Neutrino masses and flavor mixing

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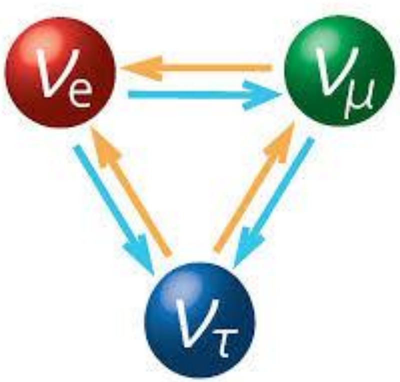
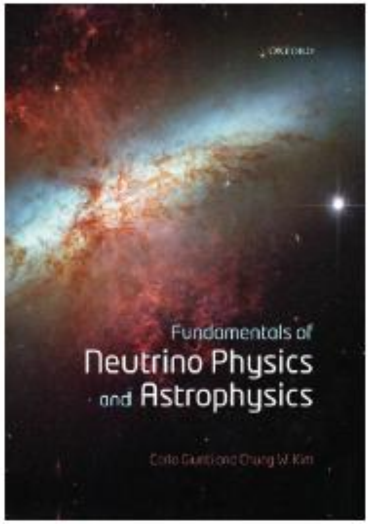
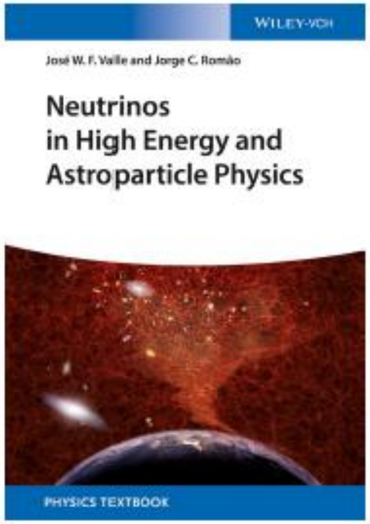
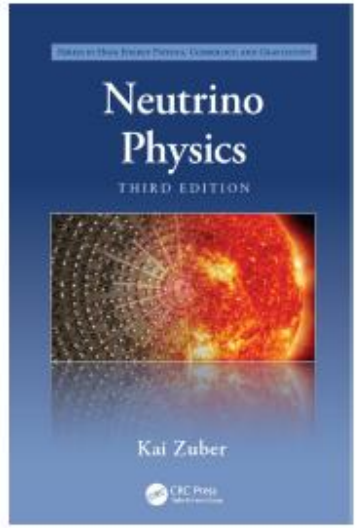
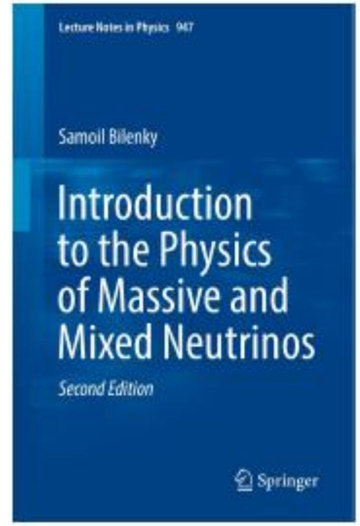


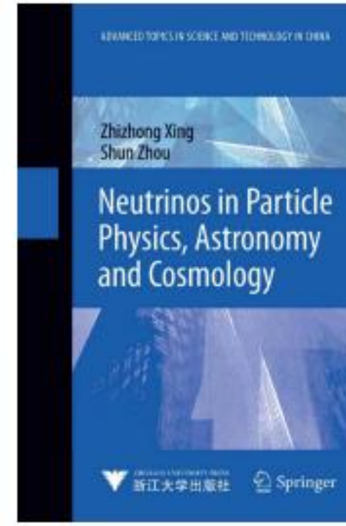
References: many excellent books…

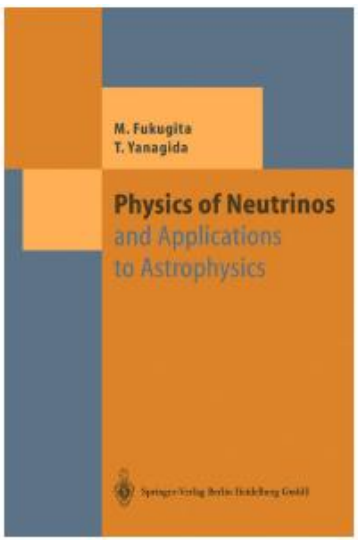


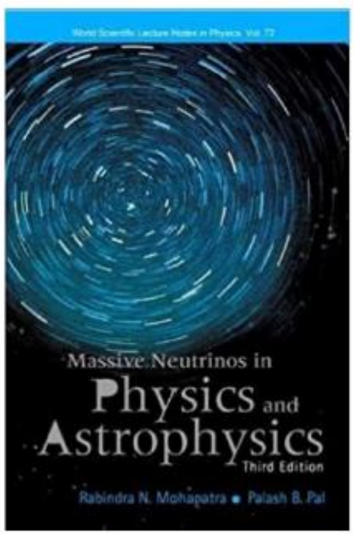


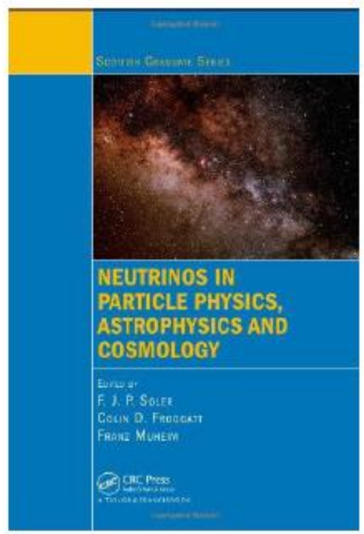


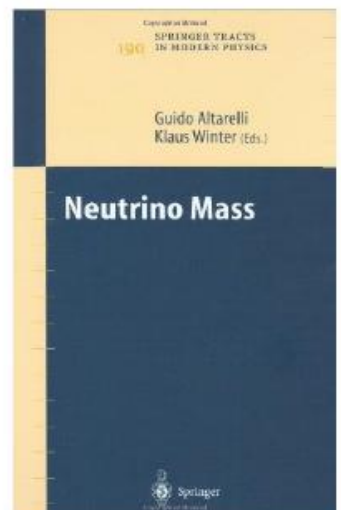


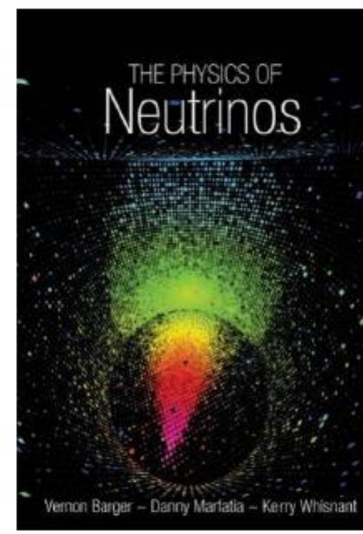






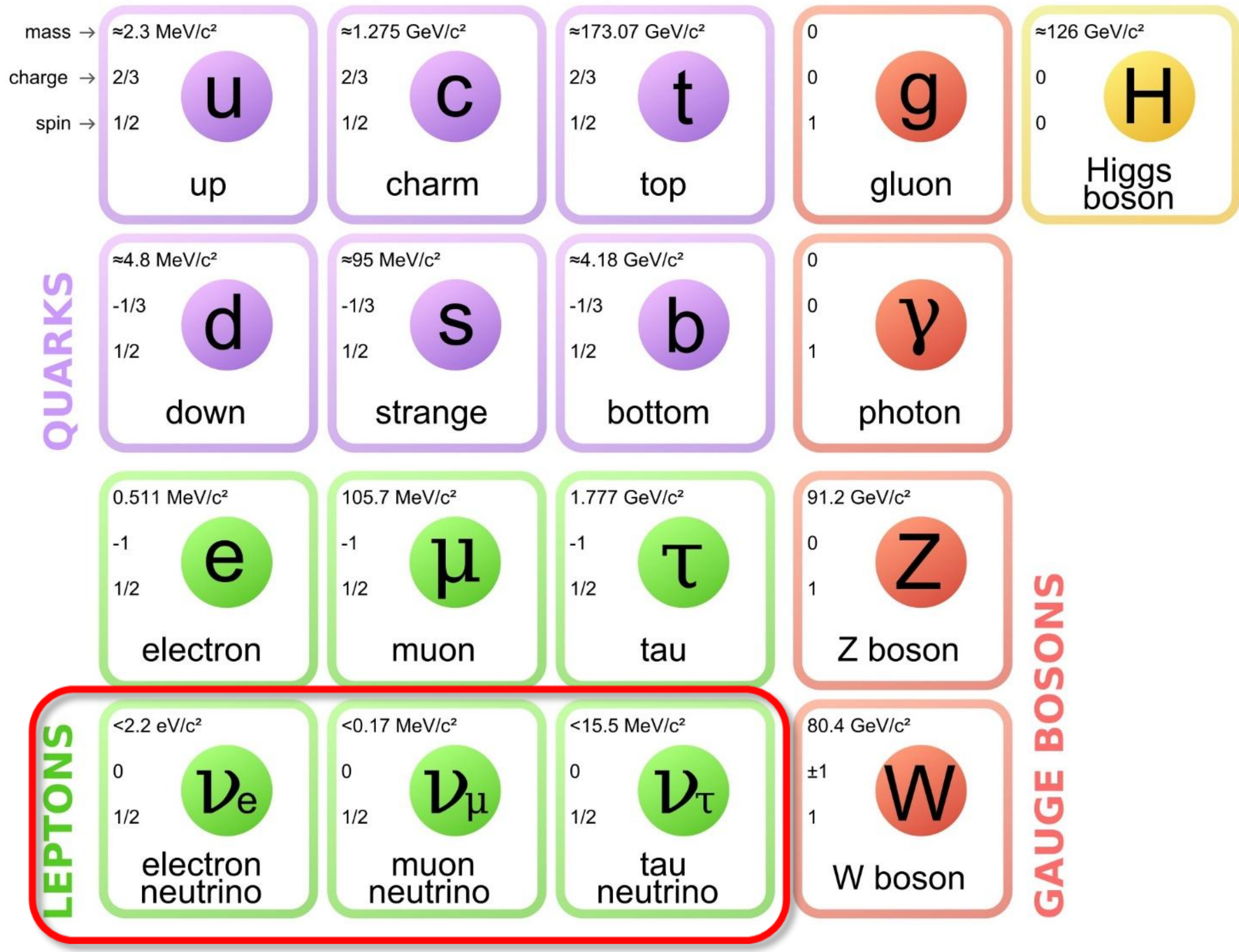






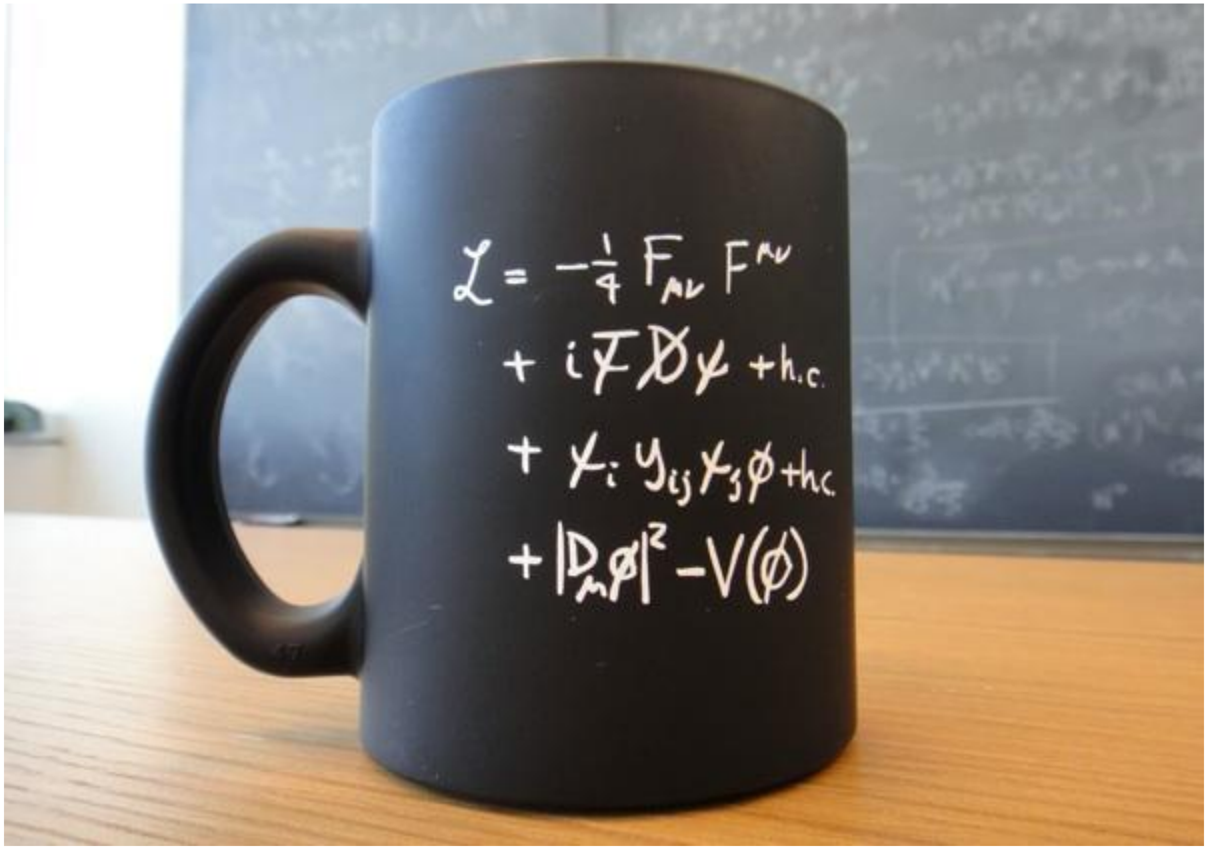
Many excellent reviews/lecture notes…

- **"Neutrino Physics",** P. Hernandez, arXiv:1708.01046
- **"2004 TASI Lectures on Neutrino Physics",** Andre de Gouvea, hep-ph/0411274
- **"TASI 2002 lectures on neutrinos",** Yuval Grossman, hep-ph/0305245
- **"Neutrino masses and mixings and…",** Alessandro Strumia, Francesco Vissani, hep-ph/0606054
- **"Neutrino Physics (CERN-2014-001)",** Zhi-Zhong Xing, arXiv:1406.7739
- **"Neutrino Mass and New Physics",** R. N. Mohapatra, A. Y. Smirnov, Ann.Rev.Nucl.Part.Sci. 56 (2006) 569-628, hep-ph/0603118
- **"Leptonic CP Violation",** G. C. Branco, R. Gonzalez Felipe, F. R. Joaquim, Rev.Mod.Phys. 84 (2012) 515-565, arXiv:1111.5332
- **"From the trees to the forest: a review of radiative neutrino mass models",** Yi Cai, Juan Herrero-García, Michael A. Schmidt, Avelino Vicente, Raymond R. Volkas, arXiv:1706.08524
- **"Lepton flavor symmetries",** Ferruccio Feruglio, Andrea Romanino, Rev.Mod.Phys. 93 (2021) 1, 015007, arXiv:1912.06028
- **"Neutrino mass and mixing with modular symmetry",** Gui-Jun Ding, Stephen F. King, Rept.Prog.Phys. 87 (2024) 8, 084201, arXiv:2311.09282



Neutrino is a portal to new physics!

CERN mug with SM Lagrangian



←vector bosons and …

←fermion interactions …

←Higgs & fermion Yukawa …

←Higgs & vector bosons
(+Higgs self-interaction
potential V)

Flavor structure of SM

SM Gauge group: $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$

i=1,2,3→generation index

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$q_{iL} = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix}$	**3**	**2**	**1/6**
$U_{iR} = u_R, c_R, t_R$	**3**	**1**	**2/3**
$D_{iR} = d_R, s_R, b_R$	**3**	**1**	**−1/3**
$\ell_{iL} = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	**1**	**2**	**−1/2**
$E_{iR} = e_R, \mu_R, \tau_R$	**1**	**1**	**−1**
$\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	**1**	**2**	**1/2**

- The fermion content is "chiral": left-handed and right-handed fermions transform differently under G_{SM}
- Explicit fermion mass term is forbidden
- No right-handed neutrinos→neutrinos are massless at renormalizable level

Quark and charged lepton masses

- The Yukawa interactions are the most general **gauge invariant** and **renormalizable** terms in the Lagrangian that involve fermions and the Higgs doublet

$$\mathcal{L}_{Yukawa} = -(Y_u)_{ij}\,\overline{q_{iL}}\,\tilde{\phi}U_{jR} - (Y_d)_{ij}\overline{q_{iL}}\phi D_{jR} - (Y_\ell)_{ij}\overline{\ell_{iL}}\phi E_{jR} + \mathrm{h.c.}$$

Up-type quark masses   down-type quark masses   charged lepton masses

Conjugated Higgs doublet $\tilde{\phi} \equiv i\sigma_2\phi^*$: $(1,2,-1/2)$ under $(SU(3)_c, SU(2)_L, U(1)_Y)$

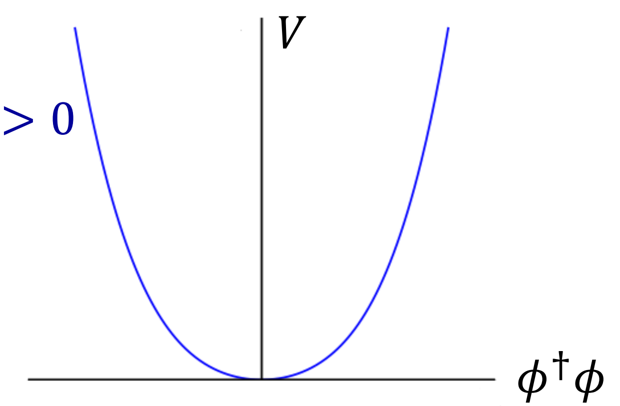
Yukawa matrices $Y_{u,d,\ell}$ are arbitrary 3×3 complex matrices in flavor space

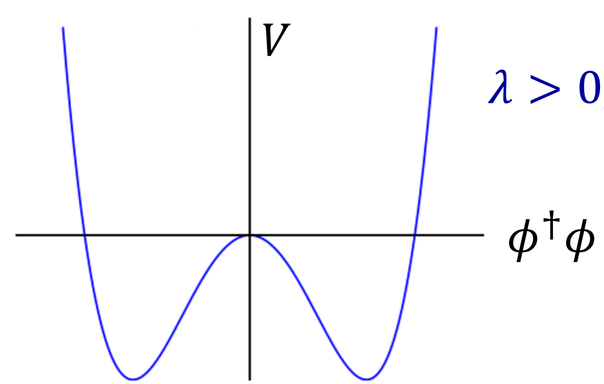
- **Higgs mechanism**

The most general scalar potential allowed by SM gauge group

$$V(\phi) = \mu^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2 \qquad \phi \equiv \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}$$

$\lambda > 0, \mu^2 > 0$



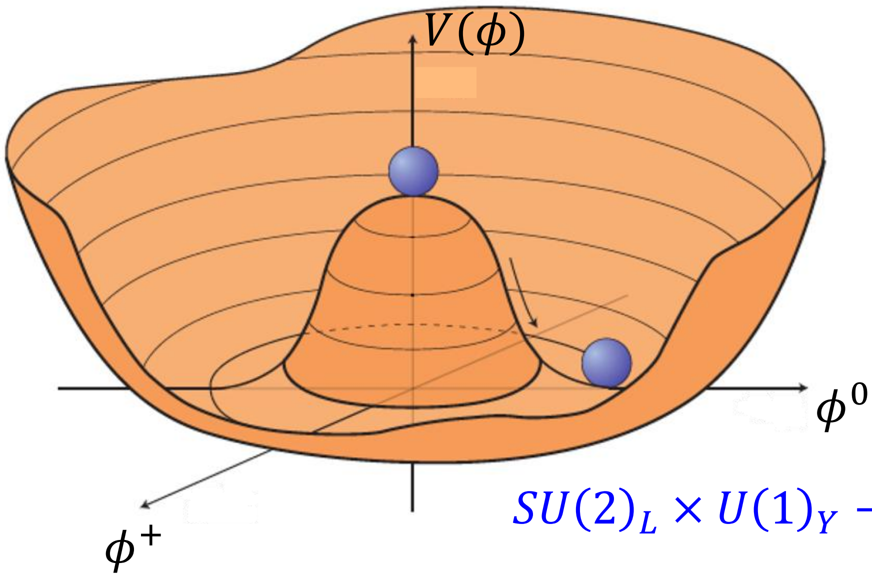


$\lambda > 0, \mu^2 < 0$

Minimizing the scalar potential, one finds the Higgs vacuum expectation value (VEV):

$$\langle \phi \rangle \equiv \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \Rightarrow \langle \tilde{\phi} \rangle = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$$

$$v = \sqrt{-\mu^2/\lambda} \simeq 246.22\text{GeV}$$



$SU(2)_L \times U(1)_Y \to U(1)_{EM}$

Unitary gauge: $$\phi \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

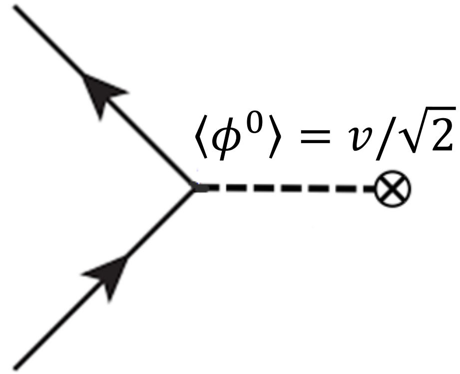
The neutral real component h is the only physical state

- After electroweak symmetry breaking, Yukawa interactions give rise to fermion mass matrices

$$(Y_u)_{ij}\, \overline{Q_{iL}}\, \tilde{\phi} U_{jR} \xrightarrow{\langle\phi\rangle} \overline{u_{iL}}(M_u)_{ij} U_{jR}, \quad M_u = \frac{v}{\sqrt{2}} Y_u$$

$$(Y_d)_{ij} \overline{Q_{iL}} \phi D_{jR} \xrightarrow{\langle\phi\rangle} \overline{d_{iL}}(M_d)_{ij} D_{jR}, \quad M_d = \frac{v}{\sqrt{2}} Y_d$$

$$(Y_\ell)_{ij} \overline{\ell_{iL}} \phi E_{jR} \xrightarrow{\langle\phi\rangle} \overline{e_{iL}}(M_\ell)_{ij} E_{jR}, \quad M_\ell = \frac{v}{\sqrt{2}} Y_\ell$$



$$\mathcal{L}_{SM} \supset -(\overline{u_L}, \overline{c_L}, \overline{t_L}) \underbrace{M_u}_{3\text{x}3} \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} - (\overline{d_L}, \overline{s_L}, \overline{b_L}) \underbrace{M_d}_{3\text{x}3} \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} - (\overline{e_L}, \overline{\mu_L}, \overline{\tau_L}) \underbrace{M_\ell}_{3\text{x}3} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$$

The fermion mass matrices $M_{u,d,\ell}$ can be diagonalized through bi-unitary transformations

$$\left.\begin{aligned} V_{Lu}^\dagger M_u V_{Ru} &= \text{diag}(m_u, m_c, m_t) \\ V_{Ld}^\dagger M_d V_{Rd} &= \text{diag}(m_d, m_s, m_b) \\ V_{L\ell}^\dagger M_d V_{R\ell} &= \text{diag}(m_e, m_\mu, m_\tau) \end{aligned}\right\}$$

Quark and charged lepton masses are proportional to the Higgs VEV v

➢ rotating from the interaction basis to the mass basis by field redefinition

LH fields:
$$\begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix} = \underbrace{V_{Lu}}_{3\text{x}3} \begin{pmatrix} u'_L \\ c'_L \\ t'_L \end{pmatrix}, \quad \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} = \underbrace{V_{Ld}}_{3\text{x}3} \begin{pmatrix} d'_L \\ s'_L \\ b'_L \end{pmatrix}, \quad \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} = \underbrace{V_{L\ell}}_{3\text{x}3} \begin{pmatrix} e'_L \\ \mu'_L \\ \tau'_L \end{pmatrix}$$

RH fields:
$$\begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} = \underbrace{V_{Ru}}_{3\text{x}3} \begin{pmatrix} u'_R \\ c'_R \\ t'_R \end{pmatrix}, \quad \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} = \underbrace{V_{Rd}}_{3\text{x}3} \begin{pmatrix} d'_R \\ s'_R \\ b'_R \end{pmatrix}, \quad \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} = \underbrace{V_{R\ell}}_{3\text{x}3} \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix}$$

The primed fields are mass eigenstates

- $V_{Lu}, \ V_{Ru}, V_{Ld}, \ V_{Rd}, V_{L\ell}, \ V_{R\ell}$ are 3-dimensional unitary matrices

$$V_{Lu}^\dagger V_{Lu} = V_{Lu} V_{Lu}^\dagger = V_{Ru}^\dagger V_{Ru} = V_{Ru} V_{Ru}^\dagger = 1$$

$$V_{Ld}^\dagger V_{Ld} = V_{Ld} V_{Ld}^\dagger = V_{Rd}^\dagger V_{Rd} = V_{Rd} V_{Rd}^\dagger = 1$$

$$V_{L\ell}^\dagger V_{L\ell} = V_{L\ell} V_{L\ell}^\dagger = V_{R\ell}^\dagger V_{R\ell} = V_{R\ell} V_{R\ell}^\dagger = 1$$

- LH up-type quarks $(u_L, c_L, t_L)^T$ and down-type quarks $(d_L, s_L, b_L)^T$ are rotated separately

Quark mixing: Cabbibo-Kobayashi-Maskawa (CKM)

basis where CC and NC diagonal ≠ mass eigenbasis

The charged current (CC) interaction in weak basis:

$$-\mathcal{L}_{CC} = \frac{g}{\sqrt{2}}\left[\overline{u_{iL}}\gamma^{\mu} d_{iL} + \overline{\nu_{iL}}\gamma^{\mu} e_{iL}\right] W_{\mu}^{+} + \text{h.c.}$$

In the mass eigenstate basis

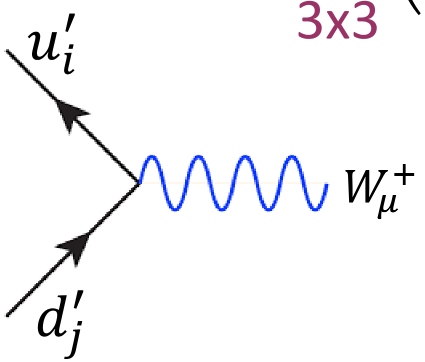
$$-\mathcal{L}_{CC} = \frac{g}{\sqrt{2}}\left[\overline{u'_{iL}}\gamma^{\mu} \underbrace{\left(V_{Lu}^{\dagger} V_{Ld}\right)_{ij}}_{\text{CKM}} d'_{jL} + \overline{\nu_{iL}}\gamma^{\mu} V_{L\ell} e'_{jL}\right] W_{\mu}^{+} + \text{h.c.}$$

$$V_{CKM} \equiv V_{Lu}^{\dagger} V_{Ld}$$

Neutrinos are massless in the SM, one can always redefine neutrino fields as

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{3L} \end{pmatrix} = \underbrace{V_{L\ell}}_{3\text{x}3} \begin{pmatrix} \nu'_{eL} \\ \nu'_{\mu L} \\ \nu'_{\tau L} \end{pmatrix}$$

$$\frac{g}{\sqrt{2}} \overline{\nu_{iL}}\gamma^{\mu} V_{L\ell} e'_{jL} W_{\mu}^{+} = \frac{g}{\sqrt{2}} \overline{\nu'_{iL}}\gamma^{\mu} e'_{jL} W_{\mu}^{+}$$



$$i\frac{g}{\sqrt{2}}\gamma^{\mu}\frac{1-\gamma_5}{2}(V_{CKM})_{ij}$$

No lepton mixing and no neutrino oscillations in SM!

CKM Parametrization

Not all entries are independent: 3 Euler angles $\theta_{12}, \theta_{13}, \theta_{23}$ and 1 complex phase δ

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$c_{ij} \equiv \cos\theta_{ij}, s_{ij} \equiv \sin\theta_{ij}$

$$|V_{ud}| \sim |V_{cs}| \sim |V_{tb}| \sim 1$$

$$|V_{us}| \sim |V_{cd}| \sim 0.22$$

$$|V_{cb}| \sim |V_{ts}| \sim 0.04$$

$$|V_{ub}| \sim |V_{td}| \sim 0.005$$

hierarchical

$s_{13} << s_{23} << s_{12} << 1$

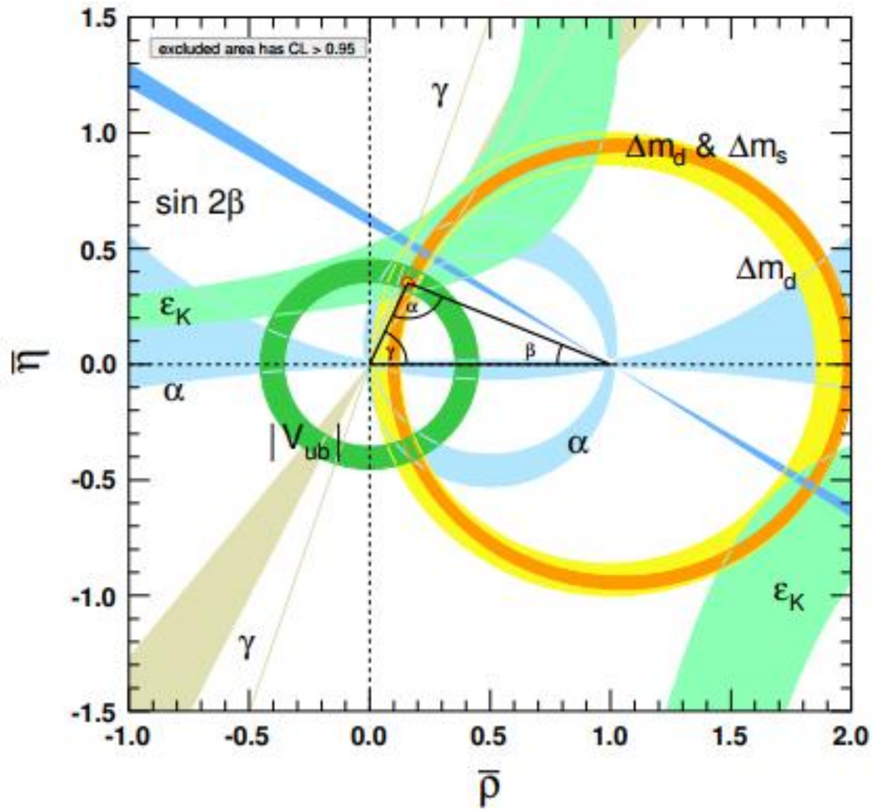
Wolfenstein parametrization

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$\lambda = s_{12}, \quad A\lambda^2 = s_{23}, \quad A\lambda^3(\rho - i\eta) = s_{13}e^{-i\delta}$$

➢ The fitting values of the CKM elements **[Particle Data Group 2024]**

$$|V_{\rm CKM}| = \begin{pmatrix} 0.97435 \pm 0.00016 & 0.22501 \pm 0.00068 & 0.003732^{+0.000090}_{-0.000085} \\ 0.22487 \pm 0.00068 & 0.97349 \pm 0.00016 & 0.04183^{+0.00079}_{-0.00069} \\ 0.00858^{+0.00019}_{-0.00017} & 0.04111^{+0.00077}_{-0.00068} & 0.999118^{+0.000029}_{-0.000034} \end{pmatrix}$$



$$\sin\theta_{12} = 0.22501 \pm 0.00068$$

$$\sin\theta_{13} = 0.003732^{+0.000090}_{-0.000085}$$

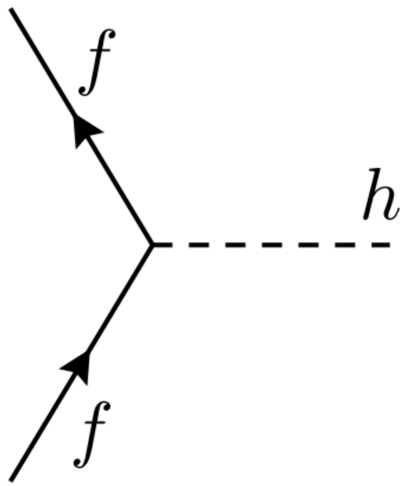
$$\sin\theta_{23} = 0.04183^{+0.00079}_{-0.00069}$$

$$\delta = 1.147 \pm 0.026$$

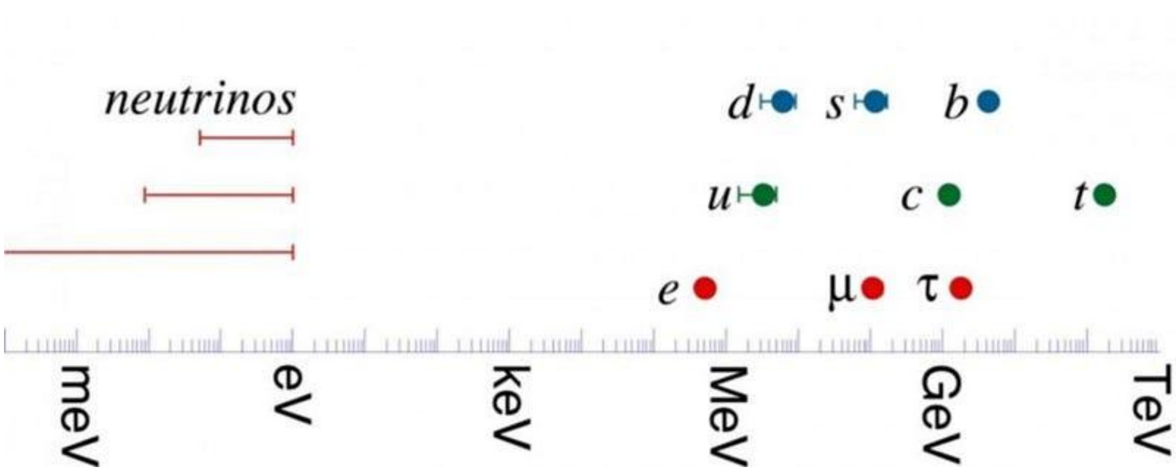
Higgs-fermion couplings

The Higgs-fermion coupling is proportional to the fermion mass

$$\left(1+\frac{h}{v}\right)\left(-m_{u_i}\overline{u_{iL}}\;u_{iR} - m_{d_i}\overline{d_{iL}}\;u_{iR} - m_{\ell_i}\overline{e_{iL}}\;e_{iR}\right) + \text{h.c.}$$

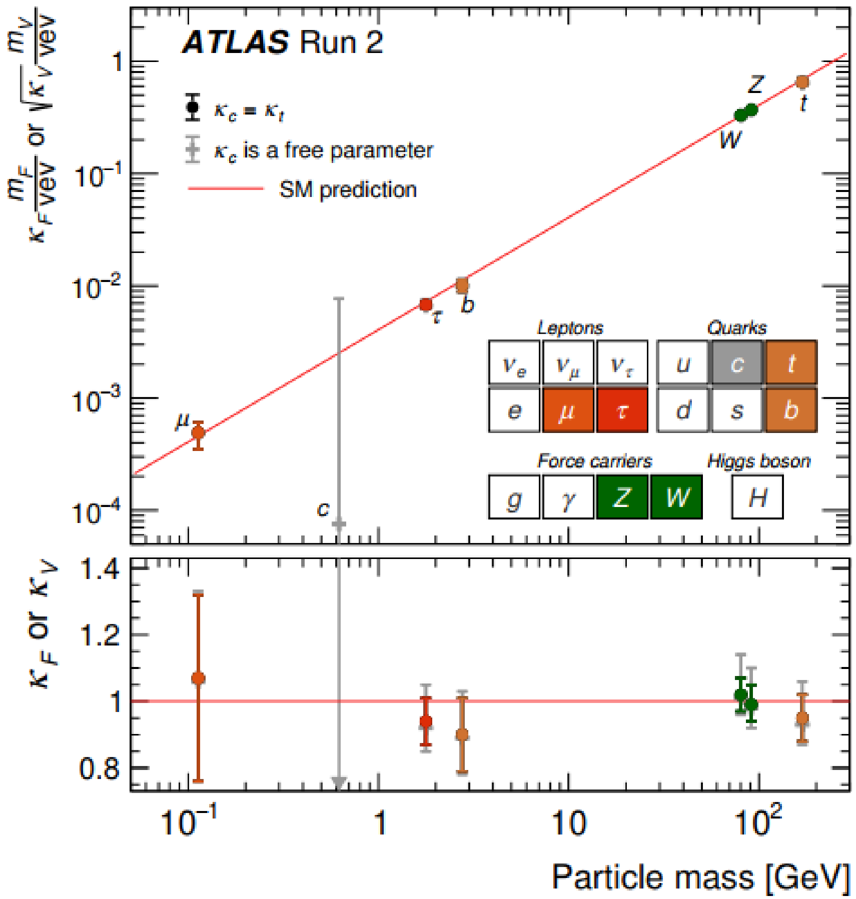


$$-i\,\frac{g}{2}\,\frac{m_f}{m_W}$$

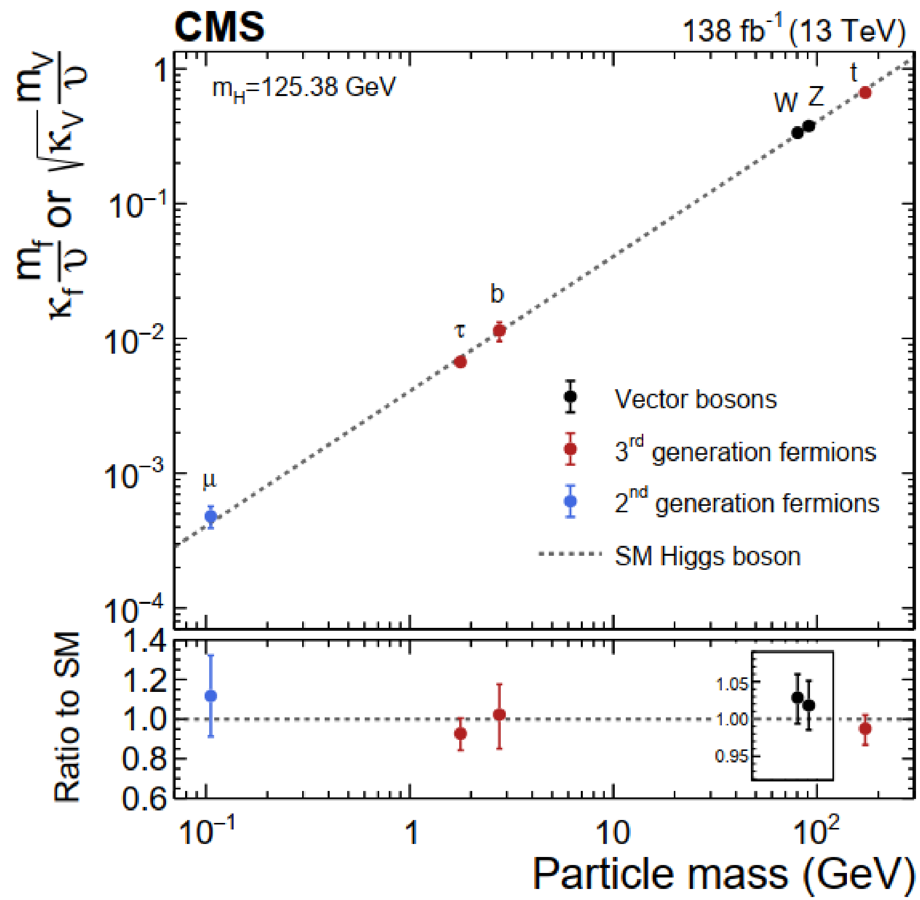


Huge hierarchies among the quark and lepton masses

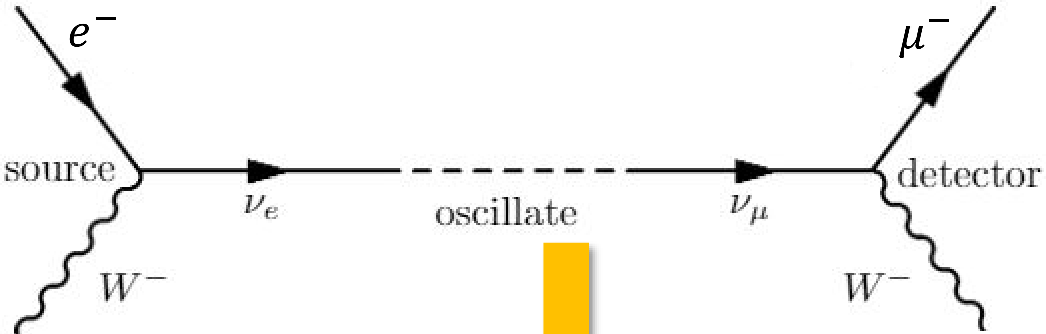
Nature 607(2022) 52, arXiv:2207.00092



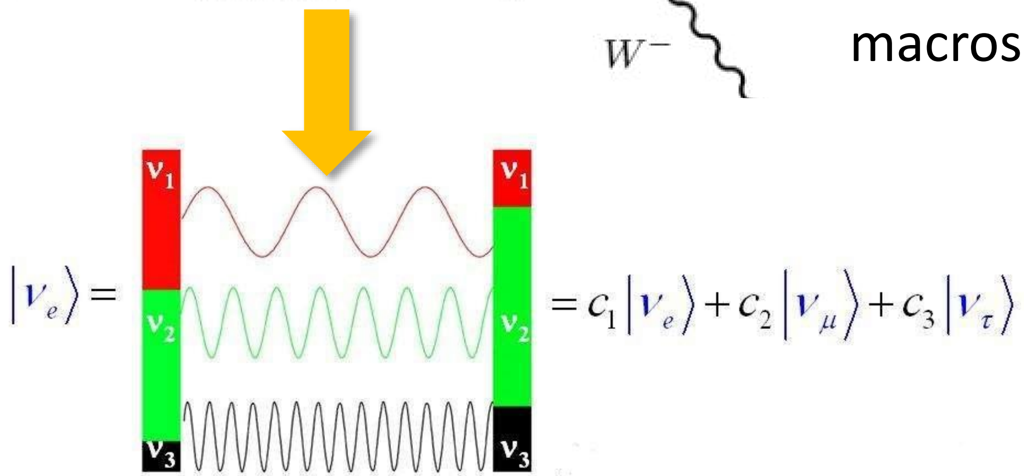
Nature 607(2022) 60, arXiv:2207.00043

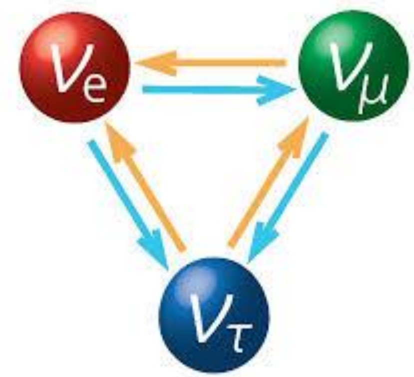


Neutrino oscillation



observed with various sources and techniques→ quantum mechanical interference on macroscopic distances





Two-flavor limit:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

See lectures by Yu-Feng Li

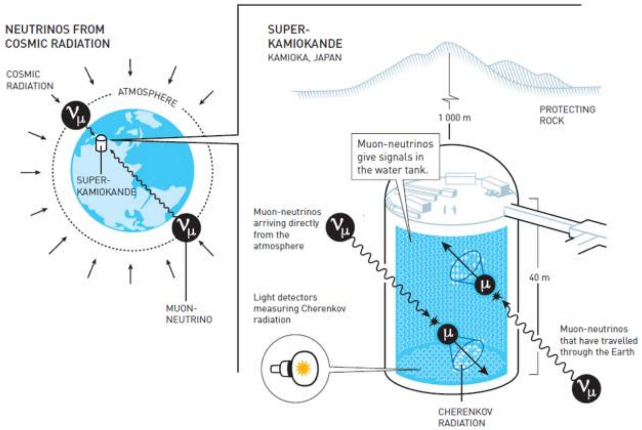
$$P(\nu_e \to \nu_\mu) = \left|\langle \nu_\mu(0) | \nu_e(t) \rangle\right|^2 = \sin^2\theta\cos^2\theta \left| e^{-im_2^2 L/(2E)} - e^{-im_1^2 L/(2E)} \right|^2$$

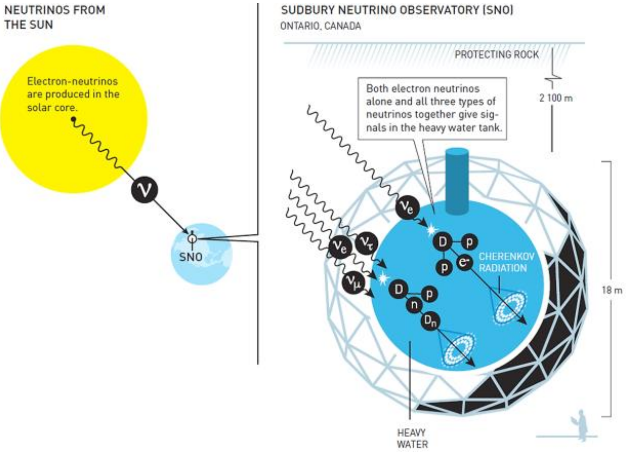
$$= \sin^2 2\theta \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right), \qquad \Delta m_{21}^2 \equiv m_2^2 - m_1^2$$

2015 Nobel Prize in Physics

Takaaki Kajita

Arthur B. McDonald





"for the discovery of neutrino oscillations, which shows that neutrinos have mass"

SM+3 massive neutrinos: global fit

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & s_{13}e^{-i\delta_{CP}}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

Global fit-Normal hierarchy

$\Delta m^2_{21}=7.49^{+0.19}_{-0.19}\times 10^{-5}\mathrm{eV}^2$

$\Delta m^2_{31}=2.513^{+0.021}_{-0.019}\times 10^{-3}\mathrm{eV}^2$

$\theta_{12}= 33.68^{+0.73}_{-0.70}(^\circ)$

$\theta_{23}= 43.3^{+1.0}_{-0.8}(^\circ)$

$\theta_{13}= 8.56^{+0.11}_{-0.11}(^\circ)$

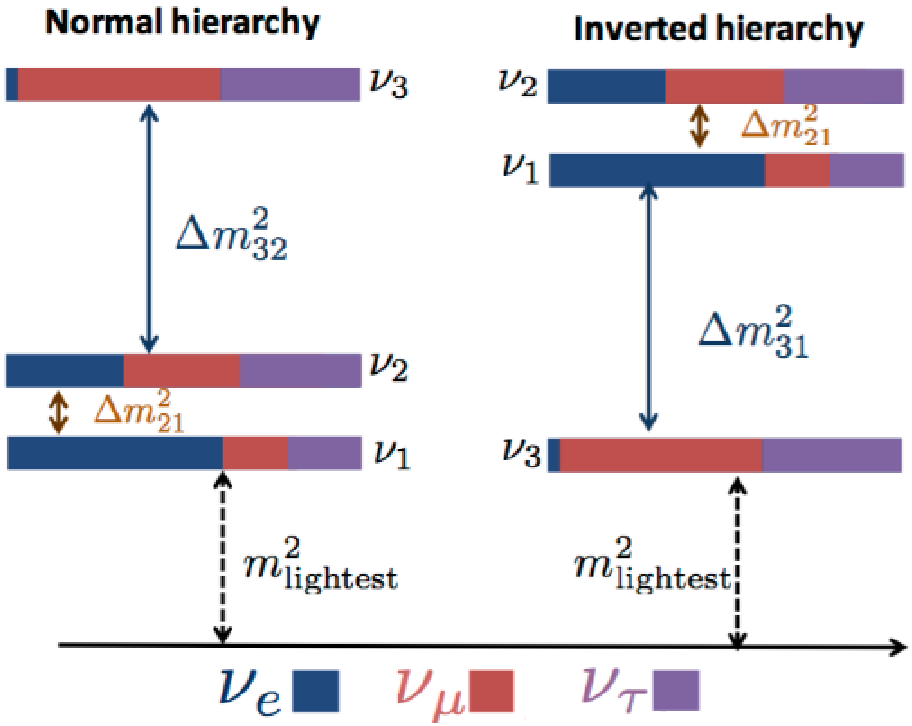
$\mathrm{sign}(\Delta m^2_{32})=?$

$\theta_{23}< 45^\circ$ or $\theta_{23}> 45^\circ$?

$\delta_{CP}=?$

$m_{\mathrm{lightest}}=?$

[Ivan Esteban et al., NuFIT6.0 (2024)]



$$\Delta m^2_{ij} = m^2_i - m^2_j$$

Massive neutrinos: Dirac versus Majorana

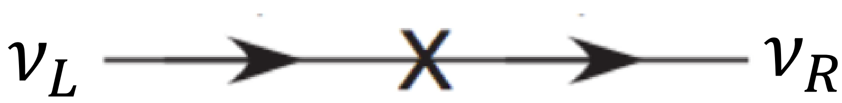
Dirac neutrino of mass m:

$$\mathcal{L}^m_{Dirac} = -m\overline{\nu_L}\nu_R - m\overline{\nu_R}\nu_L$$
$$= -m\overline{\nu_D}\,\nu_D$$

$$\nu_D \equiv \nu_L + \nu_R \Rightarrow \nu_D^c \neq \nu_D$$

Dirac: $\nu_D = \nu_L + \nu_R \rightarrow$ LH and RH components are **independent**

- break SM gauge invariance



Majorana neutrino of mass m:

$$\mathcal{L}^m_{Majorana} = -\frac{1}{2}m\overline{\nu_L^c}\nu_L - \frac{1}{2}m\overline{\nu_L}\nu_L^c$$
$$= -\frac{1}{2}m\nu_L^T C\nu_L - \frac{1}{2}m\overline{\nu_L}C\overline{\nu_L}^T$$
$$= -\frac{1}{2}m\overline{\nu_M}\nu_M$$

$$\nu_M \equiv \nu_L + \nu_L^c \Rightarrow \nu_M = \nu_M^c$$

Majorana: $\nu_M = \nu_L + \nu_L^c \rightarrow$ LH and RH components are **NOT independent**

- break SM gauge invariance

$$P_L = \frac{1-\gamma_5}{2},\quad P_R = \frac{1+\gamma_5}{2},\quad \psi_L \equiv P_L\psi,\ \psi_R \equiv P_R\psi,$$
$$\bar{\psi} \equiv \psi^\dagger\gamma^0,\quad \psi^c = C\bar{\psi}^T = C\gamma^0\psi^*,\quad \overline{\psi^c} \equiv -\psi^T C^{-1},$$
$$C = i\gamma^2\gamma^0,\quad C^\dagger = C^{-1},\quad C^T = -C$$

Massive Dirac neutrinos in SM

Introduce three right-handed neutrino ν_{iR} which are SM singlets,

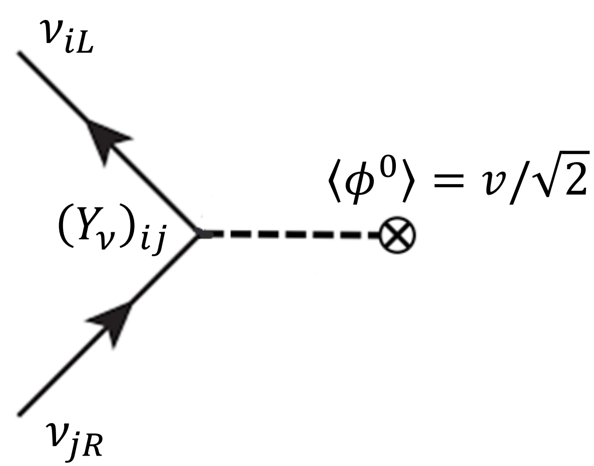
$\nu_{iR} \sim (1,1,0)$ under $(SU(3)_c, SU(2)_L, U(1)_Y)$

Massive Dirac neutrino via Yukawa coupling: SM+ν_R

$$\mathcal{L}^{\nu}_{Yukawa} = -(Y_\nu)_{ij}\, \overline{\ell_{iL}}\, \tilde{\phi}\nu_{jR} + \text{h.c.}$$

EWSB: $\langle\tilde{\phi}\rangle = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$ $\qquad M_\nu = \frac{v}{\sqrt{2}} Y_\nu$

$$\mathcal{L}^{\nu}_{Dirac} = -\,\overline{\nu_{iL}}\,(M_\nu)_{ij}\nu_{jR} + \text{h.c.}$$



Neutrino mass-eigenstate basis:

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \underbrace{V_{L\nu}}_{3\text{x}3} \begin{pmatrix} \nu'_{1L} \\ \nu'_{1L} \\ \nu'_{1L} \end{pmatrix}, \quad \begin{pmatrix} \nu_{1R} \\ \nu_{2R} \\ \nu_{3R} \end{pmatrix} = \underbrace{V_{R\nu}}_{3\text{x}3} \begin{pmatrix} \nu'_{1R} \\ \nu'_{2R} \\ \nu'_{3R} \end{pmatrix}$$

$$\Rightarrow \quad V^{\dagger}_{L\nu} M_\nu V_{R\nu} = \text{diag}(m_1, m_2, m_3)$$

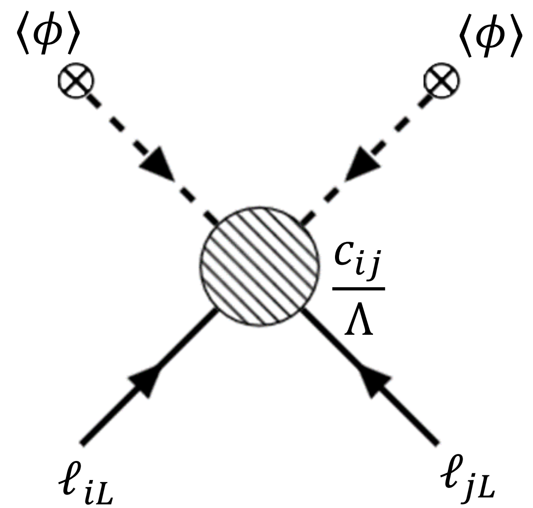
Lepton number conservation:
$\ell_{iL} \to e^{i\varphi}\ell_{iL}$,
$\nu_{iR} \to e^{i\varphi}\nu_{iR}$

tiny Yukawa couplings $\quad (Y_\nu)_{ij} \sim \frac{\sqrt{2}m_\nu}{v} \sim \frac{0.2\text{ eV}}{200\text{ GeV}} \sim 10^{-12}$

Massive Majorana neutrinos in SM

Massive Majorana neutrino via dim-5 Weinberg operator

$$\mathcal{L}^{\nu}_{\text{Weinberg}} = -\frac{1}{2}\frac{c_{ij}}{\Lambda}\left(\overline{\ell^{c}_{iL}}\,\tilde{\phi}^{*}\right)\left(\tilde{\phi}^{\dagger}\ell_{iL}\right) + \text{h. c.}$$



EWSB: $\langle\tilde{\phi}\rangle = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$

$$M_{\nu} = c_{ij}\frac{v^2}{2\Lambda}$$

$$\mathcal{L}^{m}_{\text{Majorana}} = -\frac{1}{2}\overline{\nu^{c}_{iL}}\,(M_{\nu})_{ij}\nu_{jL} + \text{h. c.}$$

Neutrino mass-eigenstate basis:

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \underbrace{V_{L\nu}}_{3\text{x}3} \begin{pmatrix} \nu'_{1L} \\ \nu'_{2L} \\ \nu'_{3L} \end{pmatrix} \Rightarrow V^{T}_{L\nu} M_{\nu}\, V_{L\nu} = \text{diag}(m_1, m_2, m_3)$$

Rough estimate: $\Lambda \sim \frac{v^2}{2m_{\nu}} \sim \frac{200\text{^}2}{2\times 0.1}\text{GeV} \sim 10^{14}\text{GeV}$

Lepton number **violation** under $\ell_{iL} \to e^{i\varphi}\ell_{iL}$:

$$\mathcal{L}^{\nu}_{\text{Weinberg}} \to e^{2i\varphi}\,\mathcal{L}^{\nu}_{\text{Weinberg}}$$

Lepton mixing: Pontecorvo–Maki–Nakagawa–Sakata (PMNS)

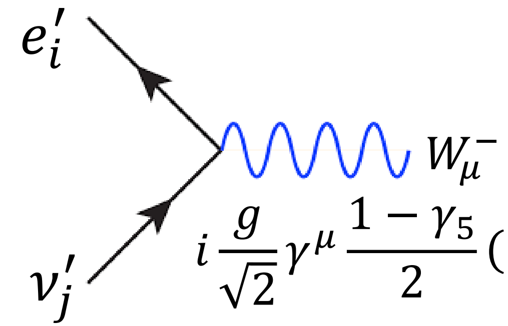
Lepton charged current (CC) interaction in weak basis:

$$-\mathcal{L}_{CC}^{\ell} = \frac{g}{\sqrt{2}} \overline{e_{iL}} \gamma^{\mu} \nu_{iL} W_{\mu}^{-} + \text{h.c.}$$

In the mass eigenstate basis $e_{iL} = (V_{Le})_{ij} e'_{jL}, \;\; \nu_{iL} = (V_{L\nu})_{ij} \nu'_{jL}$

$$-\mathcal{L}_{CC}^{\ell} = \frac{g}{\sqrt{2}} \overline{e'_{iL}} \gamma^{\mu} \underbrace{\left(V_{L\ell}^{\dagger} V_{L\nu}\right)_{ij}}_{\text{PMNS}} \nu'_{jL} W_{\mu}^{-} + \text{h.c.}$$

$$U_{PMNS} \equiv V_{L\ell}^{\dagger} V_{L\nu}$$



$$i \frac{g}{\sqrt{2}} \gamma^{\mu} \frac{1-\gamma_5}{2} (U_{PMNS})_{ij}$$

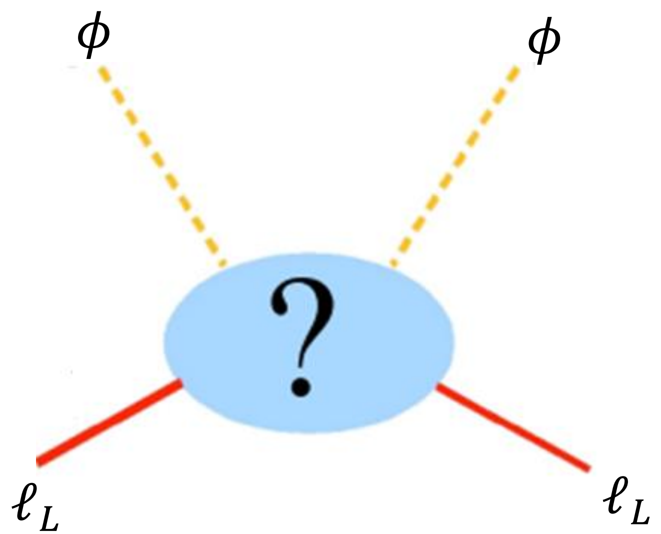
In standard parametrization

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix}$$

Atmospheric mixing	Reactor mixing & Dirac CP phase	Solar mixing	Majorana CP phases
$\theta_{23} \sim 43.3^{\circ}$	$\theta_{13} \sim 8.56^{\circ}, \;\; \delta_{CP} \sim 212^{\circ}?$	$\theta_{12} \sim 33.68^{\circ}$	$\alpha_{21}, \alpha_{31} \sim ?$

The Majorana CP violation phases α_{21}, α_{31} are unphysical for Dirac neutrinos

UV completion of Weinberg operator at tree-level



- Transformation properties:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim (2, 1/2)$$

$$\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim (2, -1/2)$$

- $SU(2)_L$ contractions

$$\mathbf{2} \otimes \mathbf{2} \otimes \mathbf{2} \otimes \mathbf{2} = (\mathbf{3} \oplus \mathbf{1}) \otimes (\mathbf{3} \oplus \mathbf{1})$$

similar to the composition of four ½-spin

$$\mathcal{O}_1 = (\ell_{iL}\phi)_1(\ell_{jL}\phi)_1 \qquad \mathcal{O}_2 = (\ell_{iL}\ell_{jL})_1(\phi\phi)_1$$

✗ **vanishing because of antisymmetry**

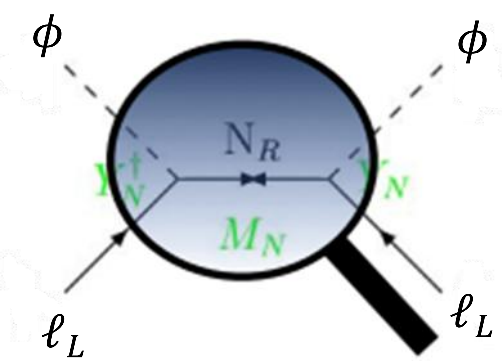
$$\mathcal{O}_3 = (\ell_{iL}\phi)_3(\ell_{jL}\phi)_3 \qquad \mathcal{O}_4 = (\ell_{iL}\ell_{jL})_3(\phi\phi)_3$$

If neutrino are Majorana particles, a **Majorana mass** can arise as the **low energy realization of a higher energy theory (new mass scale!)**

Three types of seesaw mechanism

$\mathcal{O}_1 = (\ell_{iL}\phi)_1(\ell_{jL}\phi)_1$ $\quad$ $\mathcal{O}_4 = (\ell_{iL}\ell_{jL})_3(\phi\phi)_3$ $\quad$ $\mathcal{O}_3 = (\ell_{iL}\phi)_3(\ell_{jL}\phi)_3$

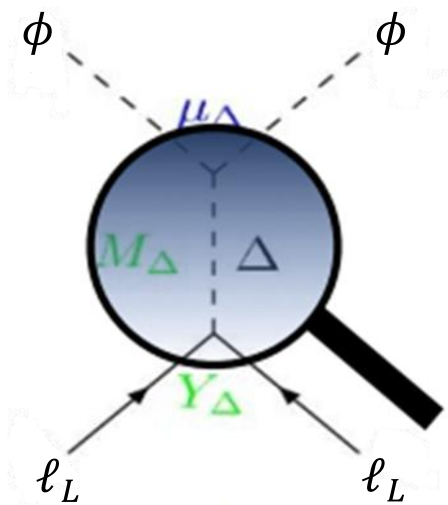
Type I see-saw:
a heavy singlet scalar



$$m_\nu = \frac{\mathcal{O} v^2}{\Lambda} \equiv Y_N^T \frac{v^2}{M_N} Y_N$$

Minkowski;
Yanagida; Glashow;
Gell-Mann, Ramond Slansky;
Mohapatra, Senjanovic...

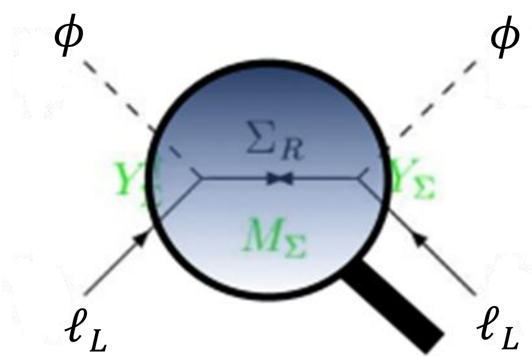
Type II see-saw:
a heavy triplet scalar



$$m_\nu = \frac{\mathcal{O} v^2}{\Lambda} \equiv Y_\Delta \frac{\mu_\Delta}{M_\Delta^2} v^2$$

Konetschny, Kummer;
Cheng, Li;
Lazarides, Shafi, Wetterich ...

Type III see-saw:
a heavy triplet fermion

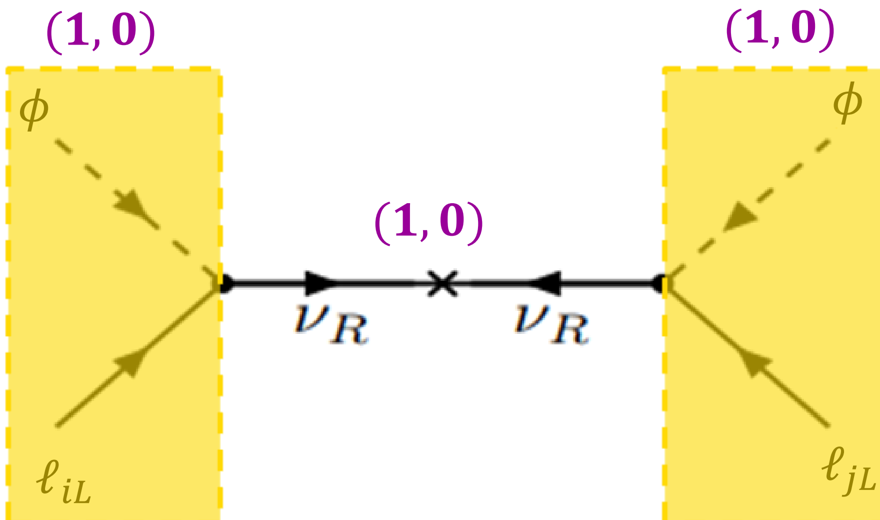


$$m_\nu = \frac{\mathcal{O} v^2}{\Lambda} \equiv Y_\Sigma^T \frac{v^2}{M_\Sigma} Y_\Sigma$$

Foot et al; Ma;
Bajc, Senjanovic...

type-I seesaw mechanism

Higgs-lepton coupling: $\overline{\ell^c_{iL}}\,\tilde{\phi}^*, \tilde{\phi}^\dagger \ell_{iL} \sim (2,-1/2) \otimes (2,1/2) = (3,0) \oplus (1,0)$



$(1,0) \otimes (1,0) = (1,0)$

Field content: SM fields+ n_R **RH neutrinos** ν_R

$$\mathcal{L}_{\rm full} = \mathcal{L}_{\rm SM} + i\overline{\nu_{iR}}\partial\!\!\!/\,\nu_{iR} - \left[(Y_\nu)_{ij}\,\overline{\ell_{iL}}\,\tilde{\phi}\,\nu_{jR} + \frac{1}{2}(M_R)_{ij}\overline{\nu^c_{iR}}\,\nu_{jR} + \text{h.c.}\right]$$

$(M_R)_{ij}$ is not protected by any symmetry, hence it can be very large

EWSB: $\langle\phi\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$ ⟹ $\mathcal{L}^\nu_{\rm mass} = -(M_D)_{ij}\overline{\nu_{iL}}\,\nu_{jR} - \frac{1}{2}(M_R)_{ij}\overline{\nu^c_{iR}}\,\nu_{jR} + \text{h.c.}$

$M_D \equiv Y_\nu \frac{v}{\sqrt{2}}$

Define a vector of LH fields with $3+n_R$ components

$$N_L = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}, \quad \nu_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}, \quad \nu_R^c = \begin{pmatrix} \nu_{1R}^c \\ \vdots \\ \nu_{n_R R}^c \end{pmatrix}$$

The neutrino Dirac-Majorana mass Lagrangian

$$\mathcal{L}_{\text{mass}}^{\nu} = -\frac{1}{2}(M_{D+M})_{ij}\overline{N_{iL}}N_{jL}^c + \text{h.c.}$$

$$M_{D+M} = \begin{pmatrix} \overbrace{0}^{3\text{x}3} & \overbrace{M_D}^{3\text{x}n_R} \\ \underbrace{M_D^T}_{n_R\text{x}3} & \underbrace{M_R}_{n_R\text{x}n_R} \end{pmatrix}$$

$(3+n_R)\times(3+n_R)$ **complex symmetric** mass matrix

In mass eigenstate basis

weak states ← $N_L = U_L N_L'$ → mass eigenstates

↓ unitary $(3+n_R)\times(3+n_R)$ complex matrix

$$N_L' = \begin{pmatrix} \nu_{1L}' \\ \vdots \\ \nu_{nL}' \end{pmatrix}$$

$n \equiv 3+n_R$

$U_L^\dagger M_{D+M} U_L^* = \text{diag}(m_1, m_2, \dots, m_n)$, $\quad m_1, m_2, \dots, m_n$ **positive** neutrino masses massive Majorana neutrinos

$$\mathcal{L}^{\nu}_{\text{mass}} = -\frac{1}{2}(M_{D+M})_{ij}\overline{N_{iL}}N^{c}_{jL} + \text{h.c.}$$

$$N'_L = \begin{pmatrix} \nu'_{1L} \\ \vdots \\ \nu'_{nL} \end{pmatrix}, \quad n \equiv 3 + n_R$$

$$U_L^{\dagger} M_{D+M} U_L^{*} = \text{diag}(m_1, m_2, \dots, m_n)$$

$$N_L = U_L N'_L$$

$$\mathcal{L}^{\nu}_{\text{mass}} = -\frac{1}{2}\overline{N'_L}\, U_L^{\dagger} M_{D+M} U_L^{*}\, N'_L + \text{h.c.} = -\frac{1}{2}\sum_{i=1}^{n} m_i \overline{\nu'_{iL}}\, \nu'^{c}_{iL} + \text{h.c.}$$

Majorana fields $\nu'_i = \nu'_{iL} + \nu'^{c}_{iL}$

$$\mathcal{L}^{\nu}_{\text{mass}} = -\frac{1}{2}\sum_{i=1}^{n} m_i \bar{\nu}'_i\, \nu'_i$$

The general Dirac-Majorana mass term leads to $3 + n_R$ massive Majorana neutrinos.

➢ Seesaw formula

In the limit $M_R \gg M_D$, there will be definitely a light and a heavy neutrino sector

Block-diagonalize M_{D+M} to separate the heavy and light sectors

$$N_L = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}, \quad N_L \to W_L N_L$$

$$W_L = \begin{pmatrix} \sqrt{1-BB^\dagger} & B \\ -B^\dagger & \sqrt{1-B^\dagger B} \end{pmatrix} \approx \begin{pmatrix} 1-\frac{1}{2}BB^\dagger & B \\ -B^\dagger & 1-\frac{1}{2}B^\dagger B \end{pmatrix}, \quad B \approx M_D M_R^{-1}$$

$$\Rightarrow \quad W_L^\dagger M_{D+M} W_L^* = \begin{pmatrix} M_\nu & 0 \\ 0 & M_N \end{pmatrix}$$

The diagonal blocks are:

$$M_\nu = -M_D M_R^{-1} M_D^T + \frac{1}{2} M_D M_R^{-1}\left(M_D^T M_D^* M_R^{-1*} + M_R^{-1*} M_D^\dagger M_D\right) M_R^{-1} M_D^T + \cdots$$

$$M_N = M_R + \frac{1}{2}\left(M_D^T M_D^* M_R^{-1*} + M_R^{-1*} M_D^\dagger M_D\right) + \cdots$$

at lowest order in the expansion, one has:

$$M_\nu = -M_D M_R^{-1} M_D^T, \quad M_N = M_R$$

The blocks M_ν, M_N are generally **non-diagonal**

$$U_\nu^\dagger M_\nu U_\nu^* = \mathrm{diag}(m_1, m_2, m_3), \quad U_N^\dagger M_N U_N^* = \mathrm{diag}\left(M_1, M_2, \dots, M_{n_R}\right)$$

$$\begin{cases} U_\nu^\dagger M_\nu U_\nu^* = \mathrm{diag}(m_1, m_2, m_3) \\ U_N^\dagger M_N U_N^* = \mathrm{diag}\left(M_1, M_2, \dots, M_{n_R}\right) \end{cases} \qquad M_i \gg m_j$$

Rotation to the **mass eigenstate** basis:

$$N_L = W_L \mathrm{diag}(U_\nu, U_N) N_L'$$

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \\ \nu_{1R}^c \\ \vdots \\ \nu_{n_R R}^c \end{pmatrix} = \begin{pmatrix} \overbrace{\sqrt{1 - BB^\dagger}\, U_\nu}^{3\mathrm{x}3} & \overbrace{BU_N}^{3\mathrm{x}n_R} \\ \underbrace{-B^\dagger U_\nu}_{n_R\mathrm{x}3} & \underbrace{\sqrt{1 - B^\dagger B}\, U_N}_{n_R\mathrm{x}n_R} \end{pmatrix} \begin{pmatrix} \nu_{1L}' \\ \nu_{2L}' \\ \nu_{3L}' \\ N_{1L}' \\ \vdots \\ N_{n_R L}' \end{pmatrix}$$

Heavy-light neutrino mixing is **small** in type-I seesaw

CC and NC interactions in type-I seesaw

- in weak basis

CC interactions: $\mathcal{L}_{CC}^{\ell} = -\frac{g}{\sqrt{2}} \overline{e_{iL}} \gamma^{\mu} \nu_{iL} W_{\mu}^{-} + \text{h.c.}$

NC interactions: $\mathcal{L}_{NC}^{\nu} = -\frac{g}{2c_W} \overline{\nu_{iL}} \gamma^{\mu} \nu_{iL} Z_{\mu}$

- in mass eigenstate basis of type-I seesaw

$$\mathcal{L}_{CC}^{\ell} = -\frac{g}{\sqrt{2}} \left[\overline{e'_{iL}} \gamma^{\mu} \left(V_{\ell L}^{\dagger} \sqrt{1 - BB^{\dagger}}\, U_{\nu} \right)_{ij} \nu'_{jL} + \overline{e'_{iL}} \gamma^{\mu} \left(V_{\ell L}^{\dagger} B U_N \right)_{ij} N'_{jL} \right] W_{\mu}^{-} + \text{h.c.}$$

$$\mathcal{L}_{NC}^{\nu,N} = -\frac{g}{2c_W} \left[\overline{\nu'_{iL}} \gamma^{\mu} \left(U_{\nu}^{\dagger} (1 - BB^{\dagger})\, U_{\nu} \right)_{ij} \nu'_{jL} + \overline{N'_{iL}} \gamma^{\mu} \left(U_N^{\dagger} B^{\dagger} B U_N \right)_{ij} N'_{jL} \right.$$
$$\left. + \overline{\nu'_{iL}} \gamma^{\mu} \left(U_{\nu}^{\dagger} \sqrt{1 - BB^{\dagger}}\, B U_N \right)_{ij} N'_{jL} + \overline{N'_{iL}} \gamma^{\mu} \left(U_N^{\dagger} B^{\dagger} \sqrt{1 - BB^{\dagger}}\, U_{\nu} \right)_{ij} \nu'_{jL} \right] Z_{\mu}$$

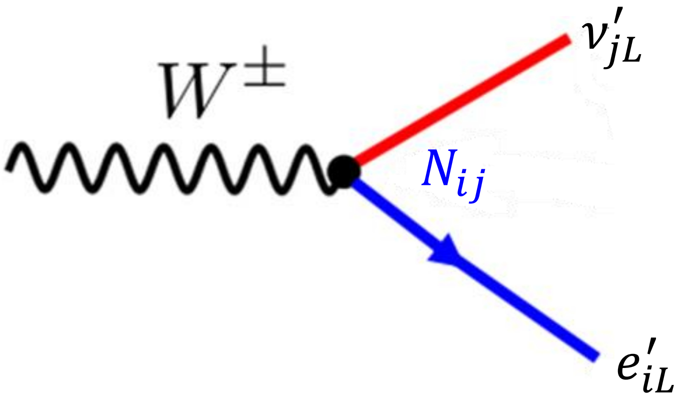
- Non-unitary (active neutrino) interactions

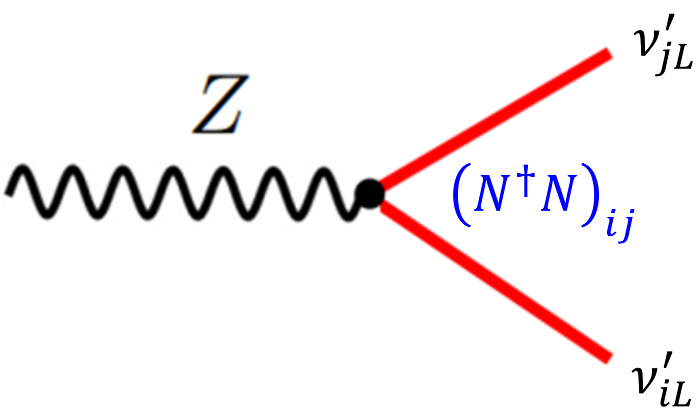
$$\mathcal{L}_{W,Z}^{\nu} = -\frac{g}{2c_W} \overline{\nu'_{iL}} \gamma^{\mu} \left(N^{\dagger} N \right)_{ij} \nu'_{jL} Z_{\mu} - \frac{g}{\sqrt{2}} \left[\overline{e'_{iL}} \gamma^{\mu} N_{ij} \nu'_{jL} W_{\mu}^{-} + \text{h.c.} \right]$$

$$N \equiv \left(1 - \frac{\varepsilon}{2} \right) U_{PMNS}, \quad U_{PMNS} = V_{\ell L}^{\dagger}\, U_{\nu}, \quad \varepsilon = V_{\ell L}^{\dagger} B B^{\dagger} V_{\ell L} \sim \frac{m_{\nu}}{M_R} \ll 1$$

Non-unitary: $N N^{\dagger} \approx 1 - \varepsilon \neq 1\,, \quad N^{\dagger} N \approx U_{PMNS}^{\dagger} (1 - \varepsilon)\, U_{PMNS} \neq 1$

- Experimental constraints on non-unitary interactions



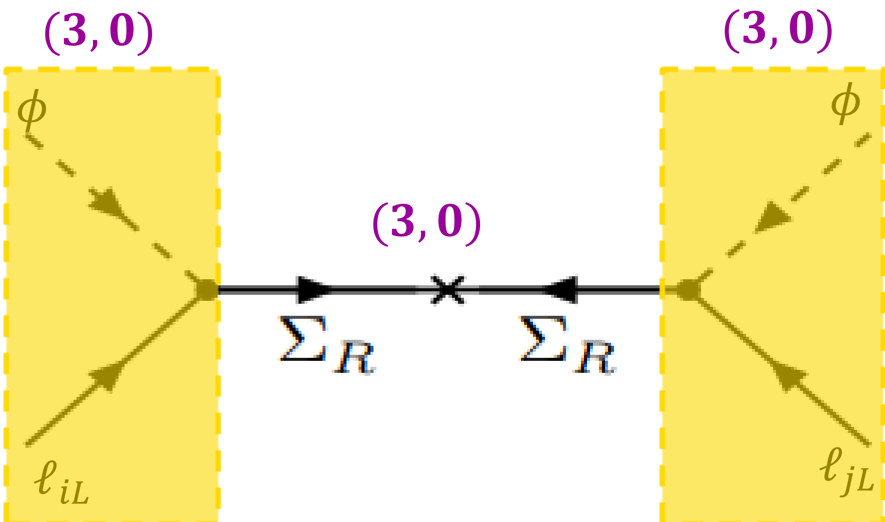


$\sqrt{2\eta_{ij}} \equiv \sqrt{\varepsilon_{ij}}$		G-SS	
		LFC	LFV
$\sqrt{2\eta_{ee}}$, $\lvert\theta_e\rvert$	1σ	$\mathbf{0.031^{+0.010}_{-0.020}}$	–
	2σ	$< \mathbf{0.050}$	–
$\sqrt{2\eta_{\mu\mu}}$, $\lvert\theta_\mu\rvert$	1σ	$< \mathbf{0.011}$	–
	2σ	$< \mathbf{0.021}$	–
$\sqrt{2\eta_{\tau\tau}}$, $\lvert\theta_\tau\rvert$	1σ	$\mathbf{0.044^{+0.019}_{-0.027}}$	–
	2σ	$< \mathbf{0.075}$	–
$\sqrt{2\eta_{e\mu}}$, $\sqrt{\lvert\theta_e\theta_\mu\rvert}$	1σ	< 0.018	$< \mathbf{4.1 \cdot 10^{-3}}$
	2σ	< 0.026	$< \mathbf{4.9 \cdot 10^{-3}}$
$\sqrt{2\eta_{e\tau}}$, $\sqrt{\lvert\theta_e\theta_\tau\rvert}$	1σ	$< \mathbf{0.045}$	< 0.107
	2σ	$< \mathbf{0.052}$	< 0.127
$\sqrt{2\eta_{\mu\tau}}$, $\sqrt{\lvert\theta_\mu\theta_\tau\rvert}$	1σ	$< \mathbf{0.024}$	< 0.115
	2σ	$< \mathbf{0.035}$	< 0.137

[Fernandez-Martinez, Hernandez-Garcia, Lopez-Pavon, arXiv:1605.08774]

type-III seesaw mechanism

Higgs-lepton coupling: $\overline{\ell^c_{iL}}\,\tilde{\phi}^*, \tilde{\phi}^\dagger \ell_{iL} \sim (2,+1/2) \otimes (2,-1/2) = (3,0) \oplus (1,0)$



$(3,0) \otimes (3,0) = (1,0) \oplus (3,0) \oplus (5,0)$

Field content: SM fields+ n_Σ **RH fermion triplets** $\vec{\Sigma}_R$

We can write each fermionic triplet as: $\vec{\Sigma}_{iR} = \left(\Sigma^{(1)}_{iR}, \Sigma^{(2)}_{iR}, \Sigma^{(3)}_{iR}\right)$

It is convenient to work with charge eigenstates and use a 2x2 representation:

$$\Sigma_{iR} = \begin{pmatrix} \frac{\Sigma^0_{iR}}{\sqrt{2}} & \Sigma^+_{iR} \\ \Sigma^-_{iR} & -\frac{\Sigma^0_{iR}}{\sqrt{2}} \end{pmatrix}, \quad \Sigma^\pm_{iR} = \frac{\Sigma^{(1)}_{iR} \mp \Sigma^{(2)}_{iR}}{\sqrt{2}}, \quad \Sigma^0_{iR} = \Sigma^{(3)}_{iR}$$

In terms of which the Lagrangian is

$$\mathcal{L}_{\rm full} = \mathcal{L}_{\rm SM} + {\rm Tr}\left[\overline{\Sigma_{iR}} i \not{D}\, \Sigma_{iR}\right] - \left[(Y_\Sigma)_{ij}\, \overline{\ell_{iL}}\, \Sigma_{jR}\, \widetilde{\phi} + \frac{1}{2} (M_\Sigma)_{ij} {\rm Tr}\left[i\sigma_2 \overline{\Sigma^c_{iR}}\, i\sigma_2\, \Sigma_{jR} \right] + {\rm h.c.} \right]$$

Similar to type-I seesaw mechanism, the neutrino mass terms read

$$\mathcal{L}^{\nu}_{\rm mass} = -\frac{1}{2} (\mathcal{M}_\nu)_{ij} \overline{N_{iL}} N^c_{jL} + {\rm h.c.}, \quad N_L = \left(\nu_{eL}\ \nu_{\mu L}\ \nu_{\tau L}\ \Sigma^{0c}_{1R} \dots \Sigma^{0c}_{n_\Sigma R}\right)^T$$

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_\Sigma \end{pmatrix}, \quad M_D = \frac{v}{\sqrt{2}} Y_\Sigma$$

The effective light neutrino mass matrix is

$$M_\nu = -M_D M_\Sigma^{-1} M_D^T$$

There are corrections to the charged-lepton masses due to the presence of the heavy fields:

$$\mathcal{M}_\ell = \begin{pmatrix} m_\ell & M_D \\ 0 & M_\Sigma \end{pmatrix}$$

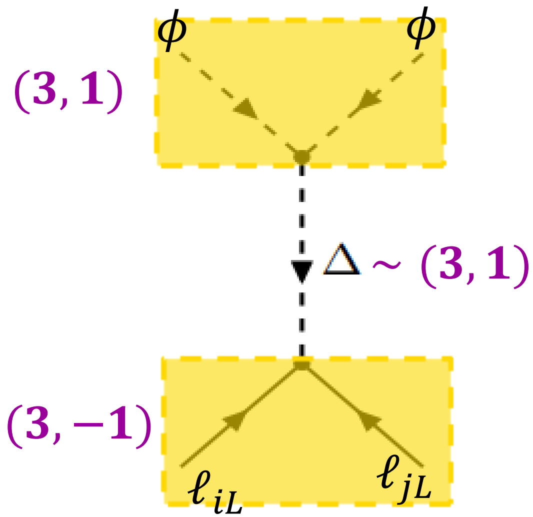
$$\mathcal{L}^{\pm}_{\rm mass} = -\frac{1}{2} (\mathcal{M}_\ell)_{ij} \overline{E_{iL}} E_{jR} + {\rm h.c.},$$

$$E_L = \left(e_L\ \mu_L\ \tau_L\ \Sigma^{+c}_{1R} \dots \Sigma^{+c}_{n_\Sigma R}\right)^T, \quad E_R = \left(e_R\ \mu_R\ \tau_R\ \Sigma^{-}_{1R} \dots \Sigma^{-}_{n_\Sigma R}\right)^T$$

- The charged and neutral current interactions are a bit more complicated but nothing drastically different with respect to the type I seesaw case....

type-II seesaw mechanism

Higgs-lepton coupling: $\overline{\ell^c_{iL}} \ell_{jL} \sim (2,-1/2) \otimes (2,-1/2) = (1,-1) \oplus (3,-1)$ ✗



The singlet combination $\overline{\nu^c_{iL}}\, e_{jL} - \overline{e^c_{iL}}\, \nu_{jL}$ doesn't lead to a neutrino mass term

Field content: SM fields+ **Y=1 scalar triplet** $\vec{\Delta} = \left(\Delta^{(1)}, \Delta^{(2)}, \Delta^{(3)}\right)$

It is convenient to work with charge eigenstates and use a 2x2 representation:

$$\Delta = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix}, \quad \Delta^{++} = \frac{\Delta^{(1)} - i\Delta^{(2)}}{\sqrt{2}}, \quad \Delta^0 = \frac{\Delta^{(1)} + i\Delta^{(2)}}{\sqrt{2}}, \quad \Delta^+ = \Delta^{(3)}$$

The Lagrangian is

$$\mathcal{L}_{\rm full} = \mathcal{L}_{\rm SM} + {\rm Tr}\left[(D^\mu\Delta)^\dagger\,(D_\mu\Delta)\right] + \left[(Y_\Delta)_{ij}\,\overline{\ell_{iL}}\,\Delta^\dagger\,i\sigma_2\ell^c_{jL} + {\rm h.c.}\right] - V(\phi,\Delta)$$

- The most general gauge-invariant scalar potential is

$$V(\phi,\Delta) = M_\Delta^2{\rm Tr}(\Delta^\dagger\Delta) + \mu\tilde{\phi}^T i\sigma_2\Delta\tilde{\phi} + \lambda\phi^\dagger\Delta^\dagger\Delta\phi + \lambda'\phi^\dagger\phi{\rm Tr}(\Delta^\dagger\Delta) + \tilde{\lambda}{\rm Tr}[(\Delta^\dagger\Delta)^2] + \hat{\lambda}\left({\rm Tr}[\Delta^\dagger\Delta]\right)^2 + {\rm h.c.}$$

In the limit $M_\Delta^2 \gg v^2$, minimize the potential

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2}\end{pmatrix},\quad \langle\Delta\rangle = \begin{pmatrix} 0 & 0 \\ u/\sqrt{2} & 0\end{pmatrix} \;\Rightarrow\; u = \frac{-\mu v^2}{\sqrt{2}M_\Delta^2}$$

The VEV of Δ^0 is supressed by the large triplet mass

- Neutrino mass

$$\mathcal{L} \supset (Y_\Delta)_{ij}\overline{\ell_{iL}}\,\Delta^\dagger i\sigma_2\ell^c_{jL} + {\rm h.c.} = -(Y_\Delta)_{ij}\overline{\nu_{iL}}\,\Delta^0\nu^c_{jL} + \cdots + {\rm h.c.}$$

$\langle\Delta^0\rangle = u/\sqrt{2}$

$$\mathcal{L}^\nu_{\rm mass} = -\frac{u}{\sqrt{2}}(Y_\Delta)_{ij}\overline{\nu_{iL}}\,\nu^c_{jL} + {\rm h.c.}$$

$$\Rightarrow\; M_\nu = \sqrt{2}\,Y_\Delta u = -\mu\frac{v^2}{M_\Delta^2}Y_\Delta$$

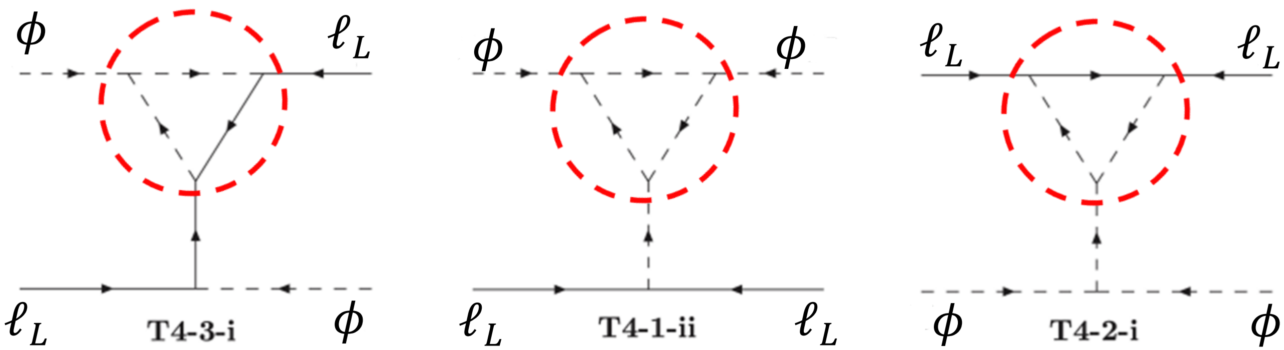
The μ term breaks lepton number and, thus, can be naturally small

Radiative origin of neutrino mass

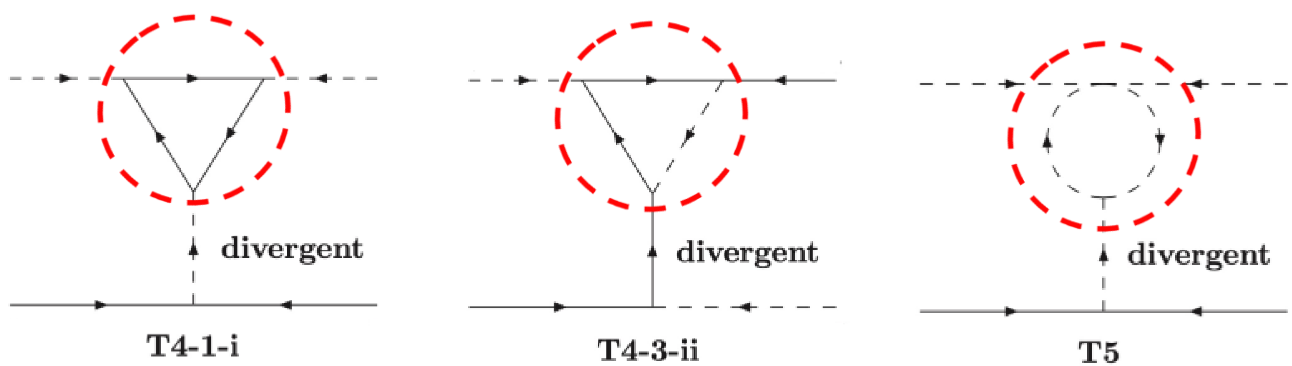
- neutrino mass vanishes at tree level, generated radiatively at n −loop order
- neutrino masses suppressed by loop factors: intermediate states can be light and probed at existing facilities such as colliders, charged lepton flavour violation. (NOTE: ν_R in seesaw at the GUT scale)

➢ **General classification of one-loop neutrino mass models**

- Topologies which are just finite corrections to tree-level seesaws

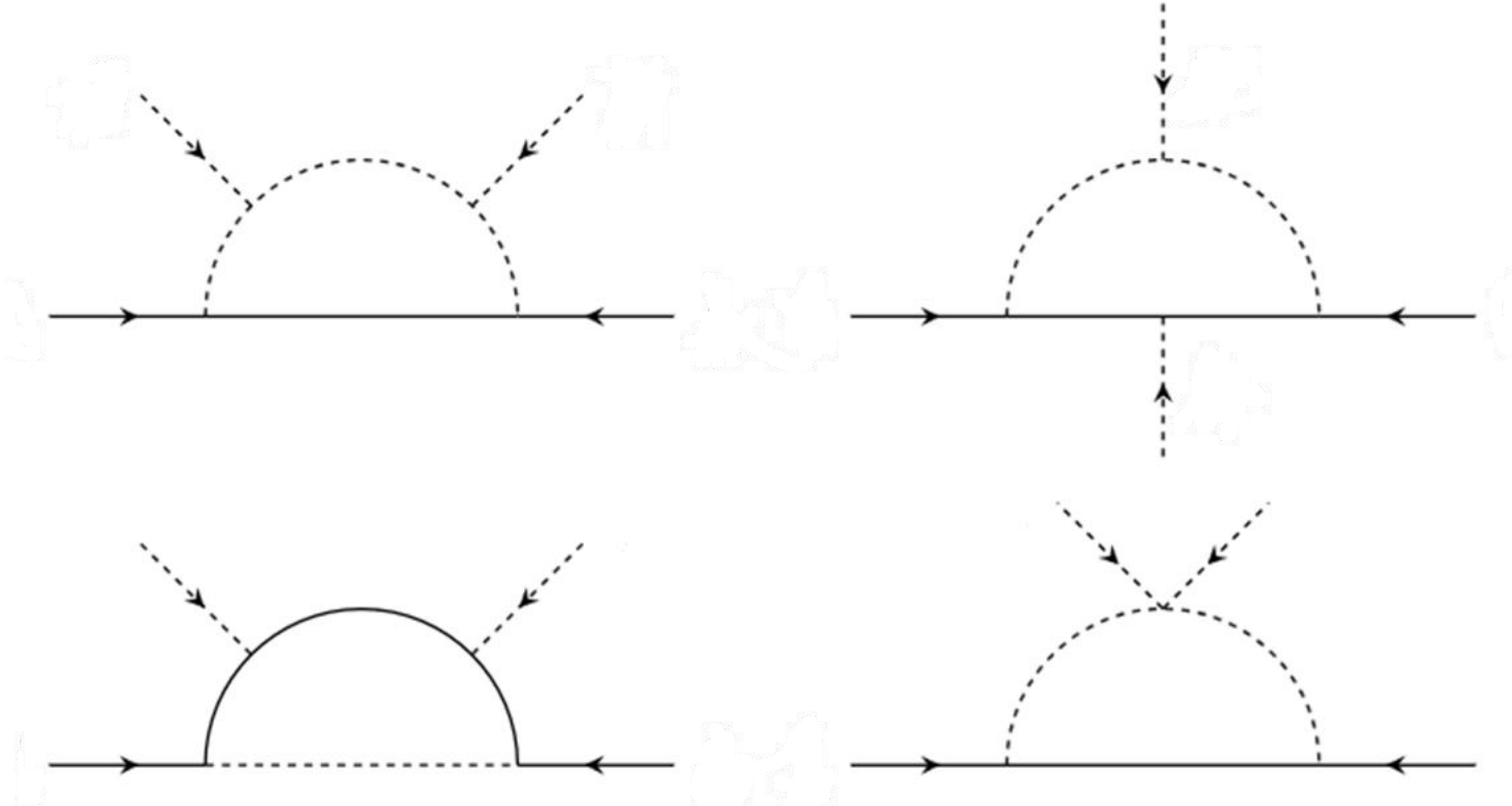


- Divergent diagrams:



• Genuine 1-loop topologies

Genuine n-loop order means that only diagrams starting from n-loop order contribute to neutrino mass. There are no lower-order terms contributing to neutrino masses.



Dashed lines denote scalars or gauge bosons; solid lines refer to fermions

- The quantum numbers of the messenger fields are fixed by SM gauge invariance
- uniqueness of tree-level seesaw lost, large (∞) number of models

[Bonnet, Hirsch, Ota, Winter, 1411.7038]

An example: Zee model

Zee model is an extension of SM with two Higgs doublets $\phi_1, \phi_2 \sim (1,2,1/2)$ and a singly-charged scalar singlet $h^+ \sim (1,1,1)$

- **Higgs basis:** only Higgs field takes a VEV

[Zee,Phys.Lett.B 93 (1980) 389; Cheng, Li, Phys. Rev. D22 (1980) 2860]

$$H_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + \varphi_1^0 + iG^0) \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(\varphi_2^0 + iA) \end{pmatrix}, \quad h^+$$

- **scalar potential** (16 independent terms)

$$\begin{aligned} V = &\mu_1^2 H_1^\dagger H_1 + \mu_2^2 H_2^\dagger H_2 - \left(\mu_3^2 H_2^\dagger H_1 + \text{H.c.}\right) + \frac{1}{2}\lambda_1 \left(H_1^\dagger H_1\right)^2 \\ &+ \frac{1}{2}\lambda_2 \left(H_2^\dagger H_2\right)^2 + \lambda_3 \left(H_1^\dagger H_1\right)\left(H_2^\dagger H_2\right) + \lambda_4 \left(H_1^\dagger H_2\right)\left(H_2^\dagger H_1\right) \\ &+ \left\{\frac{1}{2}\lambda_5 \left(H_1^\dagger H_2\right)^2 + \left[\lambda_6 \left(H_1^\dagger H_1\right) + \lambda_7 \left(H_2^\dagger H_2\right)\right] H_1^\dagger H_2 + \text{H.c.}\right\} \\ &+ \mu_h^2 \left|h^+\right|^2 + \lambda_h \left|h^+\right|^4 + \lambda_8 \left|h^+\right|^2 H_1^\dagger H_1 + \lambda_9 \left|h^+\right|^2 H_2^\dagger H_2 \\ &+ \lambda_{10} \left|h^+\right|^2 \left(H_1^\dagger H_2 + \text{H.c.}\right) + \left(\mu \epsilon_{\alpha\beta} H_1^\alpha H_2^\beta h^- + \text{H.c.}\right) \end{aligned}$$

Scalar mass spectrum

Two charged Higgs: $$m^2_{h_1^+,h_2^+} \equiv \frac{1}{2}\left[M^2_{H^+} + M^2_{33} \mp \sqrt{\left(M^2_{H^+} - M^2_{33}\right)^2 + 2v^2\mu^2}\,\right]$$

$$M^2_{H^+} = \mu_2^2 + \frac{1}{2}v^2\lambda_3\,, \quad M^2_{33} = \mu_h^2 + v^2\lambda_8$$

one CP-odd Higg $$m_A^2 = M^2_{H^+} - \frac{1}{2}v^2\left(\lambda_5 - \lambda_4\right)$$

two CP-even Higgs

$$m^2_{H,h} \equiv \frac{1}{2}\left\{m_A^2 + v^2\left(\lambda_1 + \lambda_5\right) \pm \sqrt{\left[m_A^2 + v^2\left(\lambda_5 - \lambda_1\right)\right]^2 + 4v^4\lambda_6^2}\right\}$$

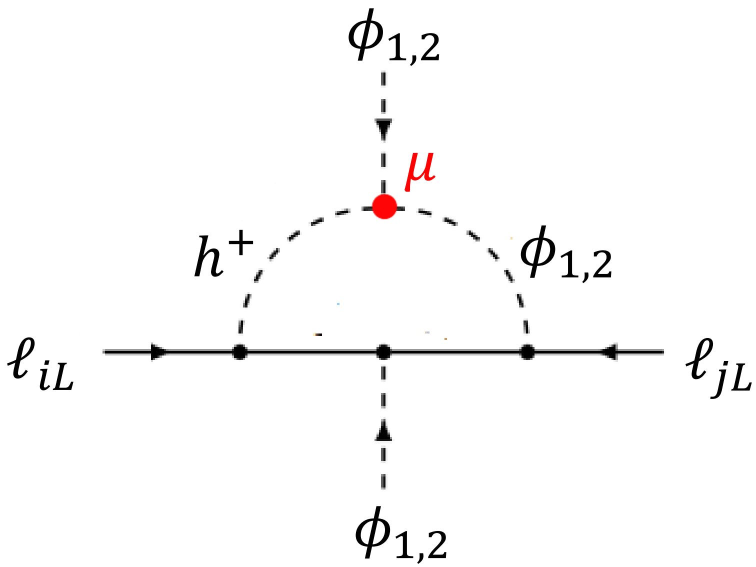
Very rich phenomenology for LHC, FCNC, LFV: stability of the potential, naturality and perturbativity, Higgs signals, Higgs lepton flavor violation…

[e.g., Herrero-Garcia,Ohlsson,Riad,Wiren,1701.05345]

➢ Lepton Yukawa interactions and neutrino mass

$f_{ij} = -f_{ji}$

$$-\mathcal{L}^{\ell}_{Yuk} = \overline{\ell_{iL}} \left[(Y_1^\dagger)_{ij}\phi_1 + (Y_2^\dagger)_{ij}\phi_2\right] E_{jR} + f_{ij}\overline{\ell^c_{iL}} i\sigma_2 \ell_{jL} h^+ + \text{h.c.}$$



In the scalar mass eigenstate basis, calculating the one-loop diagram

$$M_\nu = \frac{s_{2\varphi}\, t_\beta}{8\sqrt{2}\pi^2\, v} \ln\frac{m^2_{h_2^+}}{m^2_{h_1^+}} \left[f\, m_E^2 + m_E^2 f^T - \frac{v}{\sqrt{2}\, s_\beta}(f\, m_E\, Y_2 + Y_2^T\, m_E\, f^T)\right],$$

$$m_E \equiv \frac{v}{\sqrt{2}}\left(c_\beta Y_1^\dagger + s_\beta Y_2^\dagger\right), \quad \tan\beta \equiv v_2/v_1\,, \quad s_{2\varphi} \equiv \frac{\sqrt{2} v\mu}{m^2_{h_2^+} - m^2_{h_1^+}}$$

If $\mu = 0$ lepton number conservation is restored and neutrino masses vanish

Scotogenic model:combining neutrino mass with Dark Matter

Scotogenic comes from the Greek word skotos (σκοτος) -darkness. Then, the meaning of scotogenic is “created from darkness”. **[E. Ma, hep-ph/0601225]**

- version of inert Higgs doublet model (See lectures by Wei Su)
- SM + 2nd Higgs doublet η + RH neutrinos N
- η and N are odd under a discrete Z_2 symmetry⇒ the lightest of them is a DM candidate
- neutrino masses generated at 1-loop (Tree-level forbidden by Z_2)

Field	$SU(2)_L \times U(1)_Y$	Z_2
ℓ_{iL}	$(2, -1/2)$	$+$
e_i	$(1, -1)$	$+$
ϕ	$(2, 1/2)$	$+$
N_i	$(1, 0)$	$-$
η	$(2, 1/2)$	$-$

➢ Fermion Yukawa and mass terms

$$\mathcal{L}_N = \overline{N_i} i \partial\!\!\!/ N_i - \frac{M_{N_i}}{2} \overline{N_i^c} N_i - (Y_N)_{ij} \, \tilde{\eta}^\dagger \, \overline{N_i} \, \ell_{jL} + \text{h.c.}$$

The Z_2 symmetry does not allow couplings $\tilde{\phi}^\dagger \overline{N} \ell_L$

➢ Scalar potential and electroweak symmetry breaking

$$V(\phi,\eta) = -m_\phi^2\phi^\dagger\phi + m_\eta^2\eta^\dagger\eta + \frac{\lambda_1}{2}\left(\phi^\dagger\phi\right)^2 + \frac{\lambda_2}{2}\left(\eta^\dagger\eta\right)^2 + \lambda_3\left(\phi^\dagger\phi\right)\left(\eta^\dagger\eta\right) + \lambda_4\left(\phi^\dagger\eta\right)\left(\eta^\dagger\phi\right) + \frac{\lambda_5}{2}\left[\left(\phi^\dagger\eta\right)^2 + \left(\eta^\dagger\phi\right)^2\right]$$

If $\lambda_5 = 0$ then we can assign $L(\eta) = -1, L(N = 0), L(\ell_L) = 1$, and lepton number conservation is restored. Small λ_5 is natural in the 't Hooft sense.

We want the Z_2 symmetry to be maintained after EWSB. Thus we must guarantee that:

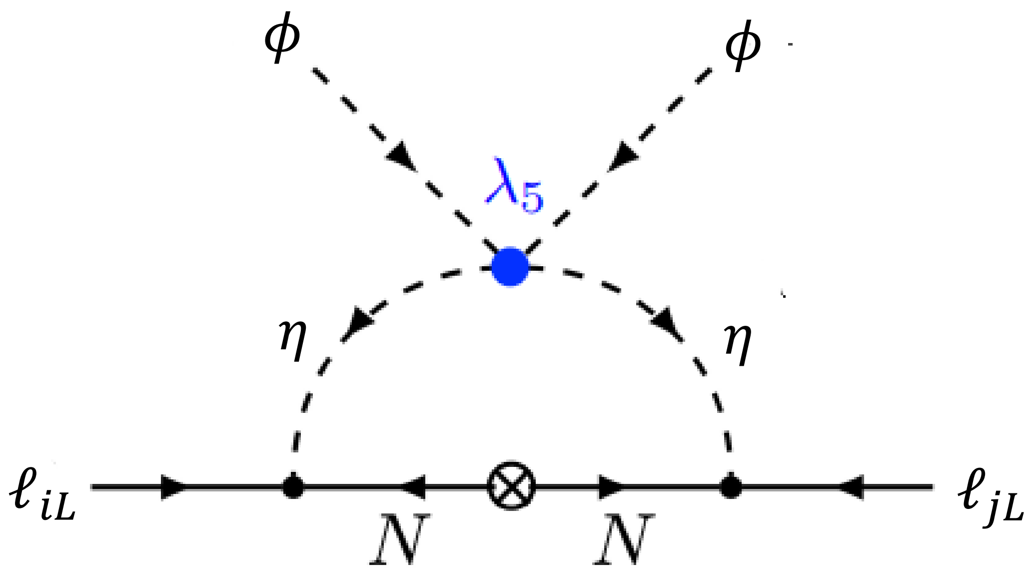
$$\langle\eta\rangle \equiv \begin{pmatrix}0\\0\end{pmatrix}, \quad \langle\phi\rangle \equiv \frac{1}{\sqrt{2}}\begin{pmatrix}0\\v\end{pmatrix}$$ ⟹ Inert scalar sector: $\eta^\pm, \quad \eta^0 = (\eta_R + i\eta_I)/\sqrt{2}$

Scalar mass spectrum:

$$\begin{cases} m_{\eta^+}^2 = m_\eta^2 + \lambda_3 v^2/2 \\ m_R^2 = m_\eta^2 + (\lambda_3+\lambda_4+\lambda_5)\,v^2/2 \\ m_I^2 = m_\eta^2 + (\lambda_3+\lambda_4-\lambda_5)\,v^2/2 \end{cases}$$ ⟹ $$\boxed{m_R^2 - m_I^2 = \lambda_5 v^2}$$

- Z_2 remains unbroken, the lightest particle η_R, η_I or N_i will be stable→dark matter candidate.

➢ Scotogenic neutrino mass matrix



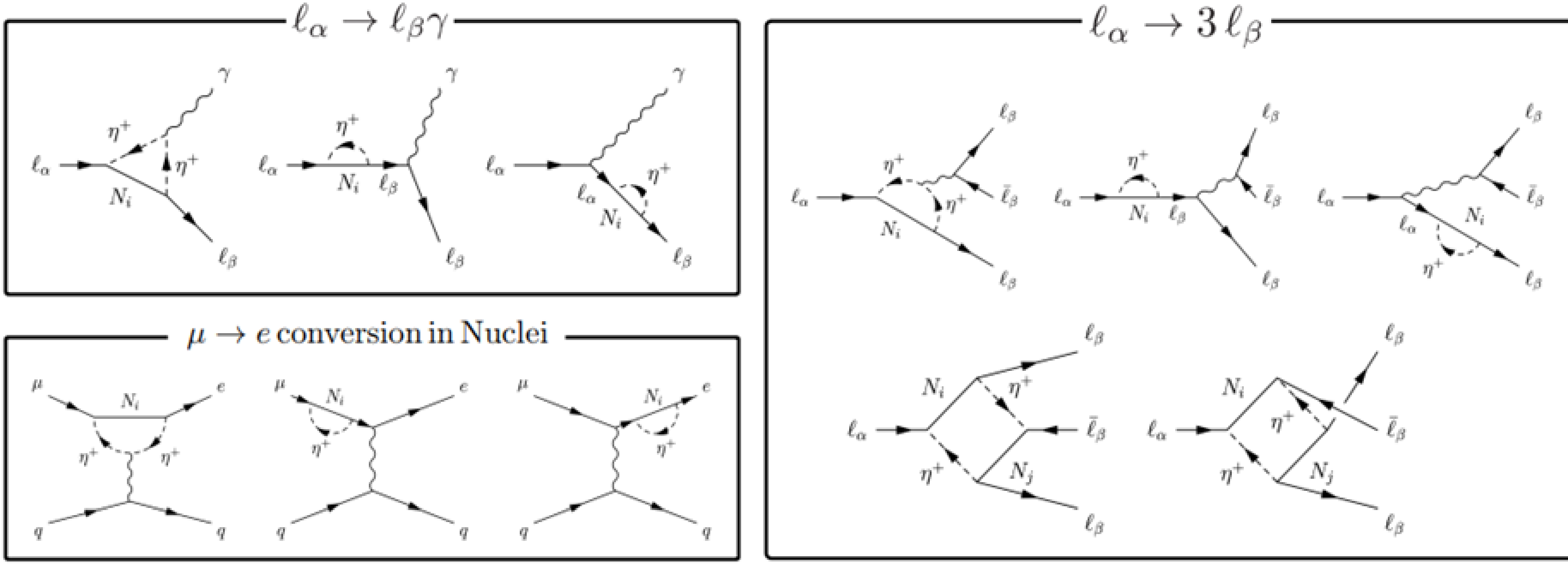
$$(M_\nu)_{\alpha\beta} = \sum_{k=1}^{3} \frac{(Y_N)_{ki}(Y_N)_{kj}}{32\pi^2} M_{N_k} \left[\frac{m_R^2}{m_R^2 - M_{N_k}^2} \log\left(\frac{m_R^2}{M_{N_k}^2}\right) - \frac{m_I^2}{m_I^2 - M_{N_k}^2} \log\left(\frac{m_I^2}{M_{N_k}^2}\right) \right]$$

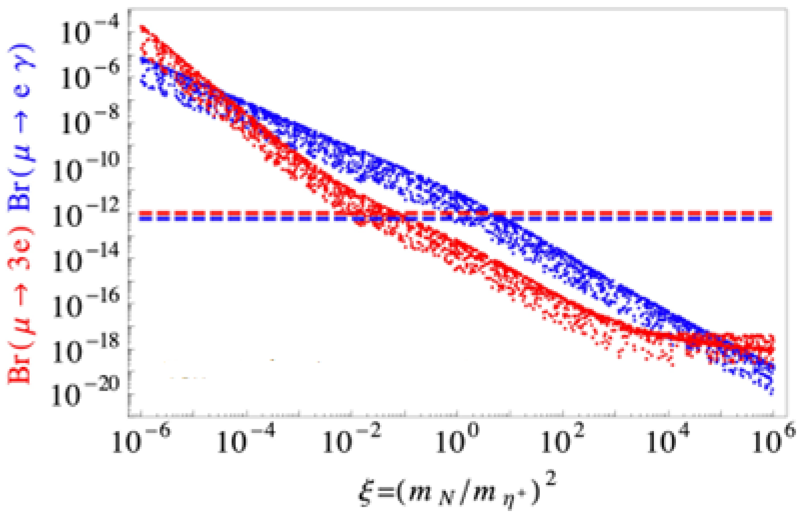
small scalar masses splitting: $m_R^2 - m_I^2 = \lambda_5 v^2 \ll m_0^2 \equiv (m_R^2 + m_I^2)/2$

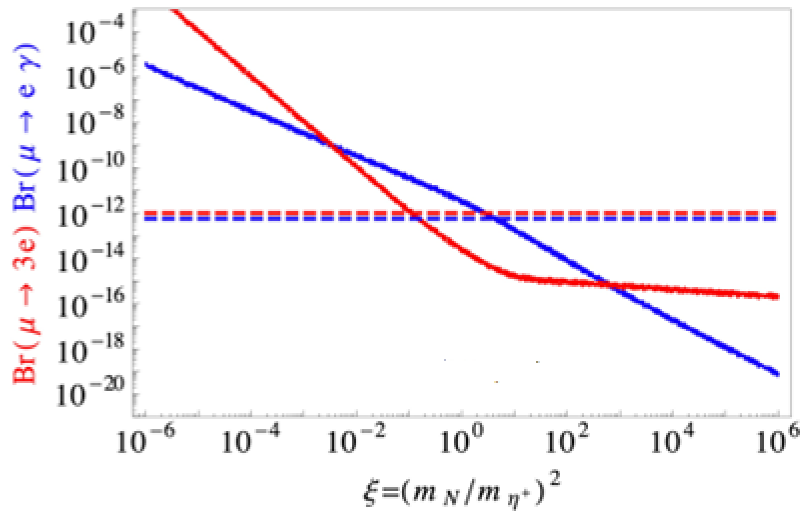
$$(M_\nu)_{\alpha\beta} \approx \frac{\lambda_5 v^2}{32\pi^2} \sum_{k=1}^{3} \frac{(Y_N)_{ki}(Y_N)_{kj} M_{N_k}}{m_0^2 - M_{N_k}^2} \left[1 + \frac{M_{N_k}^2}{m_0^2 - M_{N_k}^2} \log\left(\frac{M_{N_k}^2}{m_0^2}\right) \right]$$

Many phenomenologically interesting aspects: LHC signals, dark matter relic abundance, LFV, etc

Lepton flavor violation in Scotogenic model







[Toma, Vicente, 1312.2840]

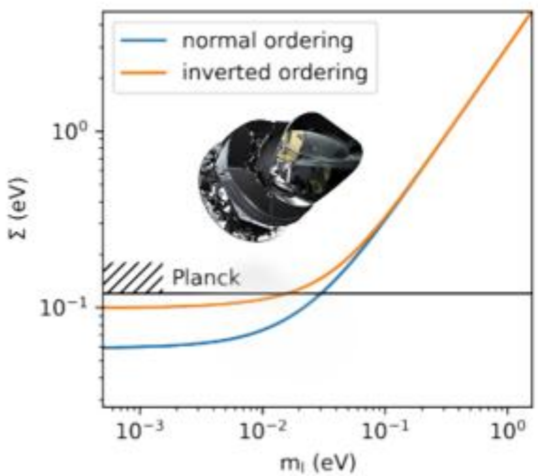
Absolute neutrino mass

neutrino oscillation determines Δm_{ij}^2, absolute mass scale is unconstrained

- Three different ways to measure absolute neutrino mass: sensitive to different quantities

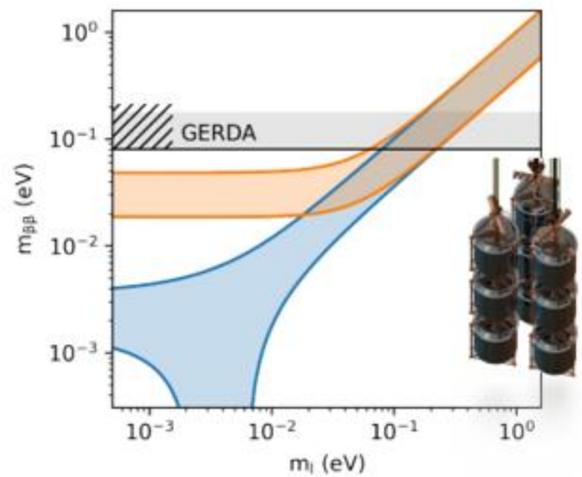
Cosmology

$$\Sigma = \sum_i m_i$$



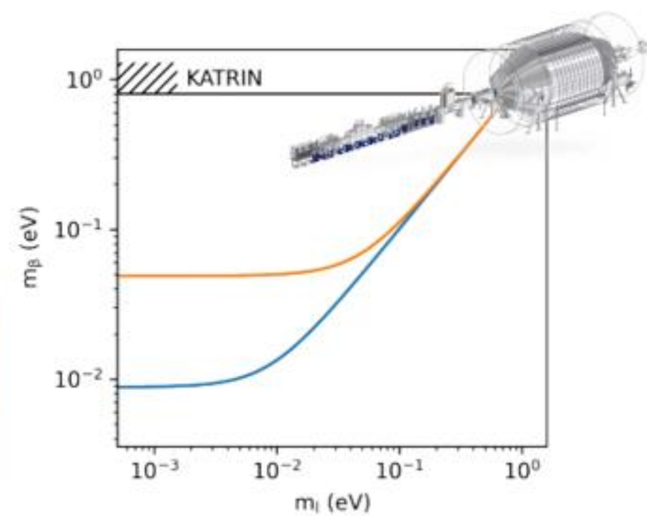
Neutrinoless ββ decay

$$|m_{\beta\beta}| = \left|\sum_i U_{ei}^2 m_i\right|$$



β-decay kinetics

$$m_\beta = \sqrt{\sum_i |U_{ei}|^2 m_i^2}$$

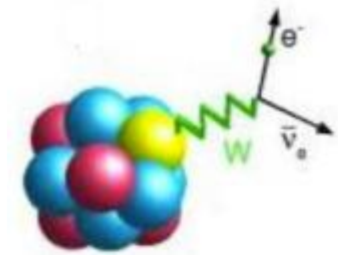


The β spectrum and neutrino mass

- β decay

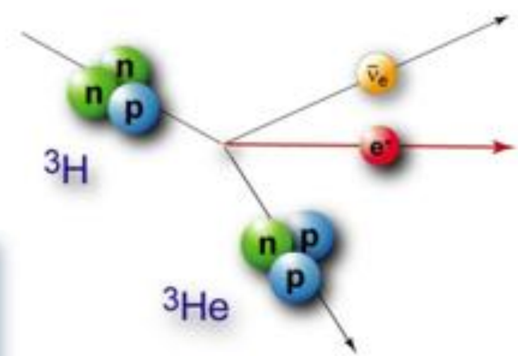
$$n \rightarrow p + e^- + \overline{\nu}_e$$

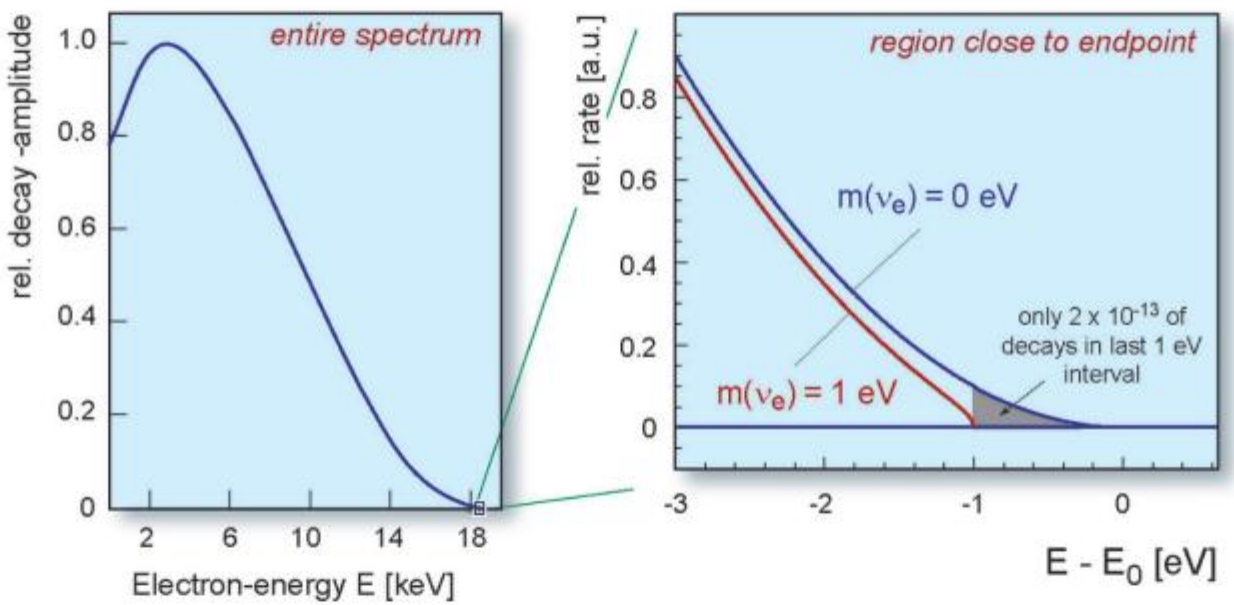
$$(A, Z) \rightarrow (A, Z+1) + e^- + \bar{\nu}_e$$



- General idea: distortion of the endpoint of the electron spectrum due to tiny $m_i \neq 0$

Tritium beta decay $\quad {}^3\mathrm{H} \rightarrow {}^3\mathrm{He} + e^- + \bar{\nu}_e$



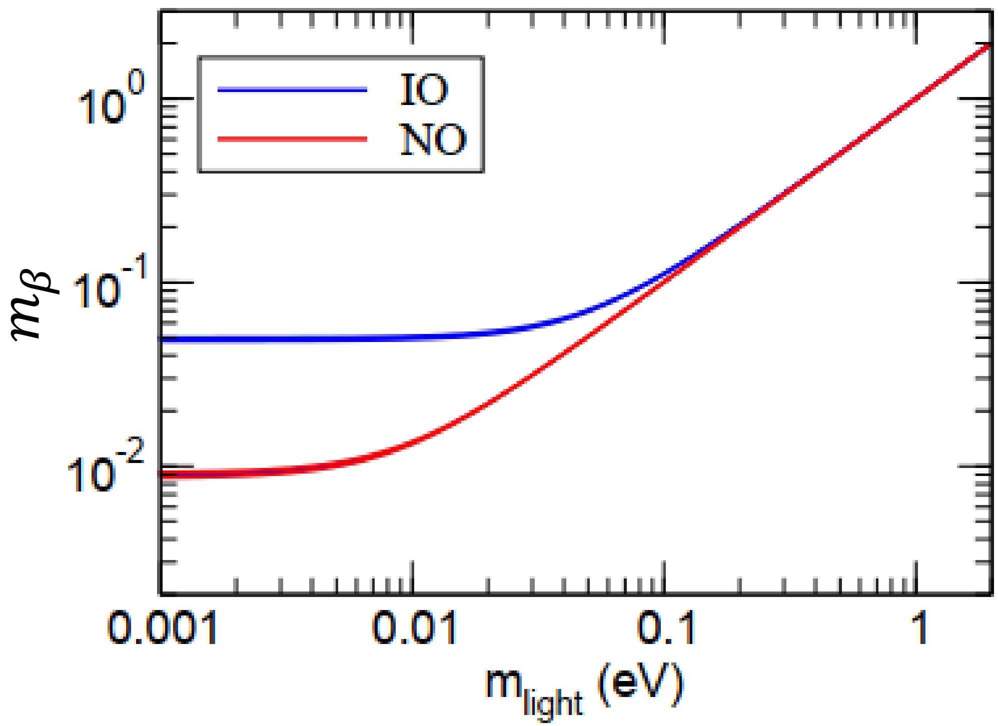


$$\frac{dN}{dE} \propto \sqrt{(E_0 - E)^2 - m_\beta^2}$$

$$m_\beta^2 = \sum_i m_i^2 |U_{ei}|^2$$

The effective mass

$$m_\beta = \sqrt{\sum_i m_i^2 |U_{ei}|^2} = \begin{cases} \sqrt{m_1^2 + \Delta m_{21}^2 (1 - c_{13}^2 c_{12}^2) + \Delta m_{32}^2 s_{13}^2} & \text{for NO} \\ \sqrt{m_3^2 - \Delta m_{21}^2 c_{13}^2 c_{12}^2 - \Delta m_{32}^2 c_{13}^2} & \text{for IO} \end{cases}$$



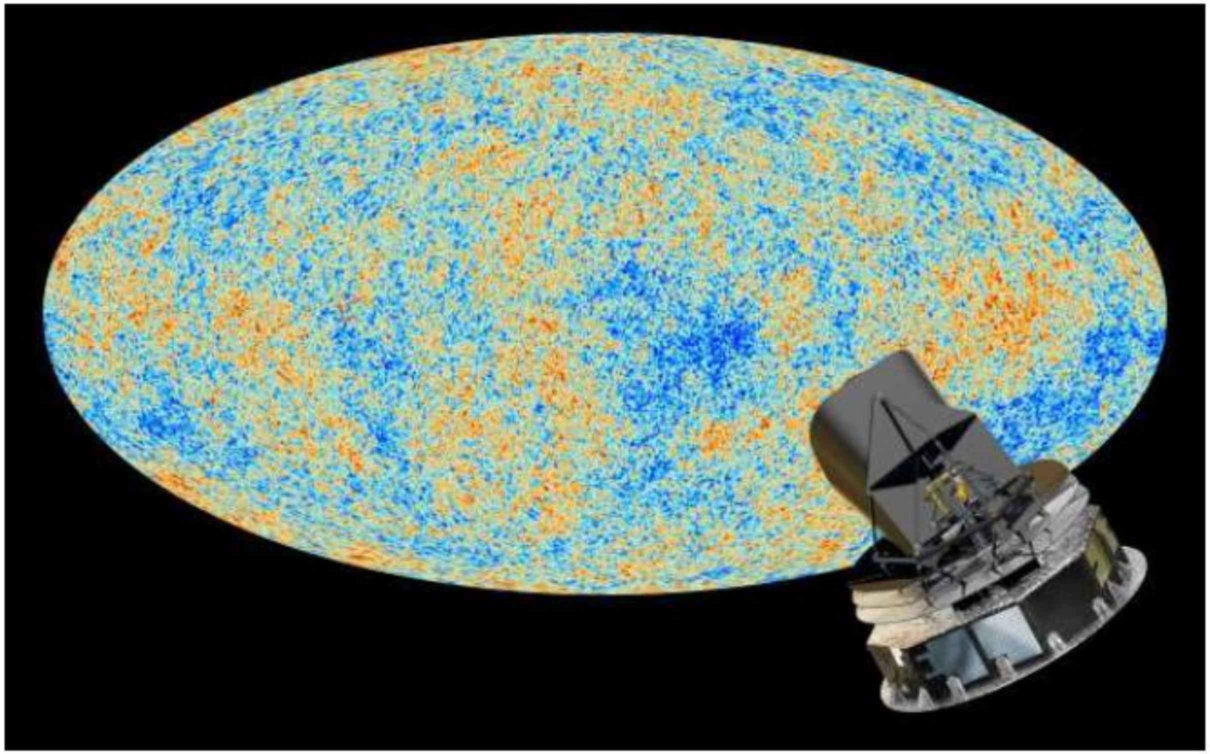
Minimum value for $m_{light} = 0$: $m_\beta^{min} = 8.5\ (48)\ \mathrm{meV}$ for NO (IO)

For $m_{light} \gg |\Delta m_{31}^2|$: $m_\beta \approx m_{light}$

KATRIN = KArlsruhe TRitium Neutrino Experiment

Sensitivity of KATRIN aims to $m_\beta \leq 0.2\text{eV}$

Cosmology and neutrinos



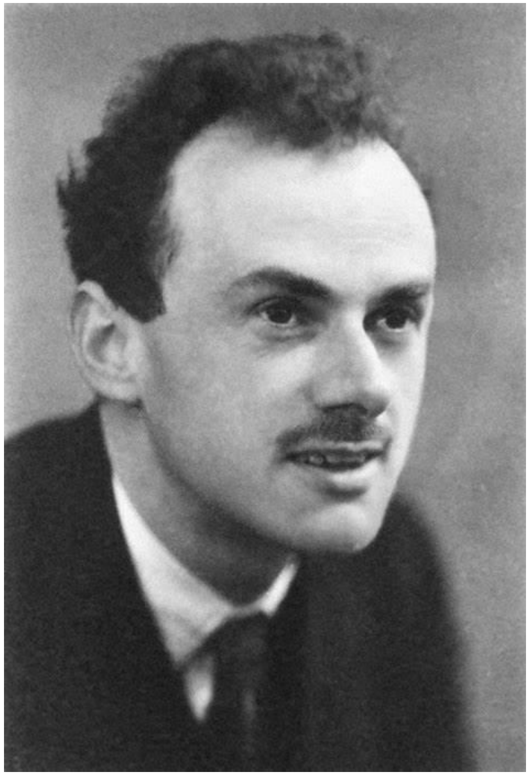
A bound on the sum over all neutrino masses is provided by the large-scale structure of the universe and the temperature fluctuations of the cosmic microwave background (CMB)

$$\Omega_\nu h^2 = \frac{\sum_i m_i}{93.5\ \mathrm{eV}}, \quad \Omega_\nu \equiv \frac{\rho_\nu}{\rho_{cr}}, h = 0.7$$

The most stringent constraint from Planck: $\sum_i m_i \leq 0.12\ \mathrm{eV}$

Massive neutrinos: Dirac or Majorana?

$\nu \neq \nu^c$



vs.

$\nu = \nu^c$

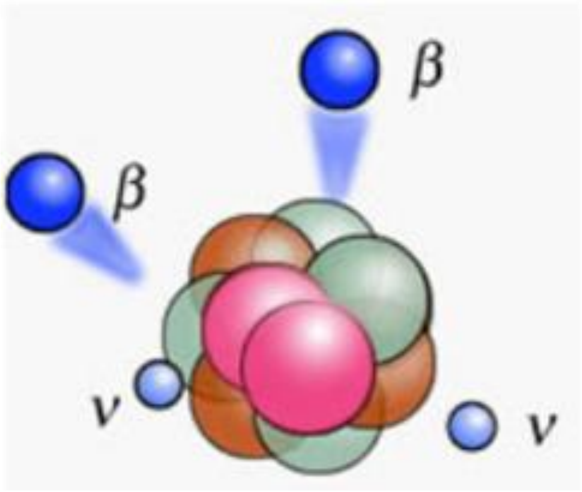
- **neutrinoless double beta decay**
- lepton number violation at collider
- cosmology

The 2β-decays

2νββ decays

$$(A,Z) \rightarrow (A,Z+2) + 2e^- + 2\overline{\nu}_e$$

$\Delta L = 0$

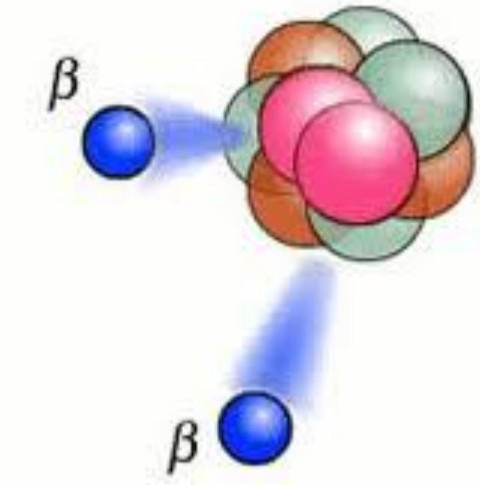


[Goeppert-Mayer, Phys. Rev.48,512(1935)]

- Allowed in SM
- second order in weak interaction
- Natural background for decay

0νββ decays

$$(A,Z) \rightarrow (A,Z+2) + 2e^-$$



$\Delta L = 2$

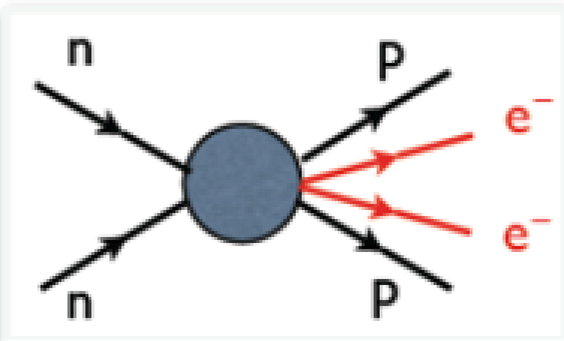
[Furry, Phys.Rev.56,1184(1939)]

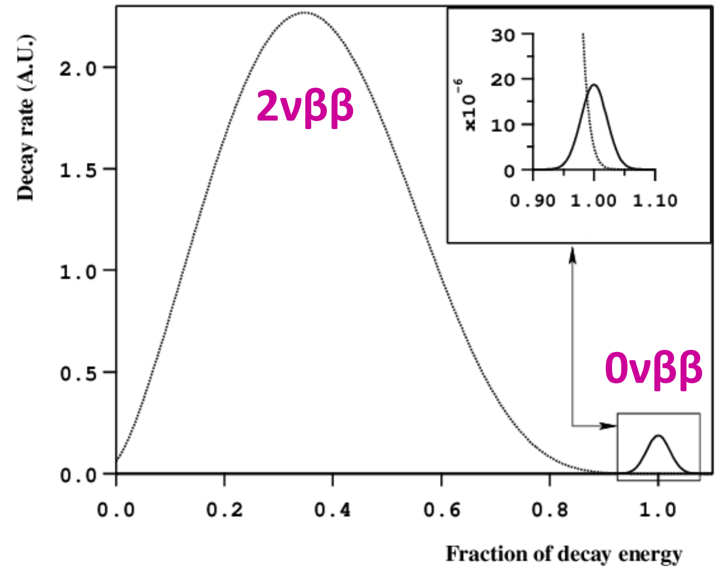
- **NOT** allowed in SM→rare
- Lepton number violation→ neutrinos are Majorana fermions

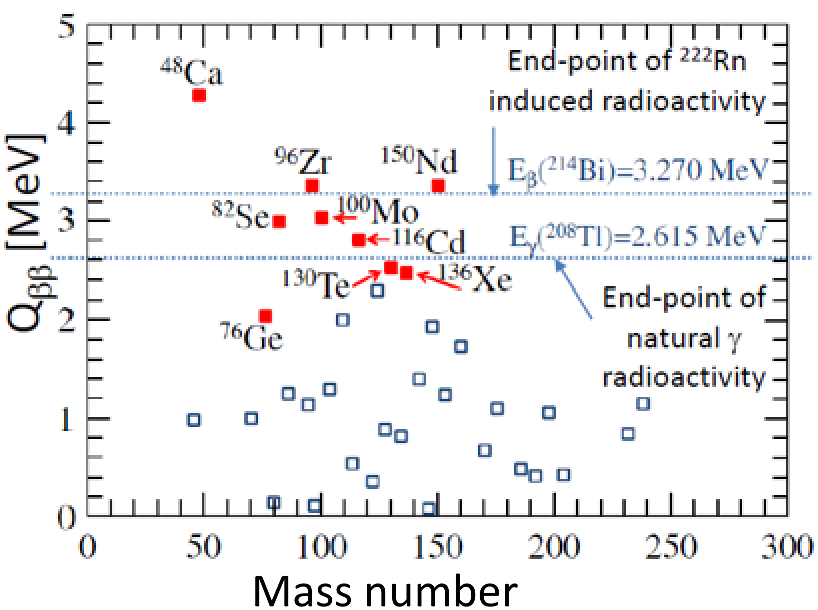
The 0νββ-decays: signature and candidate nuclei

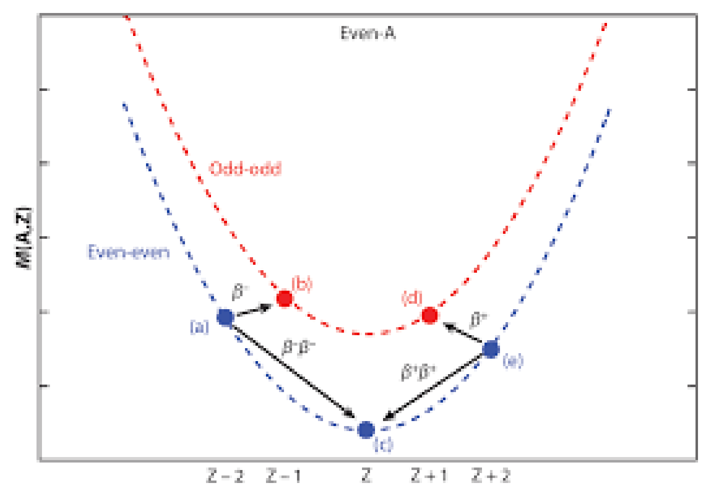
0νββ is potentially observable in certain even-even nuclei (9 isotopes including ^{48}Ca, ^{76}**Ge**, ^{100}Mo, ^{130}Te, ^{136}**Xe**) for which single beta decay is energetically forbidden. The decay rate is less than 1 event per ton and year.

$$T_{1/2} > 10^{25}\ \text{yr}$$









The 9 experimentally most feasible isotopes:

$$Q_{\beta\beta} = M(A,Z) - M(A,Z+2)$$

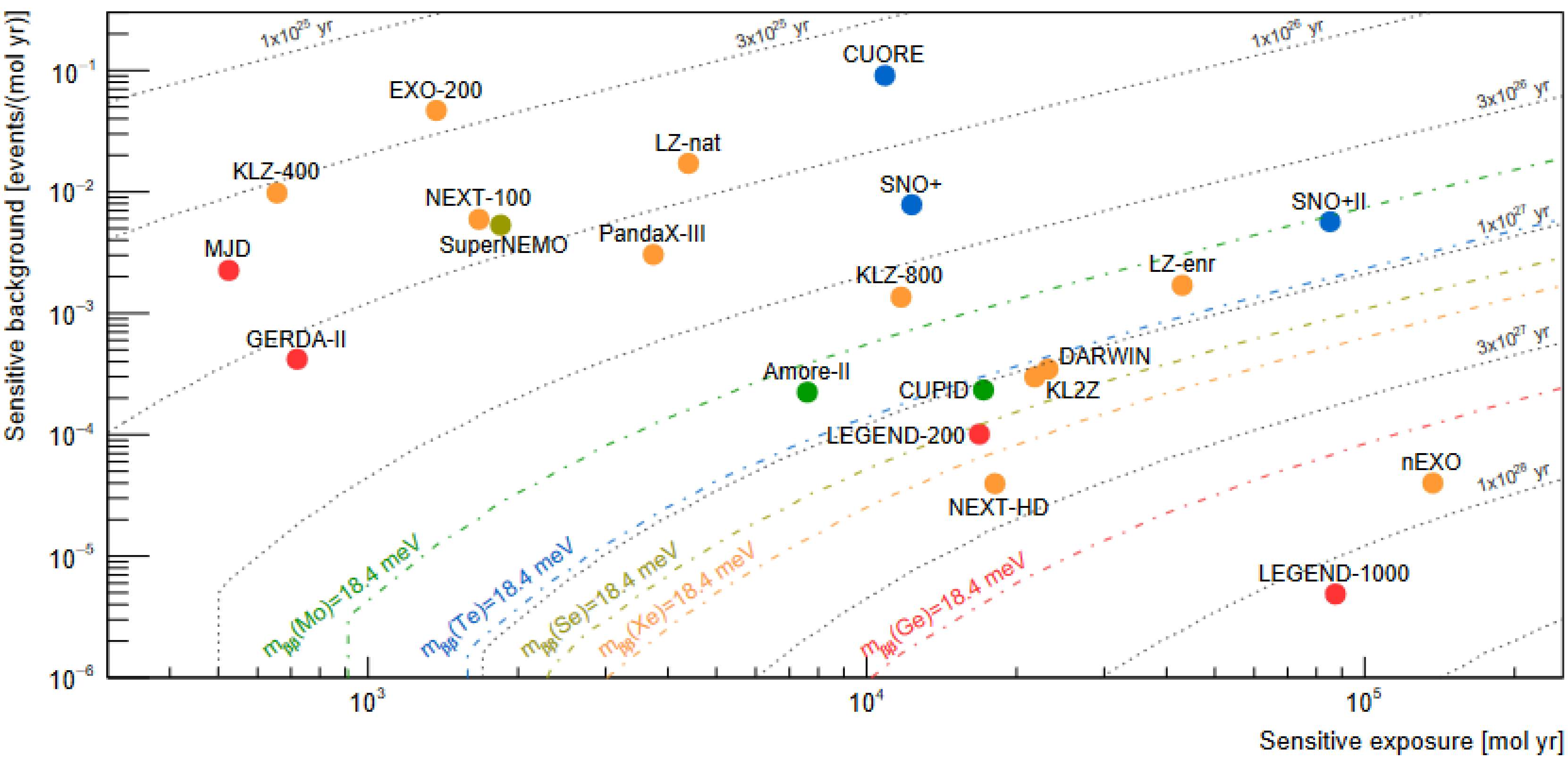
Isotope	Abundance (%)	$Q_{\beta\beta}$ (MeV)
^{48}Ca	0.187	4.263
^{76}Ge	7.8	2.039
^{82}Se	9.2	2.998
^{96}Zr	2.8	3.348
^{100}Mo	9.6	3.035
^{116}Cd	7.6	2.813
^{130}Te	34.08	2.527
^{136}Xe	8.9	2.459
^{150}Nd	5.6	3.371

Current and future experiments

Most stringent constraints on the half life:

- ^{136}Xe (KamLAND-Zen): $T_{1/2} > 2.3\times10^{26}$ yrs **[KamLAND-Zen Collaboration, 2203.02139]**
- ^{76}Ge (GERDA): $T_{1/2} > 1.8\times 10^{26}$ yrs **[GERDA collaboration, 2009.06079]**
- ^{130}Te (CUORE): $T_{1/2} > 2.2\times 10^{25}$ yrs **[CUORE collaboration, 2104.06906]**

There are many 0νββ decay experiments in plan and construction



[Agostini, Benato, Detwiler, Menendez, Vissani, 2202.01787]

Half life of 0νββ-decays

$$T_{1/2}^{-1} = |\, m_{\beta\beta} \,|^2 G^{0\nu} \,|\, M^{0\nu} \,|^2$$

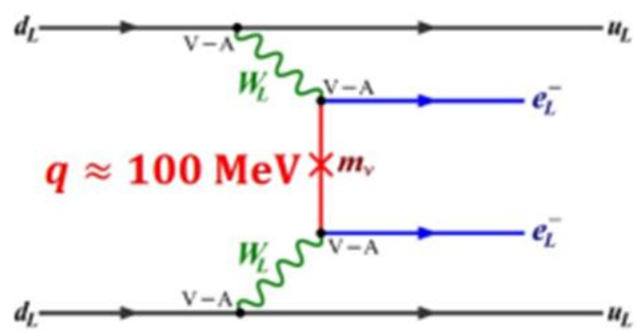
particle physics

phase-space factors

nuclear physics

$$A_{\mu\nu}^{\text{lep}} = \frac{1}{4}\sum_{i=1}^{3} U_{ei}^2 \gamma_\mu (1+\gamma_5) \frac{\not{q} + m_{\nu_i}}{q^2 - m_{\nu_i}^2} \gamma_\nu (1-\gamma_5)$$

$$\approx \frac{\gamma_\mu (1+\gamma_5)\gamma_\nu}{4q^2} \boxed{\sum_{i=1}^{3} U_{ei}^2 m_{\nu_i}} \quad m_{\beta\beta}$$



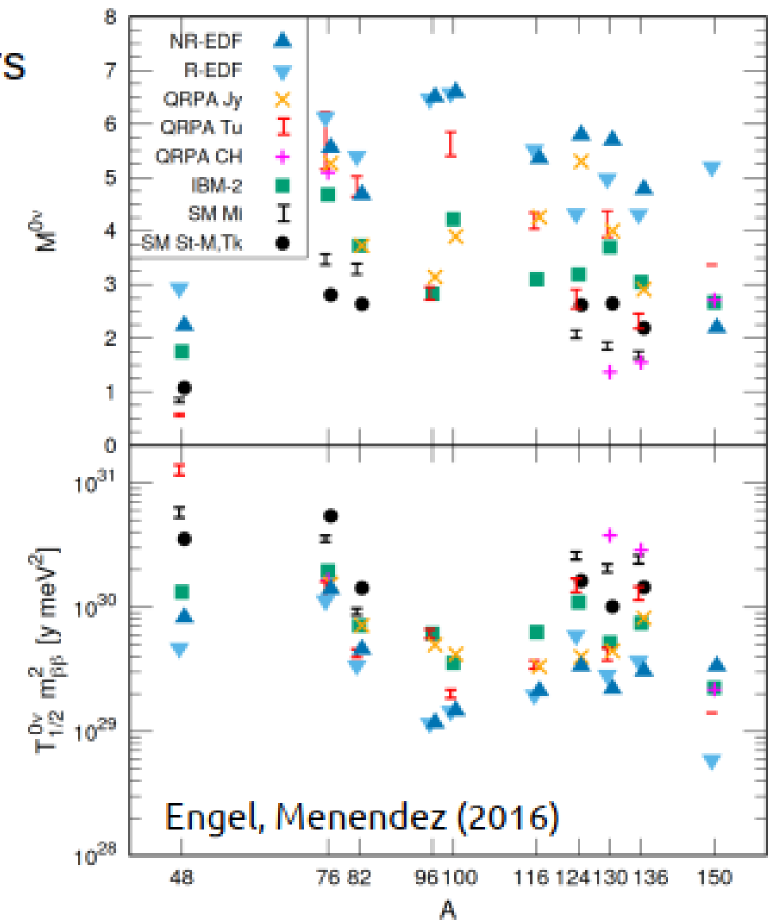
$$T_{1/2}^{0\nu} = \left(G|\mathcal{M}|^2 |m_{\beta\beta}|^2\right)^{-1} \simeq 10^{27-28} \left(\frac{0.01\text{eV}}{|m_{\beta\beta}|}\right)^2 \text{year}$$

$$T_{1/2}^{-1} = | m_{\beta\beta} |^2 \, G^{0\nu} | M^{0\nu} |^2$$

particle physics

phase-space factors

nuclear physics



- **Dependence** on isotope and specific operator
- Differences between different **nuclear models**
- "**the g_A problem**" quenching of the axial-vector coupling

Effective mass $m_{\beta\beta}$ for 0νββ-decay

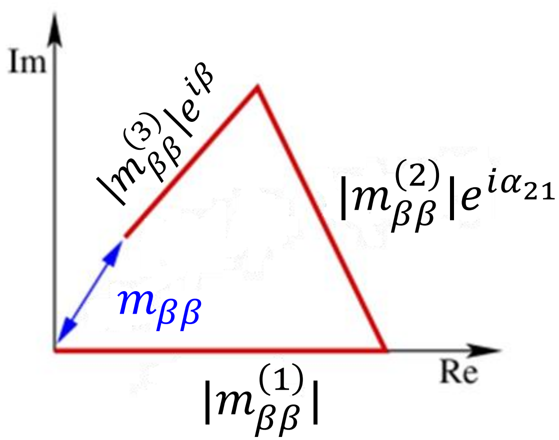
$$m_{\beta\beta} = \sum_i U_{ei}^2 m_i = |m_{\beta\beta}^{(1)}| + |m_{\beta\beta}^{(2)}| e^{i\alpha_{21}} + |m_{\beta\beta}^{(3)}| e^{i\beta}$$

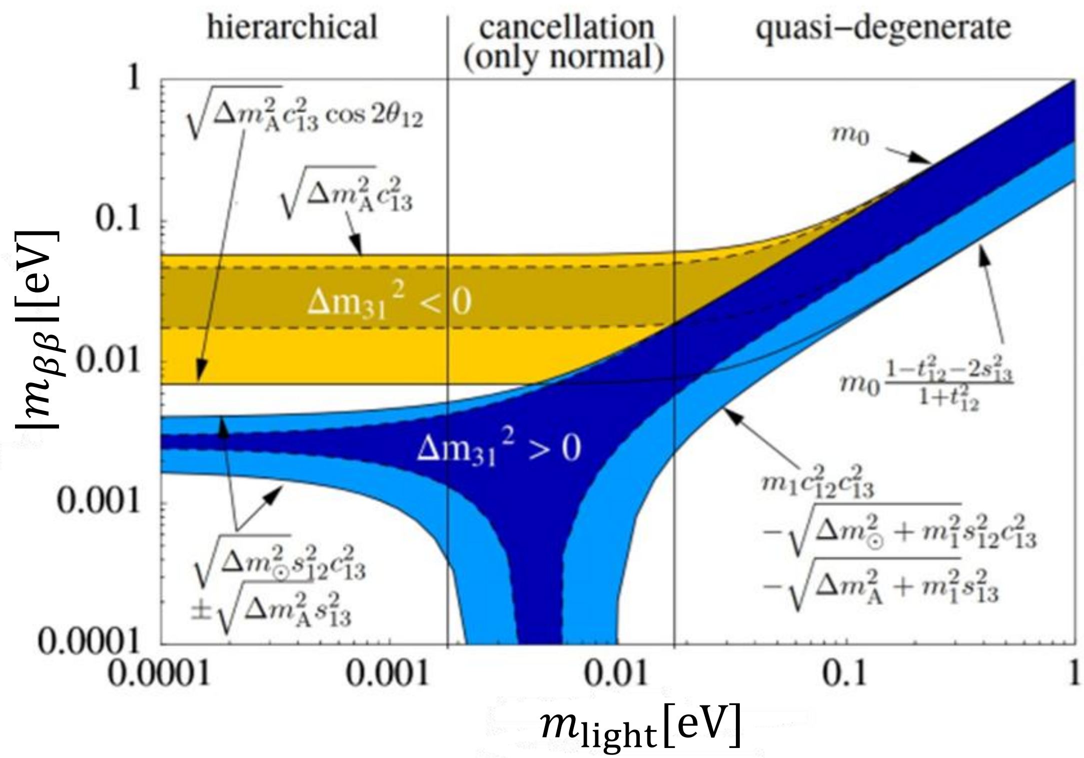
$$|m_{\beta\beta}^{(1)}| = m_1 |U_{e1}|^2 = m_1 c_{12}^2 c_{13}^2$$

$$|m_{\beta\beta}^{(2)}| = m_2 |U_{e2}|^2 = m_2 s_{12}^2 c_{13}^2$$

$$\beta \equiv \alpha_{31} - 2\delta_{CP} \qquad |m_{\beta\beta}^{(3)}| = m_3 |U_{e3}|^2 = m_3 s_{13}^2$$

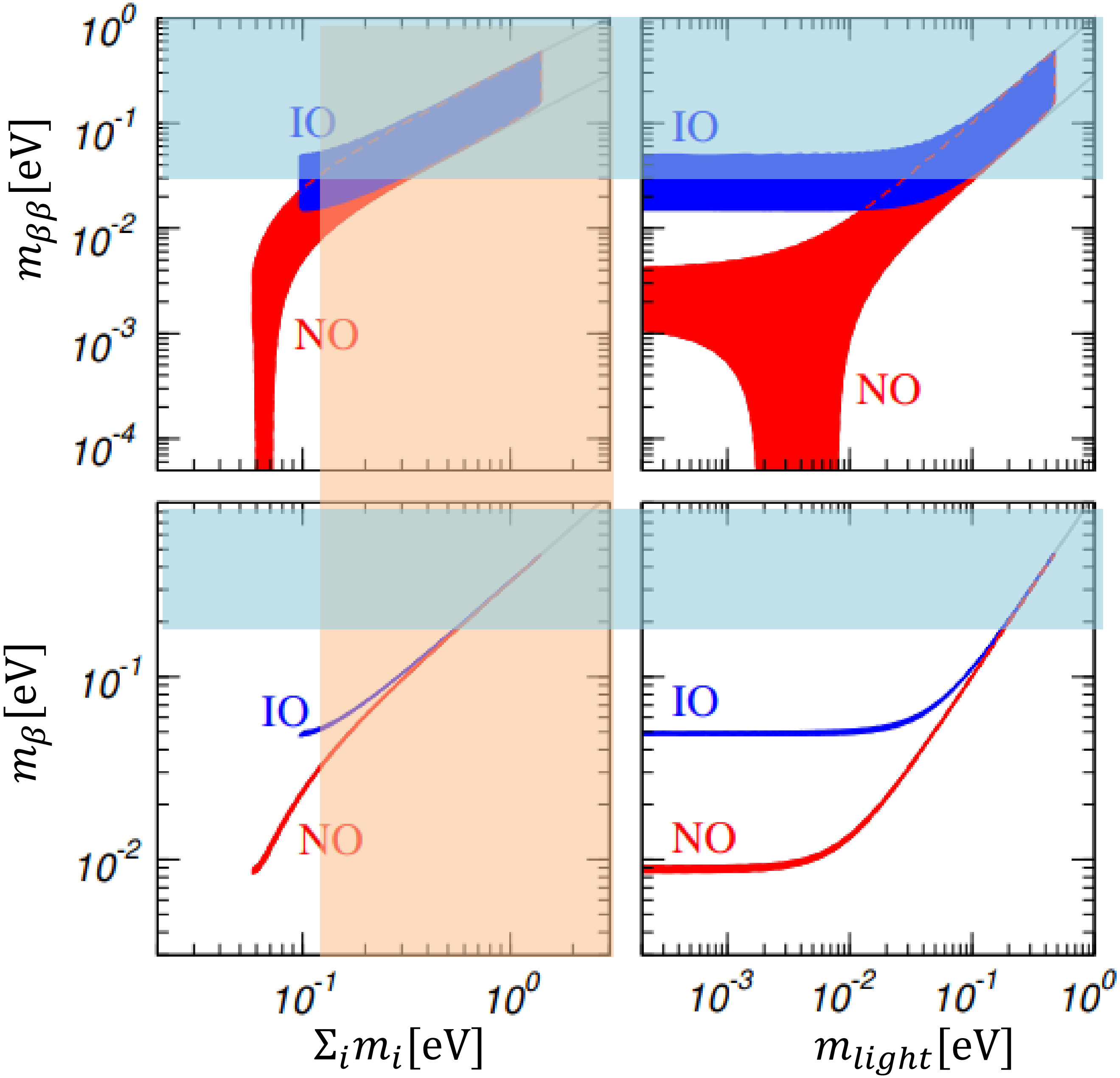
- **unknown Majorana phase and lightest neutrino**
- **Quasi-degenerate** region above 0.2 eV
- Accidental **cancellation** for NO





[Lindner, Merle, Rodejohann, hep-ph/0512143]

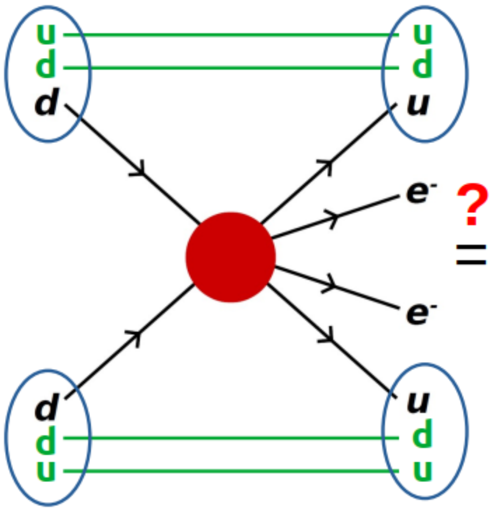
Absolute neutrino masses from synergies of neutrino facilities

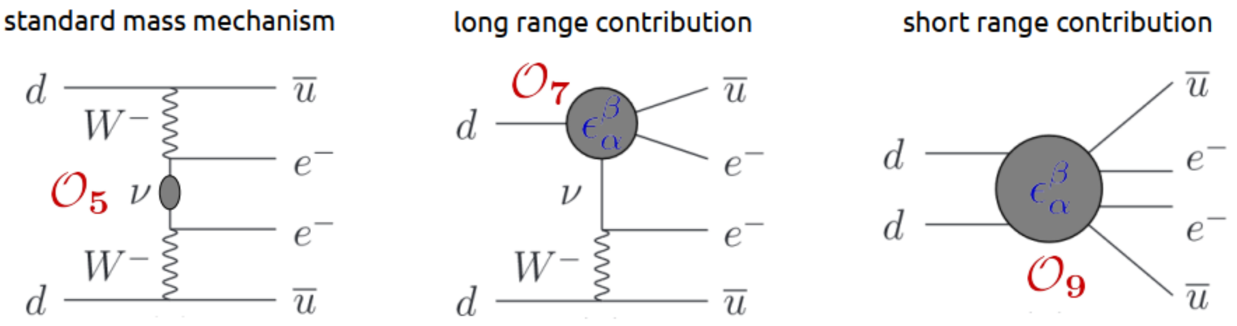


Next generation of ton scale 0νββ experiments will cover the IO region.

Possible BSM physics in 0νββ decay

The 0νββ decay is usually assumed to be dominantly mediated by light and massive Majorana neutrinos. It can also be induced by other ΔL=2 physics, there are many possible sources.





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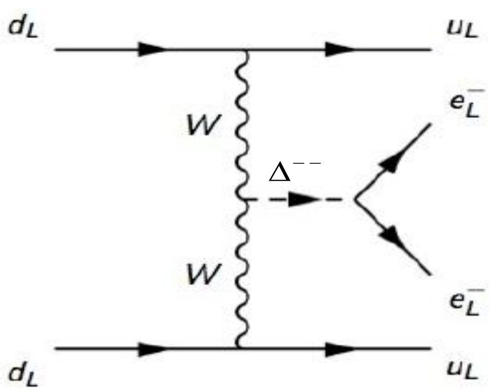
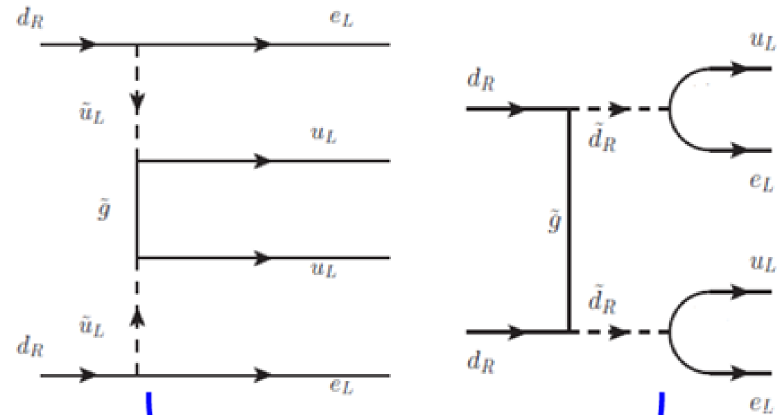
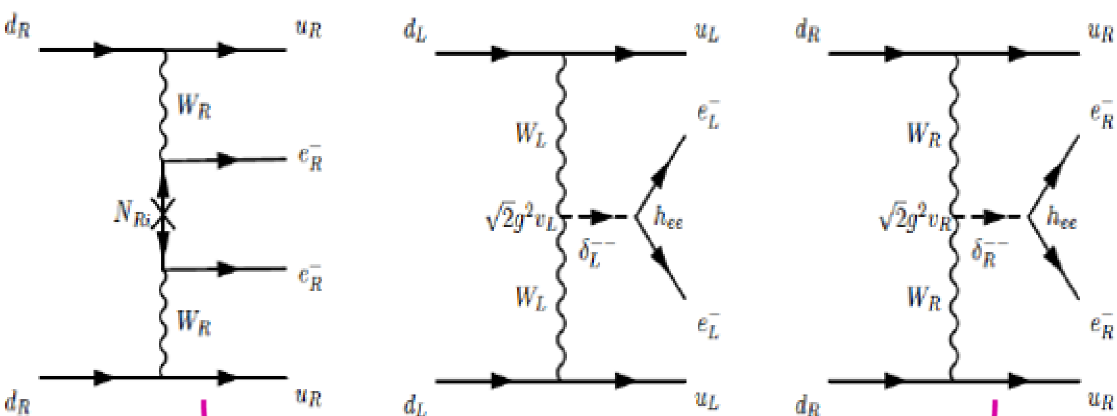


Type II seesaw



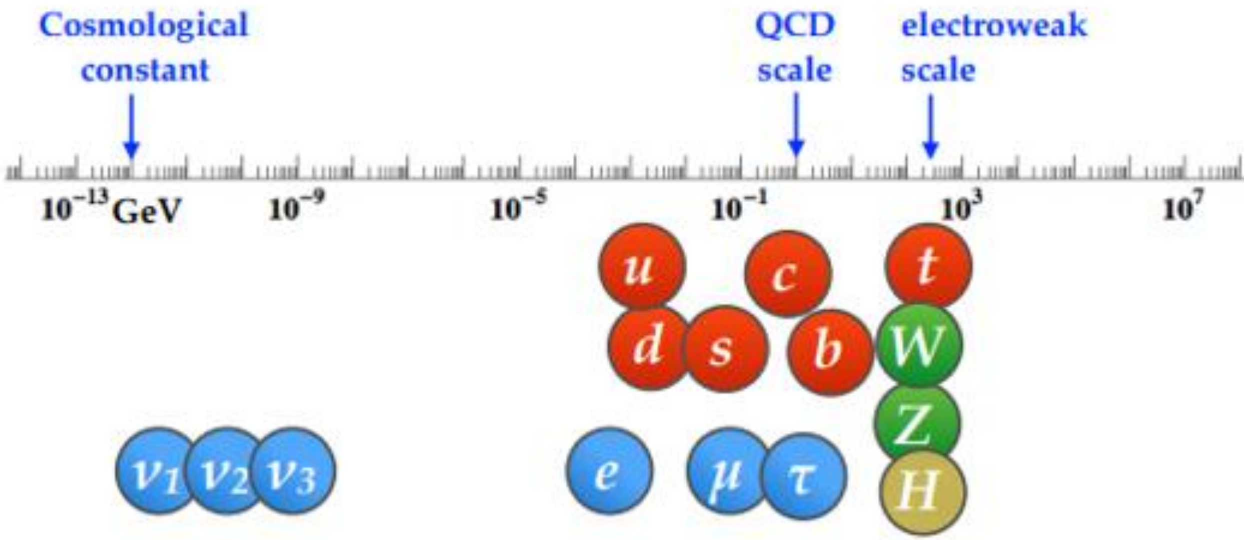
R-parity violating SUSY



Left-right model

- 0νββ decay is connected to TeV scale physics and LFV.
- **A plenty of possible new physics scenario leading to $0\nu\beta\beta$**

Flavor puzzle 1: why mass hierarchies?



Quarks: from MeV to 100 GeV

Charged leptons: from MeV to GeV

Neutrinos: $\sum_{i=1}^{3} m_i \leq 0.12$ eV from cosmology

Flavor puzzle 2: why different quark and lepton mixing?

Charged current interaction of SM

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \left[\overline{(u\ c\ t)_L}\, \gamma^\mu V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W_\mu^+ + \overline{(e\ \mu\ \tau)_L}\, \gamma^\mu U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L W_\mu^- \right] + \text{h.c.}$$

quark **CKM** mixing matrix   lepton **PMNS** mixing matrix

- Quark mixings are small **[Particle Data Group 2024]**

$$|V_{\rm CKM}| = \begin{pmatrix} 0.97435 \pm 0.00016 & 0.22501 \pm 0.00068 & 0.003732^{+0.000090}_{-0.000085} \\ 0.22487 \pm 0.00068 & 0.97349 \pm 0.00016 & 0.04183^{+0.00079}_{-0.00069} \\ 0.00858^{+0.00019}_{-0.00017} & 0.04111^{+0.00077}_{-0.00068} & 0.999118^{+0.000029}_{-0.000034} \end{pmatrix}$$

- Lepton mixings are large **[NuFIT 6.0 (2024)]**

$$|U|^{\rm IC24\ with\ SK\text{-}atm}_{3\sigma} = \begin{pmatrix} 0.801 \to 0.842 & 0.519 \to 0.580 & 0.142 \to 0.155 \\ 0.252 \to 0.501 & 0.496 \to 0.680 & 0.652 \to 0.756 \\ 0.276 \to 0.518 & 0.485 \to 0.673 & 0.637 \to 0.743 \end{pmatrix}$$

“Who orderd that ?”

Where do fermion mass hierarchy, flavor mixing, and CP violation come from?

Is there a simple organization principle?

Isidor Issac Rabi (Nobel Prize in Physics in 1944)

Pathways to flavor puzzle of SM

- **Anarchy: NO** particular structure

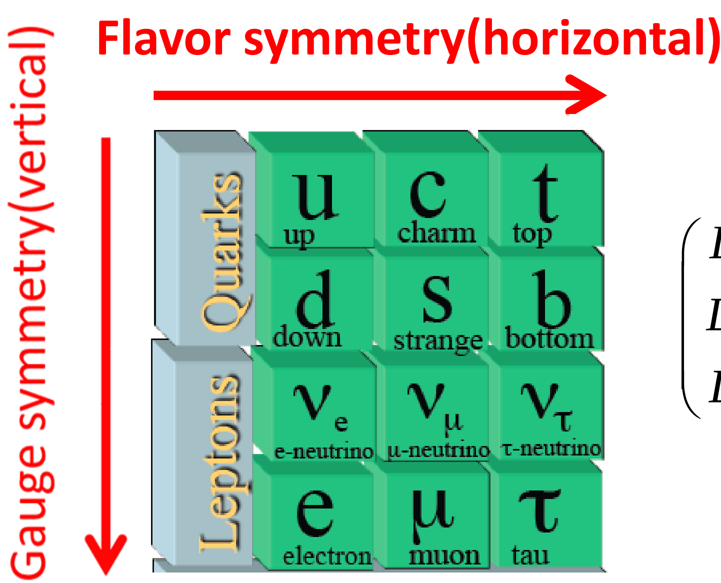
[Hall, Murayama, Weiner, hep-ph/9911341 ; Gouvea,Murayama, 1204.1249]

$$m_\nu \propto \begin{pmatrix} O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \end{pmatrix}$$

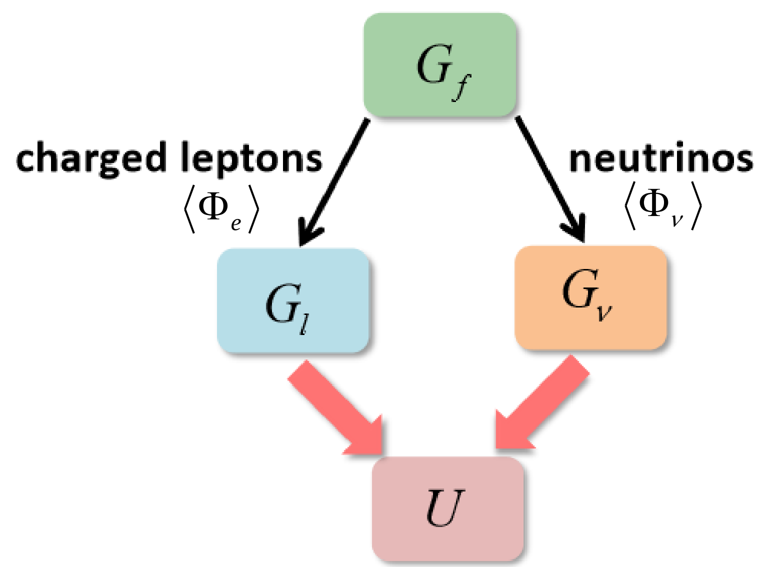
each matrix element is random

- large number of O(1) free parameters→ only statistical tests
- Anarchy is not applicable to charged lepton and quark Yukawa couplings

- **Flavor symmetry: symmetry as a guiding principle**



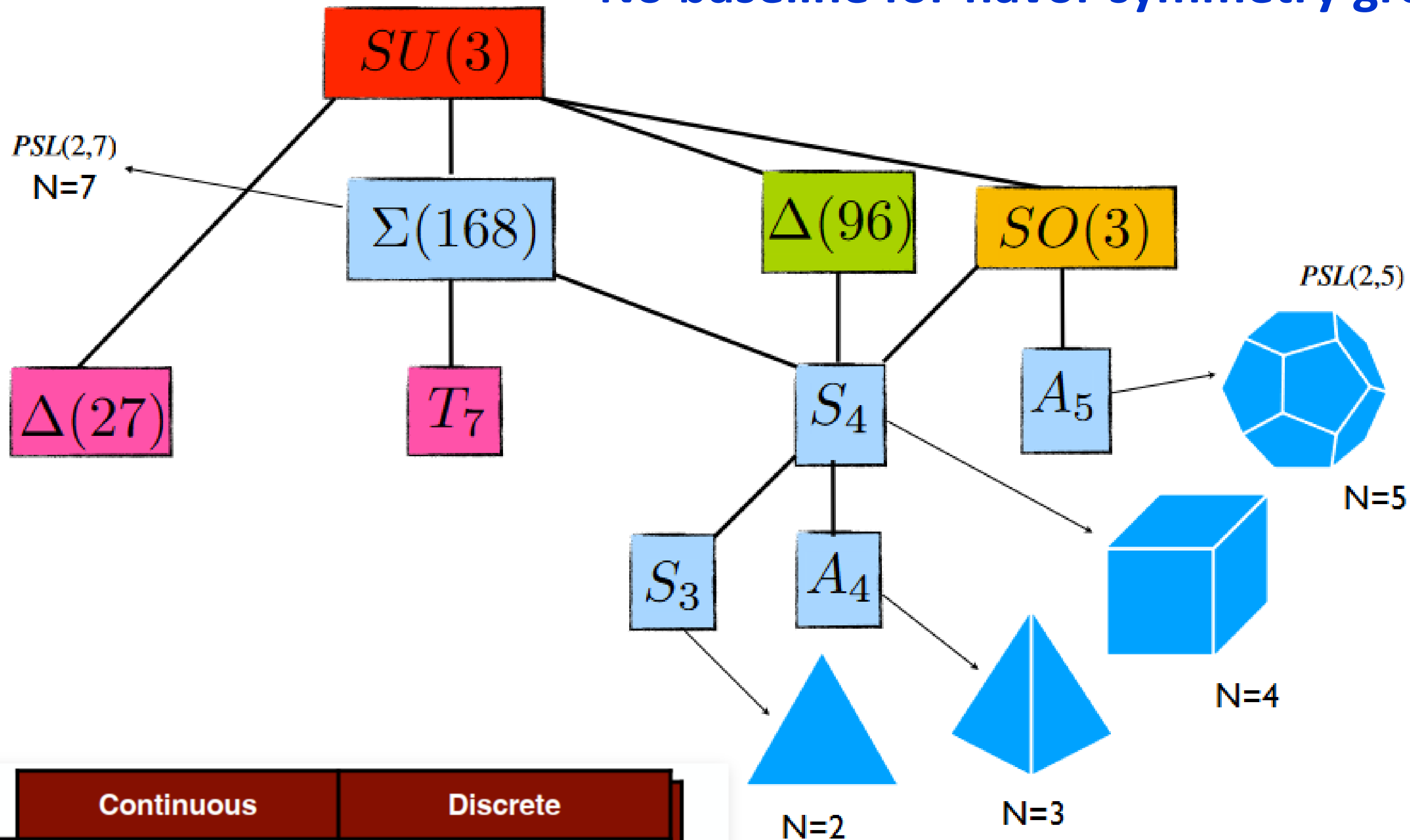
$$\begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \\ \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \\ \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \end{pmatrix} \sim \mathbf{3}$$



[Reviews: Altarelli, Feruglio,1002.0211;Tanimoto et al., 1003.3552; King and Luhn, 1301.1340; Xing,1909.09610; Feruglio,Romanino, 1912.06028; Ding, King, 2311.09282; Ding,Valle, 2402.16963]

Non-Abelian discrete flavor symmetry

No baseline for flavor symmetry group!



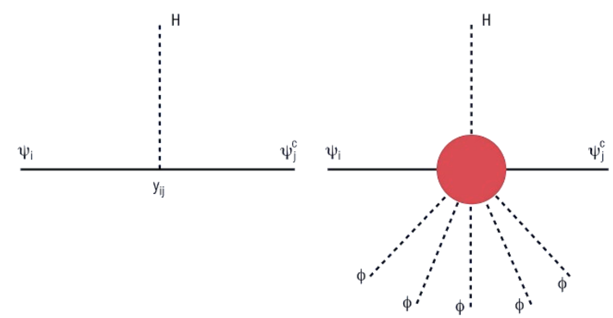
	Continuous	Discrete
Abelian	U(1)	Z_n
Non-Abelian	SU(3), SO(3),…	S_3, A_4, S_4, A_5, …

[Ding, King, 2311.09282]

Simplest flavor symmetry: Froggatt-Nielsen mechanism

➢ Flavor symmetry ***G**$_f$* **= *U*(1)**

$$\mathcal{L}_{\text{Yuk}} = y_{ij}\overline{\psi_L^i}\phi\psi_R^j \rightarrow y'_{ij}\left(\frac{\varphi}{M_F}\right)^{-FN(\psi_L^i)+FN(\psi_R^j)}\overline{\psi_L^i}\phi\psi_R^j$$



[C. D. Froggatt and H. B. Nielsen, Nucl. Phys. B 147, 277 (1979)]

The scalar field φ carrying a negative unit of FN charge, its VEV breaks the $U(1)_f$ flavor symmetry

$$FN(\varphi) = -1, \quad \lambda = \langle\varphi\rangle / M_F < 1$$

Fermion mass matrix:

$$(M_\psi)_{ij} = y'_{ij}\left(\frac{\langle\varphi\rangle}{M_F}\right)^{-FN(\psi_L^i)+FN(\psi_R^j)}$$

- For example, the quark mass hierarchies and CKM mixing matrix can be reproduced by the following $U(1)_f$ charges

$$FN(Q_L^i) = (-3,-2,0)\,, \quad FN(u_R^i) = (4,2,0)\,, \quad FN(d_R^i) = (1+r,r,r)\,, \quad r \simeq 2$$

$$M_u \sim \begin{pmatrix} \lambda^7 & \lambda^5 & \lambda^3 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}, \quad M_d \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^4 & \lambda^4 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix}$$

$\mathcal{O}(1)$ parameters→ only orders of magnitude predicted

Tri-bimaximal mixing

➢ Tri-bimaximal mixing is a particular constant lepton mixing matrix favored by neutrino oscillation data before 2012, it is **ruled out** by measurement of θ_{13}

TB mixing $$U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

⇒

$$\theta_{23} = 45^\circ$$
$$\theta_{12} \simeq 35.26^\circ$$
$$\theta_{13} = 0^\circ$$

Exp. data(NuFIT6.0)

$$41.3^\circ \leq \theta_{23}^{\text{exp}} \leq 49.9^\circ$$
$$31.63^\circ \leq \theta_{12}^{\text{exp}} \leq 35.95^\circ$$
$$8.19^\circ \leq \theta_{23}^{\text{exp}} \leq 8.89^\circ$$

In the basis of diagonal charged leptons, the neutrino mass matrix is

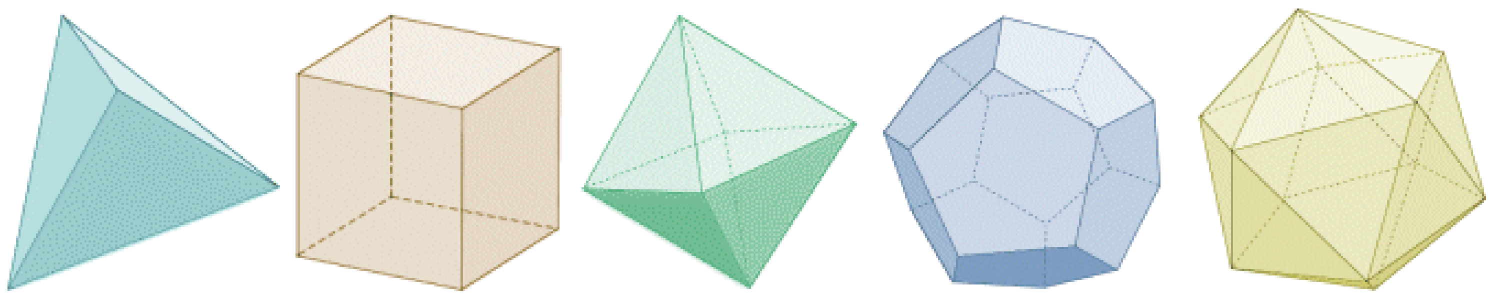
$$M_\nu = U\,\text{diag}(m_1, m_2, m_3) U^T$$

$$\Rightarrow \quad M_\nu = \frac{m_1}{6}\begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix} + \frac{m_2}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_3}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

Eigenvectors: $m_1 \to \frac{1}{\sqrt{6}}\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}, \; m_2 \to \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \; m_3 \to \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

- mixing angles are independent of neutrino masses

Platonic solids



solid	faces	vertices	Plato	Group
tetrahedron	4	4	fire	A_4
octahedron	8	6	air	S_4
icosahedron	20	12	water	A_5
hexahedron	6	8	earth	S_4
dodecahedron	12	20	?	A_5

Plato's fire A_4 can explain Tri-bimaximal mixing

[Ma and Rajasekaran, hep-ph/0106291; Babu, Ma, Valle,hep-ph/0206292; Altarelli, Feruglio, hep-ph/0504165]

Discrete flavor symmetry approach to lepton mixing

- ①Lepton mixing is **fully** determined by flavor symmetry G_f, **i.e.** $G_l > Z_2$ **&** $G_\nu > Z_2$

$$U = \frac{1}{\sqrt{3}}\begin{pmatrix} \sqrt{2}\cos\vartheta & 1 & -\sqrt{2}\sin\vartheta \\ -\sqrt{2}\cos(\vartheta-\pi/3) & 1 & \sqrt{2}\sin(\vartheta-\pi/3) \\ -\sqrt{2}\cos(\vartheta+\pi/3) & 1 & \sqrt{2}\sin(\vartheta+\pi/3) \end{pmatrix}$$

ϑ: discrete, fixed by groups G_f, G_l, G_ν

- **Lepton mixing angles:**

$$\sin^2\theta_{12} = 1/(3\cos^2\theta_{13}) \simeq 0.341,$$

$$\sin^2\theta_{23} = \frac{1}{2} \pm \frac{1}{2}\tan\theta_{13}\sqrt{2-\tan^2\theta_{13}} = 0.605 \text{ or } 0.395$$

- Dirac CP phase δ_{CP} is **conserved:** $\sin\delta_{CP}=0$

Testable at JUNO, DUNE, Hyper-K

- **Larger** groups required, for example **$|G_f|=648$** for Majorana neutrinos

➢ ②Lepton mixing is **partially** determined by flavor symmetry G_f, i.e. $G_l > Z_2$ & $G_\nu = Z_2$

TM1: $U = \begin{pmatrix} \sqrt{\frac{2}{3}} & \times & \times \\ -\frac{1}{\sqrt{6}} & \times & \times \\ -\frac{1}{\sqrt{6}} & \times & \times \end{pmatrix}$ **TM2:** $U = \begin{pmatrix} \times & \frac{1}{\sqrt{3}} & \times \\ \times & \frac{1}{\sqrt{3}} & \times \\ \times & \frac{1}{\sqrt{3}} & \times \end{pmatrix}$ **GR2:** $U = \begin{pmatrix} \times & s^\nu_{12} & \times \\ \times & \frac{c^\nu_{12}}{\sqrt{2}} & \times \\ \times & \frac{c^\nu_{12}}{\sqrt{2}} & \times \end{pmatrix}$

$\tan\theta^\nu_{12} = 2/(1+\sqrt{5}) = 1/\phi$

Sum rules: ⇓

$$\begin{cases} 3\cos^2\theta_{12}\cos^2\theta_{13} = 2, \\ \cos\delta = \dfrac{(5\sin^2\theta_{13}-1)\cot 2\theta_{23}}{2\sin\theta_{13}\sqrt{2-6\sin^2\theta_{13}}} \end{cases}$$

$$\begin{cases} 3\sin^2\theta_{12}\cos^2\theta_{13} = 1, \\ \cos\delta = \dfrac{\cos 2\theta_{13}\cot 2\theta_{23}}{\sin\theta_{13}\sqrt{2-3\sin^2\theta_{13}}} \end{cases}$$

$$\begin{cases} \sqrt{5}\,\phi\sin^2\theta_{12}\cos^2\theta_{13} = 1\,, \\ \cos\delta = \dfrac{(\phi^2\cot^2\theta_{13}-2)\cot 2\theta_{23}}{2\sqrt{\phi^2\cot^2\theta_{13}-1}} \end{cases}$$

[Albright, Rodejohann, 0812.0436; Albright, Dueck,Rodejohann, 1004.2798; Costa, King, 2307.13895; Ding,Valle,2402.16963 ....]

➢ ②Lepton mixing is **partially** determined by flavor symmetry G_f, i.e. $G_l > Z_2$ & $G_\nu = Z_2$

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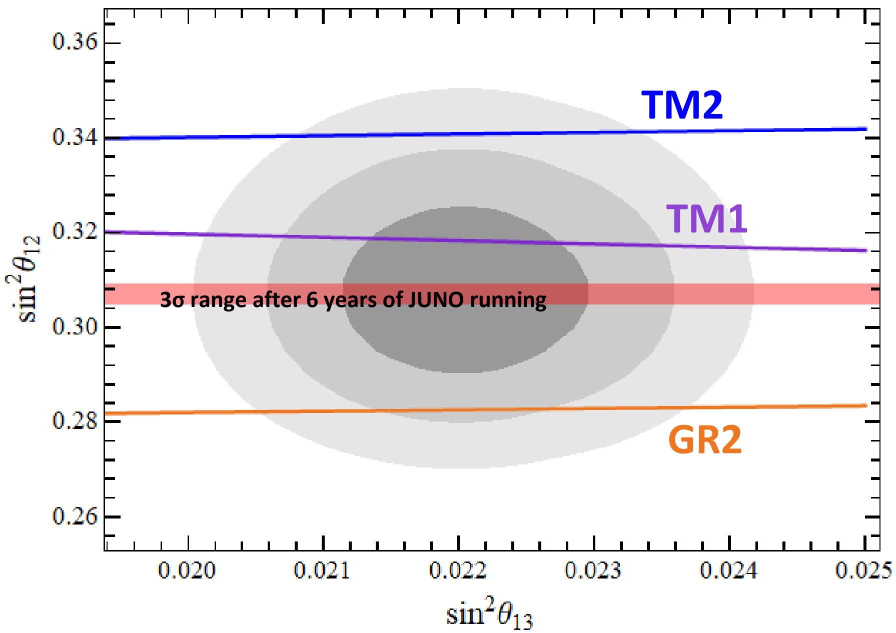
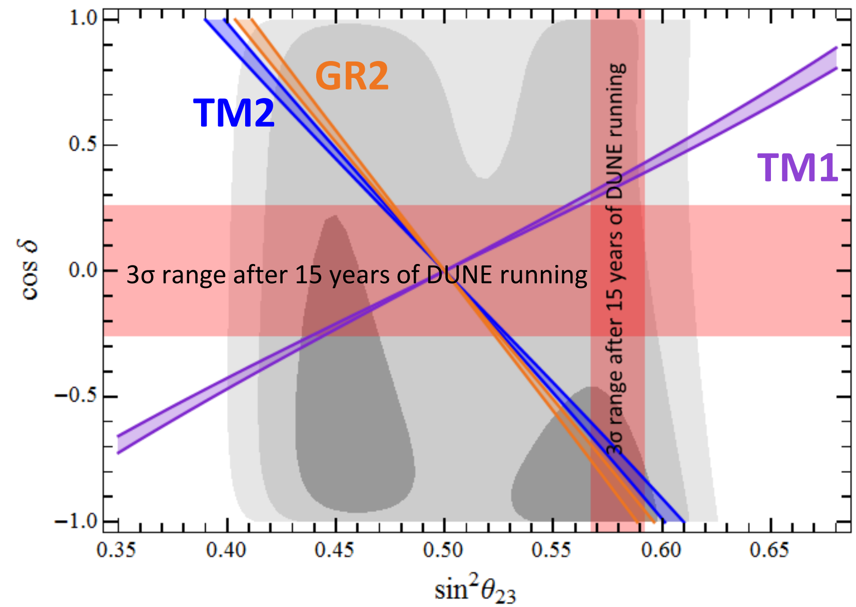
$\tan\theta^\nu_{12} = 2/(1+\sqrt{5}) = 1/\phi$

[Costa, King, 2307.13895]

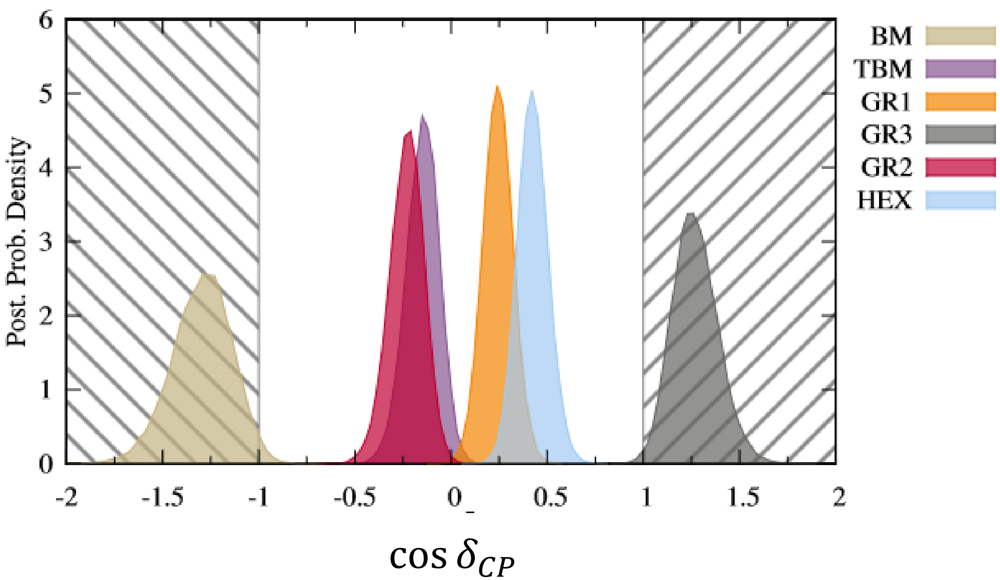


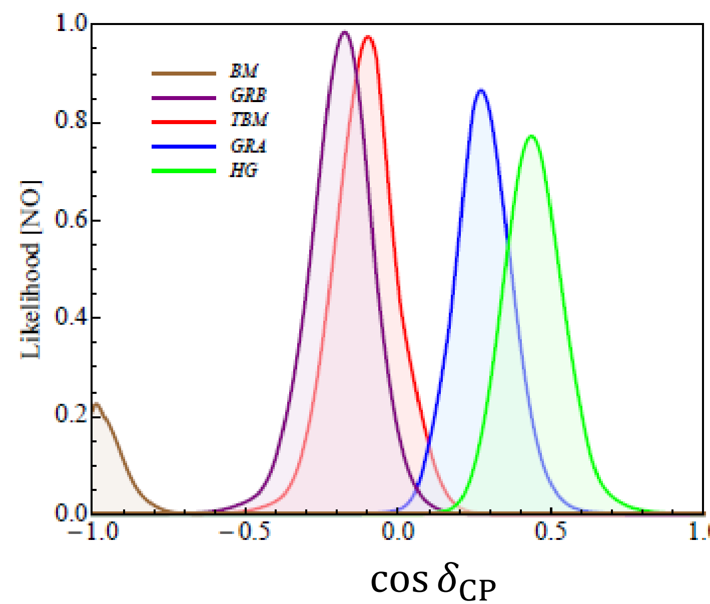


➢ ③Lepton mixing is partially determined by flavor symmetry G_f, i.e. $G_l \leq Z_2$ & $G_\nu > Z_2$

$$U = U_{12}^{e\dagger} U_{23}^{e\dagger} R_{23}^{\nu} R_{12}^{\nu}$$

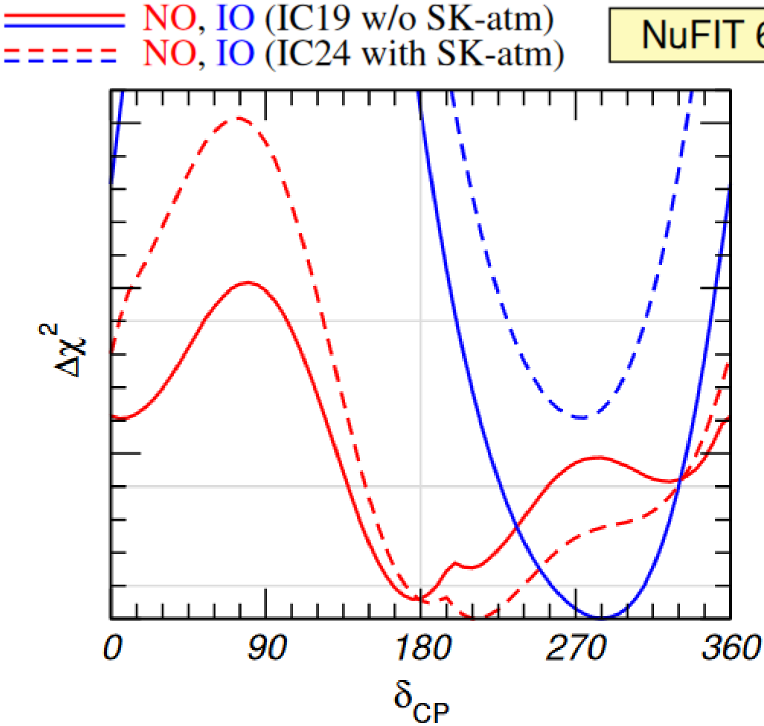
$$\cos\delta_{CP} = \frac{\tan^2\theta_{23}\sin^2\theta_{12} + \sin^2\theta_{13}\cos^2\theta_{12} - \sin^2\theta_{12}^{\nu}(\tan^2\theta_{23} + \sin^2\theta_{13})}{\sin 2\theta_{12}\sin\theta_{13}\tan\theta_{23}}$$



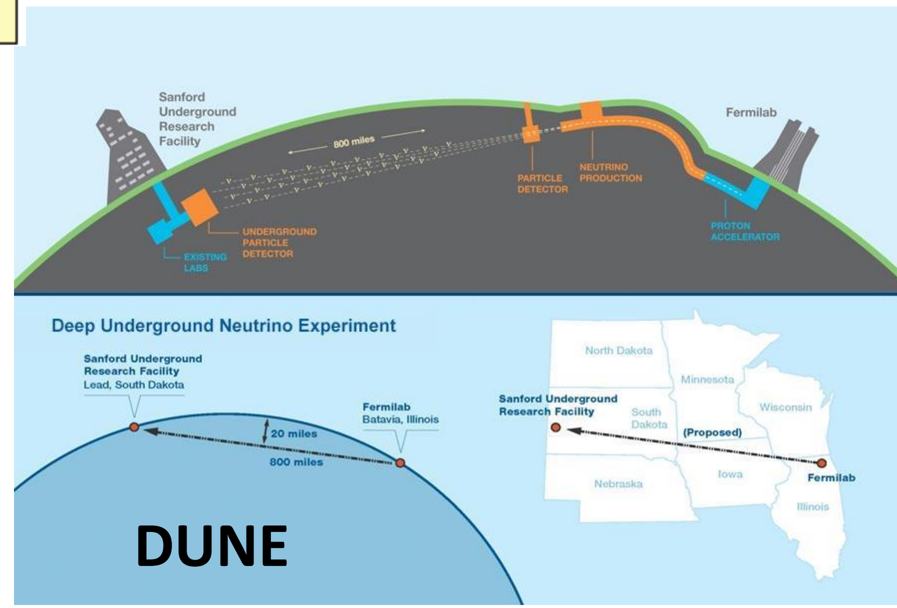


[Petcov, 1405.6006; Ballett, King, et al, 1410.7573; Girardi,Petcov,Titov,1410.8056; Ding,Valle,2402.16963 ….]

Group theoretical origin of CP violation



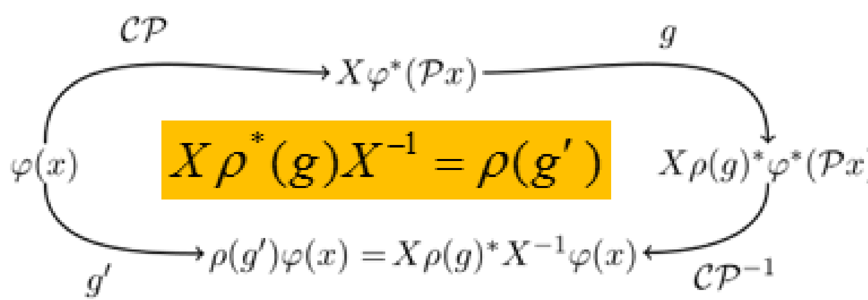
NuFIT 6.0 (2024)



- **flavor symmetry → flavor+CP symmetries**

Flavor symmetry ⇕ **CP symmetry** } **Mixing angles &CP phases**

"closure" relations have to hold!



$$X\rho^*(g)X^{-1} = \rho(g')$$

[Grimus,Rebelo,hep-ph/9506272; Feruglio et al., 1211.5560; Lindner et al., 1211.6953; Chen et al., 1402.0507]

a simple predictive CP symmetry: μτ reflection

μτ reflection =μτ exchange+canonical CP

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \rightarrow \begin{pmatrix} \nu_e^c \\ \nu_\tau^c \\ \nu_\mu^c \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \nu_e^c \\ \nu_\mu^c \\ \nu_\tau^c \end{pmatrix}$$

This CP transformation is **not** a unit matrix.

If the neutrino mass matrix is **invariant** under the μτ reflection

$$m_\nu = \begin{pmatrix} a & b & b^* \\ b & c & d \\ b^* & d & c^* \end{pmatrix}$$

$\nu_e \quad \nu_\mu \leftrightarrow \nu_\tau^c$

$\Rightarrow \quad |U| =$

$\Rightarrow \quad \theta_{23} = \dfrac{\pi}{4}, \quad \delta_{CP} = \pm\dfrac{\pi}{2}$

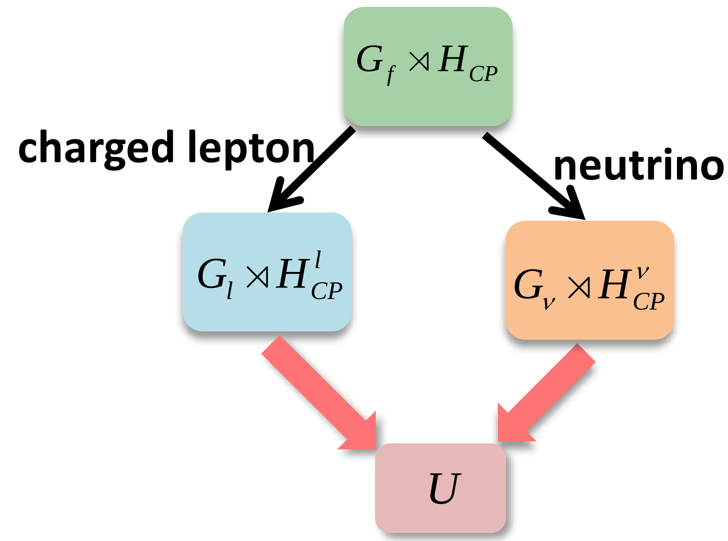
The last two rows have equal magnitudes

[Harrison, Scott, hep-ph/0210197; Grimus, Lavoura, hep-ph/0305309; Xing, Zhao, 1512.04207]

Flavor and CP symmetry to lepton mixing

- Flavor + CP symmetries have rich symmetry breaking patterns, and the resulting lepton mixing matrix is determined up to few continuous free parameters.

$G_l \rtimes H^l_{CP}$	$G_\nu \rtimes H^\nu_{CP}$	U	# parameters
Z_n	$K_4 \times CP$	$Q_l^\dagger P_l^T \Sigma_l^\dagger \Sigma_\nu P_\nu Q_\nu$	0
Z_n	$Z_2 \times CP$	$Q_l^\dagger P_l^T \Sigma_l^\dagger \Sigma_\nu R_{23}(\theta) P_\nu Q_\nu$	1
$Z_2 \times CP$	$K_4 \times CP'$	$Q_l^\dagger P_l^T R_{23}^T(\theta_l) \Sigma_l^\dagger \Sigma_\nu P_\nu Q_\nu$	
$Z_2 \times CP$	$Z_2 \times CP'$	$Q_l^\dagger P_l^T R_{23}^T(\theta_l) \Sigma_l^\dagger \Sigma_\nu R_{23}(\theta_\nu) P_\nu Q_\nu$	2
Z_2	$K_4 \times CP$	$Q_l^\dagger P_l^T U_{23}^\dagger(\theta_l, \delta_l) \Sigma_l^\dagger \Sigma_\nu P_\nu Q_\nu$	
Z_n	CP	$Q_l^\dagger P_l^T \Sigma_l^\dagger \Sigma_\nu O_3(\theta_1, \theta_2, \theta_3) Q_\nu$	3
CP	$K_4 \times CP'$	$Q_l^\dagger O_3^T(\theta_1, \theta_2, \theta_3) \Sigma_l^\dagger \Sigma_\nu Q_\nu$	
Z_2	$Z_2 \times CP$	$Q_l^\dagger P_l^T U_{23}^\dagger(\theta_l, \delta_l) \Sigma_l^\dagger \Sigma_\nu R_{23}(\theta_\nu) P_\nu Q_\nu$	



[Ding,Valle,2402.16963]

- The lepton mixing angles as well as Dirac and Majorana CP phases can be predicted by residual symmetry, neutrino masses are not constrained except in concrete models.

Universal flavor symmetry for quark and lepton mixing

[Lu, Ding, 1610.05682, 1806.02301; Li, Lu, Ding, 1706.04576; Lu, Ding, 1901.07414]

$$G_f \rtimes H_{CP}$$

charged lepton

$$Z_2^{g_l} \times X_l$$

$$X_l = \Sigma_l \Sigma_l^T, \quad \Sigma_l^\dagger \rho_3(g_l) \Sigma_l = \pm\mathrm{diag}(1,-1,-1)$$

$$U_l = \Sigma_l R_{23}(\theta_l) P_l Q_l$$

neutrino

$$Z_2^{g_\nu} \times X_\nu$$

$$X_\nu = \Sigma_\nu \Sigma_\nu^T, \quad \Sigma_\nu^\dagger \rho_3(g_\nu) \Sigma_\nu = \pm\mathrm{diag}(1,-1,-1)$$

$$U_\nu = \Sigma_\nu R_{23}(\theta_\nu) P_\nu Q_\nu$$

$$U(\theta_l,\theta_\nu) = Q_l^\dagger P_l^T R_{23}^T(\theta_l) \Sigma_l^\dagger \Sigma_\nu R_{23}(\theta_\nu) P_\nu Q_\nu$$

- All mixing angles and CP phases are expressed in terms of two free angles $\theta_{l,\nu} \in [0,\pi)$
- This scheme can be extended to quark sector, and the quark and lepton mixing can be described simultaneously in terms of totally four free angles.

Quark and lepton mixing from Dihedral group D_n and CP

Quark sector:

[Lu, Ding, 1901.07414; Ding,Valle,2402.16963]

Assignment: 2+1 $\begin{pmatrix} \begin{pmatrix} u_L \\ d_L \end{pmatrix} \\ \begin{pmatrix} c_L \\ s_L \end{pmatrix} \end{pmatrix} \sim \mathbf{2}_1, \quad \begin{pmatrix} t_L \\ b_L \end{pmatrix} \sim \mathbf{1}_1$

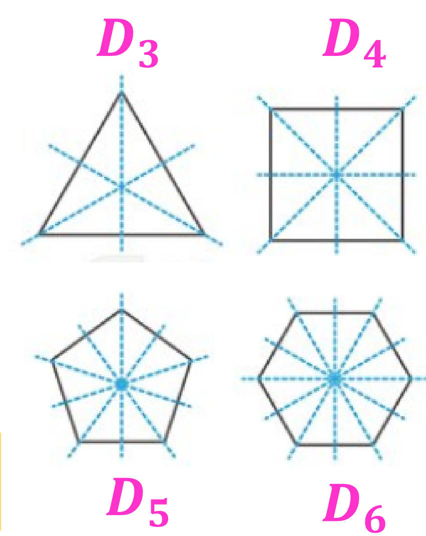
The 3rd generation is much heavier!

Residual symmetry: $Z_2^{g_u} = Z_2^{SR^x},\ X_u = SR^{n/2}, Z_2^{g_d} = Z_2^{SR^y}, X_d = S,\ x, y = 0, \ldots, n-1$

➢The CKM matrix is determined as $c_{u,d} \equiv \cos\theta_{u,d},\ s_{u,d} \equiv \sin\theta_{u,d}$

$$V_{CKM} = \begin{pmatrix} -c_d \sin\varphi_1 & \cos\varphi_1 & s_d \sin\varphi_1 \\ c_u c_d \cos\varphi_1 + i s_u s_d & c_u \sin\varphi_1 & -c_u s_d \cos\varphi_1 + i c_d s_u \\ -c_d s_u \cos\varphi_1 + i c_u s_d & -s_u \sin\varphi_1 & s_u s_d \cos\varphi_1 + i c_u c_d \end{pmatrix}$$

with $\varphi_1 = \dfrac{y-x}{n}\pi$ ⟵ **fixed by residual symmetry**



- Cabibbo angle from group: $\cos^2\theta_{13}^q \sin^2\theta_{12}^q = \cos^2\varphi_1$

- CP phase from mixing angles: $J_{CP}^q \approx \frac{1}{2}\sin 2\varphi_1 \sin\varphi_2 \sin\theta_{13}^q \sin\theta_{23}^q$

- Viable CKM matrix for $\phi_1 = 3\pi/7$ which can be achieved in D_{14} group

	θ_u^{bf}/π	θ_d^{bf}/π	$\sin\theta_{12}^q$	$\sin\theta_{23}^q$	$\sin\theta_{13}^q$	J_{CP}^q
Our	0.01326	0.00117	0.22252	0.04166	0.00357	3.223×10^{-5}
Data	—	—	0.22500 ± 0.00100	0.04200 ± 0.00059	0.003675 ± 0.000095	$(3.120\pm0.090)\times10^{-5}$

Hierarchical quark mixing angles and irregular CP phase can be accommodated.

- Lepton sector : $\phi_1 = 2\pi/7$

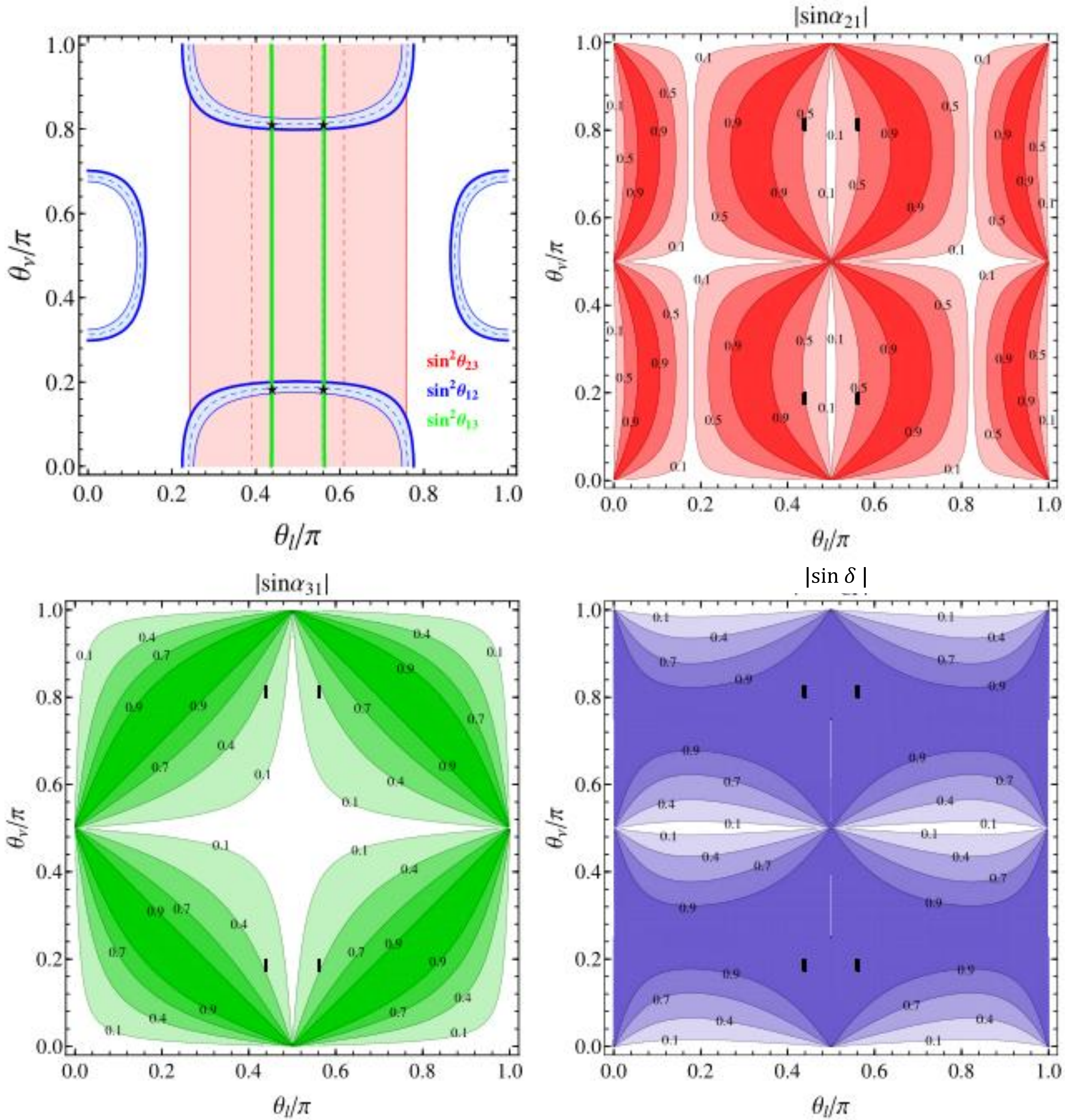
$$U_{PMNS} = R_{12}(\theta_l)\begin{pmatrix} 0 & \cos\frac{2\pi}{7} & \sin\frac{2\pi}{7} \\ i & 0 & 0 \\ 0 & -\sin\frac{2\pi}{7} & \cos\frac{2\pi}{7} \end{pmatrix} R_{13}(\theta_\nu)Q_\nu$$

$$\cos^2\theta_{23}\cos^2\theta_{13} = \cos^2\frac{2\pi}{7}$$

- Numerical benchmark

$$\theta_l = 0.439\pi,\quad \theta_\nu = 0.811\pi,\quad \chi^2_{\min} = 4.147,$$
$$\sin^2\theta_{13} = 0.0220,\quad \sin^2\theta_{12} = 0.318,\quad \sin^2\theta_{23} = 0.603,$$
$$\delta = 1.530\pi,\quad \alpha_{21}/\pi = 0.164\ (\mathrm{mod}\ 1),\quad \alpha_{31}/\pi = 0.112\ (\mathrm{mod}\ 1)$$

Atmospheric angle $\theta_{23} > 45^\circ$ and nearly maximal CP violation $\delta \approx 3\pi/2$

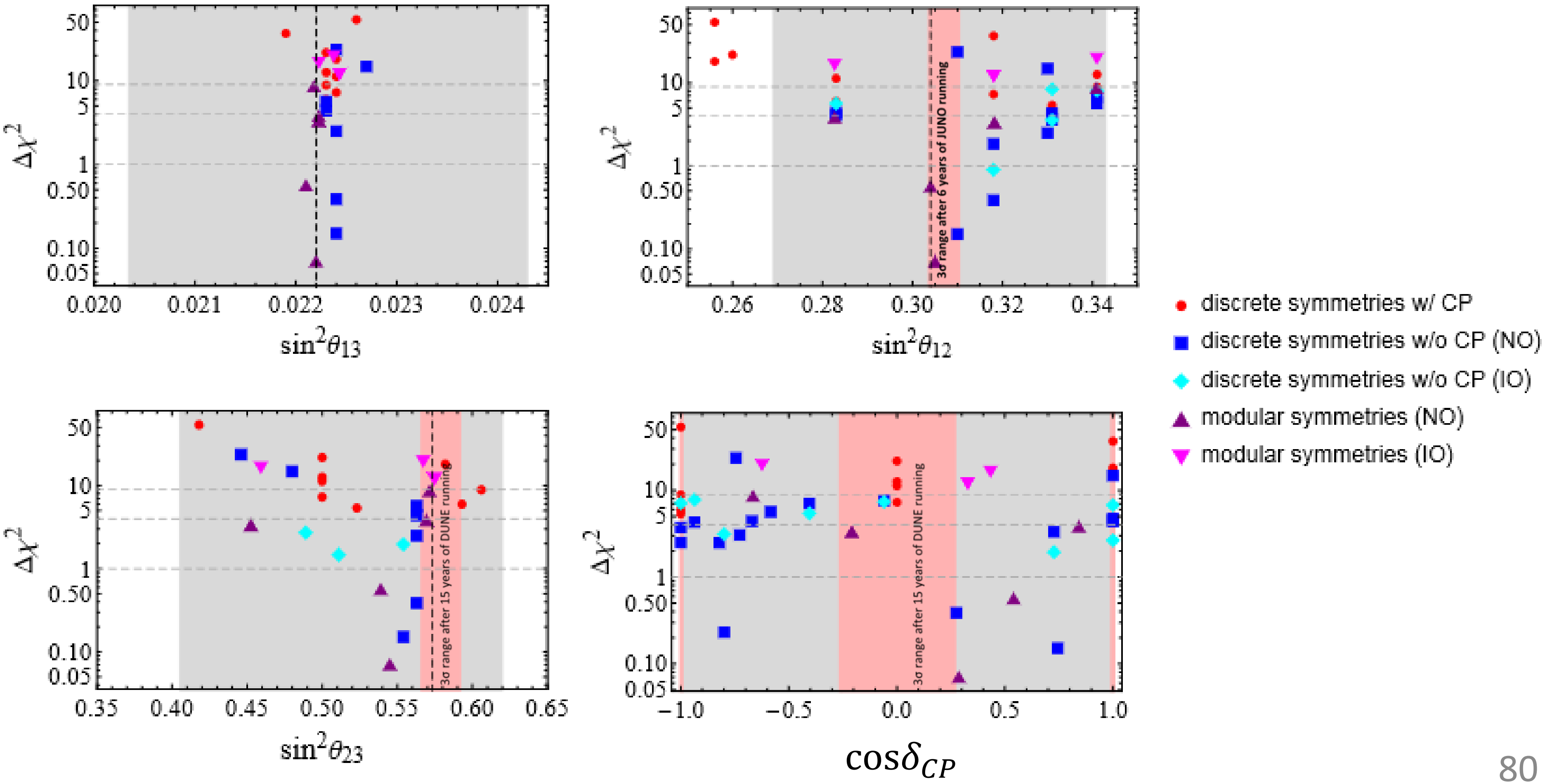


quite predictive!

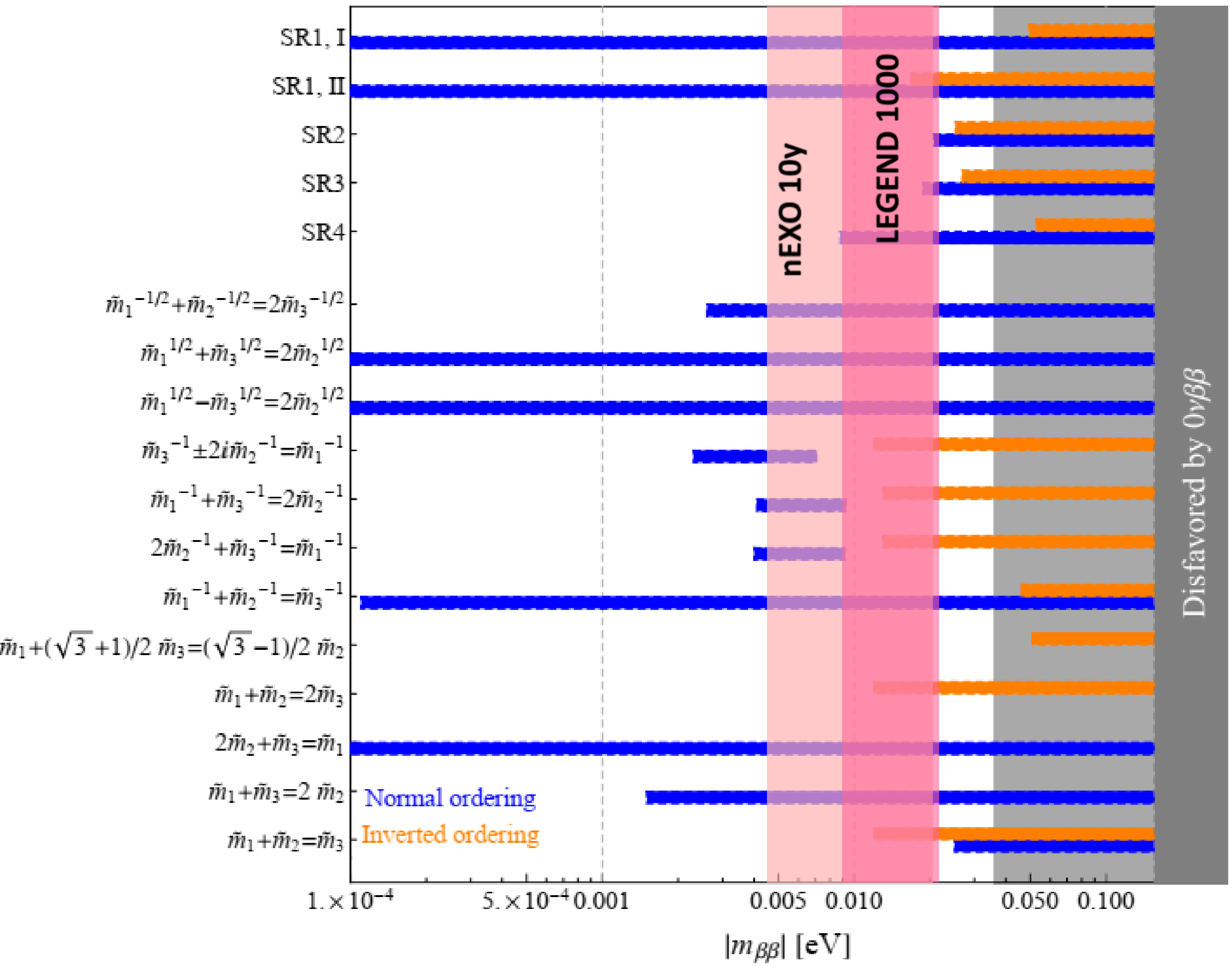
Testing flavor & CP symmetries

Flavor symmetry models can be ruled out by measuring symmetry protected correlations

- Precise measurements of mixing angles and CP phase



➢ Test neutrino mass sum rules of flavor symmetry at 0νββ decay



[Snowmass, 2203.12169]

PMNS

neutrino masses

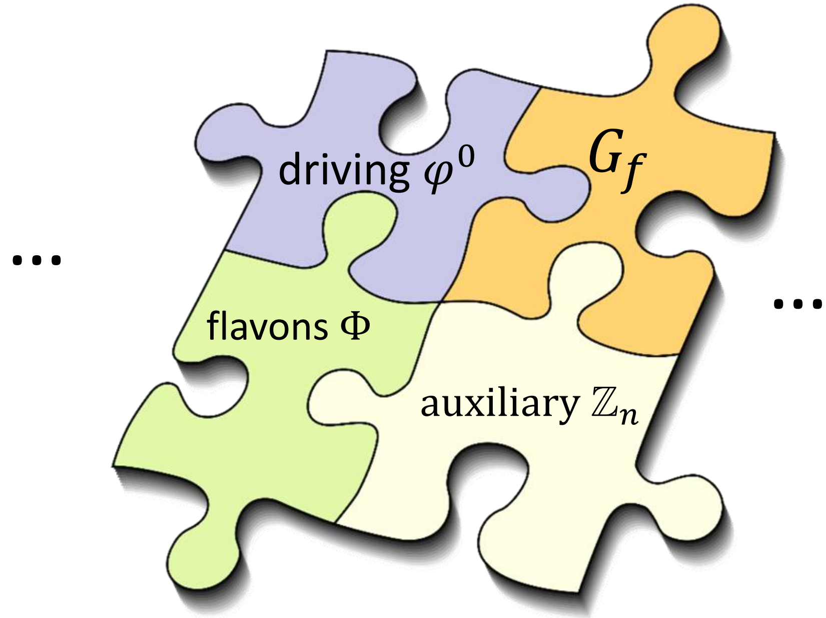
Majorana phases

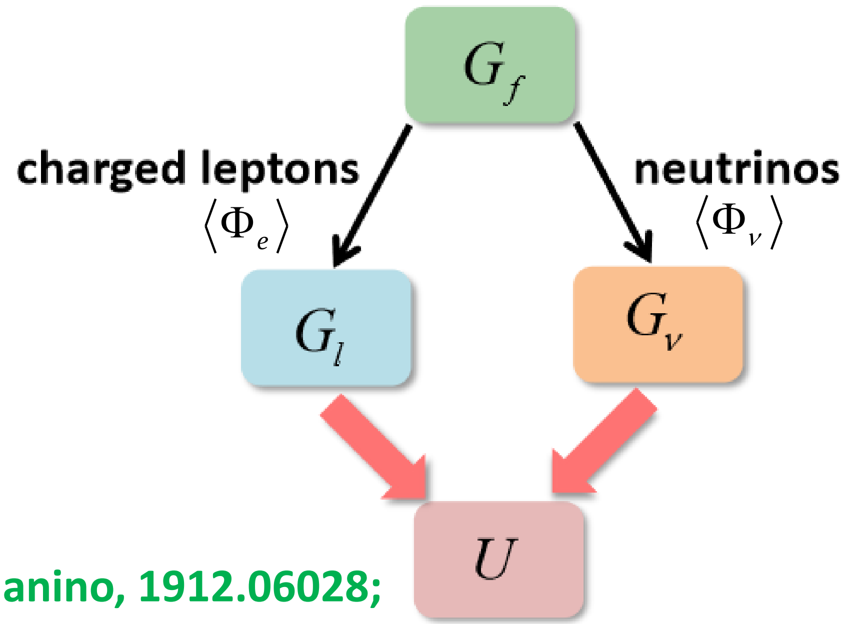
$$|m_{\beta\beta}| = \left|\sum_k U_{ek}^2\, m_k\right|$$

$$= \left|c_{12}^2\, c_{13}^2\, m_1 + s_{12}^2\, c_{13}^2\, e^{i\alpha_{21}}\, m_2 + s_{13}^2\, e^{i\alpha_{31}}\, m_3\right|$$

cosines and sines of the mixing angles

Obstacles of flavor symmetry model building





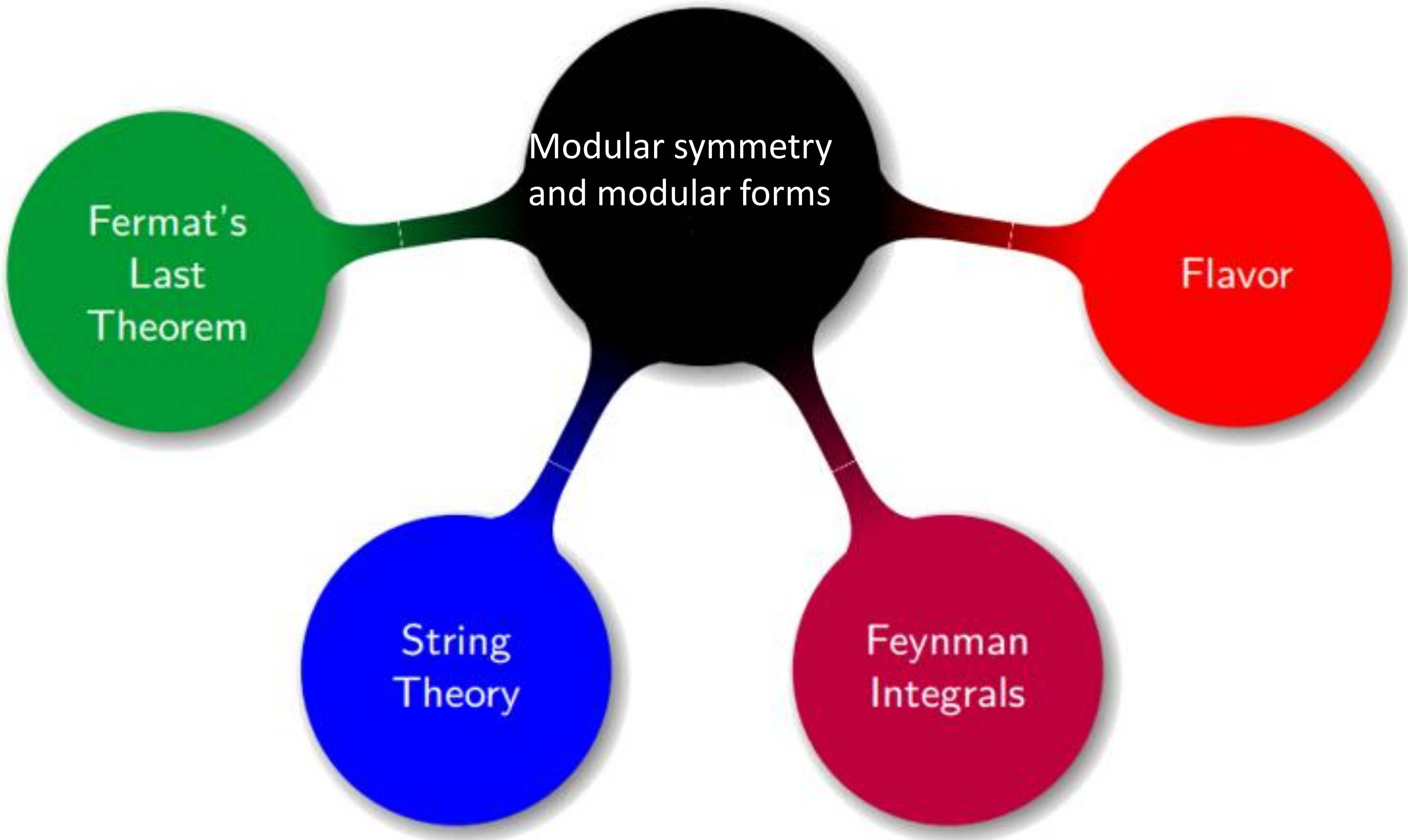
[Feruglio,Romanino, 1912.06028; Ding,Valle, 2402.16963]

➢ Realistic description of fermion masses and mixing angles requires flavor symmetry G_f be broken by Higgs-like fields "flavons" Φ_e, Φ_ν etc

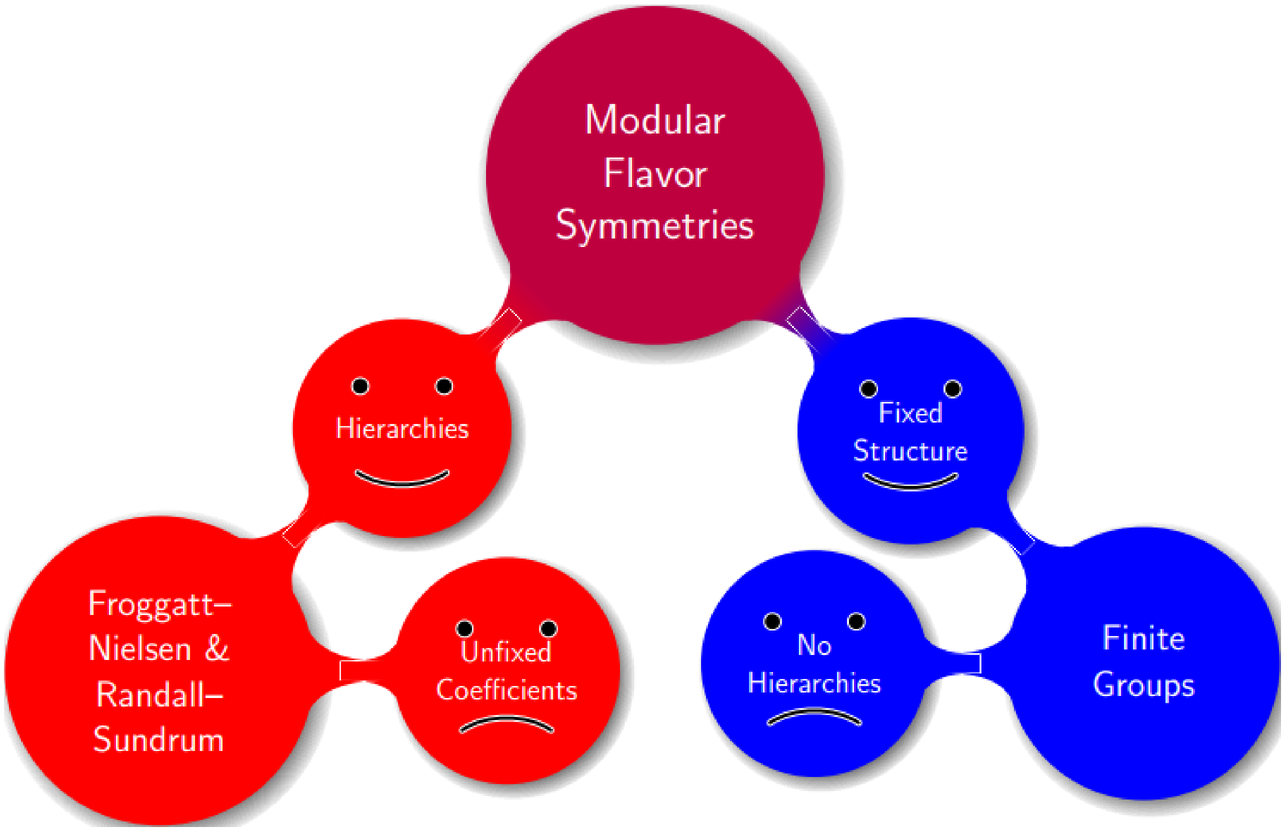
- A large number of free parameters in the scalar potential
- large shaping symmetries and many auxiliary fields

Flavor symmetry models are complicated by the symmetry breaking sector!

Modular symmetry and modular forms are well-known in Mathematics and some areas of physics.



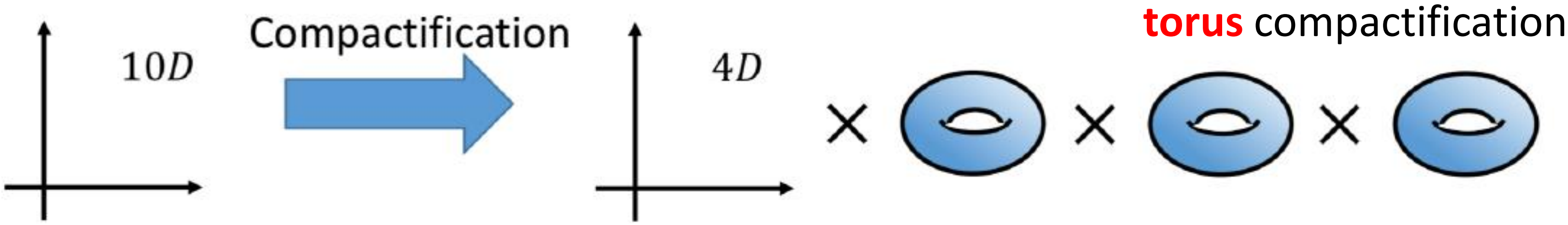
Modular flavor symmetries give rise to the attractive features of FN and RS models as well as non-Abelian flavor symmetries while avoiding their problematic aspects



[Ding, King, arXiv:2311.09282]

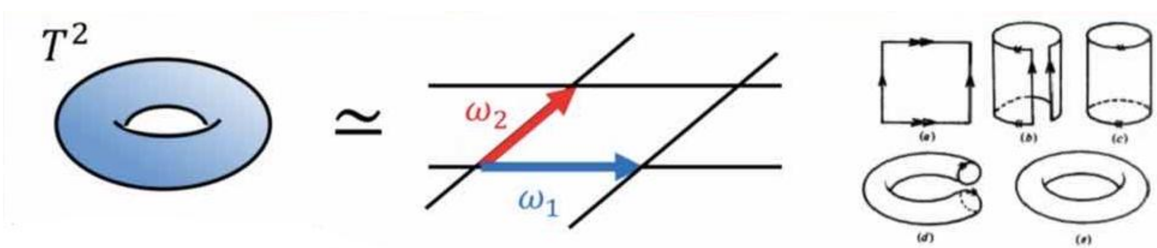
Modular symmetry

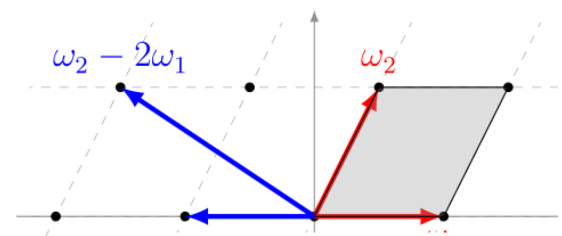
Modular invariance is motivated by more fundamental theory such as string theory at high energy scale



4D effective Lagrangian : $S = \int d^4x d^6y \, \mathcal{L}_{10D} \Rightarrow \int d^4x \, \mathcal{L}_{\text{eff}}(\varphi, \tau_i)$

The shape of a torus T^2 is characterized by a modulus $\tau = \omega_2 / \omega_1, \ \text{Im} \ \tau > 0$





$(x, y) \sim (x, y) + n_1\omega_1 + n_2\omega_2$

The torus (lattice) is left **invariant** by modular transformations

$$\begin{pmatrix} \omega_2' \\ \omega_1' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \omega_2 \\ \omega_1 \end{pmatrix} \Rightarrow \tau \to \gamma\tau = \frac{a\tau + b}{c\tau + d}$$

$SL(2,Z): \ ad - bc = 1, \ a, b, c, d$ integers

- **Modular group : generated by S (duality), T (shift)**

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \tau \mapsto -\frac{1}{\tau}$$

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad \tau \mapsto \tau + 1$$

$$S^4 = (ST)^3 = 1$$

$\mathcal{L}_{\text{eff}}(\tau, \Phi) \Rightarrow \mathcal{L}_{\text{eff}}$ **modular invariant**

Modular invariant theory

For N=1 global SUSY, the modular invariant action

[Ferrara et al, 1989; Feruglio, 1706.08749]

$$S = \int d^4x d^2\theta d^2\bar{\theta}\, K(\Phi_I, \bar{\Phi}_I, \tau, \bar{\tau}) + \int d^4x d^2\theta\, W(\Phi_I, \tau) + \text{h.c.}$$

- Minimal Kahler potential **(less constrained)**

$$K = -h\Lambda^2 \ln(-i\tau + i\bar{\tau}) + \sum_I (-i\tau + i\bar{\tau})^{-k_I} |\Phi_I|^2$$

- Modular invariant superpotential

$$W = \sum_n Y_{I_1 I_2 \ldots I_n}(\tau)\; \Phi_{I_1}\Phi_{I_2}\ldots\Phi_{I_n}$$

Modular transformation: weight k, representation $\rho(\gamma)$

$$\tau \to \gamma\tau = \frac{a\tau + b}{c\tau + d}, \quad \Phi_I \to (c\tau + d)^{-k_I} \rho_I(\gamma)\Phi_I,$$

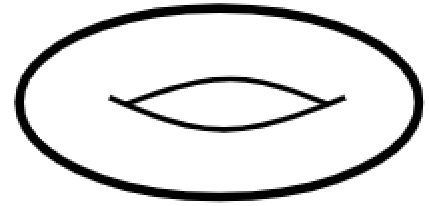
$$Y_{I_1 I_2 \ldots I_n}(\tau) \to Y_{I_1 I_2 \ldots I_n}(\gamma\tau) = (c\tau + d)^{k_Y} \rho_Y(\gamma) Y_{I_1 I_2 \ldots I_n}(\tau)$$

Modular invariance requires

$$k_Y = k_{I_1} + k_{I_2} + \cdots + k_{I_n}, \quad \rho_Y \otimes \rho_{I_1} \otimes \cdots \otimes \rho_{I_n} \supset 1$$

Yukawa couplings are modular forms $Y_{I_1 I_2 \ldots I_n}(\tau)$

Modulus can be the unique source of flavor symmetry breaking.



$SL(2,Z)$ on torus T^2

$T^N \in \Gamma(N)$

$\Gamma(N)$
congruence subgroups

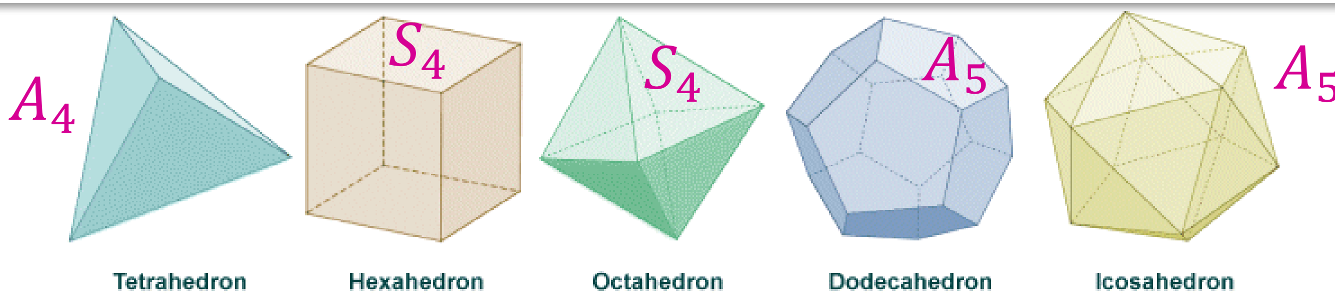
$\Gamma'_N \equiv SL(2,Z)/\Gamma(N)$
or $\Gamma_N \equiv SL(2,Z)/\pm\Gamma(N)$

finite modular groups

modular invariant flavor models

- Finite modular groups: known flavor symmetry S_3, A_4, S_4, A_5

N	2	3	4	5	6	7
Γ_N	S_3	A_4	S_4	A_5	$\Gamma_6 \cong S_3 \times A_4$	$\Gamma_7 \cong \Sigma(168)$
Γ'_N	S_3	$A'_4 = T'$	$S'_4 \cong SL(2, \mathbb{Z}_4)$	$A'_5 \cong SL(2, \mathbb{Z}_5)$	$\Gamma'_6 \cong S_3 \times T'$	$\Gamma'_7 \cong SL(2, \mathbb{Z}_7)$



- Bottom-up models for lepton and quark **[review: Ding, King, arXiv:2311.09282]**

	Γ_N/Γ'_N	leptons alone	leptons & quarks	$SU(5)$	$SO(10)$
$N=2$	S_3	Kobayashi et al, 1803.10391...	—	Kobayashi et al, 1906.10341...	—
$N=3$	A_4	Feruglio, 1706.08749,1807.01125...	Okada,Tanimoto,1905.13421; King, King, 2002.00969; Yao, Lu, Ding, 2012.13390...	Anda, King,Perdomo, 1812.05620; Chen, Ding, King,2101.12724...	Ding, King,Lu,2108.09655
	T'	Liu, Ding,1907.01488...	Lu, Liu, Ding,1912.07573...	—	—
$N=4$	S_4	Penedo,Petcov,1806.11040 ; Novichkov, Penedo et al,1811.04933...	Qu, Liu et al,2106.11659	Zhao, Zhang,2101.02266 ; Ding, King, Yao,2103.16311...	—
	S'_4	Novichkov,Penedo,Petcov,2006.03058...	Liu, Yao, Ding, 2006.10722...	—	—
$N=5$	A_5	Novichkov, Penedo et al,1812.02158; Ding, King, Liu, 1903.12588...	—	—	—
	A'_5	Wang, Yu, Zhou, 2010.10159 ...	Yao, Liu, Ding,2011.03501	—	—
$N=6$	Γ_6	—	—	Abe,Higaki et al, 2307.01419	—
	Γ'_6	Li,Liu,Ding,2108.02181	—	—	—
$N=7$	Γ_7	Ding, King et al, 2004.12662	—	—	—
	Γ'_7	—	—	—	—

Modular flavor symmetry: significant reduction of the number of parameters

Minimal modular lepton model

Modular symmetry allows to construct quite predictive lepton models. The modular flavor symmetry is modular binary octahedral group $2O$ which is the Shur double cover of S_4

	L	$E_D^c = (e^c, \mu^c)$	τ^c	N^c	$H_{u,d}$
$2O$	$\mathbf{3}$	$\widehat{\mathbf{2}}'$	$\mathbf{1}'$	$\mathbf{3}$	$\mathbf{1}$
k_I	-1	6	5	1	0

[Ding, Liu, Lu, Weng, 2307.14926]

- **Charged leptons**

$$\mathcal{W}_E = \alpha \left(E_D^c L Y^{(5)}_{\widehat{\mathbf{2}}'}\right)_{\mathbf{1}} H_d + \beta \left(E_D^c L Y^{(5)}_{\widehat{\mathbf{4}}}\right)_{\mathbf{1}} H_d + \gamma \left(\tau^c L Y^{(4)}_{\mathbf{3}'}\right)_{\mathbf{1}} H_d$$

$$M_E = \begin{pmatrix} -\alpha Y^{(5)}_{\widehat{\mathbf{2}}',2} - \sqrt{2}\beta Y^{(5)}_{\widehat{\mathbf{4}},3} & \sqrt{3}\beta Y^{(5)}_{\widehat{\mathbf{4}},1} & \sqrt{2}\alpha Y^{(5)}_{\widehat{\mathbf{2}}',1} + \beta Y^{(5)}_{\widehat{\mathbf{4}},4} \\ -\alpha Y^{(5)}_{\widehat{\mathbf{2}}',1} + \sqrt{2}\beta Y^{(5)}_{\widehat{\mathbf{4}},4} & -\sqrt{2}\alpha Y^{(5)}_{\widehat{\mathbf{2}}',2} + \beta Y^{(5)}_{\widehat{\mathbf{4}},3} & -\sqrt{3}\beta Y^{(5)}_{\widehat{\mathbf{4}},2} \\ \gamma Y^{(4)}_{\mathbf{3}',1} & \gamma Y^{(4)}_{\mathbf{3}',3} & \gamma Y^{(4)}_{\mathbf{3}',2} \end{pmatrix} v_d$$

- **Neutrino mass : seesaw mechanism**

Minimal #p: $\alpha, \beta, \gamma, g^2/\Lambda$

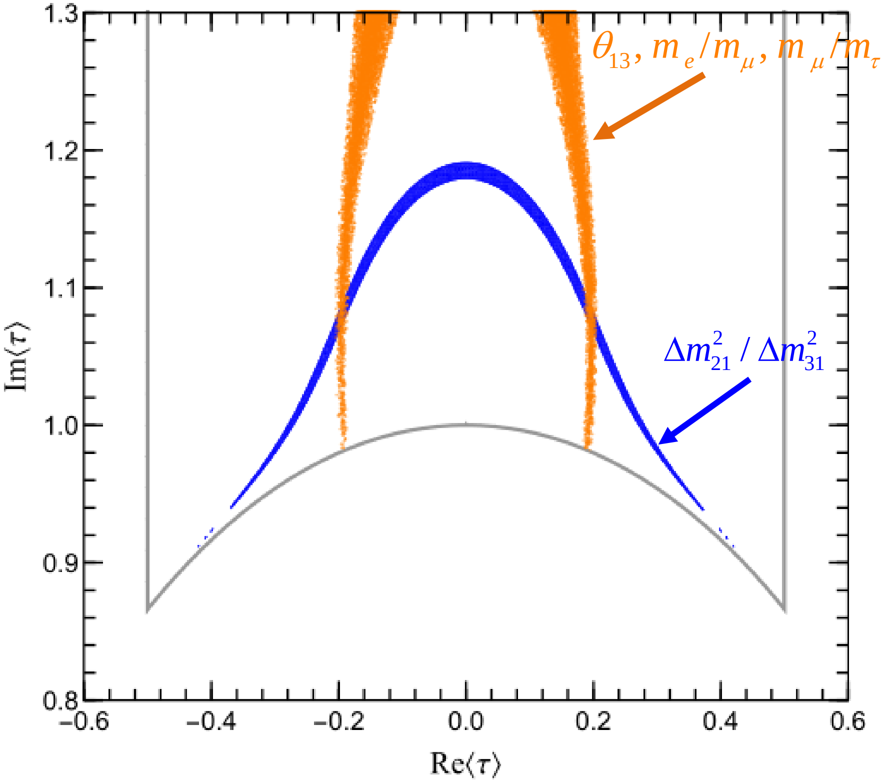
$$\mathcal{W}_\nu = g H_u (N^c L)_{\mathbf{1}} + \Lambda \left(N^c N^c Y^{(2)}_{\mathbf{2}}\right)_{\mathbf{1}}$$

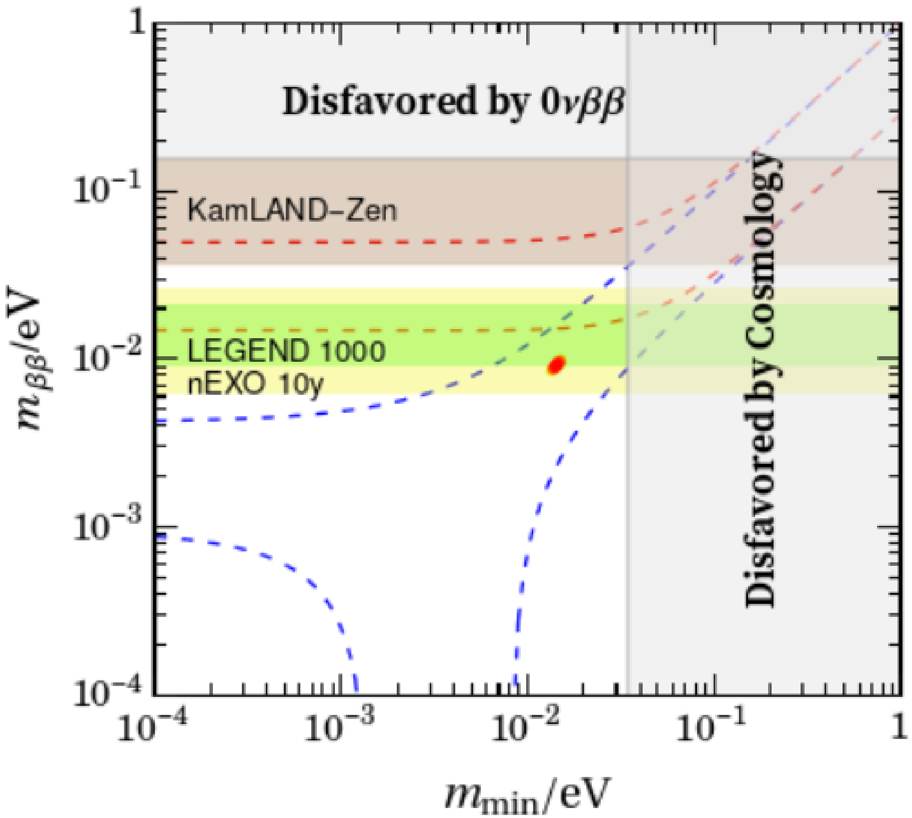
$$M_D = g \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} v_u\,, \quad M_N = \begin{pmatrix} -2Y^{(2)}_{\mathbf{2},1} & 0 & 0 \\ 0 & \sqrt{3}Y^{(2)}_{\mathbf{2},2} & Y^{(2)}_{\mathbf{2},1} \\ 0 & Y^{(2)}_{\mathbf{2},1} & \sqrt{3}Y^{(2)}_{\mathbf{2},2} \end{pmatrix} \Lambda$$

Light neutrino mass

$$m_1 = \frac{1}{|2Y_{\mathbf{2},1}^{(2)}|}\frac{g^2 v_u^2}{\Lambda}, \quad m_2 = \frac{1}{|Y_{\mathbf{2},1}^{(2)} - \sqrt{3}Y_{\mathbf{2},2}^{(2)}|}\frac{g^2 v_u^2}{\Lambda}, \quad m_3 = \frac{1}{|Y_{\mathbf{2},1}^{(2)} + \sqrt{3}Y_{\mathbf{2},2}^{(2)}|}\frac{g^2 v_u^2}{\Lambda}$$

only depends on modulus τ up to overall scale





Neutrino mass spectrum is normal ordering

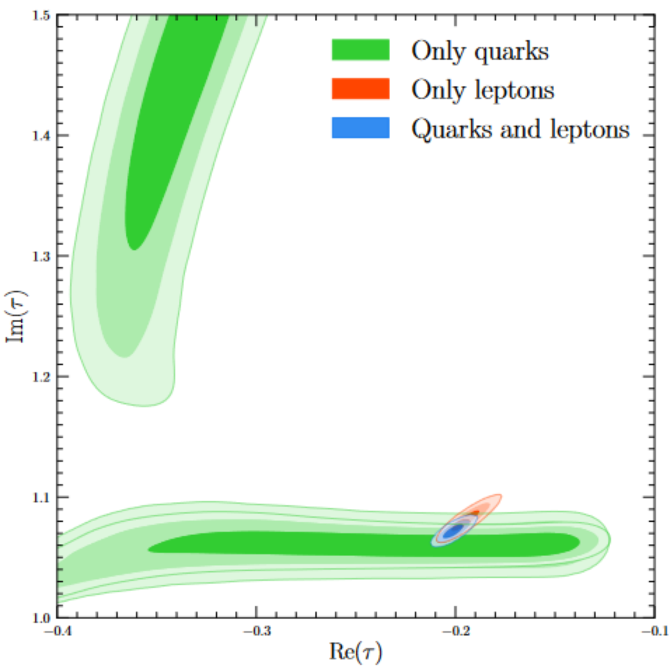
$$m_\beta = 16.76\ \text{meV}, \quad m_{\beta\beta} = 9.17\ \text{meV}$$

Extension to quark sector

	$Q_D=(Q_1,Q_2)$	Q_3	$U_D^c=(u^c,c^c)$	t^c	$D_D^c=(d^c,s^c)$	b^c
$2O$	$\mathbf{2}$	$\mathbf{1}'$	$\widehat{\mathbf{2}}'$	$\mathbf{1}'$	$\mathbf{2}$	$\mathbf{1}'$
k_I	k_{Q_D}	k_{Q_D}	$3-k_{Q_D}$	$6-k_{Q_D}$	$6-k_{Q_D}$	$-k_{Q_D}$

$$M_u=\begin{pmatrix} g_1^u Y^{(3)}_{\hat{\mathbf{4}},3} & -g_1^u Y^{(3)}_{\hat{\mathbf{4}},2} & 0 \\ g_1^u Y^{(3)}_{\hat{\mathbf{4}},4} & g_1^u Y^{(3)}_{\hat{\mathbf{4}},1} & 0 \\ -g_2^u Y^{(6)}_{\mathbf{2},2} & g_2^u Y^{(6)}_{\mathbf{2},1} & g_3^u Y^{(6)}_{\mathbf{1}} \end{pmatrix} v_u\,,\quad M_d=\begin{pmatrix} g_1^d Y^{(6)}_{\mathbf{1}}-g_3^d Y^{(6)}_{\mathbf{2},1} & g_2^d Y^{(6)}_{\mathbf{1}'}+g_3^d Y^{(6)}_{\mathbf{2},2} & -g_4^d Y^{(6)}_{\mathbf{2},2} \\ g_3^d Y^{(6)}_{\mathbf{2},2}-g_2^d Y^{(6)}_{\mathbf{1}'} & g_3^d Y^{(6)}_{\mathbf{2},1}+g_1^d Y^{(6)}_{\mathbf{1}} & g_4^d Y^{(6)}_{\mathbf{2},1} \\ 0 & 0 & g_5^d \end{pmatrix} v_d$$

8 couplings: $g_1^u, g_2^u, g_3^u, g_1^d, g_2^d, g_3^d, g_4^d, g_5^d$



The complex modulus τ is common in both quark and lepton sectors, and its value is fixed by the lepton parameters

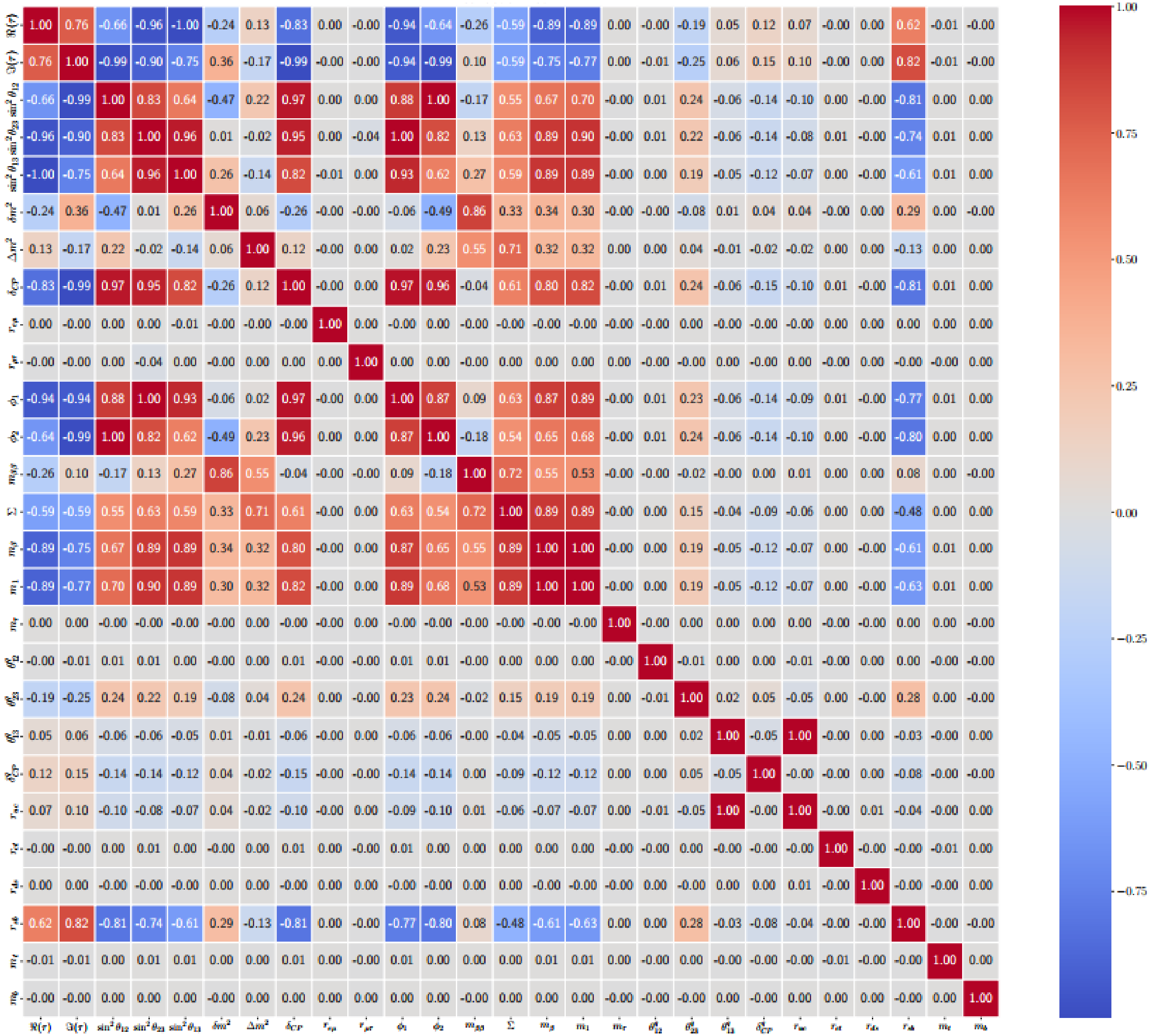
$$\langle\tau\rangle=-0.1946+1.0799i$$

- The model uses **14** parameters to describe the masses and mixing of both quark and lepton sectors: **12** masses+**6** mixing angles+**4** CP phases.

	Combined		Leptons only		Quarks only	
Observable	Best fit	χ^2 breakdown	Best fit	χ^2 breakdown	Best fit	χ^2 breakdown
$\sin^2\theta_{12}$	0.344	10.2	0.329	3.99	-	-
$\sin^2\theta_{23}$	0.508	0.0714	0.506	0.0438	-	-
$\sin^2\theta_{13}$	0.0231	1.41	0.0219	0.462	-	-
δm^2 (eV^2)	7.28×10^{-5}	0.246	7.43×10^{-5}	0.183	-	-
Δm^2 (eV^2)	0.00249	0.0636	0.00248	0.0497	-	-
$r_{e\mu}\equiv m_e/m_\mu$	0.00474	2.60×10^{-5}	0.00474	8.18×10^{-6}	-	-
$r_{\mu\tau}\equiv m_\mu/m_\tau$	0.0586	4.66×10^{-5}	0.0586	4.66×10^{-8}	-	-
m_τ (GeV)	1.29	5.52×10^{-8}	1.29	2.94×10^{-9}	-	-
$\min(\chi^2_{\text{leptons}})$	11.94		4.74		-	
θ^q_{12}	0.227	5.68×10^{-4}	-	-	0.227	1.52×10^{-7}
θ^q_{13}	0.00351	0.0146	-	-	0.00349	5.92×10^{-5}
θ^q_{23}	0.0395	0.948	-	-	0.0402	2.32×10^{-7}
δ^q_{CP}	1.23	0.221	-	-	1.21	1.63×10^{-8}
$r_{uc}\equiv m_u/m_c$	0.00212	0.100	-	-	0.00196	0.0024
$r_{ct}\equiv m_c/m_t$	0.00282	2.61×10^{-6}	-	-	0.00282	2.90×10^{-8}
$r_{ds}\equiv m_d/m_s$	0.0505	5.84×10^{-5}	-	-	0.0505	3.08×10^{-8}
$r_{sb}\equiv m_s/m_b$	0.0209	7.12	-	-	0.0183	2.62×10^{-5}
m_b	0.965	1.67×10^{-7}	-	-	0.965	1.99×10^{-8}
m_t	89.2	9.42×10^{-9}	-	-	89.2	9.52×10^{-11}
$\min(\chi^2_{\text{quarks}})$	8.40		-		0.00248	
$\min(\chi^2_{\text{comb}})$	20.3		-		-	

[Ding, Lisi, Marrone, Petcov, 2409.15823]

- **Uniqueness of modular models**: complex modulus τ is the portal connecting quarks and leptons **[Ding, Lisi, Marrone, Petcov, 2409.15823]**

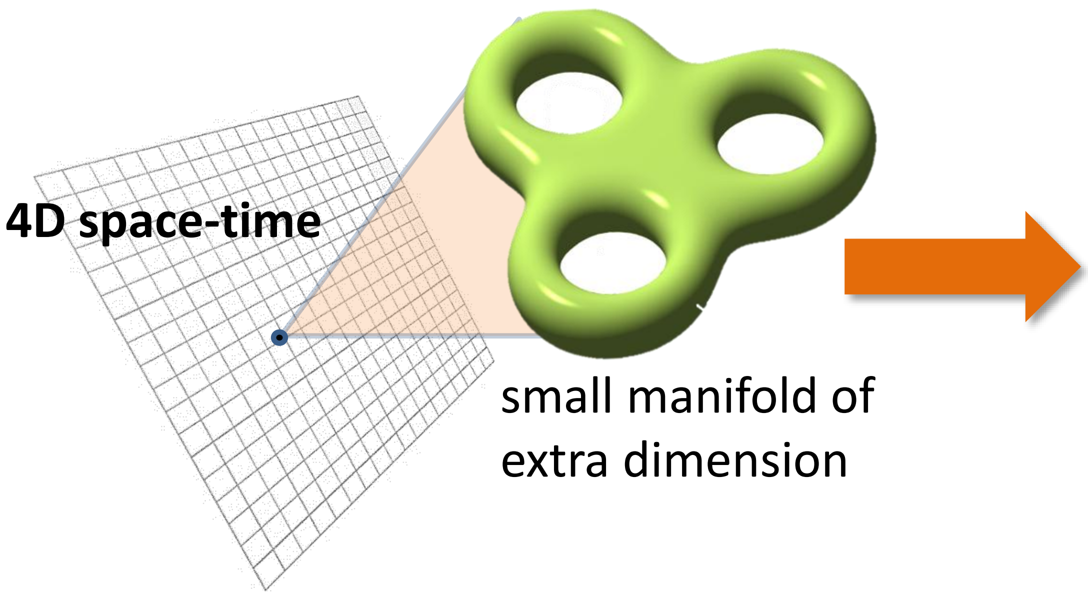


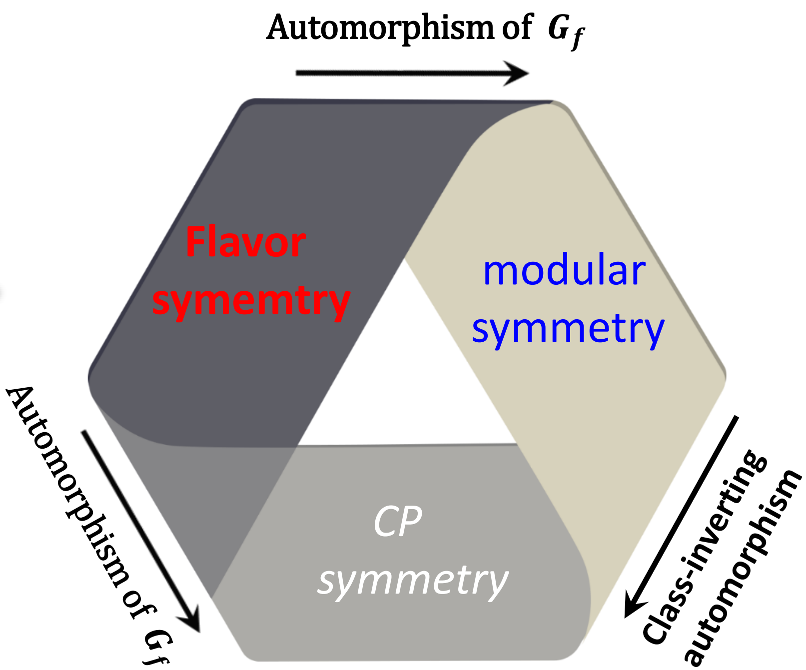
The quark mass ratio $r_{sb} \equiv m_s/m_b$ is
①strongly negatively correlated with the three lepton mixing angles;
② strongly negatively (positively) correlated with the leptonic Dirac (Majorana) CPV phase δ_{CP} (η_1, η_2);
③strongly negatively correlated with the lightest neutrino masses $m_1, m_\beta, m_{\beta\beta}$.

Unification of flavor, CP and modular symmetries

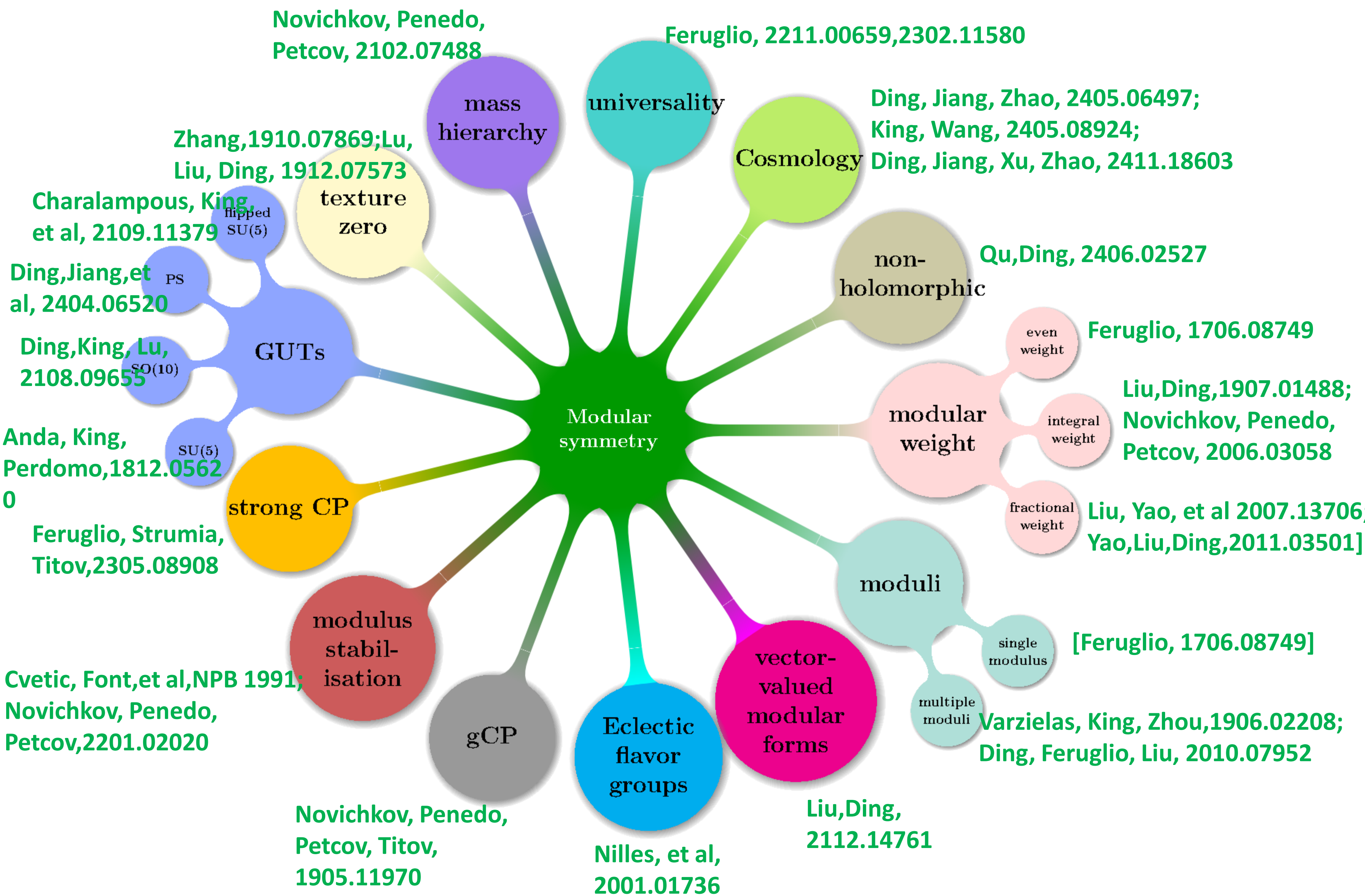
➢Top-down approach (orbifold string compactification) gives

- Normal symmetries of extra dimensions ➜traditional flavor symmetries
- String duality transformations ➜modular flavor symmetries
- CP symmetry [Nilles, Ramos-Sanchez, Vaudrevange, 2001.01736; 2004.05200]





Top-down and bottom-up approaches do not yet meet, and model building in its infancy
[Chen, Perez, Hamud, et al, 2108.02240; Baur, Nilles et al,2207.10677; Ding, King, et al, 2303.02071; Li, Ding, 2308.16901; Li, Lu, Ding,2405.13460]


Modular symmetry
mass hierarchy
Novichkov, Penedo, Petcov, 2102.07488
universality
Feruglio, 2211.00659,2302.11580
Cosmology
Ding, Jiang, Zhao, 2405.06497; King, Wang, 2405.08924; Ding, Jiang, Xu, Zhao, 2411.18603
non-holomorphic
Qu,Ding, 2406.02527
modular weight
even weight
Feruglio, 1706.08749
integral weight
Liu,Ding,1907.01488; Novichkov, Penedo, Petcov, 2006.03058
fractional weight
Liu, Yao, et al 2007.13706; Yao,Liu,Ding,2011.03501]
moduli
single modulus
[Feruglio, 1706.08749]
multiple moduli
Varzielas, King, Zhou,1906.02208; Ding, Feruglio, Liu, 2010.07952
vector-valued modular forms
Liu,Ding, 2112.14761
Eclectic flavor groups
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Conclusions

- Neutrino oscillation shows that neutrinos are massive, extension of SM is required to accommodate tiny neutrino masses.
- Many fundamental questions remain to be answered: the nature of neutrinos (Majorana vs. Dirac) , absolute neutrino mass, CP violation in lepton sector, flavor structure of quarks and lepton, the possible connection with dark matter and baryon-asymmetry…
- A rich experimental neutrino program lies ahead, complementarity of different experimental approaches: neutrino oscillation, $0\nu\beta\beta$ decay, β decay, cosmology, colliders, lepton flavor violation etc. A bright future!

Thank you for your attention!