

Nuclear structure with *ab initio* No-Core Shell Model

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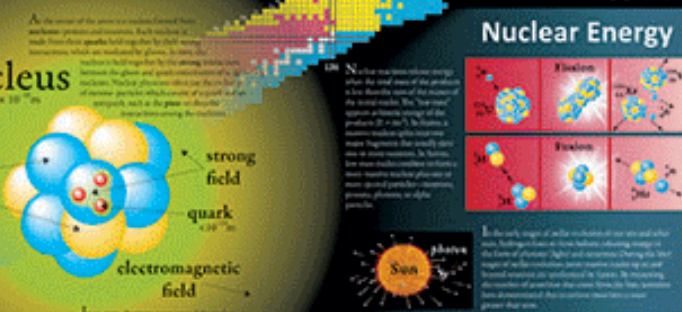
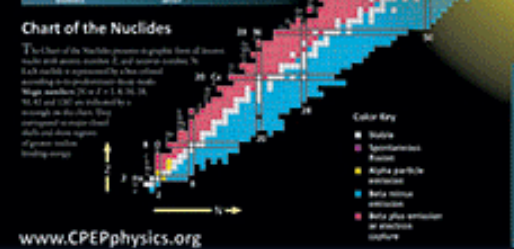
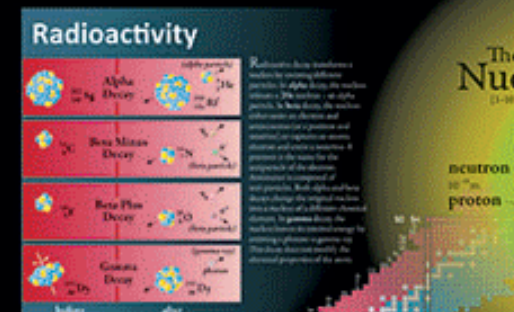
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第二届核结构与反应少体问题研讨会
惠州

2025. 1. 13-1. 19

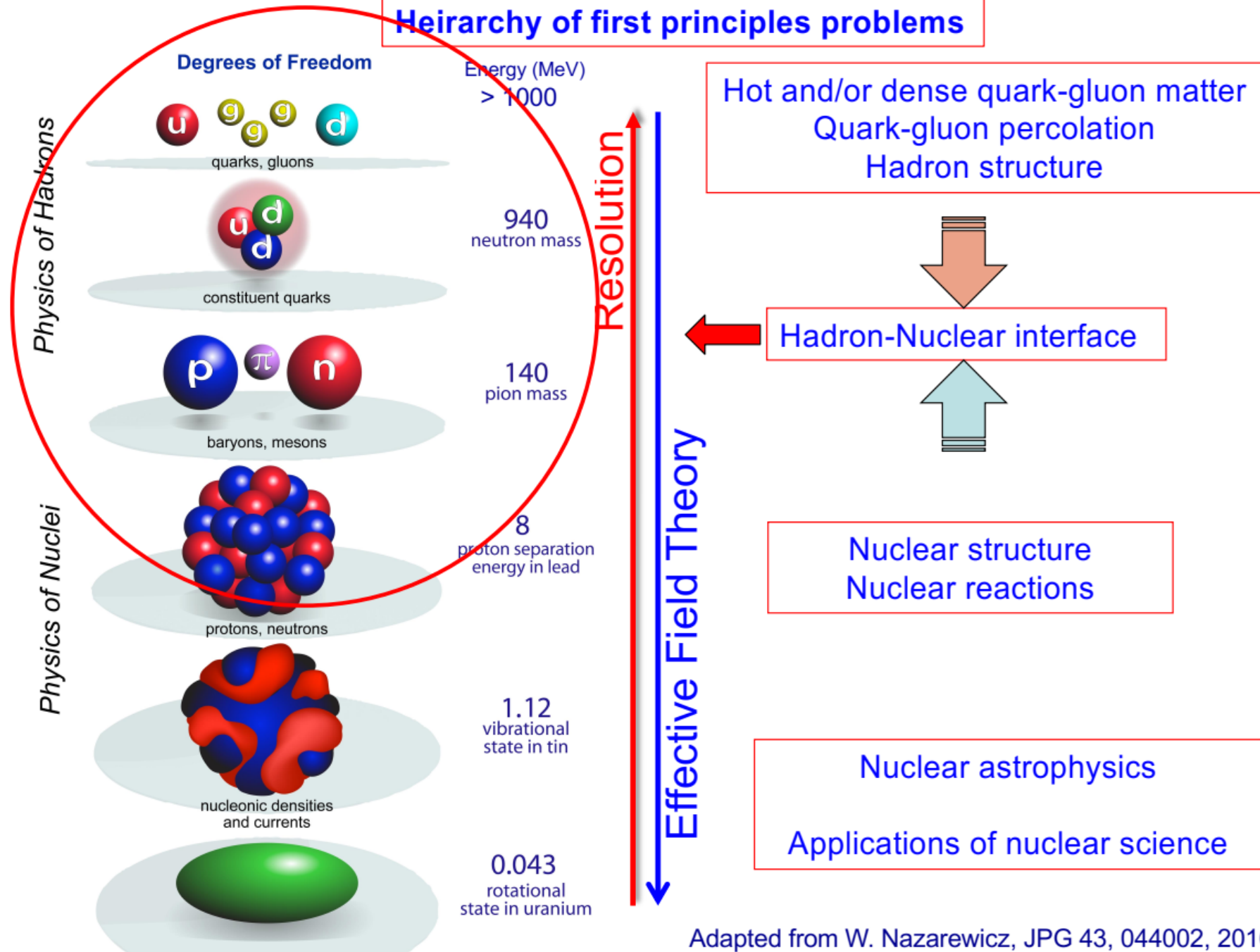
Nuclear Science

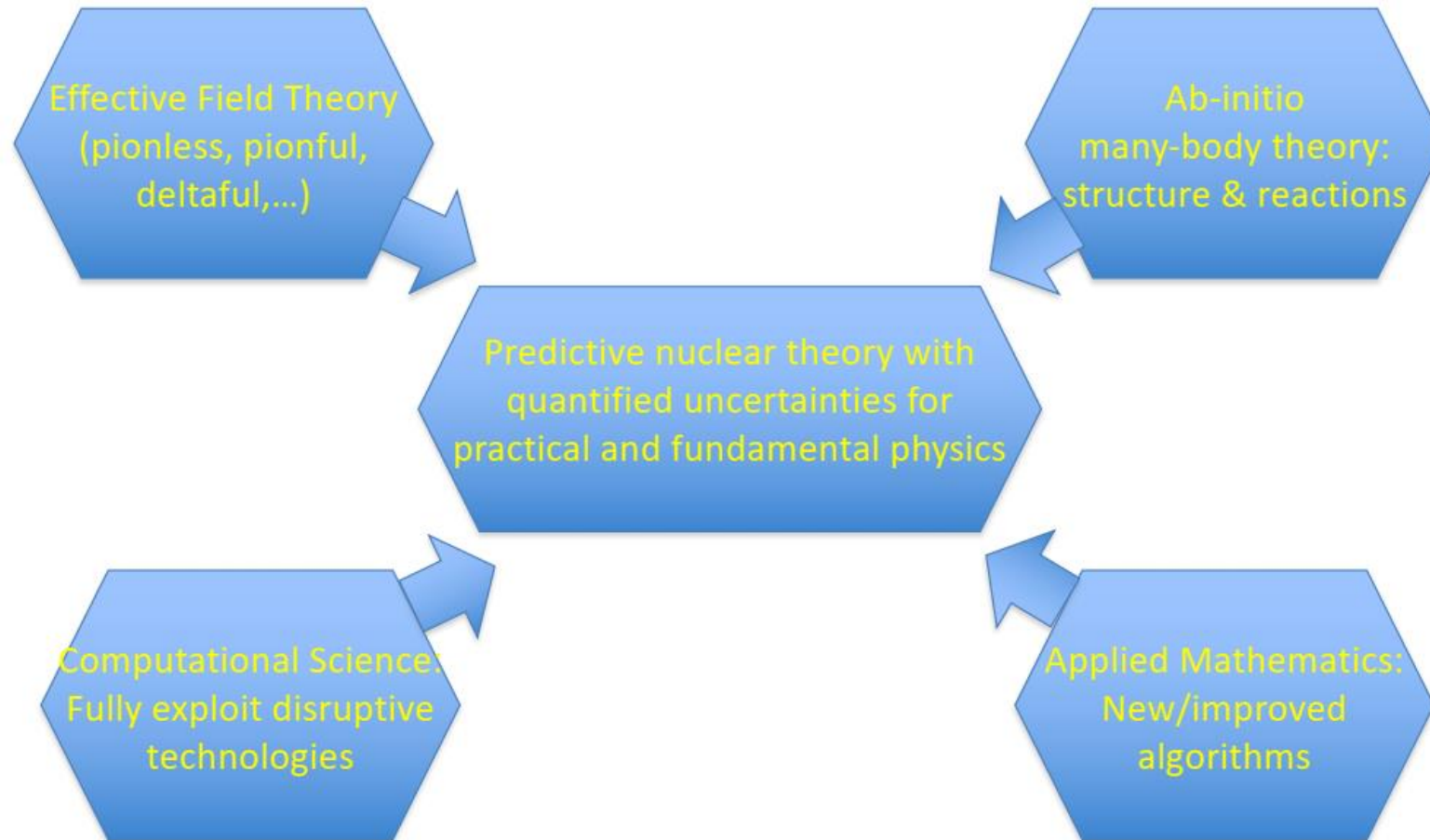


Outline

- No-Core Shell Model
- Long range operators
- Sum rules with NCSM
- Two-body currents of LENPIC interaction
- Monopole transition form factor of ^4He
- Time-dependent basis function for nuclear reactions

Heirarchy of first principles problems





Non-relativistic *ab initio* approaches

➤ Few-body methods

- Faddeev Equation for $A=3$ system
 - typically in momentum space
- Faddeev-Yakuboski Equations for $A=4$ system
 - can nowadays be pushed to $A=5$ and 6 (Lazauskas)
- Hyperspherical Harmonics $A=6$

➤ Many-body methods

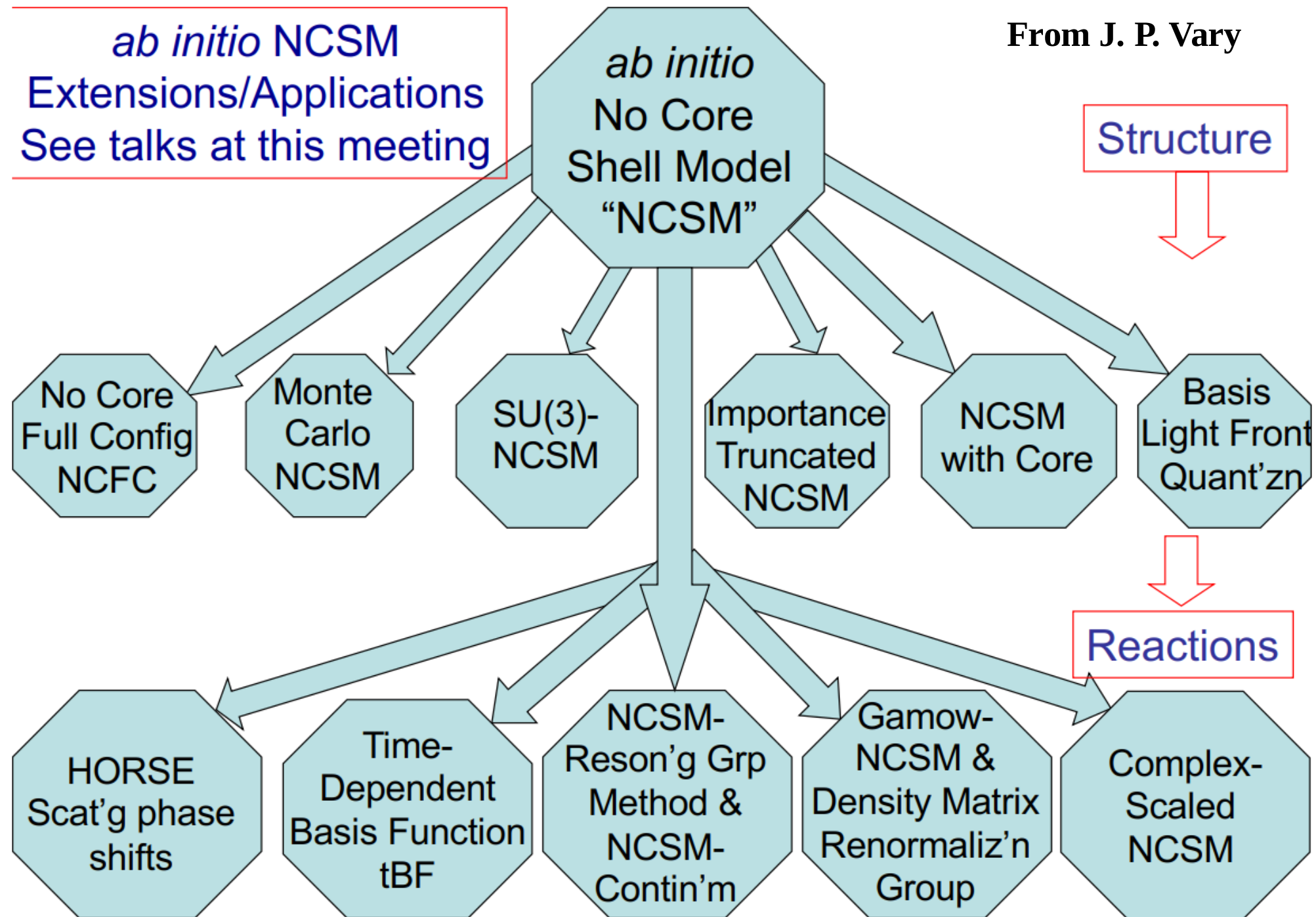
- Variational Monte-Carlo ($A \leq 12$)
- Green's Function Monte-Carlo ($A \leq 12$)
- Configuration Interaction (CI) methods (NCSM ($A \leq 20$), Coupled Cluster)
- Nuclear Lattice Simulations
- In-Medium SRG (IM-SRG)

➤ Supercomputer Era

➤ Quantum computing Era?

ab initio NCSM
Extensions/Applications
See talks at this meeting

From J. P. Vary



***Ab initio* No-Core Shell Model**

No-Core Shell model

$$\hat{\mathbf{H}}_{\text{rel}} = \hat{\mathbf{T}}_{\text{rel}} + \sum_{i < j} V_{ij} + \sum_{i < j < k} V_{ijk} + \dots$$

- Expand wavefunction in basis states $|\Psi\rangle = \sum a_i |\Phi_i\rangle$
- Express Hamiltonian in basis $\langle \Phi_j | \hat{\mathbf{H}} | \Phi_i \rangle = H_{ij}$
- Diagonalize Hamiltonian matrix H_{ij}
- No-Core: All A nucleons are treated equally
- Complete basis: exact results (Caveat: complete basis is infinite dimensional)
- In practice:
 - Truncate basis
 - Study behavior of observables as function of truncation
- Computational challenge:
 - Construct large ($10^{10} \times 10^{10}$) sparse symmetric matrix
 - Obtain lowest eigenvalues & -vectors corresponding to low-lying spectrum and eigenstates

P. Navratil, J. P. Vary and B.R. Barrett,
Phys. Rev. Lett. **84**, 5728 (2000);
Phys. Rev. C **62**, 054311 (2000)

Basis Expansion

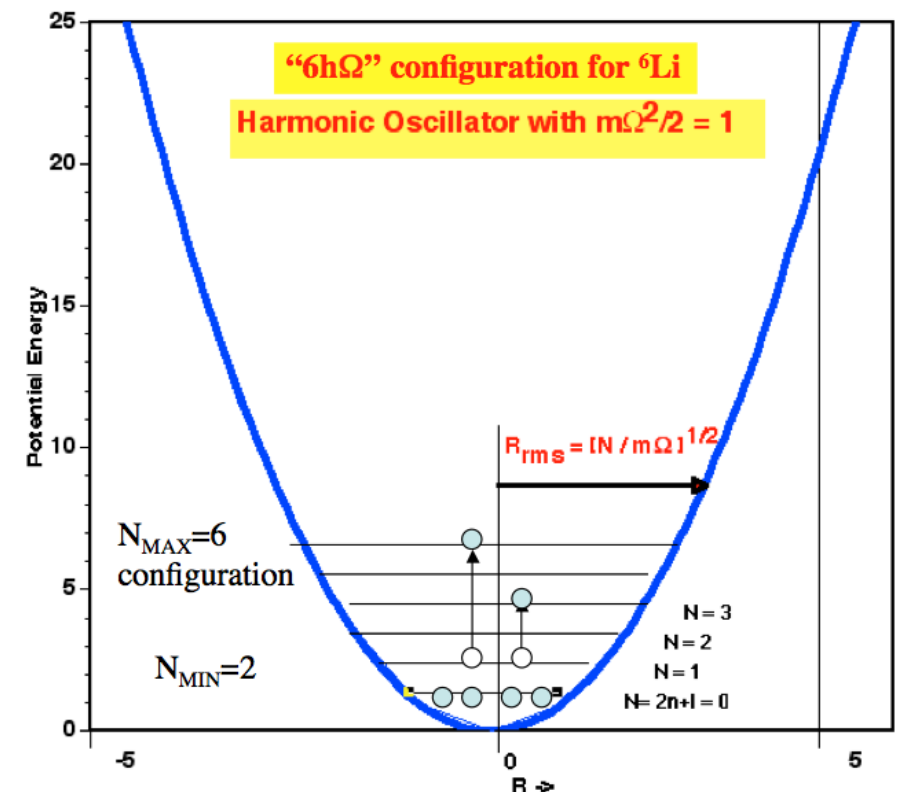
- Expand A -body wave function in basis functions

$$\Psi(r_1, \dots, r_A) = \sum a_i \Phi_i(r_1, \dots, r_A)$$

- Use basis of single Slater Determinants of Single-Particle states

$$\Phi_i(r_1, \dots, r_A) = \frac{1}{\sqrt{(A!)}} \begin{vmatrix} \phi_{i1}(r_1) & \phi_{i2}(r_1) & \dots & \phi_{iA}(r_1) \\ \phi_{i1}(r_2) & \phi_{i2}(r_2) & \dots & \phi_{iA}(r_2) \\ \vdots & \vdots & & \vdots \\ \phi_{i1}(r_A) & \phi_{i2}(r_A) & \dots & \phi_{iA}(r_A) \end{vmatrix}$$

- Single Particle basis states $\phi_{ik}(r_k)$:
 - HO, Coulomb-Sturmian, Natural orbitals
- **M-scheme**: Many-body basis states: eigenstates of J_z
- **$N_{\max}$ truncation**: $\sum_{k=1}^A (2n_{ik} + l_{ik}) \leq N_0 + N_{\max}$
 - Exact factorization of CoM in HO basis
 - Suitable for excited states



Main Challenges

- 2x [AMD EPYC 7763](#) (Milan) CPUs
- 64 cores per CPU
- AVX2 instruction set
- 512 GB of DDR4 memory total
- 204.8 GB/s memory bandwidth per CPU
- 1x [HPE Slingshot 11](#) NIC
- PCIe 4.0 NIC-CPU connection
- 39.2 GFlops per core
- 2.51 TFlops per socket
- 4 NUMA domains per socket (NPS=4)

Partition	# of nodes	CPU	GPU
GPU	1536	1x AMD EPYC 7763	4x NVIDIA A100 (40GB)
	256	1x AMD EPYC 7763	4x NVIDIA A100 (80GB)
<u>CPU</u>	<u>3072</u>	2x AMD EPYC 7763	-
Login	40	1x AMD EPYC 7713	1x NVIDIA A100 (40GB)



- Computationally demanding => needs new algorithms & high-performance computers
- Requires convergence assessments and extrapolation tools to retain predictive power
- Current limit 10^{14} (Perlmutter):
 ^{12}C ($N_{\text{max}}=12$), ^6He ($N_{\text{max}}=22$)
- Achievable for nuclei up to $A \sim 20$ with largest computers available

Scientific Computing in the USA

DOE Computer facilities for scientific computing

- ▶ National Energy Research Scientific Computing center (**NERSC**)
 - ▶ located at LBNL
 - ▶ about 9,000 users
 - ▶ many different small and medium-size projects
 - ▶ most computing time allocated to DOE Office of Science
 - ▶ 12.5% Nuclear Physics (2022)
 - ▶ 74 different Nuclear Physics projects (2022)
- ▶ Oakridge Leadership Computing Facility (**OLCF**)
 - ▶ relatively small user base
 - ▶ limited number of large projects
- ▶ Argonne Leadership Computing Facility (**ALCF**)
 - ▶ relatively small user base
 - ▶ limited number of large projects

also NSF Computer facilities,
as well as local university computing resources

Supercomputing Era from a personal perspective

P. Maris, NTSE2024, Busan

► NERSC

- Seaborg (2002-2008)
IBM SP, 416 nodes, 6,656 cores, 6,656 GB
- Franklin (2009-2012)
Cray XT4, 9,572 nodes, 38,288 cores, 76,576 GB
- Hopper (2010-2015)
Cray XE6, 6,384 nodes, 153,216 cores, 216,832 GB
- Edison (2013-2019)
Cray XC30, 5,200 nodes, 124,800 cores, 332,800 GB
- Cori-KNL (2016-2023)
Cray XC40, 9,688 nodes, 658,784 cores, 893 TB
- Perlmutter-GPU (2022-), NVIDIA GPUs
HPE Cray EX, 1,792 GPU nodes, 768 TB
- Perlmutter (2022-):
HPE Cray EX, 3,072 CPU nodes, 1,536 TB
- NERSC-10 planned for 2026

OLCF and ALCF systems

P. Maris, NTSE2024, Busan

▶ OLCF

- ▶ Jaguar (2008-2012) Cray XT5, 37,538 nodes, 150,152 cores
 - ▶ first supercomputer to run scientific application at petaflop rate
 - ▶ nuclear theory early science project: $^{14}\text{C} \rightarrow ^{14}\text{N} \beta$ decay
- ▶ ...
- ▶ Frontier (2023-), AMD GPUs, 9,402 nodes, 4.5 PB

▶ ALCF

- ▶ Intrepid (2009-2013), IBM Blue Gene/P
- ▶ Mira (2013-2019), IBM Blue Gene/Q
48 racks, 49,152 nodes, 786,432 cores, 768 TB
- ▶ Theta (2017-2023), Cray XC40
4.2k nodes, 270k cores, 874 TB
- ▶ Aurora (2024-), 63,744 Intel Data Center GPUs, > 7 PB

HPC is entering the Exascale computing era !

- ▶ General fortran77 code for (No-Core) Configuration Interaction calculations of many-fermion and many-boson systems, dating back to the early nineties
 - Vary, The Many-Fermion Dynamics Shell-Model Code, ISU, 1992 (unpublished)
 - Vary and Zheng, ISU report, 1994 (unpublished)
- ▶ Contributors in period 1996 - 2006
 - ▶ Petr Navratil (now at TRIUMF, Canada)
 - ▶ Jim Coyle (still at ISU)
 - ▶ Andreas Nogga (now at JUELICH, Germany)
- ▶ Lanczos algorithm for obtaining lowest eigenvalues and vectors
- ▶ Large scale calculations: distributed-memory systems using MPI
- ▶ Mixed precision: matrix elements and vectors in single precision, but overlaps for orthonormalization in double precision
- ▶ Observables (besides energies)
 - ▶ static: r^2 , μ , Q , occupation probabilities, ...
 - ▶ transitions: M1, E2, and GT (assuming isospin symmetry)

Effective Nucleon Interaction

Chiral Perturbation Theory (χ PT)

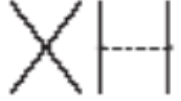
Weinberg's χ PT allows for controlled power series expansion

Expansion parameter : $\left(\frac{Q}{\Lambda_\chi}\right)^v$, Q – momentum transfer,

$\Lambda_\chi \approx 1 \text{ GeV}$, χ – symmetry breaking scale

2N Force 3N Force 4N Force

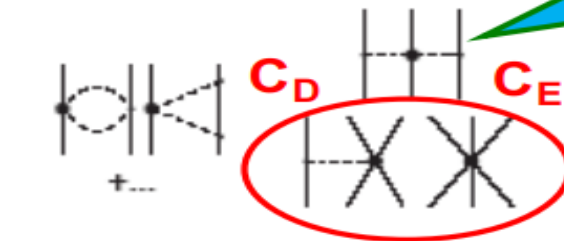
Q^0
LO



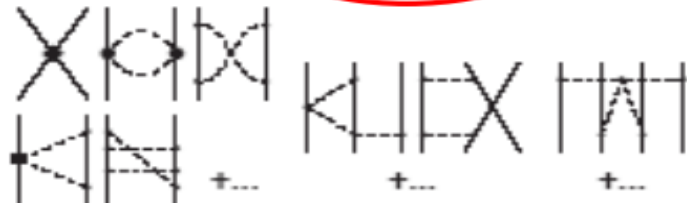
Q^2
NLO



Q^3
NNLO



Q^4
N³LO

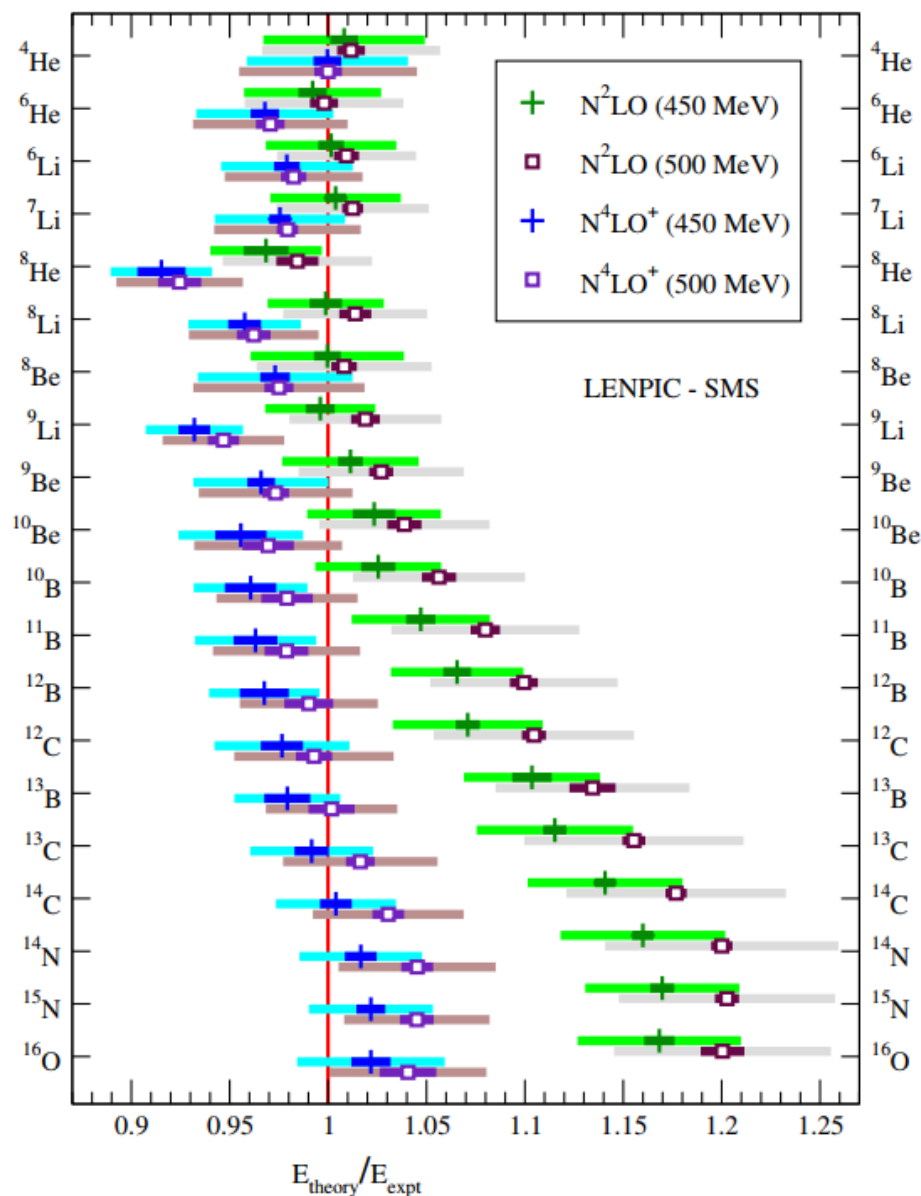


Within χ PT 2π -NNN Low Energy Constants (LEC) are related to the NN-interaction LECs $\{c_i\}$

Additional terms from χ PT with LECs specific to NNN systems

Regularization is essential, which is also implicit within the Harmonic Oscillator (HO) wave function basis (see below)

Binding Energies with LENPIC-SMS chiral EFT



P. Maris, H. Le, A. Nogga, R. Roth, J.P. Vary
Front. Phys. 11, 1098262 (2023)

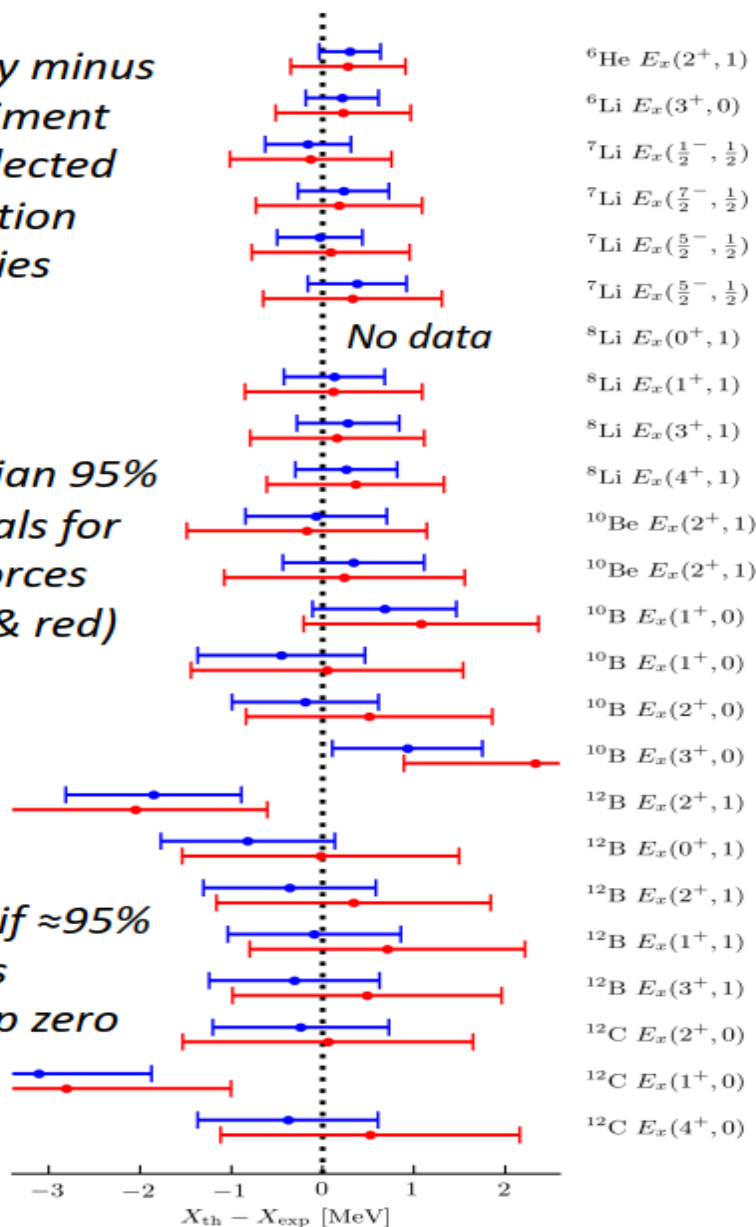
- ▶ NN potential up to $N^4\text{LO}^+$
- ▶ 3NFs at $N^2\text{LO}$
- ▶ SRG evolved to $\alpha = 0.08 \text{ fm}^4$
- ▶ LECs fitted to
 - ▶ NN scattering data
 - ▶ ^3H binding energy
 - ▶ Nd scattering
- ▶ Parameter-free predictions
- ▶ Error bars
 - ▶ numerical uncertainty
 - ▶ chiral EFT uncertainty from Bayesian analysis

Excitation energies from effective field theory with quantified uncertainties

Theory minus
experiment
for selected
excitation
energies

Bayesian 95%
intervals for
two forces
(blue & red)

Check if $\approx 95\%$
of bars
overlap zero



Objectives

- Predict properties of ground and excited states of light nuclei with robust theoretical error estimates.
- Test consistent [LENPIC](#) chiral effective field theory (EFT) interactions with 2- and 3-nucleon forces.
- Extend and test a Bayesian statistical model that learns from the order-by-order EFT convergence pattern to account for correlated excitations.

Impact

- First test of novel chiral nucleon-nucleon potentials with consistent three-nucleon forces.
- Demonstrates understanding of theoretical uncertainties due to chiral EFT expansion.
- Accounting for correlations produces agreement with experimental excitation energies (see figure).
- Exceptions in ^{12}C and ^{12}B indicate different theoretical correlations in the nuclear structure.

Accomplishments

P. Maris et al, Phys. Rev. C **103**, 054001 (2021);
Editors' Suggestion; arXiv: 2012.12396 [nucl-th]

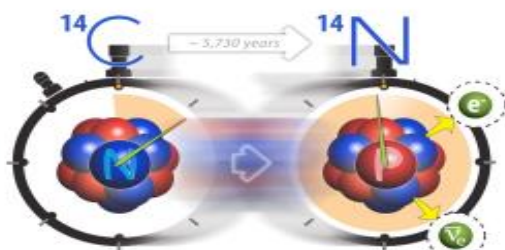


Origin of the Anomalous Long Lifetime of ^{14}C

P. Maris,¹ J. P. Vary,¹ P. Navrátil,^{2,3} W. E. Ormand,^{3,4} H. Nam,⁵ and D. J. Dean⁵

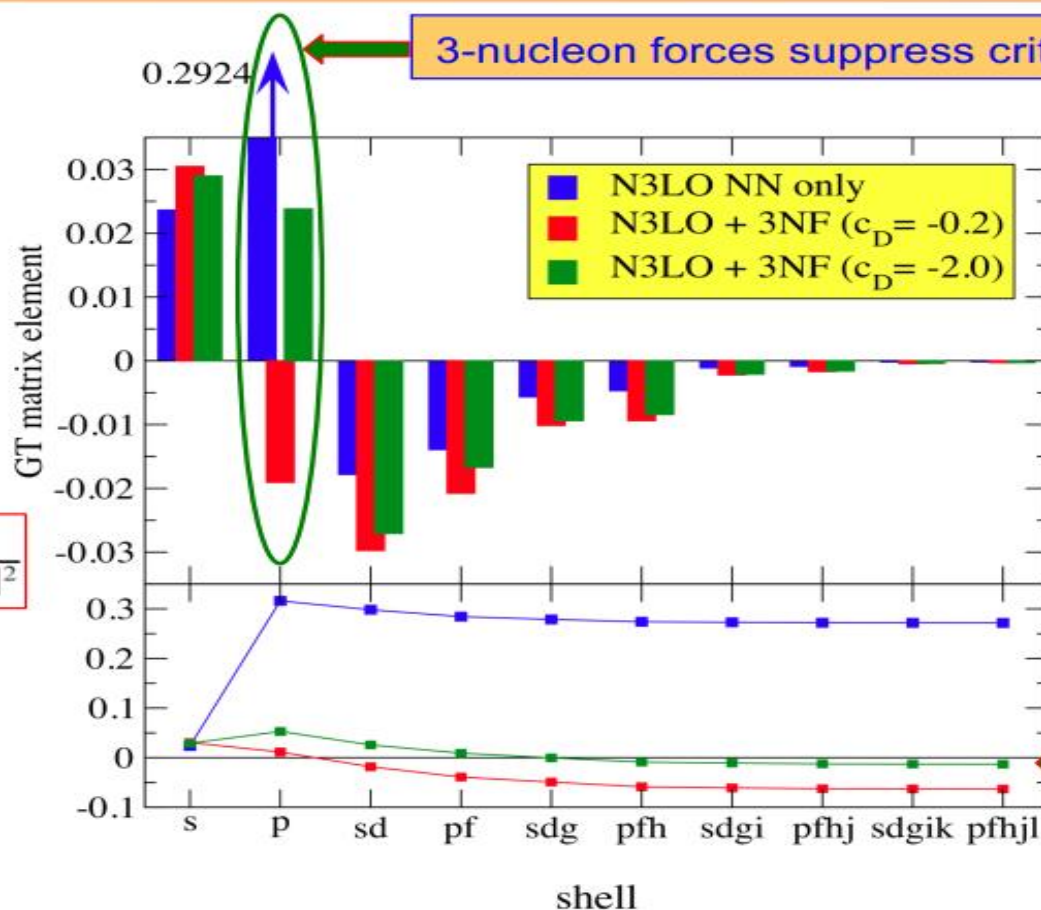


- Solves the puzzle of the long but useful lifetime of ^{14}C
- Establishes a major role for strong 3-nucleon forces in nuclei
- Strengthens foundation for guiding experiments



$$T_{1/2} = \frac{1}{f(Z, E_0)} \frac{2\pi^3 \hbar^7 \ln 2}{m_e^5 c^4 G_V^2} \frac{1}{g_A^2 |M_{GT}|^2}$$

$$M_{GT} = \sum_k \langle \Psi_f || \sigma(k) \tau_+(k) || \Psi_i \rangle$$



3-nucleon forces suppress critical component

- Dimension of matrix solved for 8 lowest states $\sim 1 \times 10^9$
- Each run takes ~ 6 hours on 215,000 cores on Cray XT5 Jaguar at ORNL
- "Scaling of *ab initio* nuclear physics calculations on multicore computer architectures," P. Maris, M. Sosonkina, J. P. Vary, E. G. Ng and C. Yang, 2010 Intern. Conf. on Computer Science, Procedia Computer Science 1, 97 (2010)

net decay rate is very small

Daejeon16 NN interaction

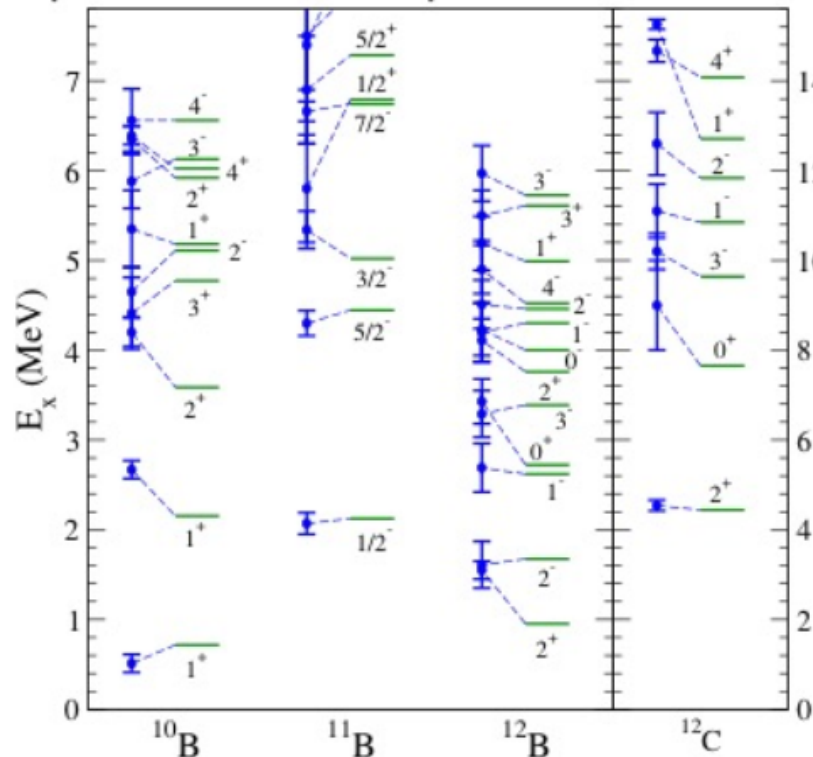
Based on SRG evolution of Entem-Machleidt “500” chiral N3LO to $\lambda = 1.5 \text{ fm}^{-1}$ followed by Phase-Equivalent Transformations (PETs) to fit selected properties of light nuclei.

A.M. Shirokov, I.J. Shin, Y. Kim, M. Sosonkina, P. Maris and J.P. Vary,
“N3LO NN interaction adjusted to light nuclei in ab exitu approach,”
Phys. Letts. B 761, 87 (2016); arXiv: 1605.00413

Application to excited states of p-shell nuclei

(Maris, Shin, Vary, in preparation)

Spectra of B isotopes and ^{12}C

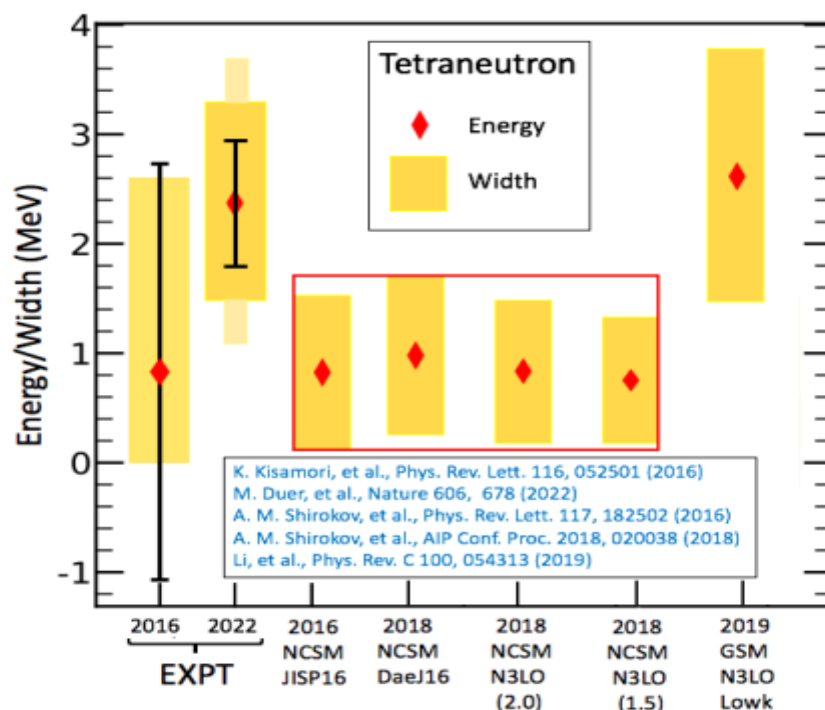


- ▶ difference of extrapolated E_b
- ▶ extrapolation uncertainties: max of E_b uncertainties
- ▶ good agreement with positive and negative parity spectra
- ▶ need large bases for 'intruder' and 'non-normal parity' states
- ▶ spectrum ^{10}B
 - ▶ correct gs 3^+ and excited 1^+
 - ▶ third 1^+ 'intruder' state
- ▶ excited 0^+ state in ^{12}C
 - ▶ Hoyle state?
 - ▶ see MCNCSM results below

Tetraneutron discovery confirms prediction

Objectives

- *Ab initio* nuclear theory aims for parameter-free predictions of nuclear properties with controlled uncertainties using supercomputer simulations
- Specific goal is to predict if the tetraneutron (4-neutron system) has a bound state, a low-lying resonance or neither



Experiment and theory for the tetraneutron's resonance energy and width. *Ab initio* No-Core Shell Model (NCSM) and Gamow Shell Model (GSM) predictions use different neutron-neutron interactions and different basis function techniques.

Impact

- Discovery in 2022 announced in Nature [1] confirms *ab initio* theory predictions from 2016 [2] of a short-lived tetraneutron resonance at low energy and the absence of a tetraneutron bound state
- Demonstrates the predictive power of *ab initio* nuclear theory since theory and experiment are within their combined uncertainties
- Sets stage for further experimental and theoretical research on new states of matter formed only of neutrons
- Shows need to anticipate a long wait time for experimental confirmation of such an exotic phenomena, ~ 6 years in this case
- Emphasizes the value of DOE supercomputer allocations (NERSC) and support for multi-disciplinary teamwork (SciDAC/NUCLEI)

Accomplishments

- [1] M. Duer, et al., Nature 606, 678 (2022)
- [2] A.M. Shirokov, G. Papadimitriou, A.I. Mazur, I.A. Mazur, R. Roth and J.P. Vary, "Prediction for a four-neutron resonance," Phys. Rev. Lett. 117, 182502 (2016)



Alpha clusters in Carbon-12 from *ab initio* theory & statistical learning

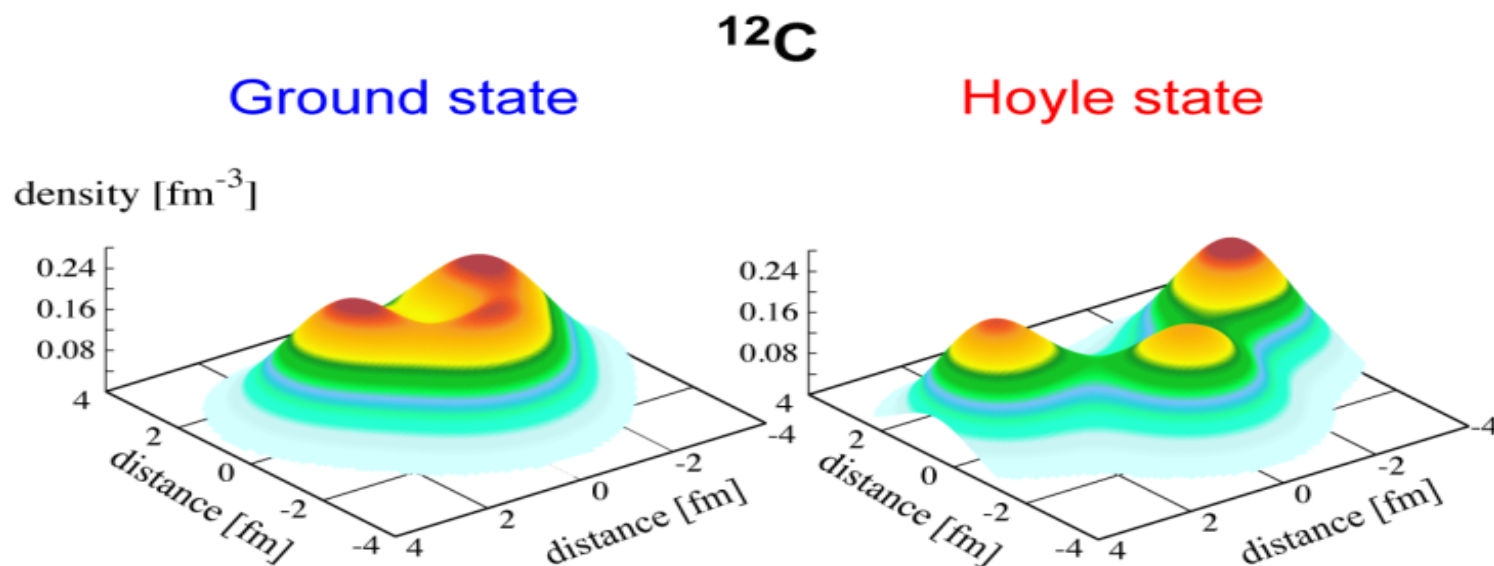
Objectives

- *Ab initio* nuclear theory aims for parameter-free predictions of critical nuclear properties with controlled uncertainties using supercomputer simulations
- Specific goal is to determine extent of alpha clustering in the Ground state and the Hoyle state of Carbon-12 (^{12}C)

Impact

- Ground state found to have 6% alpha clustering while Hoyle state discovered to be 3- α s 61% of the time
- With this high percentage of 3- α s, the Hoyle state is confirmed as a natural gateway state for the cosmic formation of ^{12}C , the key element for organic life
- Statistical learning confirms 3- α feature of Hoyle state

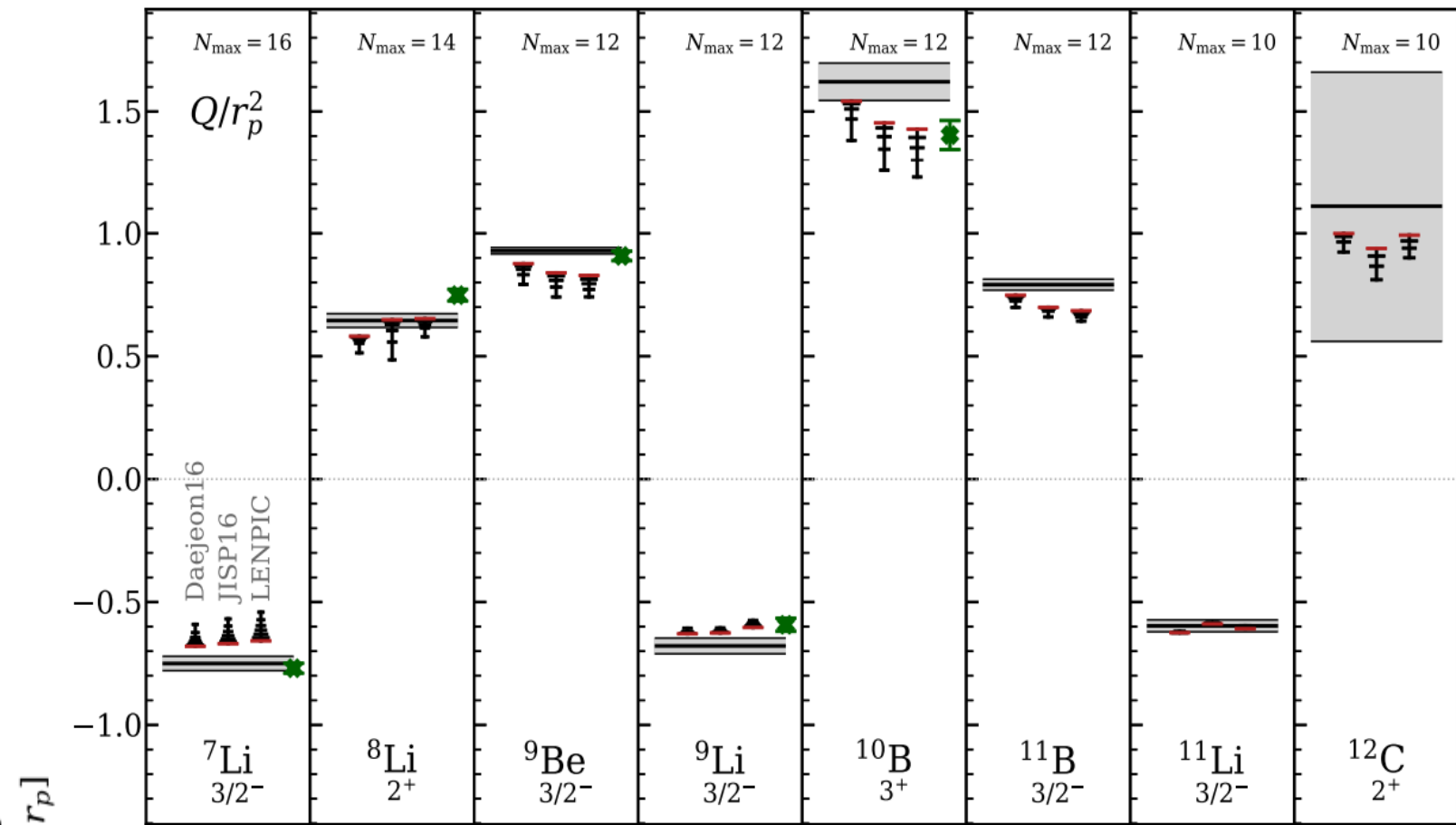
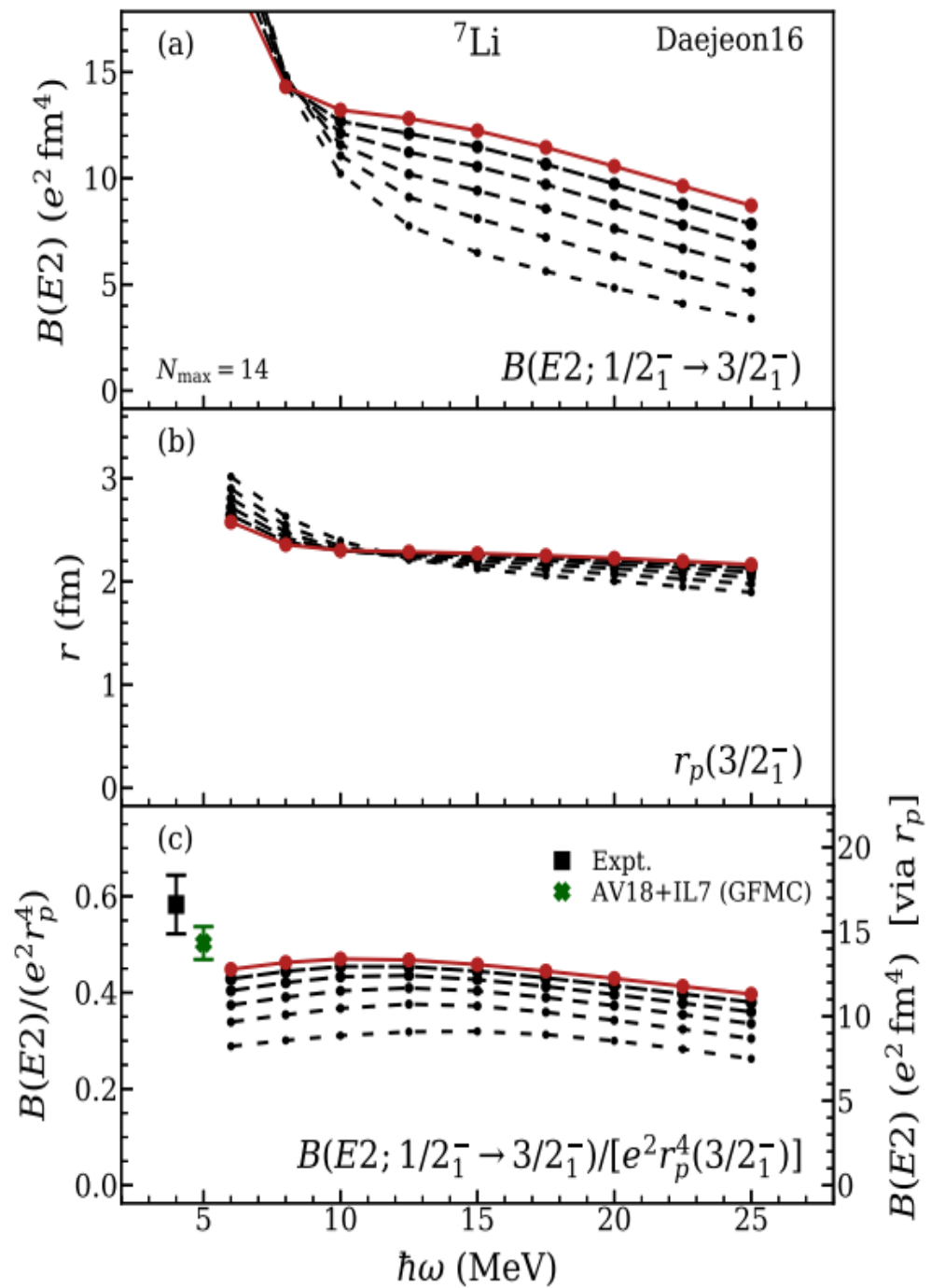
Ab initio Monte-Carlo Shell Model results for density contours of ^{12}C Ground state and first excited 0^+ (Hoyle) state using the Daejeon16 two-nucleon potential. Simulations were performed on Fugaku in Japan, the world's largest supercomputer at the time.



Accomplishments

T. Otsuka, T. Abe, T. Yoshida, Y. Tsunoda, N. Shimizu, N. Itagaki, Y. Utsuno, J. Vary, P. Maris and H. Ueno, "Alpha-Clustering in Atomic Nuclei from First Principles with Statistical Learning and the Hoyle State Character," *Nature Communications* 13:2234 (2022)

Long range operators



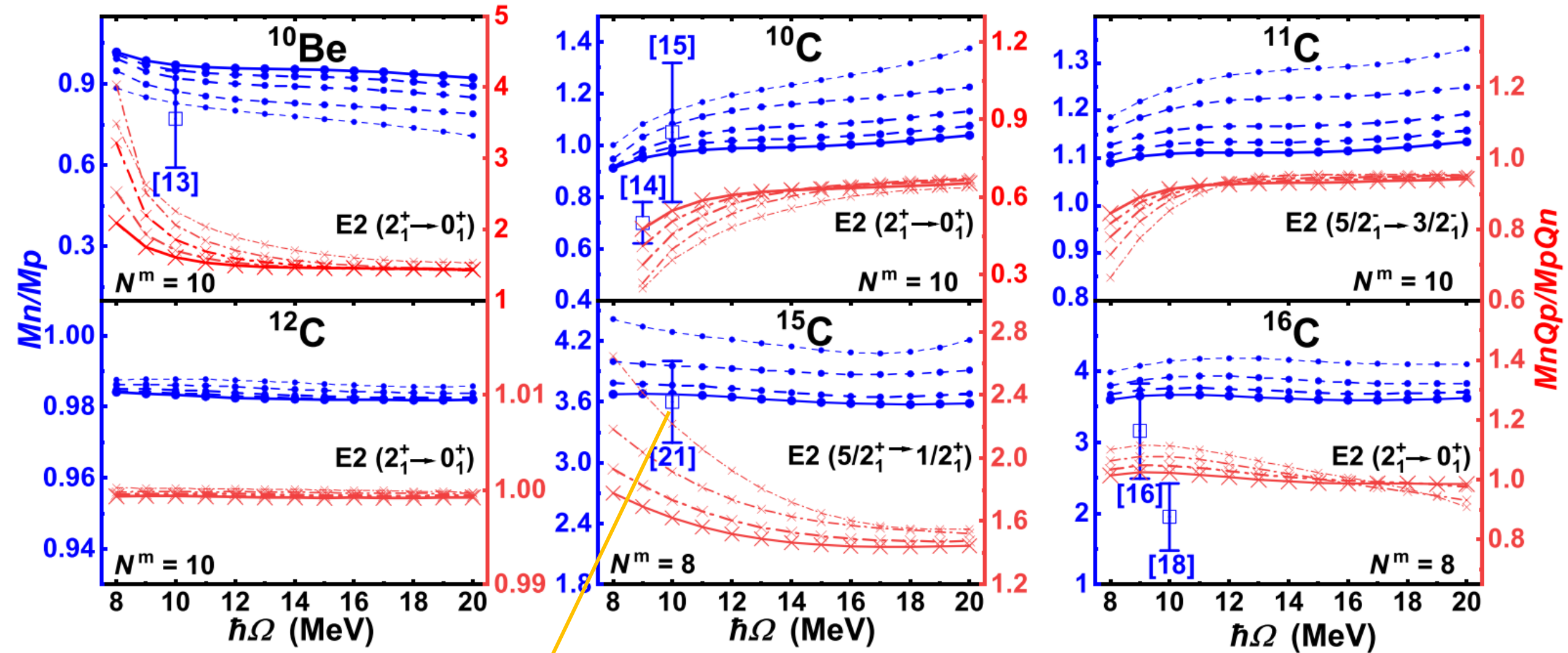
- Long-range observables converge slowly in HO basis
- Ratio's of long-range observables converge significantly better

M. Caprio *et al.*, Phys. Rev. C **105**, L061302 (2022)

¹⁵C Experiment

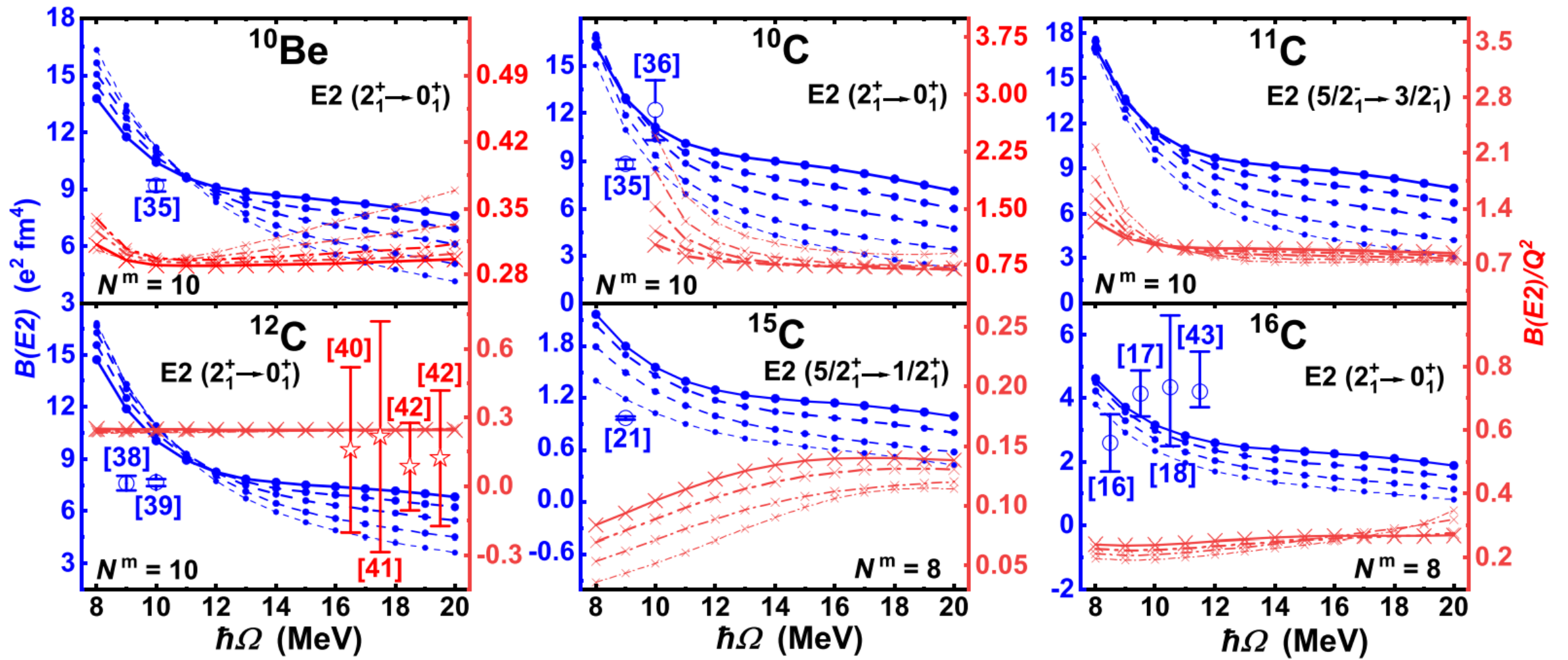
$$\frac{M_n}{M_p} = \frac{\langle J_f || \sum_{i \in n} r_i^2 Y_2(\hat{r}_i) || J_i \rangle}{\langle J_f || \sum_{i \in p} r_i^2 Y_2(\hat{r}_i) || J_i \rangle}$$

$\hbar\Omega$	17 MeV			18 MeV			Extrapolation	Experiment
N _{max}	4	6	8	4	6	8		
E _{g.s.} ($\frac{1}{2}1^+$) [MeV]	-100.034	-104.146	-106.091	-100.194	-104.134	-106.019	-107.793(45)	-106.503 [54]
E _x ($\frac{5}{2}1^+$) [MeV]	0.556	0.941	1.169	0.494	0.908	1.148	1.440(9)	0.740(15) [54]
B(E2; $\frac{5}{2}1^+\rightarrow\frac{1}{2}1^+$) [e ² fm ⁴]	0.699	0.938	1.115	0.658	0.899	1.080	2.025(30)	0.97(2) [28]
$\mu_{\text{g.s.}}$ ($\frac{1}{2}1^+$) [μ_{N}]	-1.723	-1.717	-1.711	-1.720	-1.714	-1.709	-1.633(53)	1.315(70) [55]
μ ($\frac{5}{2}1^+$) [μ_{N}]	-1.464	-1.442	-1.428	-1.467	-1.443	-1.429	-1.407(9)	1.758(30) [55]
M _n /M _p ($\frac{5}{2}1^+\rightarrow\frac{1}{2}1^+$)	3.870	3.649	3.578	3.876	3.652	3.575	3.529(6)	3.6(4)



J. Chen *et al.*, Phys. Rev. C **106**, 064312 (2022)

H. Li, D. Fang, H. J. Ong, A. M. Shirokov, J. P. Vary, P. Yin, X. Zhao, Phys. Rev. C **110**, 064325 (2024)



H. Li, D. Fang, H. J. Ong, A. M. Shirokov, J. P. Vary, P. Yin, X. Zhao, Phys. Rev. C **110** , 064325 (2024)

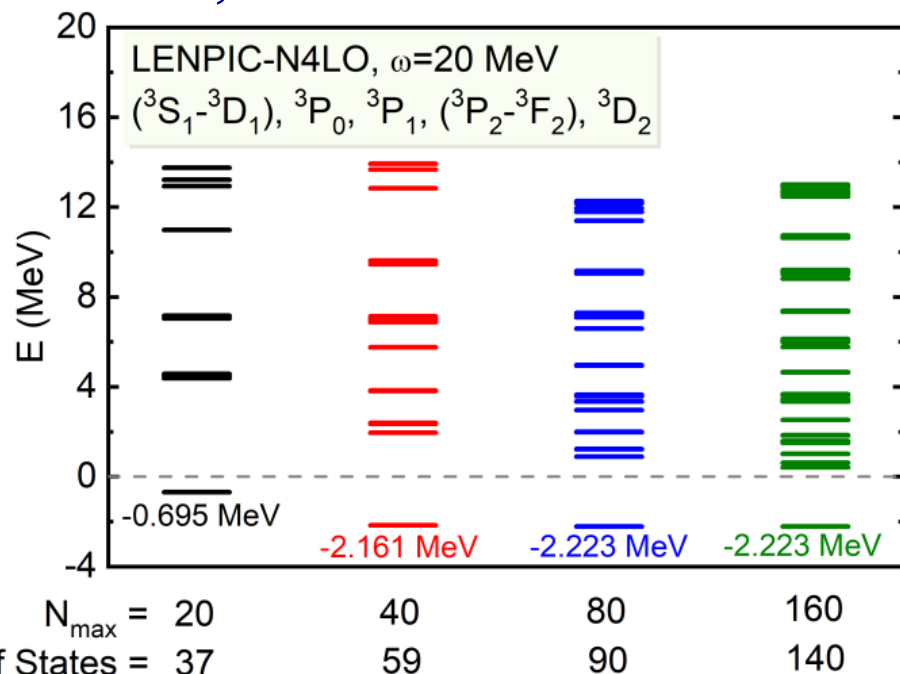
Sum rules with NCSM

Sum rules with NCSM

$$I_M = \sum_{\mu}^M |\langle \mu | \hat{O} | i \rangle|^2 g(\omega_{\mu})$$

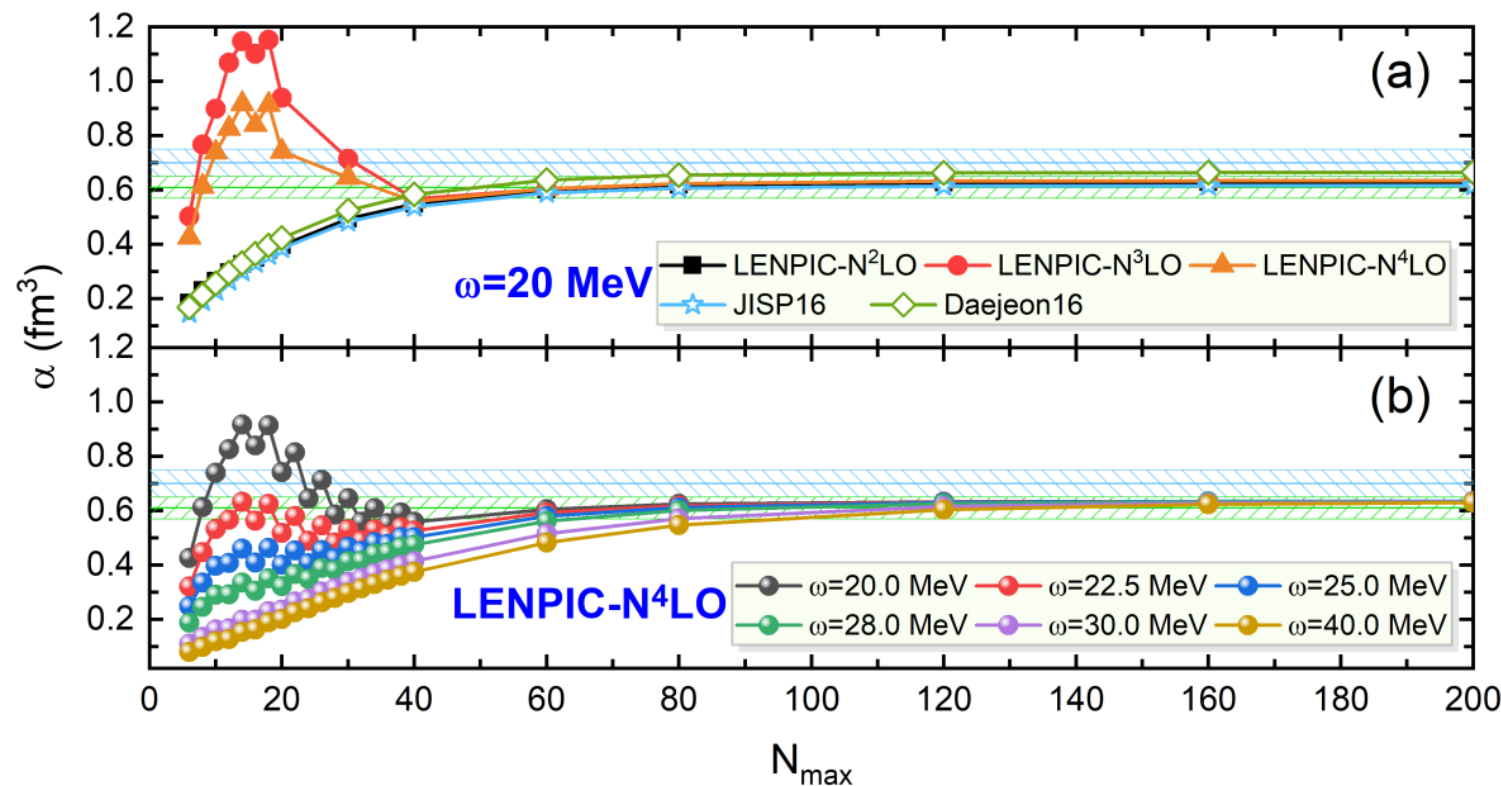
$$\alpha_E = \frac{8\pi}{9} \sum_k \frac{B(E1; J_0 \rightarrow J_k)}{E_k - E_0} = \frac{1}{2\pi^2} \int_{\omega_{\text{th}}}^{\infty} d\omega \frac{\sigma_{\gamma}^{\text{ud}}(\omega)}{\omega^2}$$

- Sum rule: response of nuclei to external field probe
- Direct techniques: only available for deuteron
- Indirect techniques: Lorentz integral transform, Lanczos sum rule method
- $A > 2$, direct calculation?



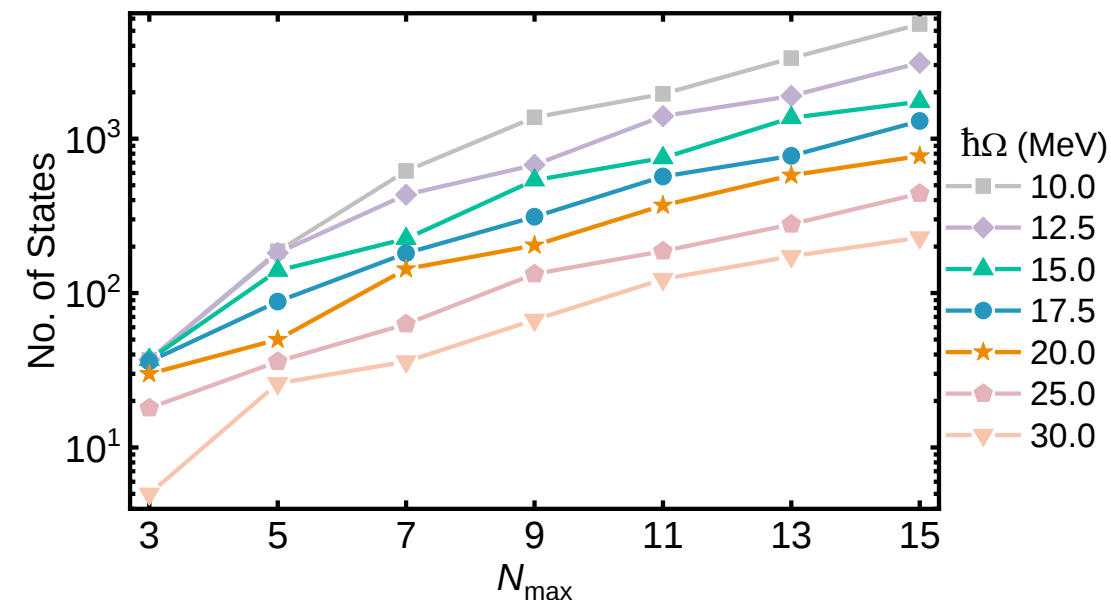
P. Yin *et al*, *J.Phys.G* 49, 125102 (2022)

P. Yin *et al*, [arXiv:2208.00267 [nucl-th]]

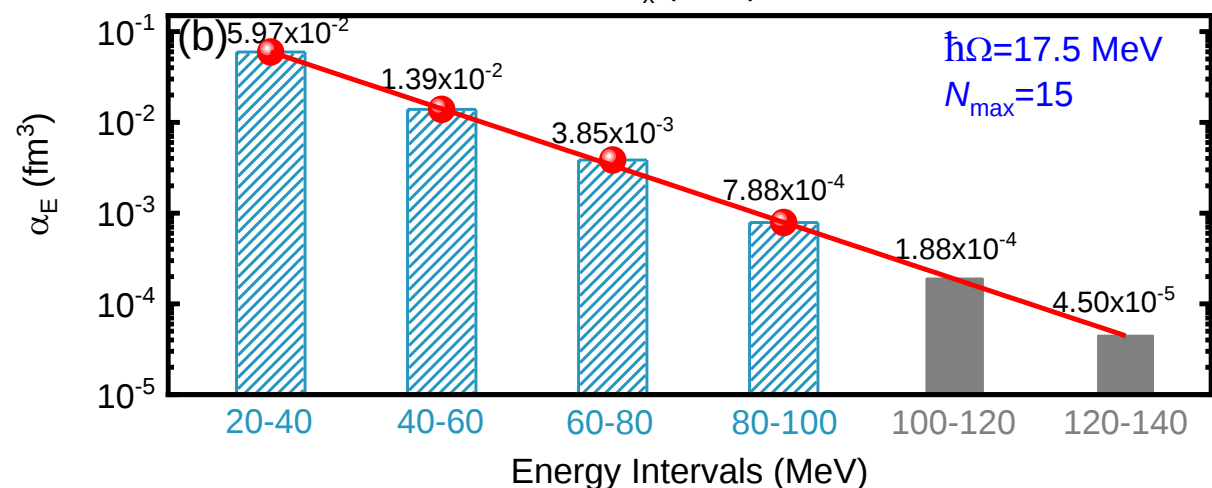
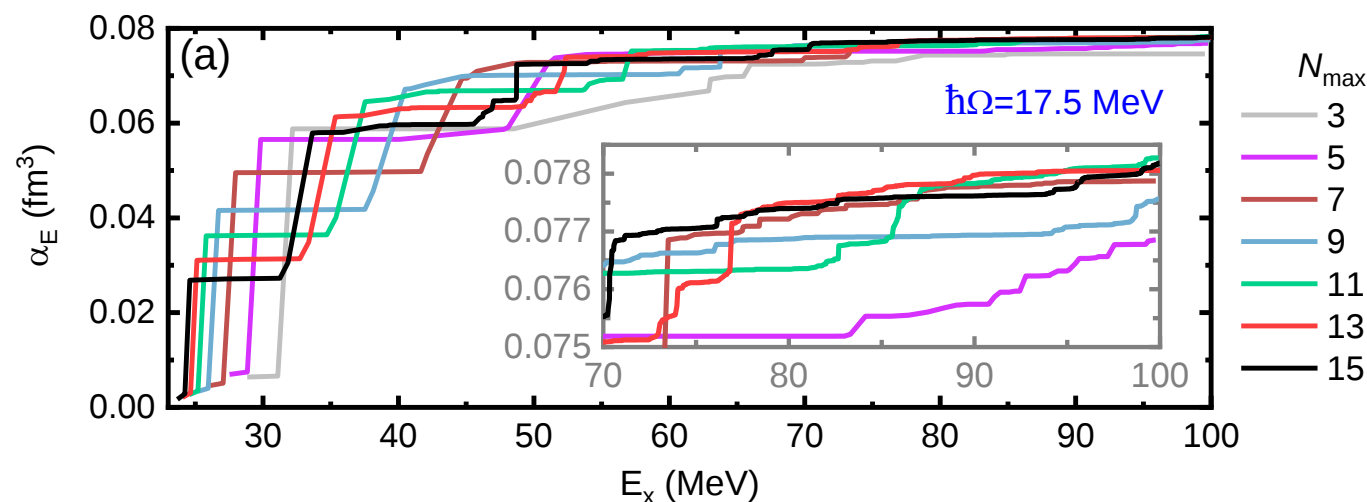


Sum rules with NCSM

- ^4He : polarizability $\alpha_E = \frac{8\pi}{9} \sum_k \frac{B(E1; J_0 \rightarrow J_k)}{E_k - E_0}$
- $E1$ selection rule: 1^- states
- Retain only 1^- states
 - M-scheme with $M=1$ ($1^-, 2^-, \dots$)
 - Lagrangian multiplier $H_{J^2} = \lambda (\mathbf{J}^2 - 2)$

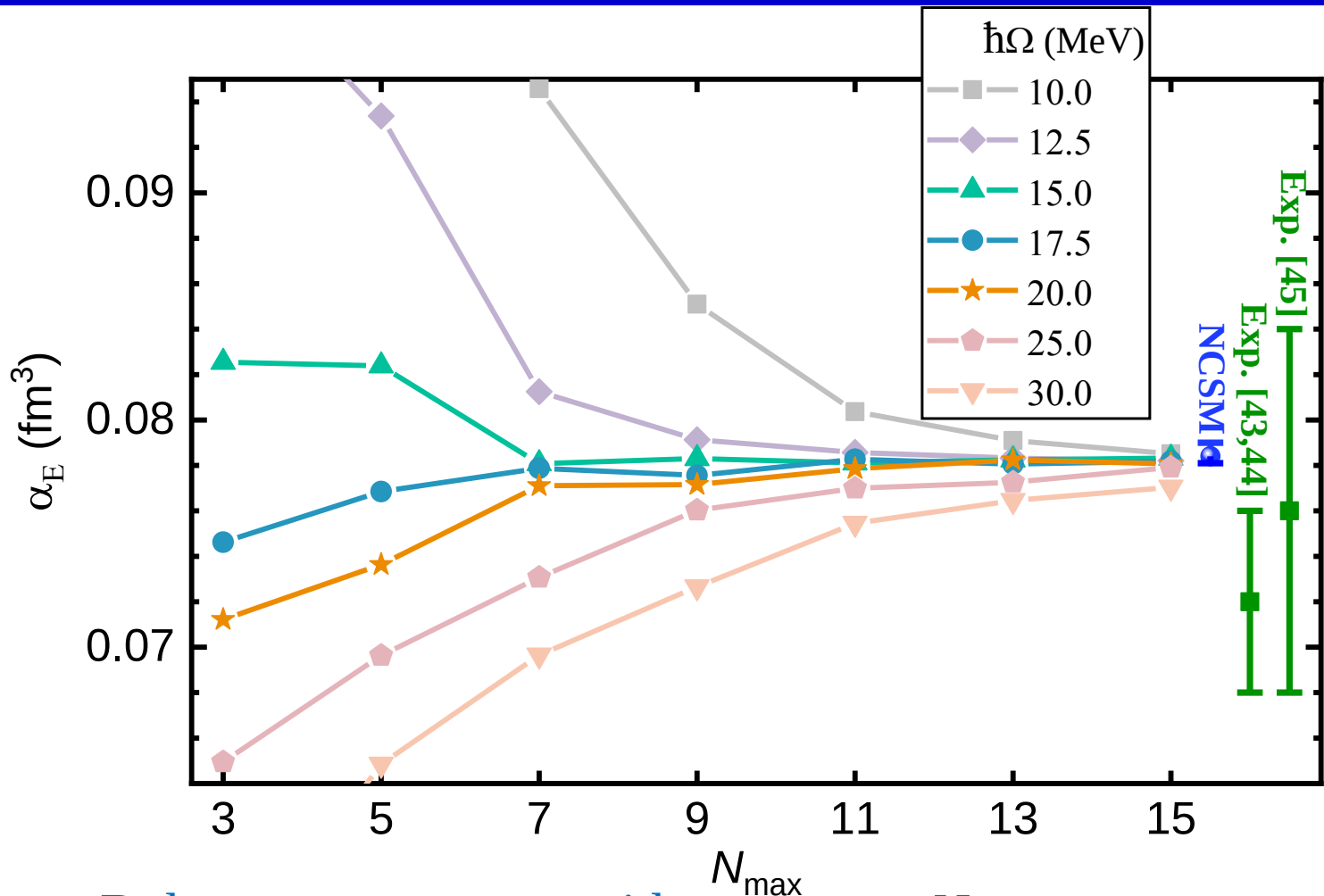


- Up through 100 MeV
- $N_{\max}=15$, $\hbar\Omega=10$ MeV
- 5522 1^- states
- 30 GPU nodes, 16800 Lanczos iterations, 21 hours



P. Yin, A. M. Shirokov, P. Maris, P. J. Fasano, M. A. Caprio, H. Li, W. Zuo, J. P. Vary, Phys. Lett. B **855**, 138857 (2024)

Sum rules with NCSM



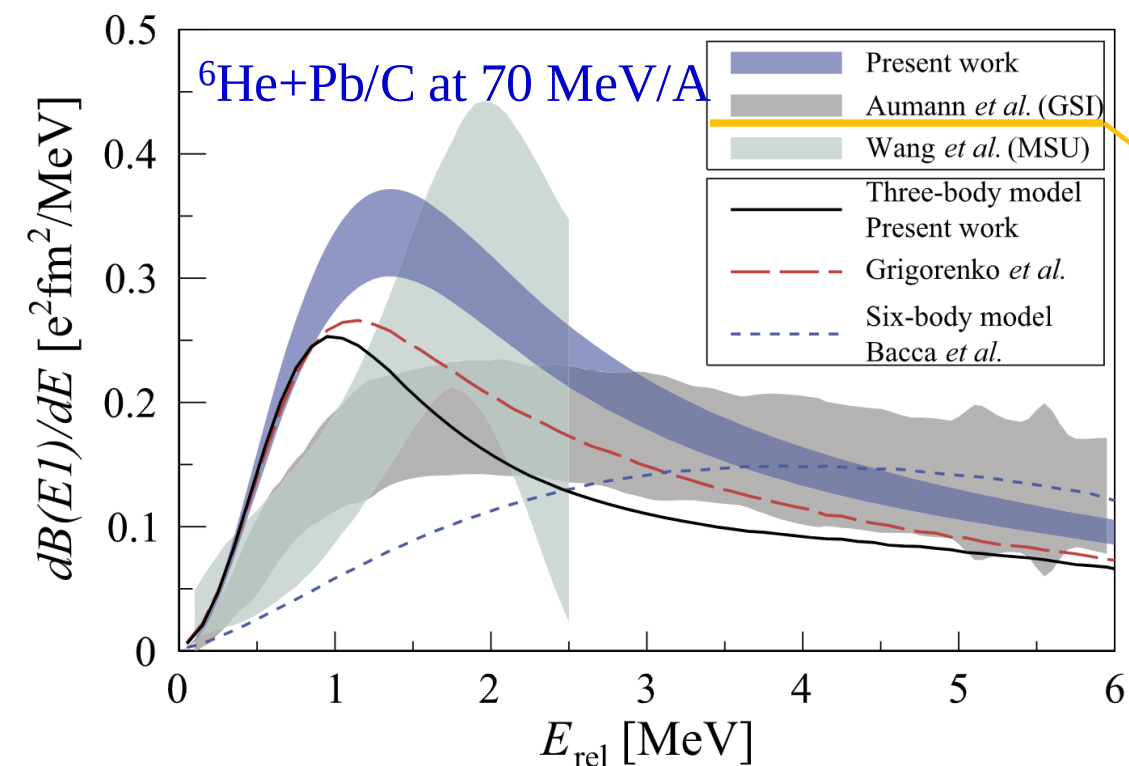
Theo./Exp.	Interaction	α_E (fm ³)
NCSM [This work]	NN (Daejeon16)	0.0782 ^{+0.0006} _{-0.0003}
HH [11]	NN (AV18) + $3N$ (UIX)	0.0655(4)
HH [12]	NN (MT-I/III)	0.076
HH [13]	$[NN (N^3LO) + 3N (N^2LO)]_{OLS}$	0.0694
NCSM [22]	$[NN (N^3LO) + 3N (N^2LO)]_{OLS}$	0.0683(14)
NCSM [23]	$[NN (N^3LO) + 3N (N^2LO)]_{1.8}$	0.07093(5)
	$[NN (N^3LO) + 3N (N^2LO)]_{3.0}$	0.06861(5)
NCSM [24]	$NN (N^3LO)$	0.084(3)
SA-NCSM [24]	$NN (N^3LO)$	0.077(3)
NCSM SA-NCSM [24]	$NN (NNLO_{opt})$	0.0680
CC [16]	$NN + 3N (NNLO_{sat})$	0.0735(1)
Exp. [43, 44]	-	0.072(4)
Exp. [45]	-	0.076(8)

- Robust convergence with respect to N_{max}
- Uncertainty due to N_{max} truncation and energy truncation
- Direct calculation is feasible and robust
- Independent of diagonalization techniques

P. Yin, A. M. Shirokov, P. Maris, P. J. Fasano, M. A. Caprio, H. Li, W. Zuo, J. P. Vary, Phys. Lett. B **855**, 138857 (2024).

Sum rule with NCSM

$$\alpha_E = \frac{8\pi\alpha}{9} \int_{E_T} \frac{dE}{E} \frac{1}{e^2} \frac{dB(E1)}{dE}$$



Y. L. Sun, et al. Phys. Lett. B 814, 136072 (2021)

Fig. 3. The $B(E1)$ distribution obtained by the present work compared with previous results from GSI [5] and MSU [23]. The experimental $B(E1)$ distributions are also compared with the three-body calculations (black solid line from this work, red dashed line from Ref. [90]) and *ab initio* six-body calculation (blue dotted line) [91].

PHYSICAL REVIEW C **86**, 064316 (2012)

Nuclear electric polarizability of ⁶He

R. Goerke,^{1,2,*} S. Bacca,^{2,†} and N. Barnea^{3,‡}

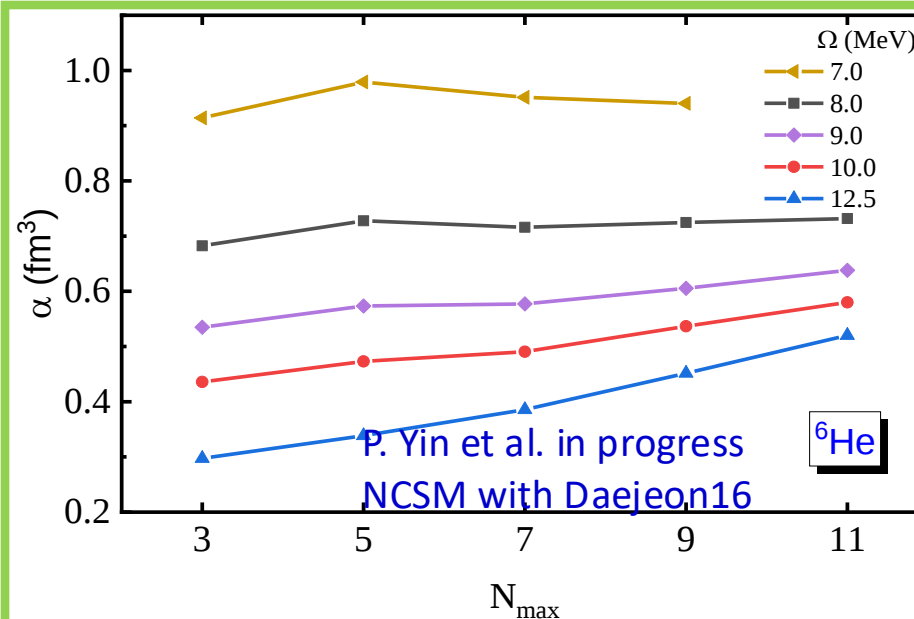
¹Department of Physics, University of Toronto, 60 St. George St., Toronto, Ontario M5S 1A7, Canada

²TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada

³Racah Institute of Physics, Hebrew University, 91904, Jerusalem, Israel

(Received 11 September 2012; revised manuscript received 19 November 2012; published 19 December 2012)

We present an estimate of the nuclear electric polarizability α_E of the ⁶He halo nucleus based on six-body microscopic calculations. Wave functions are obtained from semirealistic two-body interactions using the hyperspherical harmonics expansion method. The polarizability is calculated as a sum rule of the dipole response function using the Lanczos algorithm and also by integrating the photoabsorption cross section calculated via the Lorentz integral transform method. We obtain $\alpha_E = 1.00(14) \text{ fm}^3$, which is much smaller than the published value $\alpha_E^{\text{exp}} = 1.99(40) \text{ fm}^3$ [Pachucki and Moro, Phys. Rev. A **75**, 032521 (2007)] extracted from experimental data. This points toward a potential disagreement between microscopic theories and experimental observations.



- Slow convergence
- $\sim 0.7 \text{ fm}^3$
- Support HH results
- Larger nuclei, total-J

Photonuclear sum rules and the tetrahedral configuration of ^4He Doron Gazit,¹ Nir Barnea,¹ Sonia Bacca,² Winfried Leidemann,^{3,4,*} and Giuseppina Orlandini^{3,4,*}¹*The Racah Institute of Physics, The Hebrew University, 91904 Jerusalem, Israel*²*Gesellschaft für Schwerionenforschung, Planckstr. 1, D-64291 Darmstadt, Germany*³*Department of Physics, George Washington University, Washington DC 20052, USA*⁴*Istituto Nazionale di Fisica Nucleare, Gruppo Collegato di Trento, Italy*

(Received 15 September 2006; published 14 December 2006)

$$m_n(\bar{\omega}) \equiv \int_{\omega_{\text{th}}}^{\bar{\omega}} d\omega \omega^n \sigma_{\gamma}^{\text{E1UR}}(\omega), \quad (1)$$

$$\Sigma^{\text{TRK}} \equiv m_0(\infty) = \frac{\mathcal{G}}{2} \langle 0 | [\mathbf{D}, [H, \mathbf{D}]] | 0 \rangle \quad (4)$$

$$\Sigma^{\text{BSR}} \equiv m_{-1}(\infty) = \mathcal{G} \langle 0 | \mathbf{D} \cdot \mathbf{D} | 0 \rangle \quad (5)$$

$$\Sigma^{\text{PSR}} \equiv m_{-2}(\infty) = \mathcal{G} \sum_n (E_n - E_0)^{-1} |\langle n | \mathbf{D} | 0 \rangle|^2. \quad (6)$$

$$\Sigma^{\text{BSR}} = \mathcal{G} \left[Z^2 \langle r_p^2 \rangle - \frac{Z(Z-1)}{2} \langle r_{pp}^2 \rangle \right] \quad (14)$$

$$\langle r_{pp}^2 \rangle \equiv \frac{1}{Z(Z-1)} \langle 0 | \sum_{i,j=1}^Z (\mathbf{r}_i - \mathbf{r}_j)^2 | 0 \rangle. \quad (15)$$

 r_{pp} with NCSM: Daejeon 16 ^4He : 2.487 fm ^6He : 2.645 fm **r_p with NCSM: Daejeon 16** ^4He : 1.90 fm ^6He : 1.514 fm

M. Huang, T. Frederico, P. Maris, P. Yin, R. Basili, P. Fasano, J. P. Vary, in preparation

TABLE I
Numerical values of $\langle r_{pp}^2 \rangle^{1/2}$, $\langle r'^2 \rangle^{1/2}$ and ρ as extracted from experimental values of σ_{-1} and $\mathcal{R}$

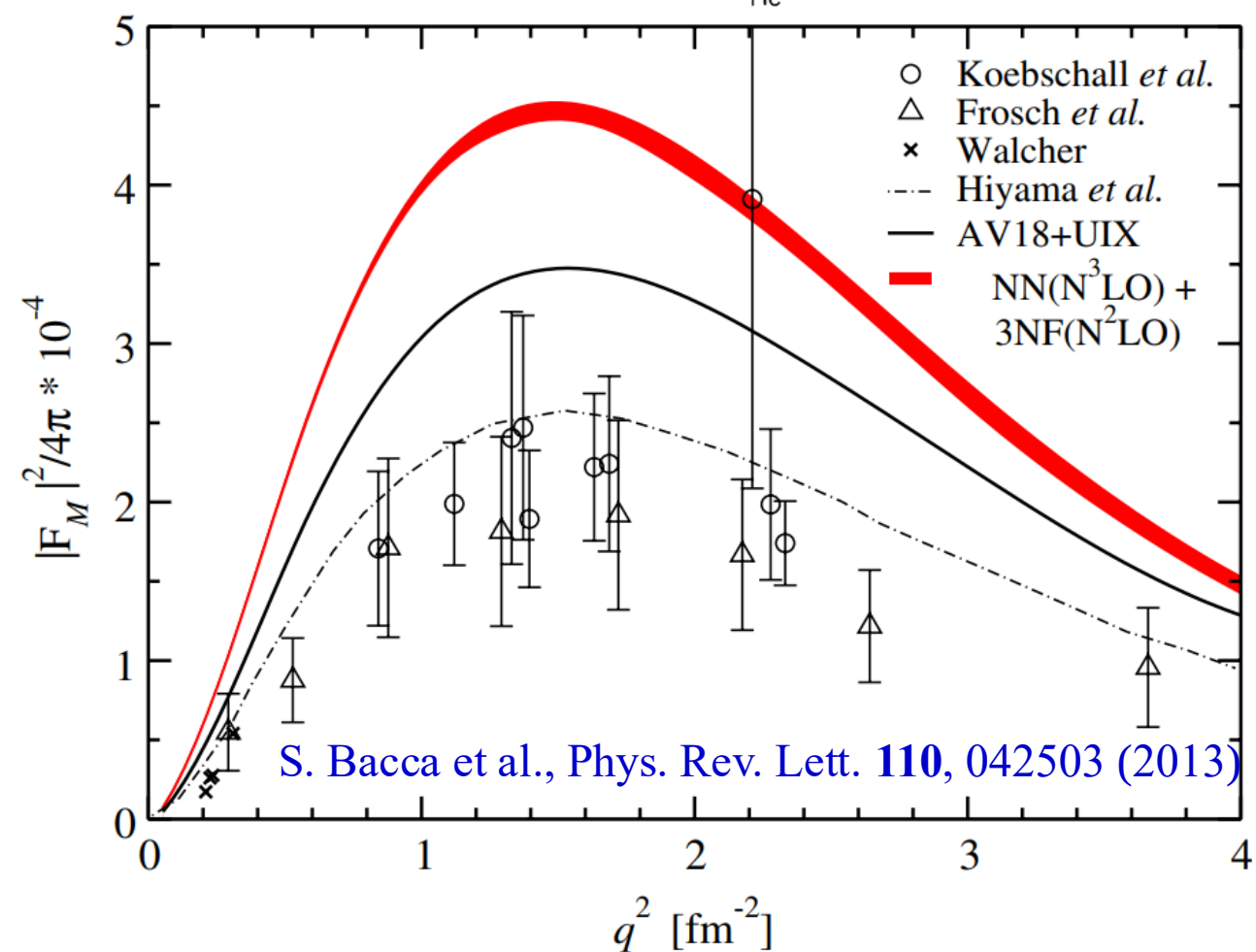
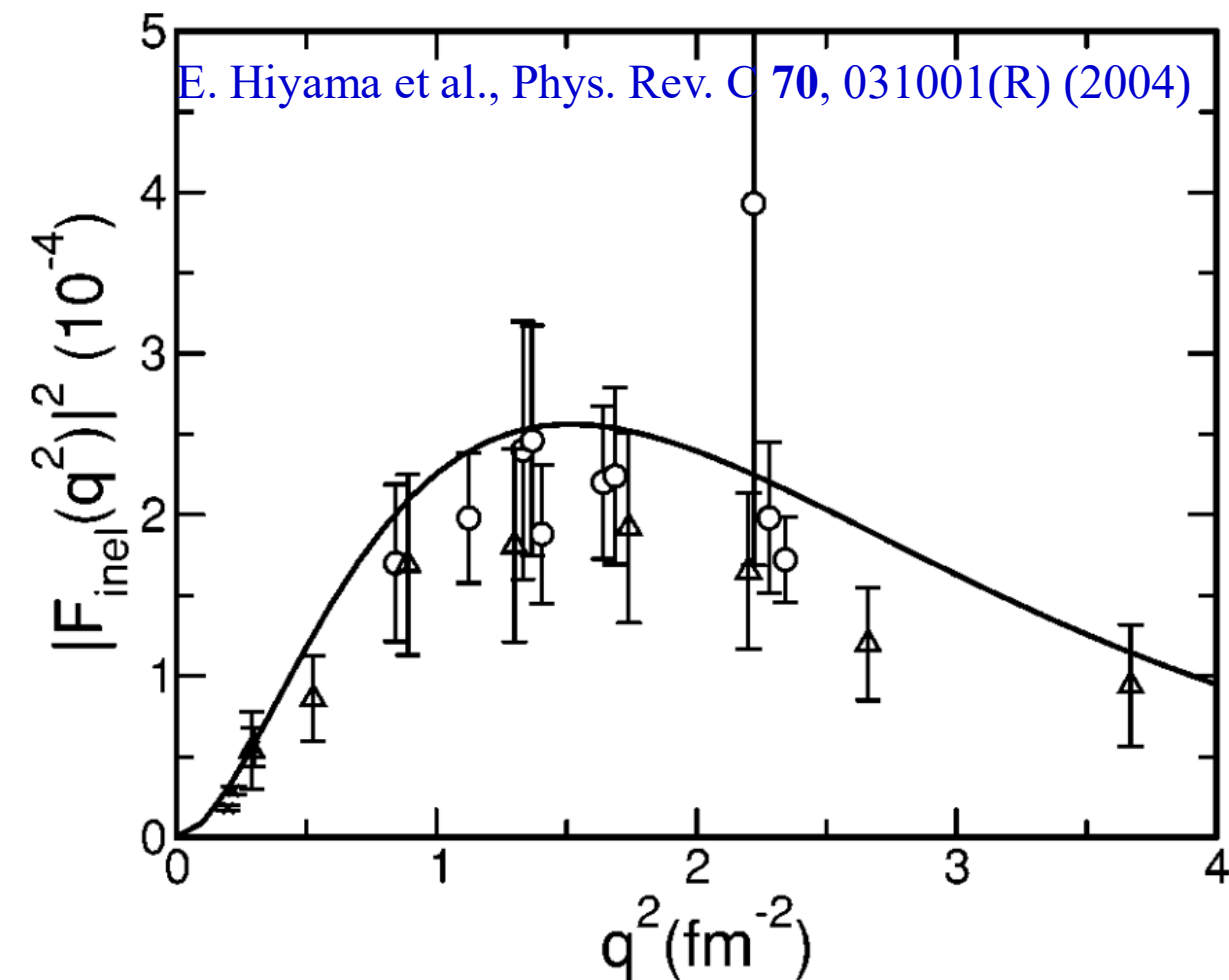
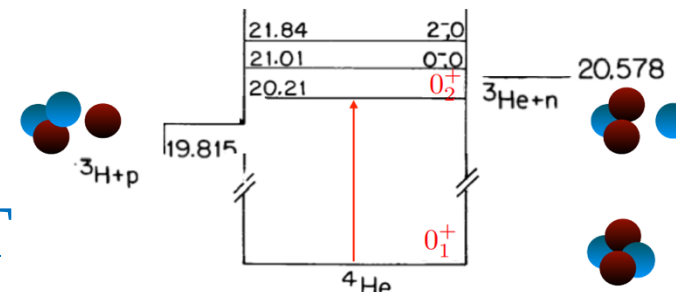
Nucleus	σ_{-1} (exp) (mb)	$\mathcal{R}$ (exp) ^a (fm)	$\langle r_{pp}^2 \rangle^{1/2}$ (fm)	$\langle r'^2 \rangle^{1/2}$ (fm)	ρ
^4He	2.3 ^a 2.1 ^b	1.63 ± 0.04	2.38 2.43	1.19 1.22	0.16 0.14
^7Li	4.64 ^c	2.41	3.73	2.15	0.05
^9Be	5.19 ^c	2.51	3.77	2.31	0.03
^{12}C	8.81 ^c	2.45	3.50	2.26	0.03
^{16}O	14.5 ^c	2.72	3.86	2.56	0.02
^{40}Ca	45.5 ^c	3.48	4.89	3.37	0.01
^{90}Zr	70.6 ^d	4.28	6.01	4.20	~0
^{208}Pb	229.2 ^d	5.50	7.74	5.43	~0

Constrain photoabsorption/strength function with nuclear structure calculations!

Monopole form factor with NCSM

Monopole transition form factor of ^4He

- Ground state: benchmark few-body methods
- Hiyama: Gaussian expansion, simplified force
- Bacca: HH+LIT, realistic $NN+3N$ from chiral EFT



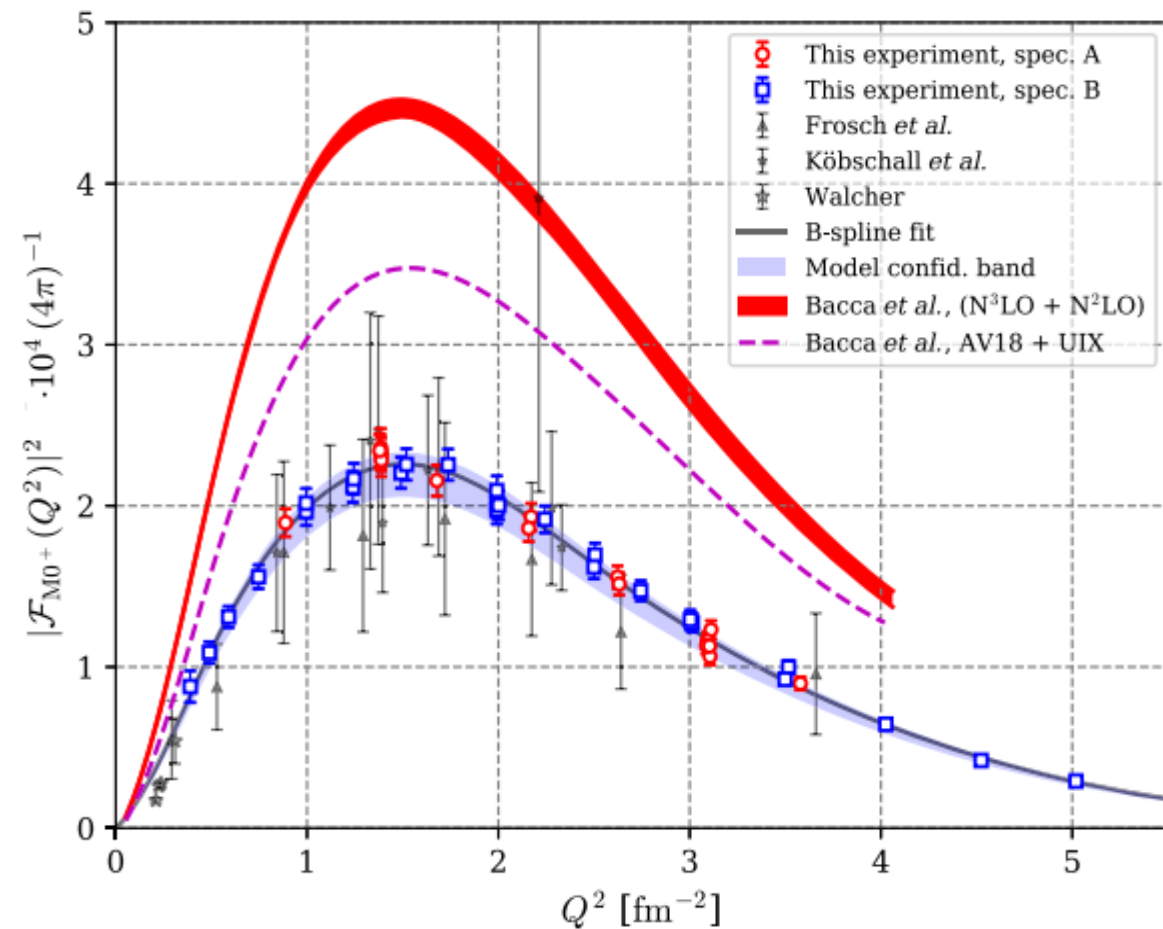
S. Bacca *et al.*, Phys. Rev. Lett. 110, 042503 (2013)

New experiment Mainz Status quo in 2023

Problem with chiral EFT
becomes stronger

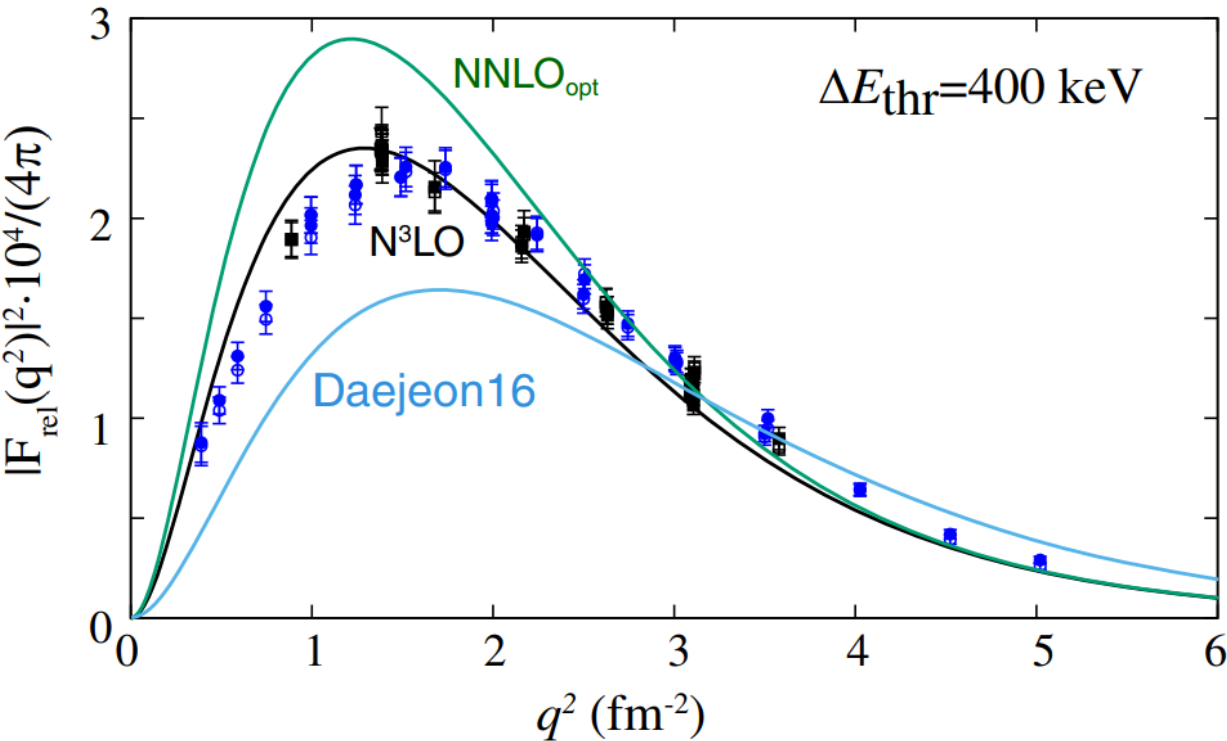
Measurement of the α -Particle Monopole Transition Form Factor Challenges Theory: A Low-Energy Puzzle for Nuclear Forces?

S. Kegel¹, P. Achenbach¹, S. Bacca^{1,2}, N. Barnea³, J. Beričič⁴, D. Bosnar⁵, L. Correa^{6,1}, M. O. Distler¹, A. Esser¹, H. Fonvieille⁶, I. Frišić⁵, M. Heilig¹, P. Herrmann¹, M. Hoek¹, P. Klag¹, T. Kolar^{7,4}, W. Leidemann^{8,9}, H. Merkel¹, M. Mihovilović^{1,4}, J. Müller¹, U. Müller¹, G. Orlandini^{8,9}, J. Pochodzalla¹, B. S. Schlimme¹, M. Schoth¹, F. Schulz¹, C. Sfienti^{1,*}, S. Širca^{7,4}, R. Spreckels¹, Y. Stöttinger¹, M. Thiel¹, A. Tyukin¹, T. Walcher¹ and A. Weber¹



Description of the Proton-Decaying 0_2^+ Resonance of the α Particle

N. Michel^{1,2,3}, W. Nazarewicz⁴, and M. Płoszajczak³



$$|F(q^2)|^2 = \left| \frac{4\pi}{Z} \int \rho_{\text{tr}}(r) j_0(qr) r^2 dr \right|^2 f_p^2(q^2)$$

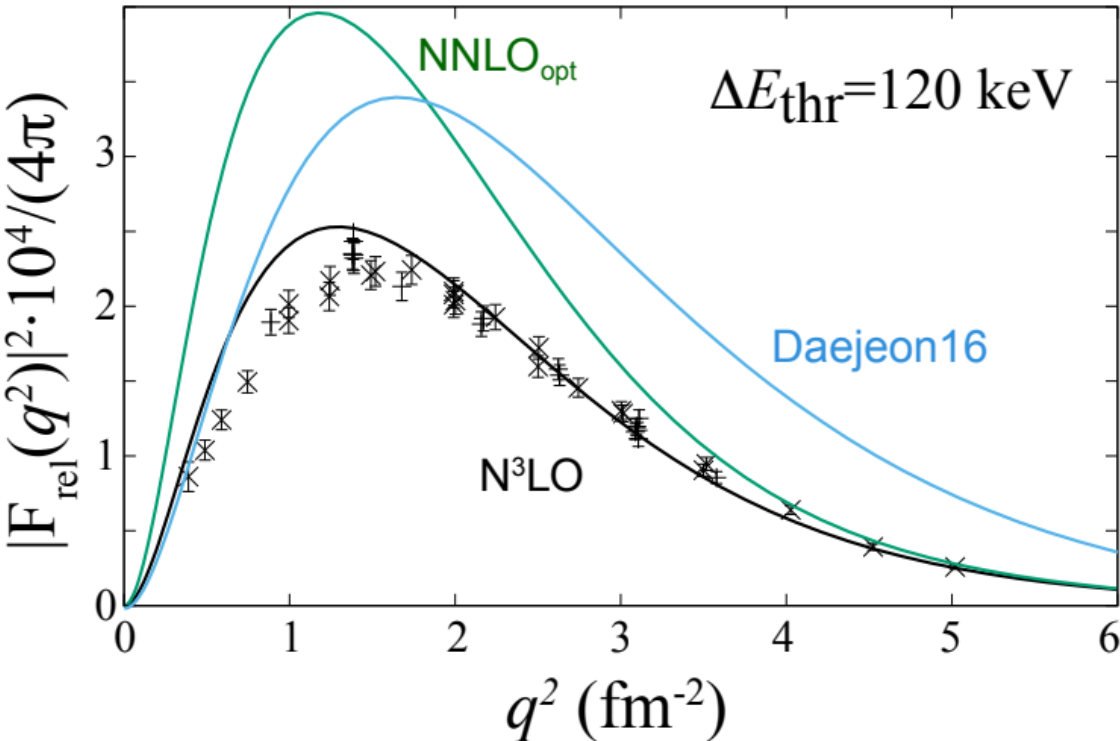
$$\rho_{\text{tr}}(r) = \langle 0_1^+ | \hat{\rho}(\vec{r}) | 0_2^+ \rangle$$

- NCGSM-CC with V_{lowk}
- $E_r=400$ keV

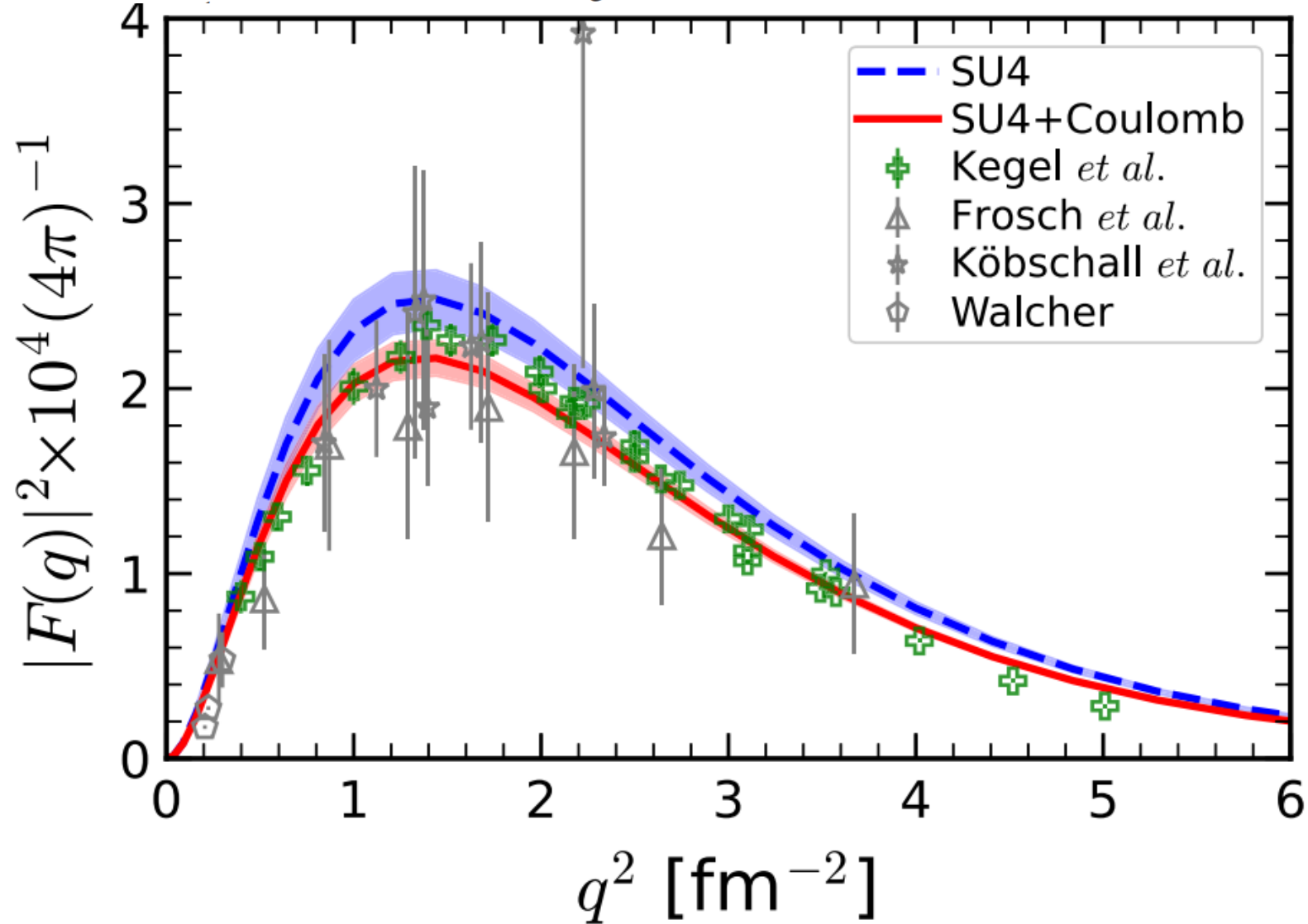
Erratum: Description of the Proton-Decaying 0_2^+ Resonance of the α Particle
[Phys. Rev. Lett. 131, 242502 (2023)]

N. Michel^{1,2,3}, W. Nazarewicz⁴, and M. Płoszajczak³

(Received 15 July 2024; revised 9 October 2024; published 4 December 2024)

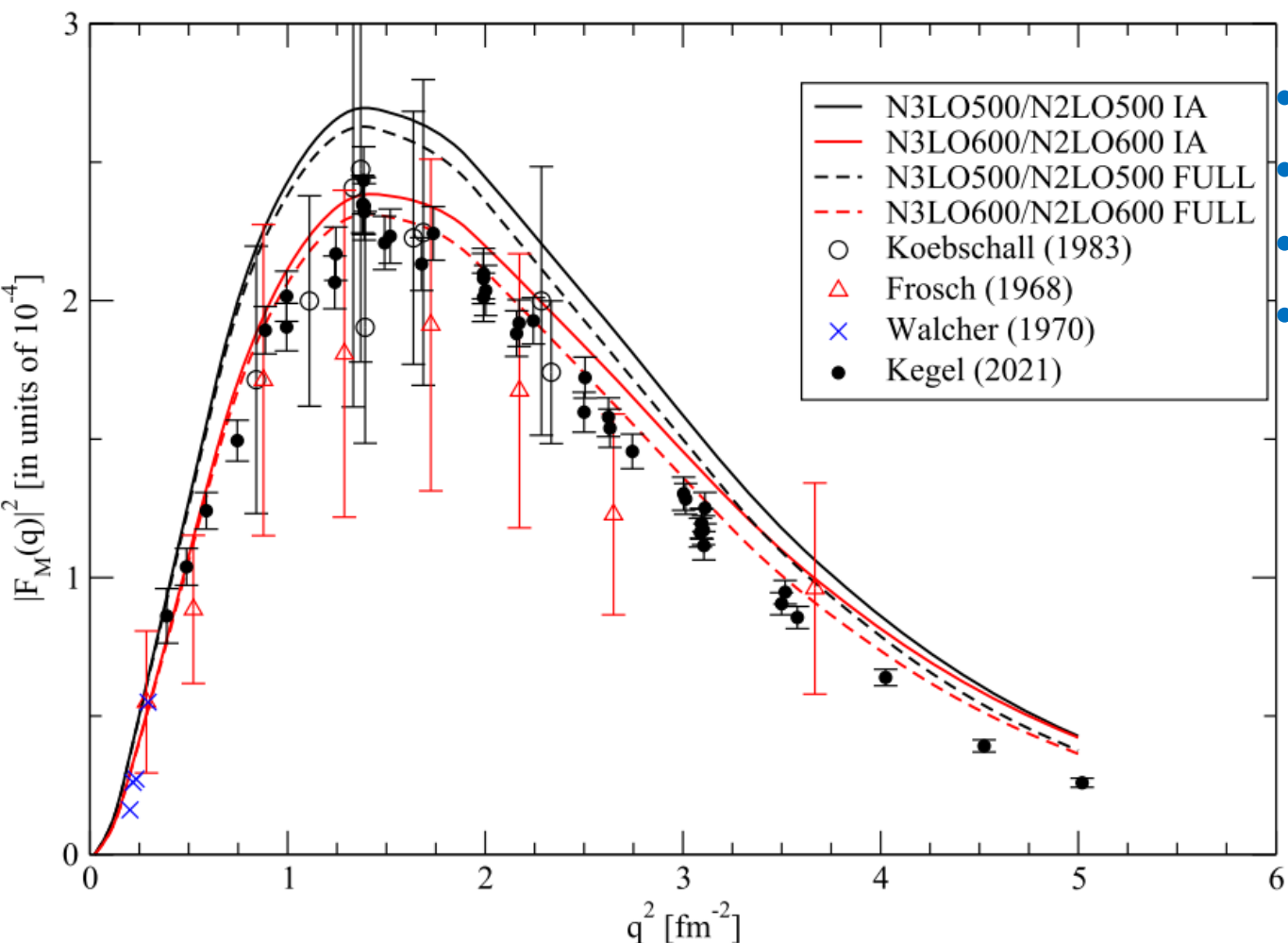


- $E_r=120$ keV

***Ab Initio* Calculation of the Alpha-Particle Monopole Transition Form Factor**Ulf-G. Meißner^{1,2,3} Shihang Shen^{4,*} Serdar Elhatisari^{5,1} and Dean Lee⁶

M. Viviani · A. Kievsky · L. E. Marcucci · L. Girlanda

Study of the Alpha-particle Monopole Transition form Factor



- Viviani: HH $^3\text{He}+n$, $^3\text{H}+p$, chiral $NN+3N$
- Two-body current
- First successful calculation with chiral EFT?
- Large $T=1$ components

K	P(T=1)	P(T=2)
28	0.35593	0.00297
30	0.45106	0.00297
32	0.56375	0.00297

In the news

From Sonia Bacca

PhysiCS ABOUT BROWSE PRESS COLLECTIONS Search article

Probing the Helium Nucleus beyond its Ground State

Evgeny Epelbaum
Physics and Astronomy, Ruhr University Bochum, Bochum, Germany

Quantamagazine Physics Mathematics Biology Computer Science Topics Archive

Das Physikportal
pro-physik.de

Rätsel um Anregung von α -Teilchen

26.04.2023 - Theoretisch bestimmte und gemessene Werte überein.

Am Mainzer Teilchenbeschleuniger „Mami“ hat die A1-Kollaboration im Rahmen der Anregung eines α -Teilchens von seinem Grundzustand zum ersten angeregten Zustand die Genauigkeit systematisch vermessen. Die Gegenüberstellung von Experiment und Theorie zeigt, dass die Anregung von α -Teilchen durch Kernkräfte nicht korrekt beschrieben wird – und wirft damit viele Fragen auf.

LIVESCIENCE

Scientists tried to solve the mystery of the helium nucleus — and ended up more confused than ever

News By Anna Demming published June 27, 2023

Helium is the simplest element in the periodic table with more than one stable isotope. But its nucleus, which is made of two protons and two neutrons, has long puzzled physicists. The mystery of the helium nucleus is a persistent enigma for nuclear physics.

PUBLIÉ LE 28 JUIN 2023 À 18H45 | MODIFIÉ LE 28 JUIN 2023
PAR LAURIE HENRY

NUCLEAR PHYSICS

A New Experiment Casts Doubt on the Leading Theory of the Nucleus

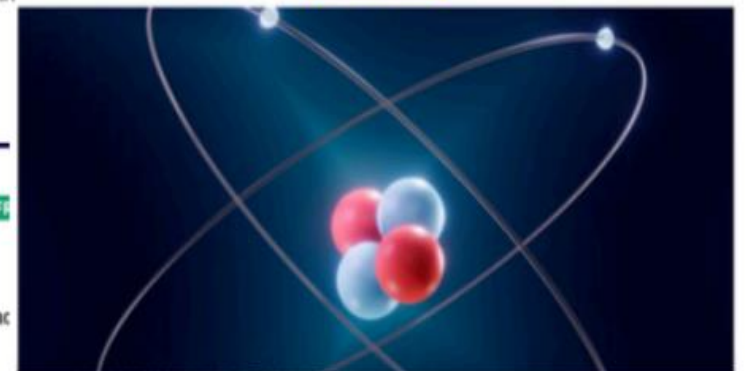
By measuring inflated helium nuclei, physicists have challenged our understanding of the force that binds protons and neutrons.

PHYSICS TODAY

Theory and experiment disagree on alpha particles

12 May 2023

Electron-scattering experiments on excited helium nuclei open questions about the accuracy of current nuclear models.



Two important comments

E. Epelbaum, Physics **16** (2023) 58

Threshold energy:

“The form factor may depend on the energy difference between the position of the resonance and the two-body breakup threshold, so any uncertainties in calculating the excitation energy would translate into relatively large uncertainties in the form factor predictions.”

⇒ investigate role of threshold energy

Kamimura, Prog. Theor. Exp. Phys. (Letter) Vol. 2023, 071D01 (2023)

Bound vs Continuum:

Claim that *“the large difference between the calculated form factors does not stem from numerical methods but from the Hamiltonian. However, the comparison between the methods was performed only for the calculation based on the bound-state approximation.”*

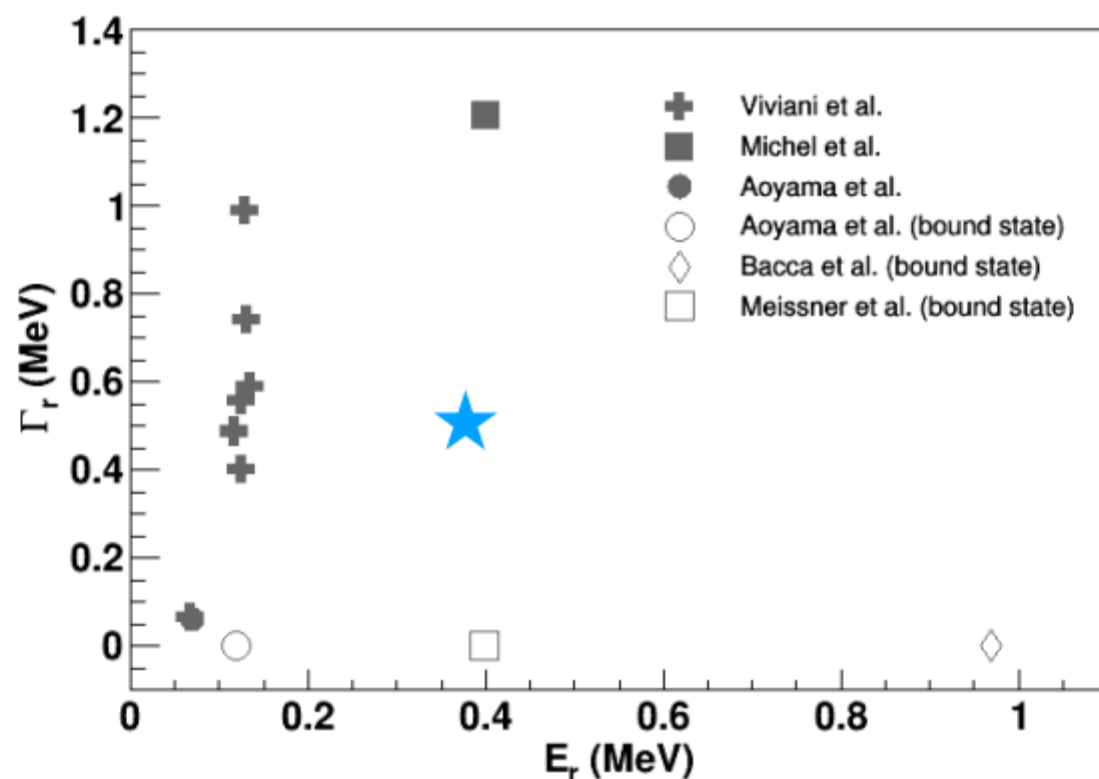
⇒ investigate role of continuum

None of the theories gets E_r and Γ_r correct

Evaluation by Tilley et al, NPA 541 (1992) 1-104



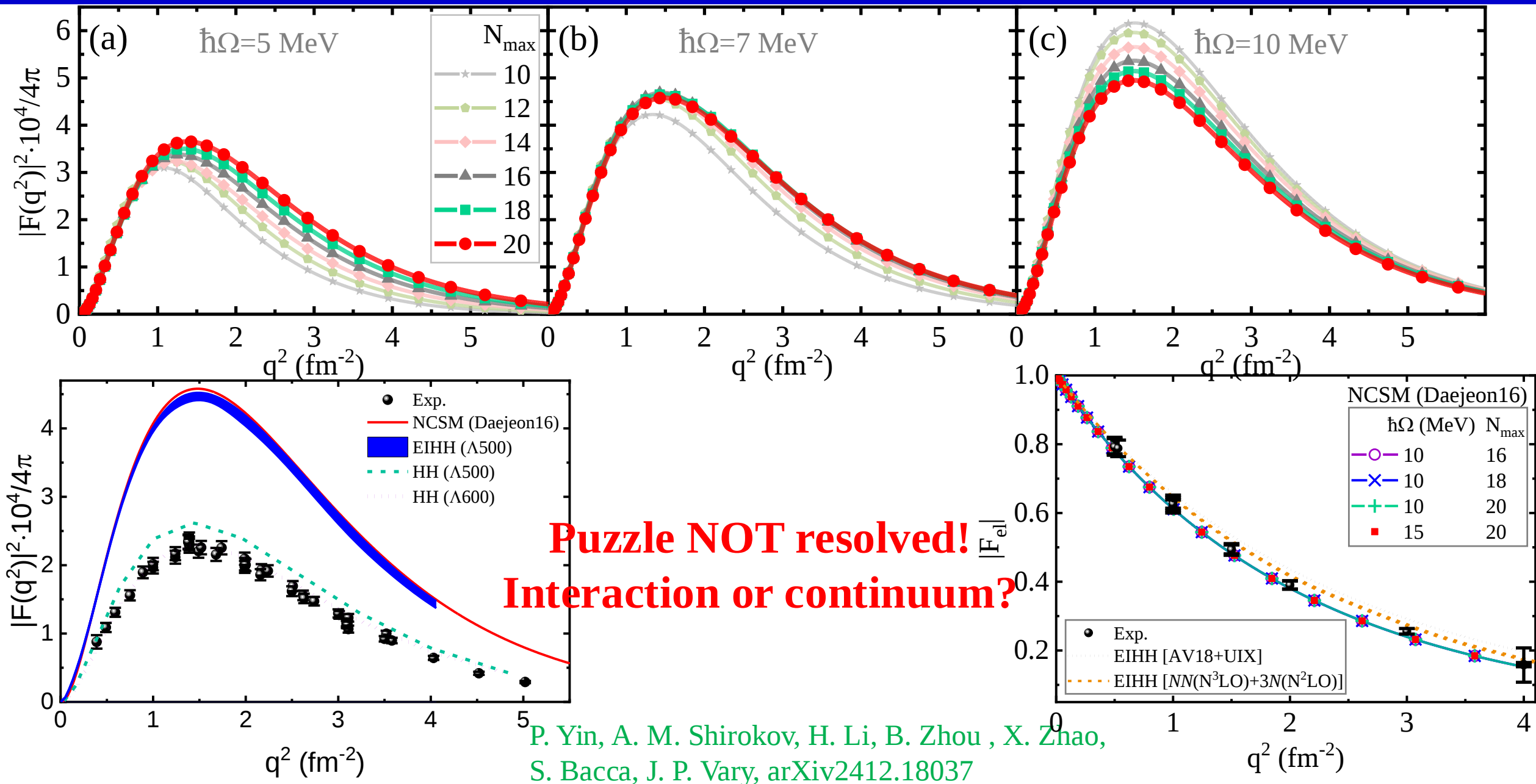
$E_r = 0.39 \text{ MeV}$ $\Gamma_r = 0.50 \text{ MeV}$



Pionless EFT with RGM: Kirscher and Griesshammer, EPJA **54**, 137 (2018)

$E_r = 0.38 \pm 0.25 \text{ MeV}$

Monopole transition form factor of ^4He



Two-body currents LENPIC interaction

$$\mu = \frac{1}{2} \int_{\mathbb{R}^3} d^3x \, \boldsymbol{x} \times \bar{\boldsymbol{j}}(\boldsymbol{x})$$

Magnetic moments of $A = 3$ nuclei obtained from chiral effective field theory operators

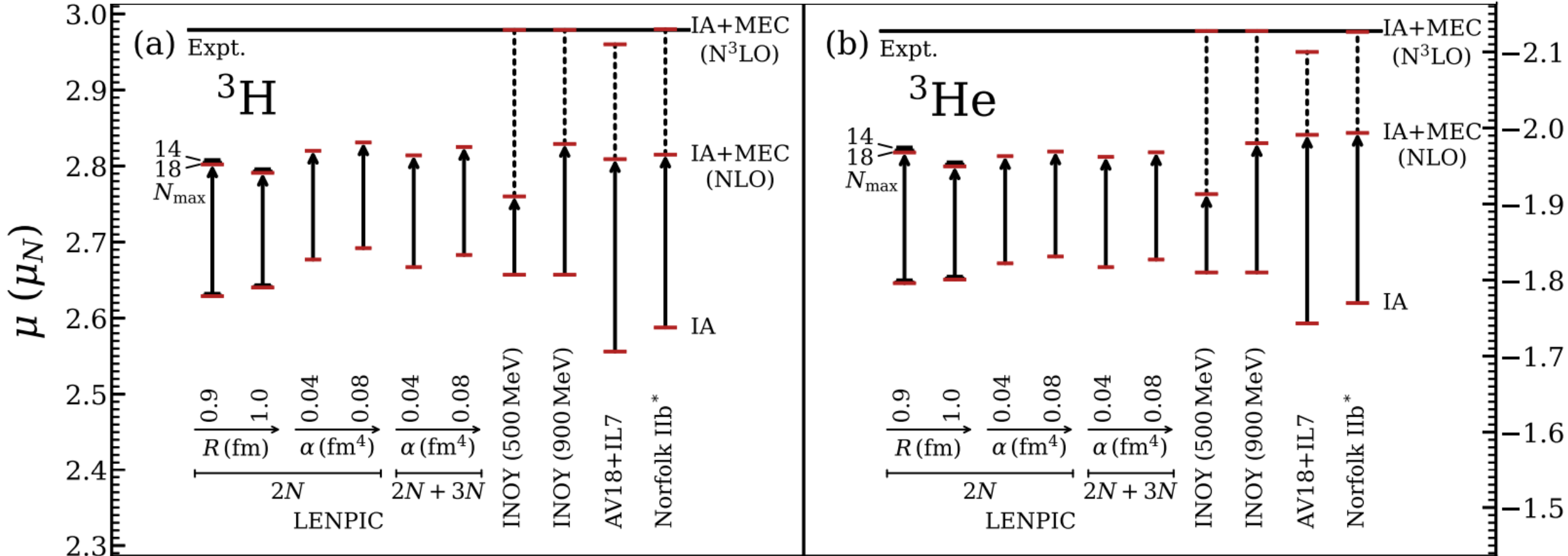
Soham Pal,^{1,*} Shiplu Sarker¹, Patrick J. Fasano^{2,†}, Pieter Maris,¹ James P. Vary¹,

Mark A. Caprio², and Robert A. M. Basili³

¹*Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011-3160, USA*

²*Department of Physics and Astronomy, University of Notre Dame, Notre Dame, Indiana 46556-5670, USA*

³*Department of Electrical and Computer Engineering, Iowa State University, Ames, Iowa 50011-3160, USA*



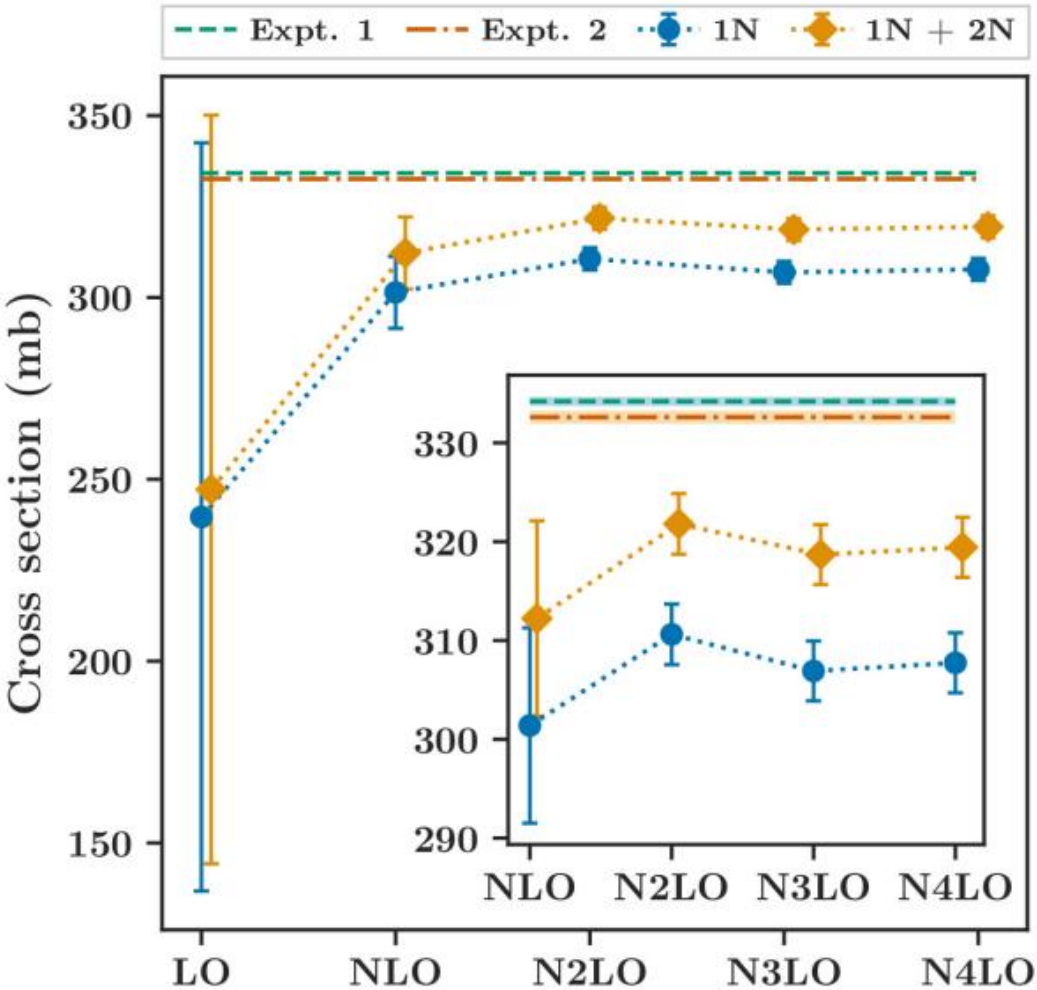
Calculations of the $np \rightarrow d\gamma$ reaction in chiral effective field theory

Weijie Du^{1,*} Soham Pal,¹ Mamoon Sharaf¹ Peng Yin^{1,2} Shiplu Sarker¹ Andrey M. Shirokov³ and James P. Vary¹

¹Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50010, USA

²Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China

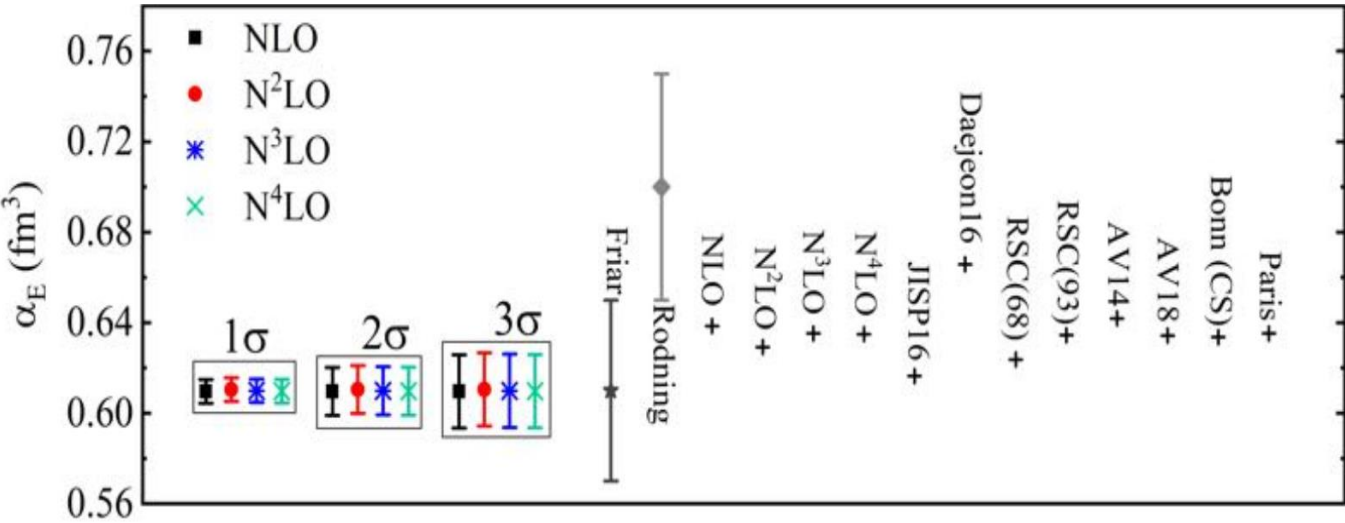
³Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, Moscow 119991, Russia



$$\bar{\sigma}_{-2} \equiv \frac{1}{2\pi^2} \int_{E_B}^{\infty} \frac{\sigma(E_\gamma)}{E_\gamma^2} dE_\gamma = \bar{\alpha} + \bar{\beta}$$

$$\bar{\alpha} = \alpha_E + \frac{Z\alpha \langle r^2 \rangle}{3Am_N} + \alpha_N + \dots$$

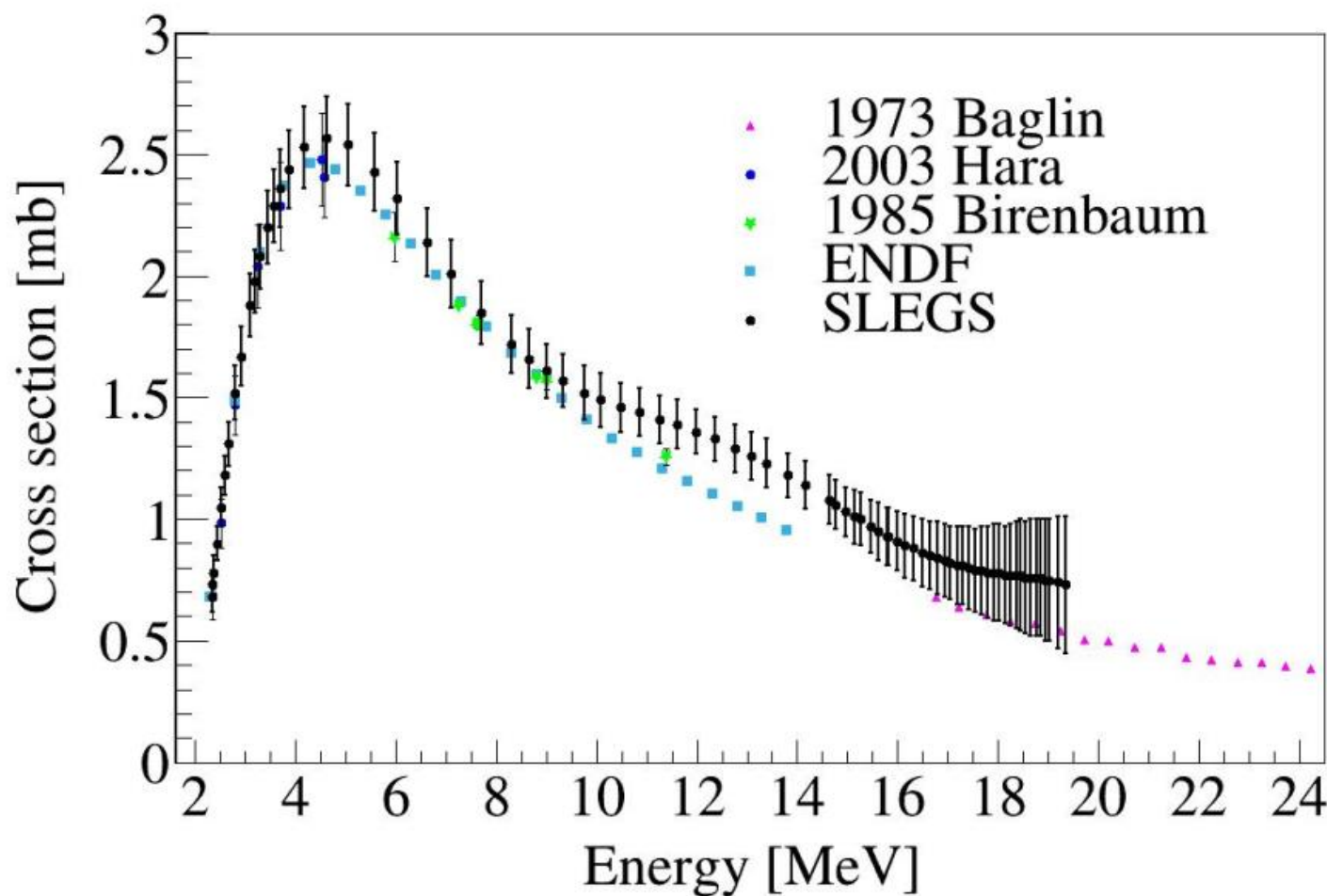
$$\bar{\beta} = \frac{2\alpha}{3} \sum_{n \neq 0} \sum_{M_n, \lambda} \frac{|\langle \Psi_n, M_n | \mu_{1, \lambda} | \Psi_0, M_0 \rangle|^2}{E_n - E_0} - \frac{Z\alpha \langle r^2 \rangle}{6m_N} - \frac{\alpha \langle D^2 \rangle}{2Am_N} + \beta_N + \beta_\pi + \dots$$



- Updated photoabsorption data
- Two-body current: 5%

Shiplu Sarker, P. Yin et al., in preparation

$$\sigma_{-2} = \bar{\alpha} + \bar{\beta} = \frac{1}{2\pi} \int_{S_n}^{\infty} \sigma(E) E^{-2} dE$$



对测量得到的 $D(\gamma, n)p$ 截面计算 σ_{-2} :

- S_n -15.66 MeV:
0.6843 fm³
 $\pm 0.0065 \text{ fm}^3(\text{stat})$
 $\pm 0.0125 \text{ fm}^3(\text{meth})$
 $\pm 0.0432 \text{ fm}^3(\text{syst})$
- S_n -19.33 MeV:
0.6943 fm³
 $\pm 0.0068 \text{ fm}^3(\text{stat})$
 $\pm 0.0137 \text{ fm}^3(\text{meth})$
 $\pm 0.0449 \text{ fm}^3(\text{syst})$

如果结合 $[2H(\gamma, p)$ absolute cross section and angular distribution at 17–25 MeV]的数据, σ_{-2} 结果为:

$$\boxed{0.7013 \pm 0.0479 \text{ fm}^3}$$

可以看出该数据的贡献仅为 0.0069 fm^3 , 与统计误差相当, 因此可以认为

$$\sigma_{-2} = \frac{1}{2\pi} \int_{S_n}^{19.34 \text{ MeV}} \sigma(E) E^{-2} dE + \Delta$$

where, $\Delta < \text{statistical uncertainty}$

Summary

- Introduction to *ab initio* NCSM approach
- Long range operators
- Sum rules with NCCI
- Monopole transition form factor of ^4He
- Two-body currents

Outlook

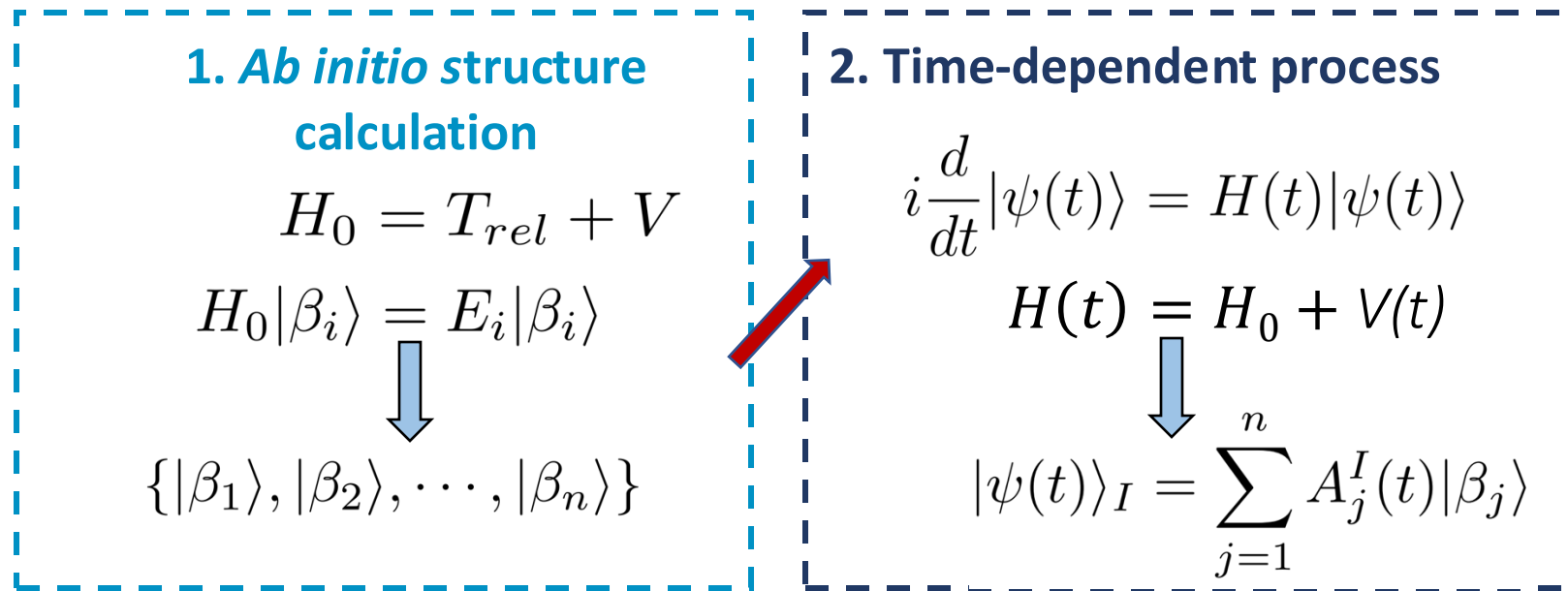
- Photoabsorption reaction
- Neutrino-nucleus reaction
- $n+\alpha$, $p+^3\text{H}$, $n+^3\text{He}$... scatterings
- Coulomb dissociation of light nuclei

NCSM with continuum (P. Navratil)

- $^3\text{He}(\alpha, \gamma)^7\text{Be}$ and $^3\text{H}(\alpha, \gamma)^7\text{Li}$
- $^7\text{Li}(p, e^+e^-)^8\text{Be}$: X17
- $\alpha + \alpha$
- $p + ^7\text{Be}$ and $p + ^7\text{Li}$
- $^8\text{Li}(n, \gamma)^9\text{Li}$, $^7\text{Be}(p, \gamma)^8\text{B}$

Time Dependent Basis Function

Scattering at sub-barrier with the tBF approach



- Natural extension of *ab initio* nuclear structure approaches, e.g., NCSM
- Non-perturbative
- Full quantal coherence

- Weijie Du, Peng Yin*, Yang Li, Guangyao Chen, Wei Zuo, Xingbo Zhao, and James P. Vary, **Phys. Rev. C** **97**, 064620 (2018);
- Weijie Du, Peng Yin, Guangyao Chen, Xingbo Zhao, and James P. Vary, in *Proceedings of the International Conference “Nuclear Theory in the Supercomputing Era–2016” (NTSE-2016), Khabarovsk, Russia, September 19–23, 2016*;
- Peng Yin, Weijie Du, Wei Zuo, Xingbo Zhao and James P. Vary, **J. Phys. G** (2022);
- Weijie Du, James P. Vary, Xingbo Zhao, Wei Zuo, **Phys. Rev. A** **104**, 012611 (2021).
- Peng Yin, Weijie Du, Wei Zuo, Xingbo Zhao and James P. Vary, arXiv 2208.00267

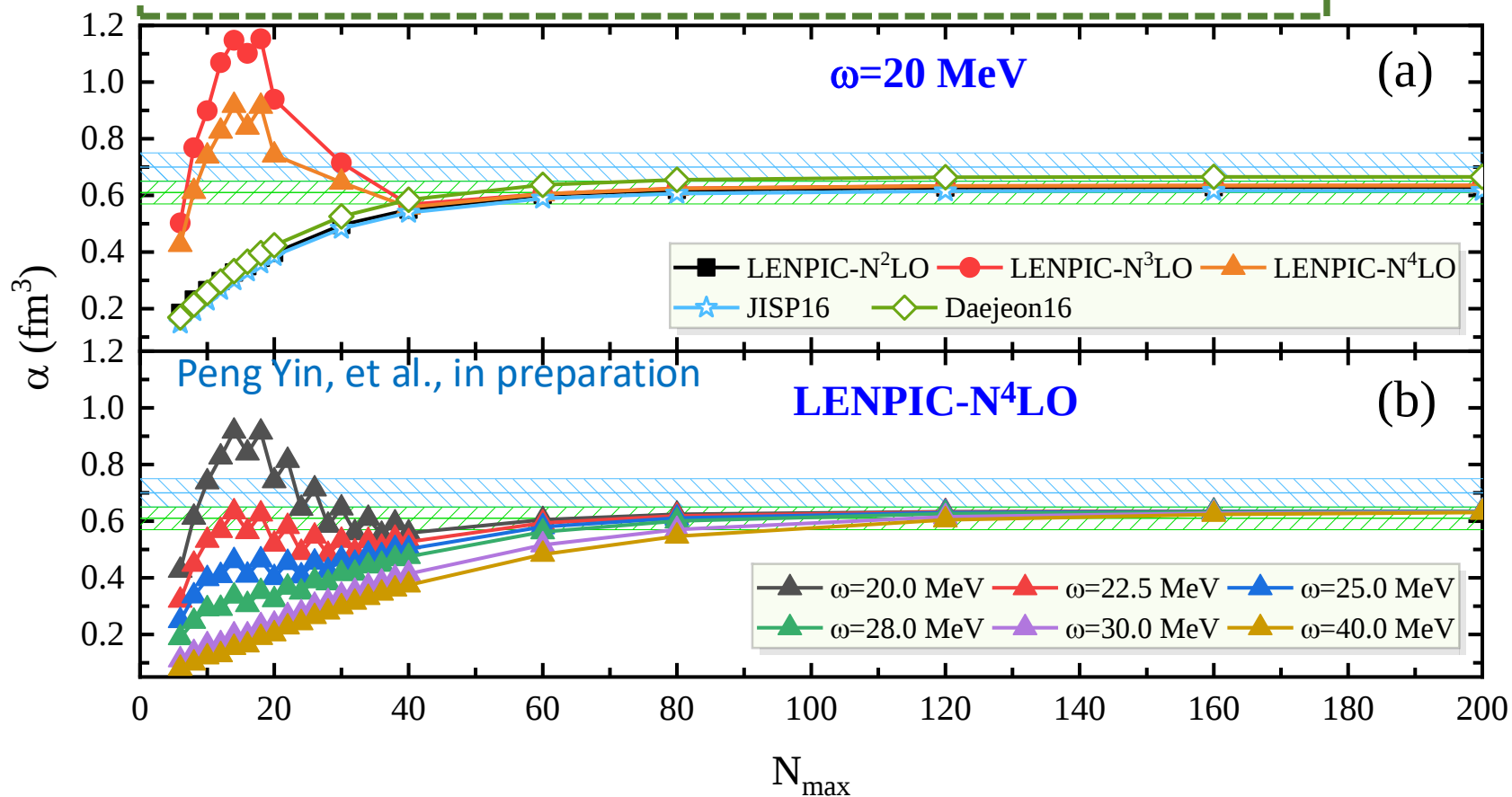
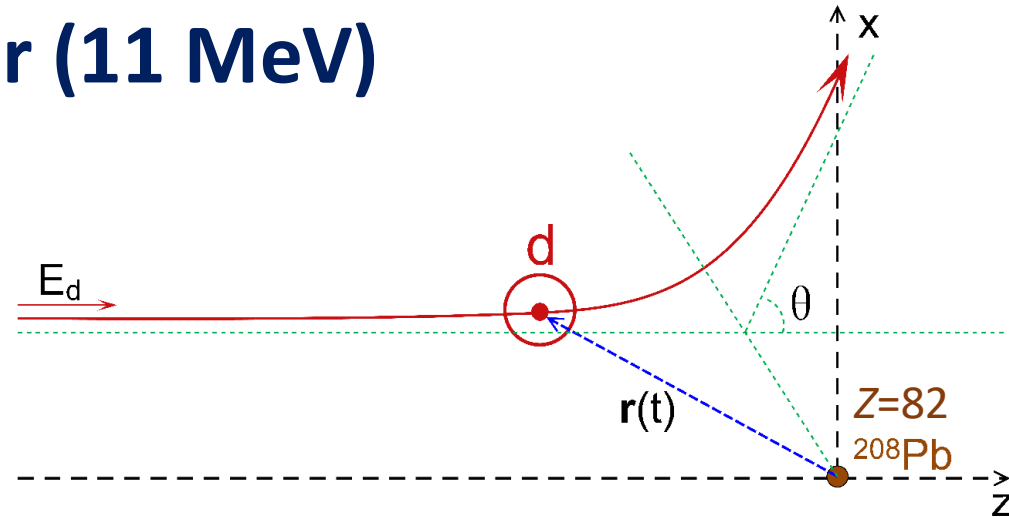
d+²⁰⁸Pb scatterings below Coulomb barrier (11 MeV)

Solving EOM for CM motion of deuteron in external field:

$$V = V_{\text{coul}} + V_{\text{pol}}$$

$$V_{\text{pol}} = -\frac{1}{2} \alpha Z^2 e^2 \frac{1}{r^4}$$

$$\alpha = \frac{8\pi}{9} \sum_{n \neq 0} \frac{B(E1; 0 \rightarrow n)}{(E_n - E_0)}$$



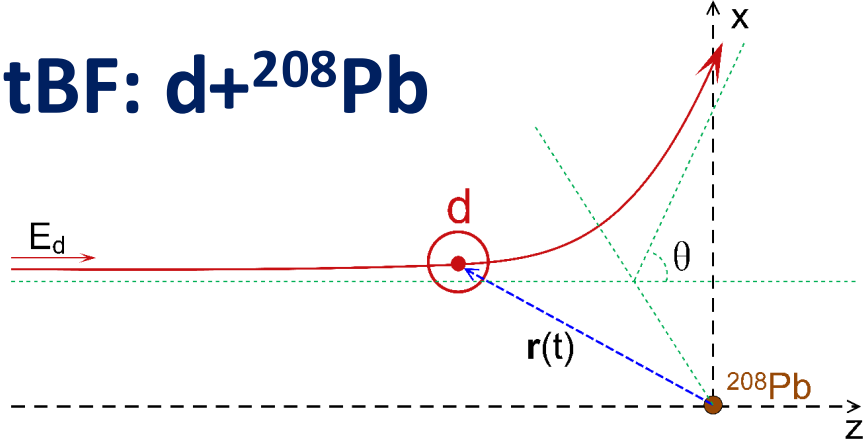
➤ Harmonic oscillator (HO) basis;

➤ Parameters of HO basis:
 N_{max} and ω , ($2n + 1 \leq N_{\text{max}}$);

➤ α is converged with respect to
 N_{max} and ω ;

➤ LENPIC interactions:
regulator $R = 1.0$ fm.

tBF: d+²⁰⁸Pb



Time-independent:

$$H_0 = T_{\text{rel}} + V_{\text{np}}$$

$$H_0 |\beta_i\rangle = E_i |\beta_i\rangle$$

Basis set $\{|\beta_j\rangle\}$

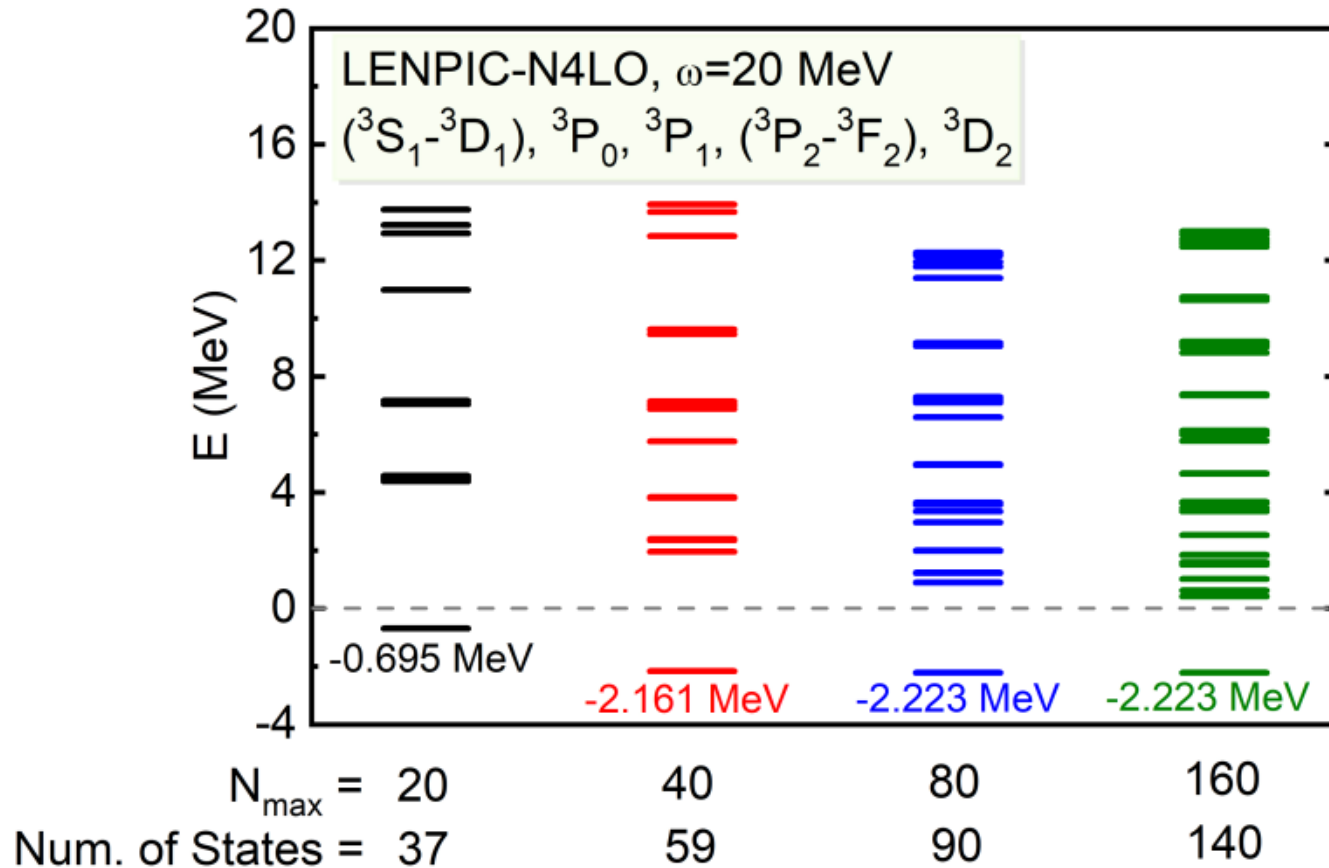
$$H(t) = H_0 + V(t)$$

E1 is dominant

Time-dependent:

$$i \frac{\partial}{\partial t} |\psi(t)\rangle_I = V_I(t) |\psi(t)\rangle_I$$

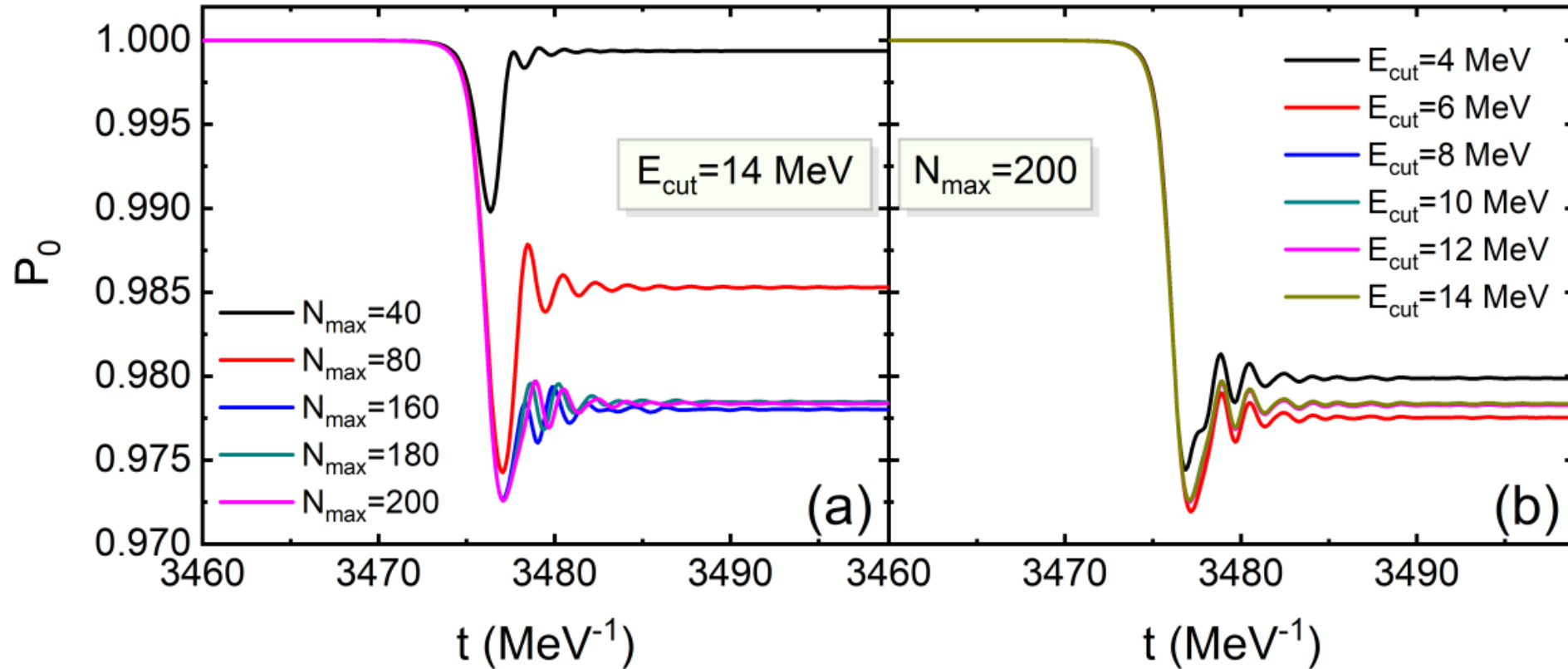
$$|\psi(t)\rangle_I = \sum_{j=1}^n A_j^I(t) |\beta_j\rangle$$



- NN interaction: LENPIC-N⁴LO
- 3DHO basis: $N_{\text{max}}, \omega = 20$ MeV
- $S=1, J_{\text{max}}=2$

d+²⁰⁸Pb scatterings at E_d=7 MeV and θ=150°

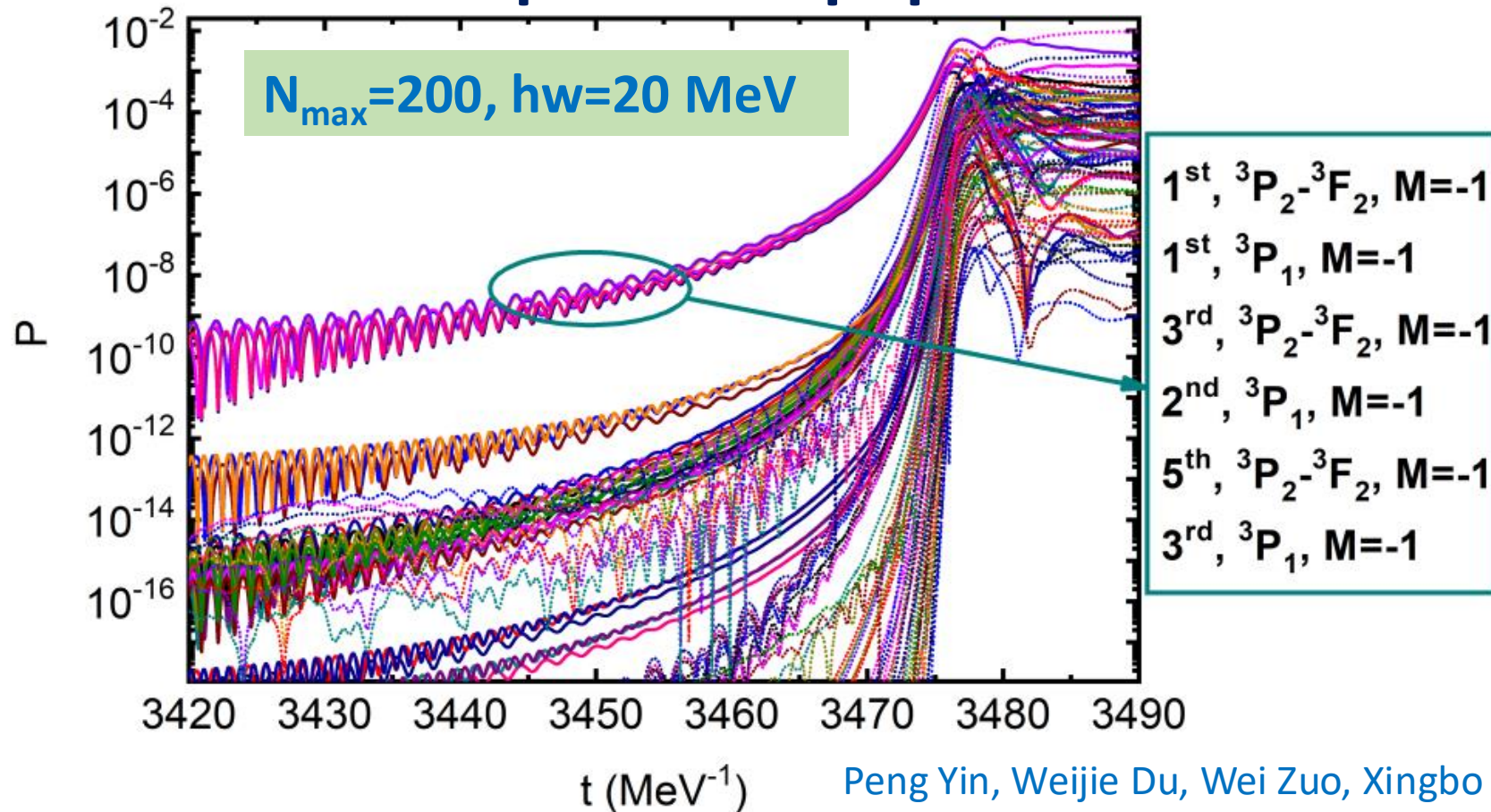
Convergence with respect to N_{max} and E_{cut}



- Initial state: deuteron ground state with M=-1;
- P_0 is the probability of the initial state;
- The asymptotic value is well converged with respect to Nmax and Ecut.

$d+^{208}\text{Pb}$ scatterings at $E_d=7$ MeV and $\theta=150^\circ$

Sequence of population

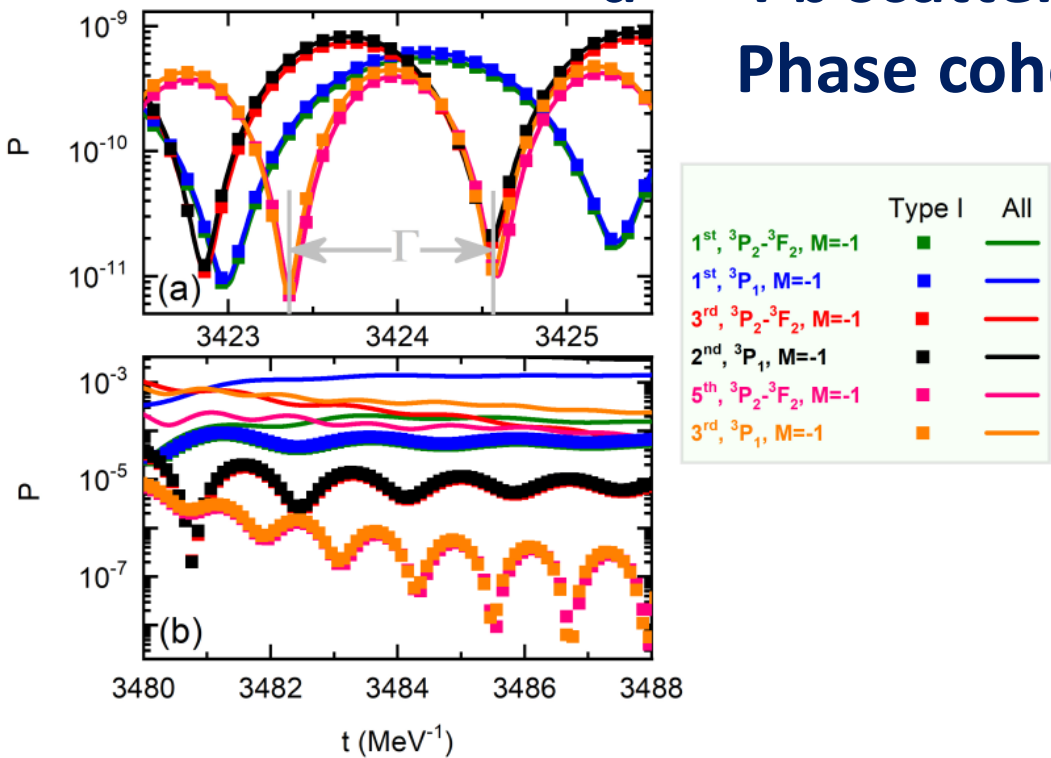


Peng Yin, Weijie Du, Wei Zuo, Xingbo Zhao and James P. Vary,
[arXiv 2208.00267](https://arxiv.org/abs/2208.00267)

- Initial state: deuteron ground state 3S_1 - 3D_1 with $M=-1$;
- P is the probability of states other than the initial state;
- Allowed states (solid lines) populate first. Forbidden states (dotted lines) populate afterward;
- 6 allowed states populate dominantly in the early stage of the time evolution.

d+²⁰⁸Pb scatterings at E_d=7 MeV and θ=150°

Phase coherence and decoherence



Transitions to forbidden states generate phase decoherence.

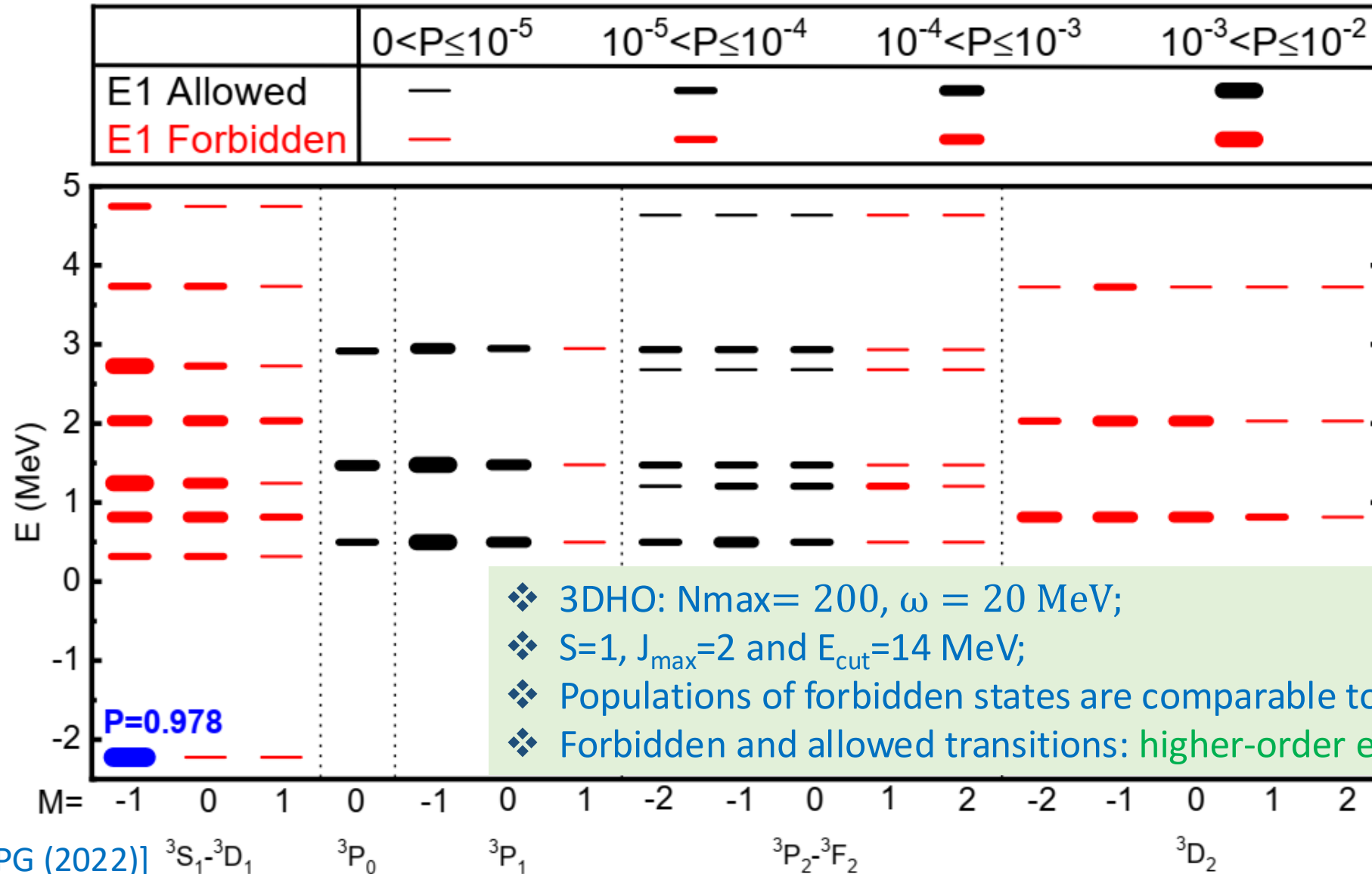
~2π ~2π ~?

	E _x (MeV)	Γ(MeV ⁻¹)				κ			
		Incoming		Outgoing		Incoming		Outgoing	
		Type I	All	Type I	All	Type I	All	Type I	All
1 st ³ P ₂ - ³ F ₂	2.72	2.31	2.31	2.30	2.10	6.28	6.28	6.26	5.71
1 st ³ P ₁	2.72	2.31	2.31	2.30	2.13	6.28	6.28	6.26	5.79
3 rd ³ P ₂ - ³ F ₂	3.69	1.70	1.70	1.69	2.16	6.27	6.27	6.24	7.97
2 nd ³ P ₁	3.70	1.70	1.70	1.69	2.12	6.29	6.29	6.25	7.84
5 th ³ P ₂ - ³ F ₂	5.15	1.22	1.22	1.17	1.32	6.28	6.28	6.03	6.80
3 rd ³ P ₁	5.17	1.22	1.22	1.17	1.31	6.31	6.31	6.05	6.77

$$\kappa = \Gamma E_x$$

Occupation probabilities of np states after $d+^{208}\text{Pb}$ scattering

$E_d=7$ MeV and $\theta=150^\circ$: LENPIC-N⁴LO



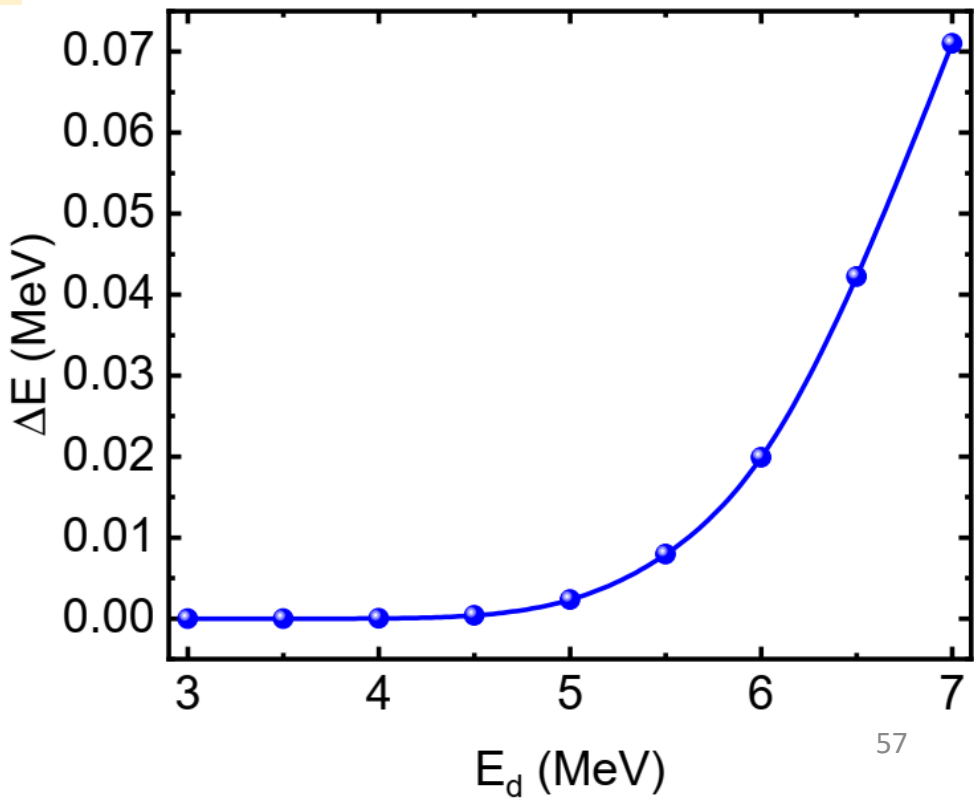
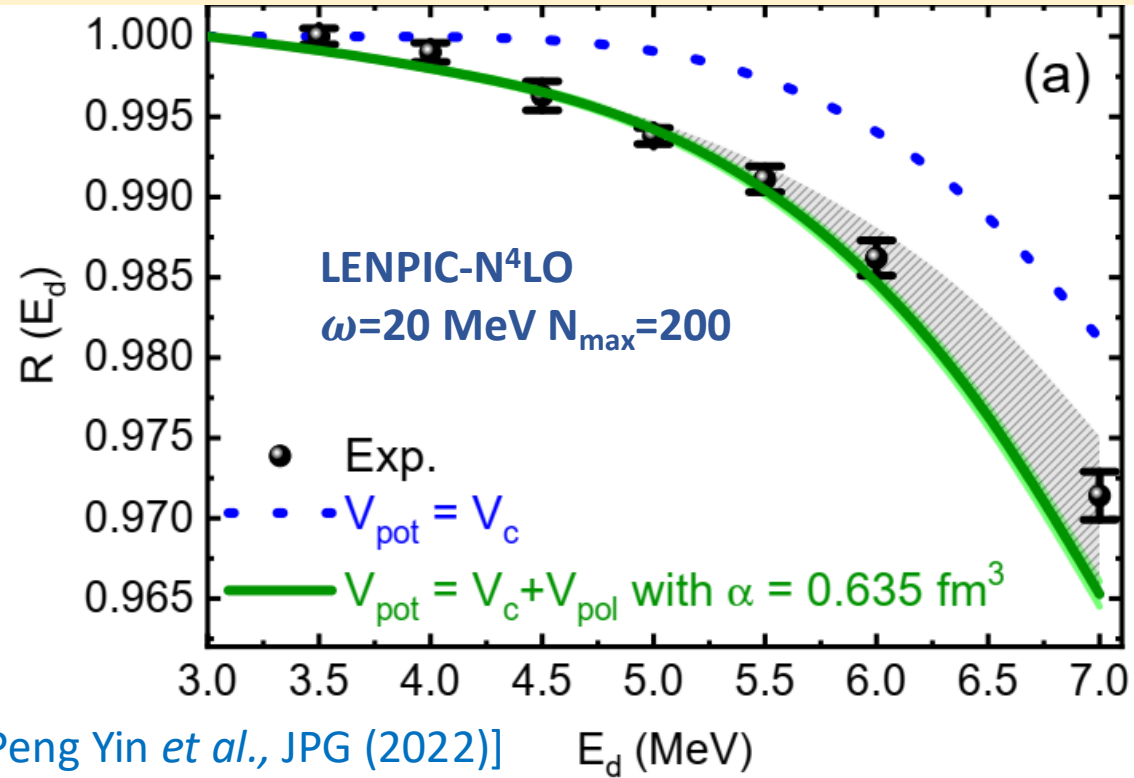
Observables for d+²⁰⁸Pb scattering

$$V_{\text{pol}} = -\frac{1}{2} \alpha Z^2 e^2 \frac{1}{r^4}$$

$$R(E_d) = \frac{\sigma(E_d = 3 \text{ MeV}, \theta_1 = 60^\circ)}{\sigma(E_d = 3 \text{ MeV}, \theta_2 = 150^\circ)} \frac{\sigma(E_d, \theta_2 = 150^\circ)}{\sigma(E_d, \theta_1 = 60^\circ)}$$

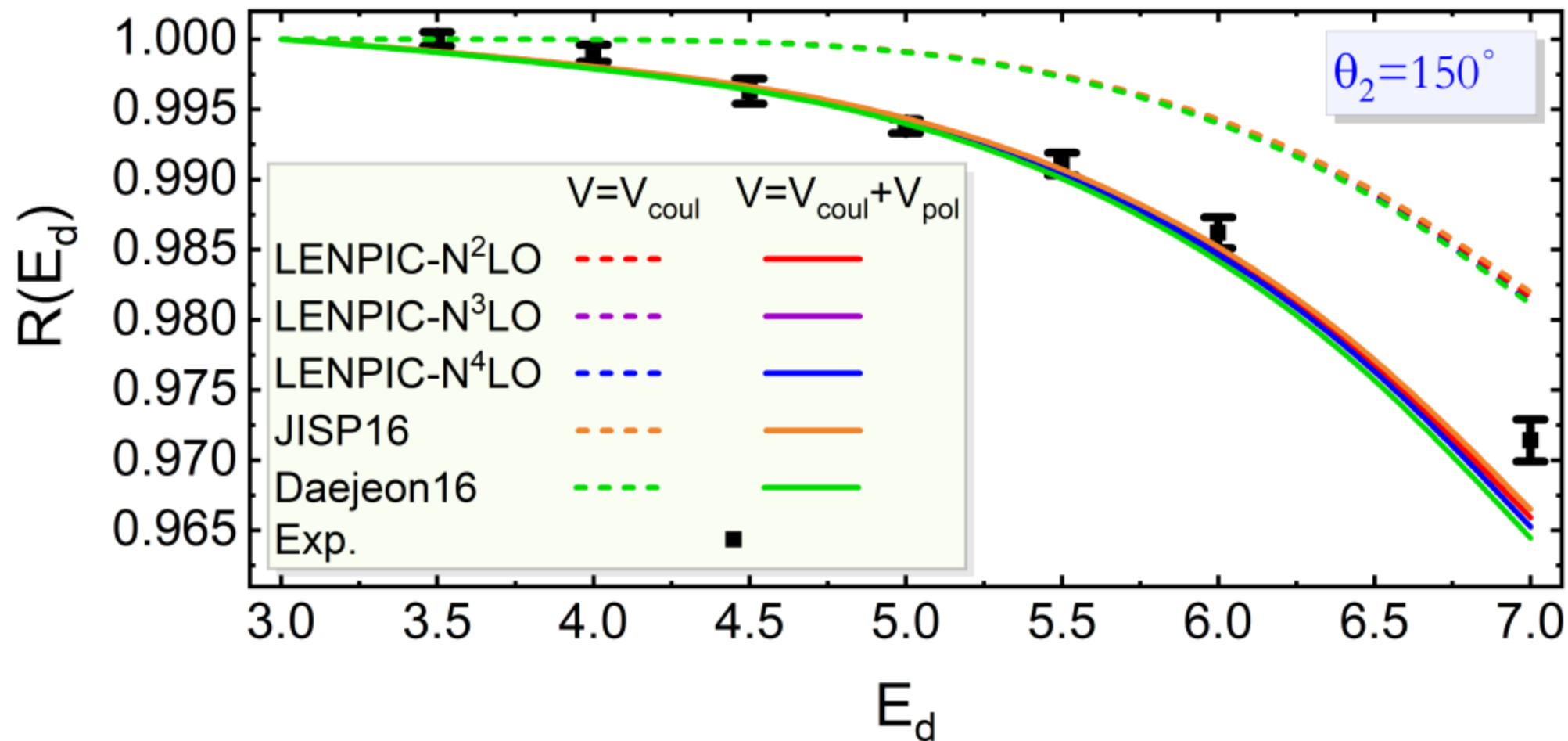
- ❖ Exp: N. L. Rodning, L. D. Knutson, W. G. Lynch and M. B. Tsang, Phys. Rev. Lett. 49, 909 (1982);
- ❖ No adjustable parameter;
- ❖ No optical potential;
- ❖ Energy loss correction.

	$\alpha \text{ (fm}^3\text{)}$
LENPIC-N ² LO	0.6292(1)
LENPIC-N ³ LO	0.6352(1)
LENPIC-N ⁴ LO	0.6349(1)
JISP16	0.6164(1)
Daejeon16	0.6658(1)



d+²⁰⁸Pb scattering $E_d=3-7$ MeV

Sensitivity to NN interactions



Conclusions/Perspectives

- ❖ Non-perturbative time-dependent basis function approach;
- ❖ Application to $d+^{208}\text{Pb}$ scattering;
- ❖ Extension of the tBF method to heavier projectiles, e.g., rare isotopes,...: using wavefunctions from NCSM.
- ❖ Extension to higher incident energies: adding strong interaction between scattering nuclei.
- ❖ Two-body electromagnetic operators from chiral effective field theory.
- ❖ Quantum consideration of center of mass motion of the projectile.
- ❖ Quantum computing