电弱相变：电弱对称性破缺与新物理

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报告内容

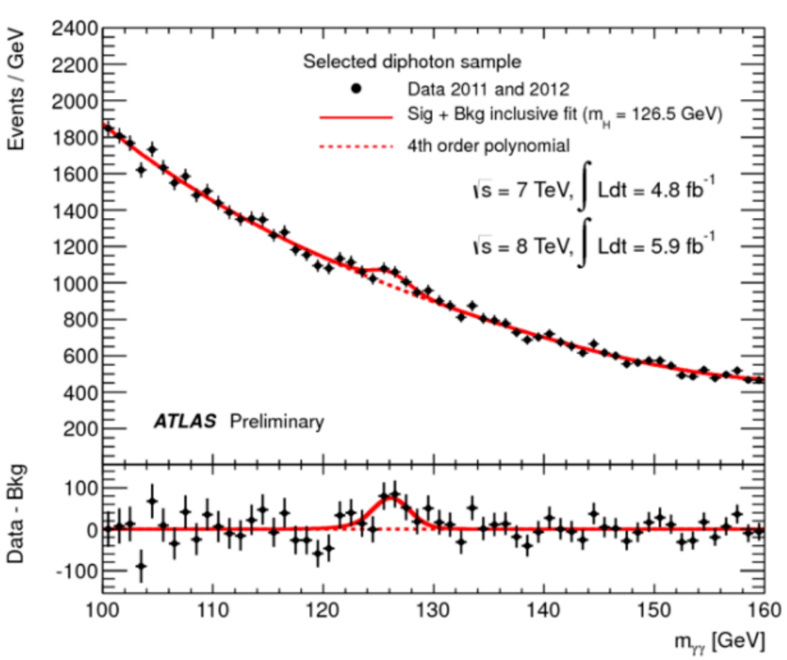
一 背景与动机

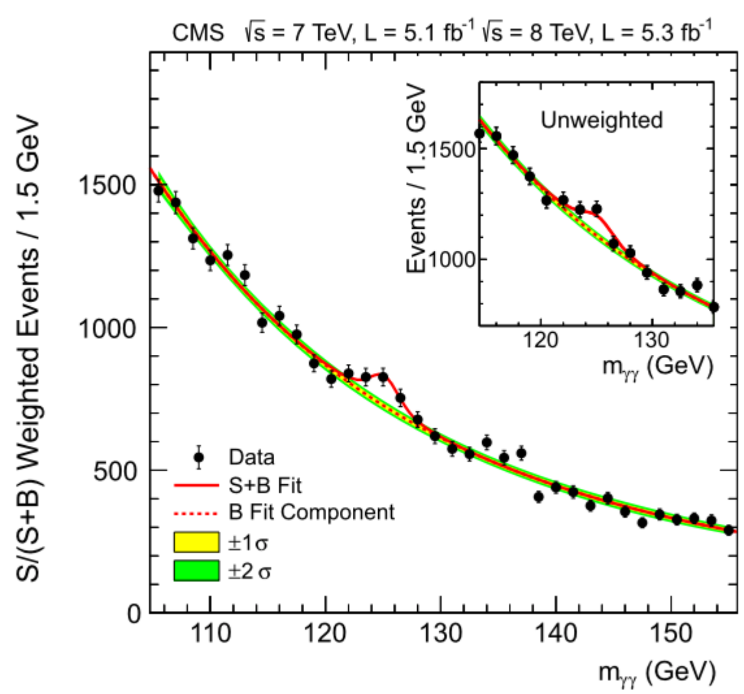
二 电弱相变理论

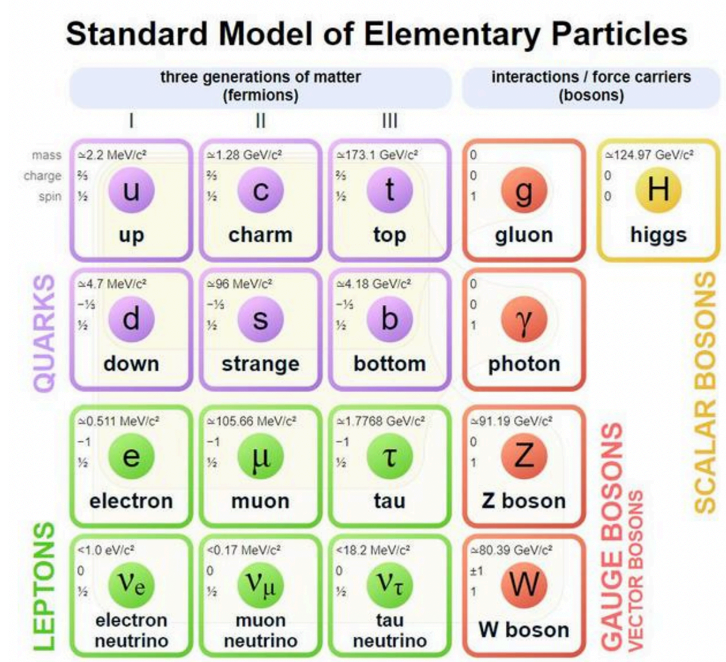
三 引力波数值模拟

四 总结与展望

125 GeV Higgs&Standard Model







Is the Standard Model complete?

Experimental Evidence?

Dark Matter

Baryogenesis

Neutrino masses

Origin of flavor

Theoretical completeness/Beauty?

Cosmological constant

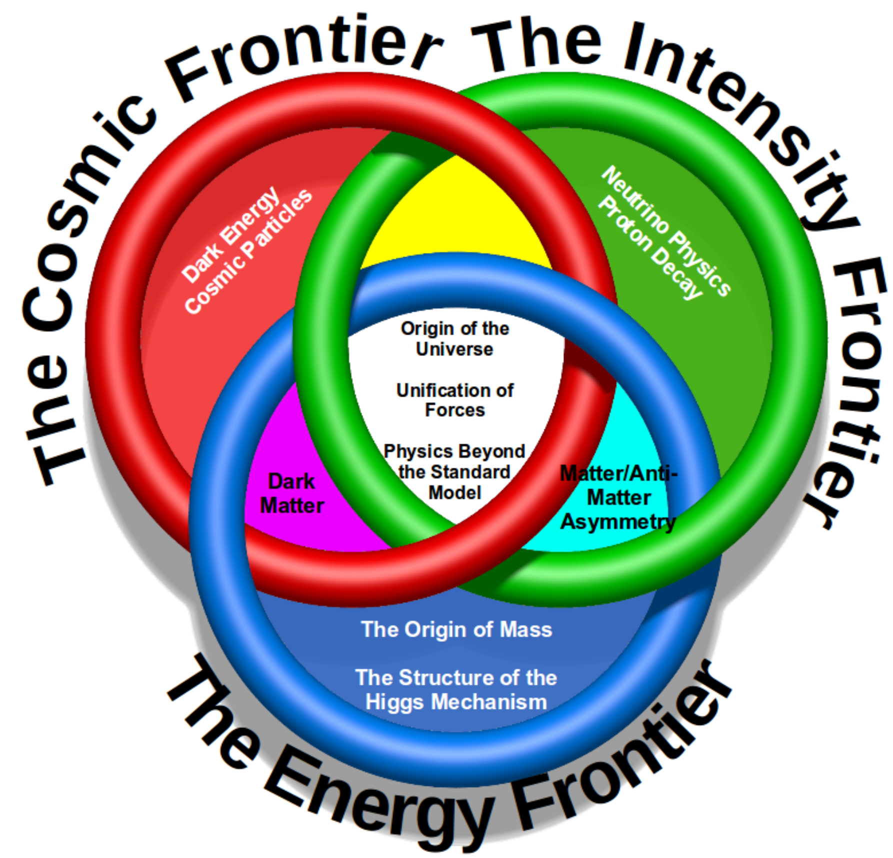
Hierarchy problem

Strong CP problem

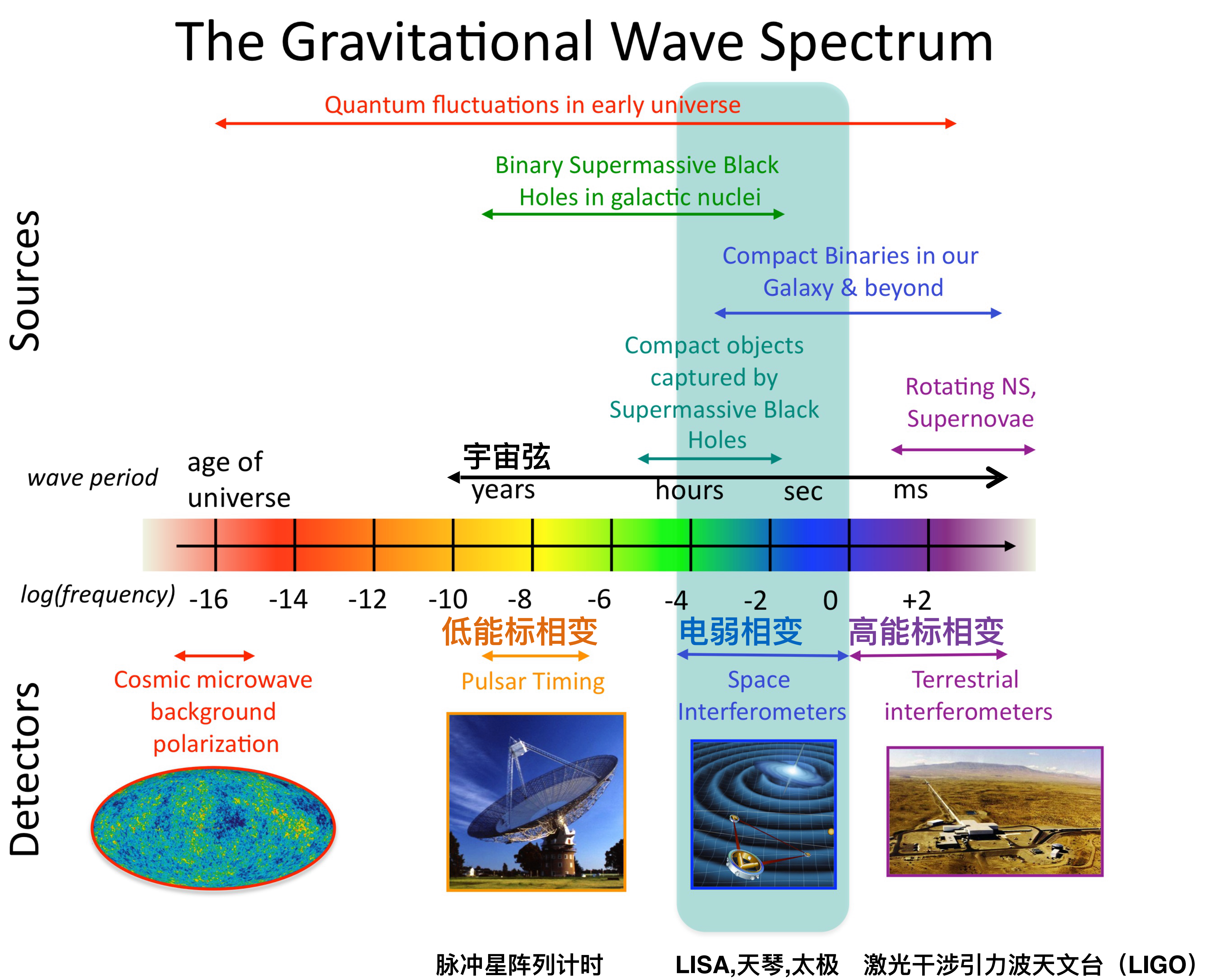
Grand Unified Theory

Quantum Gravity

Search for New physics



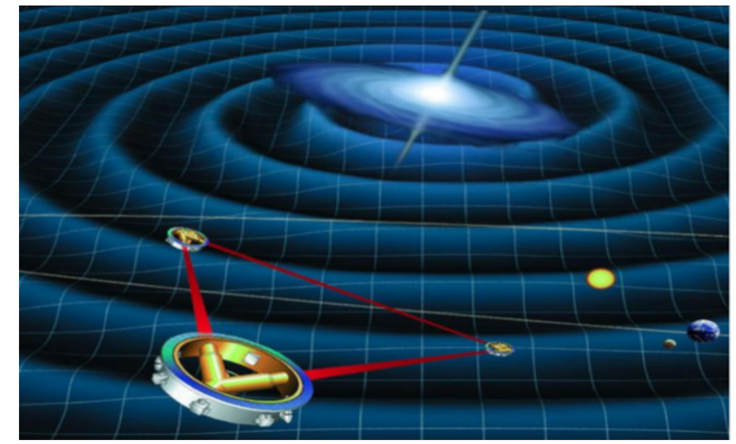
随机引力波探测开启了探索早期宇宙背后新物理的一个新的窗口



新物理&相变引力波

重要的引力波源，主要科学目标之一

PTA,LIGO,LISA,天琴,太极,…



超出粒子物理标准模型
新物理

相变参数

Effective action $\rightarrow \beta,\ H_*$

Energy budget $\rightarrow \alpha,\ \kappa(\alpha, v_{\rm w})$

Bubble wall dynamics $\rightarrow v_{\rm w}$

引力波功率谱

Numerical simulations $\rightarrow$

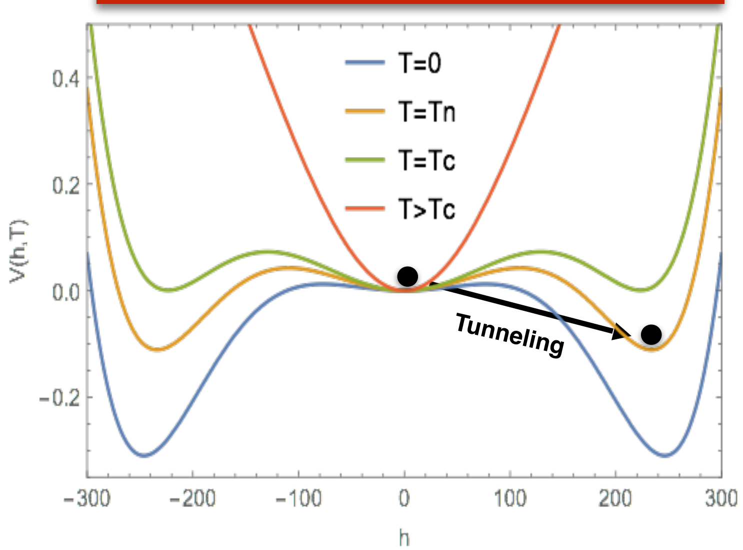
$h^2\Omega_{\rm GW}(f; H_*, \alpha, \beta, v_{\rm w})$

实验探测

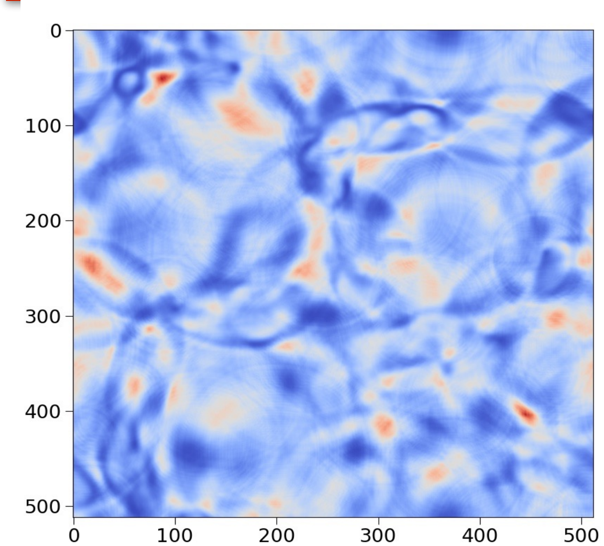
Configuration + noise level $\rightarrow$

$h^2\Omega_{\rm sens}(f)$

有限温场论计算



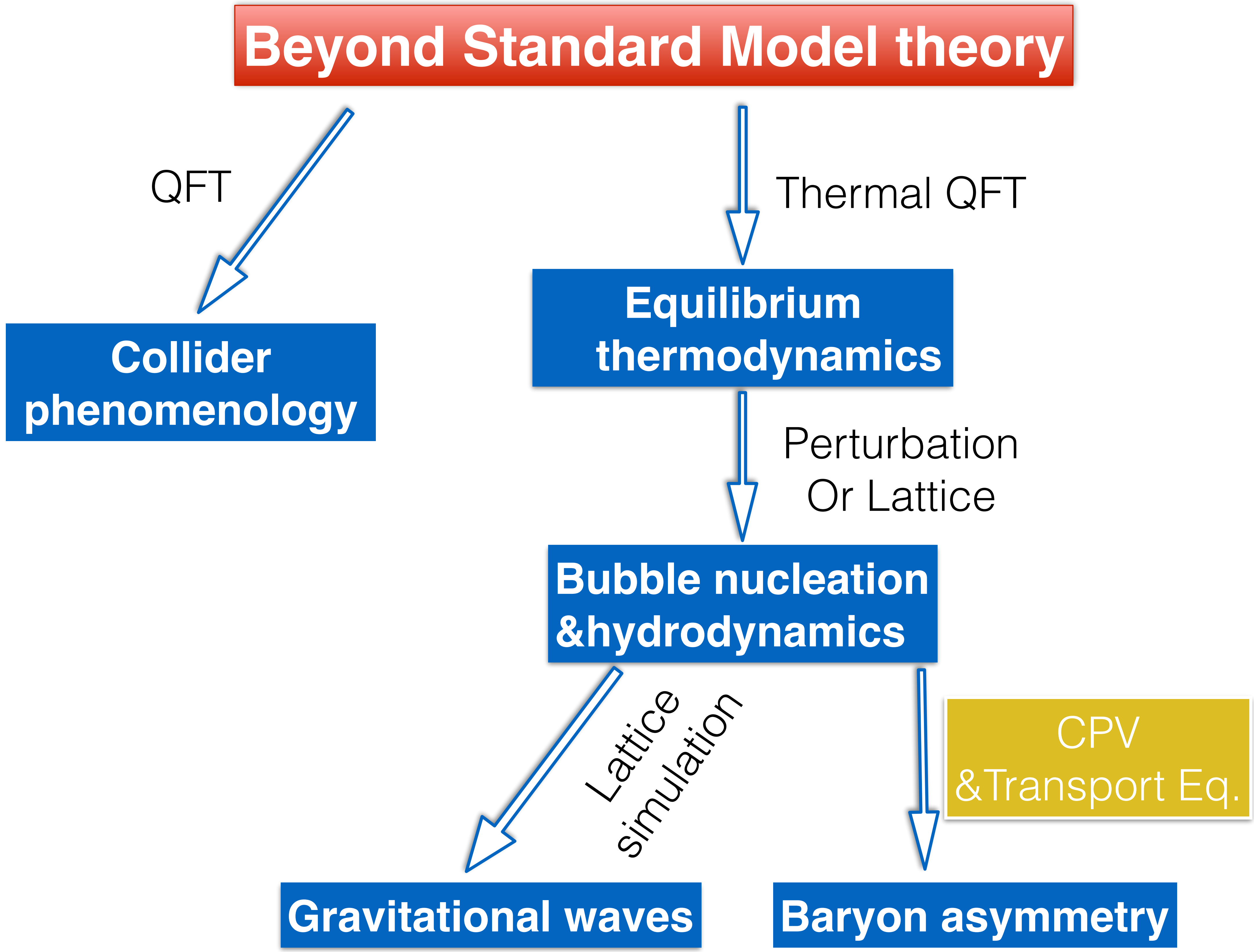
格点场论模拟建立理论和实验的桥梁



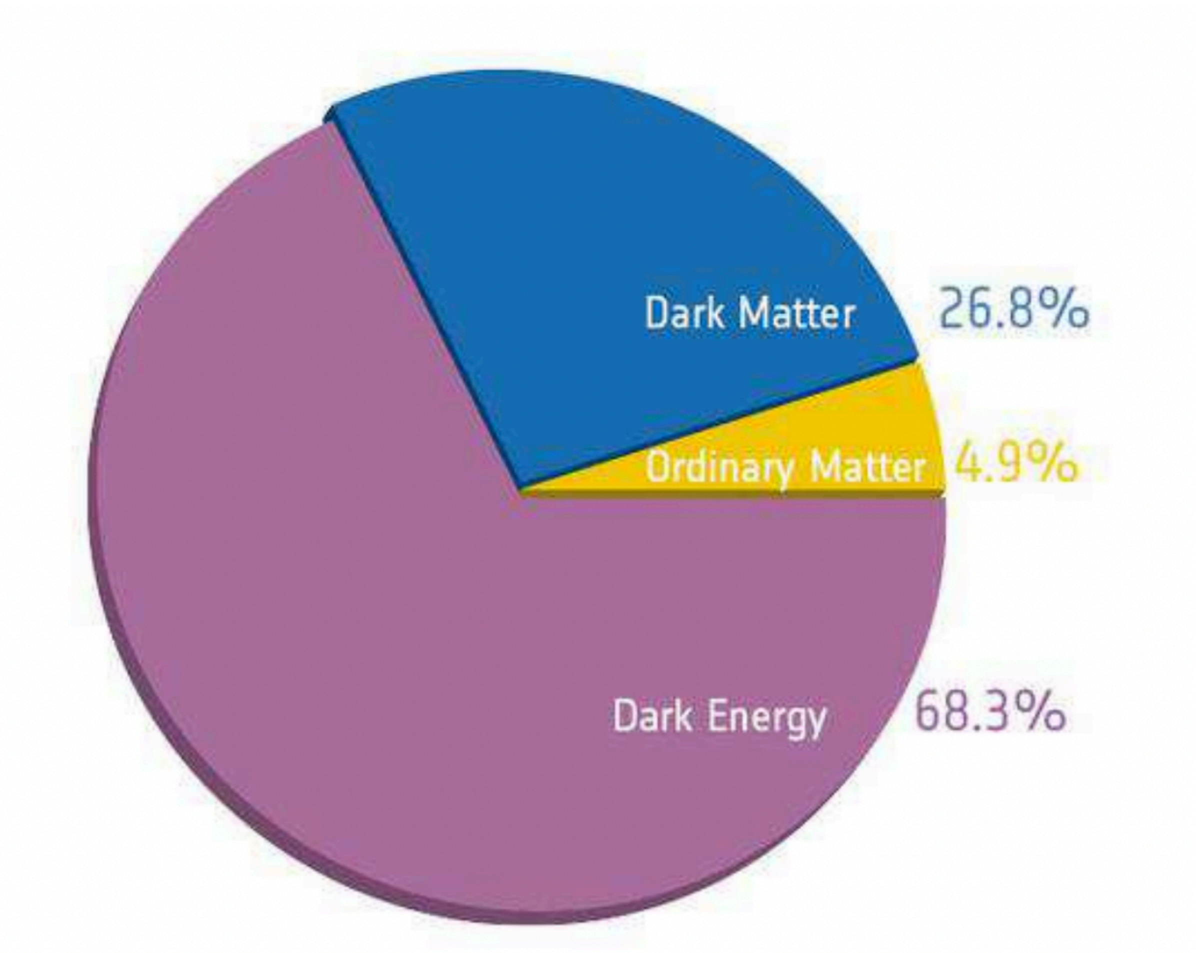
信噪比

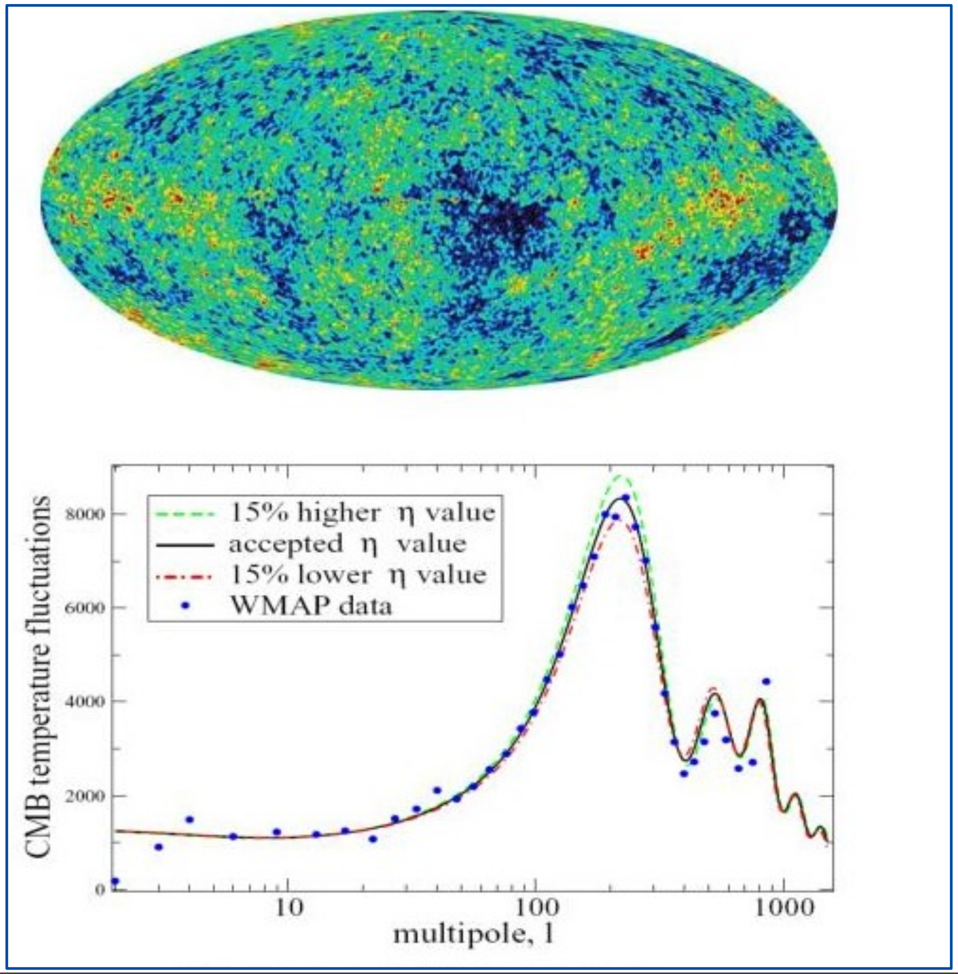
Chiara Caprini *et al* JCAP03(2020)024

电弱相变相关粒子物理宇宙学



Baryon Asymmetry of the Universe





$$\eta = 5.5 \times 10^{-10} (\Omega_b h^2 / 0.020)$$

$$n_b = \eta\, n_\gamma = \eta \times \frac{2\zeta(3)}{\pi^2} T^3$$

How to explain the Baryon Asymmetry of the Universe?

The three Sakharov conditions
for Baryon Asymmetry???

Nobel Peace Prize in 1975

- Baryon Number Violation
  - Weak Sphaleron within SM
- C&CP Violation
  - BSM physics
- Out of thermal equilibrium
  - BSM physics

BAU&CPV

CP violation arises naturally in the quark sector of the Standard Model. It's been observed in K, D, and B mesons. But that's not enough!!!

CKM matrix:

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{-i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{-i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{-i\delta_{13}} & -c_{12}c_{23}-s_{12}c_{23}s_{13}e^{-i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

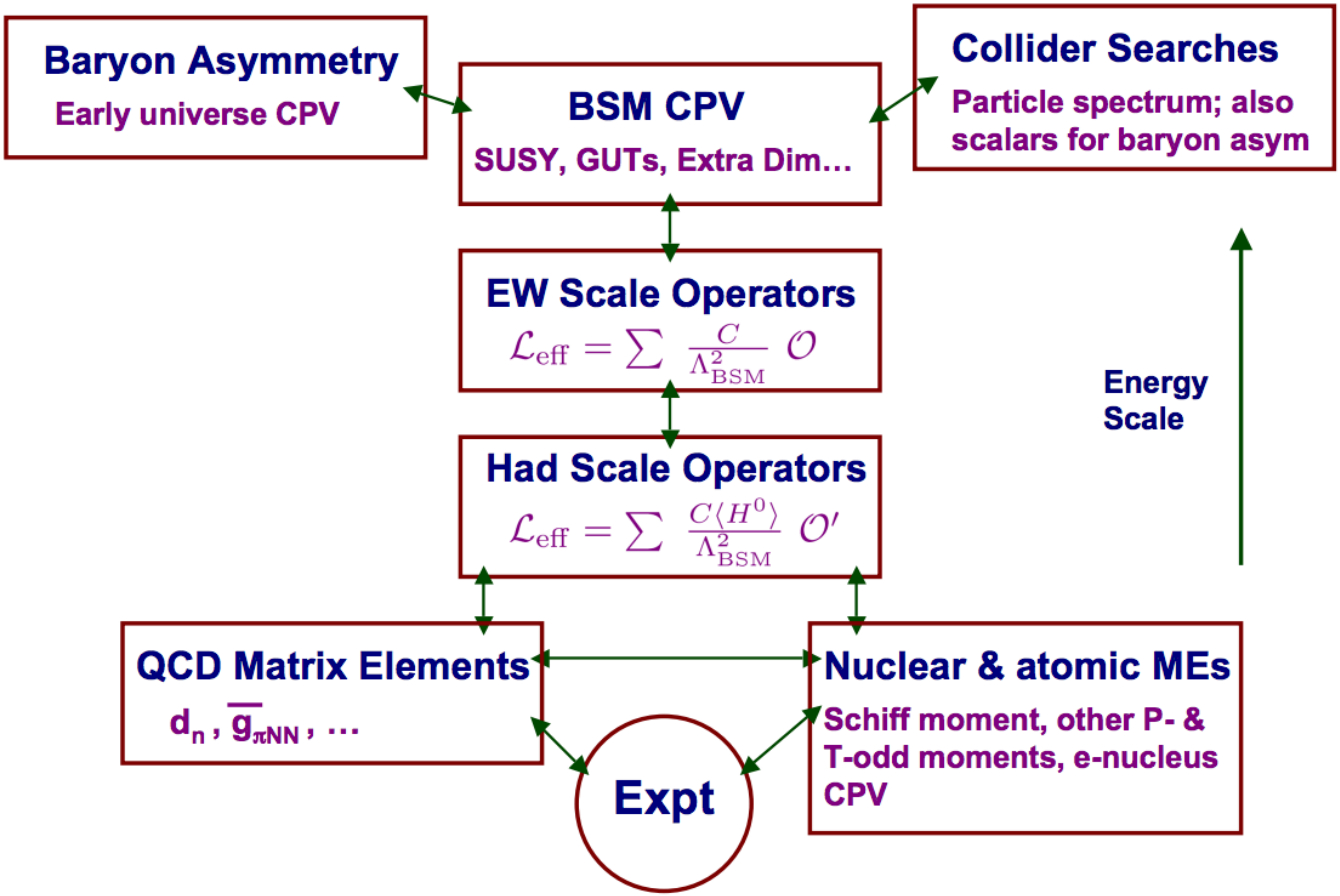
The invariant phase using Jarlskog invariant

$$\begin{aligned} J_{\mathrm{CKM}} = & \ (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2) \\ \times & \ (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)K, \end{aligned}$$

$$K = \mathrm{Im}\, V_{ii} V_{jj} V_{ij}^{\star} V_{ji}^{\star} = s_1^2 s_2 s_3 c_1 c_2 c_3 \sin\delta$$

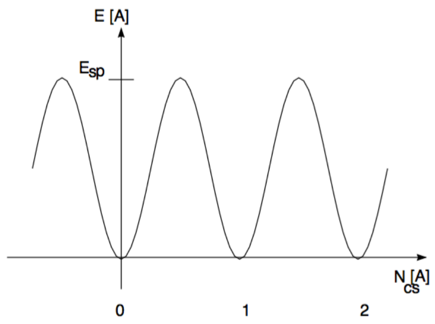
$$\frac{J_{\mathrm{CKM}}}{T_c^{\ 12}} \approx 10^{-20} \ll 10^{-11},$$ Tc is the SM cross-over temperature

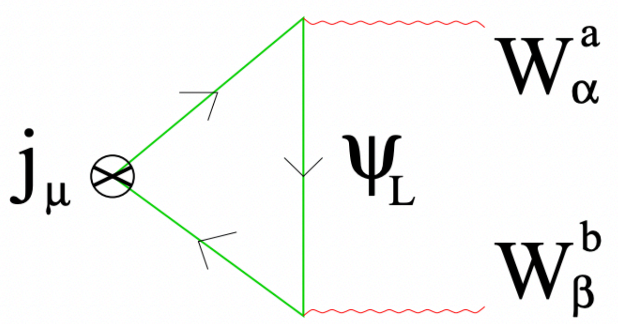
BAU& CPV



Engel etal, Prog.Part.Nucl.Phys. 71 (2013) 21-74

BAU& Electroweak Sphaleron





$$\partial_\mu J^\mu_B = i\frac{N_F}{32\pi^2}\left(-g_2^2 F^{a\mu\nu}\widetilde{F}^a_{\mu\nu} + g_1^2 f^{\mu\nu}\widetilde{f}_{\mu\nu}\right),$$

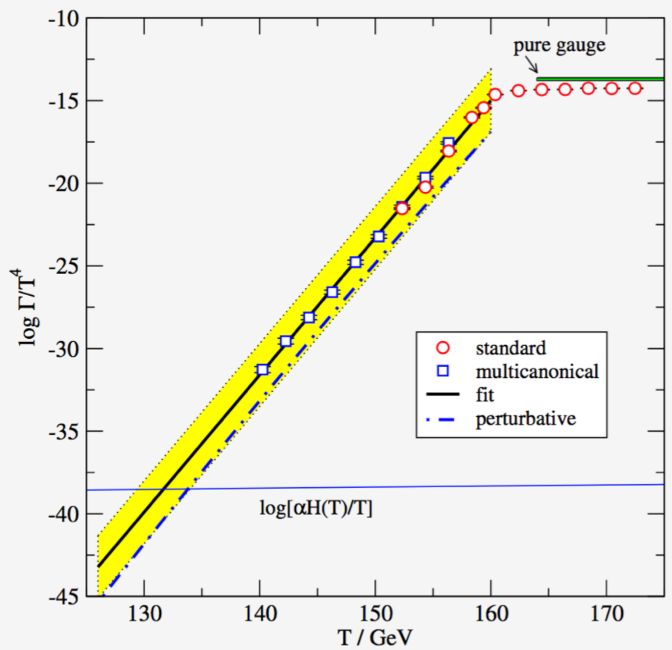
$$\Delta B = N_F(\Delta N_{\rm CS} - \Delta n_{\rm CS}),$$

$$N_{\rm CS} = -\frac{g_2^2}{16\pi^2}\int d^3x\, 2\epsilon^{ijk}\,{\rm Tr}\left[\partial_i A_j A_k + i\frac{2}{3}g_2 A_i A_j A_k\right],$$

$$n_{\rm CS} = -\frac{g_1^2}{16\pi^2}\int d^3x\,\epsilon^{ijk}\,\partial_i B_j B_k,$$

$$A_i \to U A_i U^{-1} + \frac{i}{g_2}(\partial_i U)U^{-1},$$

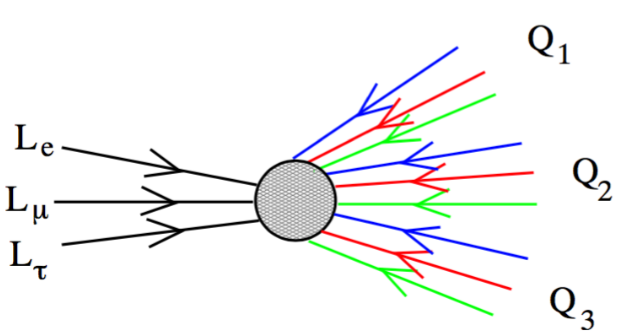
$$\delta N_{\rm CS} = \frac{1}{24\pi^2}\int d^3x\,{\rm Tr}\left[(\partial_i U)U^{-1}(\partial_j U)U^{-1}(\partial_k U)U^{-1}\right]\epsilon^{ijk}.$$



Lattice result, $T_C = (159.5 \pm 1.5)\text{GeV}$, Phys.Rev.Lett,113, 141602 (2014).

$$\Gamma^{\rm sym} \approx 6\times(18\pm3)\alpha_W^5 T^4, \qquad \Gamma^{\rm brok}\sim T^4\exp\left(-\frac{E_{\rm sph}}{T}\right)$$

The Standard Model already contains a process that violates B-number. It is known as the electroweak sphaleron ("sphaleros" is Greek for "ready to fall").



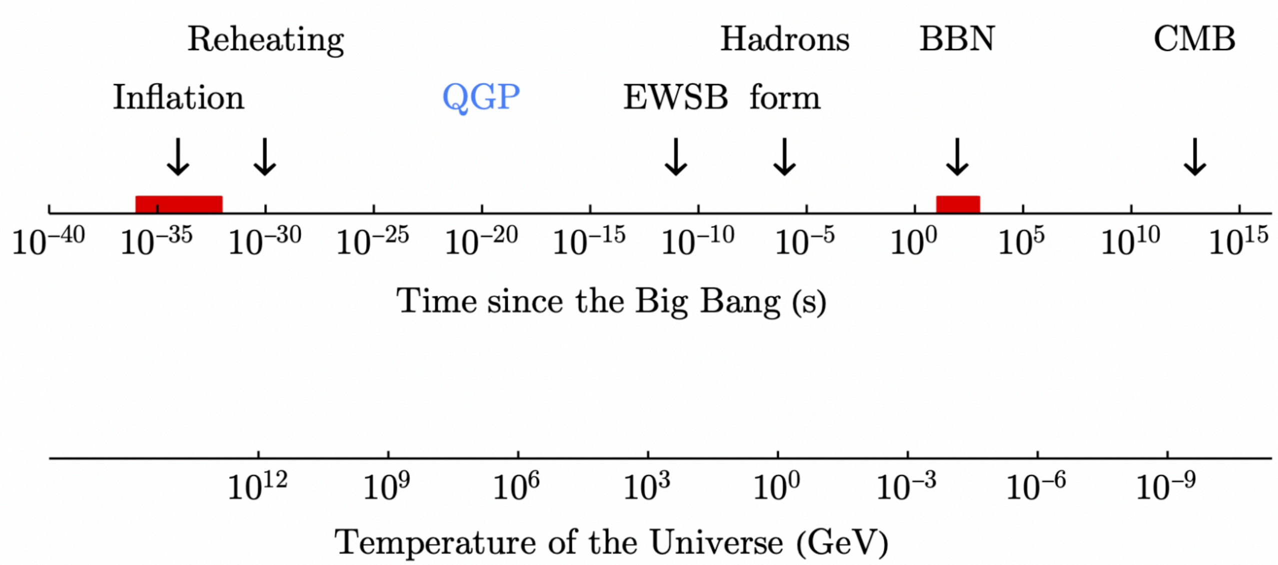
Klinkhammer & Manton (1984); Kuzmin, Rubakov, & Shaposhnikov (1985); Harvey & Turner (1990) but also identified earlier by Dashen, Hasslacher, & Neveu (1974) and Boguta (1983)

BAU mechanism

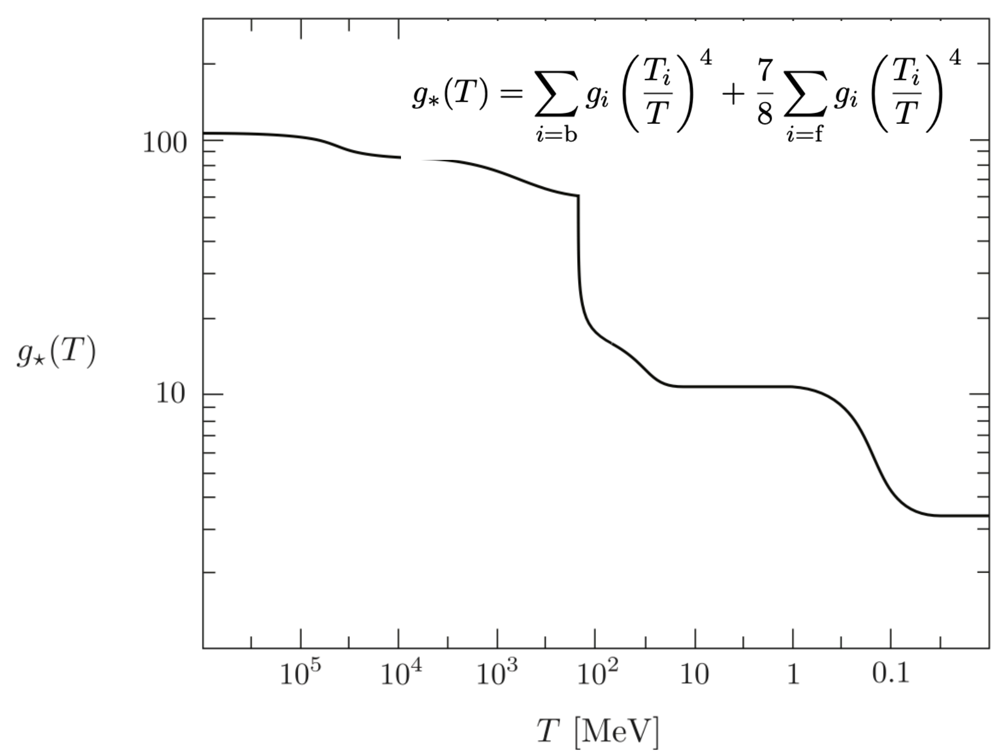
Popular Mechanisms

- Leptogenesis → BAU related to the origin of neutrino masses
- Electroweak Baryogenesis → BAU created during Electroweak phase transition
- GUT Baryogenesis → BAU from B-violating decay of heavy GUT stuff
- Affleck-Dine → BAU from rolling scalars carrying B charges
- Hidden Sector Asymmetric Baryogenesis → BAU in an exotic sector related to dark matter

Key Events in the early Universe

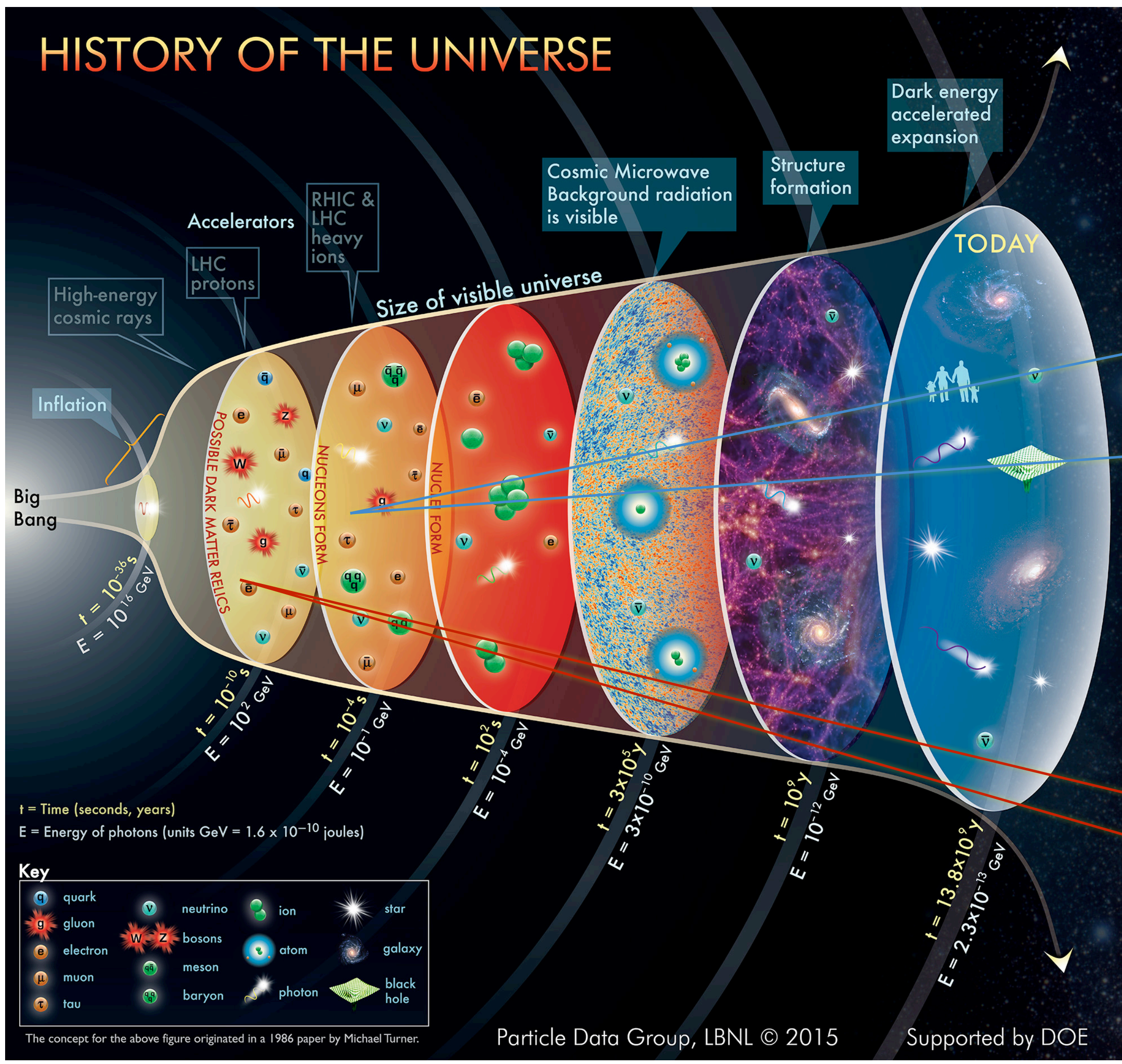


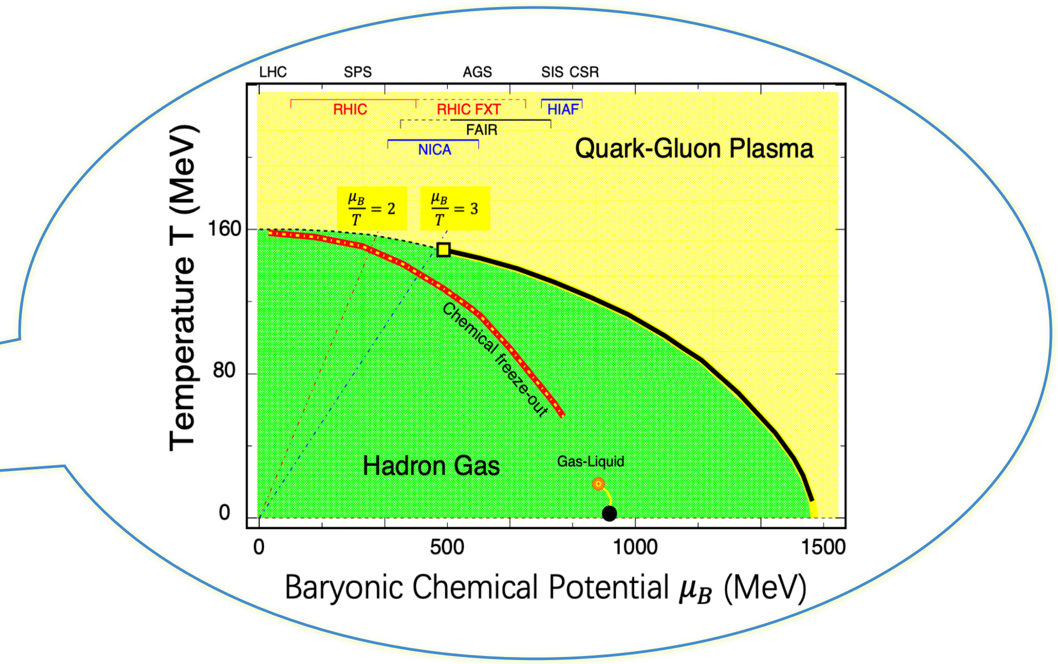
Radiation dominate Universe: $H = \frac{1}{2t} = \sqrt{\frac{8\pi}{3}\frac{\rho_{\rm rad}}{M_p^2}} = \sqrt{\frac{8\pi}{3}\frac{\pi^2}{30}g_*\frac{T^4}{M_p^2}} = \sqrt{\frac{8\pi^3}{90}g_*}\frac{T^2}{M_p}$



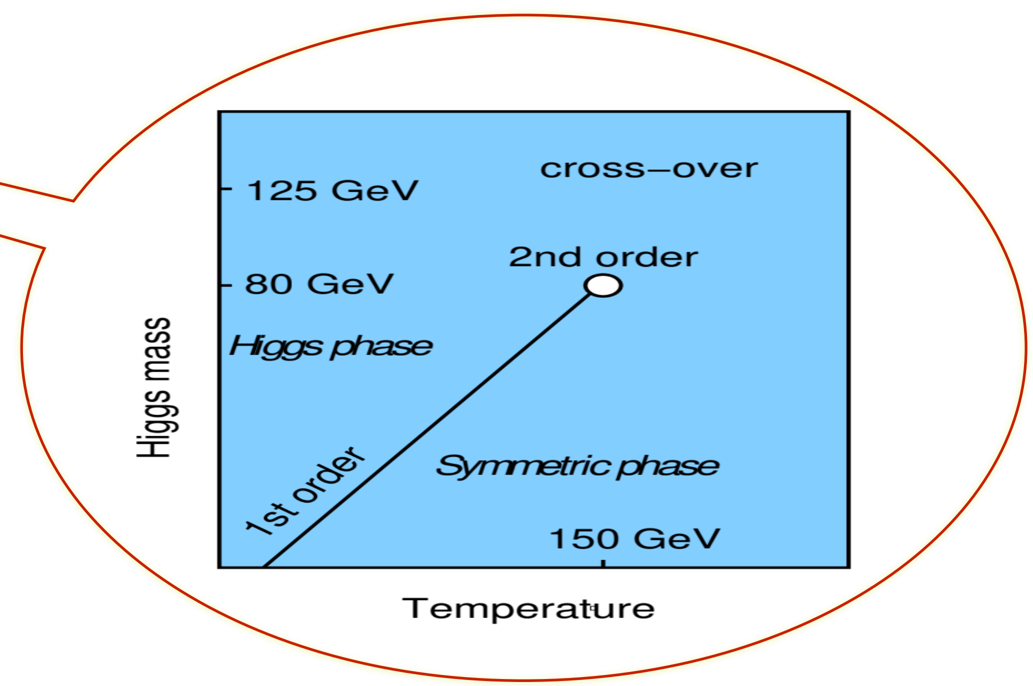
Event	time t	redshift z	temperature T
Inflation	10^{-34} s	–	–
Baryogenesis	?	?	?
EW phase transition	20 ps	10^{15}	100 GeV
QCD phase transition	20 μs	10^{12}	150 MeV
Dark matter freeze-out	?	?	?
Neutrino decoupling	1 s	6×10^9	1 MeV
Electron-positron annihilation	6 s	2×10^9	500 keV
Big Bang nucleosynthesis	3 min	4×10^8	100 keV
Matter-radiation equality	60 kyr	3400	0.75 eV
Recombination	260–380 kyr	1100–1400	0.26–0.33 eV
Photon decoupling	380 kyr	1000–1200	0.23–0.28 eV
Reionization	100–400 Myr	11–30	2.6–7.0 meV
Dark energy-matter equality	9 Gyr	0.4	0.33 meV
Present	13.7 Gyr	0	0.24 meV

BAU& Non-equilibrium



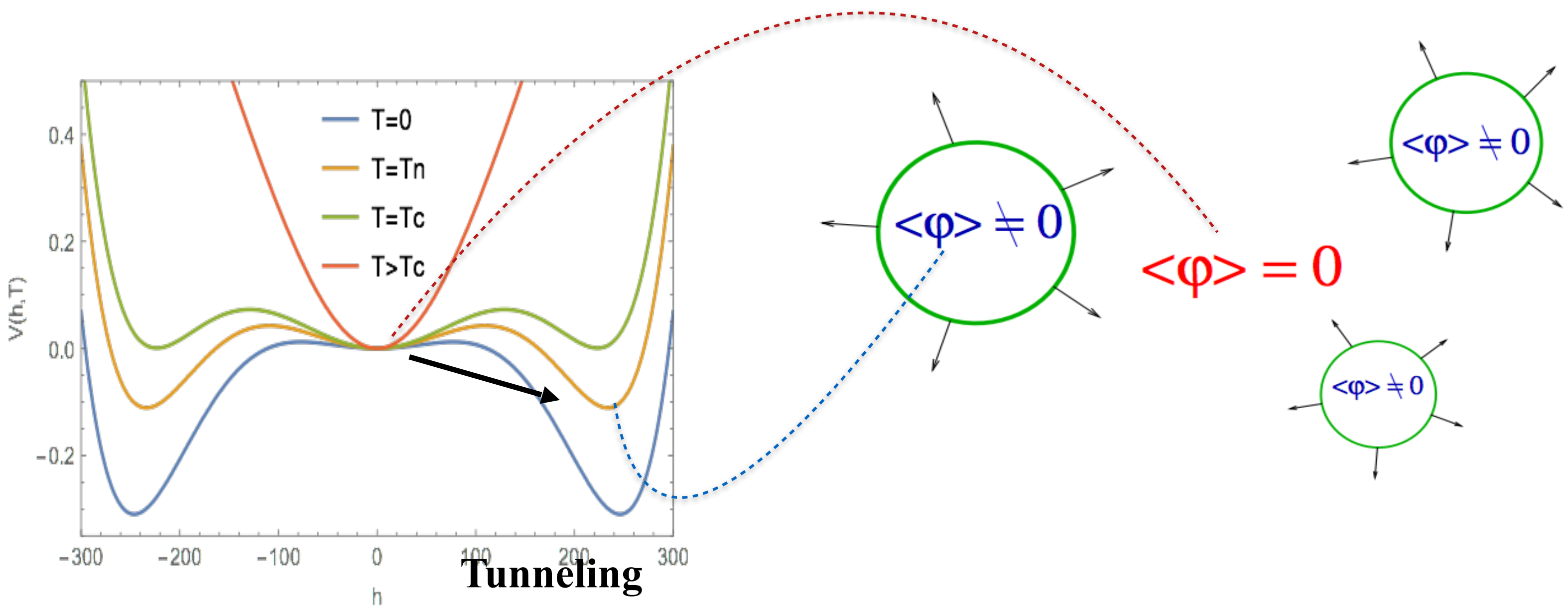


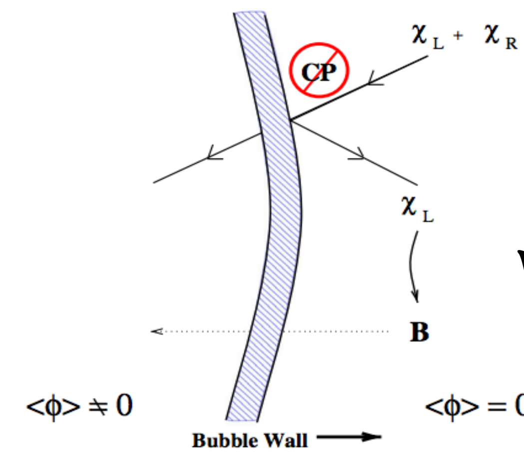
QCD
相图



电弱
相变

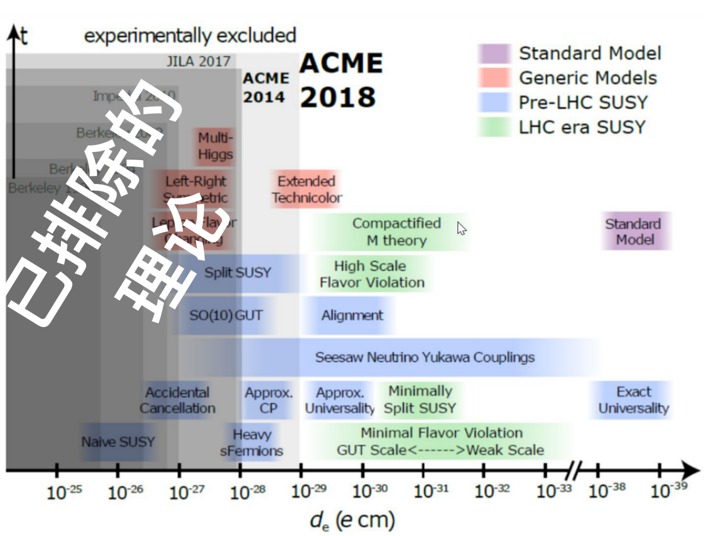
正反物质不对称&强一阶电弱相变

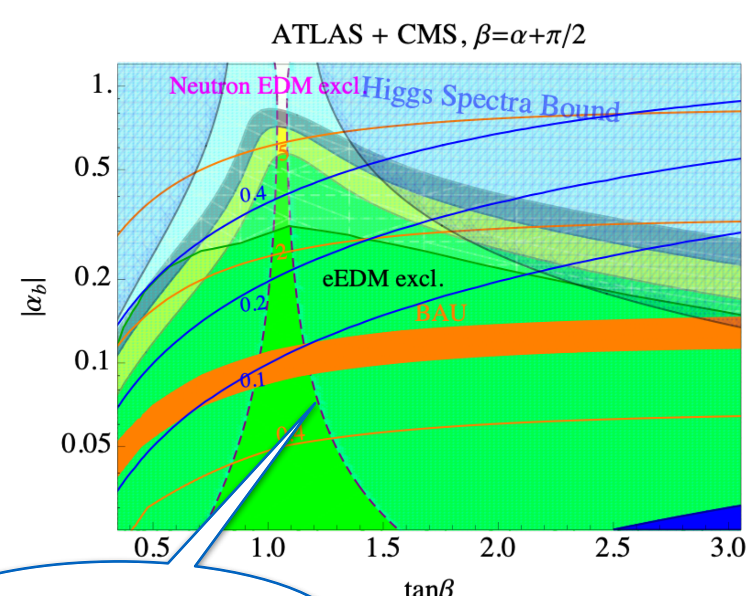




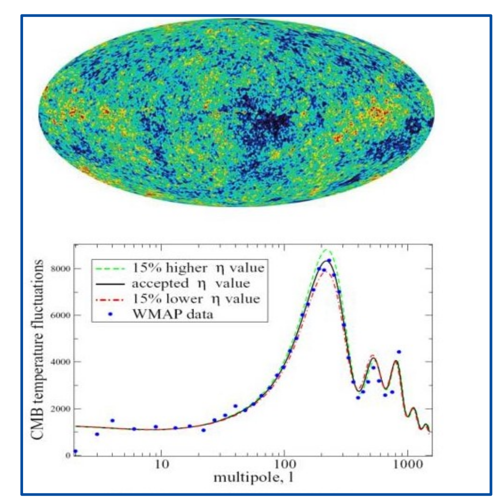
电弱相变中解释BAU

Chup etal，Rev Mod Phys.91.015001





CPV cancellation



$$\frac{n_B}{s} = (8.579 \pm 0.109) \times 10^{-11}$$

Bian, Liu, Shu, PRL115 (2015) 021801

Bubble wall velocity with the EW plasma

Wall frame

Boltzmann equation which dictates the time evolution of the particle distribution

$$\frac{df_a}{dt} = \partial_t f_a + \dot{\vec{x}} \cdot \partial_{\vec{x}} f_a + \dot{\vec{p}} \cdot \partial_{\vec{p}} f_a = C[f_a], \qquad \text{(a)}$$

The *fluid ansatz* for the distribution function is written as

$$f \approx f_v - f_v' \delta \bar{X} + \delta f_u + \mathcal{O}(\delta f^2), \qquad f_v = \frac{1}{e^{\beta\gamma(E - v p_z)} \pm 1}, \quad f_v' \equiv \frac{df_v}{d\beta\gamma E},$$

$$\delta \bar{X} = \mu + \beta\gamma\delta\tau(E - v p_z)$$ perturbations from equilibrium

μ: chemical potential, δτ: temperature perturbation, δf_u : the velocity perturbation

The force and group velocity

$$\dot{z} \equiv \frac{\partial \omega}{\partial p_z} = \frac{p_z}{E} + s\frac{m^2\theta'}{2E^2 E_z}, \qquad \dot{p_z} \equiv -\frac{\partial \omega}{\partial z} = -\frac{(m^2)'}{2E} + s\frac{(m^2\theta')'}{2E E_z}, \qquad \text{(b)}$$

ω is the energy of the WKB wave packet and $E_z^2 \equiv p_z^2 + m^2$

Inserting the force and group velocity of eq. (b) into the Boltzmann equation (a), we have

$$\left[\frac{p_z}{E}\partial_z - \frac{(m^2)'}{2E}\partial_{p_z}\right]\left(f_v - f_v'\delta\bar{X} + \delta f_u\right) = C[f]$$

PHYSICAL REVIEW D 102, 063516 (2020)

Bubble wall velocity with the EW plasma

With $q = (\mu, \delta\tau, u)^T$, the transport equations take the form

$$A_v \vec{q}\,' + \Gamma \vec{q} = S,$$

Γ: the collision term C in the above equation

$$A_v = \begin{pmatrix} C_v^{1,1} & \gamma v C_0^{-1,0} & D_v^{0,0} \\ C_v^{0,1} & \gamma(C_v^{-1,1} - v C_v^{0,2}) & D_v^{-1,0} \\ C_v^{2,2} & \gamma(C_v^{1,2} - v C_v^{2,3}) & D_v^{1,1} \end{pmatrix},$$

Integrals of the particle distribution functions.

$$S = \gamma v \frac{(m^2)'}{2T^2} \begin{pmatrix} C_v^{1,0} \\ C_v^{0,0} \\ C_v^{2,1} \end{pmatrix},$$

Source term

The Higgs EOM in the presence of out of equilibrium particle populations

$$E_h \equiv \Box\phi + \frac{dV_{\text{eff}}(\phi, T)}{d\phi} + \sum_i \frac{dm_i^2}{d\phi} \int \frac{d^3p}{(2\pi)^3} \frac{\delta f_i(p, x)}{2E} = 0,$$

$$M_1 \equiv \int dz E_h h' dz = 0,$$

The total pressure on the wall should be zero

$$M_2 \equiv \int dz E_h h' [2h(z) - h_0] dz = 0.$$

An asymmetry in the total pressure between the front and back of the wall should be zero

$$h(z) = \frac{h_0}{2}\left[\tanh\left(\frac{z}{L_h}\right) + 1\right]$$

Bubble profile

EWBG with the EW plasma

Boltzmann equation $$(v_g\partial_z + F\partial_{p_z})f = \mathcal{C}[f]$$

$$v_g = \frac{p_z}{E_w},$$

CP-violating complex mass term $$\hat{m}(z) = m(z)e^{i\gamma^5\theta(z)}$$

$$F = -\frac{(m^2)'}{2E_w} + s s_{k_0}\frac{(m^2\theta')'}{2E_w E_{wz}},$$

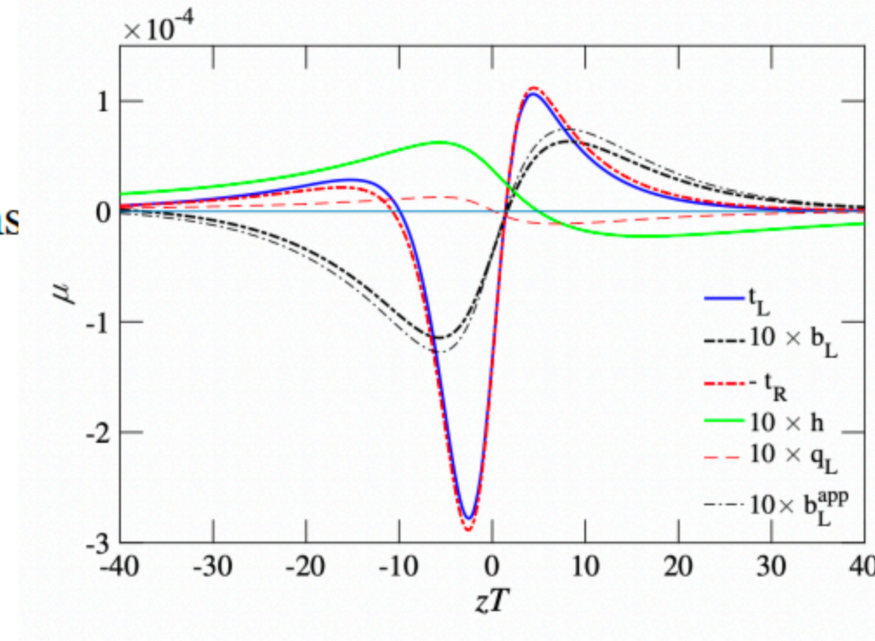
$$\mu \equiv \mu_e + s_{k_0}\mu_o,$$
$$\delta f \equiv \delta f_e + s_{k_0}\delta f_o.$$

Transport equations $$Aw' + (m^2)'Bw = S + \delta C,$$

collision terms

$w=(\mu,u)^T$

$$A = \begin{pmatrix} -D_1 & 1 \\ -D_2 & R \end{pmatrix}, \quad B = \begin{pmatrix} -v_w\gamma_w Q_1 & 0 \\ -v_w\gamma_w Q_2 & \bar{R} \end{pmatrix},$$



Source term $S = (S_1, S_2)^T$ $$S^o_{h\ell} = v_w\gamma_w s\left[(m^2\theta')'Q^{8o}_\ell - (m^2)'m^2\theta' Q^{9o}_\ell\right],$$

chemical potential for left handed baryon number $$\mu_{B_L} = \frac{1}{2}(1+4D_0^t)\mu_{t_L} + \frac{1}{2}(1+4D_0^b)\mu_{b_L} + 2D_0^t\mu_{t_R}$$

Baryon asymmetry $$\eta_B = \frac{405\,\Gamma_{\rm sph}}{4\pi^2 v_w\gamma_w g_* T}\int dz\,\mu_{B_L} f_{\rm sph}\, e^{-45\Gamma_{\rm sph}|z|/4v_w},$$

VEV- insertion source tends to predict a larger baryon asymmetry than the WKB source by a factor of ~10.

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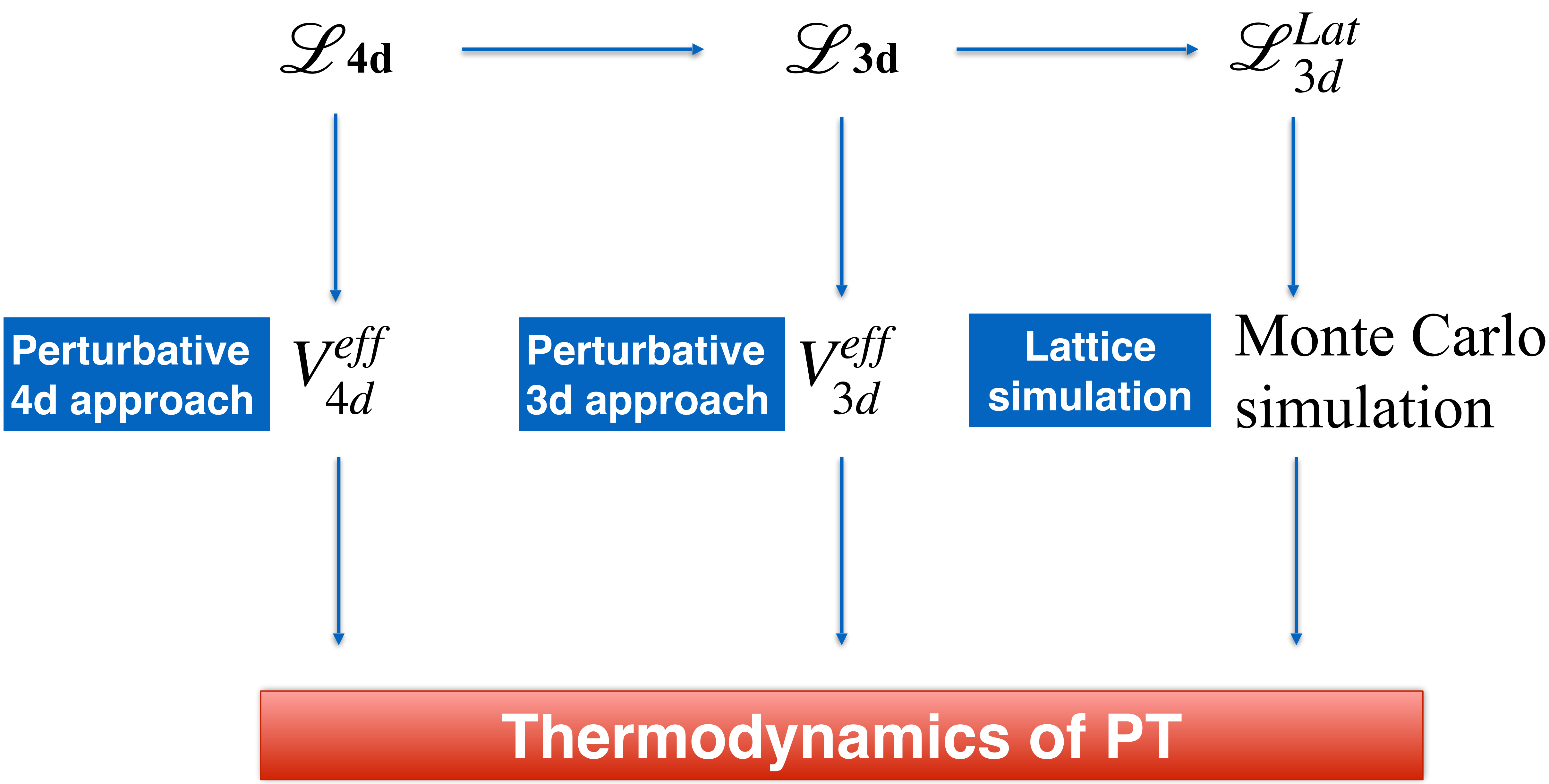
Methods for PT dynamics study



Effective potential at zero temperature

Action $$S[\phi] = \int d^4x \mathcal{L}\{\phi(x)\}$$

The generating functional (vacuum-to-vacuum amplitude):

$$Z[j] = \langle 0_{\rm out} \mid 0_{\rm in} \rangle_j \equiv \int d\phi \exp\{i(S[\phi] + \phi j)\} \qquad \phi j \equiv \int d^4x \phi(x) j(x)$$

The connected generating functional W[j] defined as:

$$Z[j] \equiv \exp\{iW[j]\}$$

The effective action Γ[ϕ] as the Legendre transformation:

$$\Gamma[\overline{\phi}] = W[j] - \int d^4x \frac{\delta W[j]}{\delta j(x)} j(x) \qquad \overline{\phi}(x) = \frac{\delta W[j]}{\delta j(x)}$$

$$\Rightarrow \quad \frac{\delta \Gamma[\overline{\phi}]}{\delta \overline{\phi}} = \frac{\delta W[j]}{\delta j} \frac{\delta j}{\delta \overline{\phi}} - j - \overline{\phi} \frac{\delta j}{\delta \overline{\phi}} = -j$$

$$\left. \frac{\delta \Gamma[\overline{\phi}]}{\delta \overline{\phi}} \right|_{j=0} = 0$$

Effective potential at zero temperature

Expand Z[j] (W[j]) in a power series of j, to obtain its representation in terms of Green functions $G_{(n)}$ (connected Green functions $G_{(n)}^c$)

$$Z[j]=\sum_{n=0}^{\infty}\frac{i^n}{n!}\int d^4x_1\ldots d^4x_n j(x_1)\ldots j(x_n)G_{(n)}(x_1,\ldots,x_n)$$

$$iW[j]=\sum_{n=0}^{\infty}\frac{i^n}{n!}\int d^4x_1\ldots d^4x_n j(x_1)\ldots j(x_n)G^c_{(n)}(x_1,\ldots,x_n)$$

The effective action can be expanded as

$$\Gamma[\overline{\phi}]=\sum_{n=0}^{\infty}\frac{1}{n!}\int d^4x_1\ldots d^4x_n\overline{\phi}(x_1)\ldots\overline{\phi}(x_n)\Gamma^{(n)}(x_1,\ldots,x_n)$$

$\Gamma^{(n)}$ are the one-particle irreducible (1PI) Green functions

Fourier transformation

$$\Gamma^{(n)}(x)=\int\prod_{i=1}^{n}\left[\frac{d^4p_i}{(2\pi)^4}\exp\{ip_ix_i\}\right](2\pi)^4\delta^{(4)}(p_1+\cdots+p_n)\Gamma^{(n)}(p)$$

$$\tilde{\phi}(p)=\int d^4xe^{-ipx}\overline{\phi}(x)$$

$$\Gamma[\overline{\phi}]=\sum_{n=0}^{\infty}\int\prod_{i=1}^{n}\left[\frac{d^4p_i}{(2\pi)^4}\tilde{\phi}(-p_i)\right](2\pi)^4\delta^{(4)}(p_1+\cdots+p_n)\Gamma^{(n)}(p_1,\ldots,p_n) \qquad (1)$$

Effective potential at zero temperature

Translationally invariant theory, with ϕc being constant $\overline{\phi}(x) = \phi_c$

Define the effective potential $V_{\text{eff}}(\phi_c)$ as

$$\Gamma[\phi_c] = -\int d^4x V_{\text{eff}}(\phi_c) \qquad (2)$$

Using the definition of Dirac δ-function

$$\delta^{(4)}(p) = \int \frac{d^4x}{(2\pi)^4} e^{-ipx}$$

We get

$$\tilde{\phi}_c(p) = (2\pi)^4 \phi_c \delta^{(4)}(p).$$

Inserting into EQ.(1), the effective action for constant field configurations recast the form of

$$\Gamma(\phi_c) = \sum_{n=0}^{\infty} \frac{1}{n!} \phi_c^n (2\pi)^4 \delta^{(4)}(0) \Gamma^{(n)}(p_i = 0) = \sum_{n=0}^{\infty} \frac{1}{n!} \phi_c^n \Gamma^{(n)}(p_i = 0) \int d^4x \qquad (3)$$

(2,3) ⇒

$$V_{\text{eff}}(\phi_c) = -\sum_{n=0}^{\infty} \frac{1}{n!} \phi_c^n \Gamma^{(n)}(p_i = 0) \qquad (4)$$

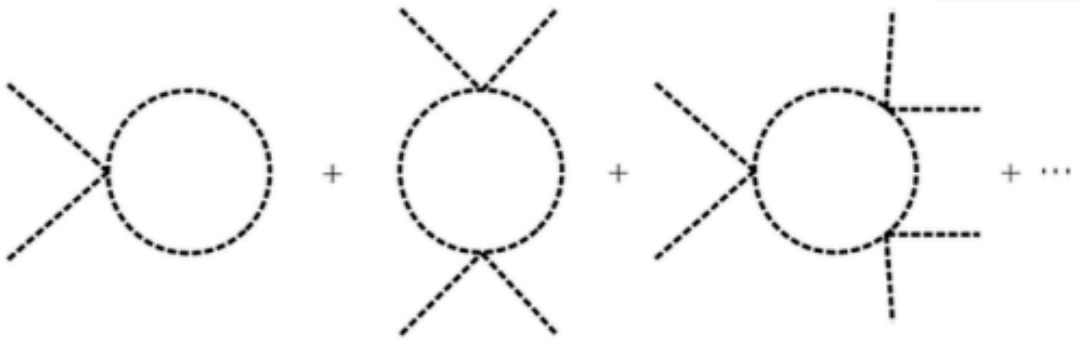
Expanding in powers of momentum, about the point where all external momenta vanish

$$\Gamma[\overline{\phi}] = \int d^4x \left[-V_{\text{eff}}(\overline{\phi}) + \frac{1}{2} (\partial_\mu \overline{\phi}(x))^2 Z(\overline{\phi}) + \cdots \right]$$

Effective potential at zero temperature

An example

Tree-level potential $V_0 = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4$



In momentum space the scalar field is $\phi_c(p) = (2\pi)^4\phi_c\delta^4(p)$

Recall EQ.(4), we get one-loop potential:

$$V_1(\phi_c) = i\sum_{n=1}^{\infty}\int\frac{d^4p}{(2\pi)^4}\frac{1}{2n}\left[\frac{\lambda\phi_c^2/2}{p^2-m^2+i\epsilon}\right]^n$$
$$= -\frac{i}{2}\int\frac{d^4p}{(2\pi)^4}\log\left[1-\frac{\lambda\phi_c^2/2}{p^2-m^2+i\epsilon}\right]$$

After Wick rotation:

$$V_1(\phi_c) = \frac{1}{2}\int\frac{d^4p_E}{(2\pi)^4}\log\left[1+\frac{\lambda\phi_c^2/2}{p_E^2+m^2}\right]$$

$$p^0 = ip_E^0,\ p_E = (-ip^0, \vec{p}\,),\ p^2 = (p^0)^2 - \vec{p}^{\,2} = -p_E^2$$

1-loop effective potential is

$$V_1(\phi_c) = \frac{1}{2}\int\frac{d^4p}{(2\pi)^4}\log\left[p^2+m^2(\phi_c)\right] \quad (5)$$

shifted mass :

$$m^2(\phi_c) = m^2 + \frac{1}{2}\lambda\phi_c^2 = \frac{d^2V_0(\phi_c)}{d\phi_c^2}$$

$$V_{\text{eff}}(\phi_c) = V_0(\phi_c) + V_1(\phi_c)$$

Effective potential at zero temperature

An example

With dimensional regularization

$$V_1(\phi_c) = \frac{1}{2}(\mu^2)^{2-\frac{n}{2}} \int \frac{d^n p}{(2\pi)^n} \log\left[p^2 + m^2\right]$$

We calculate the one-loop correction to the effective potential by first calculating it with respect to the mass and then integrating.

$$\frac{\partial V_1}{\partial m^2} = \frac{1}{2}(\mu^2)^{2-\frac{n}{2}} \int \frac{d^n p}{(2\pi)^n} \frac{1}{p^2 + m^2}$$

$$V_1 = \frac{m^4}{64\pi}\left(-\left[\frac{1}{\epsilon} - \gamma_E + \log 4\pi\right] + \log\frac{m^2}{\mu^2} - \frac{3}{2}\right)$$

The derivative is just a single disconnected bubble.

Subtracting the $1/\varepsilon - \gamma - \log 4\pi$ term, we get

$$V_1 = \frac{1}{64\pi^2} m^4\left(\log\frac{m^2}{\mu^2} - \frac{3}{2}\right)$$

$\boldsymbol{m^2 = d^2V/d\phi^2}$

Finite temperature potential and free energy

The grand canonical partition function

$$\mathcal{Z}(T) \equiv \mathrm{Tr}[e^{-\beta(\hat{H}-\mu\hat{N})}], \quad \text{where} \quad \beta \equiv \frac{1}{T} \qquad \mu_B/T \ll 1$$

$$\phi(x) = T\sum_{\omega_n}\int_{\mathbf{k}} e^{ik\cdot x}\phi(k) = T\sum_{\omega_n}\int_{\mathbf{k}} e^{i(\omega_k\tau-\mathbf{k}\cdot\mathbf{x})}\phi(k)$$

$$\omega_n = 2n\pi T \qquad\qquad k = (\omega_n, \mathbf{k})$$

$$\mathcal{Z}(T) = \int \mathcal{D}\phi \exp\left(-T\sum\int_{\mathbf{k}}\frac{1}{2}\left(\mathbf{k}^2+\omega_n^2+m^2\right)|\phi(k)|^2\right)$$

$$= \exp\left[-\frac{V}{T}\int\frac{d^3k}{(2\pi)^3}\left(\frac{\omega}{2}+T\ln\left(1-e^{-\omega/T}\right)\right)\right]$$

The free energy

$$F = -T\ln\mathcal{Z}$$

$$\lim_{V\to\infty}\frac{F}{V} = \int\frac{d^3k}{(2\pi)^3}\left(\frac{\omega}{2}+T\ln\left(1-e^{-\omega/T}\right)\right)$$
$$= \int\frac{d^3k}{(2\pi)^3}\left(\frac{\sqrt{m^2+\mathbf{k}^2}}{2}+T\ln\left(1-e^{-\sqrt{m^2+\mathbf{k}^2}/T}\right)\right)$$
$$\equiv J_0(m)+\tilde{J}_B(m,T) \qquad \tilde{J}_i = T^4/2\pi^2 J_i.$$

$$\left(\lim_{V\to\infty}\frac{F}{V}\right)_{\text{fermions}} = \int\frac{d^3k}{(2\pi)^3}\left(\frac{\omega}{2}+T\ln\left(1+e^{-\omega/T}\right)\right)$$
$$= \int\frac{d^3k}{(2\pi)^3}\left(\frac{\sqrt{m^2+\mathbf{k}^2}}{2}+T\ln\left(1+e^{-\sqrt{m^2+\mathbf{k}^2}/T}\right)\right)$$
$$\equiv J_0(m)+\tilde{J}_F(m,T)$$

$$\tilde{J}_B(m,T) = T\int\frac{d^3k}{(2\pi)^3}\ln\left(1-e^{-\sqrt{m^2+\mathbf{k}^2}/T}\right)$$
$$= \frac{T}{2\pi^2}\int d|\mathbf{k}|\,\mathbf{k}^2\ln\left(1-e^{-\sqrt{m^2+\mathbf{k}^2}/T}\right)$$
$$= \frac{T^4}{2\pi^2}\int dx\,x^2\ln\left(1-e^{-\sqrt{(m/T)^2+x^2}}\right)$$

$$\tilde{J}_F = \frac{T^4}{2\pi^2}\left(-\frac{7\pi^4}{360}+\frac{\pi^2m^2}{24T^2}-\frac{m^4}{32T^4}\left[\ln\left(\frac{e^{\gamma_E}}{\pi^2}\frac{m^2}{T^2}\right)-\frac{3}{2}\right]+\mathcal{O}\left(\frac{m^6}{T^6}\right)\right)$$

high-T expansion m ≪ T

2307.00068

Effective potential at finite temperature-imaginary time

Feynman rules for the different fields in the imaginary time formalism:

$$\text{Boson propagator} \quad : \quad \frac{i}{p^2 - m^2}; \; p^\mu = [2ni\pi\beta^{-1}, \vec{p}\,]$$

$$\text{Fermion propagator} \quad : \quad \frac{i}{\gamma \cdot p - m}; \; p^\mu = [(2n+1)i\pi\beta^{-1}, \vec{p}\,]$$

$$\text{Loop integral} \quad : \quad \frac{i}{\beta} \sum_{n=-\infty}^{\infty} \int \frac{d^3p}{(2\pi)^3}$$

$$\text{Vertex function} \quad : \quad -i\beta(2\pi)^3 \delta_{\sum \omega_i} \delta^{(3)}(\sum_i \vec{p}_i)$$

With above FR EQ.(5) becomes

$$V_1^\beta(\phi_c) = \frac{1}{2\beta} \sum_{n=-\infty}^{\infty} \int \frac{d^3p}{(2\pi)^3} \log(\omega_n^2 + \omega^2) \tag{6}$$

with $\omega^2 = \vec{p}^{\,2} + m^2(\phi_c)$

Define

$$v(\omega) = \sum_{n=-\infty}^{\infty} \log(\omega_n^2 + \omega^2)$$

We have

$$v(\omega) = 2\beta \left[\frac{w}{2} + \frac{1}{\beta} \log\left(1 - e^{-\beta\omega}\right) \right] + \omega - \text{independent terms}$$

Substituting into EQ.(6) we get

$$V_1^\beta(\phi_c) = \int \frac{d^3p}{(2\pi)^3} \left[\frac{\omega}{2} + \frac{1}{\beta} \log\left(1 - e^{-\beta\omega}\right) \right] \tag{7}$$

$$\frac{1}{2} \int \frac{d^3p}{(2\pi)^3} \omega = \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} \log[p^2 + m^2(\phi_c)]$$

$$\frac{1}{\beta} \int \frac{d^3p}{(2\pi)^3} \log\left(1 - e^{-\beta\omega}\right) = \frac{1}{2\pi^2\beta^4} J_B[m^2(\phi_c)\beta^2]$$

▸ Effective potential at finite temperature-real time

Propagators for scalar fields can be written as

$$G(p) \equiv \begin{pmatrix} G^{(11)}(p) & G^{(12)}(p) \\ G^{(21)}(p) & G^{(22)}(p) \end{pmatrix}$$

$$\begin{aligned} G^{(11)}(p) &= \Delta(p) + 2\pi n_B(\omega_p)\delta(p^2 - m^2) \\ G^{(22)}(p) &= G^{(11)*} \\ G^{(12)} &= 2\pi e^{\beta\omega_p/2} n_B(\omega_p)\delta(p^2 - m^2) \\ G^{(21)} &= G^{(12)} \end{aligned}$$

$$n_B(\omega) = \frac{1}{e^{\beta\omega} - 1}$$

The propagators for fermion fields can be written as

$$S(p)_{\alpha\beta} \equiv \begin{pmatrix} S^{(11)}_{\alpha\beta}(p) & S^{(12)}_{\alpha\beta}(p) \\ S^{(21)}_{\alpha\beta}(p) & S^{(22)}_{\alpha\beta}(p) \end{pmatrix}$$

$$\begin{aligned} S^{(11)}(p) &= (\gamma \cdot p + m)\left(\Delta(p) - 2\pi n_F(\omega_p)\delta(p^2 - m^2)\right) \\ S^{(22)}(p) &= S^{(11)*} \\ S^{(12)} &= -2\pi(\gamma \cdot p + m)[\theta(p^0) - \theta(-p^0)] e^{\beta\omega_p/2} n_F(\omega_p)\delta(p^2 - m^2) \\ S^{(21)} &= -S^{(12)} \end{aligned}$$

$$n_F(\omega) = \frac{1}{e^{\beta\omega} + 1}$$

Δ(p) is the boson/fermion propagator at zero temperature

The main feature of the real time formalism is that the propagators come in two terms:

1. one which is the same as in the zero temperature field theory(Δ(p)), and a second one where all the temperature dependence is contained.

2. (12), (21) and (22) components are unphysical since one of their time arguments has an imaginary component.

Effective potential at finite temperature-real time

Disconnected bubble diagrams

$$\frac{dV_1^\beta}{dm^2(\phi_c)} = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\left[\frac{-i}{-p^2+m^2(\phi_c)-i\epsilon} + 2\pi n_B(\omega)\delta(p^2-m^2(\phi_c))\right] \quad (8)$$

After integration on $m^2(\varphi c)$, the first part contributes to the effective potential as

$$-\frac{i}{2}\int \frac{d^4p}{(2\pi)^4}\log(-p^2+m^2(\phi_c)-i\epsilon)$$

Considering

$$-\frac{i}{2}\int_{-\infty}^{\infty}\frac{dx}{2\pi}\log(-x^2+\omega^2-i\epsilon) = \frac{\omega}{2} + \text{constant}$$

Performing the p^0 integral, we get

$$\int \frac{d^3p}{(2\pi)^3}\frac{\omega}{2} \quad (9)$$

Using the identity

$$\delta(p^2-m^2) = \frac{1}{2\omega_p}\left[\delta(p^0+\omega_p)+\delta(p^0-\omega_p)\right]$$

Integration over p^0 in the β-dependent of the EQ 8, we get

$$\int \frac{d^3p}{(2\pi)^3}\frac{1}{2\omega}n_B(\omega) \quad (10)$$

Upon integration over $m^2(\varphi c)$ leads to the second term of EQ (7)

hep-ph/9901312
1701.01554

1-loop Effective potential at finite temperature

1-loop finite-T thermal effective potential

$$V_{\rm eff}(\phi, T) = V_{\rm tree} + V_{\rm 1\text{-}loop}$$

1-loop

$$V_{\rm 1\text{-}loop} = \frac{1}{2}\sum\!\!\!\!\!\!\int_P \ln\left(P^2+m^2\right)$$

$$= \frac{1}{2}\left(\frac{\bar{\mu}^2 e^{\gamma_{\rm E}}}{4\pi}\right)^{\epsilon}\int\frac{d^Dp}{(2\pi)^D}\ln(p^2+m^2) - \int_p T\ln\Big(1\mp n_{\rm B/F}(E_p,T)\Big)$$

$$V_{\rm CW}(m) \qquad\qquad V_T \sim J_{T,b/f}\left(\frac{m^2}{T^2}\right)$$

$$= \frac{T}{2}\int_p \ln(p^2+m^2) + \frac{1}{2}\sum\!\!\!\!\!\!\int'_{P/\{P\}} \ln(P^2+m^2)$$

$$V_{\rm soft}(m) \qquad\qquad V_{\rm hard}(m)$$

Daisy/ring resummation $\quad V_{daisy} = V_{soft}^{resummed} - V_{soft}$

$$V_{\rm soft}(m) = -\frac{T}{12\pi}(m^2)^{\frac{3}{2}} \longrightarrow V_{soft}^{resummed} = -\frac{T}{12\pi}(m^2+\Pi_T)^{\frac{3}{2}}$$

Arnold-Espinosa eff potential $\quad V_{\rm eff}^{\rm A\text{-}E\ res.}(\phi, T, \bar{\mu}) = V_{\rm tree} + V_{\rm CW} + V_T + V_{\rm daisy}$

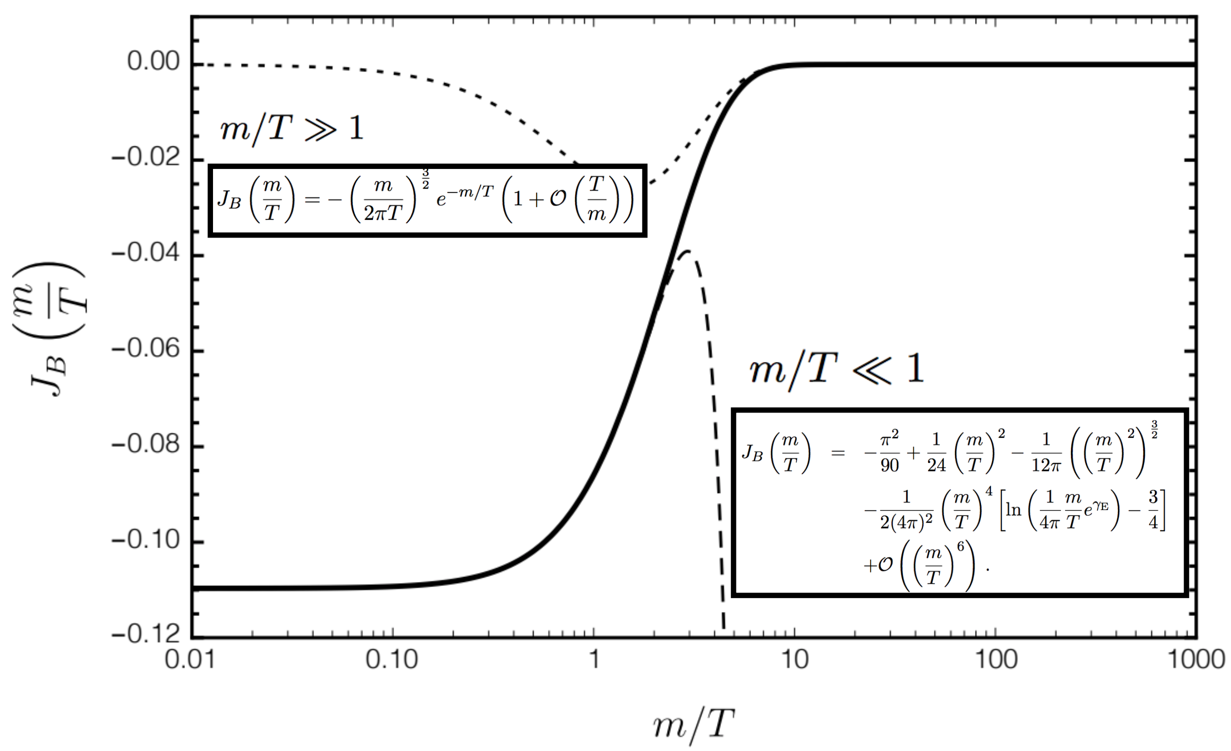
$$V_{\rm eff}^{\rm resummed}(\phi, T, \bar{\mu}) = V_{\rm tree} + V_{\rm soft}^{\rm resummed} + V_{\rm hard}$$

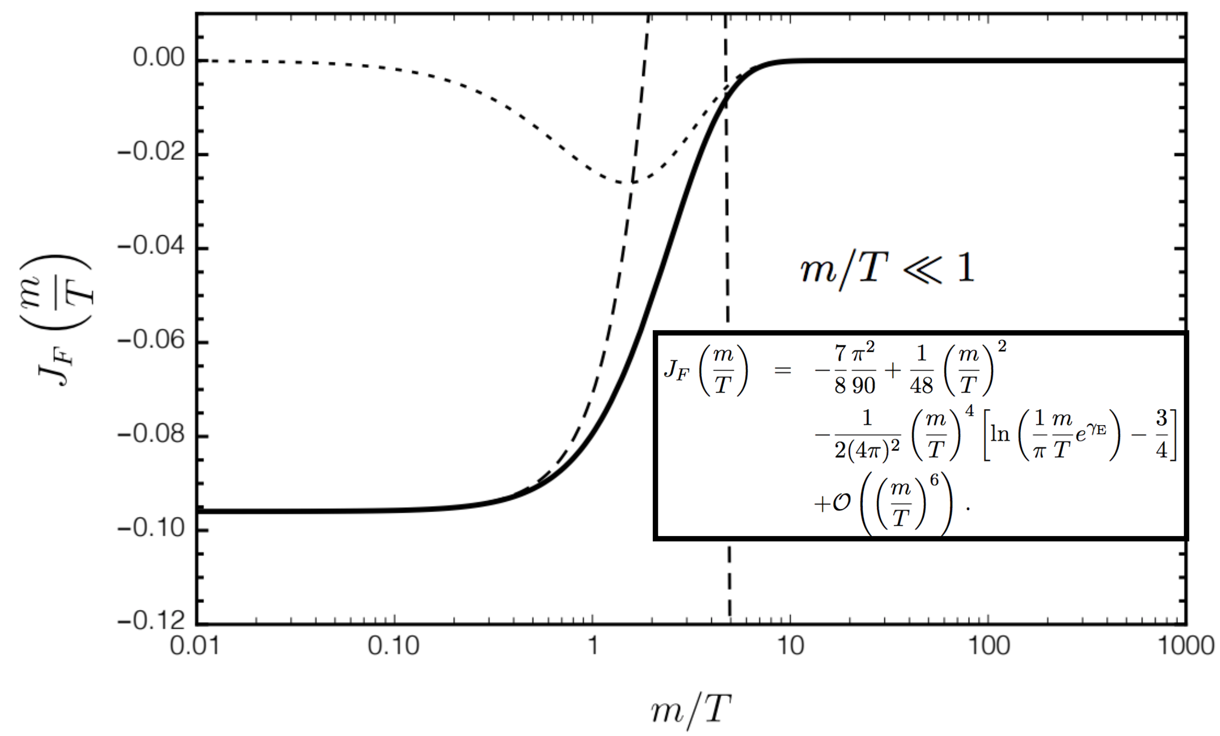
Phys. Rev. D47 (1993) 3546 [hep-ph/9212235]
See also Parwani method in Phys. Rev. D45 (1992) 4695 [hep-ph/9204216]

Thermal effective scalar potential for PT study

$$V_T(\phi,T) = V_0(\phi) + T^4\left[\sum_B J_B\left(\frac{M_B}{T}\right) + \sum_F J_F\left(\frac{M_F}{T}\right)\right]$$

all fermions F and bosons B that are relativistic at temperature T





High-T expansion

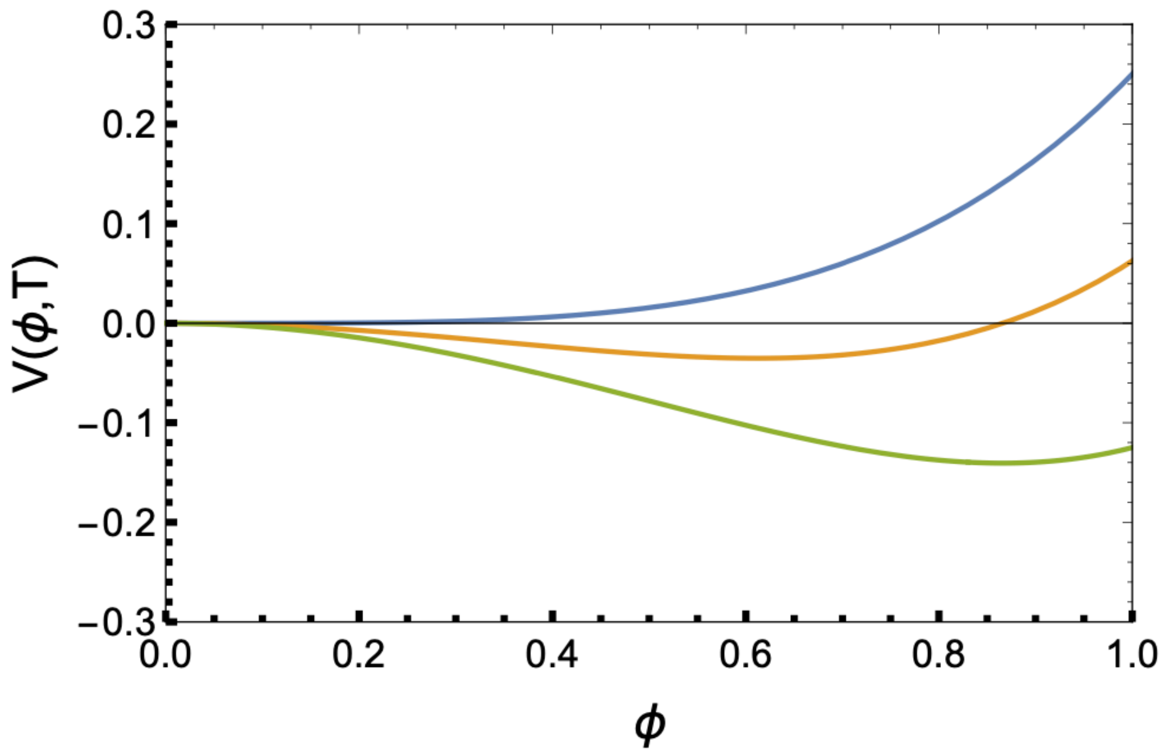
$m/T \ll 1$

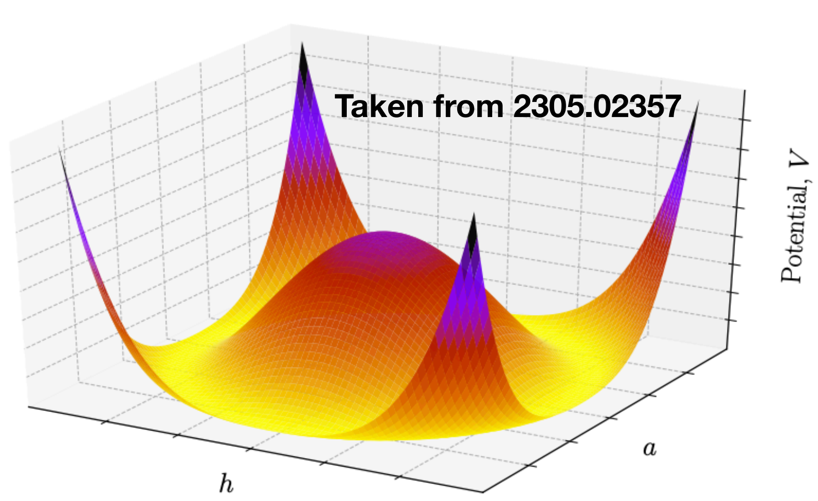
$$\begin{aligned} V_T(\phi) &= V_0(\phi) + \frac{T^2}{24}\left(\sum_S M_S^2(\phi) + 3\sum_V M_V^2(\phi) + 2\sum_F M_F^2(\phi)\right) \\ &\quad - \frac{T}{12\pi}\left(\sum_S \left(M_S^2(\phi)\right)^{\frac{3}{2}} + \sum_V \left(M_V^2(\phi)\right)^{\frac{3}{2}}\right) \\ &\quad + \text{higher order terms}. \end{aligned}$$

MS, MV , MF are the masses of the scalar fields S, vector fields V and fermonic fields F

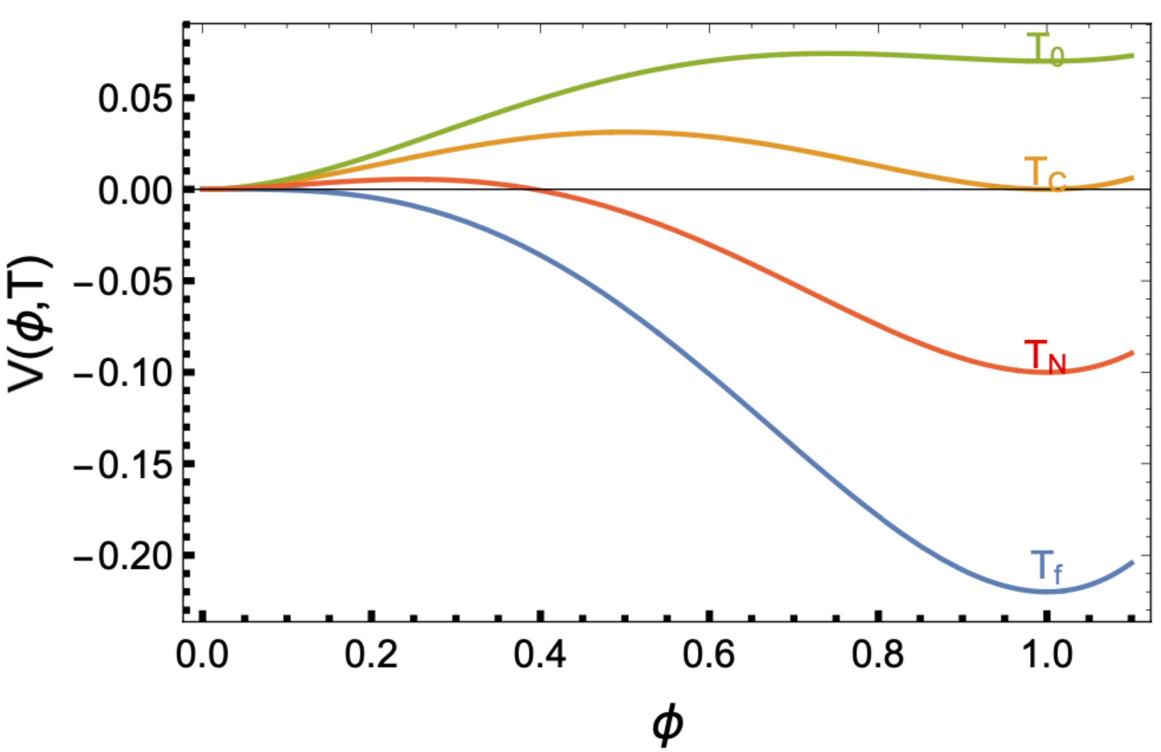
▸ Phase transition types

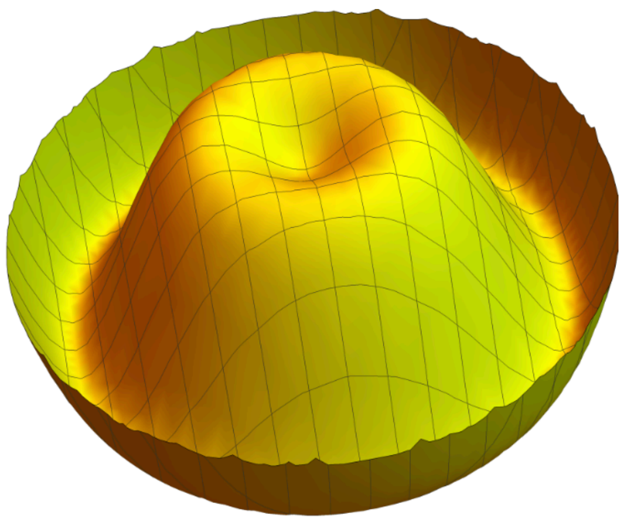
Second order





First order



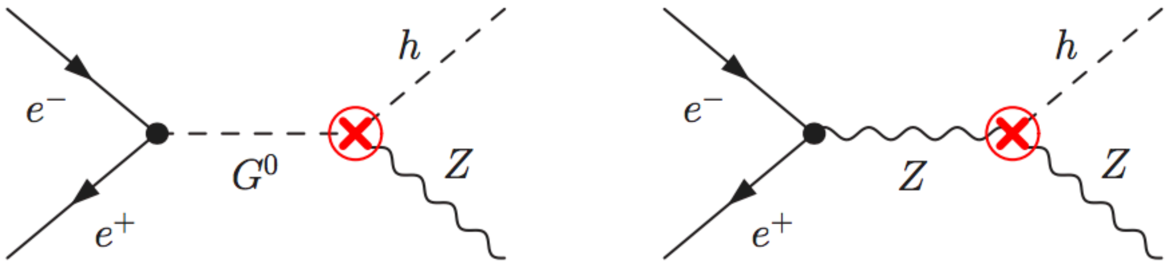


▸ Collider search for 2step FOPT

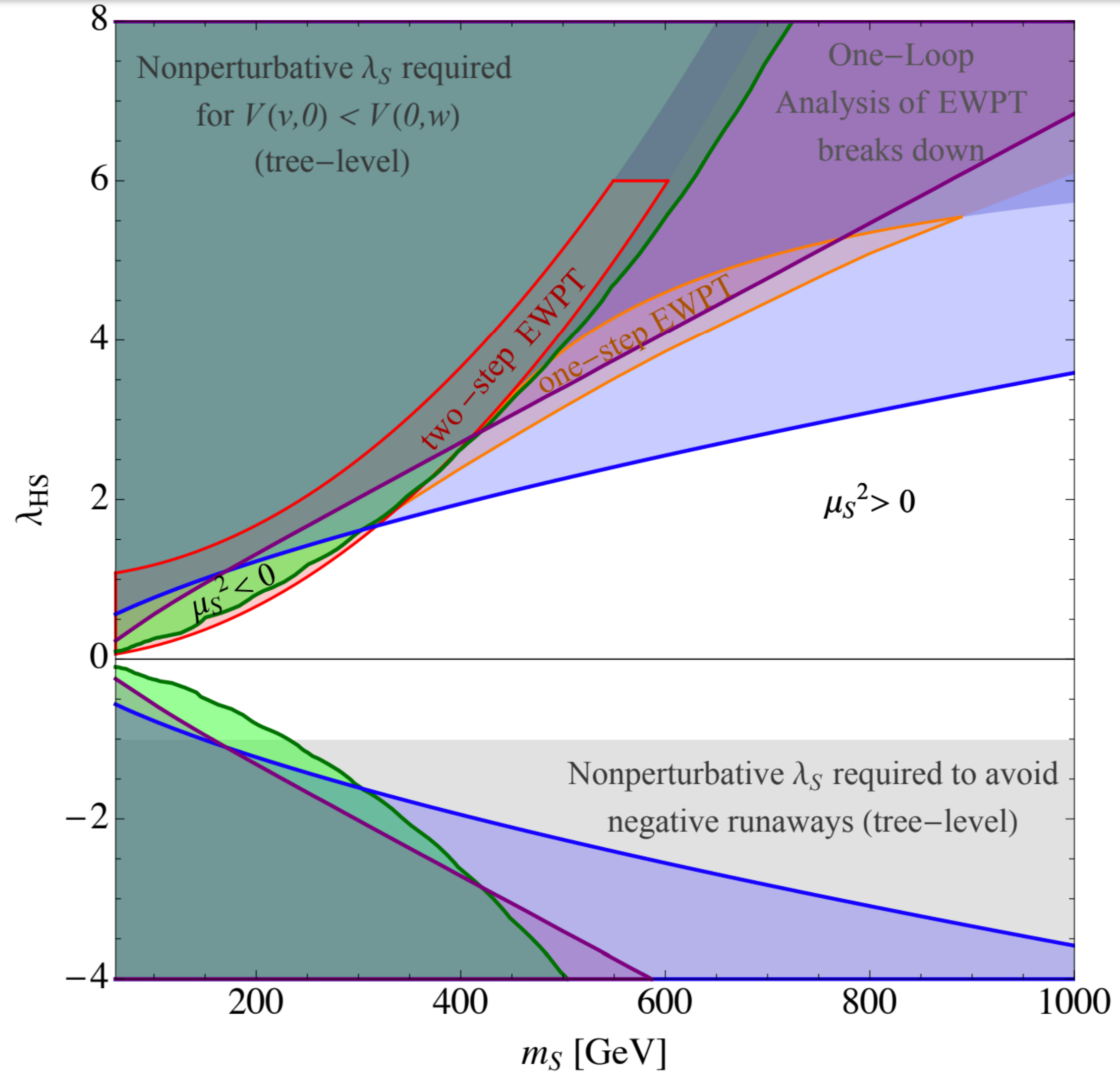
◉ Zh@ILC/CEPC

$$V_0 = -\mu^2|H|^2 + \lambda|H|^4 + \frac{1}{2}\mu_S^2 S^2 + \lambda_{HS}|H|^2 S^2 + \frac{1}{4}\lambda_S S^4$$

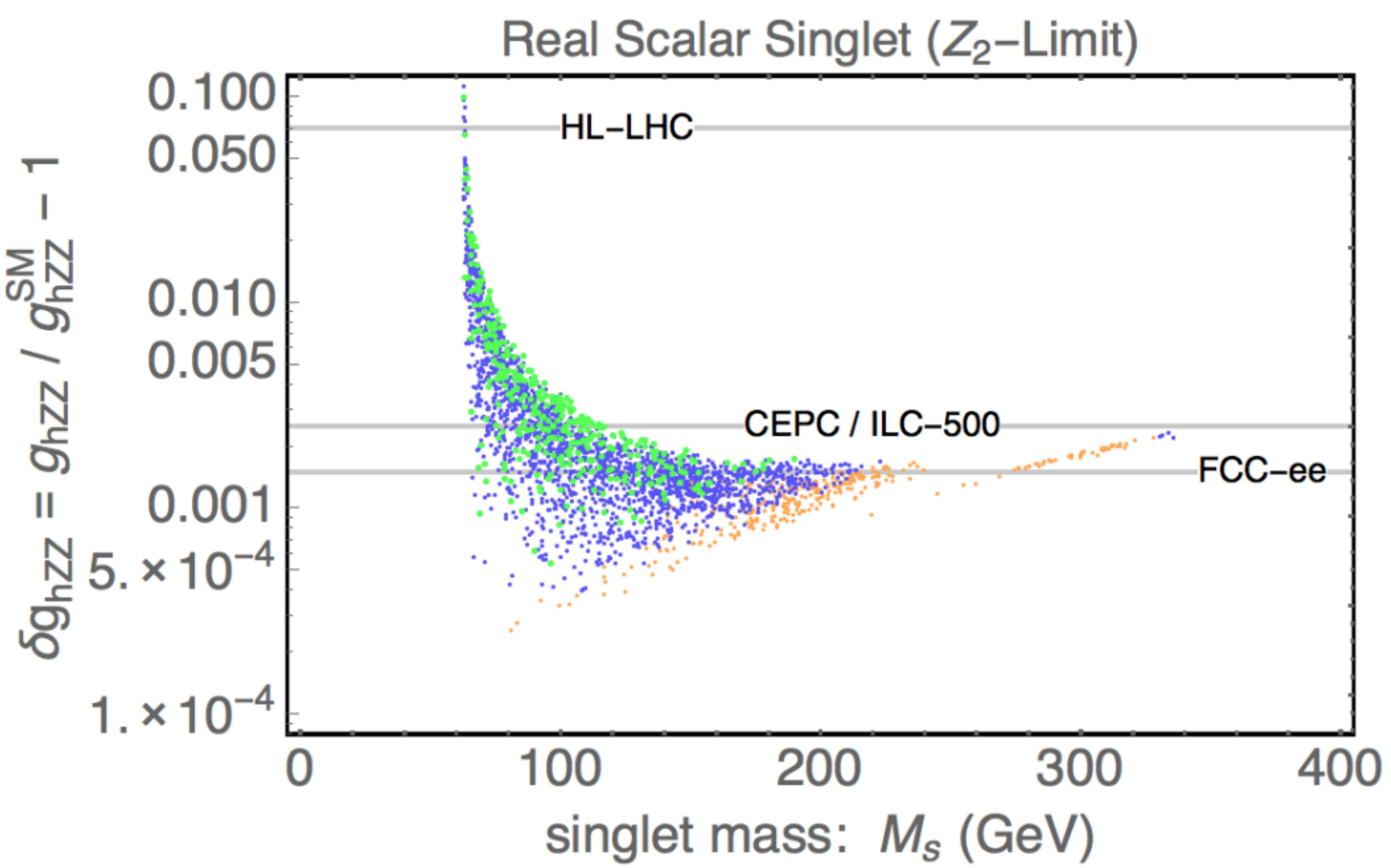
$$V_{\rm eff}(h,T) = V_0(h) + V_0^{CW}(h) + V_T(h,T) + V_r(h,T)$$



Craig,Englert, and McCullough,1305.5251



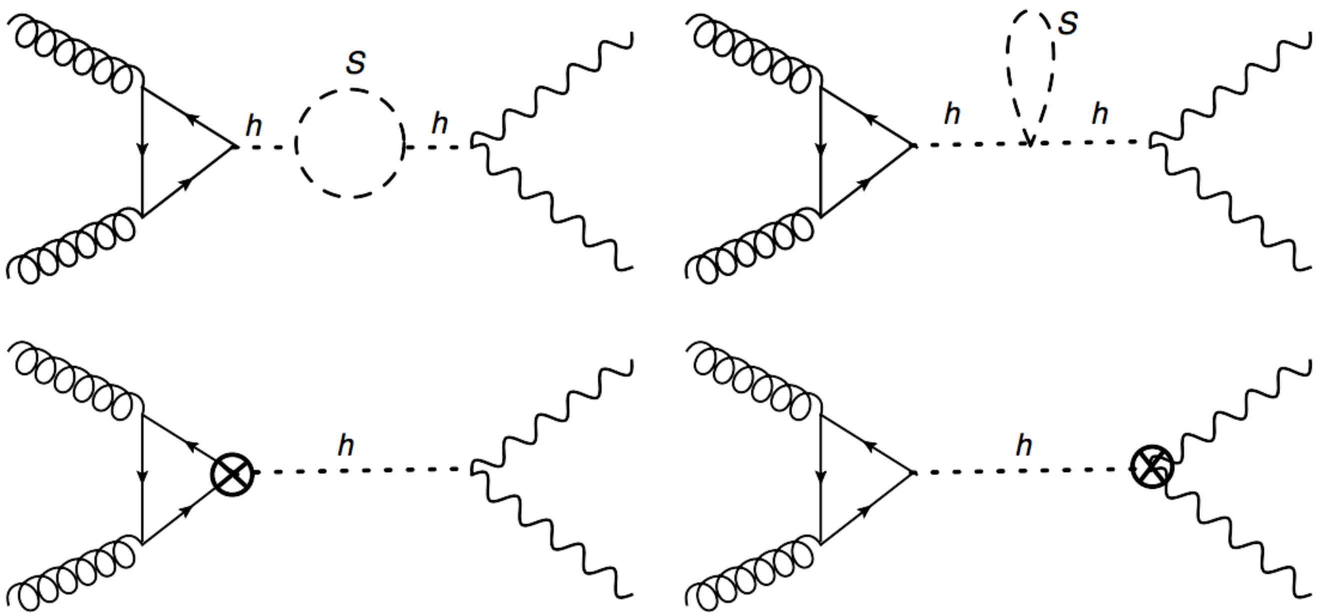
Curtin, Meade, Yu,1409.0005

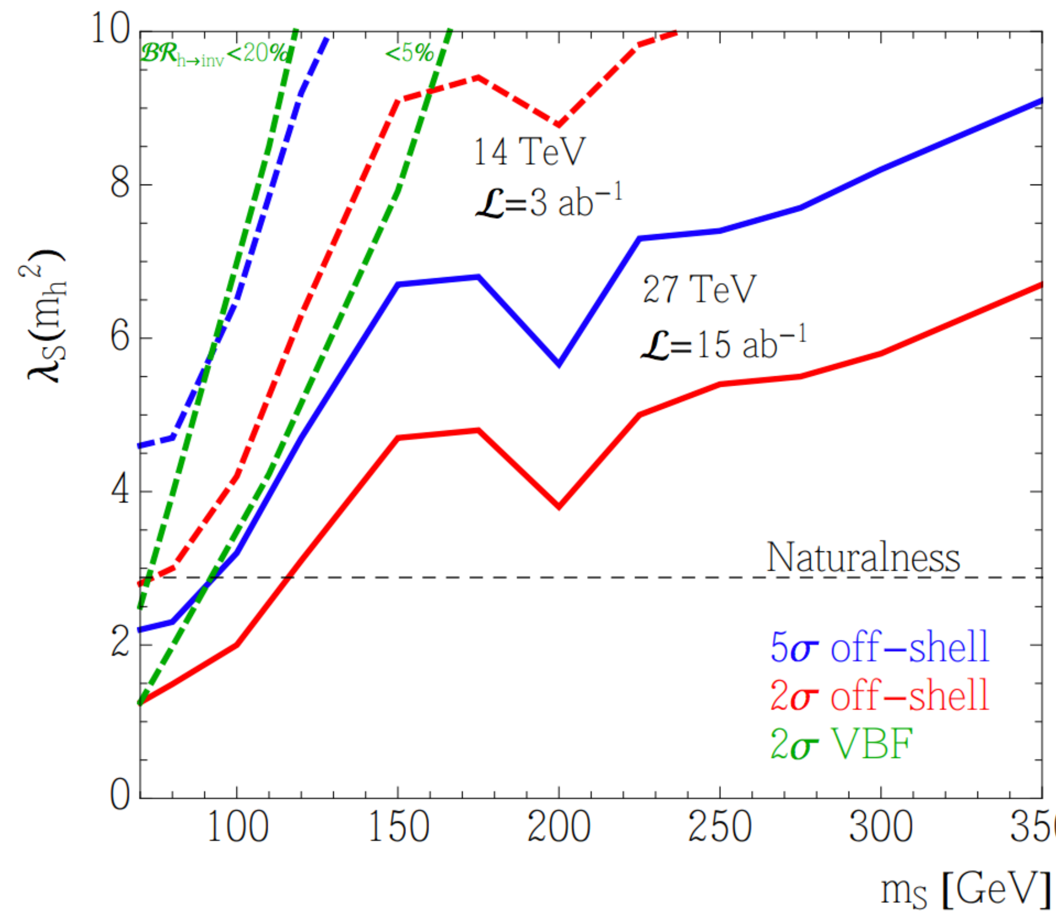


Huang, Long, and Wang,1608.06619

▸ Collider search for 2 step FOPT

◉ Off-shell Higgs@LHC

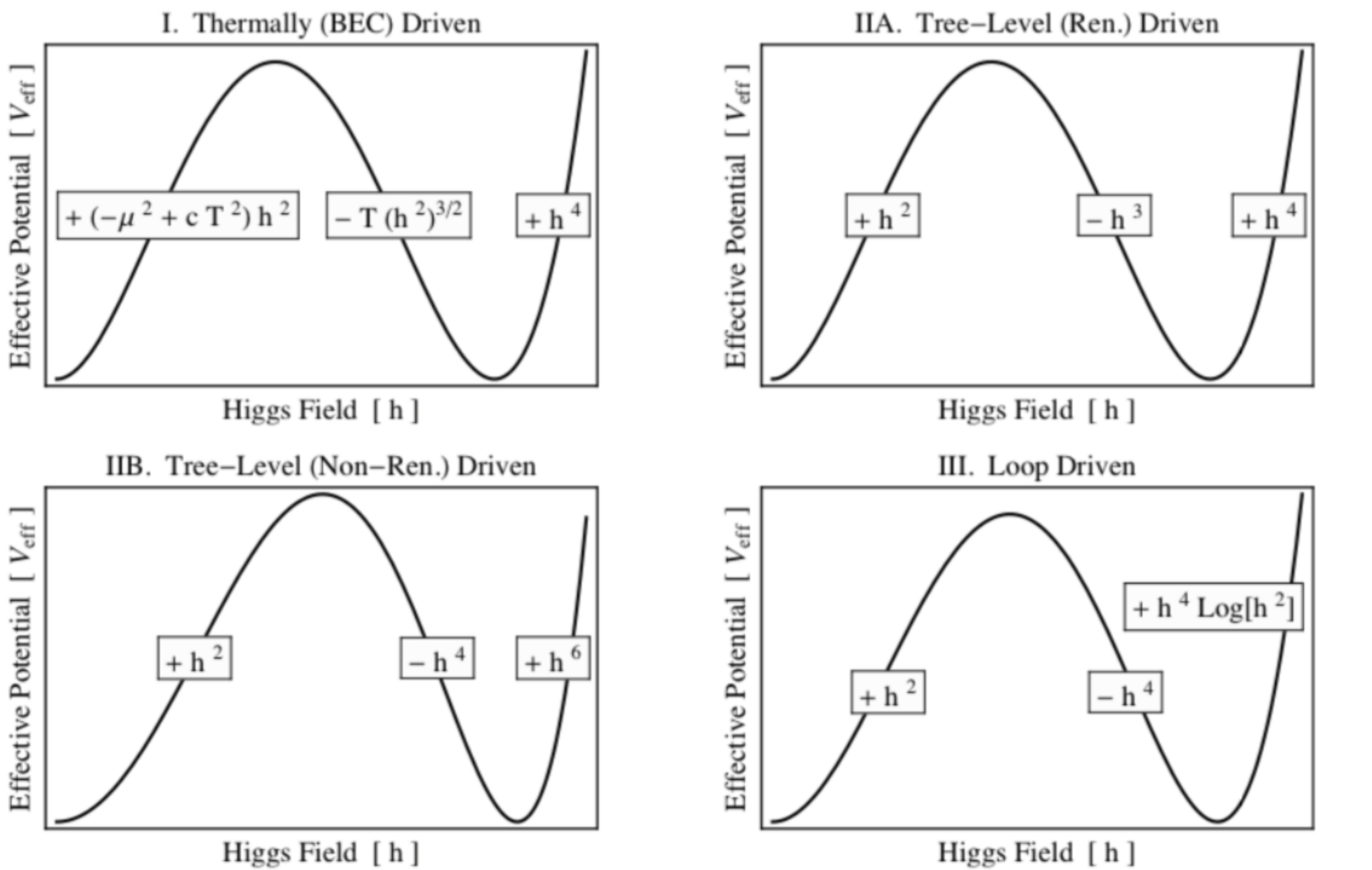




Goncalves,Han,and Mukhopadhyay, 1710.02149

See also: Lee, Park,and Qian, 1812.02679

Model classes for one-step FOPT



Chung, Long, Wang, Phys.Rev.D 87 (2013) 2, 023509

▶ Thermal driven Class-I

$$V_{\rm eff}(h,T) \approx \frac{1}{2}(-\mu^2 + cT^2)h^2 - \frac{eT}{12\pi}(h^2)^{3/2} + \frac{\lambda}{4}h^4$$

$$e \sim \sum_{\text{light bosonic fields}} (\text{degrees of freedom}) \times (\text{coupling to Higgs})^{3/2}.$$

$$\frac{v(T_c)}{T_c} \approx \frac{e}{6\pi\lambda}$$

TABLE I. Examples of models in the Thermally (BEC) Driven class. The expressions for e are calculated in the limit that the field-independent contributions to $m^2_{\rm eff}(h,T)$ are negligible (e.g., the thermal mass tuning has been performed). Here, the symbol $\tilde{A}_t$ is $\tilde{A}_t = A_t - \mu/\tan\beta$ and g_s is the number of real scalar singlet degrees of freedom coupling to the Higgs.

Model	$-\Delta\mathcal{L}$	c	e
SM [43]		$c_{\rm SM} = \frac{6m_t^2+6m_W^2+3m_Z^2+\frac{3}{2}m_H^2}{12v^2}$	$e_{\rm SM} = \frac{6m_W^3+3m_Z^3}{v^3}$
MSSM [41]		$c_{\rm SM} + \frac{6m_t^2}{12v^2}\left(1 - \frac{\tilde{A}_t^2}{m_Q^2}\right)$	$e_{\rm SM} + \frac{6m_t^3}{v^3}\left(1 - \frac{\tilde{A}_t^2}{m_Q^2}\right)^{3/2}$
Colored scalar [20]	$M_X^2\lvert X\rvert^2 + \frac{K}{6}\lvert X\rvert^4 + Q\lvert H\rvert^2\lvert X\rvert^2$	$c_{\rm SM} + \frac{6}{24}\,\frac{Q}{2}$	$e_{\rm SM} + 6(\frac{Q}{2})^{3/2}$
Singlet scalar [43,44]	$M^2\lvert S\rvert^2 + \lambda_S\lvert S\rvert^4 + 2\zeta^2\lvert H\rvert^2\lvert S\rvert^2$	$c_{\rm SM} + \frac{g_S}{24}\zeta^2$	$e_{\rm SM} + g_S\zeta^3$
Singlet Majoron [45]	$\mu_s^2\lvert S\rvert^2 + \lambda_s\lvert S\rvert^4 + \lambda_{hs}\lvert H\rvert^2\lvert S\rvert^2 + \frac{1}{2}y_i S\nu_i\nu_i + \text{H.c.}$	$c_{\rm SM} + \frac{2}{24}\,\frac{\lambda_{hs}}{2}$	$e_{\rm SM} + 2(\frac{\lambda_{hs}}{2})^{3/2}$
Two-Higgs doublets [46]	$\mu_D^2 D^\dagger D + \lambda_D(D^\dagger D)^2 + \lambda_3 H^\dagger H D^\dagger D$ $+\lambda_4\lvert H^\dagger D\rvert^2 + (\lambda_5/2)[(H^\dagger D)^2 + \text{H.c.}]$	$c_{\rm SM} + \frac{2\lambda_3+\lambda_4}{12}$	$e_{\rm SM} + 2(\frac{\lambda_3}{2})^{3/2} + (\frac{\lambda_3+\lambda_4-\lambda_5}{2})^{3/2}$ $+(\frac{\lambda_3+\lambda_4+\lambda_5}{2})^{3/2}$

▸ Tree driven-Class IIA

$$V_{\rm eff}(\varphi, T) \approx \frac{1}{2}(m^2 + cT^2)\varphi^2 - \mathcal{E}\varphi^3 + \frac{\lambda}{4}\varphi^4$$

$$T_c \approx \sqrt{\frac{m^2}{c}}\sqrt{\frac{2\mathcal{E}^2}{\lambda m^2} - 1}, \qquad \frac{v(T_c)}{T_c} \approx \sqrt{\frac{2c}{\lambda}}\frac{1}{\sqrt{1 - \frac{\lambda m^2}{2\mathcal{E}^2}}}\cos\alpha.$$

TABLE II. Examples of models that fall into Class IIA. For the non-SUSY models, corrections to the SM Lagrangian are shown, whereas for the SUSY models only the superpotential corrections are given.

Model	$\Delta\mathcal{L}$
xSM [53–56]	$\frac{1}{2}(\partial S)^2 - [\frac{b_2}{2}S^2 + \frac{b_3}{3}S^3 + \frac{b_4}{4}S^4 + \frac{a_1}{2}H^\dagger H S^2 + \frac{a_2}{2}H^\dagger H S^2]$
$\mathbb{Z}_2$xSM [14,57]	$\frac{1}{2}(\partial S)^2 - [\frac{b_2}{2}S^2 + \frac{b_4}{4}S^4 + \frac{a_2}{2}H^\dagger H S^2]$
Two-Higgs doublets [58]	$\mu_D^2\lvert D\rvert^2 + \lambda_D\lvert D\rvert^4 + \lambda_3\lvert H\rvert^2\lvert D\rvert^2 + \lambda_4\lvert H^\dagger D\rvert^2$ $+(\lambda_5/2)[(H^\dagger D)^2 + \text{H.c.}]$

Model	ΔW
NMSSM [59–61]	$\lambda H_1 H_2 N - \frac{\kappa}{3}N^3 + rN$
nMSSM [62]	$\lambda H_1 H_2 S + \frac{m_{12}^2}{\lambda}S$
$\mu\nu$MSSM [63]	$-\lambda_i H_1 H_2 \nu_i^c + \frac{\kappa_{ijk}}{3}\nu_i^c\nu_j^c\nu_k^c + Y_\nu^{ij}H_2 L_i \nu_j^c$

Class IIA (1) no extra EWSB: xSM

For the "xSM" model, the gauge invariant finite temperature effective potential is found to be:

$$V(h,s,T) = -\frac{1}{2}[\mu^2 - \Pi_h(T)]h^2 - \frac{1}{2}[-b_2 - \Pi_s(T)]s^2 + \frac{1}{4}\lambda h^4 + \frac{1}{4}a_1 h^2 s + \frac{1}{4}a_2 h^2 s^2 + \frac{b_3}{3}s^3 + \frac{b_4}{4}s^4, \quad \text{(C1)}$$

with the thermal masses given by

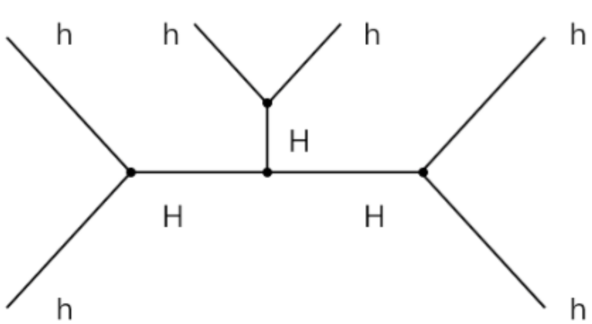
$$\Pi_h(T) = \left(\frac{2m_W^2 + m_Z^2 + 2m_t^2}{4v^2} + \frac{\lambda}{2} + \frac{a_2}{24}\right)T^2,$$

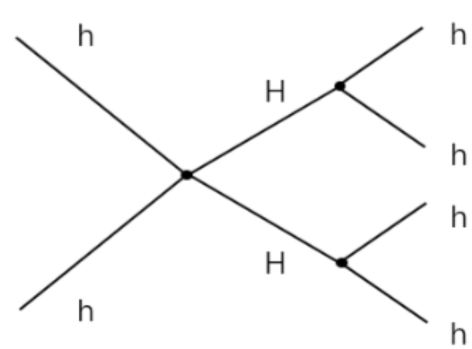
$$\Pi_s(T) = \left(\frac{a_2}{6} + \frac{b_4}{4}\right)T^2, \quad \text{(C2)}$$

PT strength

$$v^{\text{xSM}}/T \equiv \frac{v_h(T)}{T} = \frac{\sqrt{v_h^2(T) + v_s^2(T)}\cos\theta(T)}{T},$$

$$\cos\theta(T) \equiv \frac{v_h(T)}{\sqrt{v_h^2(T) + v_s^2(T)}},$$





For small mixing limit between the extra Higgs and the SM Higgs, one have

$$c_4^{xSM} = -\frac{a_1^2 - 8b_2\lambda}{32b_2} + \frac{\theta^2(a_1^2(6b_2 - \mu^2) - 8a_1b_2b_3 + 8b_2^2(a_2 - 2\lambda))}{32b_2^2} + O(\theta^3)$$

$$c_6^{xSM} = -\frac{a_1^2(a_1b_3 - 3a_2b_2)}{192b_2^3} - \frac{\theta^2 a_1}{256b_2^4}(a_1^3b_2 + 4a_1^2b_3(\mu^2 - 3b_2) + 4a_1b_2(a2(11b_2 - 2\mu^2) - 6b_2(b4 + \lambda) + 4b_3^2) - 32a_2b_2^2b_3) + O(\theta^3)$$

$$c_8^{xSM} = \frac{a_1^4 b_4}{1024b_2^4} + \frac{a_1^3\theta^2}{1024b_2^5}(a_1(a_2b_2 + 4b_4(\mu^2 - 3b_2)) + 16b_2b_3b_4) + O(\theta^3)$$

Class IIA (1) with extra EWSB: **GM model**

The most general scalar potential $V(\Phi,\Delta)$ invariant under $SU(2)_L \times SU(2)_R \times U(1)_Y$ is given by

extra EWSB

$$v_\phi^2 + 8 v_\xi^2 \equiv v^2 \approx (246\text{GeV})^2$$

$$\begin{aligned} V(\Phi,\Delta) =& \frac{1}{2}m_1^2 \mathrm{tr}[\Phi^\dagger\Phi] + \frac{1}{2}m_2^2 \mathrm{tr}[\Delta^\dagger\Delta] + \lambda_1 \left(\mathrm{tr}[\Phi^\dagger\Phi]\right)^2 \\ &+ \lambda_2 \left(\mathrm{tr}[\Delta^\dagger\Delta]\right)^2 + \lambda_3 \mathrm{tr}\left[\left(\Delta^\dagger\Delta\right)^2\right] + \lambda_4 \mathrm{tr}[\Phi^\dagger\Phi]\mathrm{tr}[\Delta^\dagger\Delta] \\ &+ \lambda_5 \mathrm{tr}\left[\Phi^\dagger \frac{\sigma^a}{2}\Phi\frac{\sigma^b}{2}\right] \mathrm{tr}[\Delta^\dagger T^a \Delta T^b] \\ &+ \mu_1 \mathrm{tr}\left[\Phi^\dagger \frac{\sigma^a}{2}\Phi\frac{\sigma^b}{2}\right] (P^\dagger \Delta P)_{ab} + \mu_2 \mathrm{tr}[\Delta^\dagger T^a \Delta T^b](P^\dagger\Delta P)_{ab}\,, \end{aligned} \quad (3)$$

Custodial ~~symmetry~~

$$v_\chi = \sqrt{2} v_\xi$$

$$\Phi \equiv (\varepsilon_2\phi^*, \phi) = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^{+*} & \phi^0 \end{pmatrix}, \quad \Delta \equiv (\varepsilon_3\chi^*, \xi, \chi) = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ -\chi^{+*} & \xi^0 & \chi^+ \\ \chi^{++*} & -\xi^{+*} & \chi^0 \end{pmatrix}, \quad (1)$$

with

$$\varepsilon_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \varepsilon_3 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad (2)$$

where the phase convention for the scalar field components is: $\chi^{--} = \chi^{++*}$, $\chi^- = \chi^{+*}$, $\xi^- = \xi^{+*}$, $\phi^- = \phi^{+*}$. Φ and Δ are transformed under $SU(2)_L \times SU(2)_R$ as $\Phi \to U_{2,L}\Phi U^\dagger_{2,R}$ and $\Delta \to U_{3,L}\Delta U^\dagger_{3,R}$ with $U_{L,R} = exp(i\theta^a_{L,R}T^a)$ and T^a being the $SU(2)$ generators.

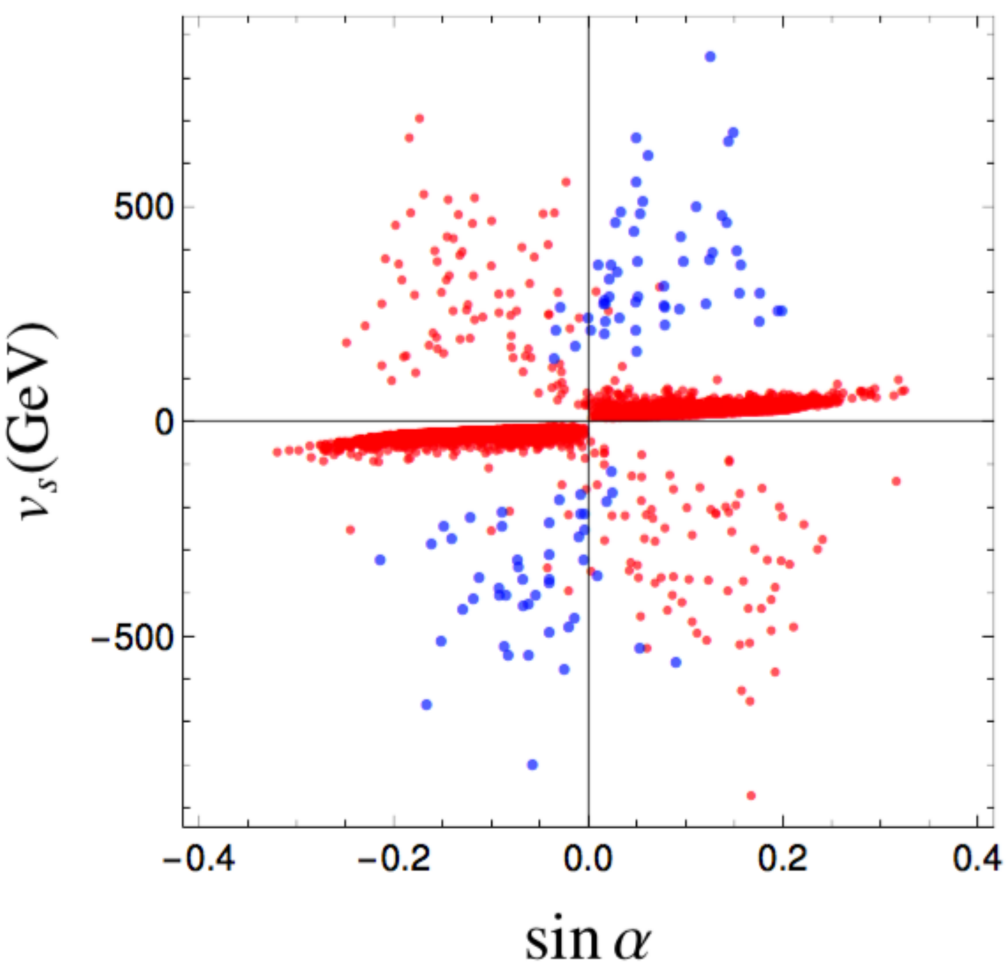
where summations over $a,b = 1,2,3$ are understood, σ's and T's are the 2×2 (Pauli matrices) 3×3 matrix representations of the $SU(2)$ generators, respectively

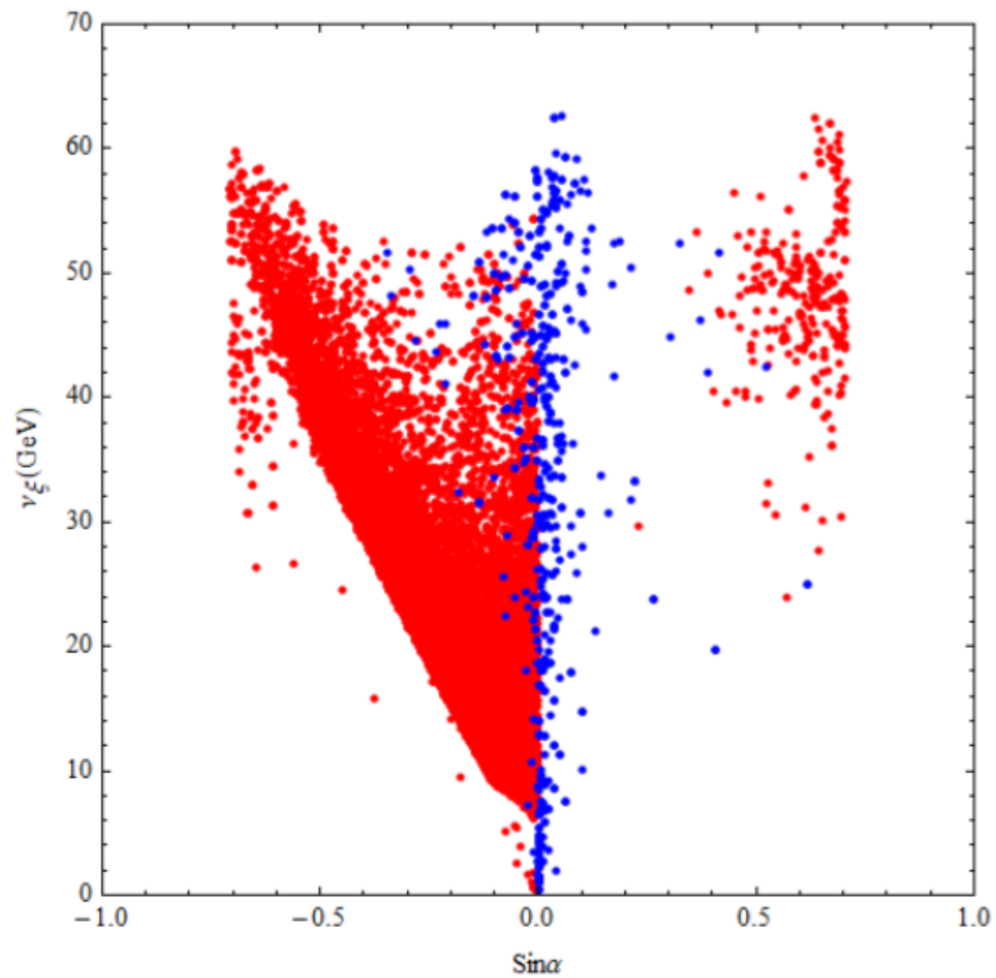
$$T_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, T_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, T_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

The P matrix, which is the similarity transformation relating the generators in the triplet and adjoint representations, is given by

$$P = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 & i & 0 \\ 0 & 0 & \sqrt{2} \\ 1 & i & 0 \end{pmatrix}.$$

xSM: without extra EWSB   GM: with extra EWSB





$$g_{hxx} = \cos\alpha\, g_{hxx}^{SM}$$

$$g_{hf\bar{f}} = \cos\alpha / \cos\theta_H g_{hf\bar{f}}^{SM},\ g_{hVV} = (\cos\alpha\cos\theta_H - \sqrt{\frac{8}{3}}\sin\alpha\sin\theta_H) g_{hf\bar{f}}^{SM},$$

$$g_{Hf\bar{f}} = \sin\alpha / \cos\theta_H g_{hf\bar{f}}^{SM},\ g_{HVV} = (\sin\alpha\cos\theta_H + \sqrt{\frac{8}{3}}\cos\alpha\sin\theta_H) g_{hVV}^{SM}.$$

▸ Tree-level driven-Class II B

< 0 causes the potential to turn over

$$V_{\rm eff}(h,T) \approx \frac{1}{2}(\mu^2 + cT^2)h^2 + \frac{\lambda}{4}h^4 + \frac{1}{8\Lambda^2}h^6$$

stabilizes the EW-broken vacuum

$$\lambda = \frac{m_H^2}{2v^2}\left(1 - \frac{\Lambda_{\rm max}^2}{\Lambda^2}\right), \qquad \Lambda_{\rm max} \equiv \sqrt{3}v^2/m_H \qquad T_c = \sqrt{\frac{\mu^2}{c}}\sqrt{\frac{\lambda^2\Lambda^2}{4\mu^2} - 1},$$

$$\mu^2 = \frac{m_H^2}{2}\left(\frac{\Lambda_{\rm max}^2}{2\Lambda^2} - 1\right), \qquad \Lambda < \Lambda_{\rm max} \qquad \frac{v(T_c)}{T_c} = \sqrt{\frac{c}{-\lambda}}\frac{2}{\sqrt{1 - \frac{4\mu^2}{\lambda^2\Lambda^2}}}.$$

$$\lambda_{HHH} \equiv \frac{m_H^2}{v}\left(1 + 2\frac{\Lambda_{\rm min}^2}{\Lambda^2}\right) \qquad \Lambda_{\rm min} = v^2/m_H$$

Model	Couplings	Wilson coefficient of H^6
$\mathbb{R}$ Singlet	$-\frac{1}{2}\lambda_{HS}\|H\|^2S^2 - g_{HS}H^\dagger HS$	$-\frac{\lambda_{HS}}{2}\frac{g_{HS}^2}{M^4}$
$\mathbb{C}$ Singlet	$-g_{HS}\|H\|^2\Phi - \frac{\lambda_{H\Phi}}{2}\|H\|^2\Phi^2 - \frac{\lambda'_{H\Phi}}{2}H^\dagger H\|\Phi\|^2 + h.c.$	$-\frac{\|g_{HS}\|^2\lambda'_{H\Phi}}{2M^4} - \frac{{\rm Re}[g_{HS}^2\lambda_{H\Phi}]}{M^4}$
2HDM	$-Z_6\|H_1\|^2H_1^\dagger H_2 - Z_6^*\|H_1\|^2H_2^\dagger H_1$	$\frac{\|Z_6\|^2}{M^2}$
$\mathbb{R}$ triplet	$gH^\dagger\tau^aH\Phi^a - \frac{\lambda_{H\Phi}}{2}\|H\|^2\|\Phi^a\|^2$	$-\frac{g^2}{M^4}\left(\frac{\lambda_{H\Phi}}{8} - \lambda\right)$
$\mathbb{C}$ triplet	$gH^Ti\sigma_2\tau^aH\Phi^a - \frac{\lambda_{H\Phi}}{2}\|H\|^2\|\Phi^a\|^2$	$-\frac{g^2}{M^4}\left(\frac{\lambda_{H\Phi}}{4} + \frac{\lambda'}{8} - 2\lambda\right)$
	$-\frac{\lambda'}{4}H^\dagger\tau^a\tau^bH\Phi^a(\Phi^b)^\dagger + h.c.$	
$\mathbb{C}$ 4−plet	$-\lambda_{H3\Phi}H_i^*H_j^*H_k^*\Phi^{ijk} + h.c.$	$\frac{\|\lambda_{H3\Phi}\|^2}{M^2}$

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Loop driven-Class III

$$V_{\rm eff}(h,T) \approx \frac{1}{2}(\mu^2 + cT^2)h^2 + \frac{\lambda}{4}h^4 + \frac{\kappa}{4}h^4 \ln\frac{h^2}{M^2}$$

$$\lambda = \frac{m_H^2}{2v^2} - \kappa\left(\ln\frac{v^2}{M^2} + \frac{3}{2}\right),$$

$$\mu^2 = -\frac{m_H^2}{2} + \kappa v^2.$$

$$T_c \approx \frac{m_H}{2\sqrt{c}}\sqrt{\epsilon}\left(1 + \frac{1}{8}\epsilon + \frac{37}{384}\epsilon^2 + \cdots\right),$$

$$\frac{v(T_c)}{T_c} \approx \frac{2v\sqrt{c}}{m_H}\frac{1}{\sqrt{\epsilon}}\left(1 - \frac{3}{8}\epsilon - \frac{103}{384}\epsilon^2 + \cdots\right).$$

$$\epsilon = 1 - \kappa v^2/m_H^2$$

TABLE III. Examples of models in the Loop Driven class.

Model	$-\Delta\mathcal{L}$
Singlet scalars [12,72]	$\sum_i^N M^2\lvert S_i\rvert^2 + \lambda_S\lvert S_i\rvert^4 + 2\zeta^2\lvert H\rvert^2\lvert S_i\rvert^2$
Singlet Majoron [73,74]	$\mu_s^2\lvert S\rvert^2 + \lambda_s\lvert S\rvert^4 + \lambda_{hs}\lvert H\rvert^2\lvert S\rvert^2 + \frac{1}{2}y_i S\nu_i\nu_i + \text{H.c.}$
Two-Higgs doublets [75–78]	$\mu_D^2 D^\dagger D + \lambda_D(D^\dagger D)^2 + \lambda_3 H^\dagger H D^\dagger D + \lambda_4\lvert H^\dagger D\rvert^2 + (\lambda_5/2)[(H^\dagger D)^2 + \text{H.c.}]$

Class III 2HDM Finite-T potential in 2HDM

$$V(h_1,h_2,T) = V_0(h_1,h_2) + V_{\rm CW}(h_1,h_2) + V_{\rm CT}(h_1,h_2) + V_{\rm th}(h_1,h_2,T) + V_{\rm daisy}(h_1,h_2,T)$$

Tree-level

$$V_0(h_1,h_2) = \frac{1}{2}m_{12}^2 t_\beta \left(h_1 - h_2 t_\beta^{-1}\right)^2 - \frac{v^2}{4}\frac{\lambda_1 h_1^2 + \lambda_2 h_2^2 t_\beta^2}{1+t_\beta^2} - \frac{v^2}{4}\frac{\lambda_{345}(h_1^2 t_\beta^2 + h_2^2)}{1+t_\beta^2} + \frac{1}{8}\lambda_1 h_1^4 + \frac{1}{8}\lambda_2 h_2^4 + \frac{1}{4}\lambda_{345} h_1^2 h_2^2$$

One-loop at zero temperature:

$$V_{\rm CW}(h_1,h_2) = \sum_i (-1)^{2s_i} n_i \frac{\hat{m}_i^4(h_1,h_2)}{64\pi^2}\left[\ln\left(\frac{\hat{m}_i^2(h_1,h_2)}{Q^2}\right) - C_i\right]$$ [Coleman, Weinberg '73]

One-loop at finite temperature:

$$V_{\rm th}(h_1,h_2,T) = \frac{T^4}{2\pi^2}\sum_i n_i J_{B,F}\left(\frac{m_i^2(h_1,h_2)}{T^2}\right)$$ [Dolan, Jackiw '74]

$$J_{B,F}(y) = \mp \sum_{l=1}^{\infty} \frac{(\pm 1)^l y}{l^2} K_2(\sqrt{y}\, l)$$ [Anderson, Halle '92]

$$V_{\rm daisy}(h_1,h_2,T) = -\frac{T}{12\pi}\sum_i n_i \left[\left(M_i^2(h_1,h_2,T)\right)^{\frac{3}{2}} - \left(m_i^2(h_1,h_2)\right)^{\frac{3}{2}}\right]$$

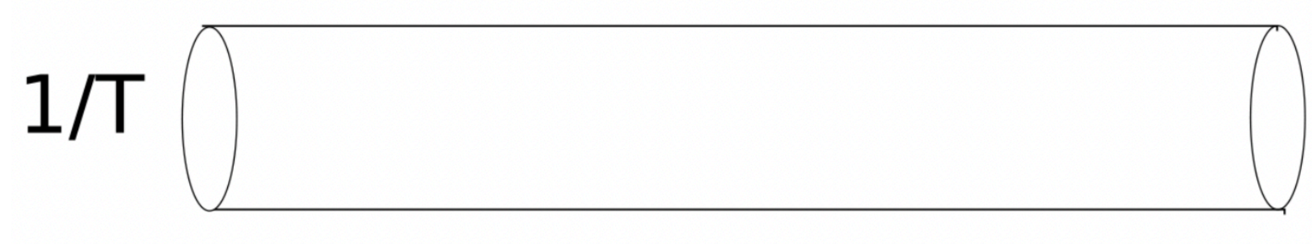
[Carrington '92; Arnold, Espinosa '93; Delaunay, Grojean, Wells '07]

Finite temperature EFT for the 3d Phase transition study

Matsubara decomposition

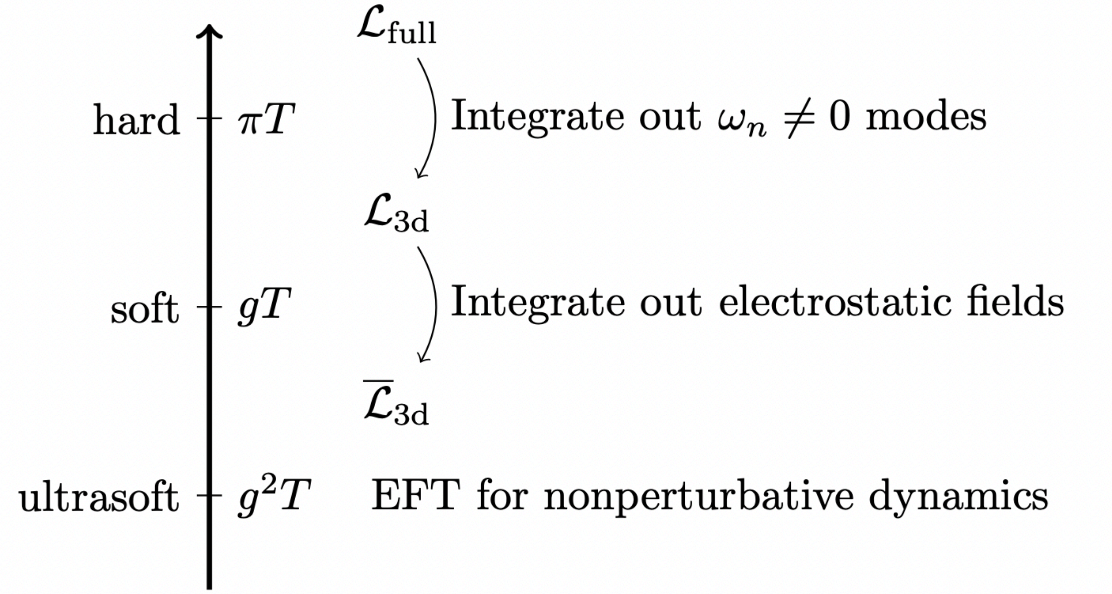
$$\phi(\tau,\mathbf{x}) = T\sum_n \tilde{\phi}(\mathbf{p})e^{i\omega_n\tau},\ \omega_n = \begin{cases} 2\pi nT & \text{bosons} \\ (2n+1)\pi T & \text{fermions} \end{cases}$$

$\omega_n \neq 0$ modes are heavy and decouple at distances ≫ 1/T, and can be integrated out



$$S = \int d^4x\ [\mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{Yukawa}}]$$

Procedure



$$S_{\text{3d}} = \int d^3x\ \Big[\frac{1}{4}F^a_{ij}F^a_{ij} + (D_i\phi)^\dagger(D_i\phi) + \bar{m}^2\phi^\dagger\phi + \bar{\lambda}(\phi^\dagger\phi)^2 + \mathcal{L}_{\text{BSM}} + \text{higher-order operators}\Big]$$

3dSMEFT for the Phase transition study

Model

$$\mathscr{L}_{4d} = \mathscr{L}_{gauge} + \left(D_\mu \Phi\right)^\dagger \left(D_\mu \Phi\right) - V(\Phi) + \bar{l}_L \gamma^\mu D_\mu l_L + \bar{e}_R \gamma^\mu D_\mu e_R + \bar{u}_R \gamma^\mu D_\mu u_R + \bar{d}_R \gamma^\mu D_\mu d_R + g_Y(\bar{q}_L \tilde{\Phi} t_R + \bar{t}_R \tilde{\Phi}^\dagger q_L)$$

$$V(\Phi) = m^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2 + c_6 \left(\Phi^\dagger \Phi\right)^3 \qquad c_6 = \Lambda^{-2}$$

$$D_\mu = \partial_\mu - ig\frac{\sigma^a}{2} A_\mu^a - ig'\frac{1}{2} B_\mu \qquad \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi_1 + i\,\chi_2 \\ \phi + \phi_0 + i\,\chi_3 \end{pmatrix}$$

Partition function

$$\mathcal{Z} = \mathrm{Tr} e^{-\beta(H - \mu_k N_k)}$$

$$= \int \mathcal{D}\varphi e^{-S + \int_0^\beta \mathrm{d}\tau \sum_i^{N_f} \mu_i N_{2,i}}, \qquad \sum_i^{N_f} \mu_i N_{2,i} = \frac{1}{N_f}(\mu_1 + \mu_2 + \mu_3) B - (\mu_1 L_1 + \mu_2 L_2 + \mu_3 L_3)$$

$$B = \frac{1}{3} \sum_{f,c} \int \mathrm{d}^3 x \bar{q}_{f,c} \gamma_0 q_{f,c}, \qquad N_f \mu_B + \sum_{i=1}^{N_f} \mu_{L_i} = 0$$

$$L_i = \int \mathrm{d}^3 x \left(\bar{e}_i \gamma_0 e_i + \frac{1}{2} \bar{\nu}_i \gamma_0 (1 - \gamma_5) \nu_i\right)$$

$$\mu_{ch} = \mu_i$$

3dSMEFT for the Phase transition study

First step

Integrate out superheavy mode

power counting

$g' \sim g, g_Y \sim g, \lambda \sim g^2, c_6 \sim g^4/\Lambda^2$

$$S_{3\mathrm{d}}^{\mathrm{heavy}} = \int \mathrm{d}^3x \left[\frac{1}{4}G_{ij}^a G_{ij}^a + \frac{1}{4}F_{ij}F_{ij} + \frac{1}{2}(D_i A_0^a)^2 + \frac{1}{2}(\partial_i B_0)^2 + \frac{1}{2}(D_i C_0^\alpha)^2 + (D_i\Phi)^\dagger(D_i\Phi) + V_{3\mathrm{d}}^{\mathrm{heavy}}\right]$$

Debye mass

$$m_D^2 = \frac{11}{6} g^2(\bar{\mu})T^2 + \frac{g^2}{4\pi^2}\left(\mu_B^2 + \sum_{i=1}^{N_f} \mu_{L_i}^2\right)$$

$$m_D'^2 = \frac{11}{6} g'^2(\bar{\mu})T^2 + \frac{g'^2}{4\pi^2}\left(\frac{11}{9}\mu_B^2 + 3\sum_{i=1}^{N_f} \mu_{L_i}^2\right)$$

$$m_D''^2 = 2g_s^2 T^2$$

3d&4d fields

$$\phi_{3d}^2 = \frac{1}{T} Z_\phi \phi_{4d}^2\,,$$

$$A_{i,3d}^2 = \frac{1}{T} Z_{A_i} A_{i,4d}^2\,,$$

$$A_{0,3d}^2 = \frac{1}{T} Z_{A_0} A_{0,4d}^2\,,$$

3dSMEFT for the Phase transition study

$$g_3'^2 = g'^2(\bar{\mu})T\left[1 + \frac{g'^2}{(4\pi)^2}\left(-\frac{1}{6}L_b - \frac{20}{3}L_f\right)\right]$$

$$g_3^2 = g^2(\bar{\mu})T\left[1 + \frac{g^2}{(4\pi)^2}\left(\frac{43}{6}L_b + \frac{2}{3} - 4L_f\right)\right] + \frac{g^4}{48\pi^2}\left[9\mathscr{A}\left(\frac{\mu_B}{3}\right) + \sum_{i=1}^{N_f}\mathscr{A}\left(\mu_{L_i}\right)\right]$$

$$L_b = 2\ln\left(\frac{\bar{\mu}}{T}\right) - 2\ln 4\pi - 2\gamma_E$$

$$L_f = L_b + 4\ln 2$$

$$\lambda_3 = T\left(\lambda(\bar{\mu}) + \frac{1}{(4\pi)^2}\left(\frac{1}{8}(3g^4 + g'^4 + 2g^2g'^2) + 3L_f(g_Y^4 - 2\lambda g_Y^2) - L_b\left(\frac{3}{16}(3g^4 + g'^4 + 2g^2g'^2) - \frac{3}{2}\lambda(3g^2 + g'^2 - 8\lambda)\right)\right)\right)$$
$$+ T^3c_6 - \frac{3g_Y^2T}{16\pi^2}(g_Y^2 - 2\lambda)\mathscr{A}\left(\frac{\mu_B}{3}\right)$$

$$\mathscr{A}(x) = \psi\left(\frac{1}{2} + \frac{ix}{2\pi T}\right) + \psi\left(\frac{1}{2} - \frac{ix}{2\pi T}\right) + 2\gamma_E + 2\ln 4$$

⋮

$$\psi(z) = \partial_z \ln\Gamma(z)$$

$$\bar{\mu}\frac{d}{d\bar{\mu}}g'^2 = \frac{41g'^4}{48\pi^2}$$

$$\bar{\mu}\frac{d}{d\bar{\mu}}g^2 = -\frac{19g^4}{48\pi^2}$$

$$\bar{\mu}\frac{d}{d\bar{\mu}}\lambda = \frac{1}{(4\pi)^2}\left(\frac{3}{8}(3g^4 + g'^4 + 2g^2g'^2) + 24\lambda^2 - 6g_Y^4 - 3\lambda\left(3g^2 + g'^2 - 4g_Y^2\right)\right)$$

⋮

3dSMEFT for the Phase transition study

Second step

Integrate out heavy mode

$$\bar{\mathscr{L}}_{3d} = \frac{1}{4}G^a_{ij}G^a_{ij} + \frac{1}{4}F_{ij}F_{ij} + \left(D_i\Phi_i\right)^\dagger\left(D_i\Phi_i\right) + \frac{1}{2}\bar{m}_3^2\Phi_i^\dagger\Phi_i + \bar{\lambda}_3\left(\Phi_i^\dagger\Phi_i\right)^2 + \bar{c}_{6,3}\left(\Phi_i^\dagger\Phi_i\right)^3$$

$$\bar{g}_3'^2 = g_3'^2$$

$$\bar{g}_3^2 = g_3^2\left(1 - \frac{g_3^2}{64\pi\, m_D}\right)$$

$$\bar{\lambda}_3 = \lambda_3 - \frac{1}{8\pi}\left(\frac{3h_1^2}{m_D} + \frac{h_2^2}{m_D'} + \frac{h_3^2}{m_D + m_D'}\right)$$

$$\bar{c}_{6,3} = c_{6,3} + \frac{1}{2(4\pi)}\frac{h_1^3}{m_D^3}$$

$$\bar{m}_3^2 = m_3^2 + \bar{\mu}_3\ terms + \cdots$$

$$h_1 = \frac{1}{4}g^2(\bar{\mu})T + \cdots$$

$$h_2 = \frac{1}{4}g'^2(\bar{\mu})T + \cdots$$

$$h_3 = \frac{1}{2}g\,(\bar{\mu})g'(\bar{\mu})T + \cdots$$

$$\bar{\mu}_3 = g^2T$$

3dSMEFT for the Phase transition study

Effective potential

$$V_{eff}^{3d} = V_{tree} + \hbar V_{1loop} + \hbar^2 V_{2loop}$$

$\mathcal{O}(g^2)$ $\quad$ $\mathcal{O}(g^3)$ $\quad$ $\mathcal{O}(g^4)$

$$V_{tree} = \frac{1}{2}\bar{m}_3^2\phi^2 + \frac{1}{4}\bar{\lambda}_3\phi^4 + \frac{1}{8}\bar{c}_{6,3}\phi^6$$

$$V_{1loop} = -\left(\frac{m_\phi^3}{12\pi} + \frac{3m_\chi^3}{12\pi} + 2\left(\frac{2m_W^2}{12\pi} + \frac{m_Z^2}{12\pi}\right)\right)$$

$$m_\phi^2 = \bar{m}_3^2 + 3\bar{\lambda}_3\phi^2 + \frac{15}{4}\bar{c}_{6,3}\phi^4$$

$$m_\chi^2 = \bar{m}_3^2 + \bar{\lambda}_3\phi^2 + \frac{3}{4}\bar{c}_{6,3}\phi^4$$

$$m_W^2 = \frac{1}{2}\bar{g}_3^2\phi^2$$

$$m_Z^2 = \frac{1}{4}\left(\bar{g}_3^2 + \bar{g}_3'^2\right)\phi^2$$

$$V_{2loop} = -\left((VVV) + (VGG) + (VVS) + (VSS) + (SSS) + (SS) + (VS) + (VV)\right)$$

S-scalar V-vector boson G-ghost

3dSMEFT for the Phase transition study

T_c

$$\begin{cases} V_{eff}^{3d}(\phi_c, T_c) = V_{eff}^{3d}(0, T_c) \\ \left.\dfrac{\partial V_{eff}^{3d}(\phi, T)}{\partial \phi}\right|_{\phi=\phi_c} = 0 \end{cases}$$

T_n

$$\Gamma = Ae^{-S_3} \qquad A \sim T^4$$

$$S_3 = 4\pi \int dr r^2 \left[\frac{1}{2}\left(\frac{d\phi(r)}{dr}\right)^2 + V_{eff}^{3d}(\phi, T)\right]$$

$\Gamma \sim H$

↓

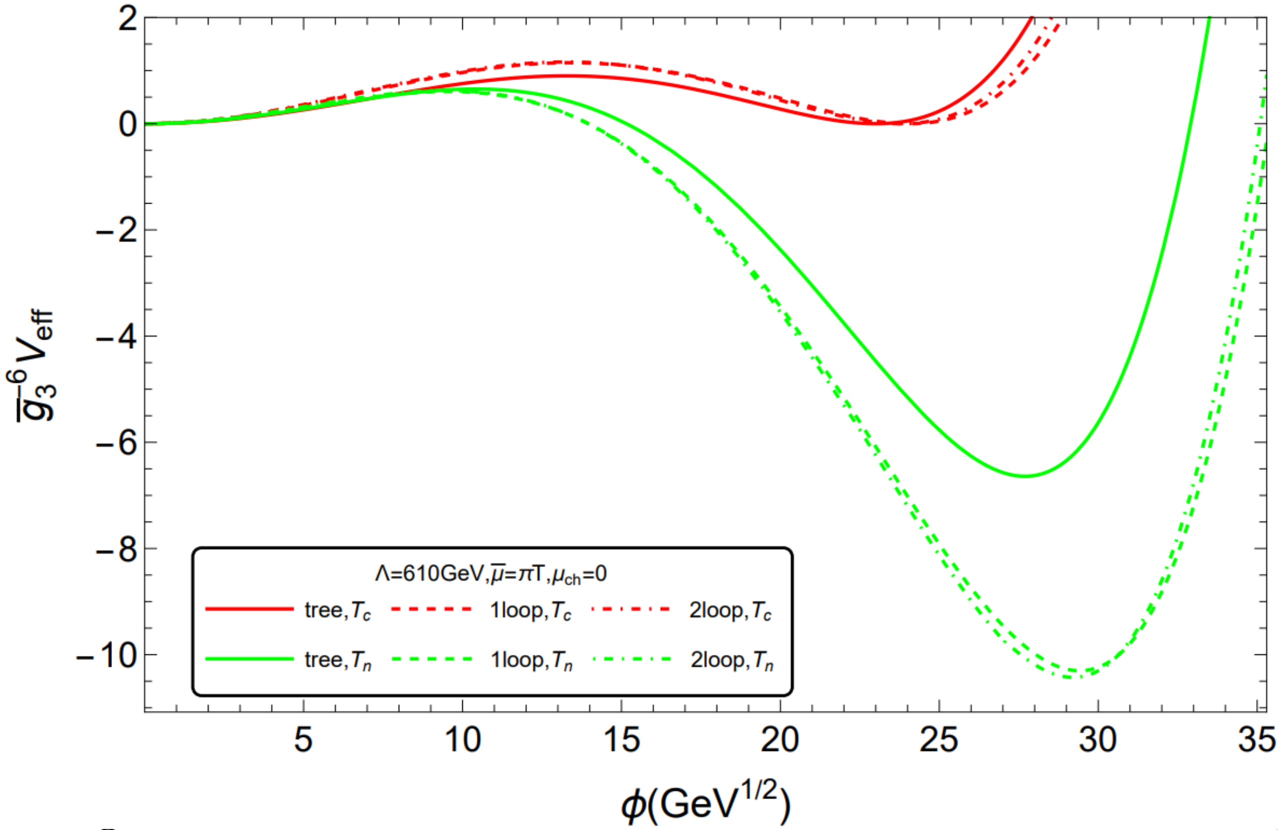
$$S_3 \approx 140$$

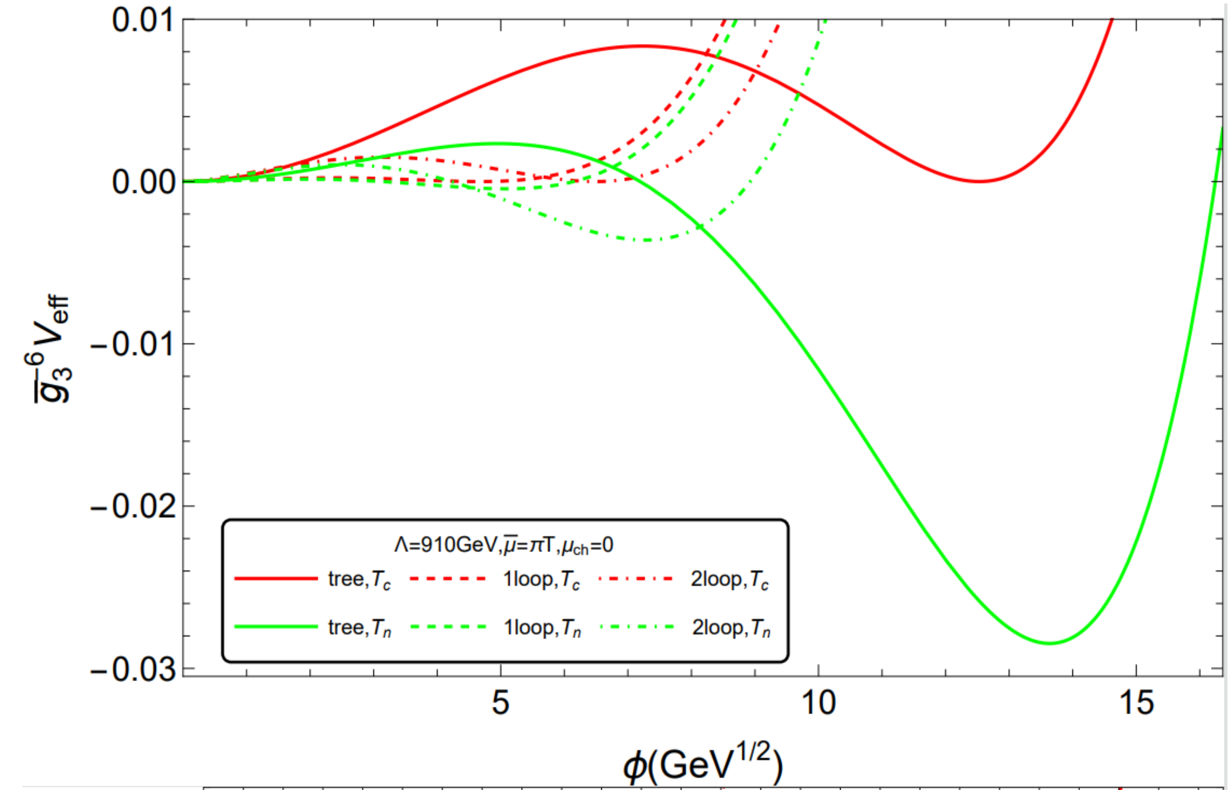
$\bar{\mu} = \pi T$ the lowest fermionic mode

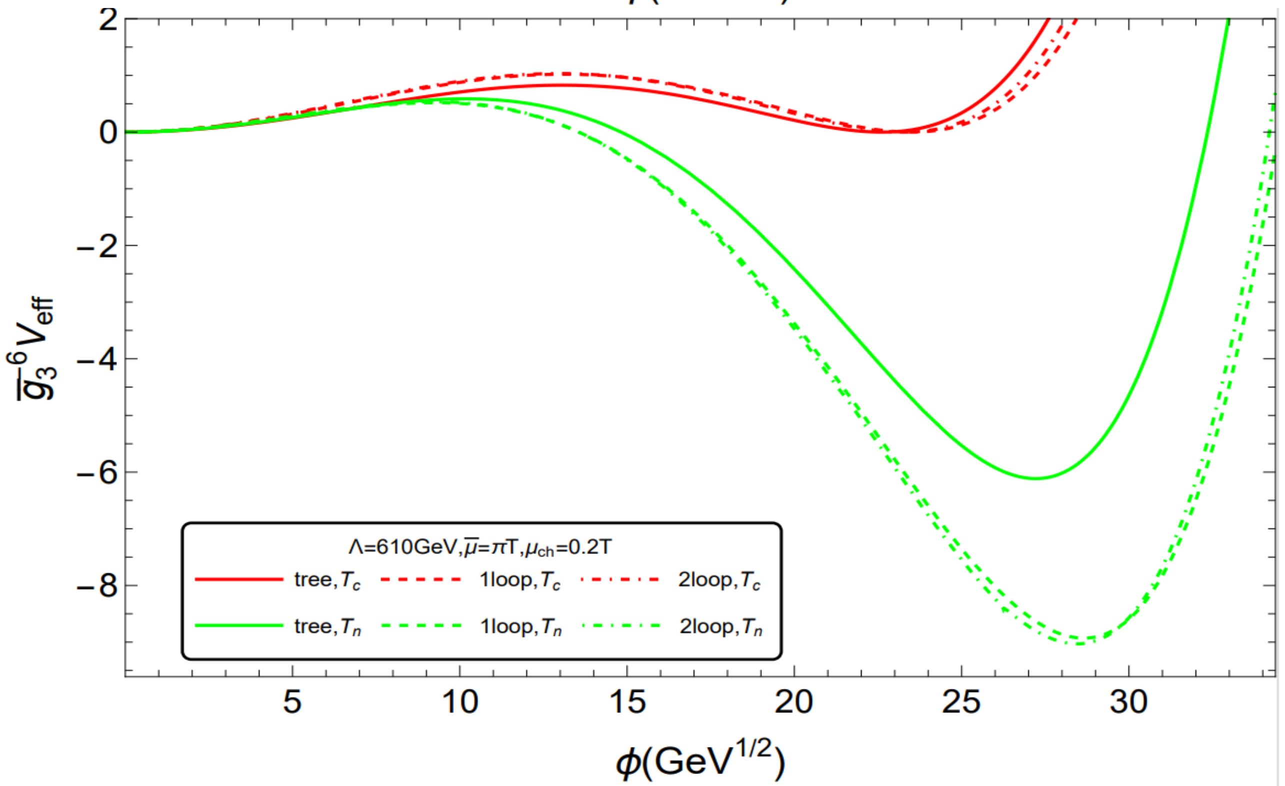
$\bar{\mu} = 2\pi T$ the lowest nonzero bosonic mode

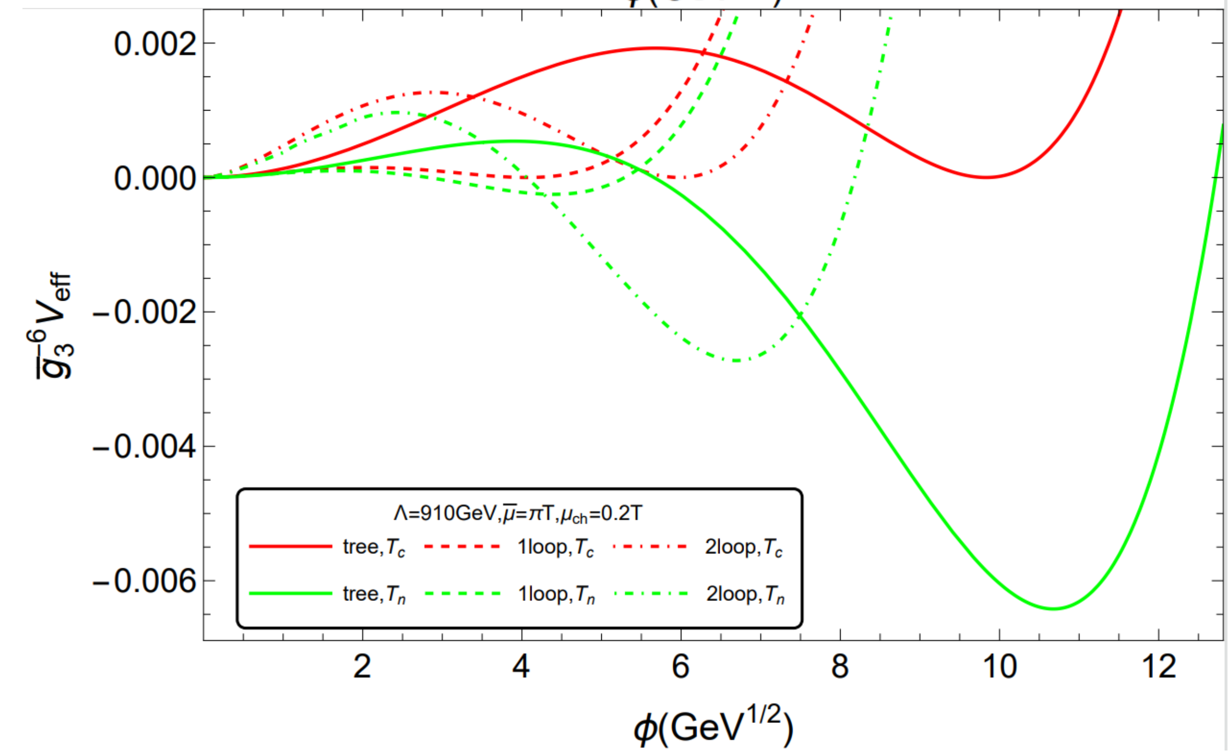
$\bar{\mu} = 4\pi e^{-\gamma_E} T$ the lowest logarithmic contribution

3dSMEFT for the Phase transition study







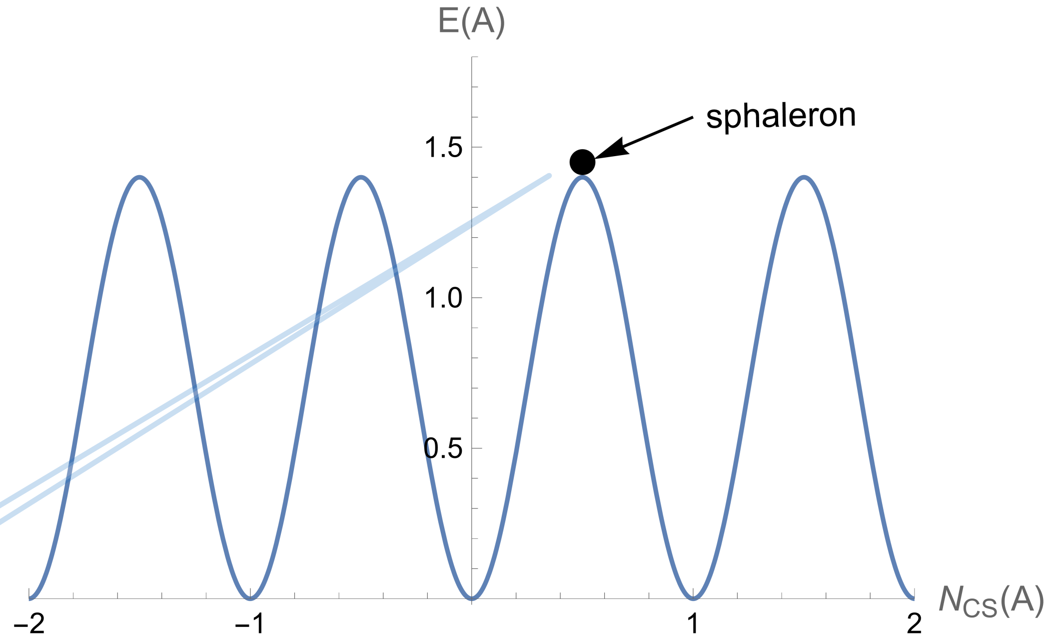


BNPC & Strongly First-order EWPT

Chern-Simons number

$$\Delta B = \Delta L = 3N_{CS}(t)$$

$$\Gamma_{sph} = \lim_{V,t\to\infty} \frac{\langle N_{CS}(t)^2 \rangle}{Vt}$$



$\Gamma_{sph} < H$

$$PT_{sph} = \frac{E_{sph}(T)}{T} - 7\ln\frac{v(T)}{T} + \ln\frac{T}{100GeV} > (35.9 - 42.8)$$

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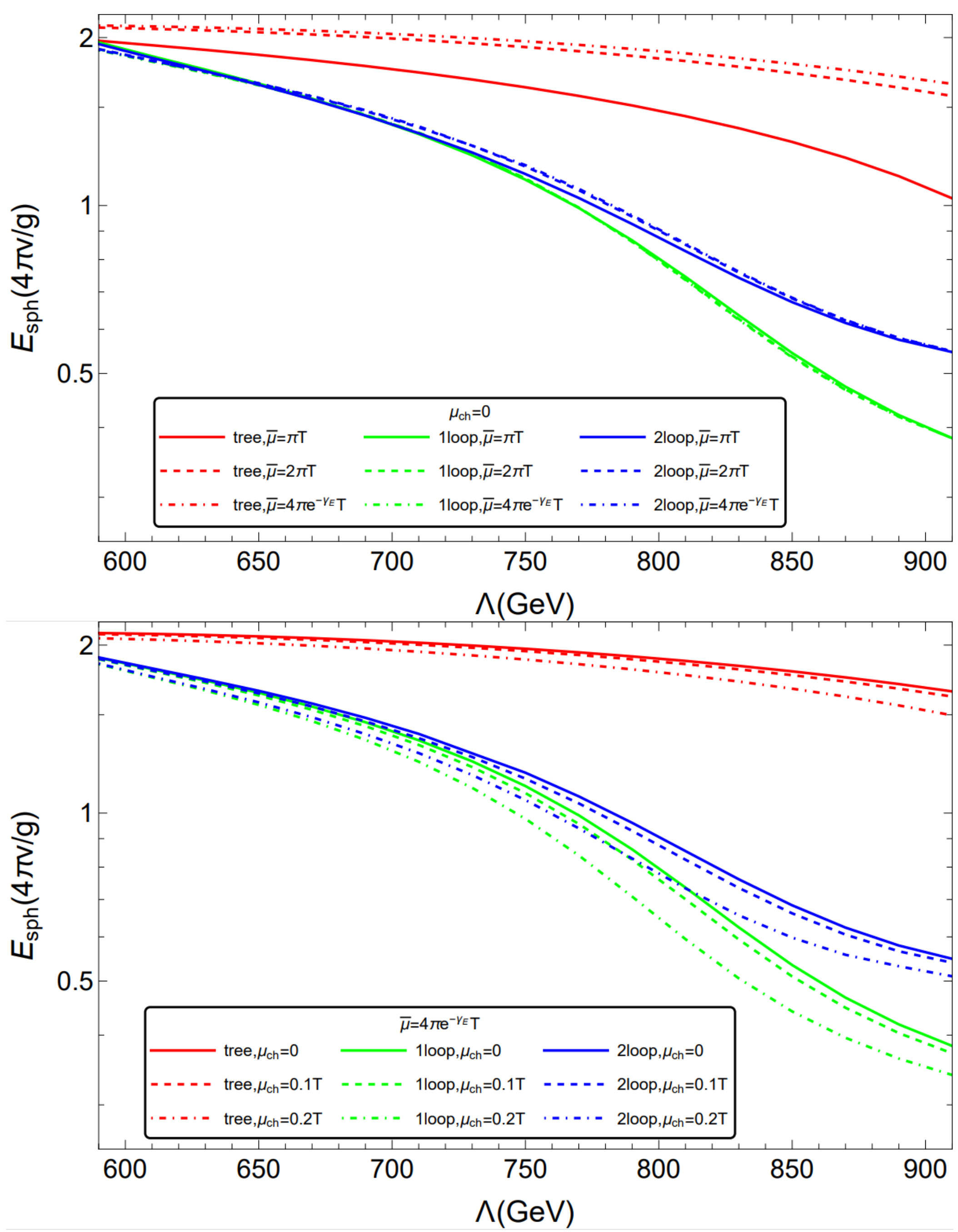
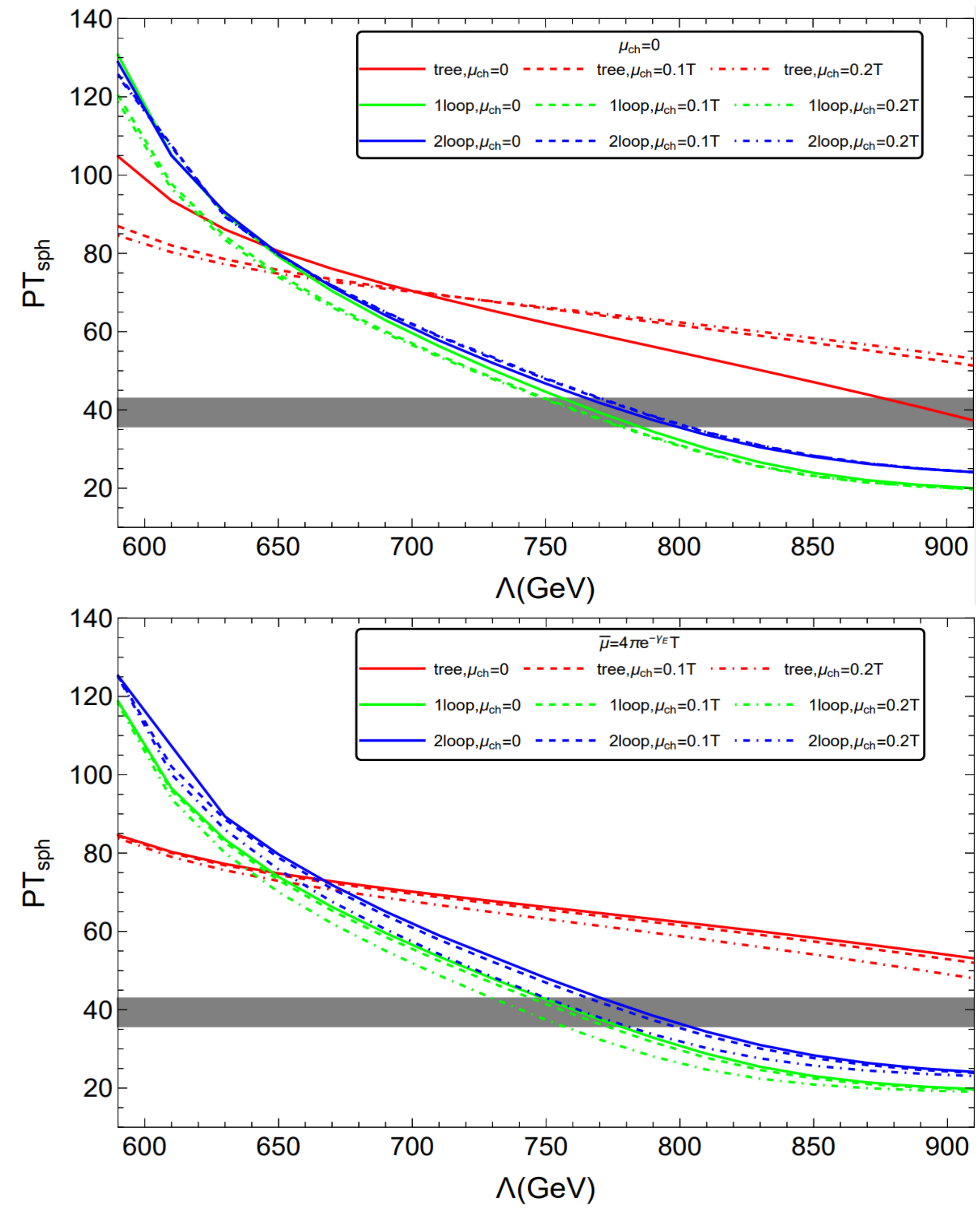
Baryon Number Preserving Condition (BNPC)

Sphaleron energy



PT_{sph}



Renhui Qin, **Ligong Bian**, 2407.01981

GW parameters and FOPT

Bounce solution

$$S_3(T) = \int 4\pi r^2 dr \left[\frac{1}{2}\left(\frac{d\phi_b}{dr}\right)^2 + V(\phi_b, T)\right]$$

$$\lim_{r\to\infty} \phi_b = 0 \,, \qquad \frac{d\phi_b}{dr}\Big|_{r=0} = 0$$

Bubble nucleation

$$\Gamma \approx A(T) e^{-S_3/T} \sim 1$$

PT strength

$$\alpha \equiv \frac{1}{\rho_r}\left(\Delta V_{\rm eff}(\phi, T) - \frac{T}{4}\Delta\frac{\partial V_{\rm eff}(\phi, T)}{\partial T}\right)$$

Phase transition inverse duration

$$\frac{\beta}{H_n} = T\frac{d(S_3(T)/T)}{dT}\Big|_{T=T_n}$$

GW parameters and FOPT

The probability, that a randomly chosen point is still in the false vacuum, given by

$$P(t) = e^{-I(t)} \qquad I(t) = \frac{4\pi}{3}\int_{t_c}^{t} dt'\Gamma(t')a(t')^3 r(t,t')^3$$

The fraction of the space which has already been converted to the broken phase

$$r(t,t') = \int_{t'}^{t} \frac{v_w(\tilde{t})d\tilde{t}}{a(\tilde{t})}$$

$r(t,t')$: the comoving radius of a bubble nucleated at t' propagated until a subsequent time t

a(t): the scale factor, $v_w(t)$: the wall velocity.

Using temperature T instead of time variable t, we have

$$I(T) = \frac{4\pi}{3}\int_{T}^{T_c} \frac{dT'}{H(T')}\Gamma(T')\frac{r(T,T')^3}{T'^4}$$

The transition completes when $P(t) \approx 0.7$, which leads to a percolation temperature T_p when

$$I(T_p) = 0.34.$$

GW spectrum from FOPT

- **Bubble collisions**

$$\Omega_{\rm col}h^2 = 1.67\times10^{-5}\left(\frac{H_*}{\beta}\right)^2\left(\frac{\kappa\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{1/3}\left(\frac{0.11v_b^3}{0.42+v_b^2}\right)\frac{3.8(f/f_{\rm env})^{2.8}}{1+2.8(f/f_{\rm env})^{3.8}}$$

peak frequency: $f_{\rm env} = 16.5\times10^{-6}\left(\frac{f_*}{H_*}\right)\left(\frac{T_*}{100{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6}{\rm Hz}$

- **Sound Wave**

$$\Omega h^2_{\rm sw}(f) = 2.65\times10^{-6}(H_*\tau_{sw})\left(\frac{\beta}{H}\right)^{-1}v_b\left(\frac{\kappa_\nu\alpha}{1+\alpha}\right)^2\left(\frac{g_*}{100}\right)^{-\frac{1}{3}}\left(\frac{f}{f_{\rm sw}}\right)^3\left(\frac{7}{4+3\left(f/f_{\rm sw}\right)^2}\right)^{7/2}$$

phase transition duration: $\tau_{sw} = min\left[\frac{1}{H_*},\frac{R_*}{\bar{U}_f}\right],\ H_*R_* = v_b(8\pi)^{1/3}(\beta/H)^{-1}$

Root-mean-square four-velocity of the plasma: $\bar{U}_f^2 \approx \frac{3}{4}\frac{\kappa_\nu\alpha}{1+\alpha}$

peak frequency: $f_{\rm sw} = 1.9\times10^{-5}\frac{\beta}{H}\frac{1}{v_b}\frac{T_*}{100}\left(\frac{g_*}{100}\right)^{\frac{1}{6}}{\rm Hz}$

- **MHD turbulence**

$$\Omega h^2_{\rm turb}(f) = 3.35\times10^{-4}\left(\frac{\beta}{H}\right)^{-1}\left(\frac{\epsilon\kappa_\nu\alpha}{1+\alpha}\right)^{\frac{3}{2}}\left(\frac{g_*}{100}\right)^{-\frac{1}{3}}v_b\frac{(f/f_{\rm turb})^3(1+f/f_{\rm turb})^{-\frac{11}{3}}}{[1+8\pi f a_0/(a_*H_*)]}$$

peak frequency: $f_{\rm turb} = 2.7\times10^{-5}\frac{\beta}{H}\frac{1}{v_b}\frac{T_*}{100}\left(\frac{g_*}{100}\right)^{\frac{1}{6}}{\rm Hz}$

▶ 对撞机&引力波探测强一阶电弱相变

■ Higgs&GWs

☑ **SM+Scalar Singlet**

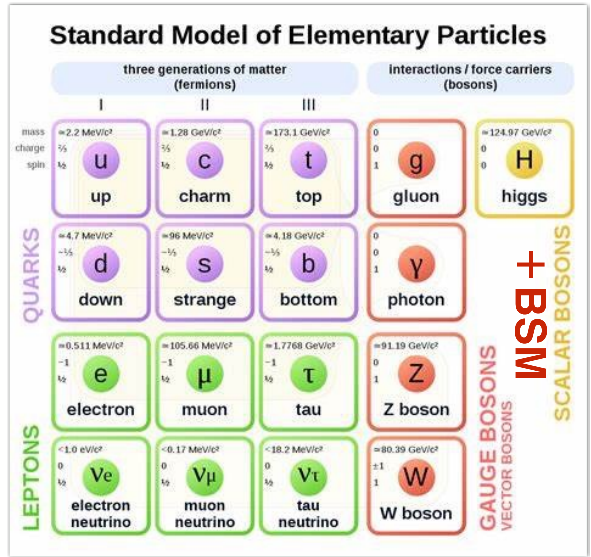
Profumo, Ramsey-Musolf, Wainwright, Winslow 14, **Bian**, Huang, Shu 15， Cheng, **Bian** 17, **Bian**, Tang 18,Chen, Li, Wu, **Bian**,19…

☑ **SM+Scalar Doublet**

Dorsch, Huber, Mimasu,No.14,Bernon, **Bian,** Jiang 17, **Bian**, Liu 18, Huang, Yu, 18,…

☑ **SM+Scalar Triplet**

Zhou, Cheng, Deng, **Bian**, Wu 18,Zhou, **Bian**, Guo,Wu 19, Ramsey-Musolf etal 21, Zhou, **Bian**,Du,22,…



☑ **Composite Higgs**

Bruggisser, Harling, Matsedonskyi, Servant,18, **Bian**,Wu,Xie 19, **Bian**,Wu,Xie 20,…

☑ **NMSSM**

Bi, **Bian**, Huang, Shu, Yin 15, **Bian,** Guo, Shu 17,Baum, Carena, Shah, Wagner, Wang 20, …

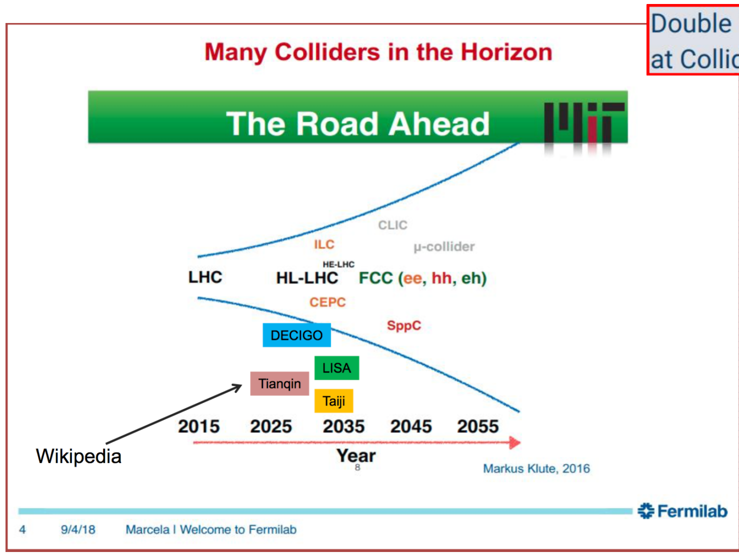
☑ **SMEFT**

Cao, Huang, Xie, & Zhang 17, Zhou, **Bian**, Guo 19, Cai,Hashino,Wang,Yu,22…

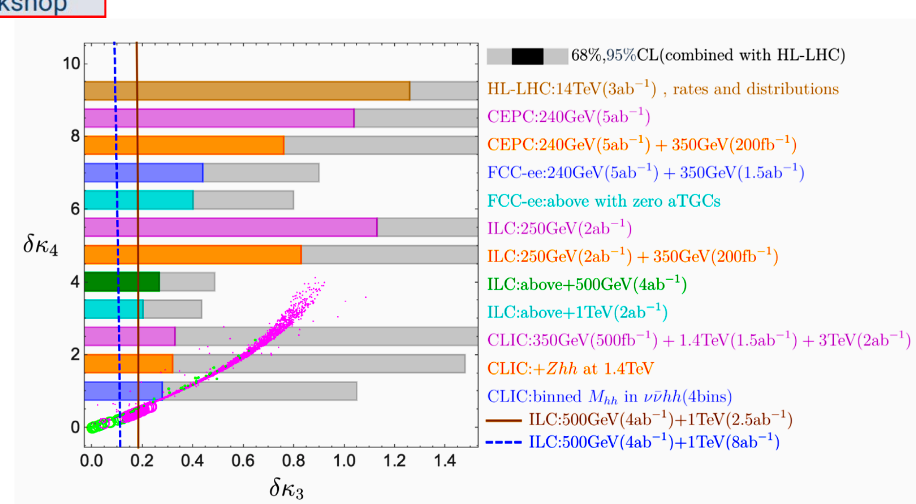
Symmetry breakdown

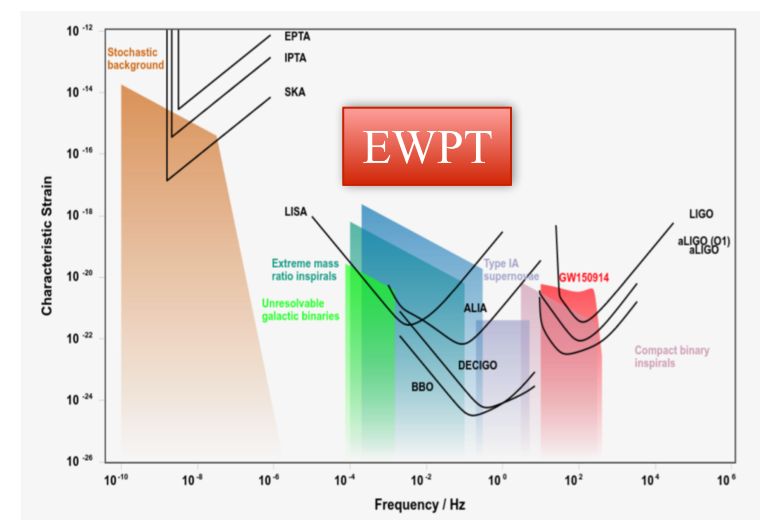
Symmetry breaking process

Double Higgs Production at Colliders Workshop



$$\Delta\mathscr{L} = -\frac{1}{2}\frac{m_h^2}{v}(1+\delta\kappa_3)h^3 - \frac{1}{8}\frac{m_h^2}{v^2}(1+\delta\kappa_4)h^4$$



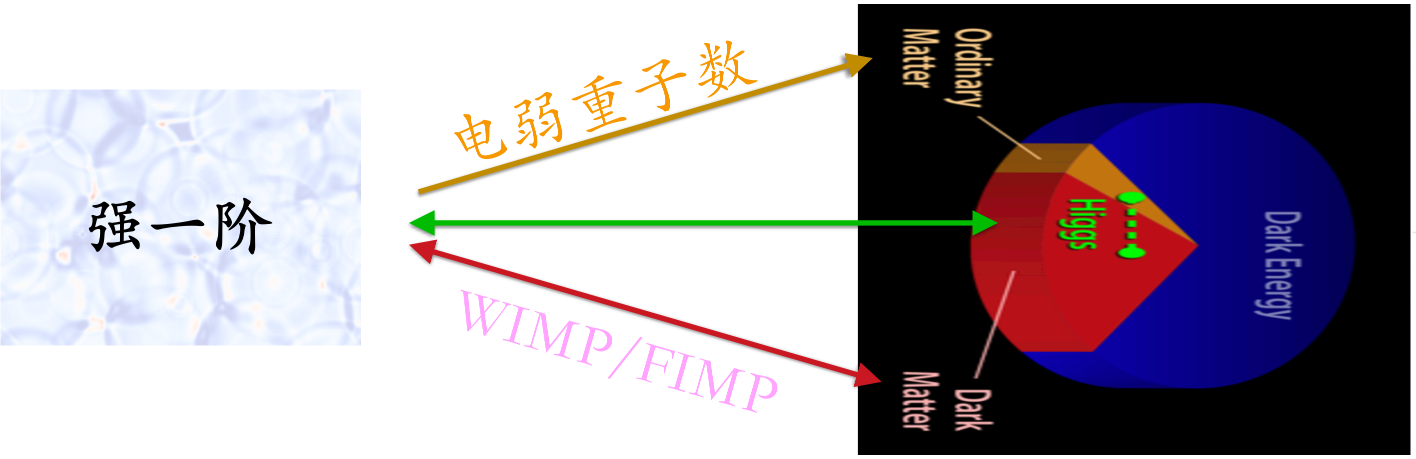


C J Moore et al. Class. Quantum Grav. 32 (2015) 015014.

SNR > 10 for two-step and one-step SFOEWPT

PTA,LIGO,LISA,天琴,太极,…

一阶相变效应



Impact of a complex singlet: Electroweak baryogenesis and dark matter
#2

Minyuan Jiang (Beijing, Inst. Theor. Phys. and Beijing, KITPC and Nanjing U.), Ligong Bian (Beijing, Inst. Theor. Phys. and Beijing, KITPC), Weicong Huang (Beijing, Inst. Theor. Phys. and Beijing, KITPC), Jing Shu (Beijing, Inst. Theor. Phys. and Beijing, KITPC)
(Feb 26, 2015)

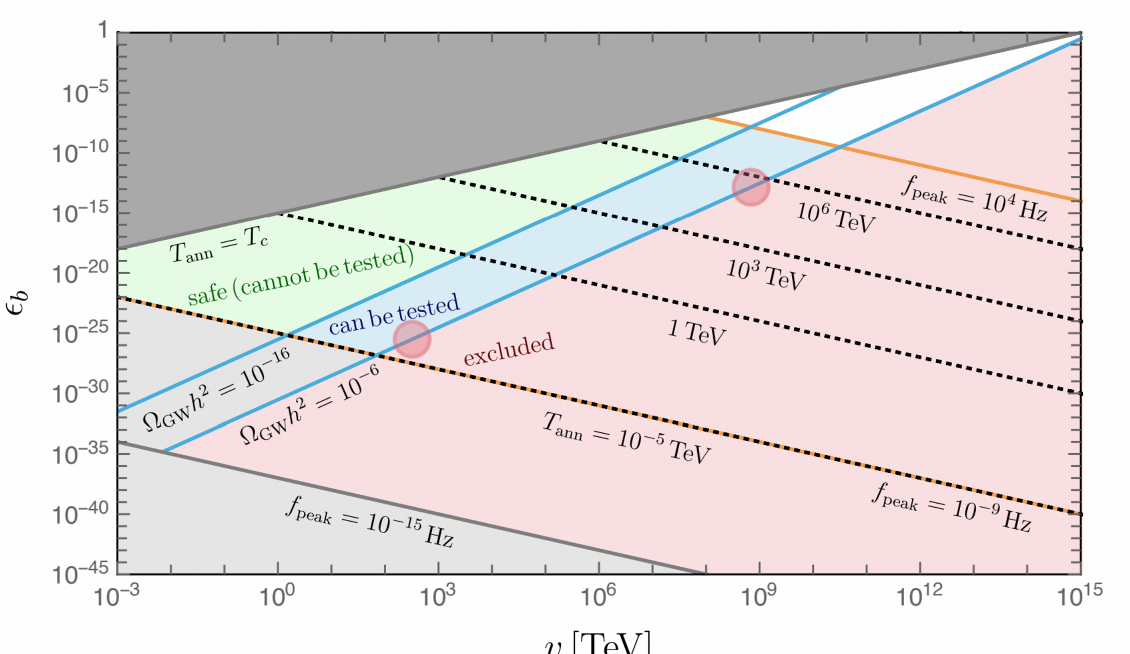
Published in: *Phys.Rev.D* 93 (2016) 6, 065032 • e-Print: 1502.07574 [hep-ph]

Thermally modified sterile neutrino portal dark matter and gravitational waves from phase transition: The Freeze-in case
#3

Ligong Bian (Chongqing U. and Chung-Ang U.), Yi-Lei Tang (Korea Inst. Advanced Study, Seoul) (Oct 7, 2018)

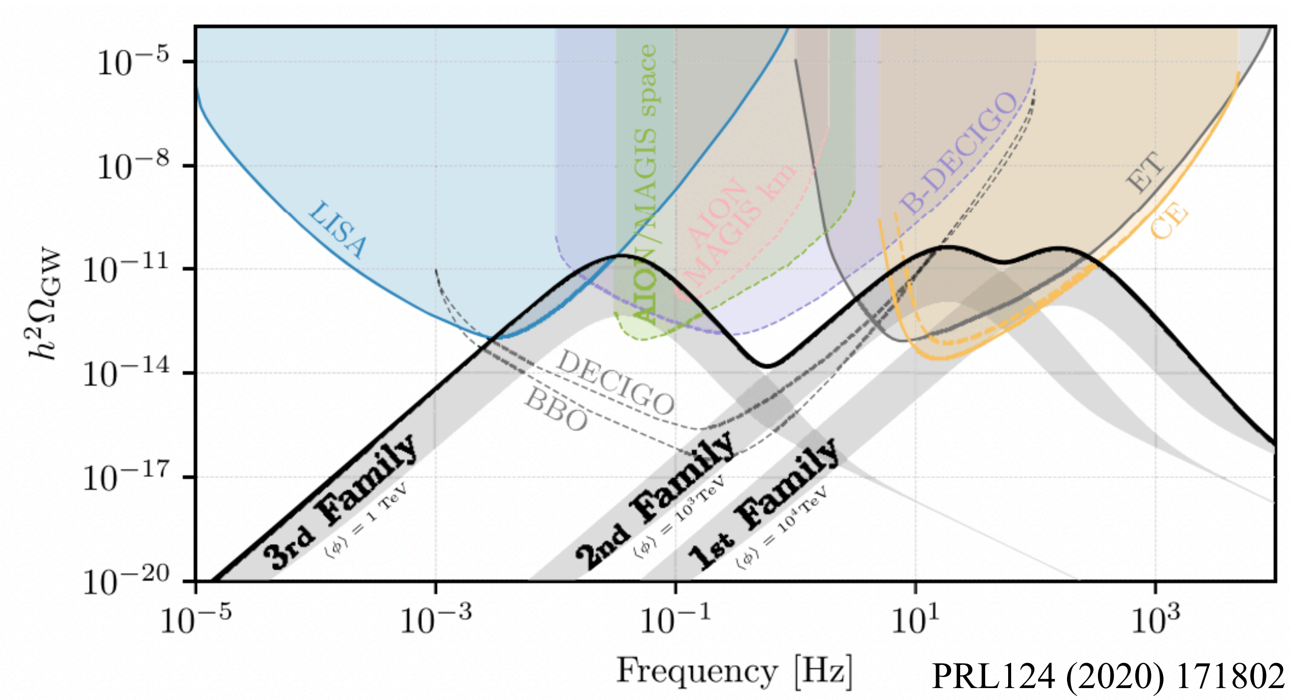
Published in: *JHEP* 12 (2018) 006 • e-Print: 1810.03172 [hep-ph]

离散的味对称性A4 与畴壁（DW）



JCAP 02 (2021) 032

三代夸克和轻子质量等级问题与FOPT



PRL124 (2020) 171802

一阶相变与Seesaw scale

Gravitational waves from first-order phase transitions in Majoron models of neutrino mass #9

Pasquale Di Bari (Southampton U.), Danny Marfatia (Hawaii U.), Ye-Ling Zhou (Southampton U. and HIAS, UCAS, Hangzhou and ICTP-AP, Beijing) (May 31, 2021)

Published in: *JHEP* 10 (2021) 193 • e-Print: 2106.00025 [hep-ph]

pdf DOI cite claim reference search 23 citations

Gravitational Waves from First-Order Phase Transitions: LIGO as a Window to Unexplored Seesaw Scales #1

Vedran Brdar (Heidelberg, Max Planck Inst.), Alexander J. Helmboldt (Heidelberg, Max Planck Inst.), Jisuke Kubo (Heidelberg, Max Planck Inst. and Toyama U.) (Oct 29, 2018)

Published in: *JCAP* 02 (2019) 021 • e-Print: 1810.12306 [hep-ph]

pdf DOI cite claim reference search 93 citations

Gravitational wave imprints of left-right symmetric model with minimal Higgs sector #1

Lukáš Gráf (Heidelberg, Max Planck Inst. and UC, Berkeley and UC, San Diego), Sudip Jana (Heidelberg, Max Planck Inst.), Ajay Kaladharan (Oklahoma State U.), Shaikh Saad (Basel U.) (Dec 22, 2021)

Published in: *JCAP* 05 (2022) 05, 003 • e-Print: 2112.12041 [hep-ph]

pdf DOI cite claim reference search 7 citations

Prospects of gravitational waves in the minimal left-right symmetric model #19

Mingqiu Li (Beijing, GUCAS), Qi-Shu Yan (Beijing, GUCAS and Beijing, Inst. High Energy Phys.), Yongchao Zhang (Southeast U., Nanjing and Washington U., St. Louis), Zhijie Zhao (Beijing, Inst. High Energy Phys.) (Dec 26, 2020)

Published in: *JHEP* 03 (2021) 267 • e-Print: 2012.13686 [hep-ph]

pdf DOI cite claim reference search 14 citations

Gravitational waves from neutrino mass and dark matter genesis #16

Pasquale Di Bari (Southampton U.), Danny Marfatia (Hawaii U.), Ye-Ling Zhou (Southampton U.) (Jan 21, 2020)

Published in: *Phys.Rev.D* 102 (2020) 9, 095017 • e-Print: 2001.07637 [hep-ph]

pdf DOI cite claim reference search 14 citations

Gravitational wave pathway to testable leptogenesis #2

Arnab Dasgupta (Pittsburgh U.), P.S. Bhupal Dev (Washington U., St. Louis and McDonnell Ctr. Space Sci.), Anish Ghoshal (Warsaw U.), Anupam Mazumdar (U. Groningen, VSI) (Jun 14, 2022)

Published in: *Phys.Rev.D* 106 (2022) 7, 075027 • e-Print: 2206.07032 [hep-ph]

pdf DOI cite claim reference search 23 citations

Cosmological implications of a B – L charged hidden scalar: leptogenesis and gravitational waves #5

Ligong Bian (Chongqing U.), Wei Cheng (Beijing, Inst. Theor. Phys.), Huai-Ke Guo (Oklahoma U.), Yongchao Zhang (Washington U., St. Louis and Peking U., CHEP) (Jul 31, 2019)

Published in: *Chin.Phys.C* 45 (2021) 11, 113104 • e-Print: 1907.13589 [hep-ph]

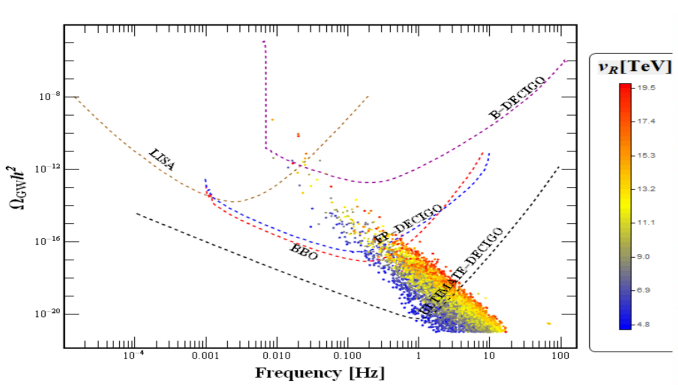
pdf DOI cite claim reference search 27 citations

Electroweak phase transition and gravitational waves in the type-II seesaw model #12

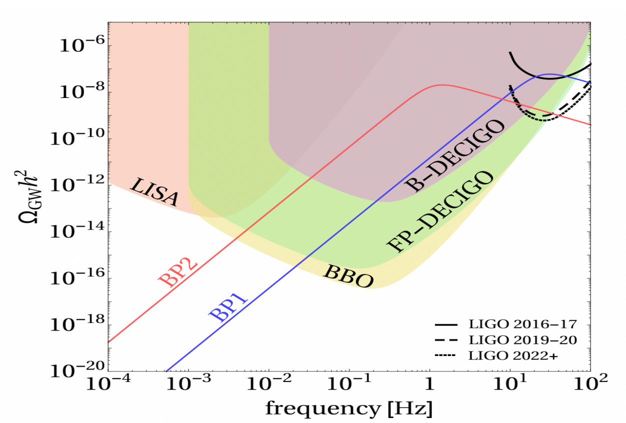
Ruiyu Zhou (CUPT, Chongqing), Ligong Bian (Chongqing U. and Peking U., CHEP), Yong Du (Beijing, Inst. Theor. Phys.) (Mar 3, 2022)

Published in: *JHEP* 08 (2022) 205 • e-Print: 2203.01561 [hep-ph]

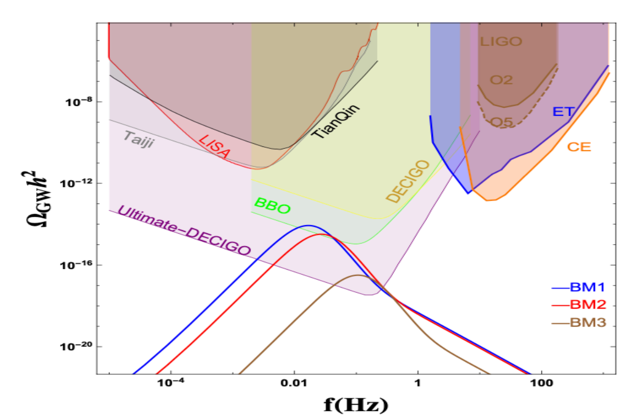
pdf DOI cite claim reference search 15 citations



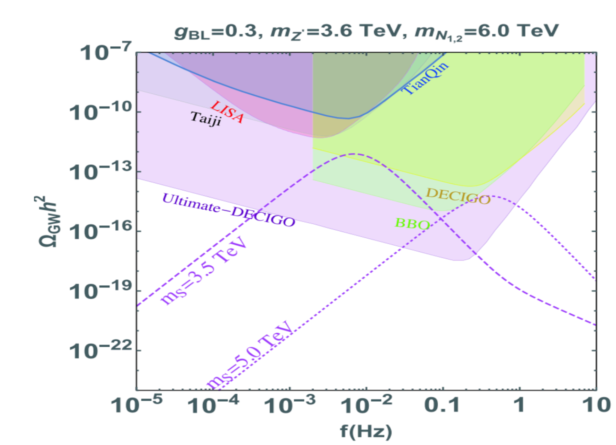
L-R



CSB



Type-II

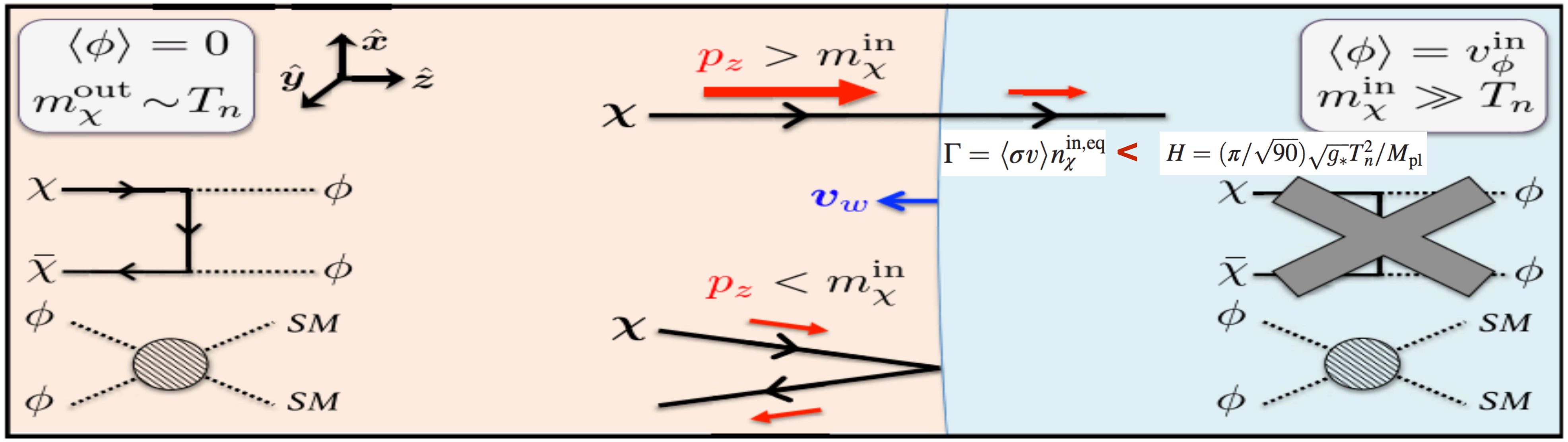


Type-I

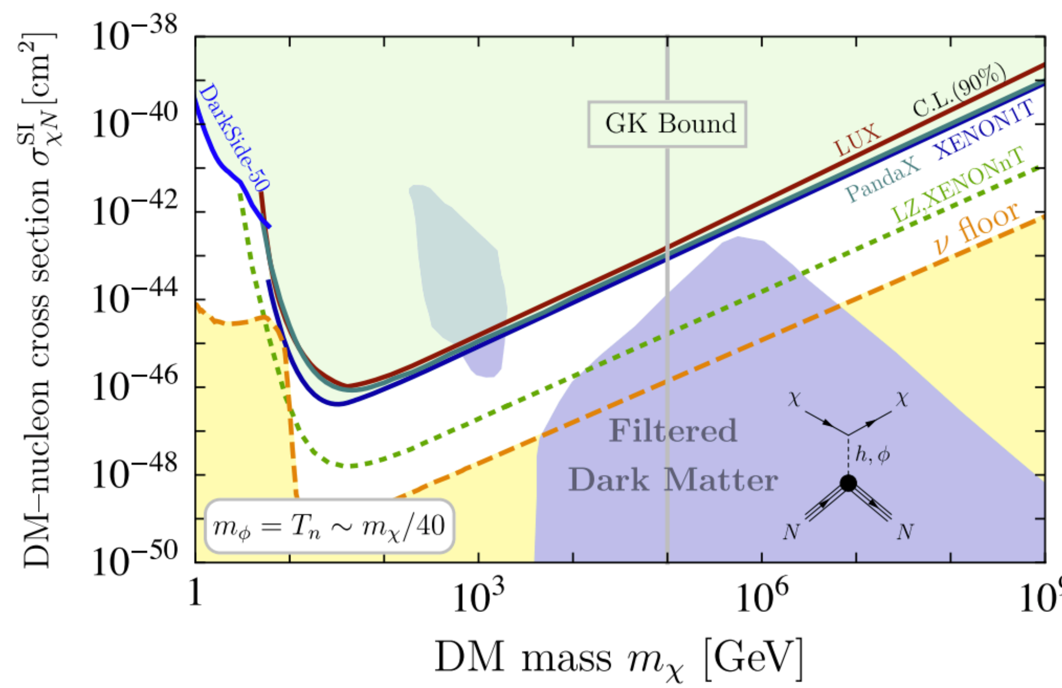
WIMP 暗物质与强一阶相变

$$\mathcal{L} \supset -V(\phi) - y_\chi \phi \bar{\chi}\chi - \beta\phi^2 H^\dagger H$$

过滤暗物质



暗区一阶相变

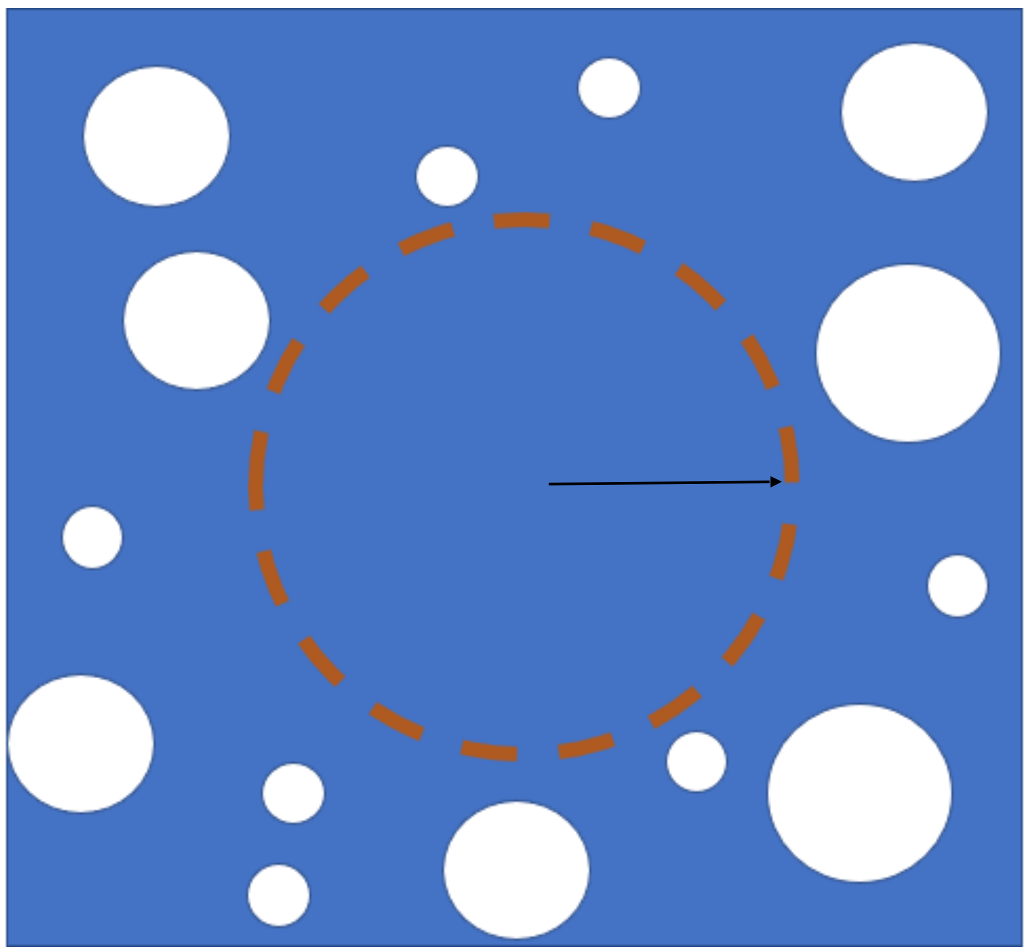


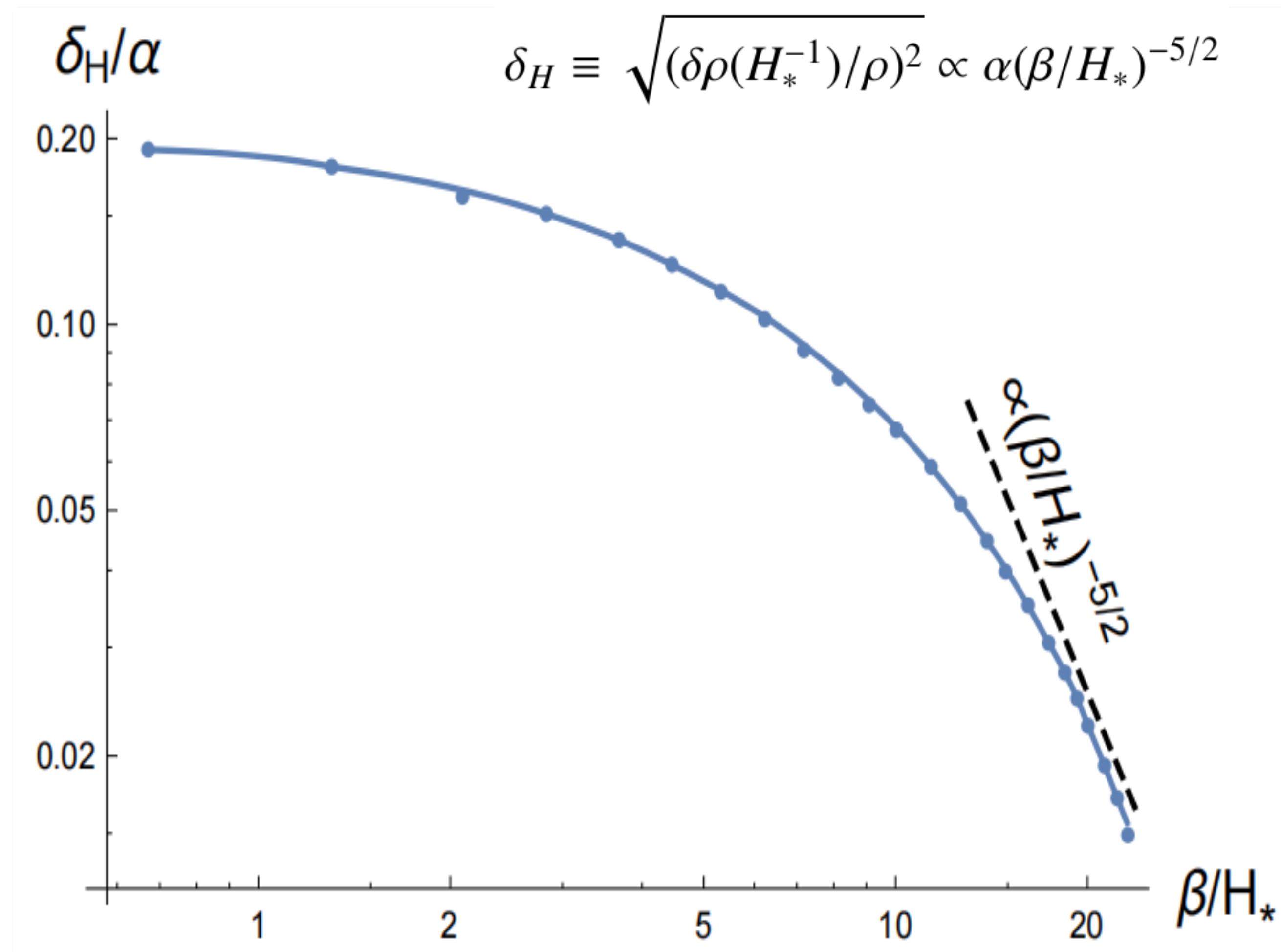
Baker , Kopp, and Long, Phys.Rev.Lett. 125 (2020) 15, 151102

see also: Chao, Li, Wang, JCAP 06 (2021) 038

真空延迟衰变与曲率扰动限制相变

Hubble-sized perturbations

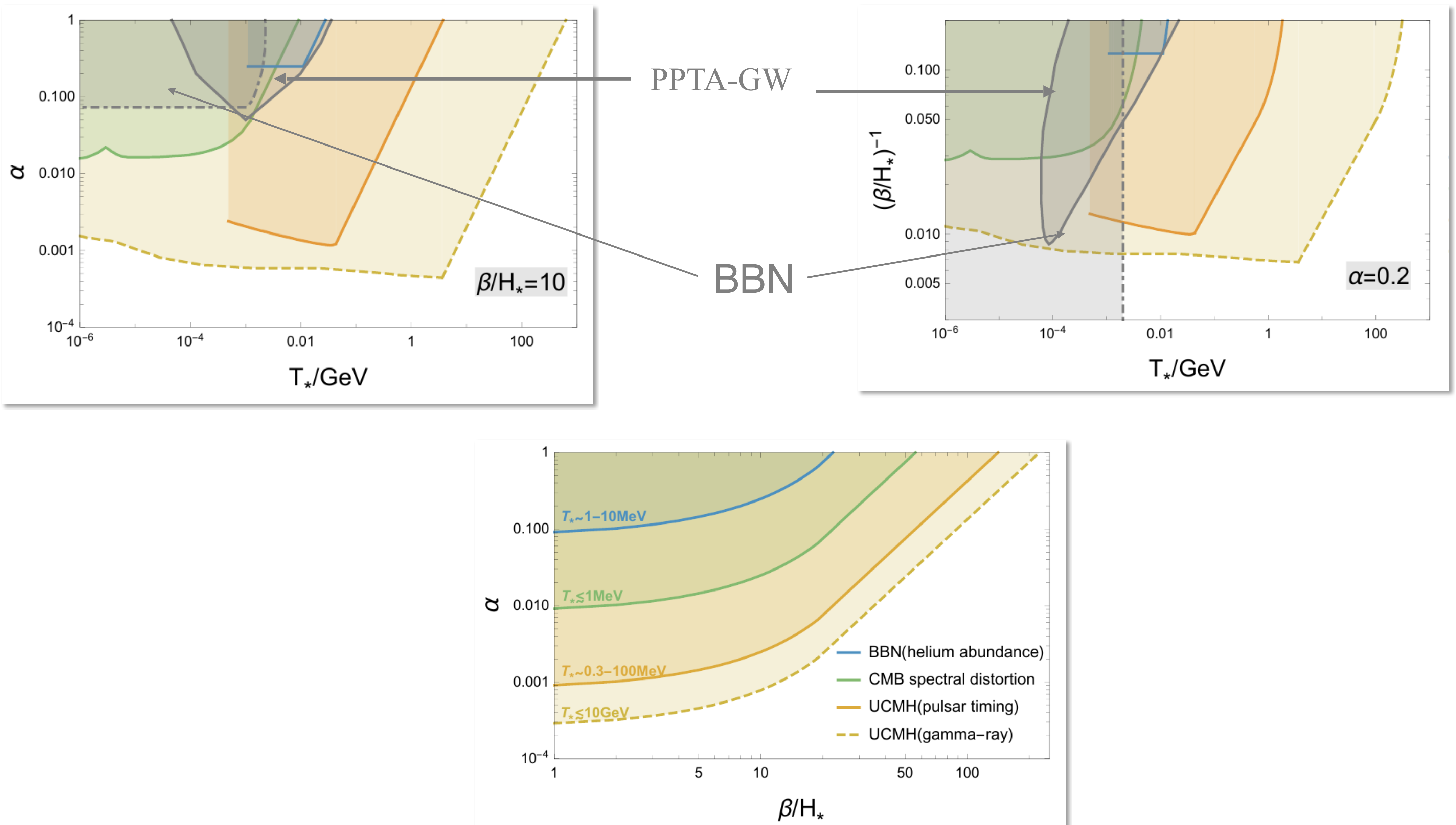




Liu, **Bian**, Cai, Guo, Wang, PRL130 (2023) 051001

真空延迟衰变与曲率扰动限制相变

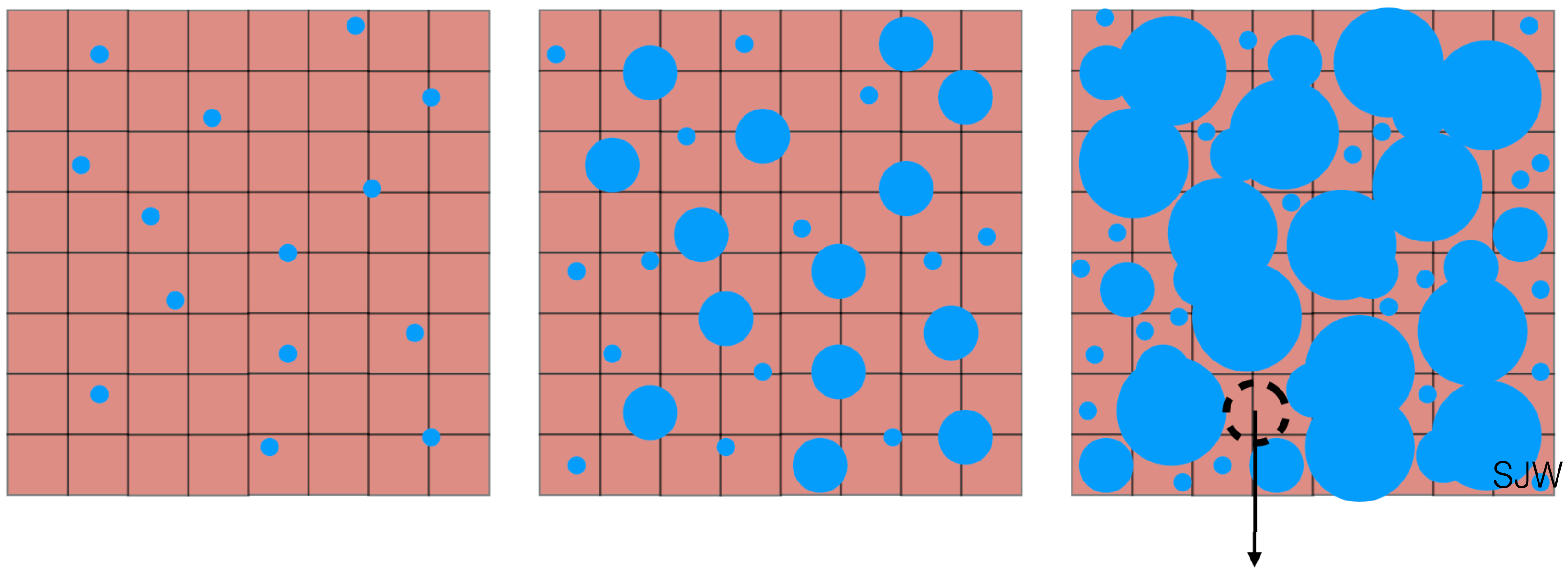
low-scale and slow 1st PTs motived for dark PT and BAU



Liu, **Bian**, Cai, Guo, Wang, PRL130 (2023) 051001

PBH 暗物质和一阶相变

PBH from postponed vacuum decay



Postponed Hubble volume V_H

↓

Un-diluted false vacuum energy

↓

PBH

Probability for a Hubble volume not to decay until time t_n

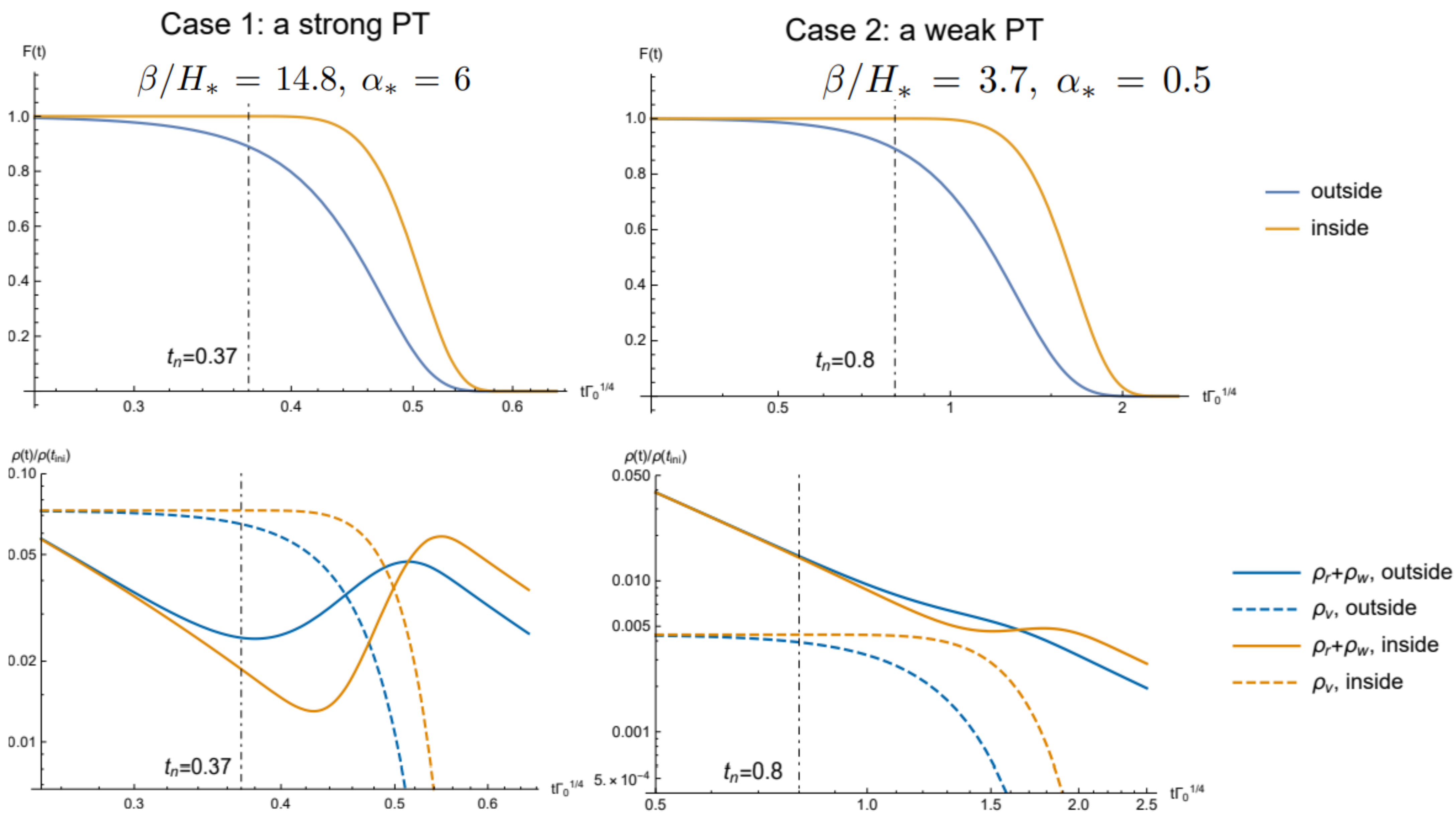
$$V_H(t) = \frac{4}{3}\pi H(t_{\rm PBH})^{-3}\frac{a(t)^3}{a(t_{\rm PBH})^3}$$

$$P(t_n) = \exp\left[-\frac{4\pi}{3}\int_{t_i}^{t_n}\frac{a^3(t)}{a^3(t_{\rm PBH})}H^{-3}(t_{\rm PBH})\Gamma(t)dt\right]$$

PBH abundance $\Omega_{\rm PBH}^{\rm form} = P(t_n)$

Collapse of the Hubble horizon

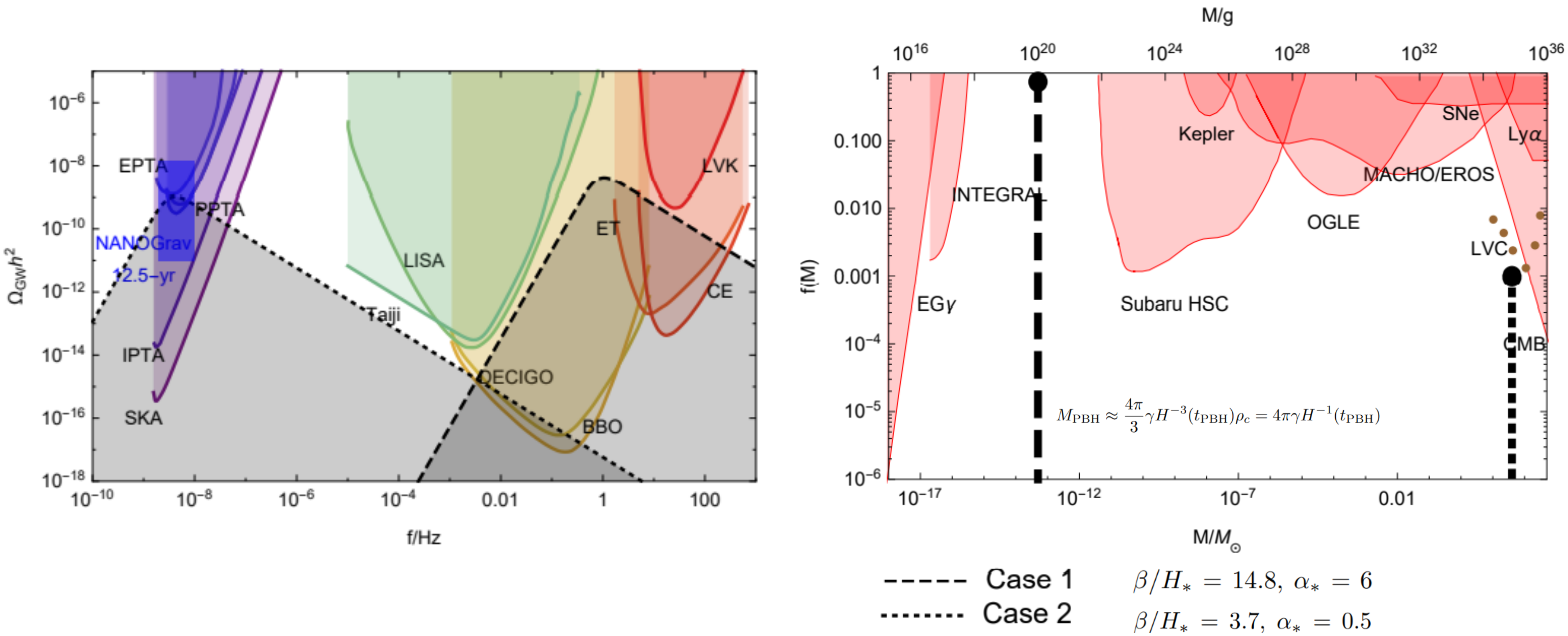
PBH 暗物质和一阶相变



$$\delta(t_{\rm PBH}) = \frac{\rho_v(t_{\rm PBH}; t_n) + \rho_r(t_{\rm PBH}; t_n)}{\rho_v(t_{\rm PBH}; t_i) + \rho_r(t_{\rm PBH}; t_i)} - 1 \geq \delta_c \Rightarrow t_{\rm PBH}$$

PBH 暗物质和一阶相变

PBH is more abundant in strong and slow first-order PTs.



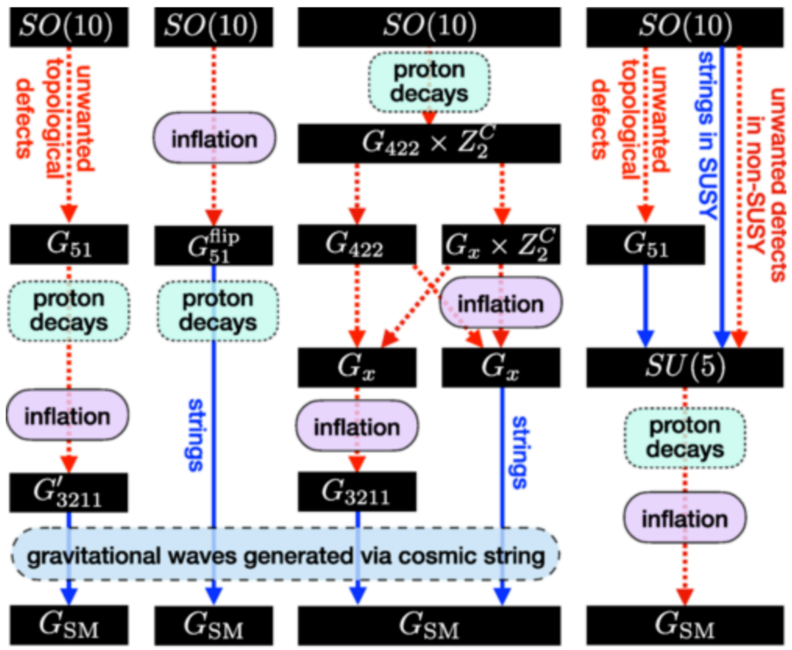
➢ Case 1: PBHs constitute all dark matter, Ω_{GW} to be probed CE,ET

➢ Case 2: GWs explain the CPL observed by NANOGrav, PBHs explain the coalescence events observed by the LIGO-Virgo collaboration

Liu，**Bian**, Cai, Guo, Wang，PRD **105**（**2022**）L021303

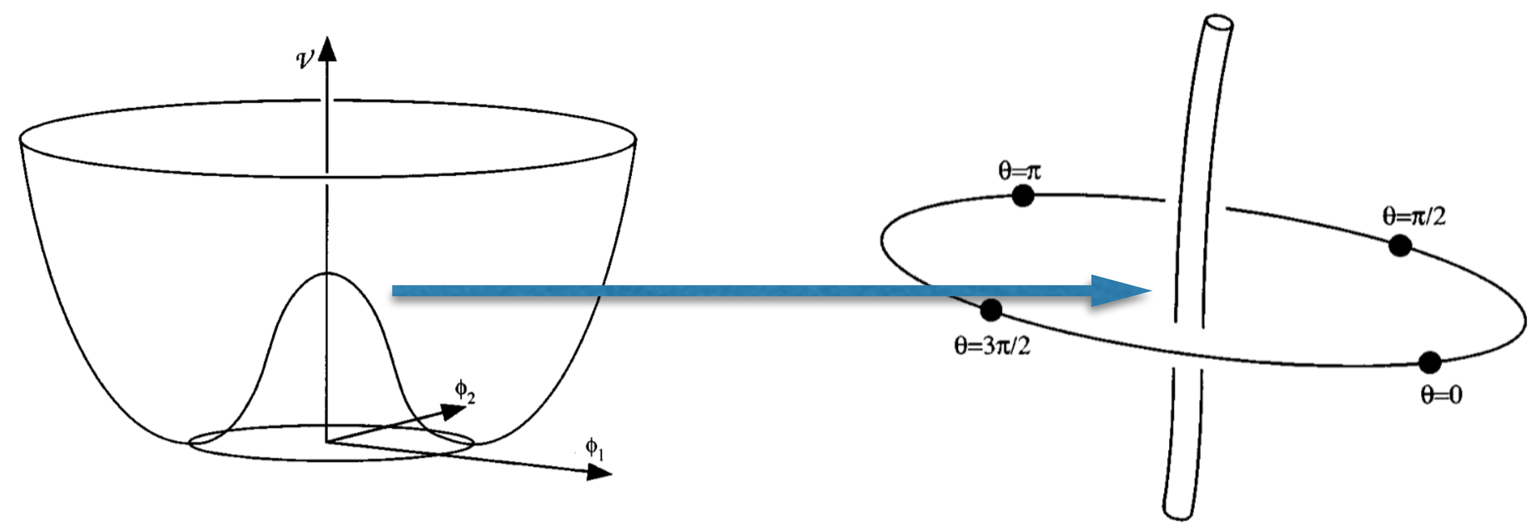
▸ 宇宙弦

通常形成于GUTs



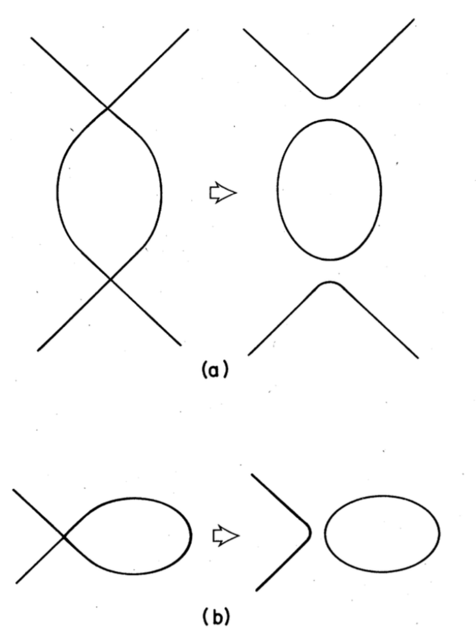
Phys. Rev. Lett. **126**, 021802

相变后U(1) 对称性自发破缺　　　宇宙弦



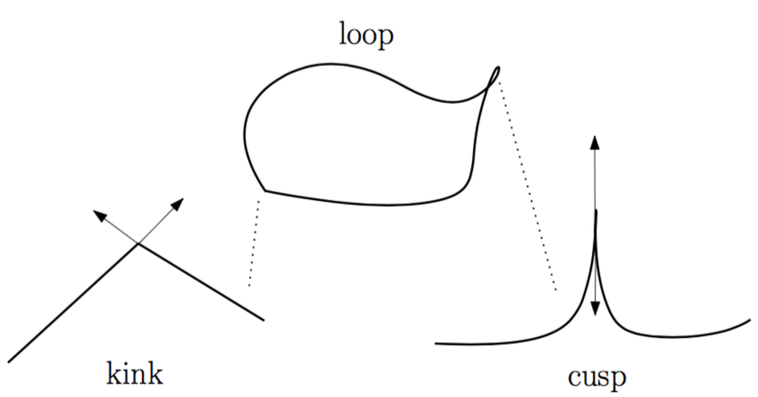
T. W. B. Kibble

宇宙弦成圈



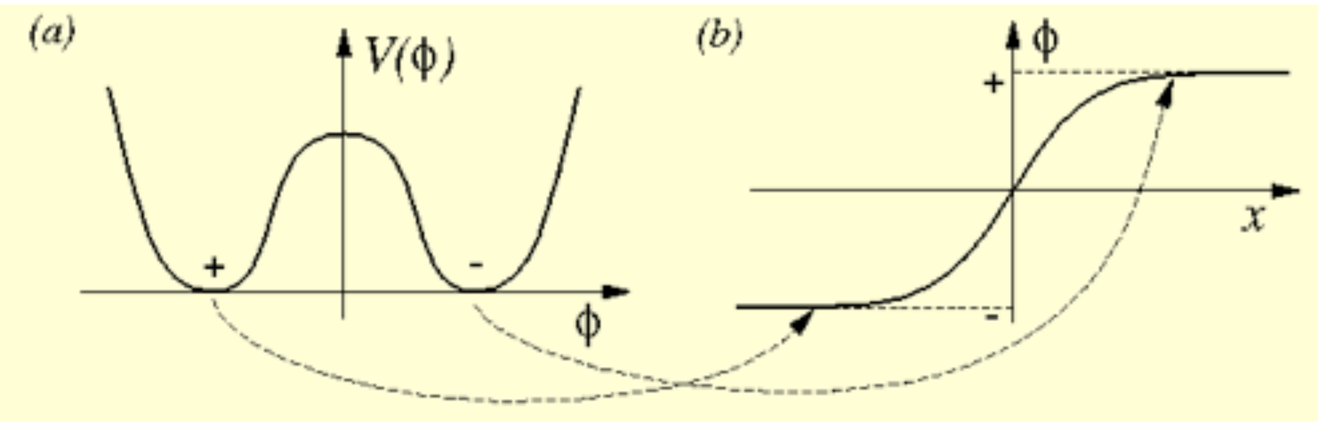
Phys.Rev.D 30 (1984) 2036

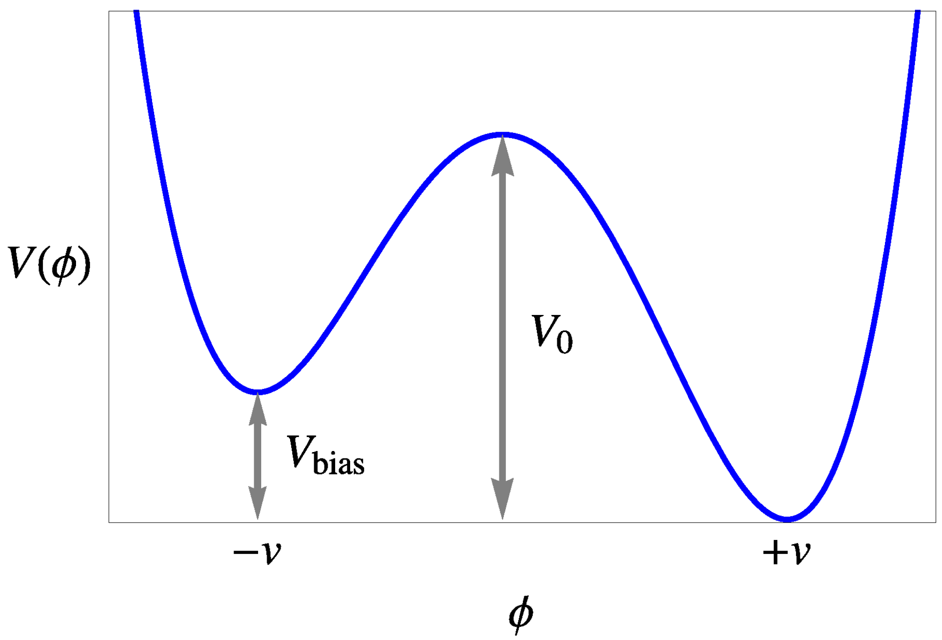
引力波辐射

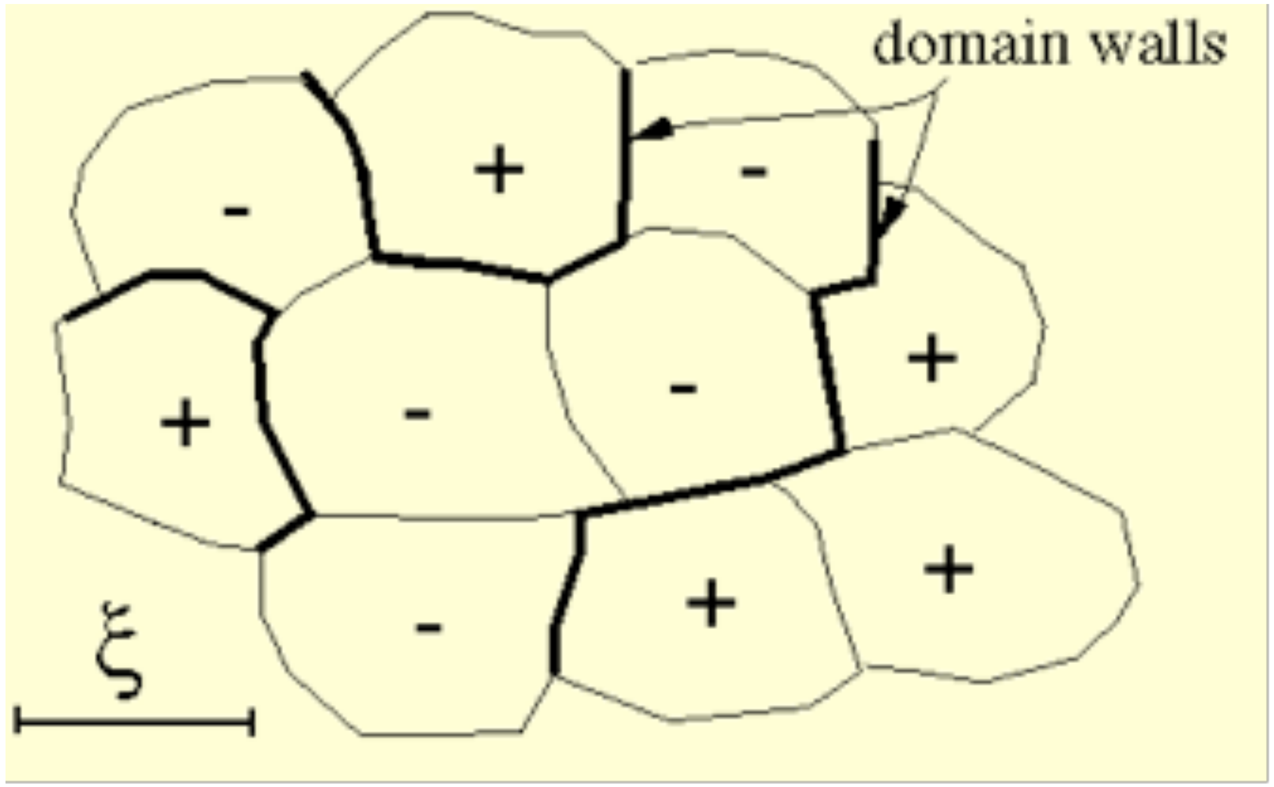


Yann Gouttenoire *et al* JCAP07(2020)032

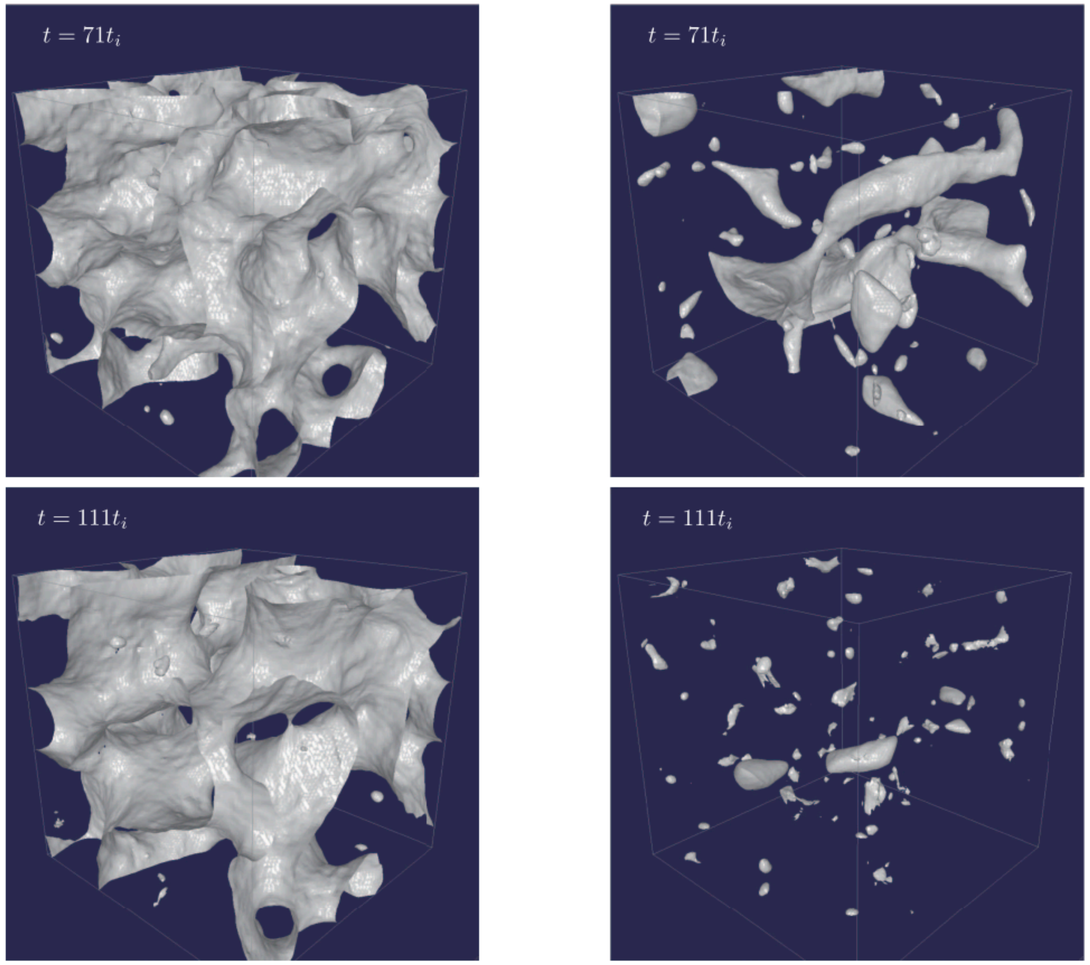
畴壁







Kibble mechanism



1002.1555

GW sources

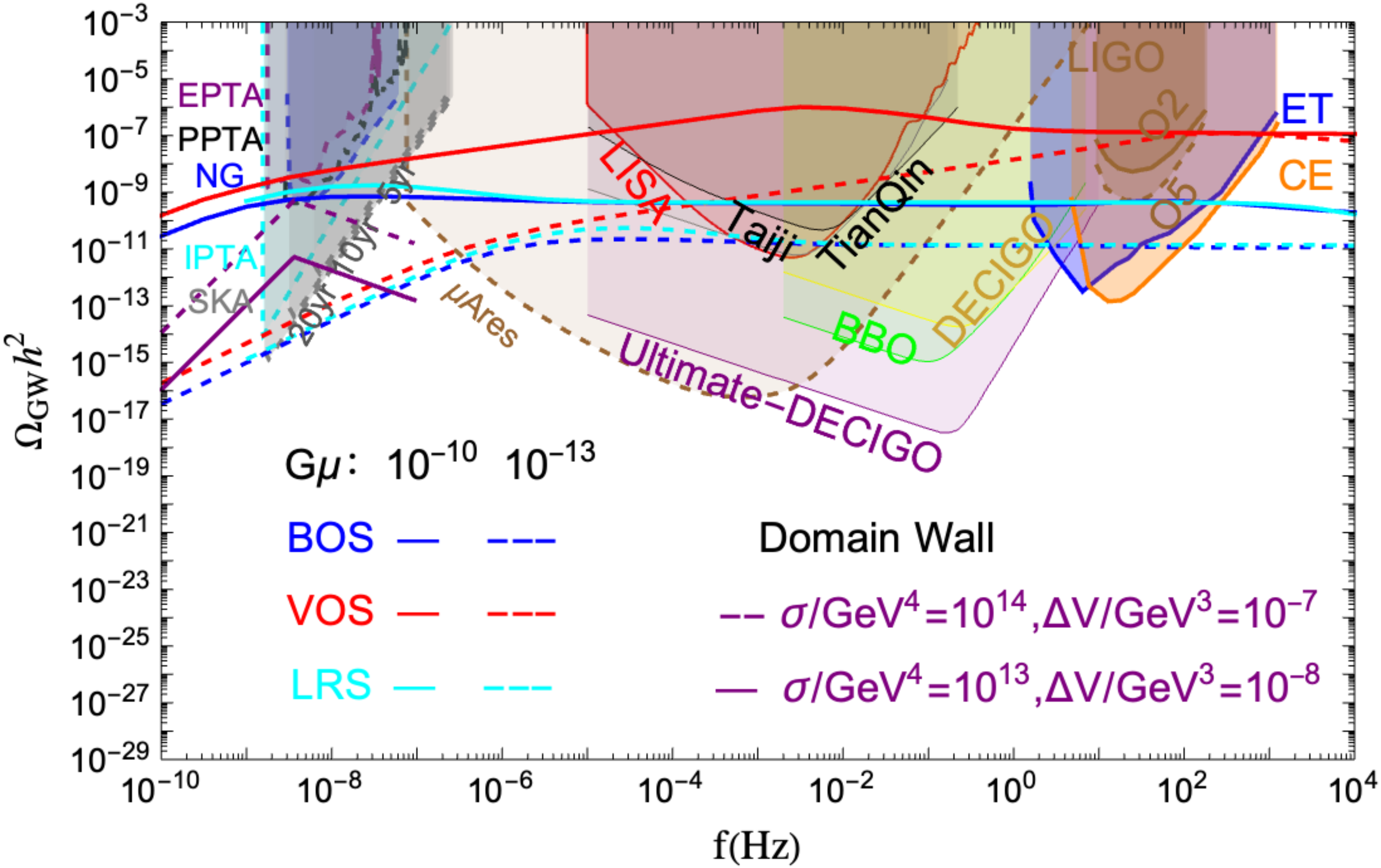
$$\Omega_{\rm GW}(f) = \begin{cases} \Omega_{\rm GW*}\left(\frac{f}{f_*}\right)^{n_{\rm GW1}} & \text{for} \quad f < f_*, \\ \Omega_{\rm GW*}\left(\frac{f}{f_*}\right)^{n_{\rm GW2}} & \text{for} \quad f > f_*, \end{cases}$$

Table 1. Cosmological GW sources

source	$n_{\rm GW1}$	$n_{\rm GW2}$	f_* [Hz]	$\Omega_{\rm GW}$
Phase transition (bubble collision)	2.8	-2	$\sim 10^{-5}\left(\frac{f_{\rm PT}}{\beta}\right)\left(\frac{\beta}{H_{\rm PT}}\right)\left(\frac{T_{\rm PT}}{100\ {\rm GeV}}\right)$	$\sim 10^{-5}\left(\frac{H_{\rm PT}}{\beta}\right)^2\left(\frac{\kappa_\phi\alpha}{1+\alpha}\right)^2\left(\frac{0.11v_w^3}{0.42+v_w^2}\right)$
Phase transition (turbulence)	3	$-5/3$	$\sim 3\times10^{-5}\left(\frac{1}{v_w}\right)\left(\frac{\beta}{H_{\rm PT}}\right)\left(\frac{T_{\rm PT}}{100\ {\rm GeV}}\right)$	$\sim 3\times10^{-4}\left(\frac{H_{\rm PT}}{\beta}\right)\left(\frac{\kappa_{\rm turb}\alpha}{1+\alpha}\right)^{3/2} v_w$
Phase transition (sound waves)	3	-4	$\sim 2\times10^{-5}\left(\frac{1}{v_w}\right)\left(\frac{\beta}{H_{\rm PT}}\right)\left(\frac{T_{\rm PT}}{100\ {\rm GeV}}\right)$	$\sim 3\times10^{-6}\left(\frac{H_{\rm PT}}{\beta}\right)\left(\frac{\kappa_v\alpha}{1+\alpha}\right)^2 v_w$
Preheating ($\lambda\phi^4$)	3	cutoff	$\sim 10^7$	$\sim 10^{-11}\left(\frac{g^2/\lambda}{100}\right)^{-0.5}$
Preheating (hybrid)	2	cutoff	$\sim \frac{g}{\sqrt{\lambda}}\lambda^{1/4}10^{10.25}$	$\sim 10^{-5}\left(\frac{\lambda}{g^2}\right)^{1.16}\left(\frac{v}{M_{\rm pl}}\right)^2$
Cosmic strings (loops 1)	$[1, 2]$	$[-1, -0.1]$	$\sim 3\times10^{-8}\left(\frac{G\mu}{10^{-11}}\right)^{-1}$	$\sim 10^{-9}\left(\frac{G\mu}{10^{-12}}\right)\left(\frac{\alpha_{\rm loop}}{10^{-1}}\right)^{-1/2}$ (for $\alpha_{\rm loop} \gg \Gamma G\mu$)
Cosmic strings (loops 2)	$[-1, -0.1]$	0	$\sim 3\times10^{-8}\left(\frac{G\mu}{10^{-11}}\right)^{-1}$	$\sim 10^{-9.5}\left(\frac{G\mu}{10^{-12}}\right)\left(\frac{\alpha_{\rm loop}}{10^{-1}}\right)^{-1/2}$ (for $\alpha_{\rm loop} \gg \Gamma G\mu$)
Cosmic strings (infinite strings)	$[0, 0.2]$	$[0, 0.2]$	—	$\sim 10^{-[11,13]}\left(\frac{G\mu}{10^{-8}}\right)$
Domain walls	3	-1	$\sim 10^{-9}\left(\frac{T_{\rm ann}}{10^{-2}{\rm GeV}}\right)$	$\sim 10^{-17}\left(\frac{\sigma}{1{\rm TeV}^3}\right)^2\left(\frac{T_{\rm ann}}{10^{-2}{\rm GeV}}\right)^{-4}$
Self-ordering scalar fields	0	0	—	$\sim \frac{511}{N}\Omega_{\rm rad}\left(\frac{v}{M_{\rm pl}}\right)^4$
Self-ordering scalar + reheating	0	-2	$\sim 0.4\left(\frac{T_R}{10^7\ {\rm GeV}}\right)$	$\sim \frac{511}{N}\Omega_{\rm rad}\left(\frac{v}{M_{\rm pl}}\right)^4$
Magnetic fields	3	α_B+1	$\sim 10^{-6}\left(\frac{T_*}{10^2{\rm GeV}}\right)$	$\sim 10^{-16}\left(\frac{B}{10^{-10}{\rm G}}\right)$
Inflation+reheating	~ 0	-2	$\sim 0.3\left(\frac{T_R}{10^7\ {\rm GeV}}\right)$	$\sim 2\times10^{-17}\left(\frac{r}{0.01}\right)$
Inflation+kination	~ 0	1	$\sim 0.3\left(\frac{T_R}{10^7\ {\rm GeV}}\right)$	$\sim 2\times10^{-17}\left(\frac{r}{0.01}\right)$
Particle prod. during inf.	-2ϵ	$-4\epsilon(4\pi\xi-6)(\epsilon-\eta)$	—	$\sim 2\times10^{-17}\left(\frac{r}{0.01}\right)$
2nd-order (inflation)	1	drop-off	$\sim 7\times10^5\left(\frac{T_{\rm reh}}{10^9\ {\rm GeV}}\right)^{1/3}\left(\frac{M_{\rm inf}}{10^{16}\ {\rm GeV}}\right)^{2/3}$	$\sim 10^{-12}\left(\frac{T_{\rm reh}}{10^9\ {\rm GeV}}\right)^{-4/3}\left(\frac{M_{\rm inf}}{10^{16}\ {\rm GeV}}\right)^{4/3}$
2nd-order (PBHs)	2	drop-off	$\sim 4\times10^{-2}\left(\frac{M_{\rm PBH}}{10^{20}\ {\rm g}}\right)^{-1/2}$	$\sim 7\times10^{-9}\left(\frac{\mathcal{A}^2}{10^{-3}}\right)^2$
Pre-Big-Bang	3	$3-2\mu$	—	$\sim 1.4\times10^{-6}\left(\frac{H_s}{0.15M_{\rm pl}}\right)^4$

1807.00786

引力波源



EPTA
PPTA
NG
IPTA
SKA
20yr
10yr
5yr
μAres
LISA
Taiji
TianQin
BBO
DECIGO
Ultimate-DECIGO
LIGO
O2
O5
ET
CE
Gμ: 10^−10 10^−13
BOS
VOS
LRS
Domain Wall
σ/GeV^4=10^14,ΔV/GeV^3=10^−7
σ/GeV^4=10^13,ΔV/GeV^3=10^−8
Ω_GW h^2
f(Hz)

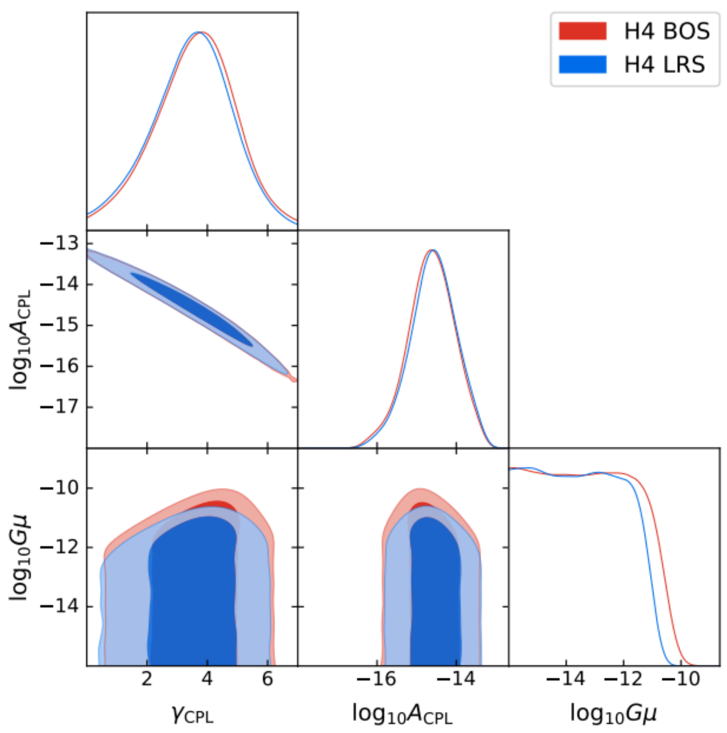
▸ PPTA 数据 & 宇宙弦

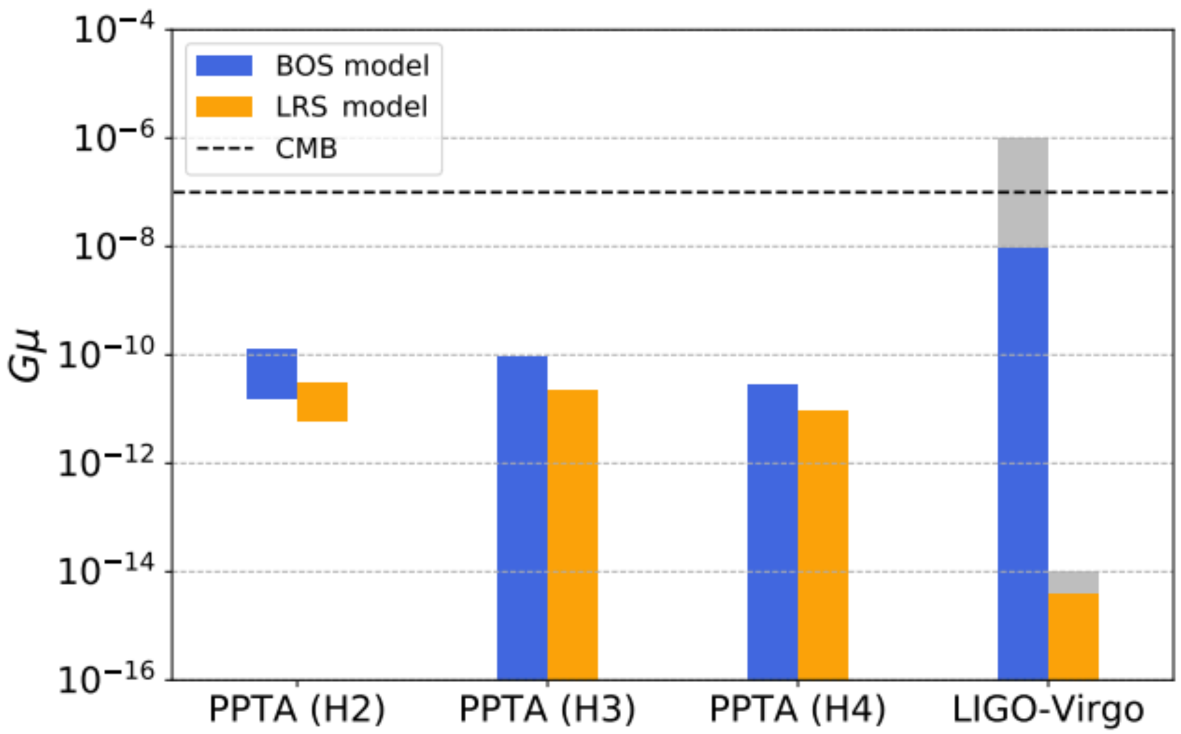
TABLE I: Hypotheses, Bayes factors, and estimated model parameters for the BOS model.

Hypothesis	Pulsar Noise	CPL Process	HD process CS spectrum	Bayes Factors	Parameter Estimation (1σ interval) $\log_{10} G\mu$	Parameter Estimation (1σ interval) $\log_{10} A_{\rm CPL}, \gamma_{\rm CPL}$
H0:Pulsar Noise	✓					
H1:CPL	✓	✓		$10^{3.2}$ (/H0)		$-14.48^{+0.62}_{-0.64}, 3.34^{+1.37}_{-1.53}$
H2:CS	✓		✓(full HD)	$10^{3.1}$ (/H0)	$-10.38^{+0.21}_{-0.21}$	
H3:CS1	✓	✓	✓(full HD)	1.96 (/H1)	< -10.02 (95% C.L.)	$-15.58^{+1.21}_{-1.64}, 3.11^{+1.95}_{-2.02}$
H4:CS2	✓	✓	✓(no-auto HD)	0.60 (/H1)	< -10.54 (95% C.L.)	$-14.61^{+0.58}_{-0.59}, 3.63^{+1.24}_{-1.40}$

TABLE II: Hypotheses, Bayes factors, and estimated model parameters for the LRS model.

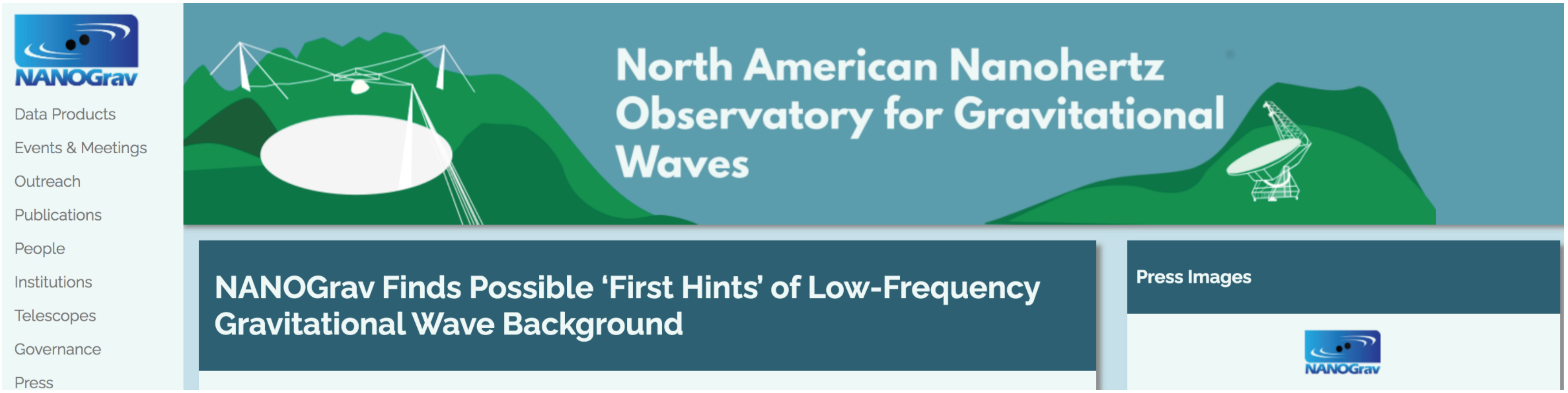
Hypothesis	Pulsar Noise	CPL process	HD process CS spectrum	Bayes Factors	Parameter Estimation (1σ interval) $\log_{10} G\mu$	Parameter Estimation (1σ interval) $\log_{10} A_{\rm CPL}, \gamma_{\rm CPL}$
H0:Pulsar Noise	✓					
H1:CPL	✓	✓		$10^{3.2}$ (/H0)		$-14.48^{+0.62}_{-0.64}, 3.34^{+1.37}_{-1.53}$
H2:CS	✓		✓(full HD)	$10^{3.3}$ (/H0)	$-10.89^{+0.14}_{-0.17}$	
H3:CS1	✓	✓	✓(full HD)	1.62 (/H1)	< -10.64 (95% C.L.)	$-15.44^{+1.18}_{-1.74}, 3.08^{+1.94}_{-1.99}$
H4:CS2	✓	✓	✓(no-auto HD)	0.55 (/H1)	< -11.04 (95% C.L.)	$-14.57^{+0.58}_{-0.59}, 3.54^{+1.24}_{-1.41}$



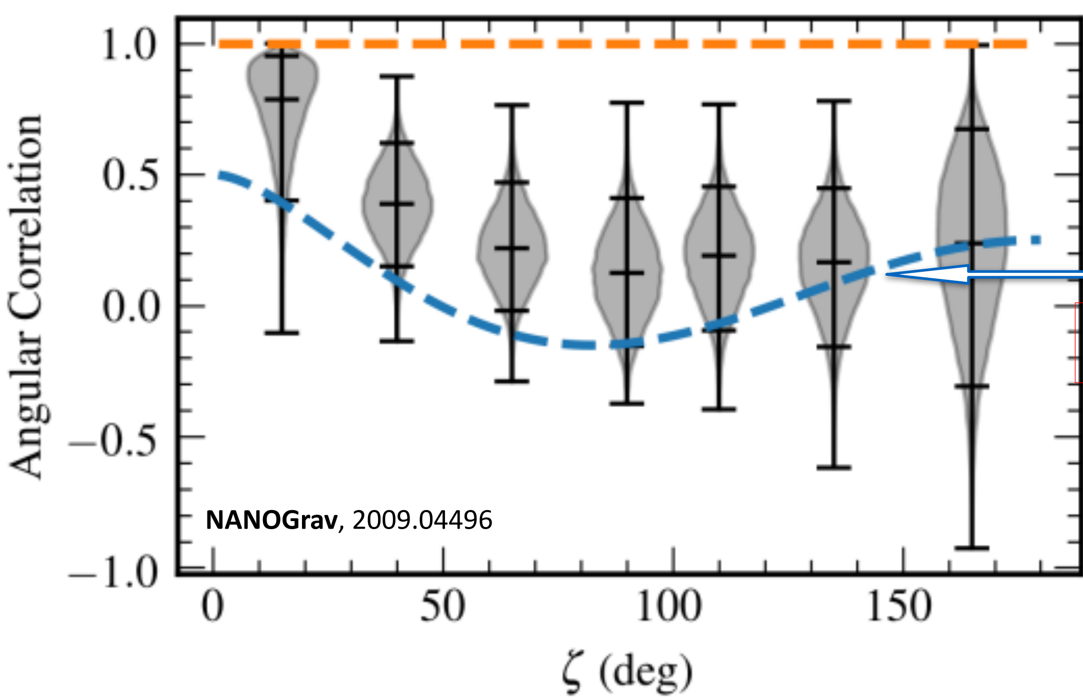


Bian*, Shu*, Wang, Yuan*, Zong, 2205.07293, PRD (Letter)

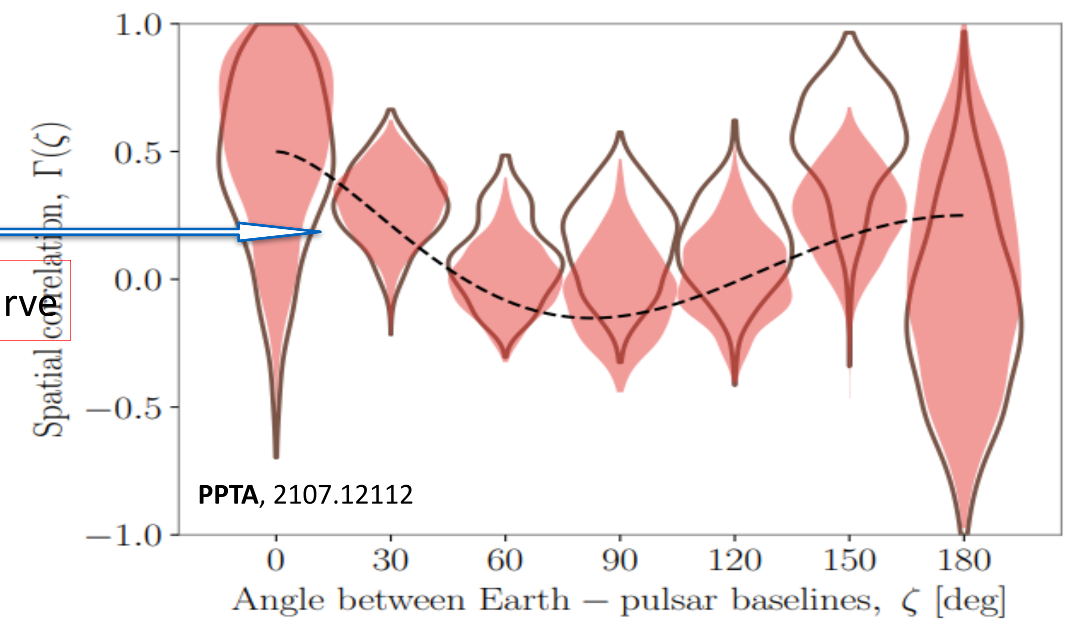
▸ 脉冲星计时阵列实验与随机引力波



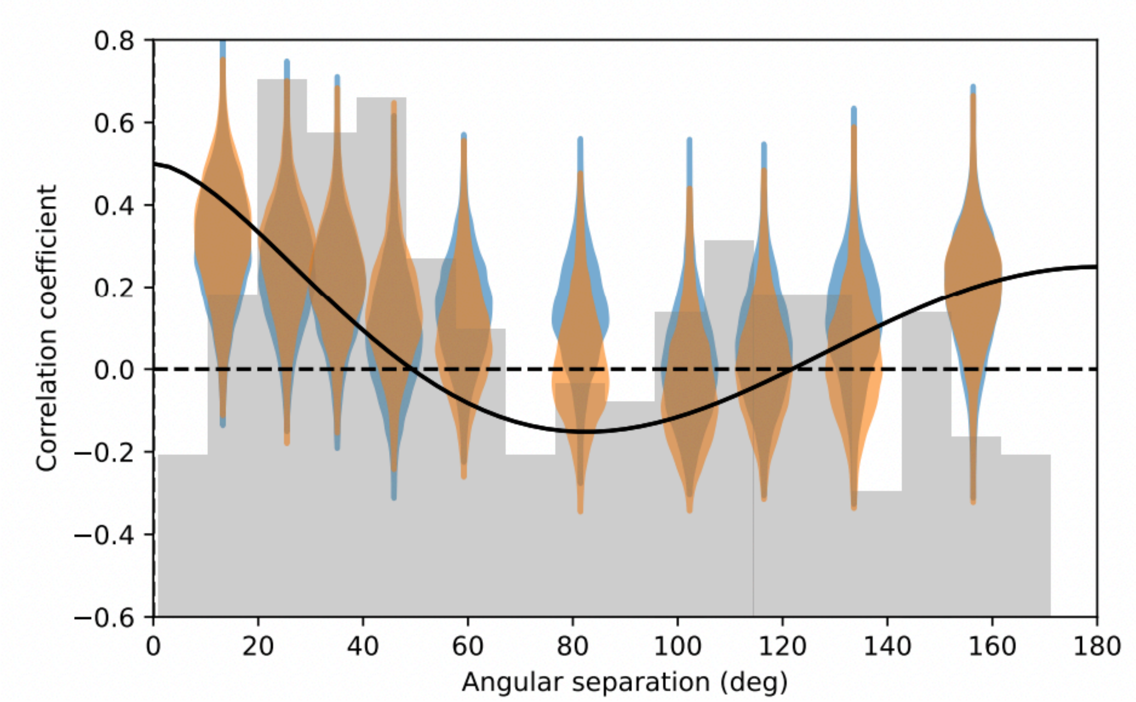
Some CPL signals, SGWB？？？



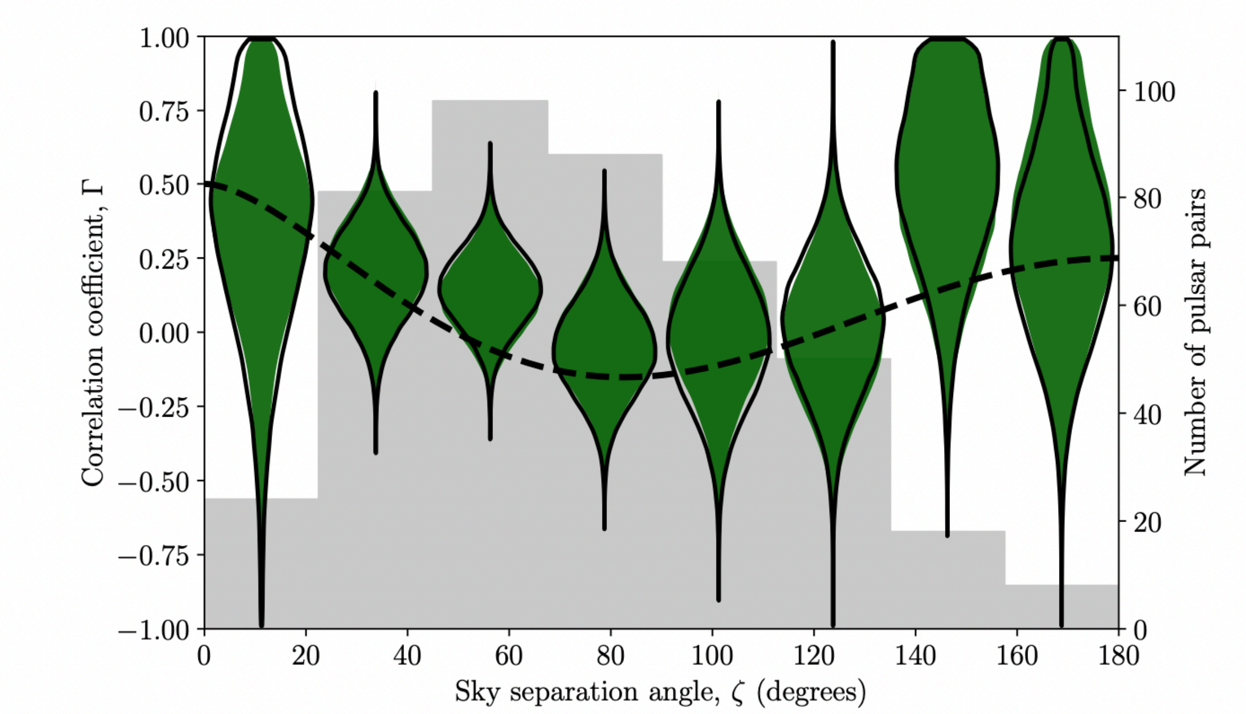
Hellings-Downs Curve



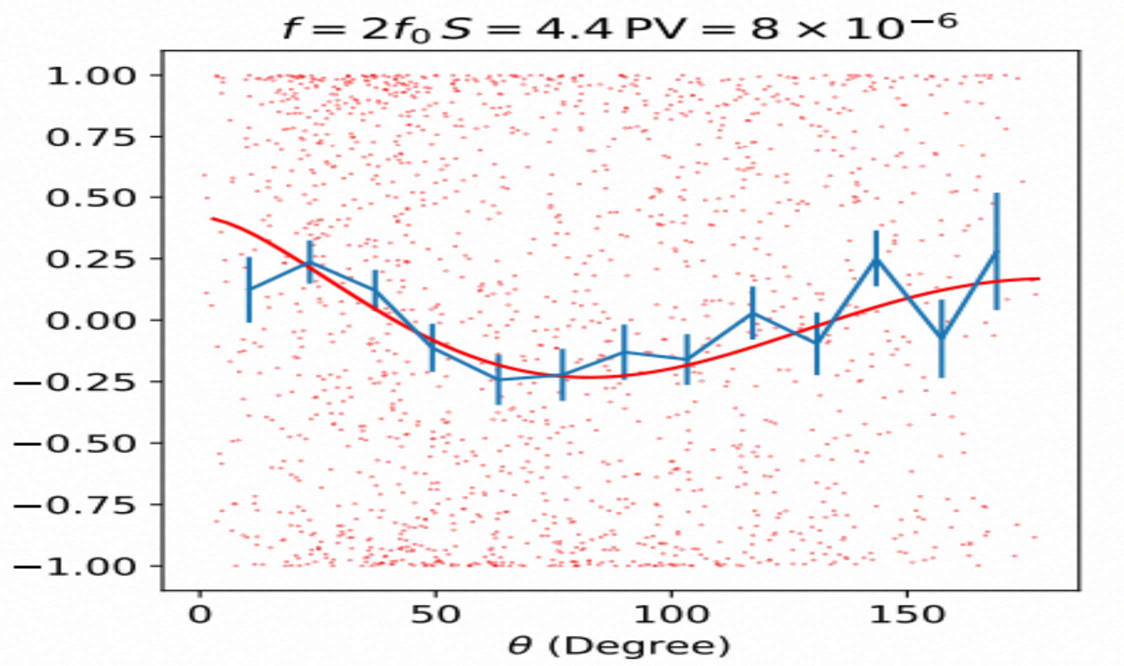
New dataset from PTAs



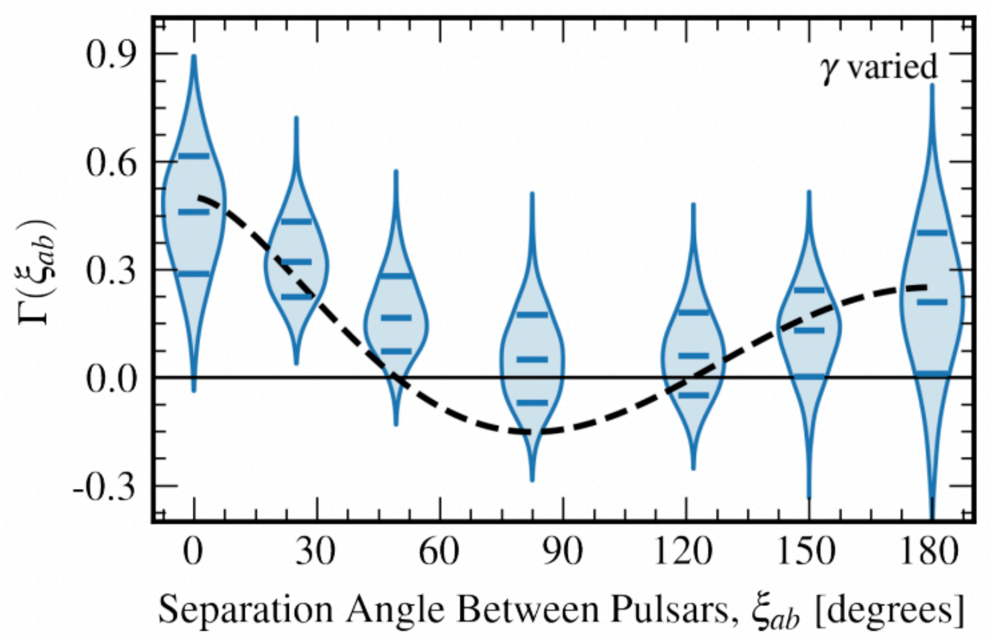
EPTA,2306.16214



PPTA,2306.16215

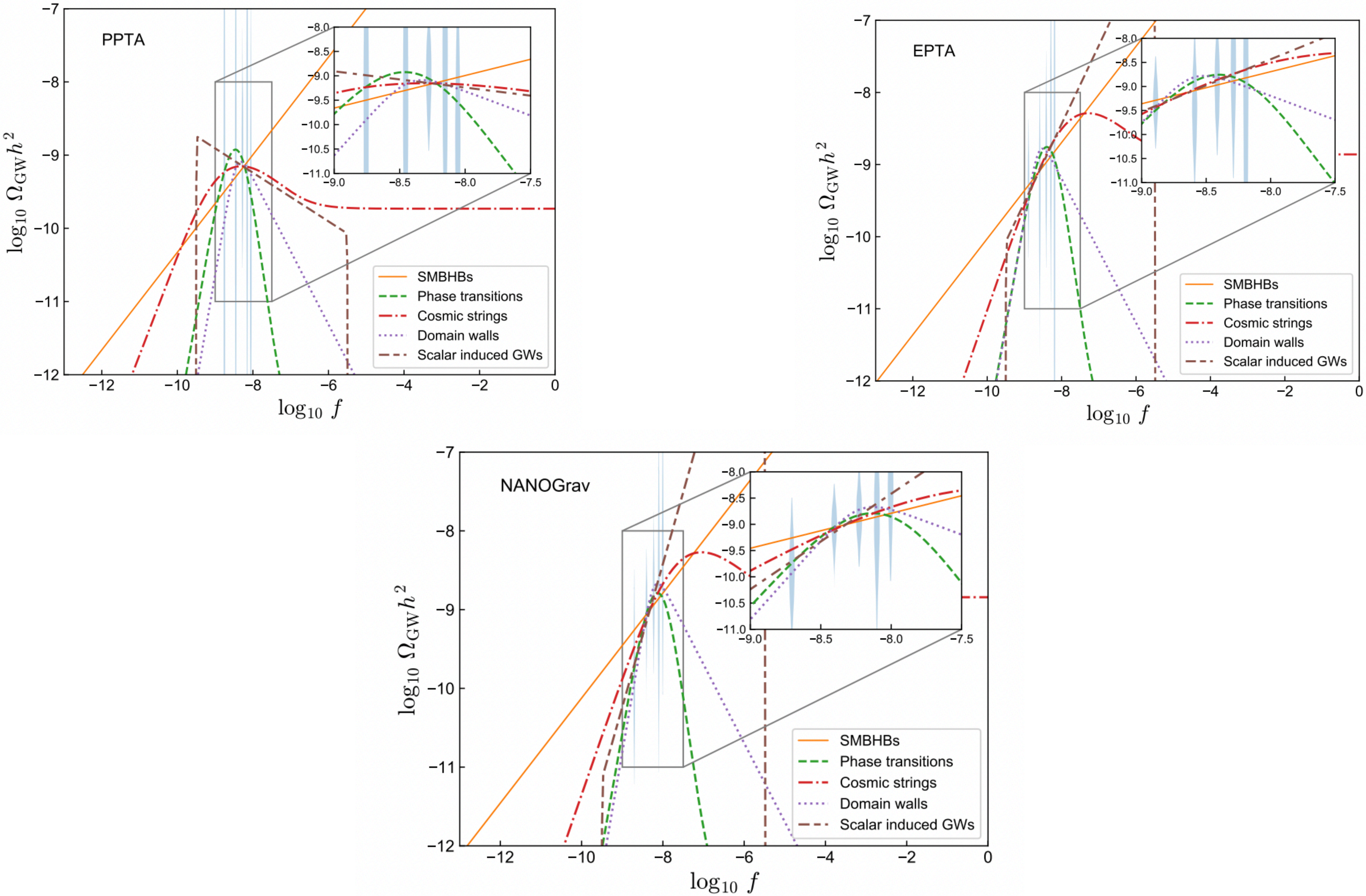


CPTA ,2306.16216



NANOGrav,2306.16213

Gravitational wave sources for Pulsar Timing Arrays



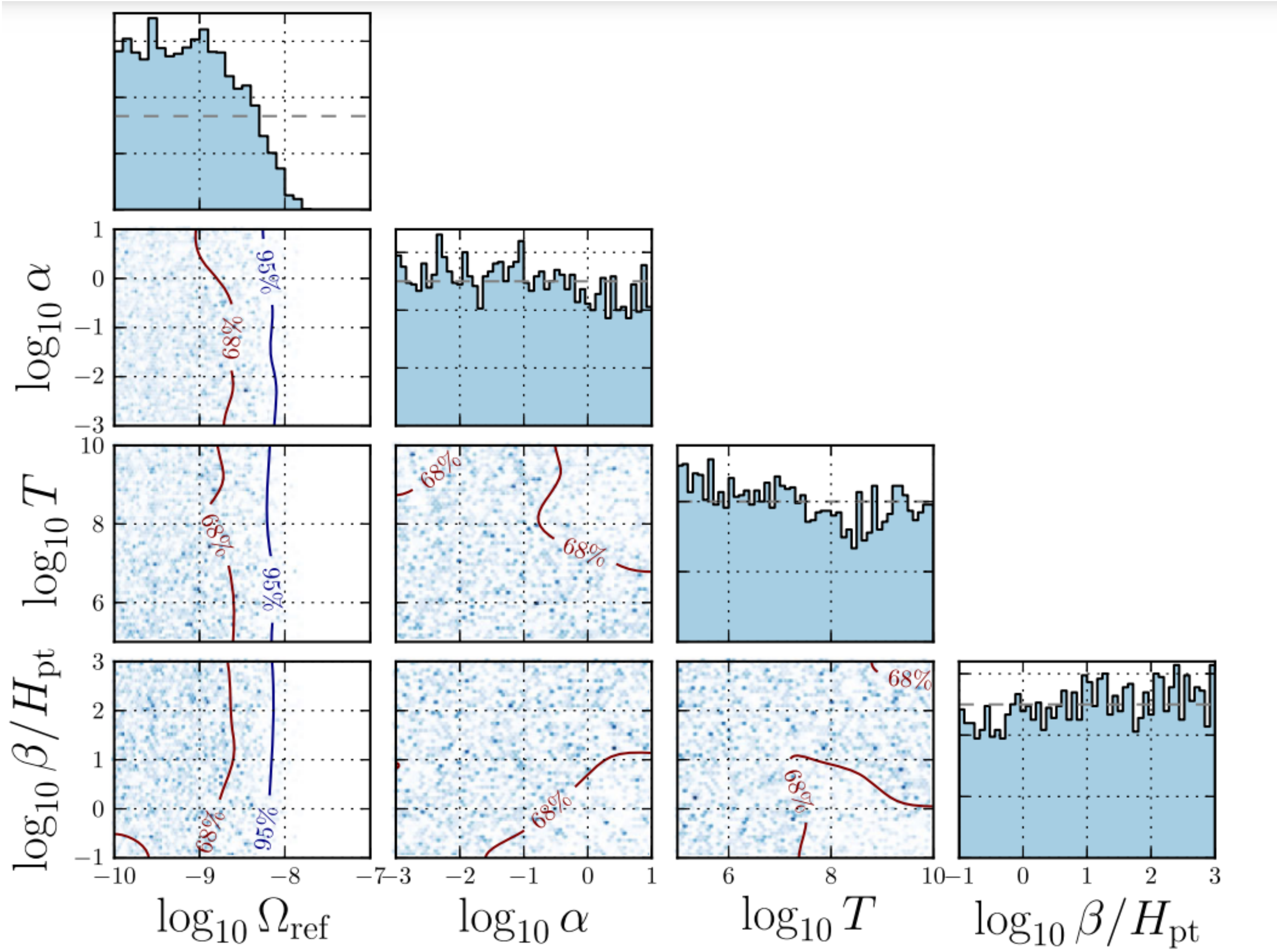
Bian,Ge,Shu,Wang,Yang,Zong,2307.02376

▸ LIGO-Virgo search for FOPT

High-scale PT

Romero, Martinovic,Callister, Guo, et al., Phys.Rev.Lett. 126 (2021) 15, 151301

LIGO-Virgo O3



▸ PPTA search for FOPT

- **PPTA DR2 dataset constrain low-scale phase transition, dark sector and QCD scale FOPT**

PHYSICAL REVIEW LETTERS **127,** 251303 (2021)

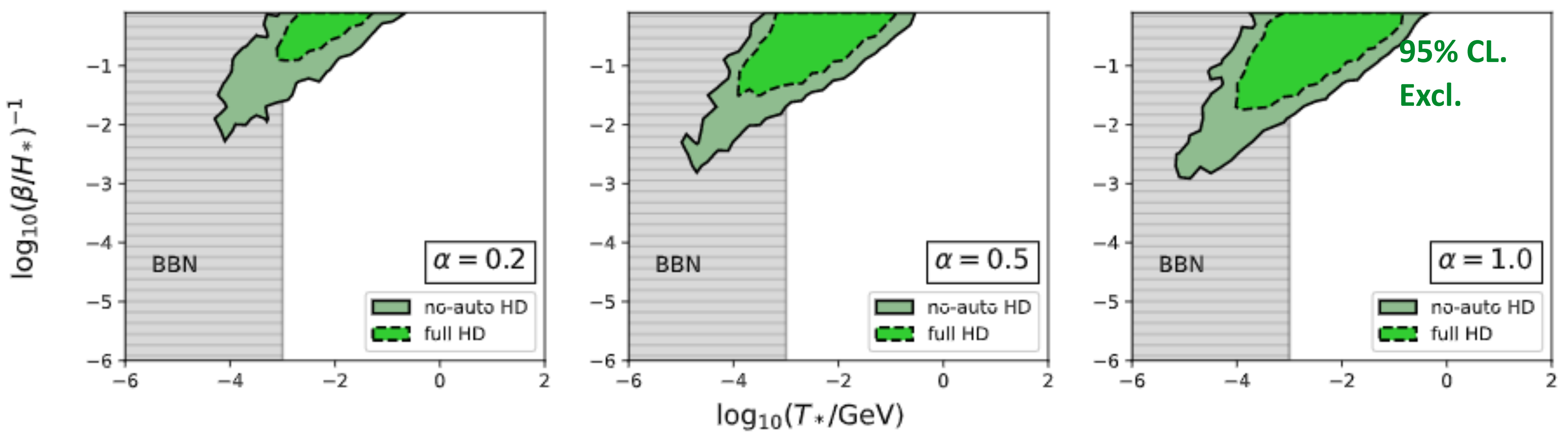
Editors' Suggestion | Featured in Physics

Constraining Cosmological Phase Transitions with the Parkes Pulsar Timing Array

Xiao Xue,[1,2,3] Ligong Bian,[4,5,*] Jing Shu,[1,2,6,7,8,†] Qiang Yuan,[9,10,7,‡] Xingjiang Zhu,[11,12,13,§] N. D. Ramesh Bhat,[14] Shi Dai,[15] Yi Feng,[16] Boris Goncharov,[11,12] George Hobbs,[17] Eric Howard,[17,18] Richard N. Manchester,[17] Christopher J. Russell,[19] Daniel J. Reardon,[12,20] Ryan M. Shannon,[12,20] Renée Spiewak,[21,20] Nithyanandan Thyagarajan,[22] and Jingbo Wang[23]

TABLE I: Description of hypotheses tested in this work and the Bayes factors between them.

Hypothesis	Pulsar noise	Common red process	HD process FOPT spectrum	Bayes Factors	Parameter Estimation (median and 1-σ interval)	
					T_*/MeV, $\alpha \times 10^3$, β/H_*	$A_{\rm comred}$, $\gamma_{\rm comred}$
H0:Pulsar Noise	yes	no	no			
H1:Common Red	yes	yes	no	$10^{3.5}$ (against H0)		$-14.45^{+0.62}_{-0.64}$, $3.31^{+1.36}_{-1.53}$
H2:FOPT	yes	no	yes (full HD)	$10^{1.8}$ (against H0)	$7.4^{+11.9}_{-4.7}$, 271^{+165}_{-92}, $9.9^{+11.4}_{-5.4}$	
H3:FOPT1	yes	yes	yes (full HD)	1.04 (against H1)	$9.6^{+232.2}_{-9.2}$, $3.8^{+27.9}_{-3.4}$, 854^{+9622}_{-782}	$-14.51^{+0.64}_{-0.68}$, $3.36^{+1.39}_{-1.54}$
H4:FOPT2	yes	yes	yes (no-auto HD)	0.96 (against H1)	$10.9^{+290.5}_{-10.6}$, $3.2^{+19.9}_{-2.8}$, 1053^{+11256}_{-962}	$-14.45^{+0.62}_{-0.64}$, $3.27^{+1.37}_{-1.54}$



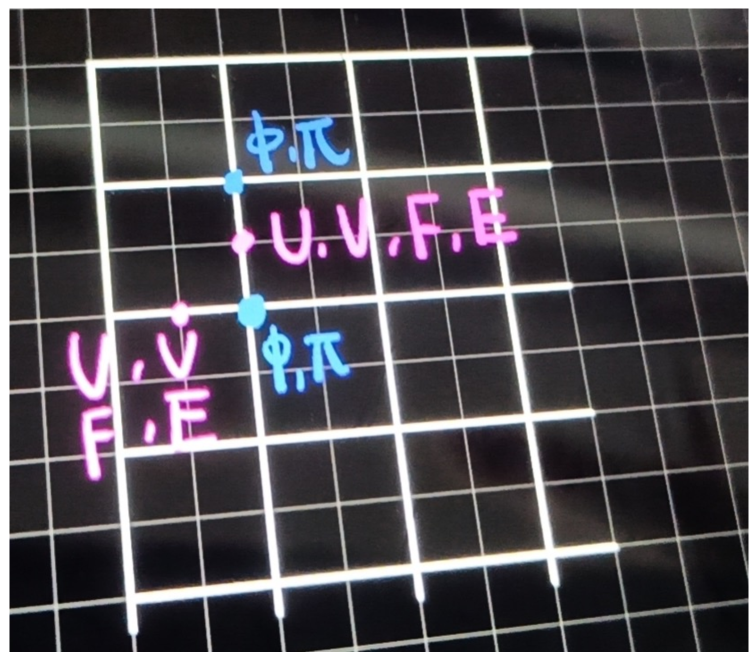
格点电弱理论

$\Phi(t, x)$： Higgs field doublet defined on sites;

$U_i(t, x)$ and $V_i(t, x)$： SU(2) and U(1) link fields, defined on the link between the neighboring sites x and x + i， $\Phi(t, x)$, $U_i(t, x)$ and $V_i(t, x)$ are defined at time steps t + Δt, t + 2Δt, . . .;

Conjugate momentum fields： $\Pi(t+\Delta t/2, x)$, $F(t+\Delta t/2, x)$ and $E(t+\Delta t/2, x)$, are defined at time steps t + Δt/2, t + 3Δt/2.

$$U_i(t,x) = \exp\left(-\frac{i}{2} g \Delta x \sigma^a W_i^a\right)$$
$$U_0(t,x) = \exp\left(-\frac{i}{2} g \Delta t \sigma^a W_0^a\right)$$
$$V_i(t,x) = \exp\left(-\frac{i}{2} g \Delta x B_i\right)$$
$$V_0(t,x) = \exp\left(-\frac{i}{2} g \Delta t B_0\right).$$



$$D_i\Phi = \frac{1}{\Delta x}\left[U_i(t,x)V_i(t,x)\Phi(t,x+i) - \Phi(t,x)\right]$$
$$D_0\Phi = \frac{1}{\Delta t}\left[U_0(t,x)V_0(t,x)\Phi(t+\Delta t,x) - \Phi(t,x)\right].$$

$$\Phi(t+\Delta t,x) = \Phi(t,x) + \Delta t \Pi(t+\Delta t/2,x)$$
$$V_i(t+\Delta t,x) = \frac{1}{2} g' \Delta x \Delta t E_i(t+\Delta t/2,x) V_i(t,x)$$
$$U_i(t+\Delta t,x) = g \Delta x \Delta t F_i(t+\Delta t/2,x) U_i(t,x),$$

Temporal gauge
$U_0(t, x) = I_2$, $V_0(t, x) = 1$

leapfrog

Di, Wang, Zhou, **Bian***, Cai*,Liu*, Phys.Rev.Lett. 126 (2021) 251102

一阶电弱相变模拟

Field basis equation of motion

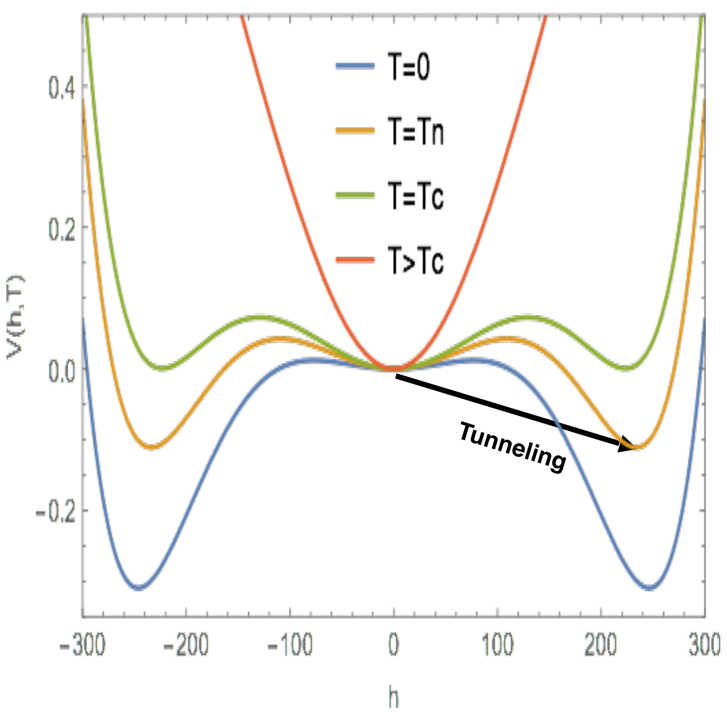
$$
\begin{aligned}
\partial_0^2\Phi =& D_iD_i\Phi - \frac{dV(\Phi)}{d\Phi},\\
\partial_0^2 B_i =& -\partial_j B_{ij} + g'\,\mathrm{Im}[\Phi^\dagger D_i\Phi],\\
\partial_0^2 W_i^a =& -\partial_k W_{ik}^a - g\,\epsilon^{abc}W_k^bW_{ik}^c + g\,\mathrm{Im}[\Phi^\dagger\sigma^a D_i\Phi]\,,
\end{aligned}
$$

$$
\begin{aligned}
&\partial_0\partial_j B_j - g'\,\mathrm{Im}[\Phi^\dagger\partial_0\Phi] = 0,\\
&\partial_0\partial_j W_j^a + g\,\epsilon^{abc}W_j^b\partial_0 W_j^c - g\,\mathrm{Im}[\Phi^\dagger\sigma^a\partial_0\Phi] = 0.
\end{aligned}
$$

Lattice implementation

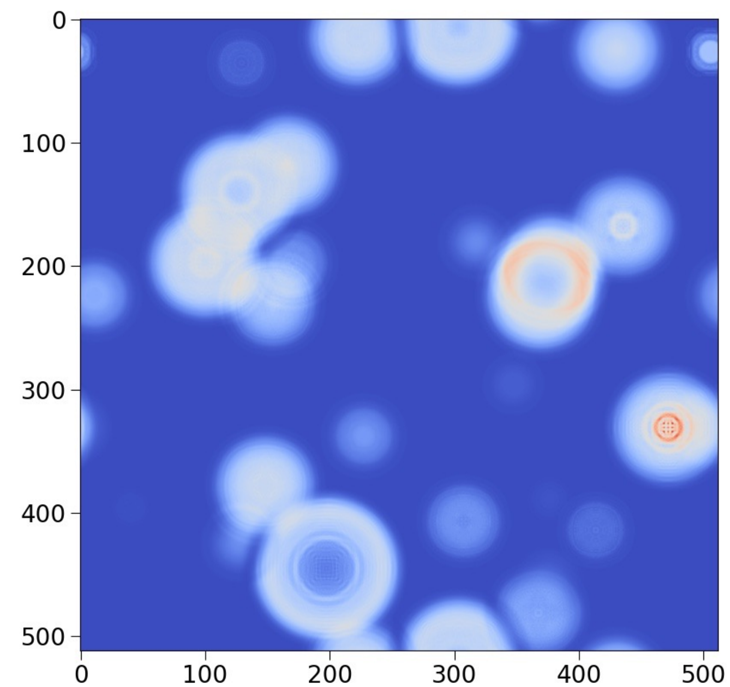
$$
\begin{aligned}
\Pi(t+\Delta t/2,x) =& \Pi(t-\Delta t/2,x) + \Delta t\Big\{\frac{1}{\Delta x^2}\sum_i[U_i(t,x)V_i(t,x)\Phi(t,x+i)\\
&-2\Phi(t,x) + U_i^\dagger(t,x-i)V_i^\dagger(t,x-i)\Phi(t,x-i)] - \frac{\partial U}{\partial\Phi^\dagger}\Big\}\\
\mathrm{Im}[E_k(t+\Delta t/2,x)] =& \mathrm{Im}[E_k(t-\Delta t/2,x)] + \Delta t\Big\{\frac{g'}{\Delta x}\mathrm{Im}[\Phi^\dagger(t,x+k)U_k^\dagger(t,x)V_k^\dagger(t,x)\Phi(t,x)]\\
&-\frac{2}{g'\Delta x^3}\sum_i \mathrm{Im}[V_k(t,x)V_i(t,x+k)V_k^\dagger(t,x+i)V_i^\dagger(t,x)\\
&+V_i(t,x-i)V_k(t,x)V_i^\dagger(t,x+k-i)V_k^\dagger(t,x-i)]\Big\}\\
\mathrm{Tr}[i\sigma^m F_k(t+\Delta t/2,x)] =& \mathrm{Tr}[i\sigma^m F_k(t-\Delta t/2,x)] + \Delta t\Big\{\frac{g}{\Delta x}\mathrm{Re}[\Phi^\dagger(t,x+k)U_k^\dagger(t,x)V_k^\dagger(t,x)i\sigma^m\Phi(t,x)]\\
&-\frac{1}{g\Delta x^3}\sum_i \mathrm{Tr}[i\sigma^m U_k(t,x)U_i(t,x+k)U_k^\dagger(t,x+i)U_i^\dagger(t,x)\\
&+i\sigma^m U_k(t,x)U_i^\dagger(t,x+k-i)U_k^\dagger(t,x-i)U_i(t,x-i)]\Big\},
\end{aligned}
$$

Finite-T Veff

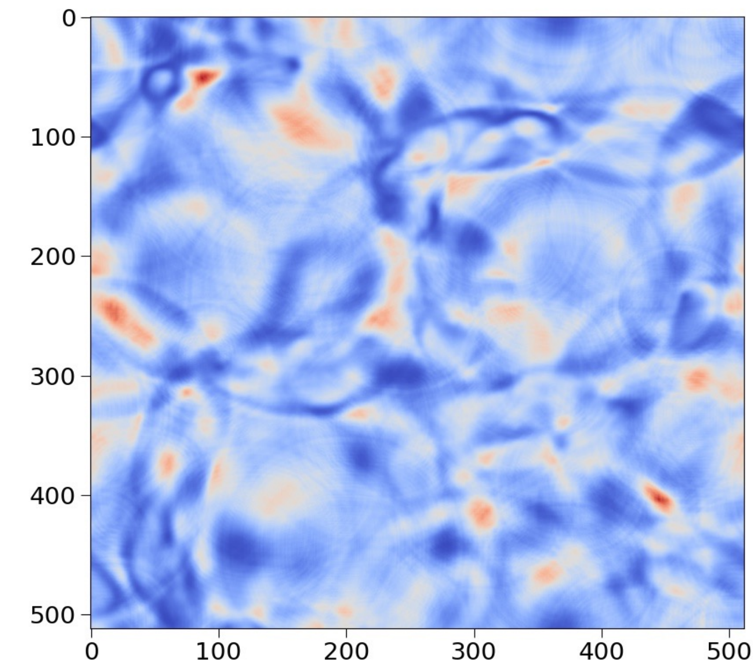


PT temperature（T_*）
duration（β）
strength（α）

Nucleation



Expansion&Percolation



Finite-T calculation

Lattice Simulation

Di, Wang, Zhou, **Bian***, Cai*,Liu*, Phys.Rev.Lett. 126 (2021) 251102

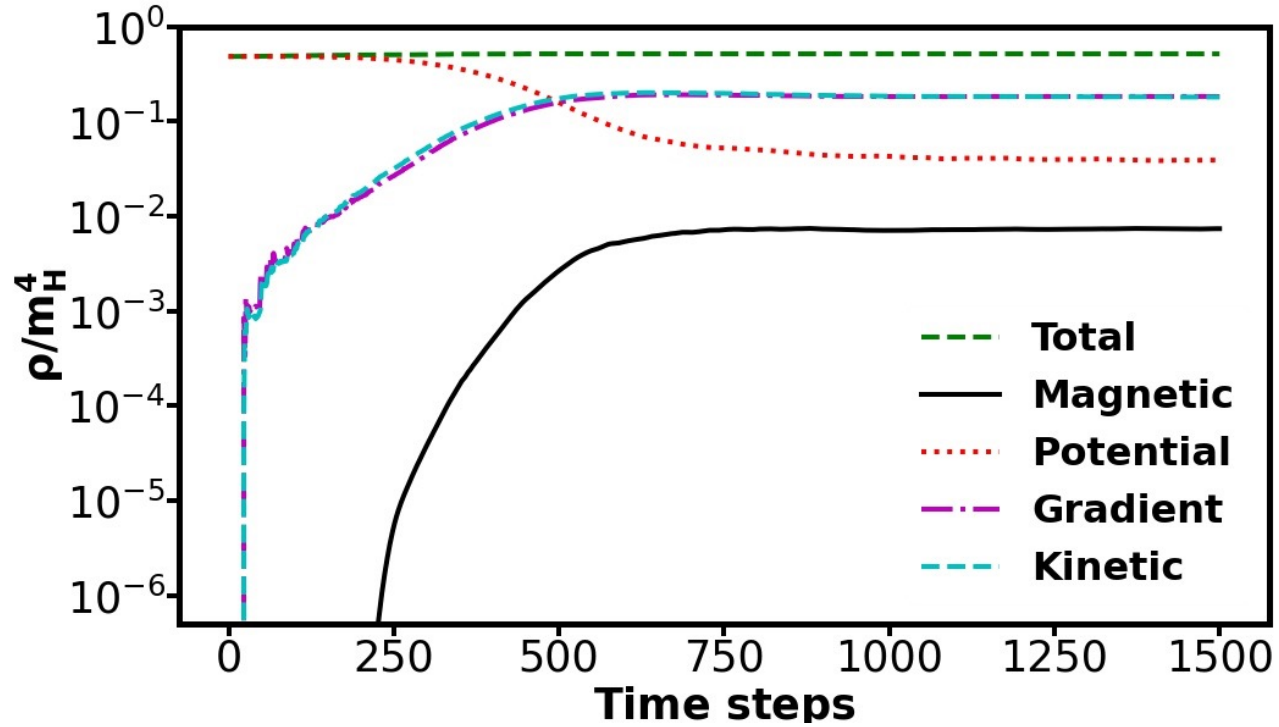
一阶电弱相变真空泡碰撞、合并产生引力波

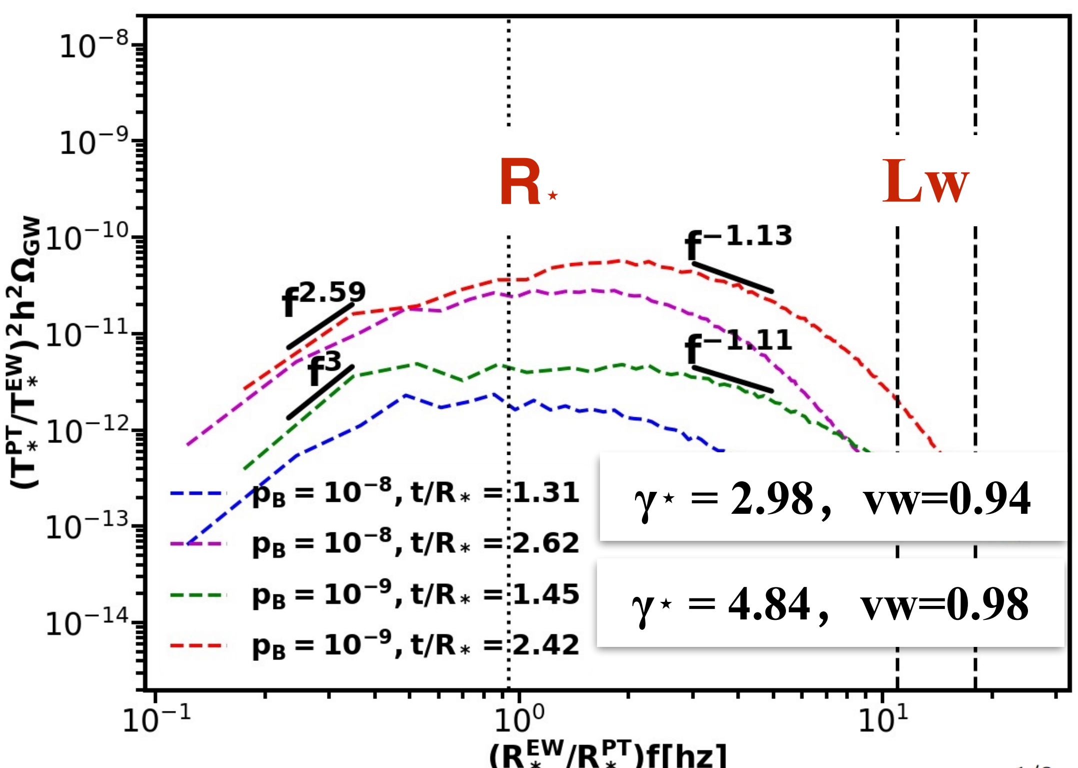
$$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{\mathrm{TT}}$$

$$T_{\mu\nu} = \partial_\mu \Phi^\dagger \partial_\nu \Phi - g_{\mu\nu} \frac{1}{2} \mathrm{Re}[(\partial_i \Phi^\dagger \partial^i \Phi)^2]$$

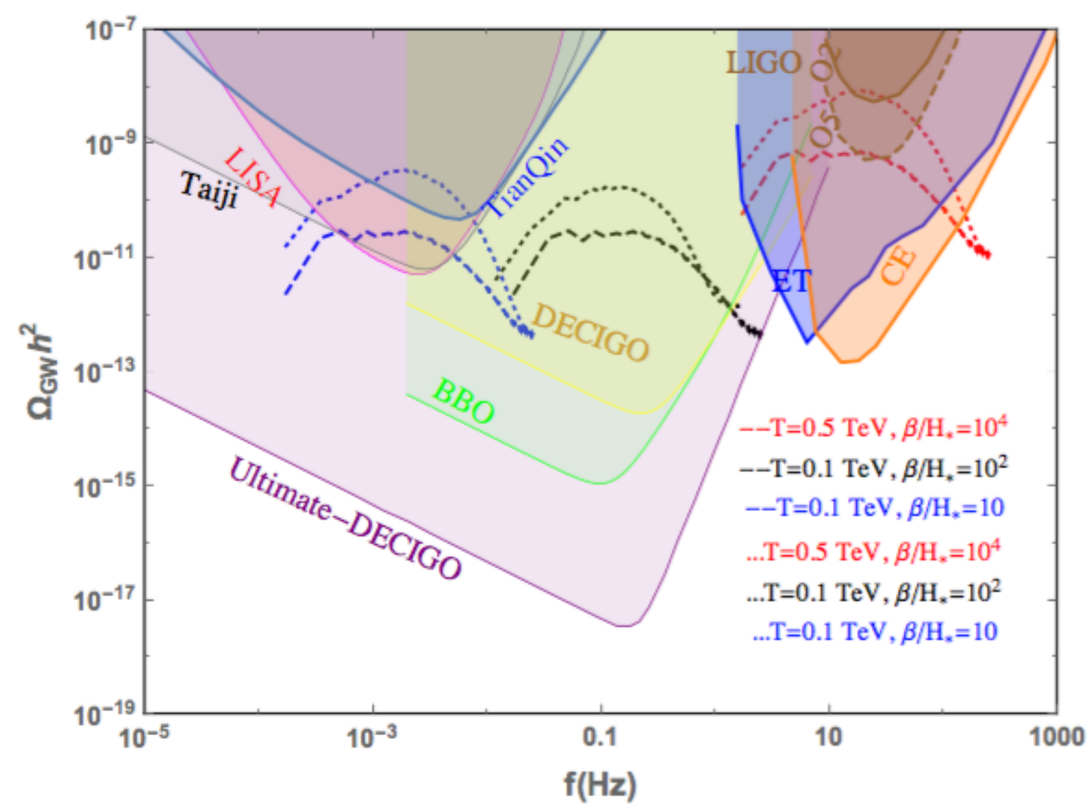
$$\langle \dot{h}_{ij}^{TT}(\mathbf{k}, t) \dot{h}_{ij}^{TT}(\mathbf{k}', t) \rangle = P_{\dot{h}}(\mathbf{k}, t)(2\pi)^3 \delta(\mathbf{k} + \mathbf{k}')$$

$$\frac{d\Omega_{\mathrm{gw}}}{d\ln(k)} = \frac{1}{32\pi G \rho_c} \frac{k^3}{2\pi^2} P_{\dot{h}}(\mathbf{k}, t)$$





$\gamma_\star = R_\star/(2Rc)$ $\quad R_* = \left(\frac{\mathcal{V}}{N_b}\right)^{1/3}$



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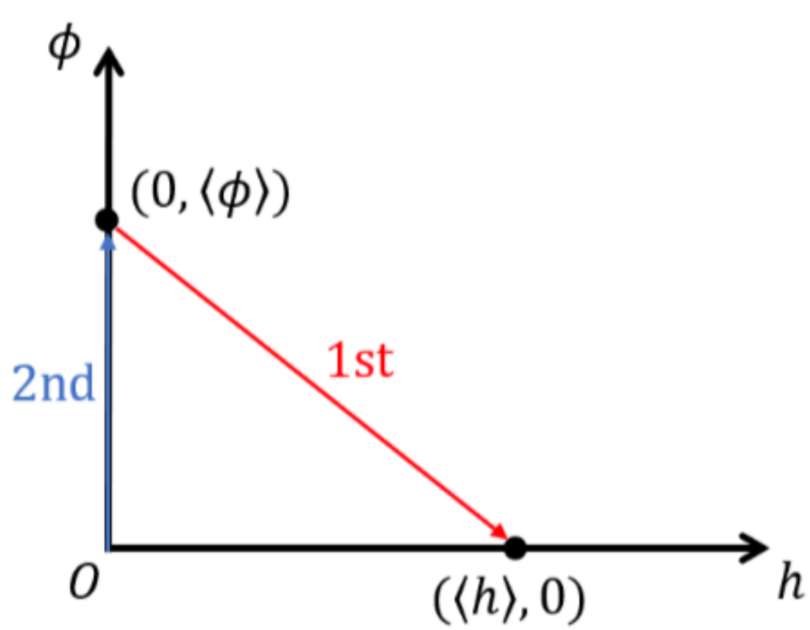
▸ Two-step FOPT potential

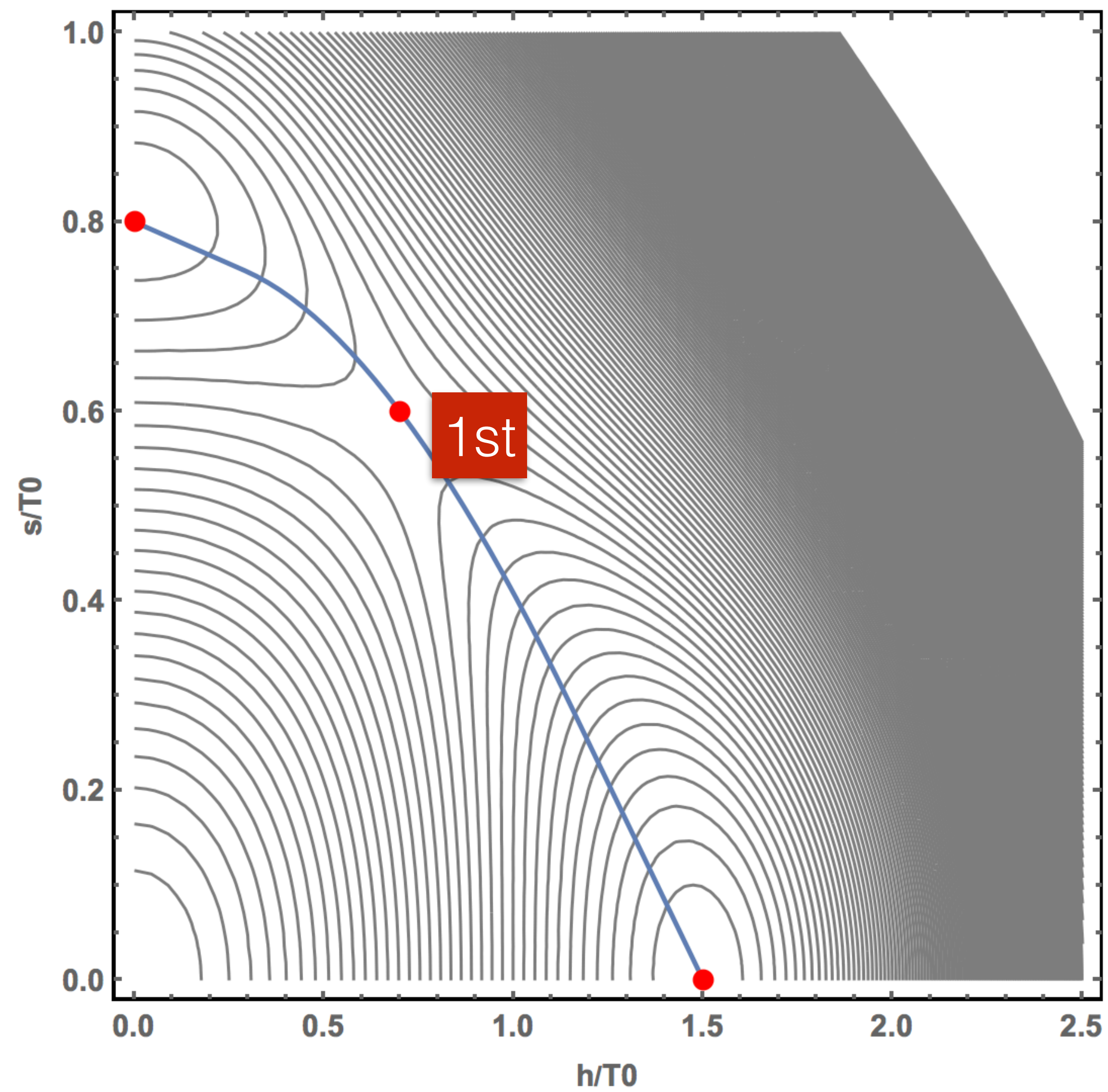
Type-a



$$V_a(\phi,h,T)=\frac{1}{2}(\mu_\phi^2+c_\phi T^2)\phi^2+\frac{1}{2}\lambda_{h\phi}h^2\phi^2+\frac{1}{4}\lambda_\phi\phi^4$$
$$+\frac{1}{2}(-\mu_h^2+c_h T^2)h^2+\frac{1}{4}\lambda_h h^4$$

$c_\phi=\lambda_\phi/4+\lambda_{h\phi}/3$

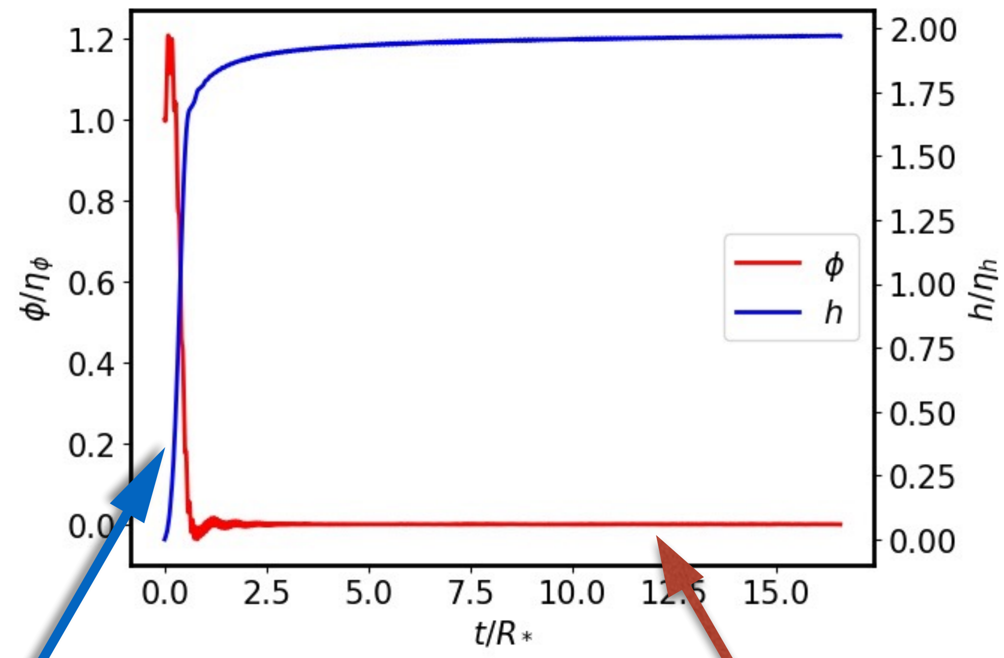
$c_h=(2m_W^2+m_Z^2+2m_t^2)/(4v^2)+\lambda_h/2+\lambda_{h\phi}/12$



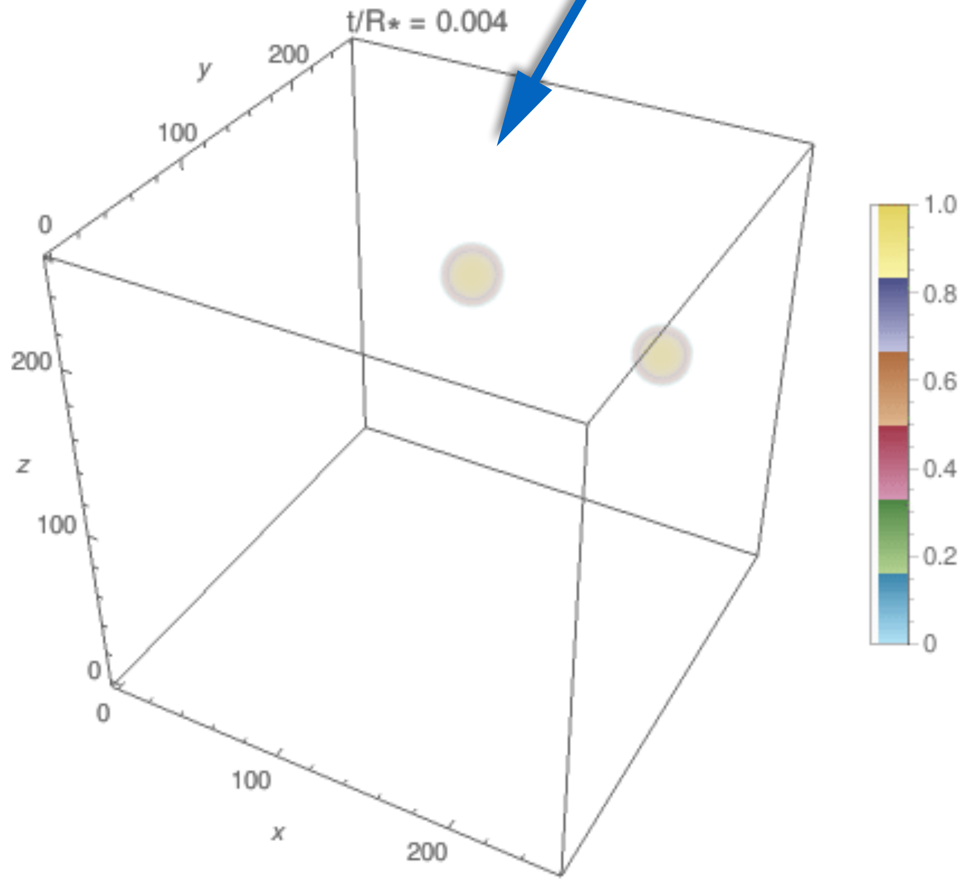
Motivated for DM&EWBG, see:1804.06813,1702.06124,1609.07143, 1605.08663, 1605.08663,etc

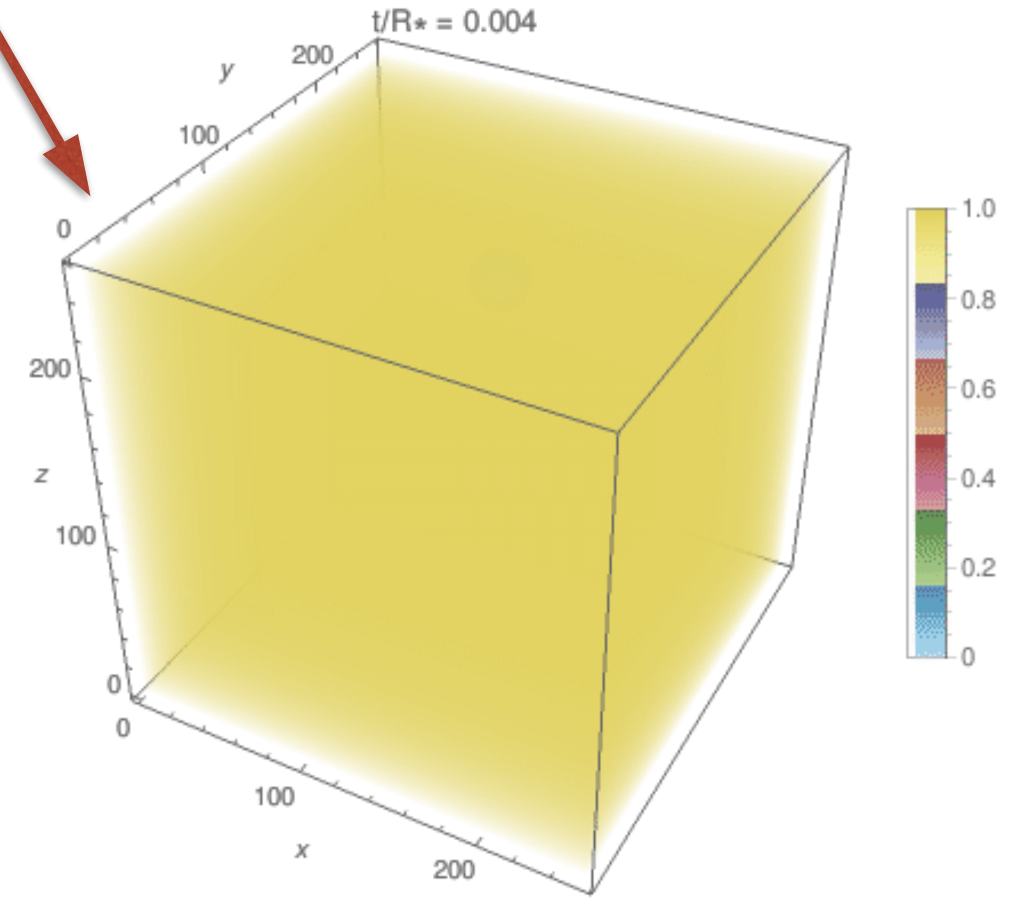
▸ Two-step PT with the second-step being FOPT

Type-a



$$h(t=0,r) = \eta_h/2\left[1-\tanh\left(\frac{r-R_0}{L_w}\right)\right]$$
$$\phi(t=0,r) = \eta_\phi/2\left[1+\tanh\left(\frac{r-R_0}{L_w}\right)\right]$$

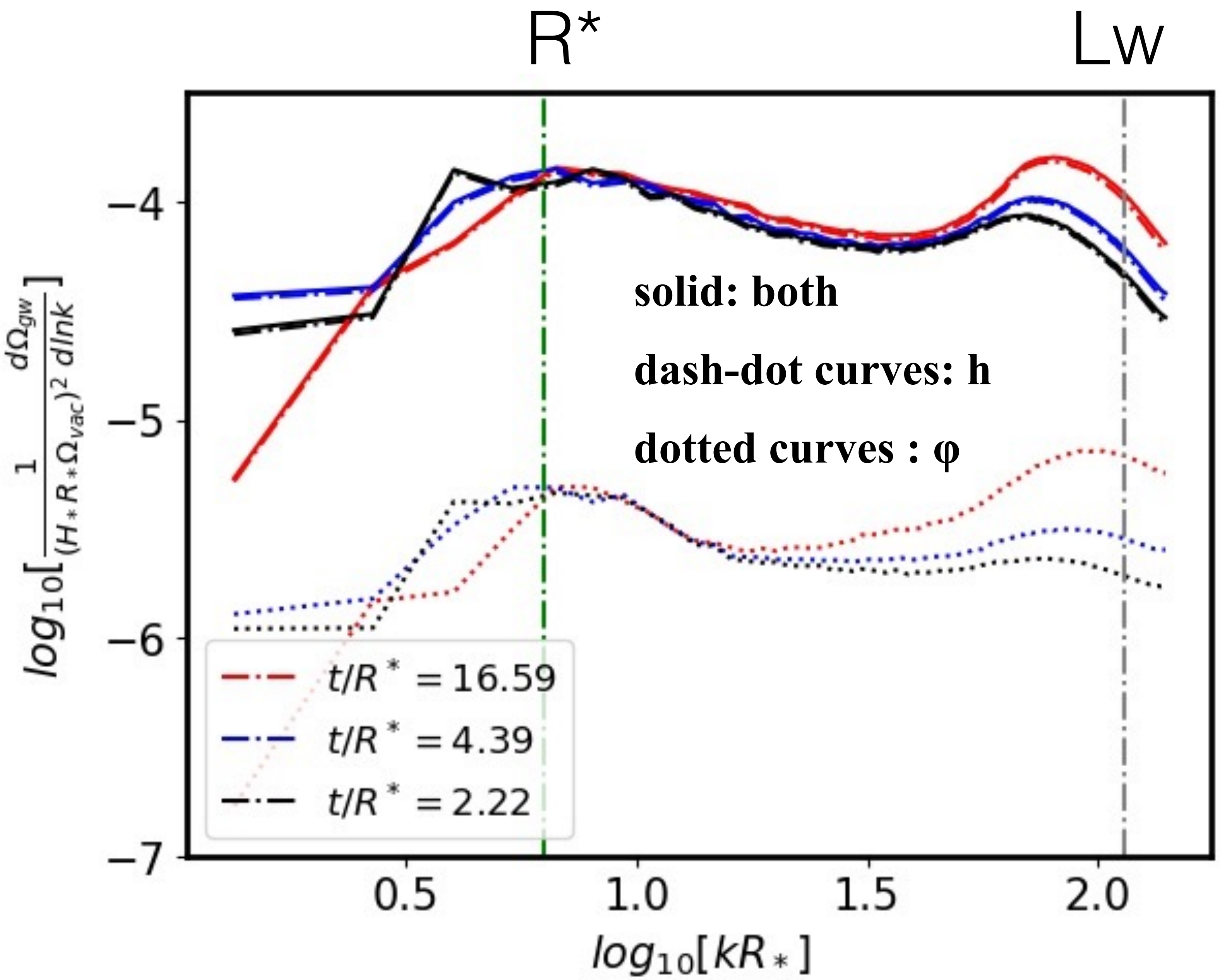




Zhao, Di, **Bian**, Cai,2204.04427

▸ Two-step PT with the second-step being FOPT

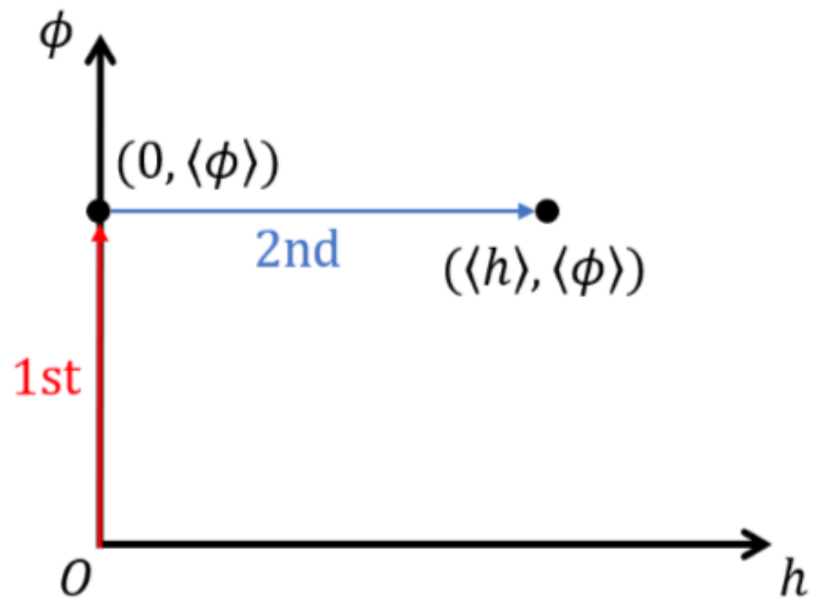
Type-a



Zhao, Di, **Bian**, Cai,2204.04427

▸ Two-step PT with first-step being FOPT

Type-b Without Global U(1)



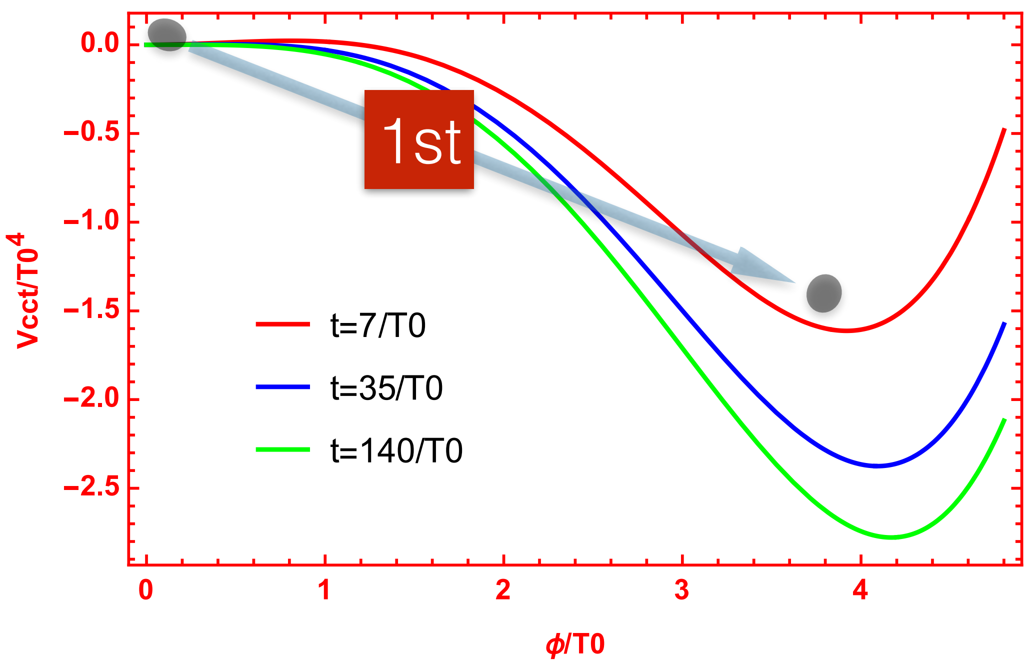
$$V_{cct}(\phi, T) = a\phi^4(\log[|\phi|^2/v_\phi^2] - 1/4) + bT^2|\phi|^2$$

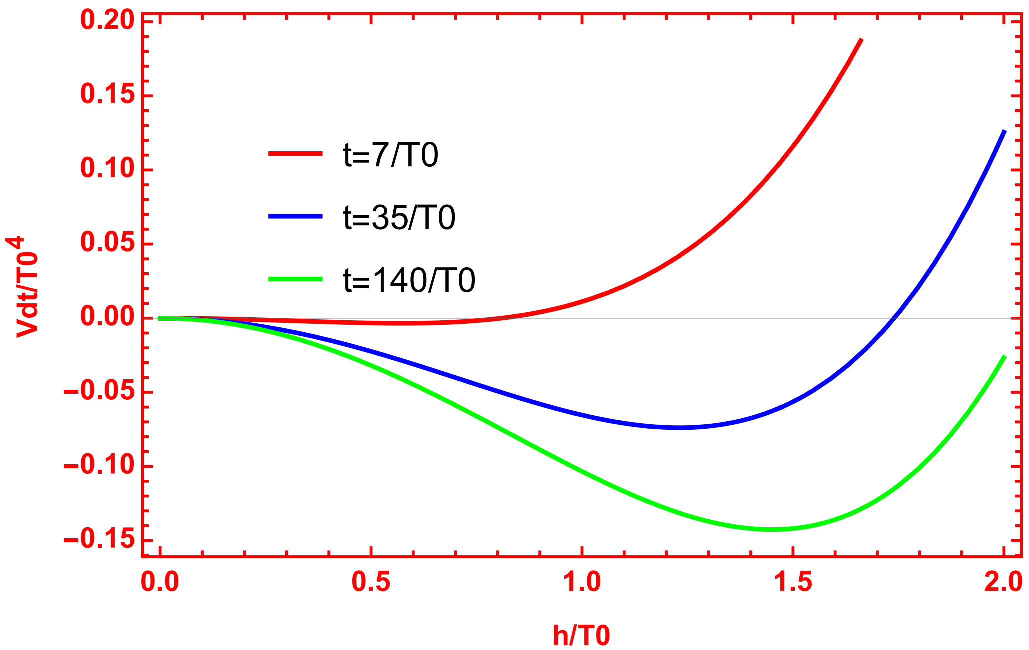
$$V_{dt}(\phi, h, T) = \frac{1}{2}c_h' T^2 h^2 + \frac{1}{4}\lambda_h h^4 - \frac{\lambda_p}{4} h^2\phi^2$$

$$c_h' = (2m_W^2 + m_Z^2 + 2m_t^2)/(4v^2) + \lambda_h/2 + \lambda_p/24$$

$$\langle h\rangle = \sqrt{(\lambda_p \eta^2 - 2c_h' T^2)/(2\lambda_h)}$$

Classical conformal + Dimensional transmutation



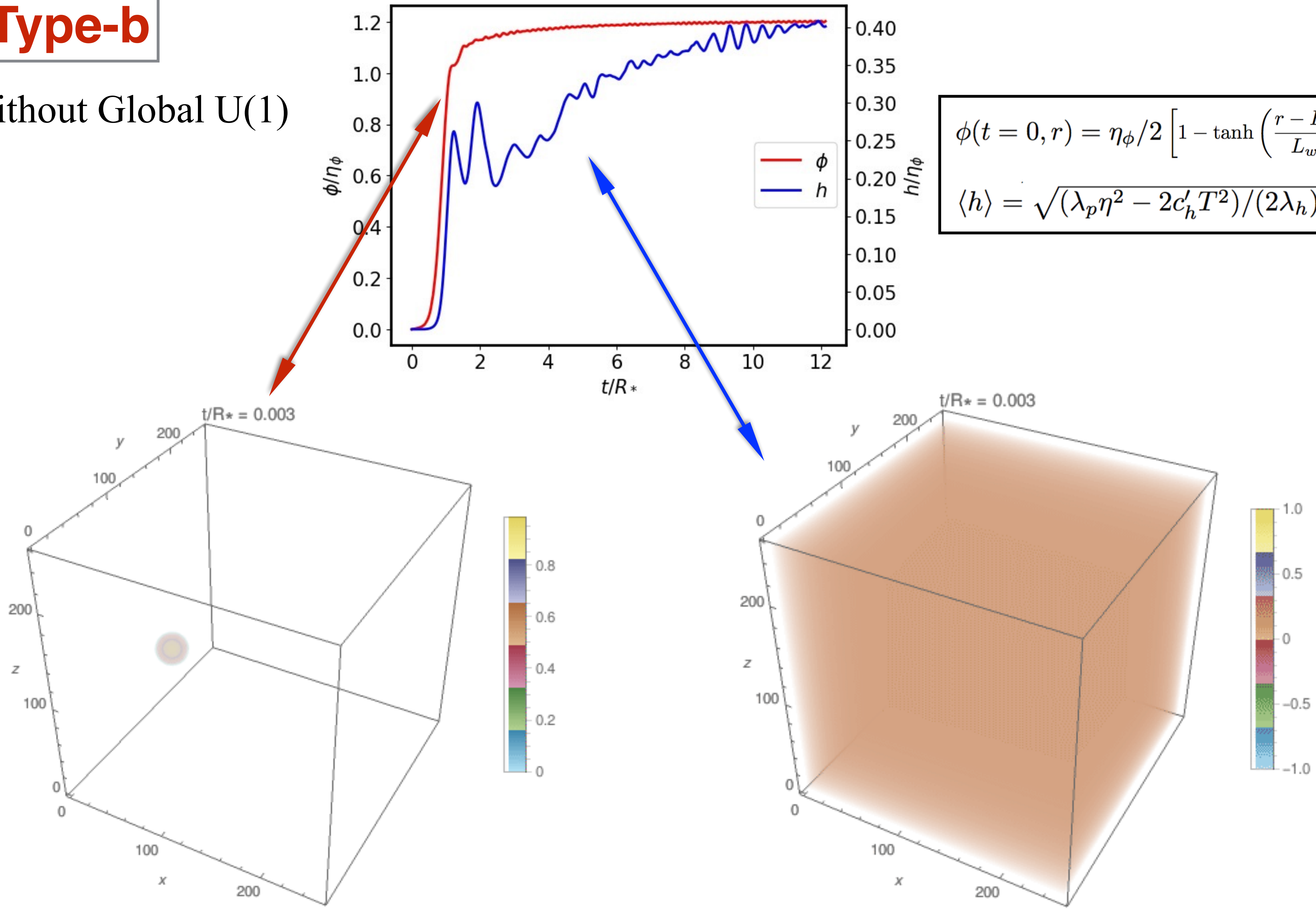


Zhao, Di, **Bian**, Cai,2204.04427

▸ Two-step PT with first-step being FOPT

Type-b

Without Global U(1)



$$\phi(t=0,r) = \eta_\phi/2\left[1-\tanh\left(\frac{r-R_0}{L_w}\right)\right]$$

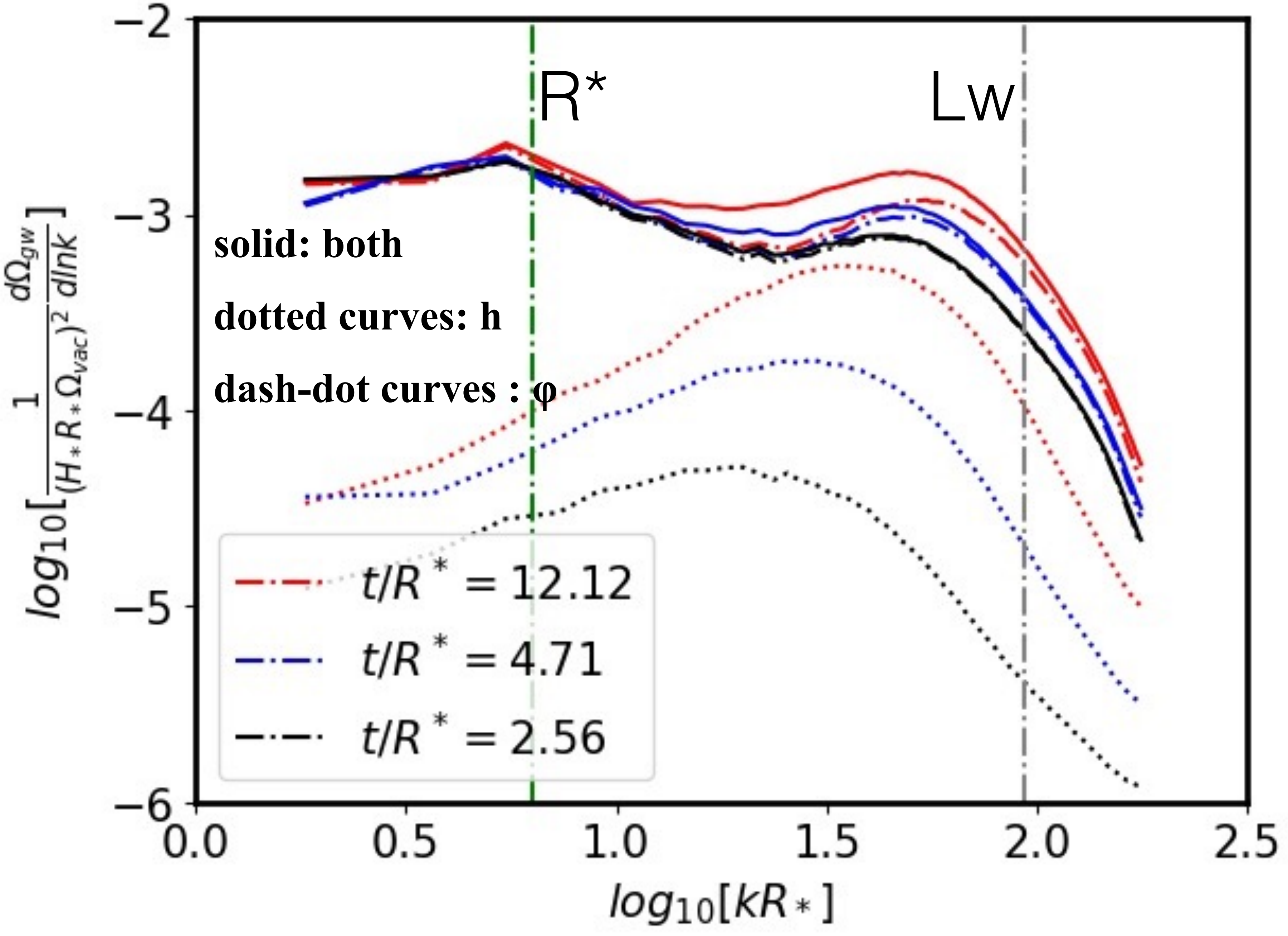
$$\langle h\rangle = \sqrt{(\lambda_p\eta^2 - 2c_h'T^2)/(2\lambda_h)}$$

Zhao, Di, **Bian**, Cai,2204.04427

▸ Two-step PT with first-step being FOPT

Type-b

Without Global U(1)



Zhao, Di, **Bian**, Cai,2204.04427

scalar field + fluid system

The full energy-momentum tensor into two components, one for the fluid $T^{\mu\nu}$ and one for the Higgs $T^{\mu\nu}$

$$T_{\mathrm{f}}^{\mu\nu} = (e+p_1)u^\mu u^\nu + p_1 g^{\mu\nu},$$

$$T_{\phi}^{\mu\nu} = \partial^\mu\phi\,\partial^\nu\phi - g^{\mu\nu}\left(\frac{1}{2}(\partial\phi)^2 + V_0(\phi)\right),$$

Fluid pressure is the total contribution from all particles

$$p_1(\phi,T) = \frac{\pi^2}{90} g_{\mathrm{eff}} T^4 - V_1(\phi,T) = -\sum_B f_B(m(\phi),T) - \sum_F f_F(m(\phi),T)$$

Thermally corrected potential $\quad V_T(\phi) = V_0(\phi) + V_1(\phi,T)$

$$\partial_\mu T_\phi^{\mu\nu} = +\partial^\nu\phi \frac{\mathrm{d}m^2}{\mathrm{d}\phi}\int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}} f(p,x). \quad \Rightarrow \quad \Box\phi - V_T'(\phi) = -\frac{\mathrm{d}m^2}{\mathrm{d}\phi}\int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\delta f(p,x)$$

Boltzmann equation with collisions and external forces

$$f(p,x) = f^{\mathrm{eq}}(p,x) + \delta f(p,x)$$

This leaves us with the equation of motion

$$\Box\phi - V_T'(\phi) = -\tilde{\eta}\frac{\mathrm{d}m^2}{\mathrm{d}\phi} u^\mu \partial_\mu\phi \qquad \text{where} \qquad \int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\delta f(p,x) = \tilde{\eta} u^\mu \partial_\mu \phi,$$

Local equilibrium and perfect fluid

SciPost Phys. Lect.Notes 24 (2021)

scalar field + fluid system

With $p(\phi,T)=p_1(\phi,T)-V_0(\phi)$, we have $T_f^{\mu\nu}=(e+p)u^\mu u^\nu+pg^{\mu\nu}$,

The full energy-momentum tensor is conserved $\partial_\mu\left(T_{\rm f}^{\mu\nu}+T_\phi^{\mu\nu}\right)=0$

which yields

$$\partial_\mu T_{\rm f}^{\mu\nu}+V_T'(\phi)\partial^\nu\phi=-\partial^\nu\phi\,\frac{{\rm d}m^2}{{\rm d}\phi}\int\frac{{\rm d}^3p}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\delta f(p,x)$$

Consider the scalar product of u with both sides

$$u_\nu\partial_\mu(wu^\mu u^\nu+pg^{\mu\nu})+V_T'(\phi)u\cdot\partial\phi=\tilde{\eta}(u\cdot\partial\phi)^2$$

Here, $w=e+p=Ts$ is the enthalpy density, and $s={\rm d}p/{\rm d}T$ is the entropy density

真空泡碰撞、合并、流体演化产生引力波

有限温度有效势能

$$V(\phi,T)=\frac{1}{2}\gamma(T^2-T_0^2)\phi^2-\frac{1}{3}AT\phi^3+\frac{1}{4}\lambda\phi^4$$

新物理

标量场-相对论流体运动方程

$$-\ddot{\phi}+\nabla^2\phi-\frac{\partial V}{\partial\phi}=\eta W(\dot{\phi}+V^i\partial_i\phi)$$

η：粒子和真空泡壁相互作用

$$\dot{E}+\partial_i(EV^i)+p[\dot{W}+\partial_i(WV^i)]-\frac{\partial V}{\partial\phi}W(\dot{\phi}+V^i\partial_i\phi)=\eta W^2(\dot{\phi}+V^i\partial_i\phi)^2$$

$$\dot{Z}_i+\partial_j(Z_iV^j)+\partial_i p+\frac{\partial V}{\partial\phi}\partial_i\phi=-\eta W(\dot{\phi}+V^j\partial_j\phi)\partial_i\phi$$

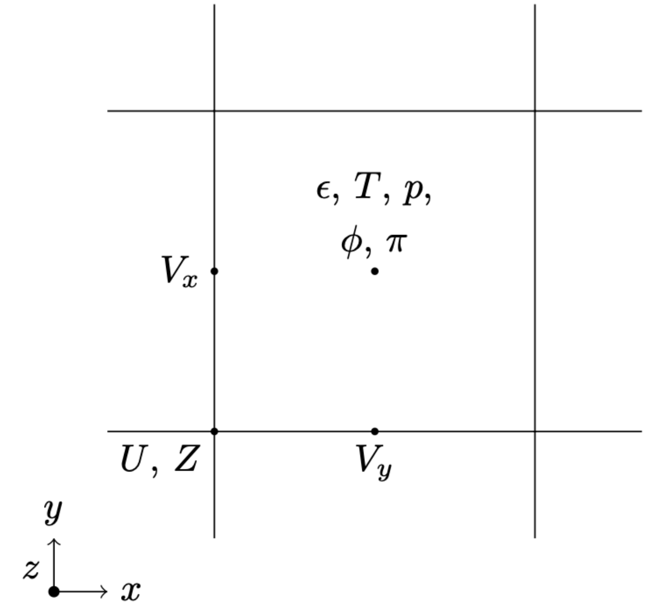
equation of state

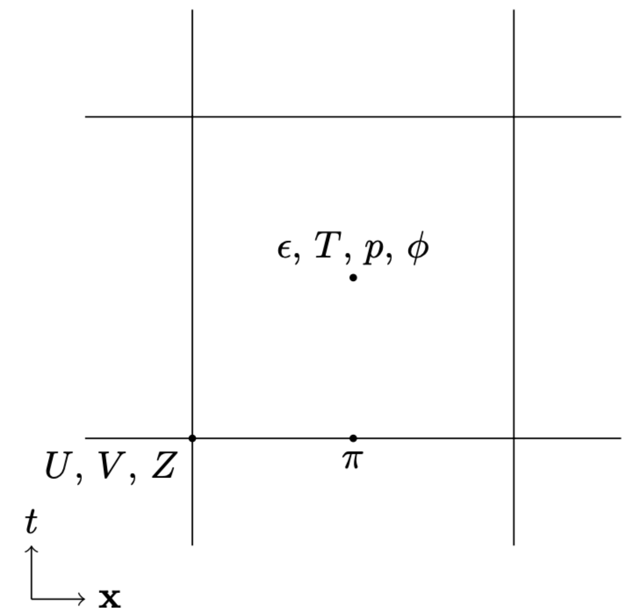
$$\epsilon(T,\phi)=3aT^4+V(\phi,T)-T\frac{\partial V}{\partial T},$$

$$p(T,\phi)=aT^4-V(\phi,T)$$

fluid momentum density $Z_i=W(\epsilon+p)U_i$

fluid energy density E = $W\epsilon$

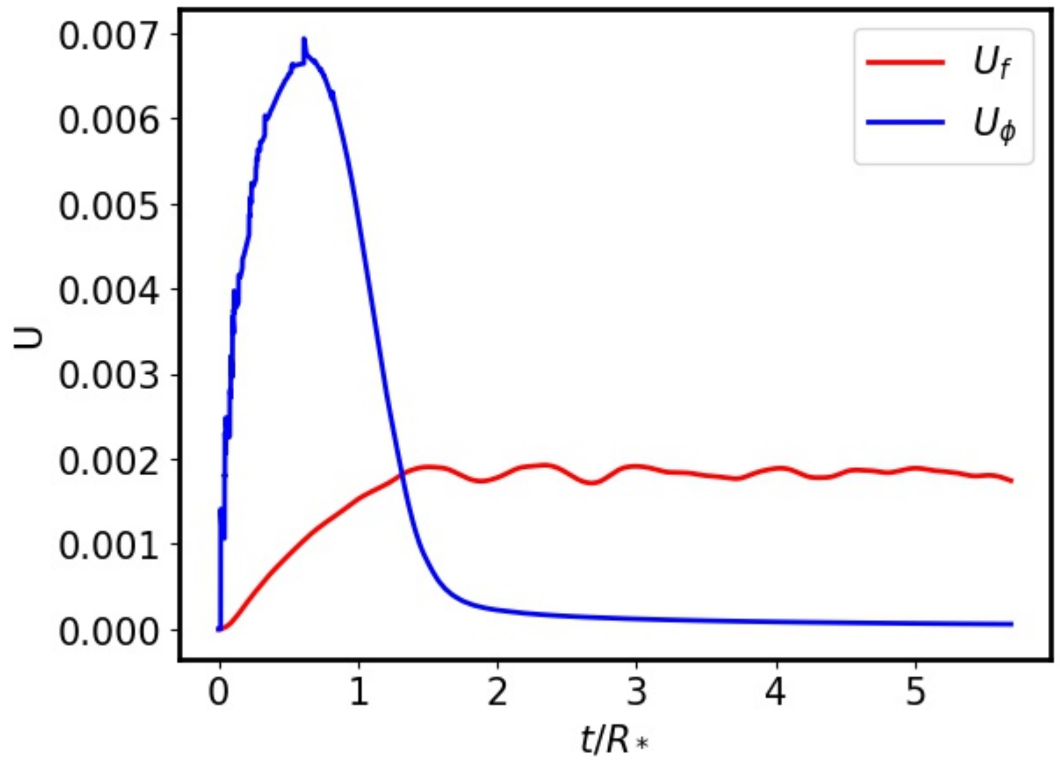




V^i is the fluid 3-velocity

$U^i = WV^i$， W： relativistic γ-factor

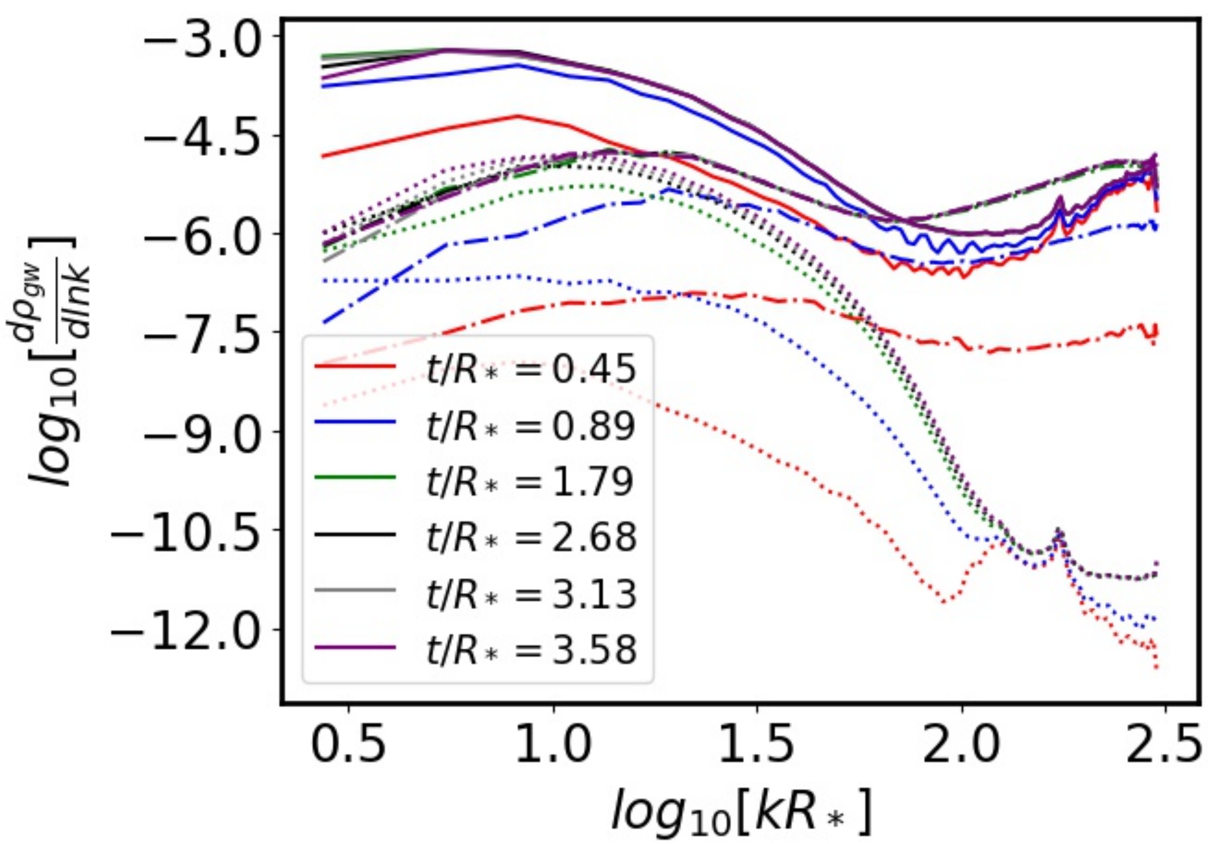
真空泡碰撞、合并、流体演化产生引力波



$$\tau_{ij}^{\phi} = \partial_i \phi \partial_j \phi, \quad \tau_{ij}^{\mathrm{f}} = W^2(\epsilon + p) V_i V_j$$

$$(\bar{\epsilon} + \bar{p}) \overline{U}_{\mathrm{f}}^2 = \frac{1}{\mathcal{V}} \int_{\mathcal{V}} d^3 x \tau_{ii}^{\mathrm{f}}$$

$$(\bar{\epsilon} + \bar{p}) \overline{U}_{\phi}^2 = \frac{1}{\mathcal{V}} \int_{\mathcal{V}} d^3 x \tau_{ii}^{\phi}$$



$$\ddot{u}_{ij} - \nabla^2 u_{ij} = 16\pi G(\tau_{ij}^{\phi} + \tau_{ij}^{\mathrm{f}})$$

$$h_{ij}(\mathbf{k}) = \lambda_{ij,lm}(\hat{\mathbf{k}}) u_{lm}(t, \mathbf{k})$$

Bian,Jia,Zhao,Zhu,arxiv:230X.XXXXX

DFSZ轴子模型

The action in expanding universe with spatially flat FLRW metric:

$$S=-\int d^4x\sqrt{-g}\left(g^{\mu\nu}\frac{1}{2}\partial_\mu\varphi^*\partial_\nu\varphi+V(\varphi)\right)$$

PQ complex scalar: $\varphi=\phi_1+i\phi_2$

$$V(\varphi)=\frac{1}{4}\lambda(|\varphi|^2-v^2)^2+\frac{\lambda}{6}T^2|\varphi|^2+\frac{m^2(T)v^2}{N_{DW}^2}(1-\cos(N_{DW}\theta))-\Xi v^3(\varphi e^{-i\delta}+\text{h.c.})$$

$m^2(T)$: $\min\left[\frac{\alpha_a\Lambda^4}{f_a^2(T/\Lambda)^{6.68}},m_a^2\right]$; θ: axion field; Ξ term: bias term

PQ era, PQ symmetry broken, second order phase transition, $T_c\sim 10^9$-10^{11}GeV

→ Axion(global) strings form and enters the scaling regime

QCD era, axion acquires a non-zero mass due to the QCD non-perturbative effect, $T\sim 100$MeV

→ String-domain wall hybrid networks form and eventually decay

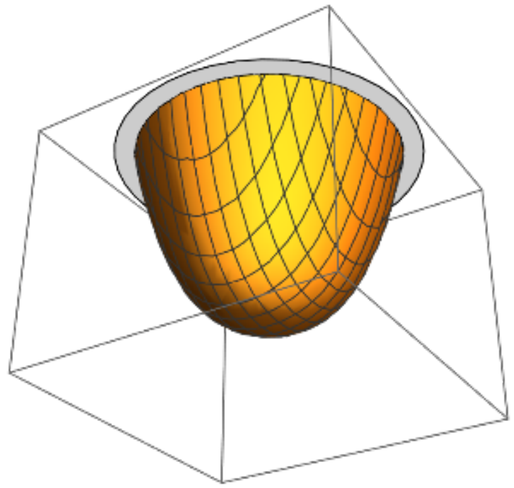
Gravitational waves and axion radiated by topological defects of two eras → Detection of axion dark matter

Yang Li, **Ligong Bian**, Rong-Gen Cai, Jing Shu, 2311.02011

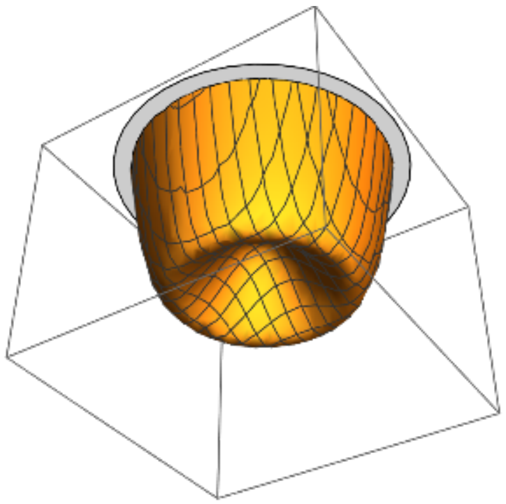
Shape of potential

PQ era:

before PQ transition

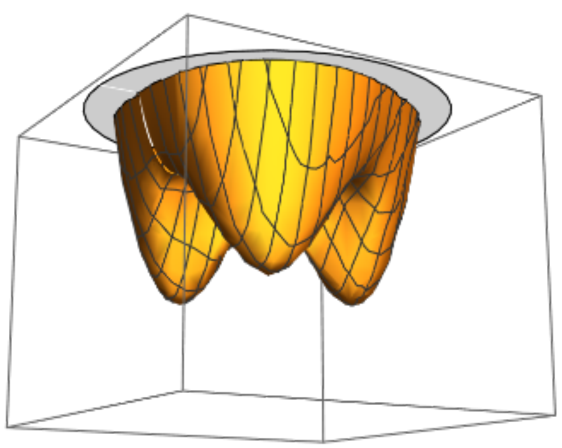


after PQ transition

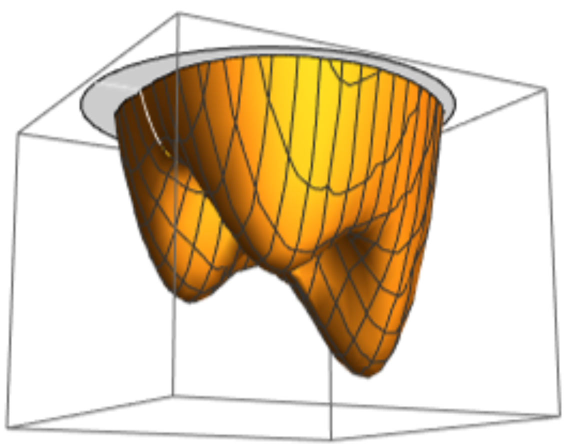


QCD era:

with nonzero axion mass without bias term



with bias term



▸ 模拟方案

Equations of motion

PQ era-the first stage

$$\begin{cases} \phi_1'' + 2\frac{a'}{a}\phi_1' - \nabla^2\phi_1 = -a^2[\lambda\phi_1(\phi_1^2+\phi_2^2-v^2+\frac{1}{3}T^2) - \frac{m^2(T)v^2}{N_{\rm DW}^2}(\cos\theta\cos N_{\rm DW}\theta + N_{\rm DW}\sin\theta\sin N_{\rm DW}\theta) - 2\Xi v^3\cos\delta] \\ \phi_2'' + 2\frac{a'}{a}\phi_2' - \nabla^2\phi_2 = -a^2[\lambda\phi_2(\phi_1^2+\phi_2^2-v^2+\frac{1}{3}T^2) - \frac{m^2(T)v^2}{N_{\rm DW}^2}(\sin\theta\cos N_{\rm DW}\theta - N_{\rm DW}\cos\theta\sin N_{\rm DW}\theta) - 2\Xi v^3\sin\delta] \end{cases}$$

Initial condition

thermal spectrum

$$\mathcal{P}_{\phi_1}(k) = \mathcal{P}_{\phi_2}(k) = \frac{n_k}{w_k} = \frac{1}{w_k}\frac{1}{e^{w_k/T}-1}, \quad \mathcal{P}_{\dot\phi_1}(k) = \mathcal{P}_{\dot\phi_2}(k) = n_k w_k = \frac{w_k}{e^{w_k/T}-1}$$

$$w_k = \sqrt{k^2/R^2 + m_{\rm eff}^2} \qquad m_{\rm eff}^2 = \lambda(T^2/3 - v^2)$$

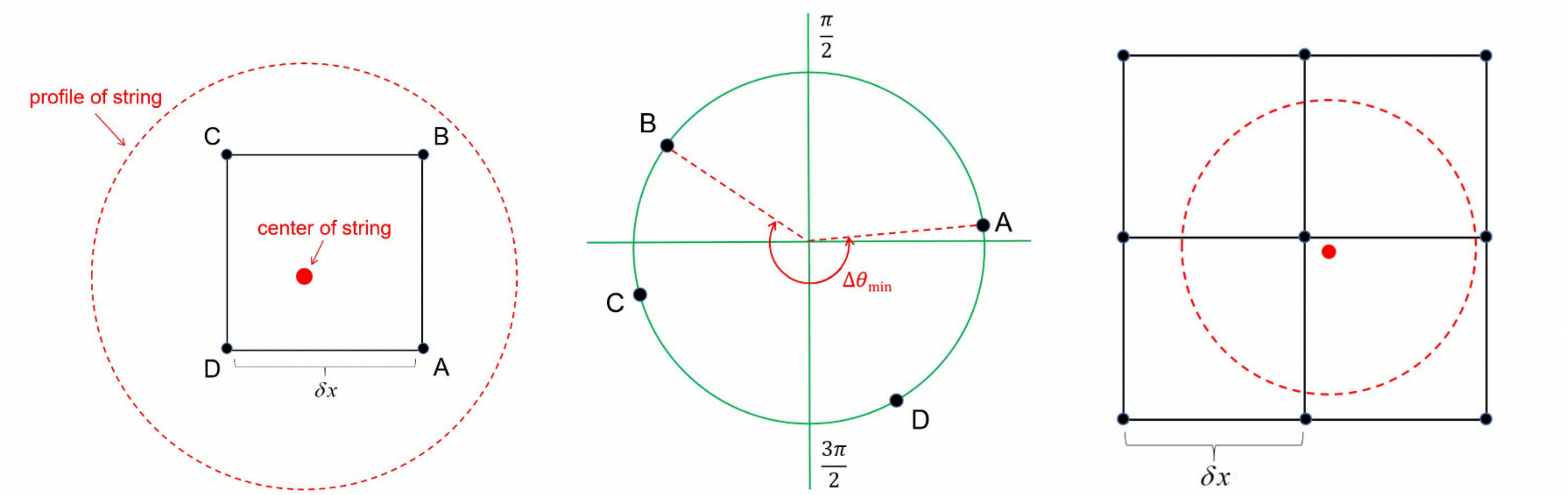
two-point correlation functions

$$\langle\phi_i(\mathbf{k})\phi_j(\mathbf{k}')\rangle = (2\pi)^3\mathcal{P}_\phi(k)\delta(\mathbf{k}-\mathbf{k}')\delta_{ij},$$
$$\langle\dot\phi_i(\mathbf{k})\dot\phi_j(\mathbf{k}')\rangle = (2\pi)^3\mathcal{P}_{\dot\phi}(k)\delta(\mathbf{k}-\mathbf{k}')\delta_{ij},$$
$$\langle\phi_i(\mathbf{k})\dot\phi_j(\mathbf{k}')\rangle = 0\,.$$

$$\langle|\phi_i(\mathbf{k})|^2\rangle = \left(\frac{N}{\delta x_{\rm phy}}\right)^3\mathcal{P}_{\phi_i}(k), \qquad \langle\phi_i(\mathbf{k})\rangle = 0,$$
$$\langle|\dot\phi_i(\mathbf{k})|^2\rangle = \left(\frac{N}{\delta x_{\rm phy}}\right)^3\mathcal{P}_{\dot\phi_i}(k), \qquad \langle\dot\phi_i(\mathbf{k})\rangle = 0,$$

▸ 宇宙弦识别

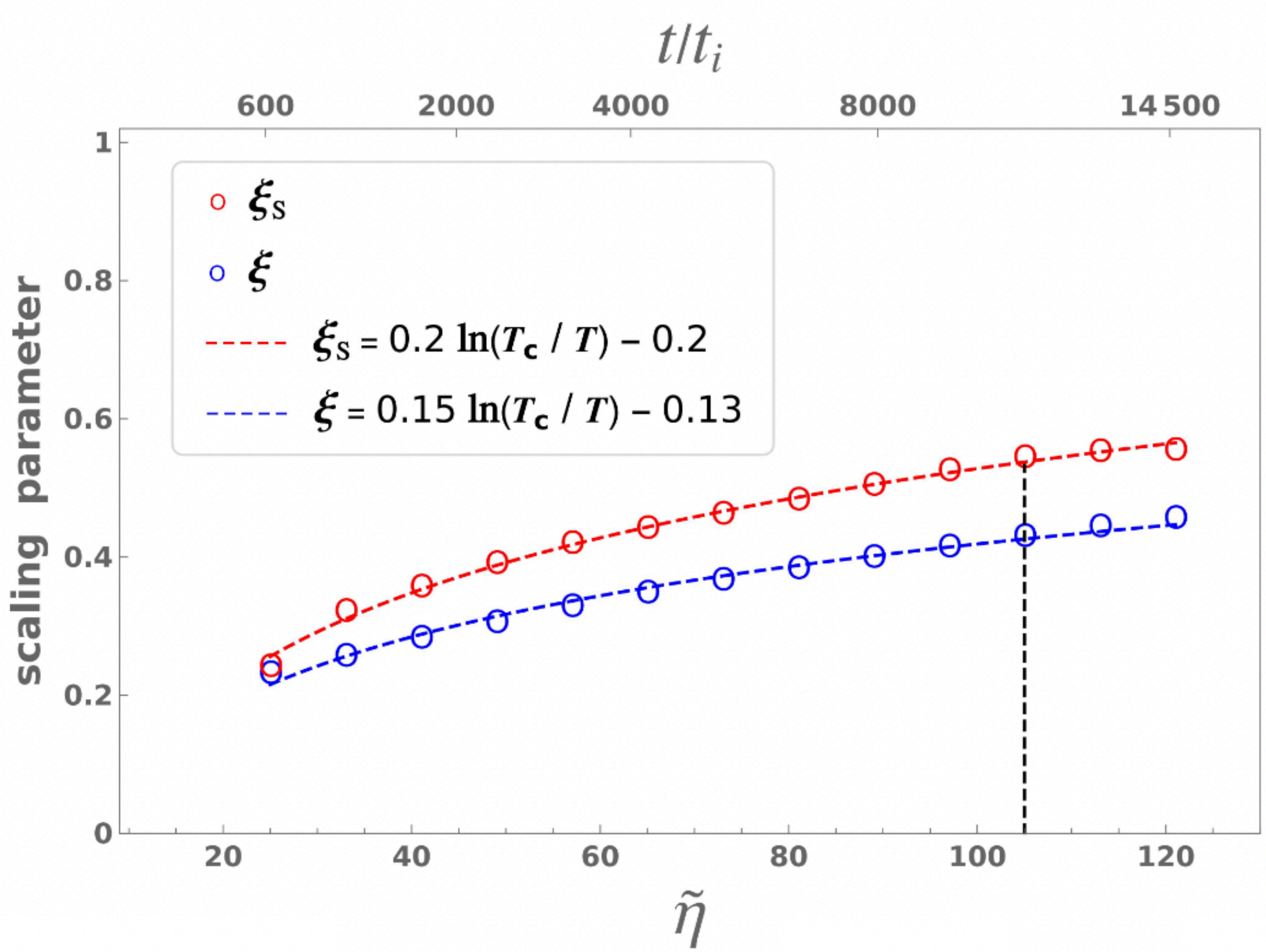
String penetrates the square loop if the minimum phase range which contains the four points is greater than π and the phase changes continuously



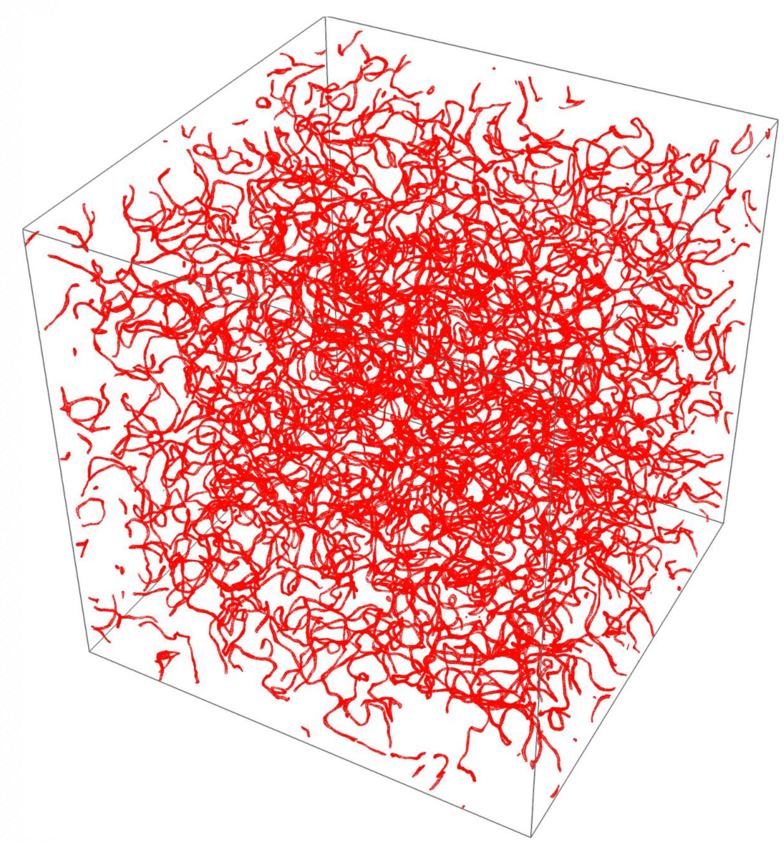
For a specific square loop, assuming that the minimum phase at four points is θmin

(1) $\theta_{\min} < \pi$.
(2) There exists at least one phase at another point minus $\theta_{\min}$ is greater than π.
(3) There exists at least one phase at another point minus $\theta_{\min}$ is smaller than π.
(4) Denote the phase closest to π in all phases greater than π as θ_a, and denote the phase closest to π in all phases smaller than π as θ_b, it is required to meet $\theta_a - \theta_b < \pi$.
(5) Calculate the difference between the phases at each of two adjacent points in a counterclockwise direction, the multiplication of the four differences is required to be negative.

▸ 宇宙弦的scaling



Axion string network in the final moment of PQ era



$$\xi = \frac{\rho_{st} t^2}{\mu_{st}}, \quad \text{with } \rho_{st} = \frac{\mu_{st} l}{R^2 V} \Longrightarrow \xi = \frac{l t^2}{R^2 V}$$

$$\xi_s = \frac{l_{phy} t^2}{V_{phy}} = \frac{t^2}{L_m^2} = \frac{1}{\kappa^2} \frac{t^2}{(t-t_0)^2} \to \frac{1}{\kappa^2}$$

string tension: $\mu_{st} \simeq \pi v^2 \ln(t/\delta_{st})$

string core width: $\delta_{st} = 1/\sqrt{\lambda(v^2 - \frac{1}{3}T^2)}$

l : comoving string length

V: comoving volume

Mean string separation $L_m = \sqrt{V_{phy}/l_{phy}}$

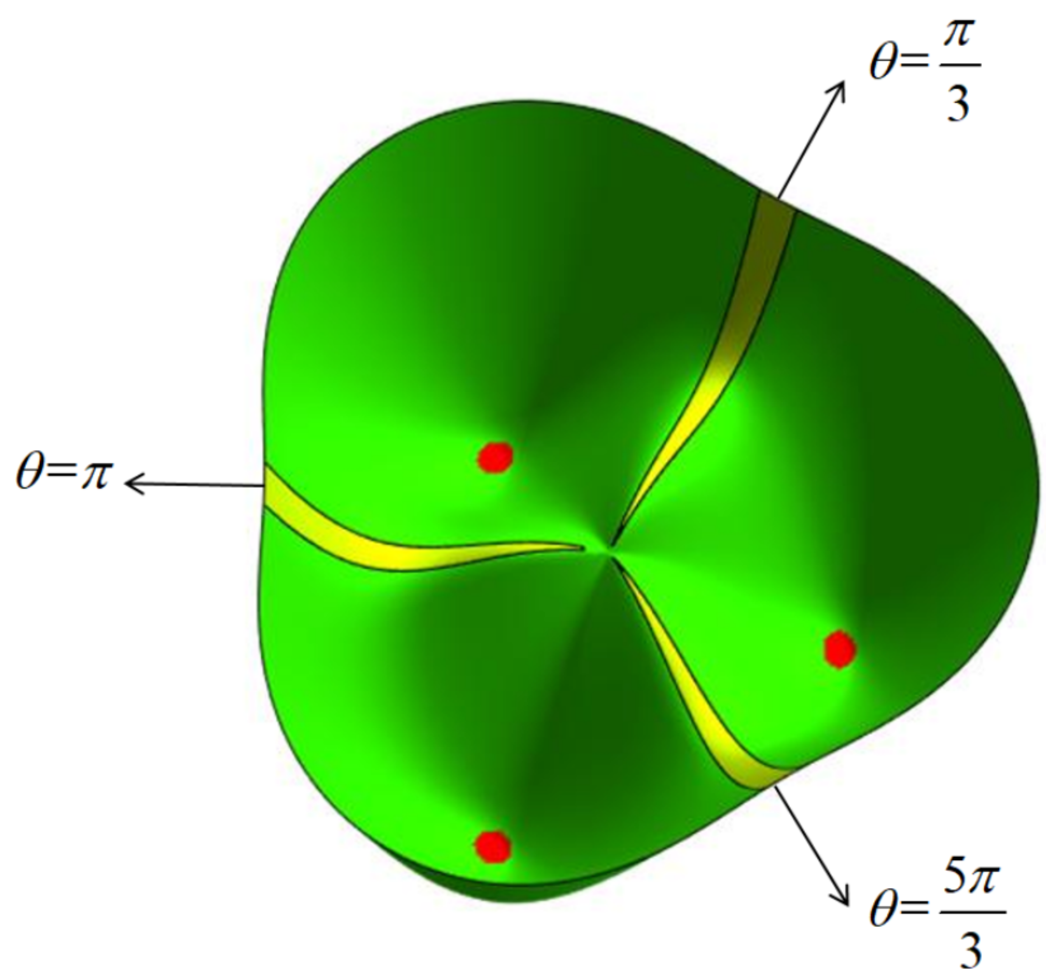
physical string length $l_{phy} = (2/3) n_c (R\delta x)$

nc: squre loops

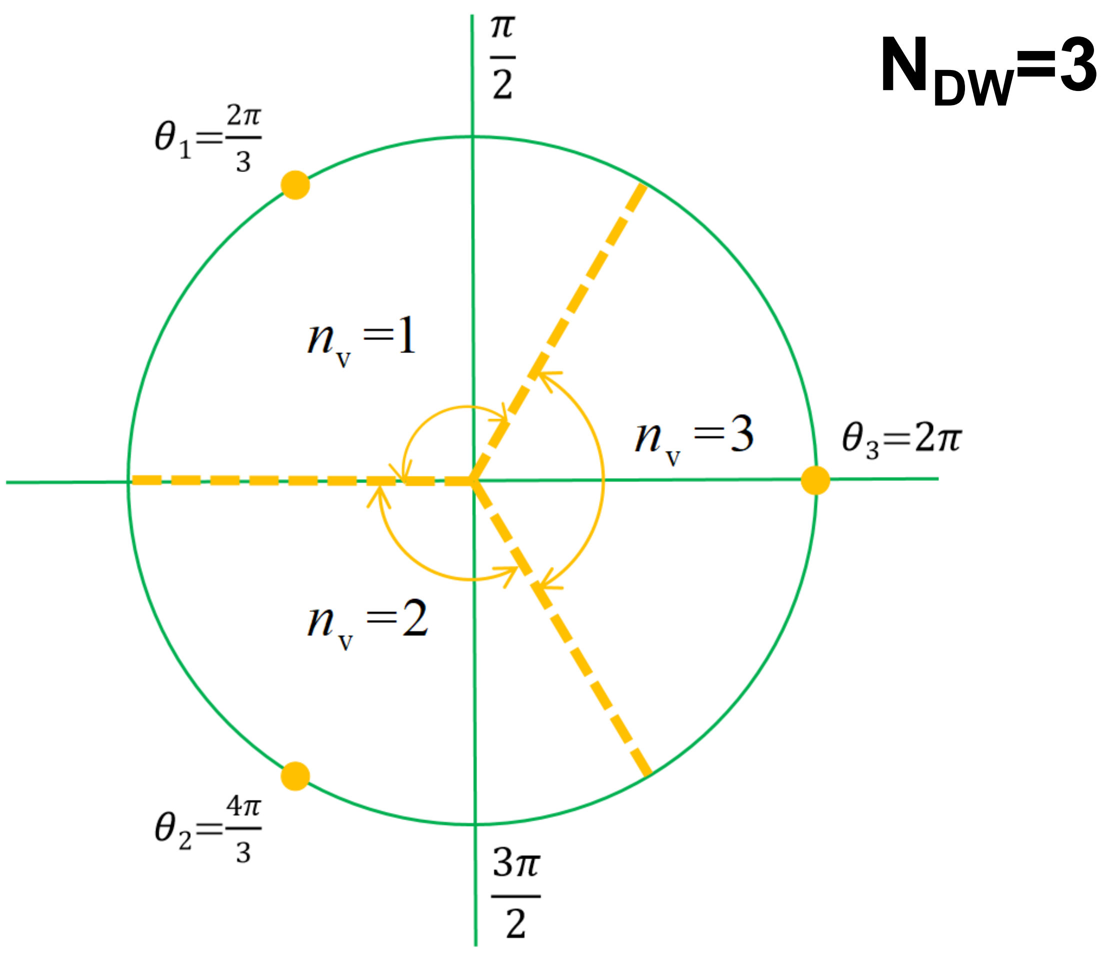
In the scaling regime, Lm increases linearly with t, and the scaling parameter tends to be a constant.

We found the scaling parameters exhibited logarithmic increase behavior, as 1809.0924, 1906.00967, 1806.0467,1806.05566

Top view of the shape of potential energy



The distribution of fields in phase space



Comoving area density

$$A/V = C \sum_{\text{links}} \delta \frac{|\nabla\theta|}{|\theta_{,x}| + |\theta_{,y}| + |\theta_{,z}|}$$

θ,i (i = x, y, z)： spatial derivatives of the dimensionless axion field θ(x)

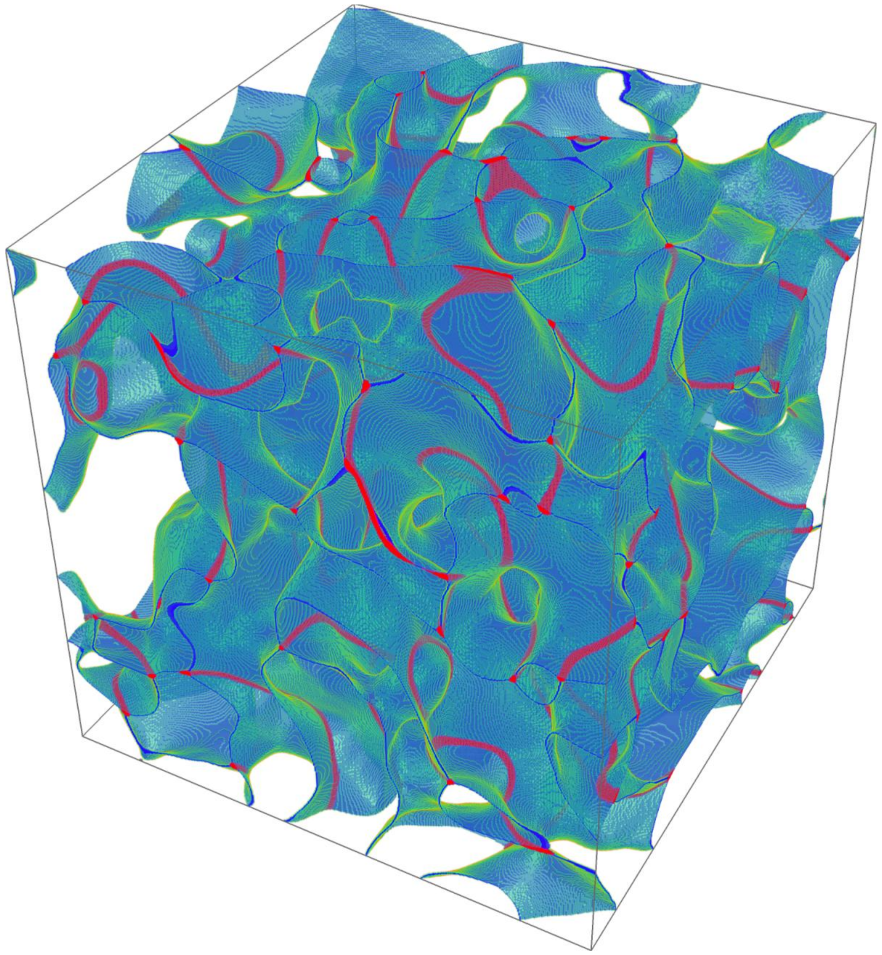
Area parameter A of DW (scaling parameter of DW) $\xi_{\rm dw} \equiv \mathcal{A} = \frac{\rho_{\rm wall}}{\sigma_{\rm wall}} t$, with $\rho_{\rm wall} = \frac{\sigma_{\rm wall} A}{R(t) V}$

$$A = \Delta A \sum_{\text{links}} \delta \frac{|\nabla\theta|}{|\theta_{,x}| + |\theta_{,y}| + |\theta_{,z}|}$$

$\Delta A = (\delta x)^2$ is the comoving area of one grid surface

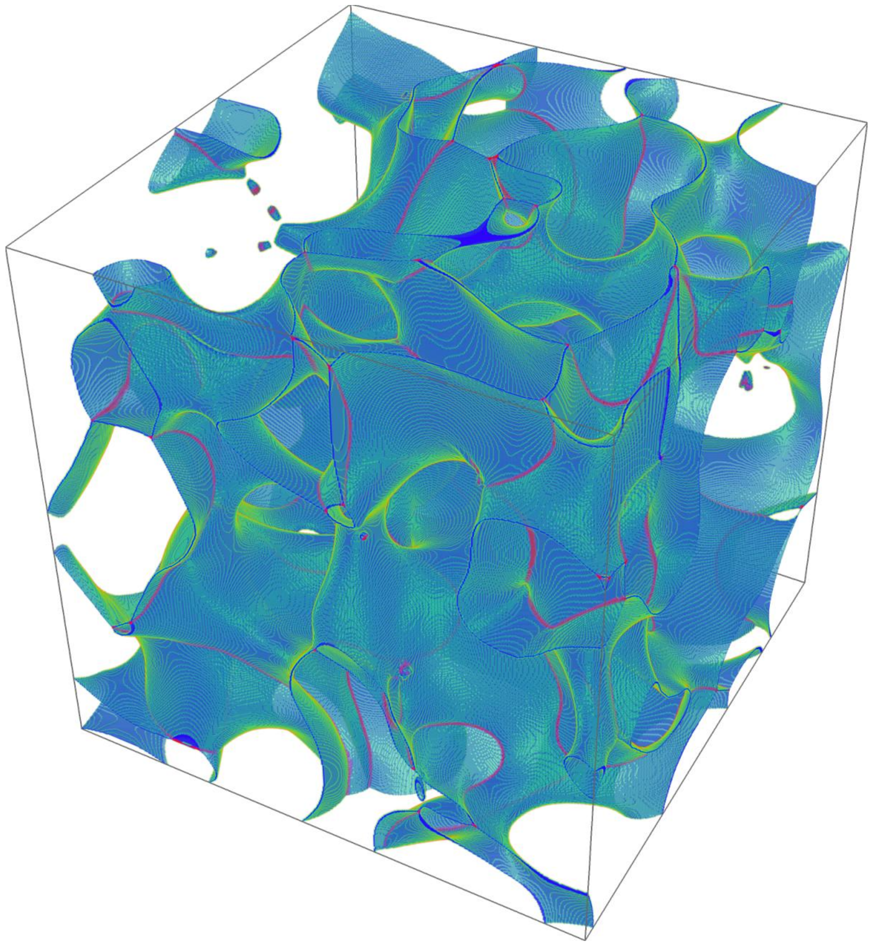
▸ String-wall evolution

Axion string-domain wall hybrid network in our simulation



Red region —— Axion string
Blue region —— Domain wall

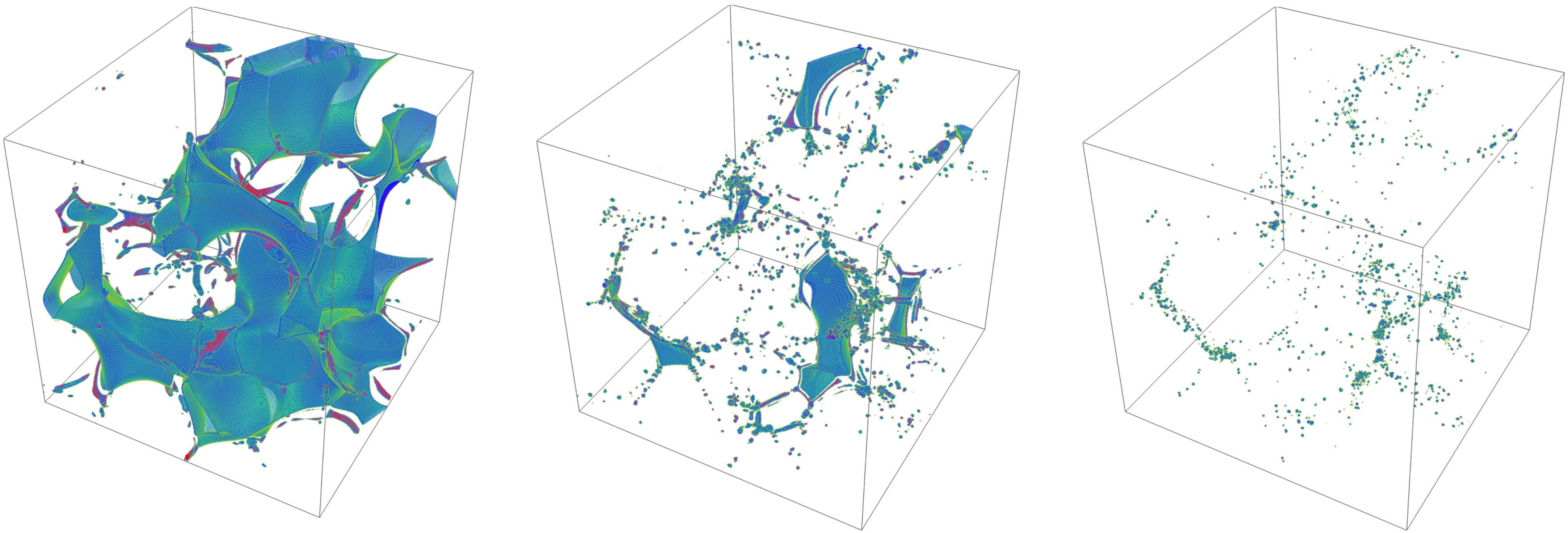
$N_{\mathrm{DW}} = 3$



$\eta = 4.6$

$\eta = 10$

▸ String-wall evolution



$\eta=15.4$ $\eta=20.8$ $\eta=26.2$

Axion string-domain wall hybrid network destruction with gravitational waves and axions emitted during this process

KSVZ轴子模型

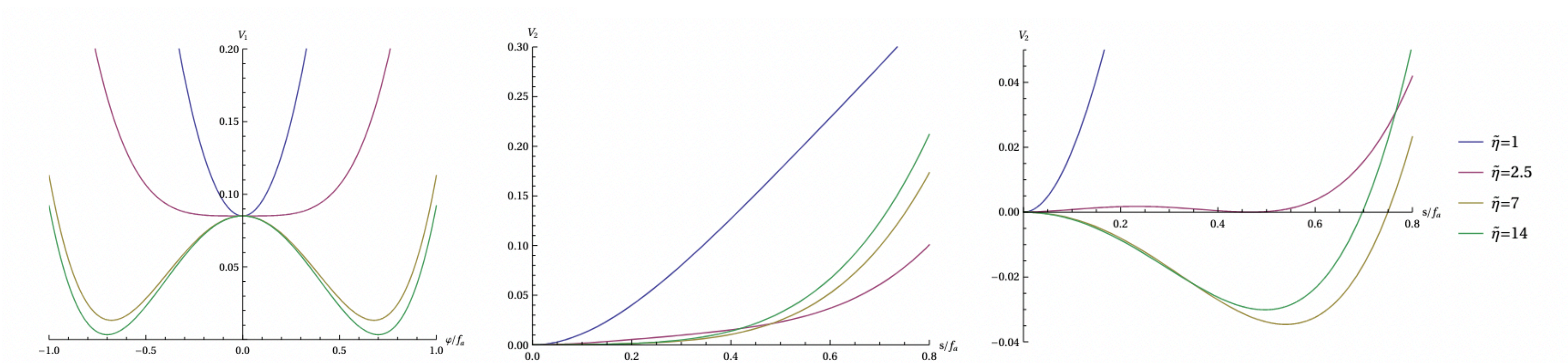
The action in expanding universe with spatially flat FLRW metric:

$$S = -\int \mathrm{d}x^4 \sqrt{-g}\left(\partial_\mu \varphi^* \partial^\mu \varphi + \frac{1}{2}\partial_\mu h \partial^\mu h + V(\varphi, h, T)\right)$$

Thermal effective potential

$$V(\varphi, h, T) = V_1(\varphi, T) + V_2(\varphi, h, T)$$

$$V_1(\varphi, T) = \lambda_\varphi \left(|\varphi|^2 - \frac{f_a^2}{2}\right)^2 + \left(\frac{\lambda_\varphi}{3} + \frac{\lambda_{\varphi h}}{6}\right) T^2 |\varphi|^2 \qquad V_2(\varphi, s, T) = \frac{1}{2}\gamma\left(T^2 - T_0^2\right) h^2 + \frac{1}{3} A T h^3 + \frac{1}{4}\lambda_h h^4 + \frac{1}{2}\lambda_{\varphi h} |\varphi|^2 h^2$$



$$T_c = \sqrt{\lambda_\phi / (\lambda_\phi / 3 + \lambda_{\phi h})} f_a$$

Potential Shape

▶ 模拟方案

Equations of motion

Only PQ era

$$\begin{cases} \tilde{\varphi}'' - \tilde{\nabla}^2\tilde{\varphi} + 2\frac{a'}{a}\tilde{\varphi}' = -a^2\frac{\partial \tilde{V}}{\partial|\tilde{\varphi}|}\frac{\tilde{\varphi}}{2|\tilde{\varphi}|}\,, \\ \tilde{h}'' - \tilde{\nabla}^2\tilde{h} + 2\frac{a'}{a}\tilde{h}' = -a^2\frac{\partial \tilde{V}}{\partial \tilde{h}}\,, \end{cases}$$

$$\tilde{\varphi} = \frac{\varphi}{f_*} \quad \tilde{h} = \frac{h}{f_*} \qquad f_* = f_a$$

$$\tilde{V} = \frac{V\left(f_*\tilde{\varphi},, f_*\tilde{h}, T\right)}{f_*^2\omega_*^2} \qquad \omega_* = a_i H_i$$

Initial condition

thermal spectrum

$$\mathcal{P}_{\phi_i}(k) = \frac{1}{\omega_k(e^{\omega_k/T}-1)} \quad \mathcal{P}_{\pi_{\phi_i}}(k) = \frac{\omega_k}{e^{\omega_k/T}-1}$$

$$w_k = \sqrt{k^2/R^2 + m_{\rm eff}^2} \qquad m_\varphi^2 = (\lambda_\varphi/3 + \lambda_{\phi h}/6)T^2 - \lambda_\phi v_\phi^2$$

two-point correlation functions

$$\langle\phi_i(\mathbf{k})\phi_j(\mathbf{k}')\rangle = (2\pi)^3\mathcal{P}_\phi(k)\delta(\mathbf{k}-\mathbf{k}')\delta_{ij},$$
$$\langle\dot{\phi}_i(\mathbf{k})\dot{\phi}_j(\mathbf{k}')\rangle = (2\pi)^3\mathcal{P}_{\dot{\phi}}(k)\delta(\mathbf{k}-\mathbf{k}')\delta_{ij},$$
$$\langle\phi_i(\mathbf{k})\dot{\phi}_j(\mathbf{k}')\rangle = 0\,.$$

$$\langle|\phi_i(\mathbf{k})|^2\rangle = \left(\frac{N}{\delta x_{\rm phy}}\right)^3\mathcal{P}_{\phi_i}(k), \qquad \langle\phi_i(\mathbf{k})\rangle = 0,$$
$$\langle|\dot{\phi}_i(\mathbf{k})|^2\rangle = \left(\frac{N}{\delta x_{\rm phy}}\right)^3\mathcal{P}_{\dot{\phi}_i}(k), \qquad \langle\dot{\phi}_i(\mathbf{k})\rangle = 0,$$

▸ 场的空间构型

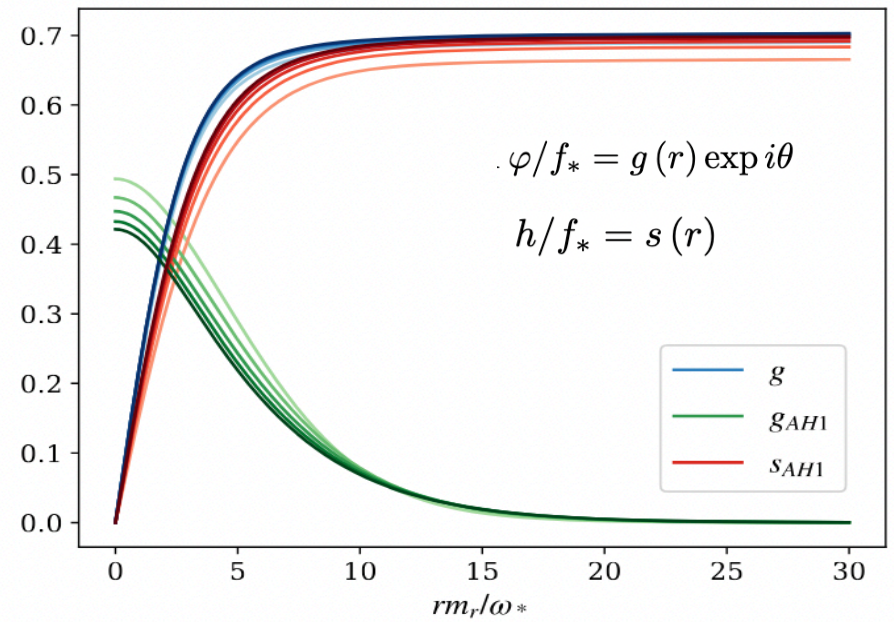
$$g''(r)+\frac{1}{r}g'(r)-\frac{1}{r^2}g(r)=\lambda_\varphi\frac{f_*^2}{\omega_*^2}g(g^2-\frac{1}{2})+(\frac{\lambda_\varphi}{3}+\frac{\lambda_{\varphi h}}{6})\frac{T^2}{\omega_*^2}g+\frac{\lambda_{\phi h}}{2}\frac{f_*^2}{\omega_*^2}gs^2\,,$$

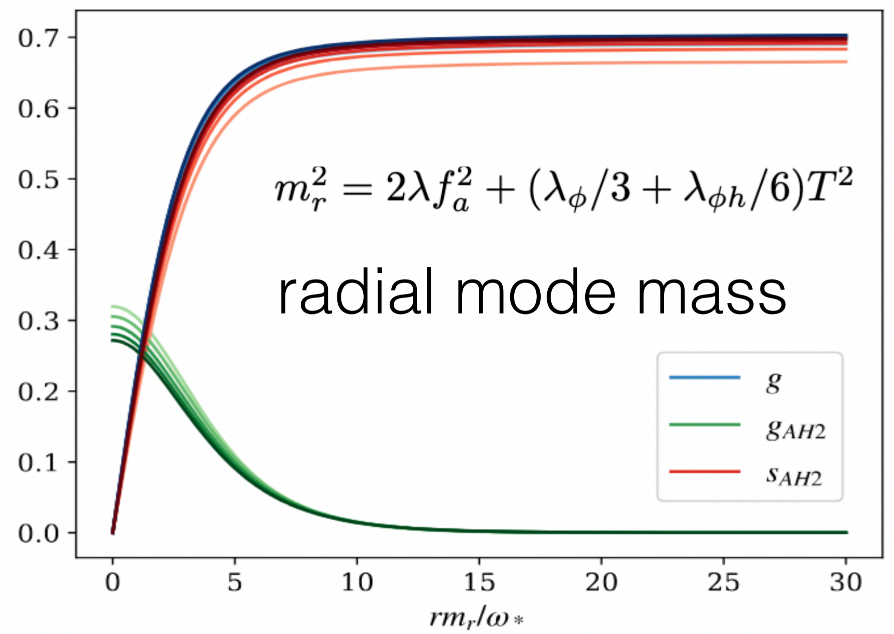
$$s''(r)+\frac{1}{r}s'(r)=\lambda_{\phi h}\frac{f_*^2}{\omega_*^2}g^2s+\gamma\frac{T^2-T_0^2}{\omega_*^2}s+AT\frac{f_*}{\omega_*^2}s^2+\lambda_h\frac{f_*^2}{\omega_*^2}s^3\,.$$

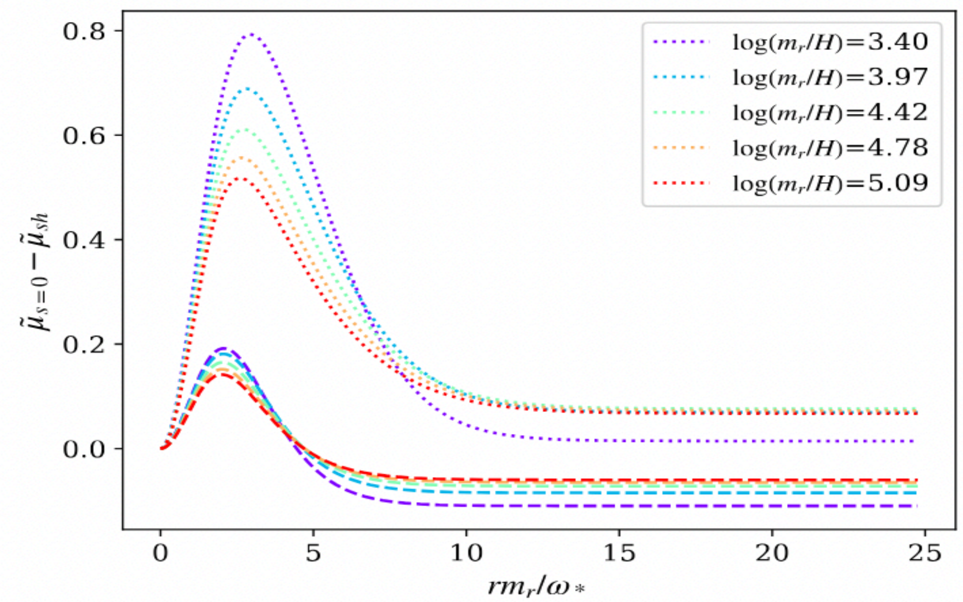
Surface tension

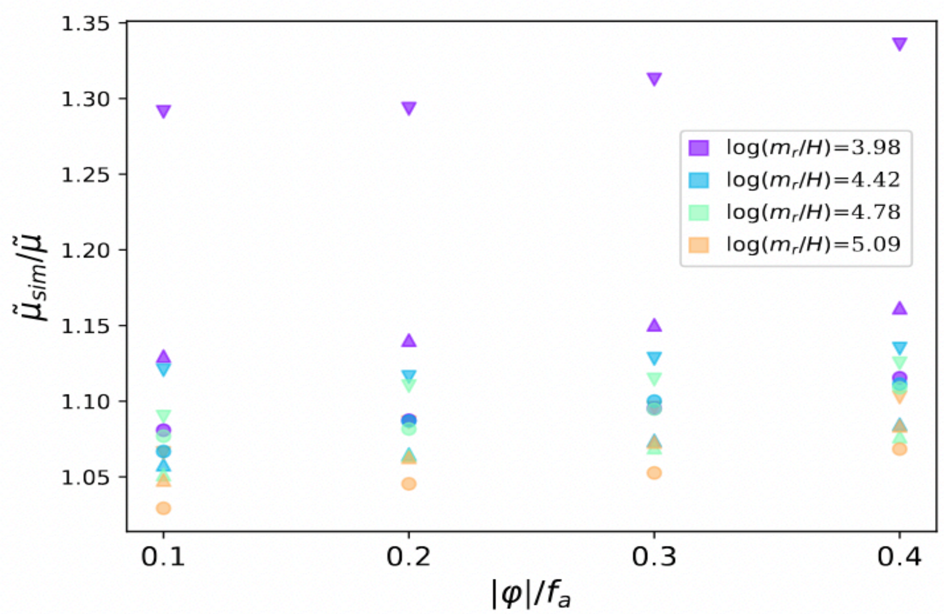
$$\tilde{\mu}=\mu/f_*^2=\int\tilde{\rho}r\mathrm{d}r\mathrm{d}\theta$$

$$\tilde{\mu}=2\pi\int\left((g'^2+\frac{g^2}{r^2}+(s'^2+\tilde{V}(g,s)\right)r\mathrm{d}r$$



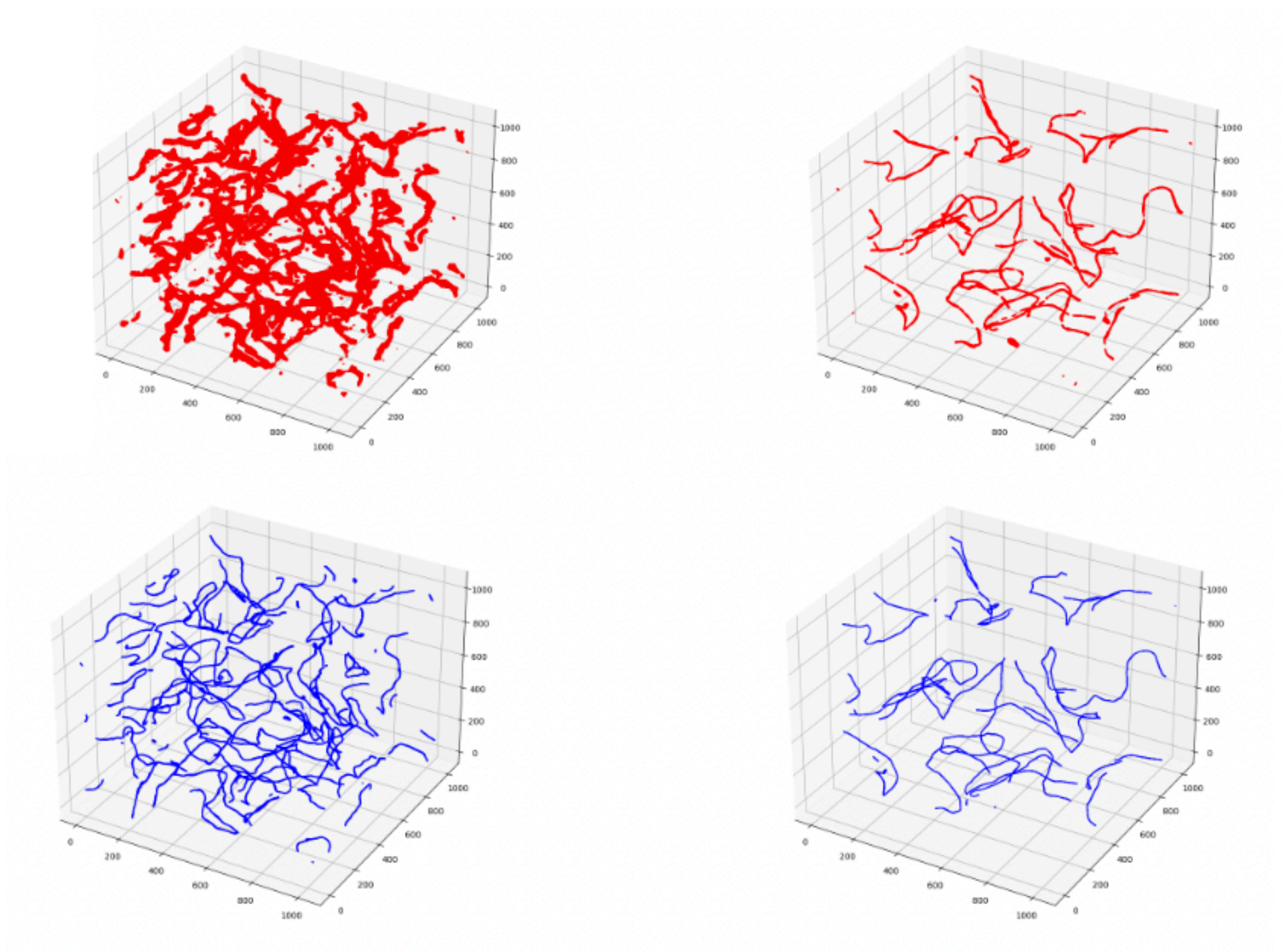






▸ Axion-Higgs String evolution

$\tilde{\eta} = 9.0$ $\tilde{\eta} = 14.0$



$s > 0.35 f_a$

$|\varphi| < 0.1 f_a$

▸ Related topics

❖Lattice simulation

- **PT GW simulation, Electroweak sphaleron, PT dynamics**
- **Topological defects: Magnetic monopoles, cosmic strings, domain walls**

❖Pheno

1. EWSB and GW from FOPT

- **Probing the Higgs Potential shape and EWPT patterns with GW production and Colliders complementarily**

2. BAU and GW from FOPT

- **Sphaleron process, bubble dynamics**

3. DM and GW from FOPT

- **DM and high/low-scale PT, DM out-of-equilibrium & FOPT, PBH DM&FOPT**

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谢谢！