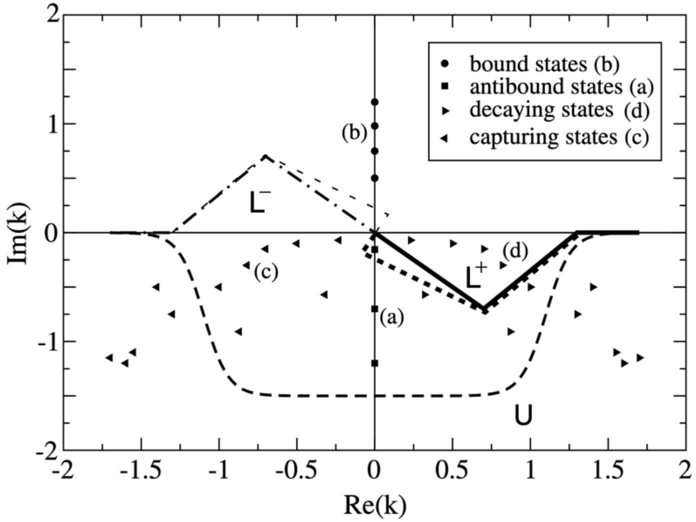
T. Berggren, Nucl. Phys. A 109, 265 (1968)

Berggren basis

Gamow Shell Model

N. Michel *et al*, Phys. Rev. Lett. 89, 042502 (2002)

Berggren Basis



N. Michel *et al.*, JPG: NPP 36, 013101 (2009)

Berggren basis completeness relation:

$$\sum_n u_n(E_n,r)u_n(E_n,r') + \int_{L^+} dE u(E,r)u(E,r') = \delta(r-r')$$

T. Berggren, Nucl. Phys. A 109, 265 (1968)

Gamow Hartree-Fock method: with three nucleon force

$$H_{int} = \sum_{i=1}^{A}(1-\frac{1}{A})\frac{\boldsymbol{p}_i^2}{2m} + \sum_{i<j}^{A}(V_{NN,ij} - \frac{\boldsymbol{p}_i\cdot\boldsymbol{p}_i}{mA}) + \sum_{i<j<k}^{A} V_{NNN,ijk}$$

1. 迭代HF方程和提取HF单体势

$$\hat{h}_{pq} = (t_{pq} + \sum_i V_{piqi}^{\mathrm{NN}} + \frac{1}{2}\sum_{ij} V_{pij,qij}^{3N}) : \hat{a}_p^\dagger \hat{a}_q :$$

2. 将U_{HF}解析延拓到复k平面并对角化

$$\langle k|\, h\, |k'\rangle = \frac{\hbar^2 k^2}{2\mu}\delta(k-k') + \sum_{\alpha\beta}\langle\alpha|\, U_{\mathrm{HF}}\, |\beta\rangle \langle k\,|\,\alpha\rangle \langle\beta\,|\,k'\rangle$$

S. Zhang *et al.*, Phys. Rev. C 108, 064316 (2023)

Many-Body Pertubation Theory

$$\begin{bmatrix} PHP & PHQ \\ QHP & QHQ \end{bmatrix} \xrightarrow[\text{Decoupling Equation}]{\text{Similarity Transformation}} \begin{bmatrix} P\mathcal{H}P & P\mathcal{H}Q \\ 0 & Q\mathcal{H}Q \end{bmatrix}$$

Similarity Transformation

Decoupling Equation

$Q\mathcal{H}P = 0$

Projection Operator: $P = \sum_{i \in P} |\Phi_i\rangle\langle\Phi_i|$

Similarity Transformation: $\mathcal{H} = X^{-1}HX,\ X = e^{\omega} \approx 1 + \omega,\ \omega = Q\omega P, P\omega P = Q\omega Q = P\omega Q = 0$

Decoupling Equation: $QHP + QHQ\omega - \omega PHP - \omega PHQ\omega = 0$

Decoupled Hamiltonian: $H_{\text{eff}}(\omega) = P\mathcal{H}P = PHP + PHQ\omega$

$\hat{Q}$-box: $\hat{Q}(\epsilon) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P$

$$H_{\text{eff}}(\omega) = \hat{Q}(\epsilon_0) + \sum_{k=1}^{\infty} \hat{Q}_k(\epsilon_0) H_{\text{eff}}(\omega)^k$$

L. Coraggio and N. Itaco, Front. Phys. 8, 345 (2020)

With EKK method, an H_{eff} the within non-degenerate model spaces can be derived.

K. Takayanagi et al. Nucl. Phys. A 852, 61 (2005)
K. Takayanagi et al. Nucl. Phys. A 864, 91 (2011)

MBPT effective operator

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Model space and full space operator:

$$\langle\tilde{\psi}_\lambda^r|\Theta|\psi_\mu^r\rangle = \langle\tilde{\psi}_\lambda^m|\Theta_{\text{eff}}|\psi_\mu^m\rangle$$

Same projection get wavefunction :

$$|\psi_\lambda^r\rangle = X|\psi_\lambda^m\rangle = (1+\omega)|\psi_\lambda^m\rangle = (P+\omega)|\psi_\lambda^m\rangle$$

and

$$\langle\tilde{\psi}_\lambda^r| = \langle\psi_\lambda^m|(P+\omega^\dagger\omega)^{-1}(P+\omega^\dagger)$$

Effective operator：

$$\Theta_{\text{eff}} = (P+\omega^\dagger\omega)^{-1}\hat{\Theta}$$

$\hat{\Theta}$ defined as：

$$\hat{\Theta} = (P+\omega^\dagger)\Theta(P+\omega) = P\Theta P + P\Theta Q\omega + \omega^\dagger Q\Theta P + \omega^\dagger Q\Theta Q\omega$$

Define $\hat{\Theta}$-box:

$$\hat{\Theta}(\epsilon) = P\Theta Q\frac{1}{\epsilon - QHQ}QH_1P$$

$$\hat{\Theta}(\epsilon_1,\epsilon_2) = PH_1Q\frac{1}{\epsilon - QHQ}Q\Theta Q\frac{1}{\epsilon - QHQ}QH_1P$$

Define:

$$\chi_0 = P\Theta P + (\hat{\Theta}_0 + h.c.) + \hat{\Theta}_{00}$$

$$\chi_1 = (\hat{\Theta}_1\hat{Q} + h.c.) + (\hat{\Theta}_{01}\hat{Q} + h.c.)$$

$$\chi_2 = (\hat{\Theta}_1\hat{Q}_1\hat{Q} + h.c.) + (\hat{\Theta}_2\hat{Q}\hat{Q} + h.c.) + (\hat{Q}_{02}\hat{Q}\hat{Q} + h.c.) + \hat{Q}\hat{\Theta}_{11}\hat{Q}$$

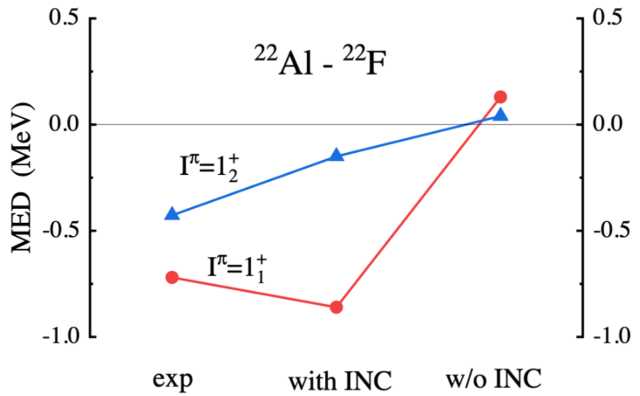
With ω:

$$\omega^\dagger\omega = -\hat{Q}_1 + (\hat{Q}_2\hat{Q} + h.c.) + (\hat{Q}_3\hat{Q}\hat{Q} + h.c.) + \cdots$$

$$\boxed{\Theta_{\text{eff}} = (P + \hat{Q}_1 + \hat{Q}_1\hat{Q}_1 + \hat{Q}_2\hat{Q} + \hat{Q}\hat{Q}_2 + \cdots) \times (\chi_0 + \chi_1 + \chi_2 + \cdots)}$$

K. Suzuki, R. Okamoto. Prog. Theor. Phys. 93,905 (1995)

Mirror symmetry breaking in dripline nuclei



mirror energy differences:
$\mathrm{MED} = \mathrm{E}_x(I,T,T_z=-T) - \mathrm{E}_x(I,T,T_z=T)$

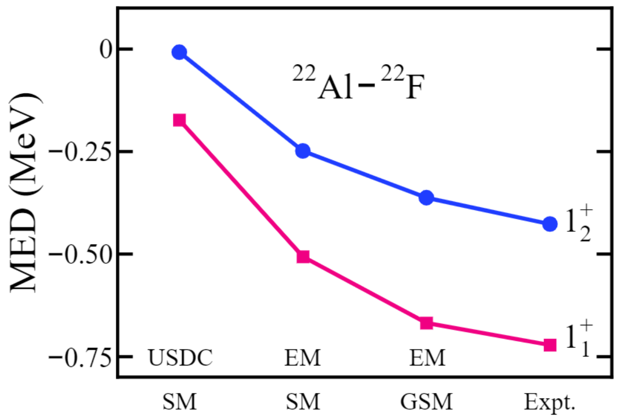
	^{22}Si → ^{22}Al		^{22}O → ^{22}F	
	Experiment	Calculations	Experiment	Calculations
I_i^{π}	$\lvert M_{\mathrm{GT}}^{+}\rvert^2$	$\lvert M_{\mathrm{GT}}^{+}\rvert^2$	$\lvert M_{\mathrm{GT}}^{-}\rvert^2$	$\lvert M_{\mathrm{GT}}^{-}\rvert^2$
1_1^+	0.0310 (58)	0.0587 [0.1138]	0.096 (20)	0.1831 [0.1059]
1_2^+	0.563 (61)	0.7449 [0.7193]	0.60 (12)	0.7196 [0.7948]

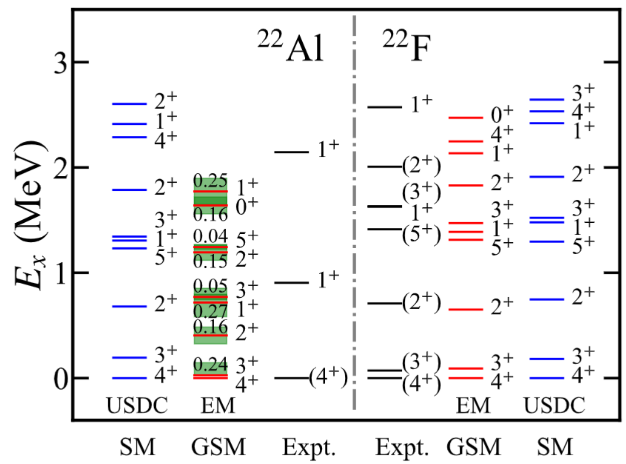
J. Lee, *et al.* Phys. Rev. Lett. 125,192503 (2020)

- Mirror symmetry breaking in ^{22}Al-^{22}F mirror pair:
  - largest MED (first 1^+ state) ever found in *sd*-shell.
  - Beta decay Gamow-Teller transition.

- USDA with isospin-nonconserving (INC) forces related to the $s_{1/2}$ orbit
  - The MSB are attributed to the significant proton occupation and loosely bound nature of the wave functions of the proton $s_{1/2}$ orbit.

Mirror symmetry breaking in dripline nuclei

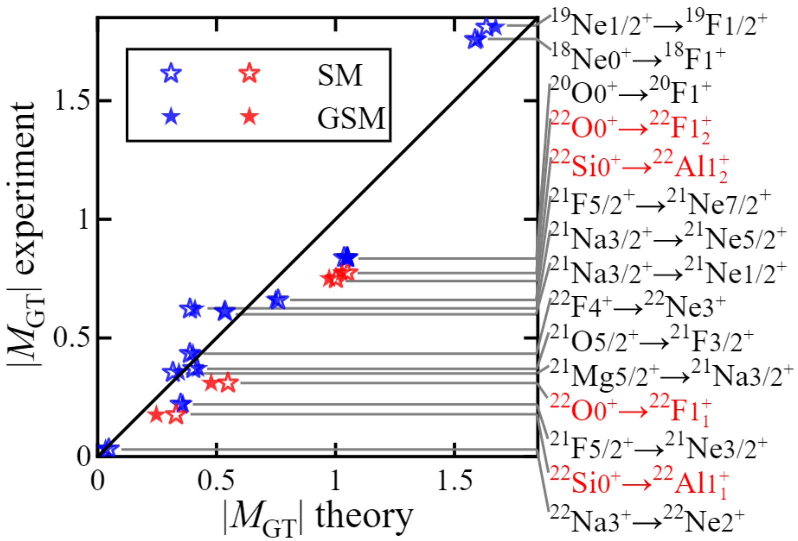




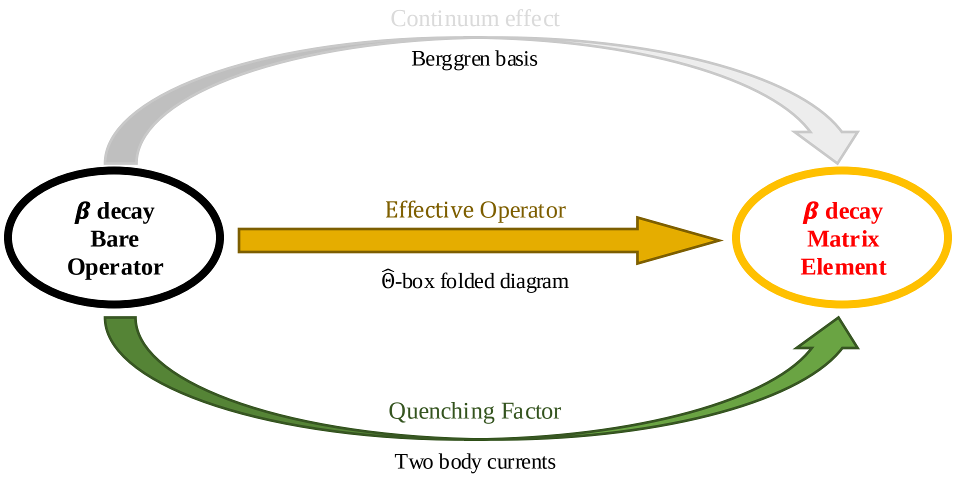
- MED and spectrum of ^{22}Al-^{22}F pair:
  - USDC interaction cannot give a good descripation.
  - EM1.8/2.0 interaction with (GSM) and without (SM) continuum

Z. C. Xu, *et al.* Phys. Rev. C. 108, L031301(2023)

Mirror symmetry breaking in dripline nuclei



Z. C. Xu, *et al.* Phys. Rev. C. 108, L031301(2023)



Beta decay calculation with ab initio GSM

- $|M_{GT}|$ of some sd-shell nuclear with EM1.8/2.0
  - With meson exchange current
  - The continuum effect
  - The MSB in A=22 nuclei

Mirror symmetry breaking in dripline nuclei

		SM		GSM	Ref. [54]	
		USDC	EM	EM	Expt.	Cal.
$^{22}Si \rightarrow {}^{22}Al$	1_1^+	0.236	0.343	0.257	0.176(16)	0.242
	1_2^+	0.721	1.042	1.012	0.750(41)	0.863
$^{22}O \rightarrow {}^{22}F$	1_1^+	0.198	0.569	0.497	0.310(32)	0.428
	1_2^+	0.719	1.092	1.068	0.775(77)	0.848

Z. C. Xu, *et al.* Phys. Rev. C. 108, L031301(2023)

- $|M_{GT}|$ of ^{22}Si & ^{22}O mirror pair
  - The USDA with INC (Cal.) change the proton *s* wave to mimic the effect of continuum
  - The $s_{1/2}$ orbit continuum change the $|M_{GT}|$
  - The realistic force & continuum are vital for MSB near dripline

EM transition in ^{36}Ca and ^{38}Ca

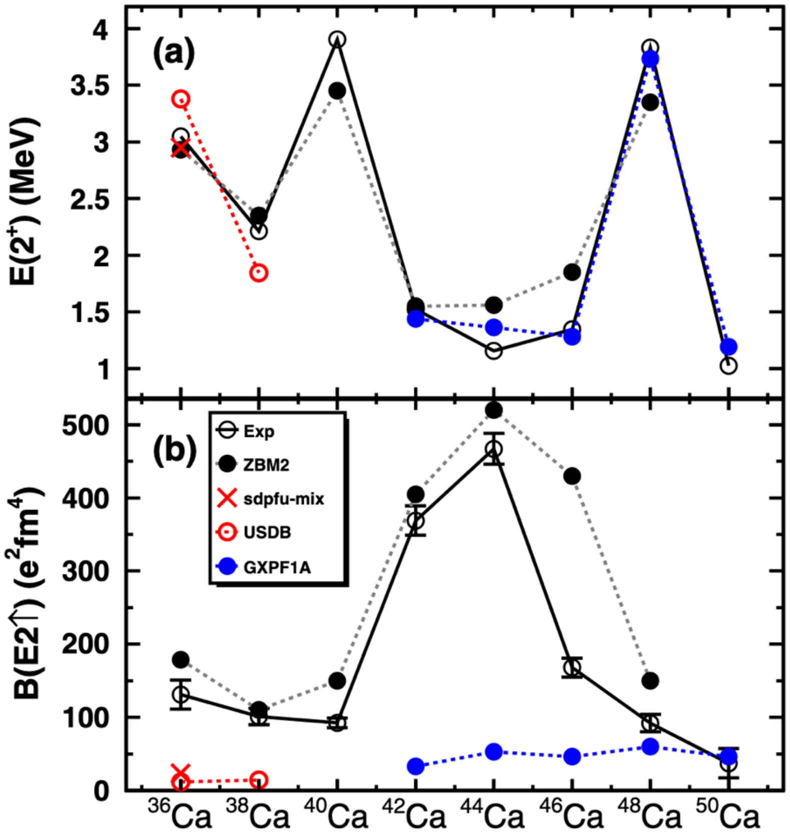


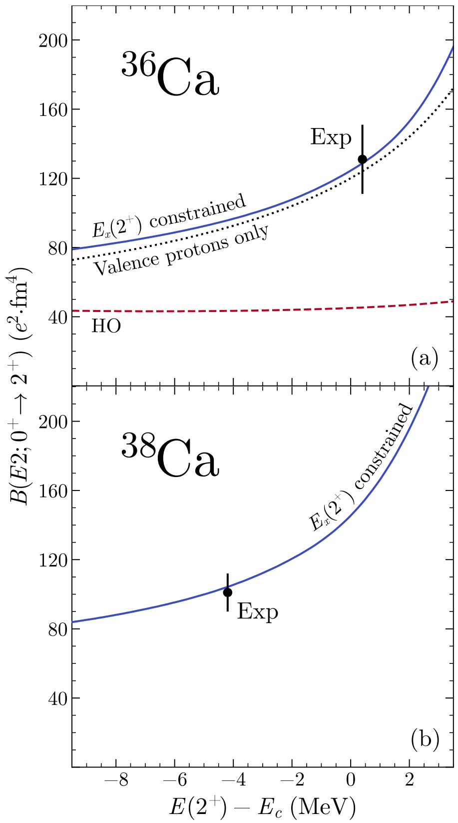
TABLE II. Comparison of $B(E2; 0^+ \to 2_1^+)$ values between experiment and theory. The ZBM2 and USDB results use effective charges of $e_p = 1.36$ and $e_n = 0.45$. The *sdpfu*-mix result [43] uses $e_p = 1.31$ and $e_n = 0.46$.

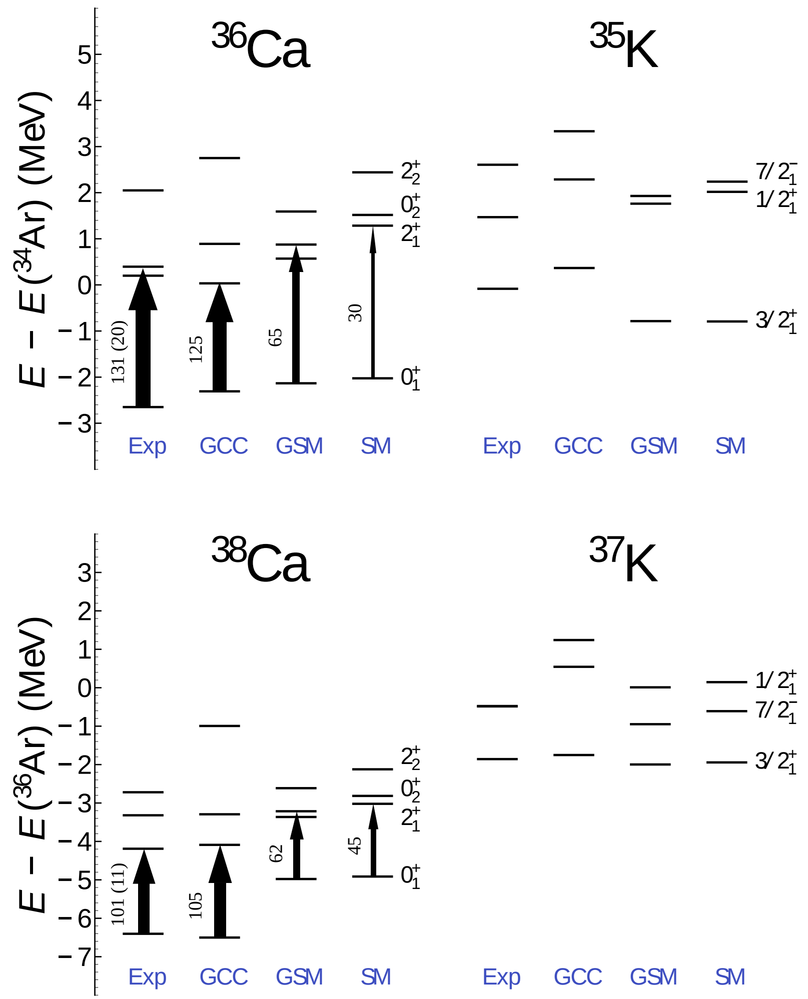
	$B(E2\uparrow)$ (e^2fm^4)			
	exp	ZBM2 [2]	USDB [42]	*sdpfu*-mix [43]
^{36}Ca	131(20)	179	11.8	23.5
^{36}S	89(9)	116	108	98
^{38}Ca	101(11)	110	14.0	-
^{38}Ar	125(4)	179	128	-

- Abnormal E(2$^+$) and B(E2) trend
- Proton cross shell?

N. Dronchi, *et al.* Phys. Rev. C. 107, 034306(2023)

EM transition in ^{36}Ca and ^{38}Ca





- B(E2) from Gamow Coupled-Channel & GSM
  - Valence protons
  - Behavior near threshold
  - GSM consistent with GCC

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Thank you for your attention!

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