第一届新物理冬季学校：
中微子物理（第一讲）

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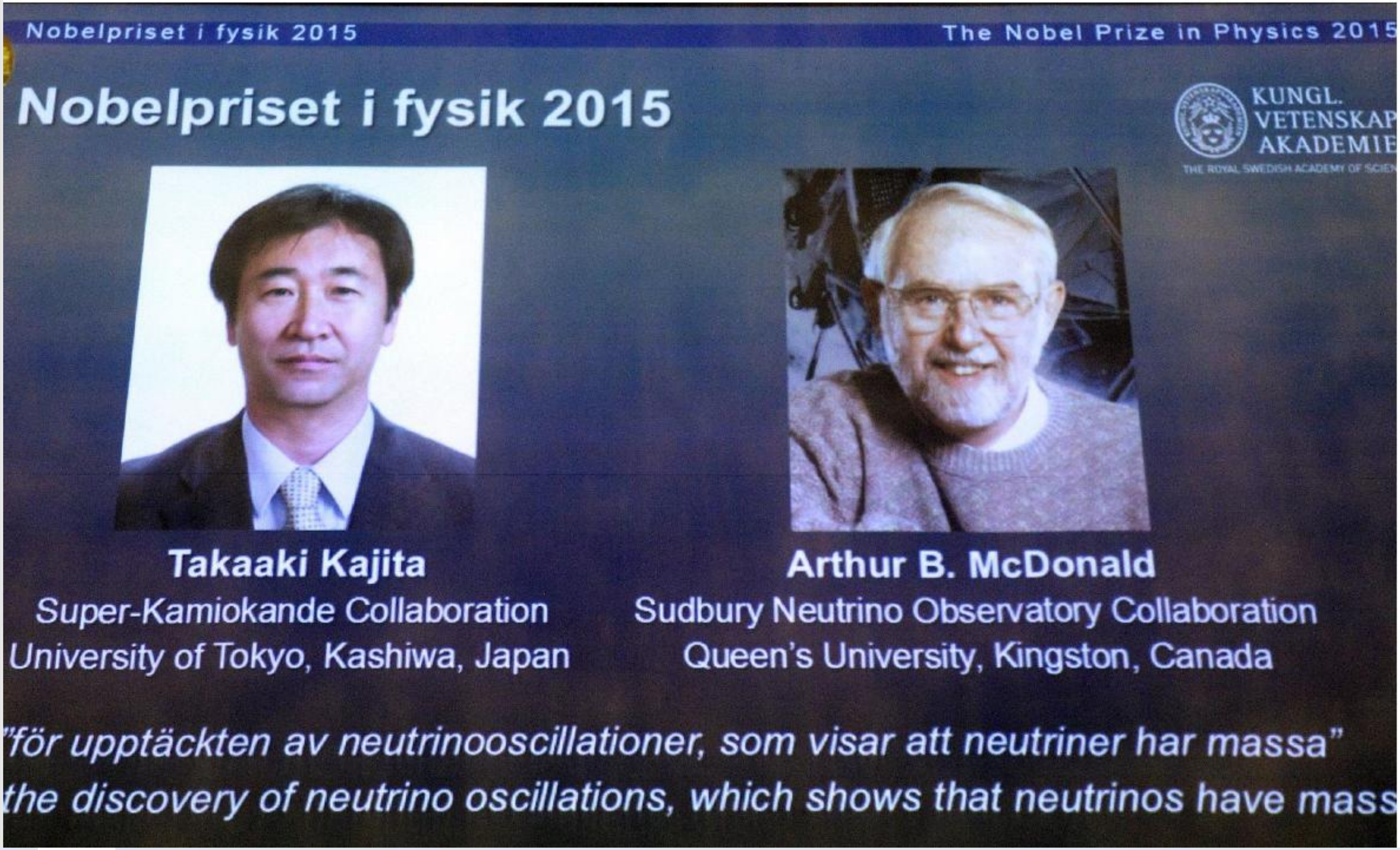
2015年10月6日，瑞典皇家科学院宣布将2015年诺贝尔物理学奖授予日本东京大学Takaaki Kajita教授和加拿大女王大学Arthur B. McDonald教授，以表彰其*发现中微子振荡现象，该现象显示中微子有质量*。



Nobelpriset i fysik 2015

The Nobel Prize in Physics 2015

Nobelpriset i fysik 2015

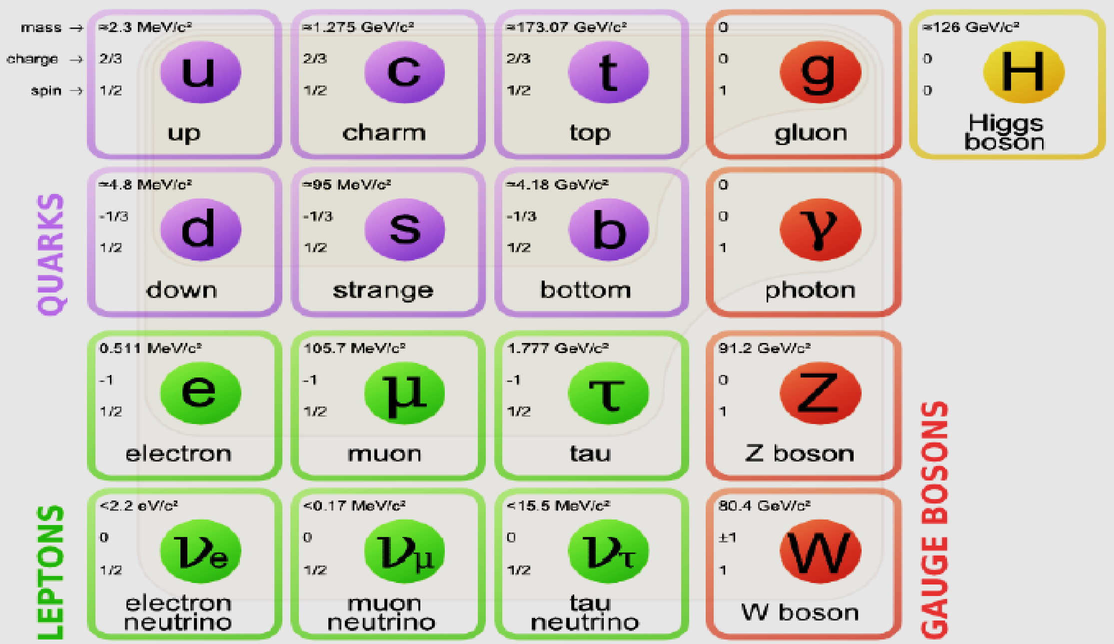
Takaaki Kajita
Super-Kamiokande Collaboration
University of Tokyo, Kashiwa, Japan

Arthur B. McDonald
Sudbury Neutrino Observatory Collaboration
Queen's University, Kingston, Canada

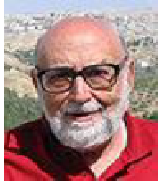
”för upptäckten av neutrinooscillationer, som visar att neutriner har massa”

the discovery of neutrino oscillations, which shows that neutrinos have mass

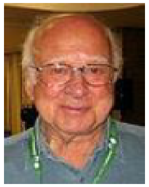
Neutrino as Fundamental Particle



2013年诺贝尔物理学奖



F. Englert



P. Higgs

其理论有助于我们理解基本粒子质量起源，并通过ATLAS和CMS实验中发现该理论预言的新的基本粒子被证实

粒子物理标准模型是目前描述强、弱和电磁相互作用最成功的理论，几乎通过所有实验的精确检验，唯一的例外是中微子振荡

1956年，发现核反应堆产生的反电子中微子 [Reines, 1995年获奖]

1962年，发现muon中微子 [Lederman/Schwartz/Steinberg, 1988年获奖]

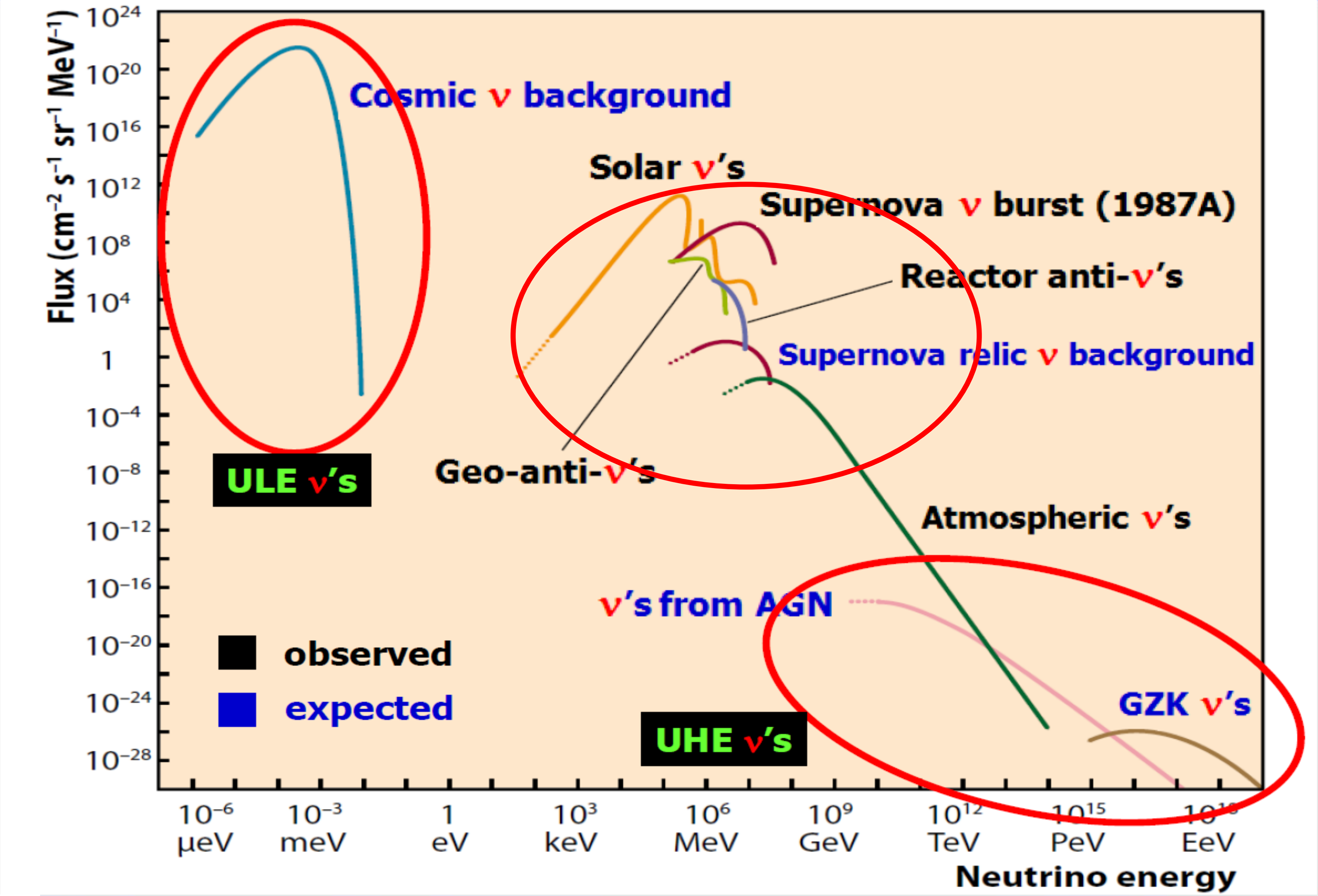
1968年，发现太阳内部核聚变产生的电子中微子 [Davis, 2002年获奖]

1987年，发现超新星SN1987A爆发产生的中微子 [Koshiba, 2002年获奖]

1998年，发现大气中微子振荡 [Kajita, 2015年获奖]

2002年，发现太阳中微子振荡 [McDonald, 2015年获奖]

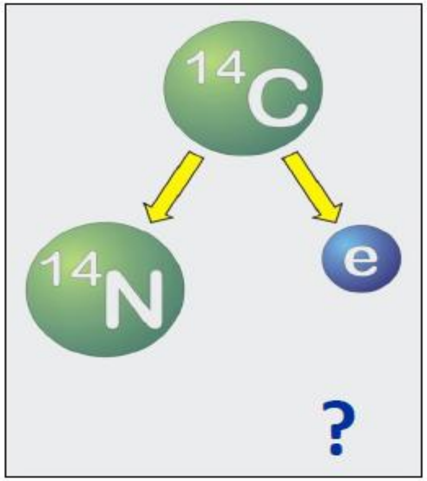
Neutrino as Cosmic Messenger

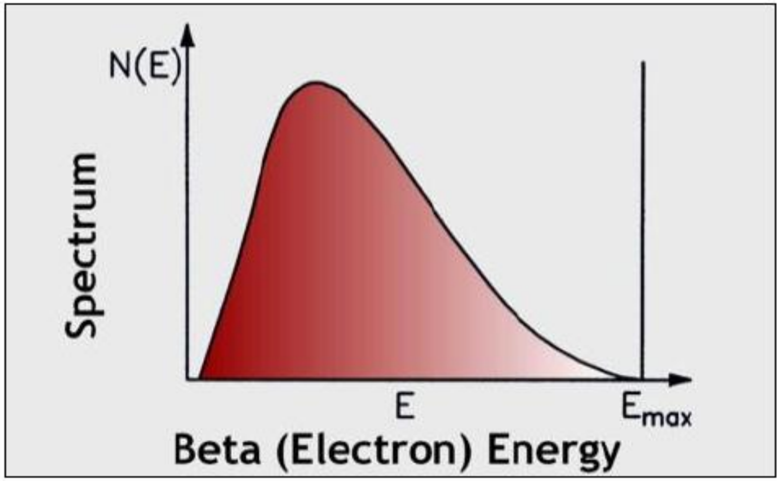


Part I:

中微子的提出与发现

1930年: 泡利的中微子假设





1914年，查德威克发现贝塔衰变末态电子连续能谱，与二体衰变过程不符

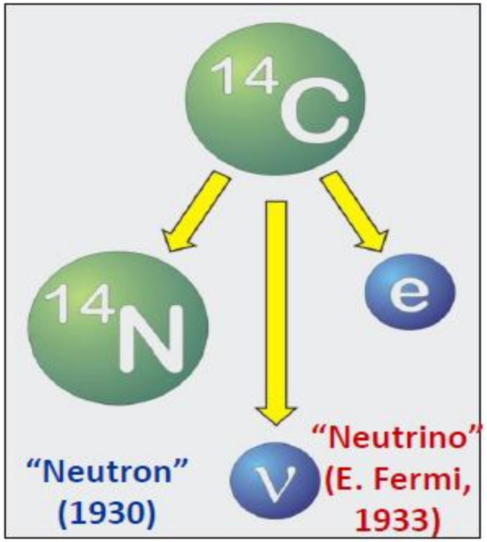
James Chadwick
(1891-1974)
Nobel Prize
1935

Niels Bohr
(1885-1962)
Nobel Prize
1922

Wolfgang Pauli
(1900-1958)
Nobel Prize
1945

1930年，泡利提出原子核里存在质量小且自旋为1/2的电中性粒子(中子)

一石二鸟：

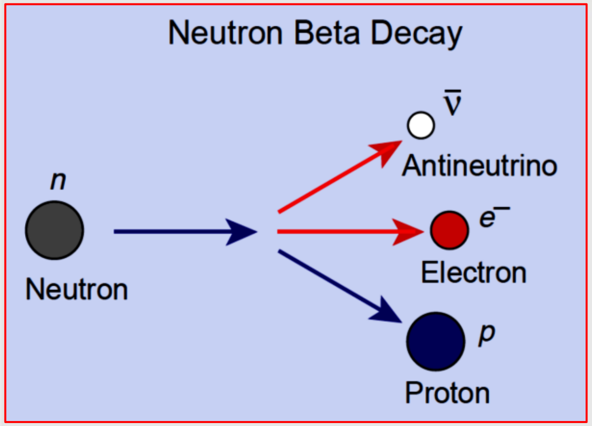


Enrico Fermi
(1901-1954)
Nobel Prize
1938

能量不守恒？存在新粒子？

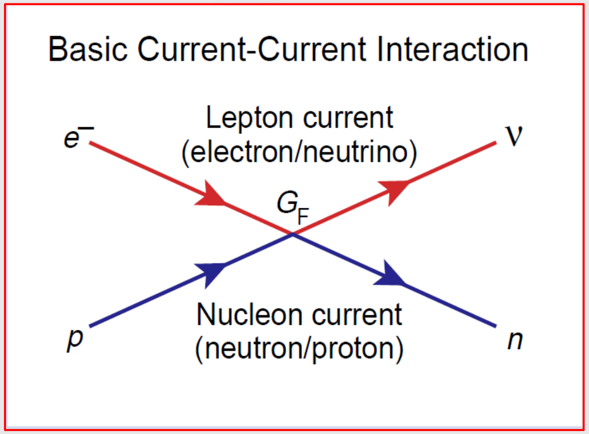
可同时解释连续能谱和氮原子核自旋

1934年：费米的贝塔衰变理论



Basis of Fermi's Theory

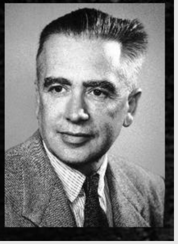
- **Pauli's hypothesis: existence of neutrino**
- **Dirac-Jordan-Klein's theory: creation of particles**
- **Heisenberg's idea: neutron and proton as two states of nucleon**



> **Fermi was fully aware of the importance of his accomplishment and said that he will be remembered for this paper…**
>
> **--- E. Segre, in "Enrico Fermi, Physicist"**

Enrico Fermi
(1901-1954)
Nobel Prize
In 1938

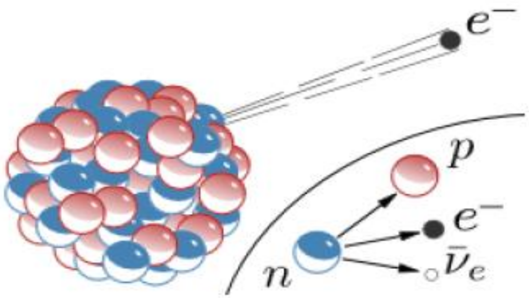
Emilio Segre
(1905-1989)
Nobel Prize
In 1959



Fermi's paper on β decays (2 months)

- **Letter submitted to *Nature***
  Editor: speculations too remote from physical reality
- **Published in 1933 in *La Ricerca Scientifica* in Italian**
- **Another one published in 1934 in *Z. Phys.* in German**
- **Translated into English in *Am. J. Phys.* in 1968**

1934年: 中微子无法探测?

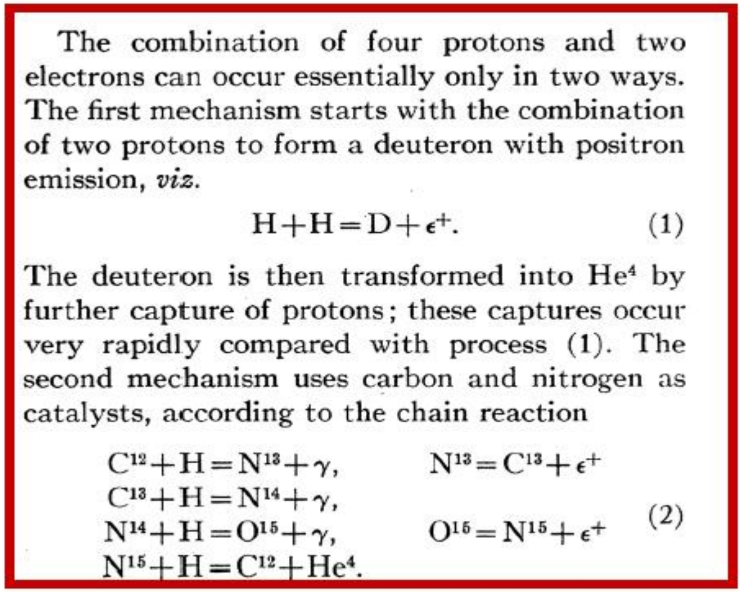


Bethe & Peierls (1934)

$\bar{\nu}_e + p \rightarrow n + e^+$ Cross section $\sigma < 10^{-44}$ cm^2

"It is therefore absolutely impossible to observe the processes of this kind with the neutrinos created in nuclear transformations."

Hans A. Bethe
(1906–2005)
Nobel Prize 1967



The combination of four protons and two electrons can occur essentially only in two ways. The first mechanism starts with the combination of two protons to form a deuteron with positron emission, *viz.*

$$H+H=D+\epsilon^+. \qquad (1)$$

The deuteron is then transformed into He^4 by further capture of protons; these captures occur very rapidly compared with process (1). The second mechanism uses carbon and nitrogen as catalysts, according to the chain reaction

$$\begin{array}{ll} C^{12}+H=N^{13}+\gamma, & N^{13}=C^{13}+\epsilon^+ \\ C^{13}+H=N^{14}+\gamma, & \\ N^{14}+H=O^{15}+\gamma, & O^{15}=N^{15}+\epsilon^+ \\ N^{15}+H=C^{12}+He^4. & \end{array} \qquad (2)$$

1939年, 贝特在太阳内部核聚变反应的著名文章中忽略了中微子

1951年, 莱因斯与费米讨论探测核爆放出的中微子的可能性

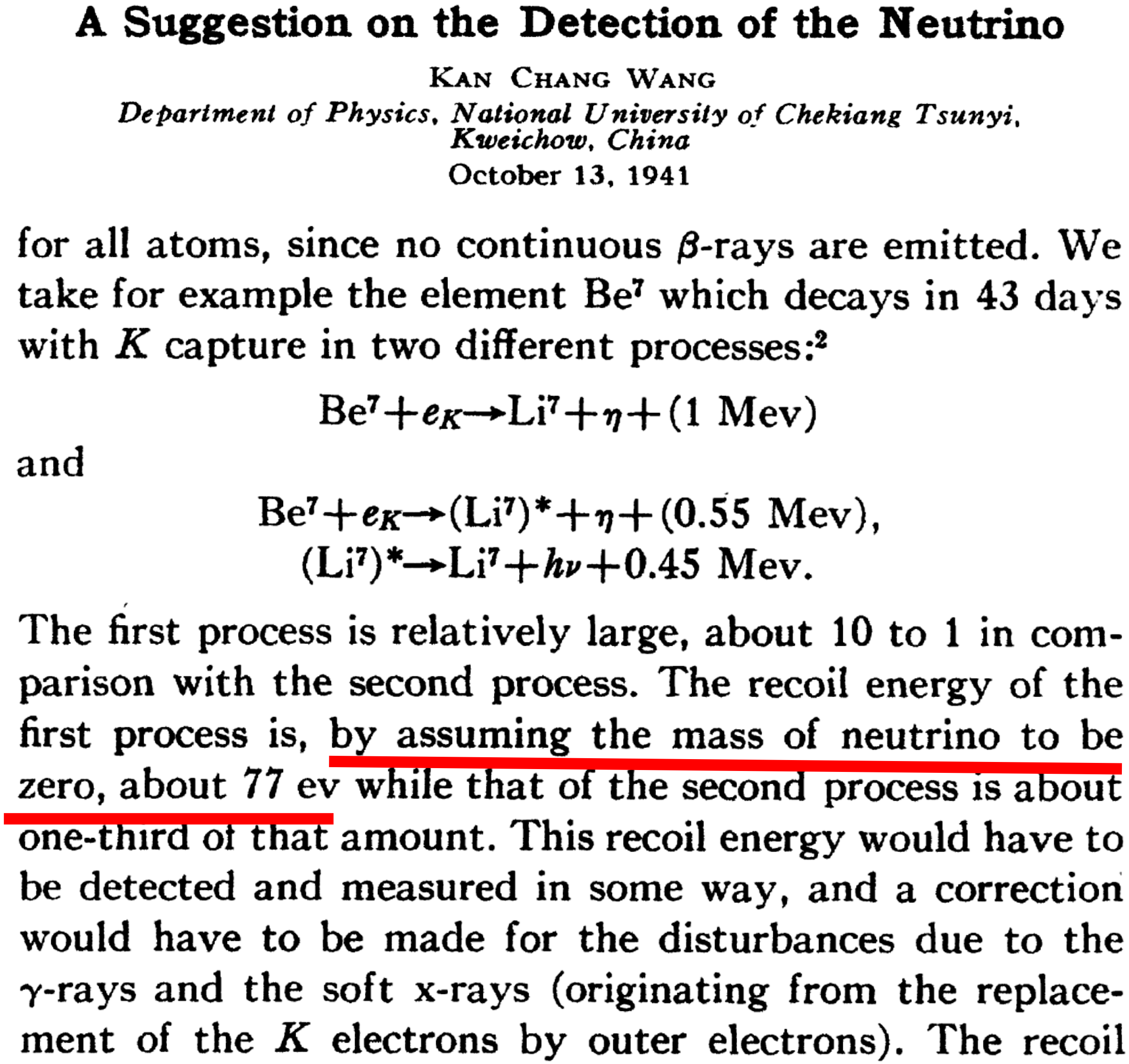
1941年：A Suggestion by Kan Chang Wang

作业：

查询 ^{7}Be和 ^{7}Li原子核质量, 并估算中微子质量为零时末态原子核的反冲动能

王淦昌
(1907-1998)

Phys. Rev. 61 (1941) 97



A Suggestion on the Detection of the Neutrino

KAN CHANG WANG
Department of Physics, National University of Chekiang Tsunyi, Kweichow, China
October 13, 1941

for all atoms, since no continuous β-rays are emitted. We take for example the element Be^7 which decays in 43 days with K capture in two different processes:[2]

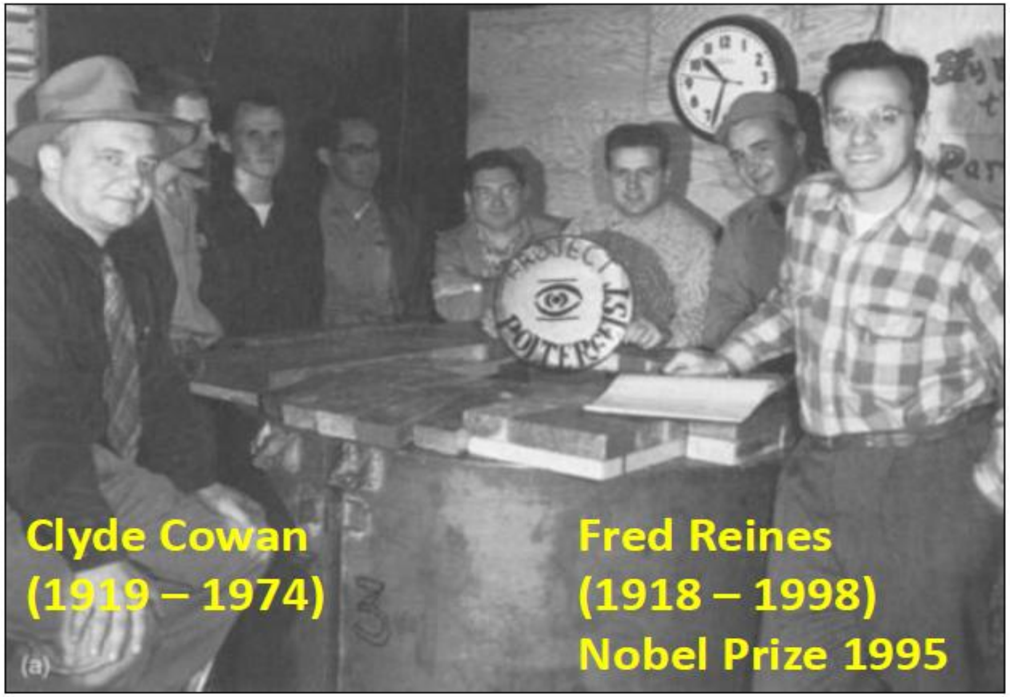
$$Be^7+e_K\rightarrow Li^7+\eta+(1\ \mathrm{Mev})$$

and

$$Be^7+e_K\rightarrow(Li^7)^*+\eta+(0.55\ \mathrm{Mev}),$$
$$(Li^7)^*\rightarrow Li^7+h\nu+0.45\ \mathrm{Mev}.$$

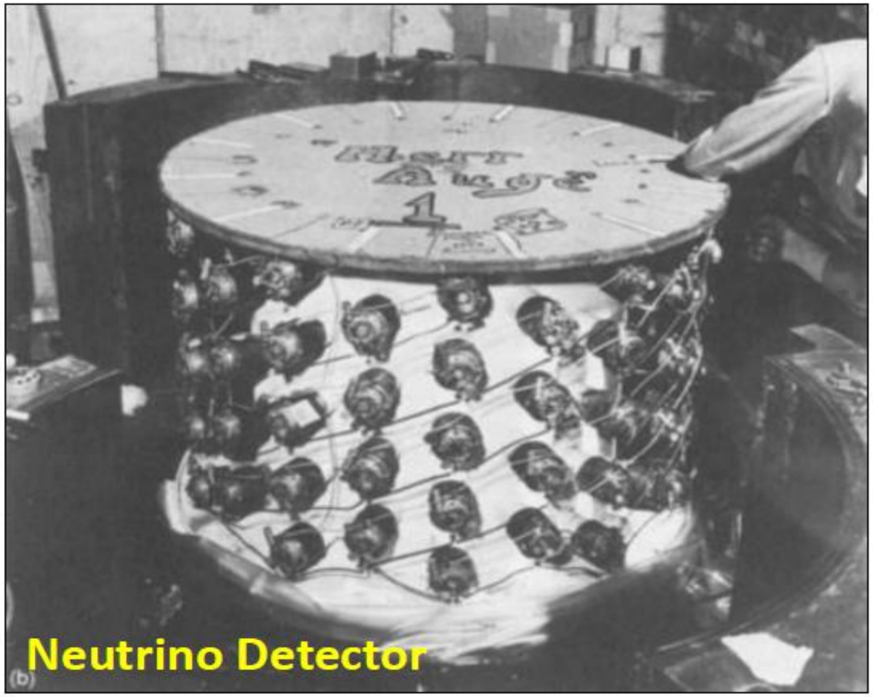
The first process is relatively large, about 10 to 1 in comparison with the second process. The recoil energy of the first process is, by assuming the mass of neutrino to be zero, about 77 ev while that of the second process is about one-third of that amount. This recoil energy would have to be detected and measured in some way, and a correction would have to be made for the disturbances due to the γ-rays and the soft x-rays (originating from the replacement of the K electrons by outer electrons). The recoil

1956年: 发现反应堆中微子



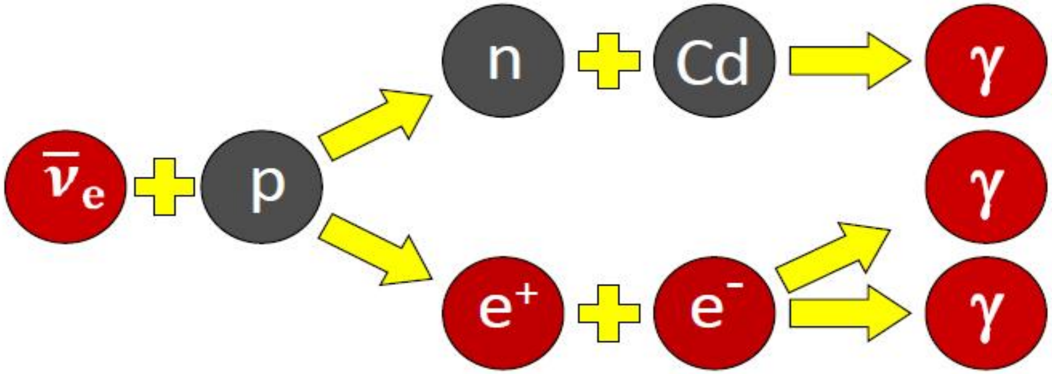
Clyde Cowan
(1919 – 1974)

Fred Reines
(1918 – 1998)
Nobel Prize 1995



Neutrino Detector

1953年, 首次实验看到信号，但是发现即使核反应堆关闭，探测器中仍有信号(主要来自宇宙线背景)

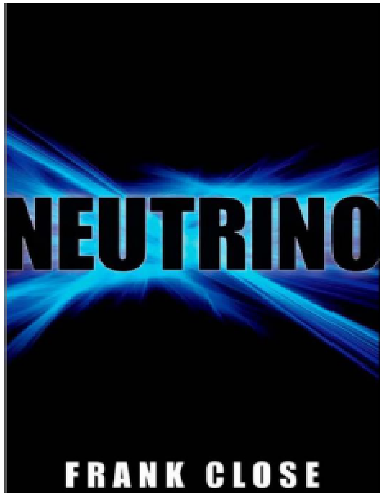


中子被镉俘获，产生 9 MeV γ

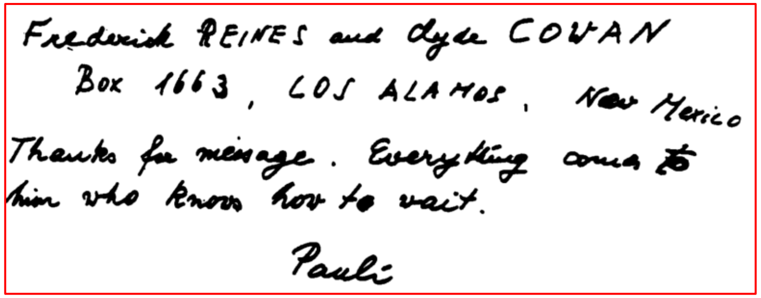
正负电子湮灭产生 0.51 MeV γ

5 微秒时间符合窗口可压低背景

1956年: 发现反应堆中微子



6月14日，柯万和莱因斯给泡利发去电报，告知


Frederick REINES and Clyde COWAN
Box 1663, LOS ALAMOS, New Mexico

Thanks for message. Everything comes to him who knows how to wait.

Pauli

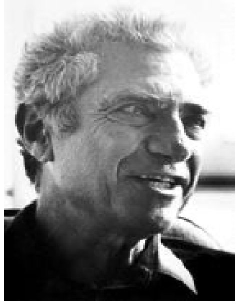
几年后，莱因斯向贝特提起后者1934年的文章中说 “there is no practically possible way of observing the neutrino.”

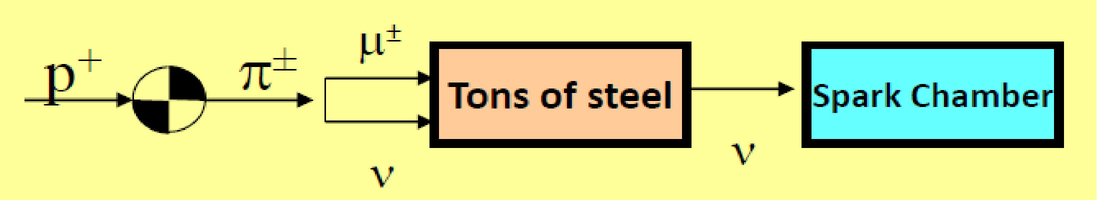
贝特回答 “Well, you shouldn't believe everything you read in the papers.”

1962年: 发现muon中微子


Leon Lederman


Melvin Schwartz


Jack Steinberg



预期: 观测显示中微子总与电子和muon一起产生

结论: 探测器中只有muon出现，所以证明$\nu_\mu \neq \nu_e$

2000年，美国费米实验室的DONUT实验发现第三种τ中微子！

Brief History of ν Oscillation: Part A

- 1957: Pontecorvo proposed Neutrino Oscillations in analogy with $K^0 \leftrightarrows \bar{K}^0$ oscillations (Gell-Mann and Pais, 1955) $\Longrightarrow$ $\nu \leftrightarrows \bar{\nu}$
- In 1957 only one neutrino type $\nu = \nu_e$ was known! The possible existence of ν_μ was discussed by several authors. Maybe the first have been Sakata and Inoue in 1946 and Konopinski and Mahmoud in 1953. Maybe Pontecorvo did not know. He discussed the possibility to distinguish ν_μ from ν_e in 1959. Lee & Yang (1960)
- 1962: Maki, Nakagava, Sakata proposed a model with ν_e and ν_μ and Neutrino Mixing:
  "weak neutrinos are not stable due to the occurrence of a virtual transmutation $\nu_e \leftrightarrows \nu_\mu$"
- 1962: Lederman, Schwartz and Steinberger discover ν_μ
- 1967: Pontecorvo: intuitive $\nu_e \leftrightarrows \nu_\mu$ oscillations with maximal mixing. Applications to reactor and solar neutrinos ("prediction" of the solar neutrino problem).
- 1969: Gribov and Pontecorvo: $\nu_e - \nu_\mu$ mixing and oscillations. But no clear derivation of oscillations with a factor of 2 mistake in the phase (misprint?).

Brief History of ν Oscillation: Part B

- 1975-76: Start of the "Modern Era" of Neutrino Oscillations with a general theory of neutrino mixing and a rigorous derivation of the oscillation probability by Eliezer and Swift, Fritzsch and Minkowski, and Bilenky and Pontecorvo. [Bilenky, Pontecorvo, Phys. Rep. (1978) 225]
- 1978: Wolfenstein discovers the effect on neutrino oscillations of the matter potential ("Matter Effect")
- 1985: Mikheev and Smirnov discover the resonant amplification of solar $\nu_e \to \nu_\mu$ oscillations due to the Matter Effect ("MSW Effect")
- 1998: the Super-Kamiokande experiment observed in a model-independent way the Vacuum Oscillations of atmospheric neutrinos ($\nu_\mu \to \nu_\tau$).
- 2002: the SNO experiment observed in a model-independent way the flavor transitions of solar neutrinos ($\nu_e \to \nu_\mu, \nu_\tau$), mainly due to adiabatic MSW transitions. [see: Smirnov, arXiv:1609.02386]
- 2015: Takaaki Kajita (Super-Kamiokande) and Arthur B. McDonald (SNO) received the Physics Nobel Prize "for the discovery of neutrino oscillations, which shows that neutrinos have mass".

Part II:

中微子振荡理论

真空振荡

物质振荡

中微子物理: standard (minimal) NP framework

$$N_{\rm obs} = \left[\int \mathcal{F}(E_\nu)\sigma(E_\nu,\ldots)\epsilon(E_\nu,\ldots)dE_\nu d\ldots\right]\frac{M}{A\ m_N}T$$

$\times\ P(\nu_\beta \rightarrow \nu_\alpha)$

N_{obs} : number of neutrino events recorded
$\mathcal{F}$: Flux of neutrinos ($\#/{\rm cm}^2/{\rm s}$)
σ : neutrino cross section per nucleon $\simeq 0.7\frac{E_\nu}{[{\rm GeV}]}\times 10^{-38}{\rm cm}^2$
ϵ : detection efficiency
M : total detector mass
A : effective atomic number of detector
m_N : nucleon mass
T : exposure time

typical "super-beam" flux at 1000 km

typical accelerator up time in one year

$$N_{\rm obs} = \left[\frac{1}{{\rm cm}^2{\rm s}}\right]\left[0.7\times10^{-38}\frac{E_\nu}{\rm GeV}{\rm cm}^2\right][\epsilon]\,[1\ {\rm GeV}]\left[\frac{M}{20\cdot 1.67\times10^{-27}\ {\rm kg}}\right][2\times10^7\ {\rm s}]$$

- **强大的中微子源：太阳、大气、反应堆、加速器**
- **足够大的中微子反应截面：~E**
- **足够大的探测器效率和靶质量**
- **足够大的有效运行时间**
- **振荡信号最优位置：E，L**

$$\boxed{N_{\rm obs} = 4\times10^{-6}\frac{E_\nu}{[{\rm GeV}]}\epsilon\frac{M}{\rm kg}}$$

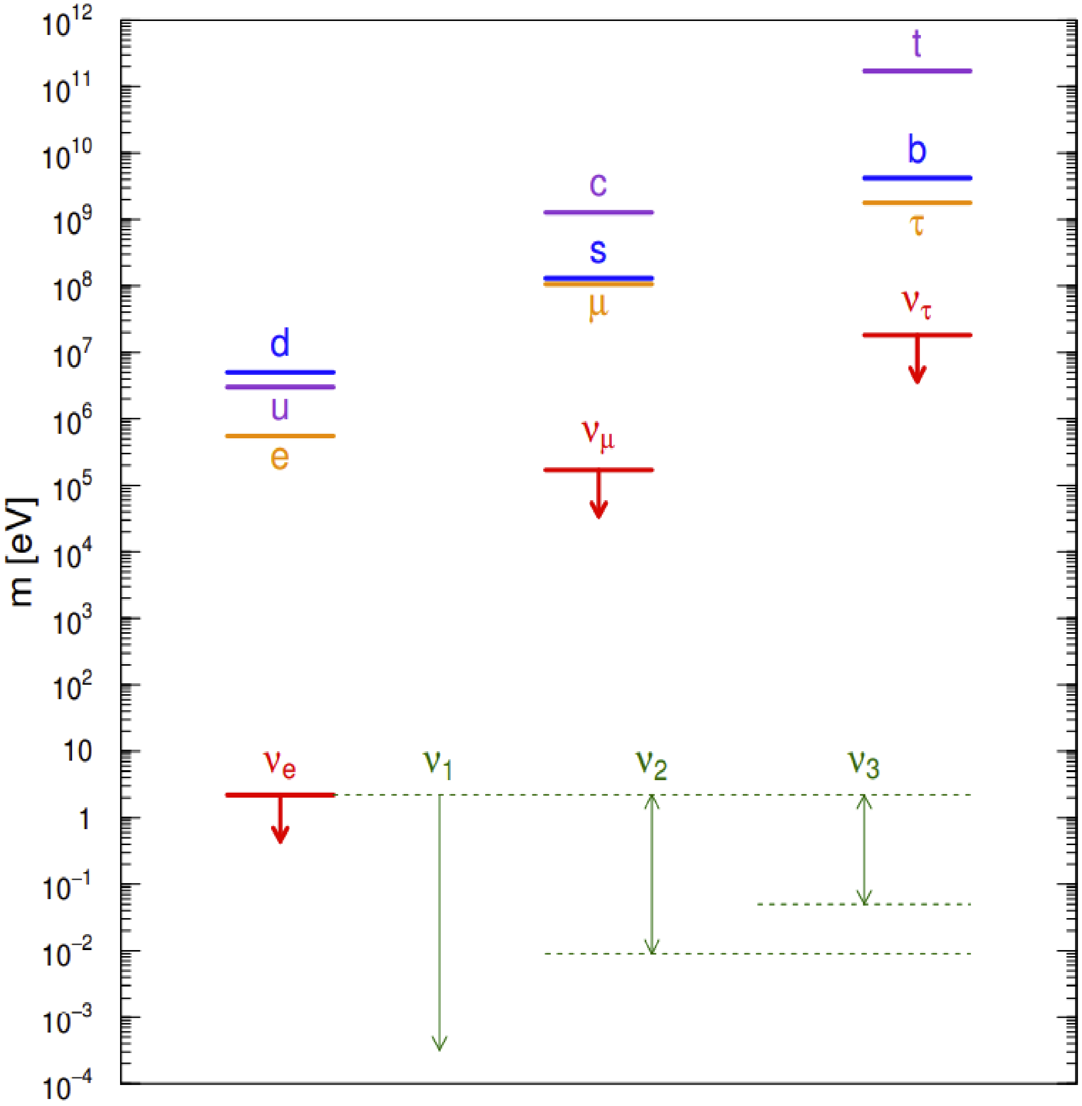
need detector masses of 10^6 kg = 1 kton to get in the game

work at high energies if you can

push this as high as you can

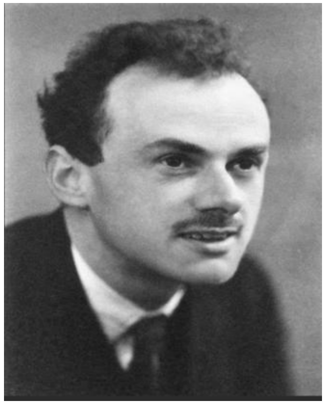
Challenge to the experimentalist: maximize efficiency and detector mass while minimizing cost

ν masses: Dirac versus Majorana



Two possibilities to define neutrino mass:

- **Dirac mass**



Left & right handed ν's

Lepton number conservation

- **Majorana mass**

Only left handed ν's

Lepton number violation

ν masses: Dirac versus Majorana

	1st Generation	2nd Generation	3rd Generation
Quarks:	$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{matrix} u_R \\ d_R \end{matrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix} \quad \begin{matrix} c_R \\ s_R \end{matrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix} \quad \begin{matrix} t_R \\ b_R \end{matrix}$
Leptons:	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \quad \begin{matrix} \boxed{\nu_{eR}} \\ e_R \end{matrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \quad \begin{matrix} \boxed{\nu_{\mu R}} \\ \mu_R \end{matrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} \quad \begin{matrix} \boxed{\nu_{\tau R}} \\ \tau_R \end{matrix}$

Standard Model extension: $\nu_R \Rightarrow$ Dirac mass Lagrangian

$$\mathscr{L}_\mathrm{D} \sim m_\mathrm{D}\,\overline{\nu_L}\nu_R$$

$$\mathscr{L}_\mathrm{Y} \sim y\,\overline{L_L}\widetilde{\Phi}\nu_R \xrightarrow[\text{Breaking}]{\text{Symmetry}} y\,v\,\overline{\nu_L}\nu_R$$

Extremely small Yukawa couplings are needed to get $m_\mathrm{D} \lesssim 1\,\mathrm{eV}$:

$$y \lesssim 10^{-11}$$

It is considered unnatural, unless there is a protecting BSM symmetry.

ν masses: Dirac versus Majorana

	1st Generation	2nd Generation	3rd Generation
Quarks:	$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{matrix} u_R \\ d_R \end{matrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix} \quad \begin{matrix} c_R \\ s_R \end{matrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix} \quad \begin{matrix} t_R \\ b_R \end{matrix}$
Leptons:	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \quad \begin{matrix} \boxed{\nu_{eR}} \\ e_R \end{matrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix} \quad \begin{matrix} \boxed{\nu_{\mu R}} \\ \mu_R \end{matrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} \quad \begin{matrix} \boxed{\nu_{\tau R}} \\ \tau_R \end{matrix}$

Majorana Mass Lagrangian for SM ν_L

$$\mathcal{L}_L^{\mathrm{M}} \sim m_L\, \overline{\nu_L^c} \nu_L = -\nu_L^T\, \mathcal{C}^\dagger\, \nu_L$$

No Majorana Neutrino Mass in the SM

$$\nu_L^T\, \mathcal{C}^\dagger\, \nu_L \text{ has } I_3 = 1 \text{ and } Y = -2 \Longrightarrow$$

Needs Y= +2 Higgs triplet (type II), or (type I)

The introduction of ν_R leads

$$\mathcal{L}_R^{\mathrm{M}} \sim m_R\, \overline{\nu_R^c} \nu_R \qquad \text{singlet under SM symmetries!}$$

Dirac + Majorana mass Lagrangian

$$\mathscr{L}^{\mathrm{D+M}}_{\mathrm{mass}} = \mathscr{L}^{\mathrm{D}}_{\mathrm{mass}} + \mathscr{L}^{R}_{\mathrm{mass}}$$

$$= -\,m_{\mathrm{D}}\left(\overline{\nu_L}\,\nu_R + \overline{\nu_R}\,\nu_L\right) - \frac{1}{2}\,m_R\left(\overline{\nu_R^c}\,\nu_R + \overline{\nu_R}\,\nu_R^c\right)$$

$$= -\frac{1}{2}\begin{pmatrix}\overline{\nu_L} & \overline{\nu_R^c}\end{pmatrix}\begin{pmatrix}0 & m_{\mathrm{D}} \\ m_{\mathrm{D}} & m_R\end{pmatrix}\begin{pmatrix}\nu_L^c \\ \nu_R\end{pmatrix} - \frac{1}{2}\begin{pmatrix}\overline{\nu_L^c} & \overline{\nu_R}\end{pmatrix}\begin{pmatrix}0 & m_{\mathrm{D}} \\ m_{\mathrm{D}} & m_R\end{pmatrix}\begin{pmatrix}\nu_L \\ \nu_R^c\end{pmatrix}$$

Seesaw $m_R \gg m_{\mathrm{D}}$

$$\mathscr{L}^{\mathrm{D+M}}_{\mathrm{mass}} = -\frac{1}{2}\,\overline{n_L^c}\,U^T\,M\,U\,n_L + \mathrm{H.c.}$$

$$U^T\,M\,U = \begin{pmatrix}m_1 & 0 \\ 0 & m_2\end{pmatrix}$$ with real $m_k \geq 0$

$\nu \simeq -i\left(\nu_L - \nu_L^c\right)$ $N \simeq \nu_R + \nu_R^c$

$$\mathscr{L}^{\mathrm{D+M}}_{\mathrm{mass}} = -\frac{1}{2}\sum_{k=1,2} m_k\left(\overline{\nu_{kL}^c}\,\nu_{kL} + \overline{\nu_{kL}}\,\nu_{kL}^c\right) = -\frac{1}{2}\sum_{k=1,2} m_k\,\overline{\nu_k}\,\nu_k$$

$\nu_k = \nu_{kL} + \nu_{kL}^c \quad \Longrightarrow \quad \boxed{\nu_k = \nu_k^c}$ Massive neutrinos are Majorana!

Lepton Flavor Mixing

Charged-Current Weak Interaction Lagrangian

$$\mathscr{L}_{\mathrm{I}}^{(\mathrm{CC})} = -\frac{g}{2\sqrt{2}} j_W^\rho W_\rho + \mathrm{H.c.}$$

Weak Charged Current: $j_W^\rho = j_{W,\mathrm{L}}^\rho + j_{W,\mathrm{Q}}^\rho$

Leptonic Weak Charged Current

$$j_{W,\mathrm{L}}^{\rho\dagger} = 2 \sum_{\alpha=e,\mu,\tau} \overline{\ell'_{\alpha L}} \gamma^\rho \nu'_{\alpha L} = 2 \overline{\boldsymbol{\ell}'_L} \gamma^\rho \boldsymbol{\nu}'_L$$

$$\underline{\boldsymbol{\ell}'_L = V_L^\ell \boldsymbol{\ell}_L} \qquad \underline{\boldsymbol{\nu}'_L = V_L^\nu \boldsymbol{n}_L}$$

$$j_{W,\mathrm{L}}^{\rho\dagger} = 2 \overline{\boldsymbol{\ell}_L} V_L^{\ell\dagger} \gamma^\rho V_L^\nu \boldsymbol{n}_L = 2 \overline{\boldsymbol{\ell}_L} \gamma^\rho V_L^{\ell\dagger} V_L^\nu \boldsymbol{n}_L = 2 \overline{\boldsymbol{\ell}_L} \gamma^\rho U \boldsymbol{n}_L$$

Mixing Matrix: $\boxed{U = V_L^{\ell\dagger} V_L^\nu}$

Low Energy 3ν (Majorana) Mixing

$$\nu_\alpha = \sum_{k=1}^{3} U_{\alpha k} \nu_k \quad \text{for} \quad \alpha = e, \mu, \tau \quad \text{with} \quad \boxed{\nu_k = \nu_k^c}$$

Standard Parameterization of Mixing Matrix (as CKM)

$$U = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$$c_{ab} \equiv \cos\vartheta_{ab} \qquad s_{ab} \equiv \sin\vartheta_{ab} \qquad 0 \le \vartheta_{ab} \le \frac{\pi}{2} \qquad 0 \le \delta_{13}, \lambda_{21}, \lambda_{31} < 2\pi$$

OSCILLATION PARAMETERS
- 3 Mixing Angles: ϑ_{12}, ϑ_{23}, ϑ_{13}
- 1 CPV Dirac Phase: δ_{13}
- 2 independent $\Delta m^2_{kj} \equiv m^2_k - m^2_j$: Δm^2_{21}, Δm^2_{31}

2 CPV Majorana Phases: λ_{21}, λ_{31} $\Longleftrightarrow$ $\boxed{|\Delta L| = 2}$ processes

Mixing of Flavor States

Left-handed Flavor Neutrinos produced in Weak Interactions

$$|\nu_e, -\rangle \qquad |\nu_\mu, -\rangle \qquad |\nu_\tau, -\rangle$$

$$\mathcal{H}_{CC} = \frac{g}{\sqrt{2}} W_\rho \left(\overline{\nu_{eL}} \gamma^\rho e_L + \overline{\nu_{\mu L}} \gamma^\rho \mu_L + \overline{\nu_{\tau L}} \gamma^\rho \tau_L \right) + \text{H.c.}$$

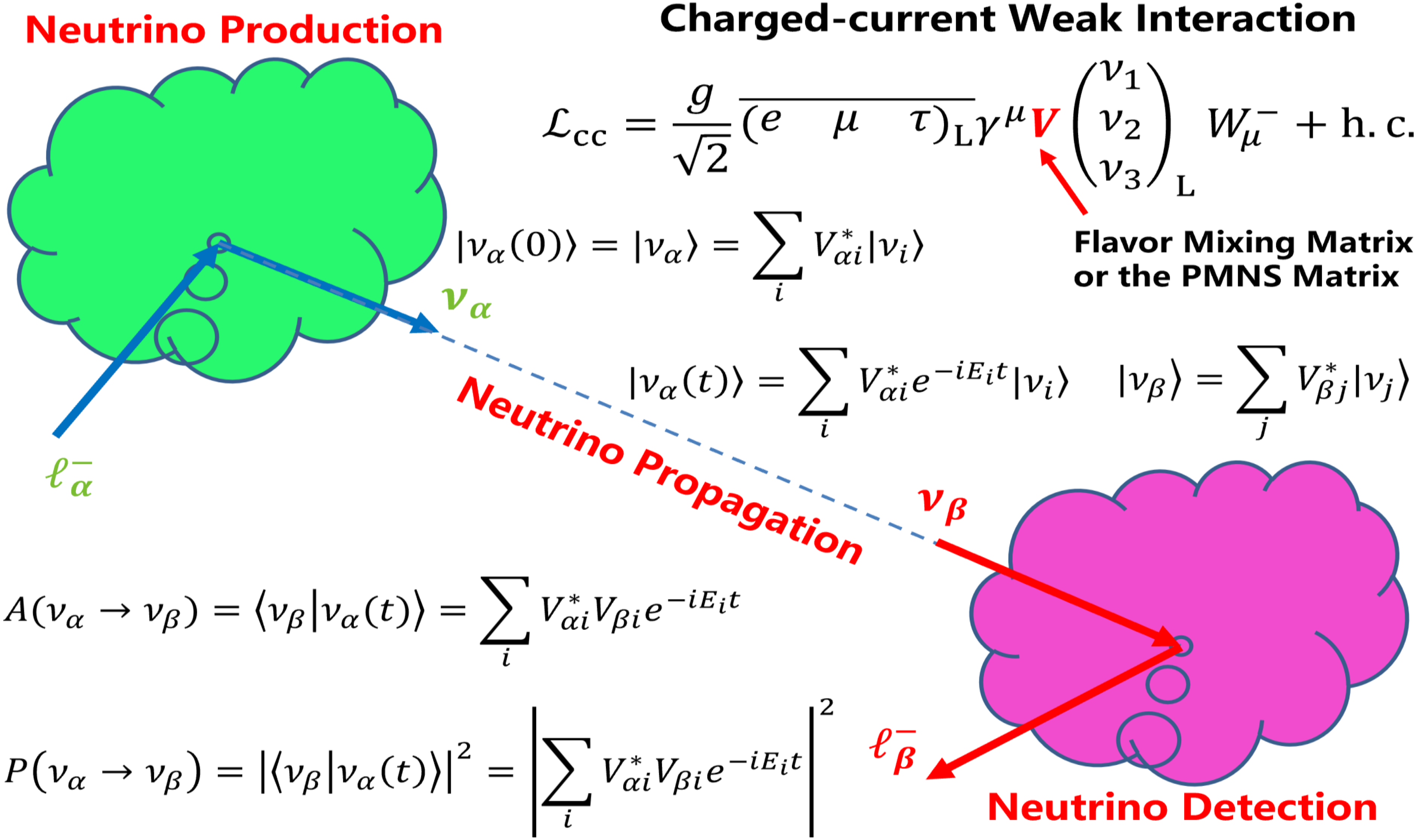
Fields $\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL} \quad \Longrightarrow \quad |\nu_\alpha, -\rangle = \sum_k U^*_{\alpha k} |\nu_k, -\rangle$ States

$$|\nu_1, -\rangle \qquad |\nu_2, -\rangle \qquad |\nu_3, -\rangle$$

Left-handed Massive Neutrinos propagate from Source to Detector

3 × 3 Unitary Mixing Matrix: $U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$

ν Oscillation: A QM phenomenon



Charged-current Weak Interaction

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{(e \quad \mu \quad \tau)_{\rm L}} \gamma^\mu \mathbf{V} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} W_\mu^- + \rm h.c.$$

Flavor Mixing Matrix or the PMNS Matrix

$$|\nu_\alpha(0)\rangle = |\nu_\alpha\rangle = \sum_i V_{\alpha i}^* |\nu_i\rangle$$

$$|\nu_\alpha(t)\rangle = \sum_i V_{\alpha i}^* e^{-iE_i t} |\nu_i\rangle \qquad |\nu_\beta\rangle = \sum_j V_{\beta j}^* |\nu_j\rangle$$

$$A(\nu_\alpha \to \nu_\beta) = \langle \nu_\beta | \nu_\alpha(t) \rangle = \sum_i V_{\alpha i}^* V_{\beta i} e^{-iE_i t}$$

$$P(\nu_\alpha \to \nu_\beta) = |\langle \nu_\beta | \nu_\alpha(t) \rangle|^2 = \left| \sum_i V_{\alpha i}^* V_{\beta i} e^{-iE_i t} \right|^2$$

Ultra-relativistic Approximation

Only neutrinos with energy $\gtrsim 0.1 MeV$ are detectable!

Charged-Current Processes: Threshold

$$\boxed{\begin{gathered}\nu + A \to B + C \\ \Downarrow \\ s = 2Em_A + m_A^2 \geq (m_B + m_C)^2 \\ \Downarrow \\ E_{\text{th}} = \frac{(m_B + m_C)^2}{2m_A} - \frac{m_A}{2}\end{gathered}}$$

$\nu_e + {}^{71}\text{Ga} \to {}^{71}\text{Ge} + e^-$ $\quad E_{\text{th}} = 0.233\,\text{MeV}$

$\nu_e + {}^{37}\text{Cl} \to {}^{37}\text{Ar} + e^-$ $\quad E_{\text{th}} = 0.81\,\text{MeV}$

$\bar{\nu}_e + p \to n + e^+$ $\quad E_{\text{th}} = 1.8\,\text{MeV}$

$\nu_\mu + n \to p + \mu^-$ $\quad E_{\text{th}} = 110\,\text{MeV}$

$\nu_\mu + e^- \to \nu_e + \mu^-$ $\quad E_{\text{th}} \simeq \frac{m_\mu^2}{2m_e} = 10.9\,\text{GeV}$

Elastic Scattering Processes: Cross Section $\propto$ Energy

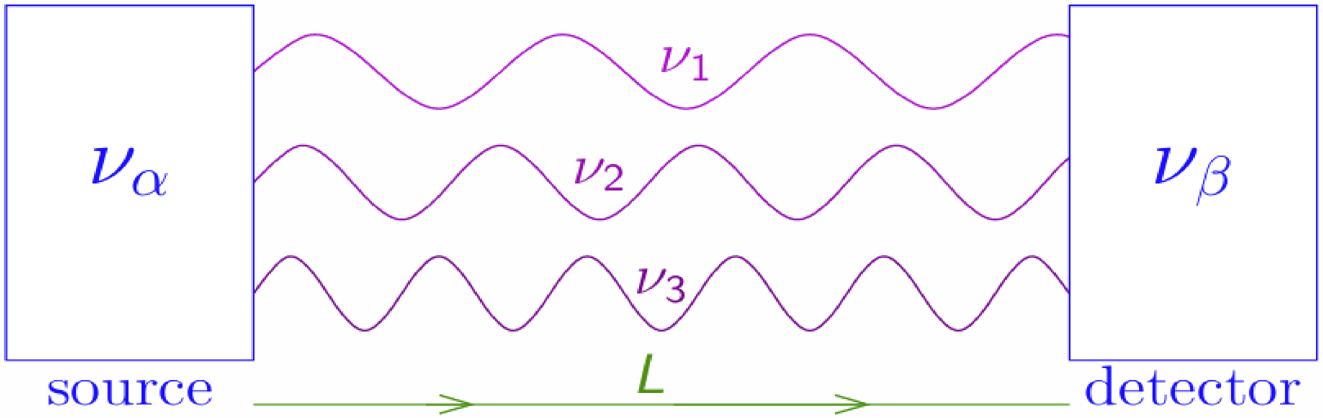
$$\nu + e^- \to \nu + e^- \qquad \sigma(E) \sim \sigma_0\, E/m_e \qquad \sigma_0 \sim 10^{-44}\,\text{cm}^2$$

Background $\Longrightarrow$ $E_{\text{th}} \simeq 5\,\text{MeV}$ (SK, SNO) , $0.25\,\text{MeV}$ (Borexino)

Laboratory and Astrophysical Limits $\Longrightarrow$ $\boxed{m_\nu \lesssim 1\,\text{eV}}$

ν Oscillation: A QM phenomenon

$$|\nu(t=0)\rangle = |\nu_\alpha\rangle = U^*_{\alpha 1}\,|\nu_1\rangle + U^*_{\alpha 2}\,|\nu_2\rangle + U^*_{\alpha 3}\,|\nu_3\rangle$$



$$|\nu(t>0)\rangle = U^*_{\alpha 1}\,e^{-iE_1 t}\,|\nu_1\rangle + U^*_{\alpha 2}\,e^{-iE_2 t}\,|\nu_2\rangle + U^*_{\alpha 3}\,e^{-iE_3 t}\,|\nu_3\rangle \neq |\nu_\alpha\rangle$$

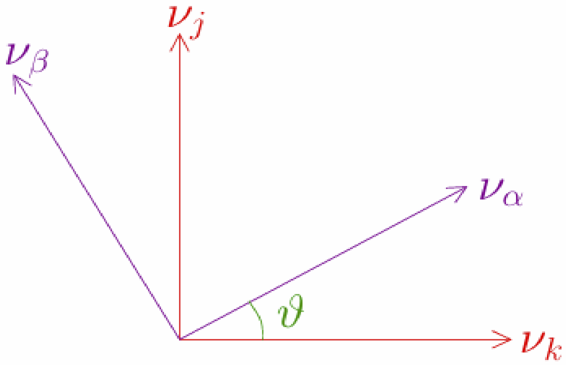
$$E_k^2 = p^2 + m_k^2 \qquad\qquad t = L$$

$$P_{\nu_\alpha \to \nu_\beta}(L) = |\langle \nu_\beta | \nu(L)\rangle|^2 = \sum_{k,j} U_{\beta k} U^*_{\alpha k} U^*_{\beta j} U_{\alpha j} \exp\left(-i\frac{\Delta m^2_{kj} L}{2E}\right)$$

the oscillation probabilities depend on U and $\Delta m^2_{kj} \equiv m_k^2 - m_j^2$

ν Oscillations: 2-flavor approximation

$$\boxed{\begin{aligned} |\nu_\alpha\rangle &= \cos\vartheta\,|\nu_k\rangle + \sin\vartheta\,|\nu_j\rangle \\ |\nu_\beta\rangle &= -\sin\vartheta\,|\nu_k\rangle + \cos\vartheta\,|\nu_j\rangle \end{aligned}}$$



$$U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

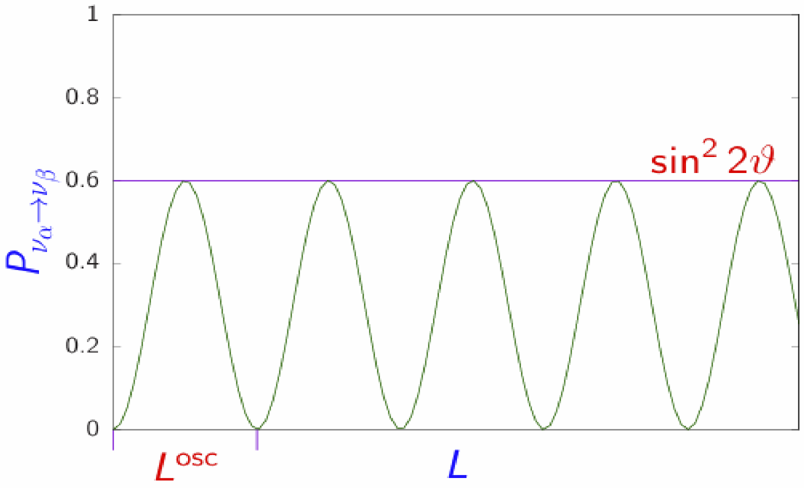
$$\Delta m^2 \equiv \Delta m^2_{kj} \equiv m^2_k - m^2_j$$

Transition Probability: $$P_{\nu_\alpha\to\nu_\beta} = P_{\nu_\beta\to\nu_\alpha} = \sin^2 2\vartheta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Survival Probabilities: $$P_{\nu_\alpha\to\nu_\alpha} = P_{\nu_\beta\to\nu_\beta} = 1 - P_{\nu_\alpha\to\nu_\beta}$$

ν Oscillations: 2-flavor approximation

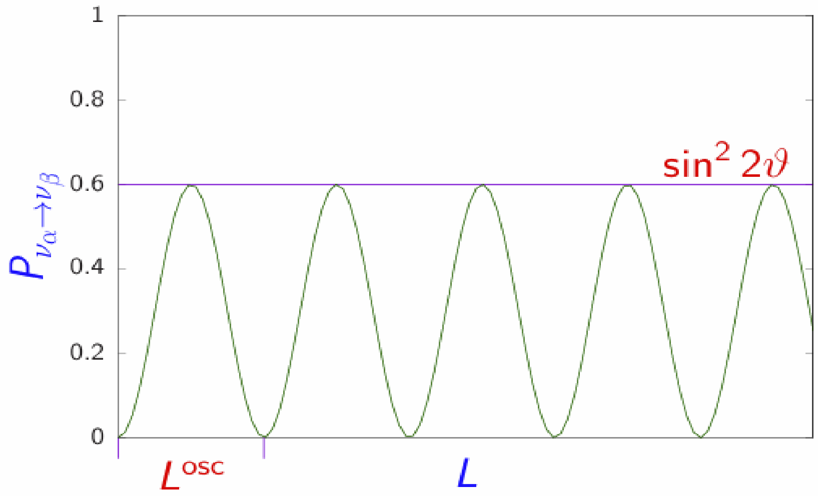
2ν-mixing: $$P_{\nu_\alpha \to \nu_\beta} = \sin^2 2\vartheta \, \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \Longrightarrow L^{\text{osc}} = \frac{4\pi E}{\Delta m^2}$$



- The effect of a tiny Δm^2 can be amplified by a large distance L.
- A tiny Δm^2 generates oscillations observable at macroscopic distances!
- Neutrino oscillations are the optimal tool to reveal tiny neutrino masses!

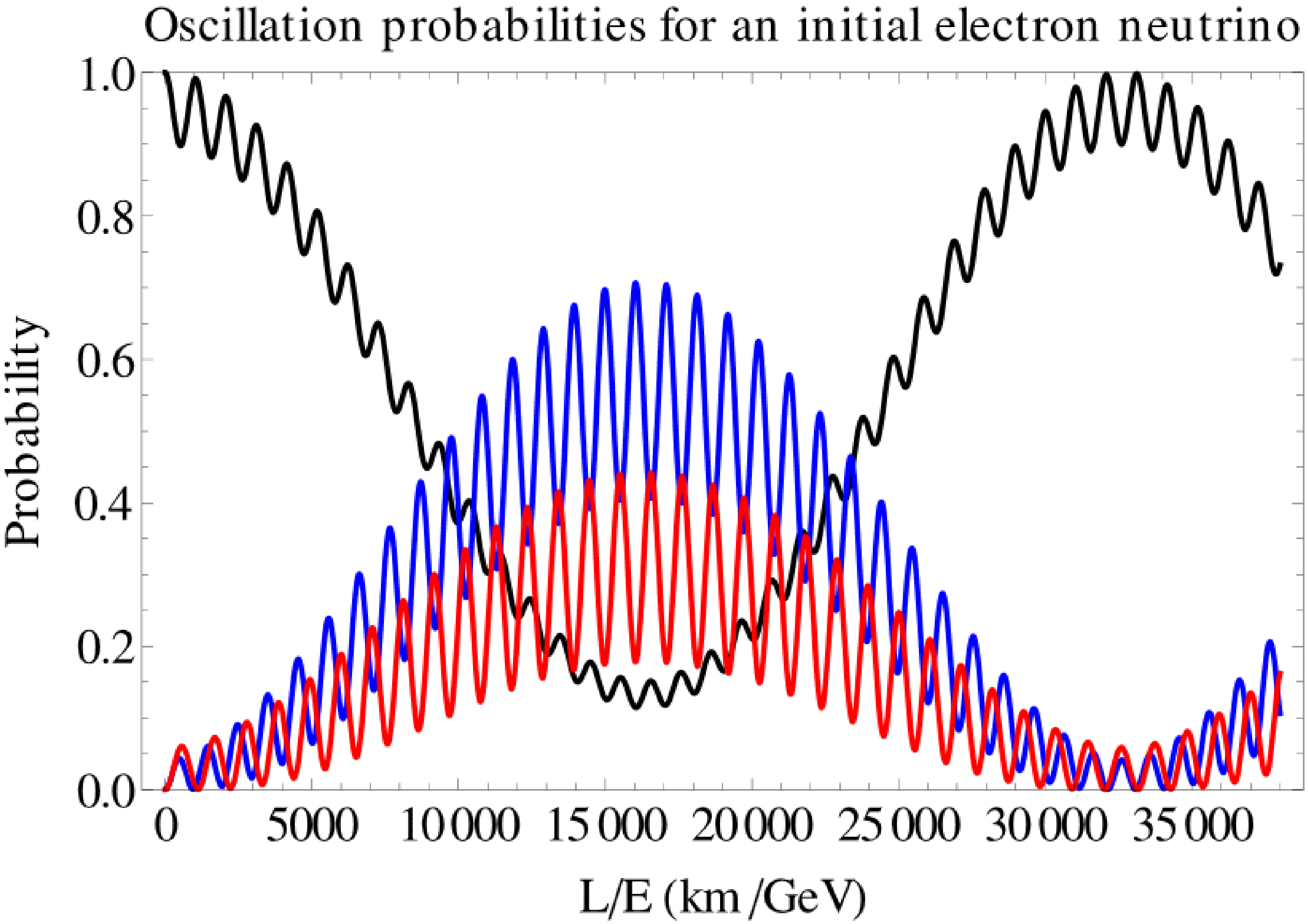
ν Oscillations: 2-flavor approximation

2ν-mixing: $$P_{\nu_\alpha\to\nu_\beta} = \sin^2 2\vartheta\ \sin^2\left(1.27\,\frac{\Delta m^2[\mathrm{eV}^2]\ L[\mathrm{km}]}{E[\mathrm{GeV}]}\right)$$



$$\frac{L}{E} \lesssim \left\{ \begin{array}{lll} 10\ \frac{\mathrm{m}}{\mathrm{MeV}}\ \left(\frac{\mathrm{km}}{\mathrm{GeV}}\right) & \text{short-baseline experiments} & \Delta m^2 \gtrsim 10^{-1}\ \mathrm{eV}^2 \\ 10^3\ \frac{\mathrm{m}}{\mathrm{MeV}}\ \left(\frac{\mathrm{km}}{\mathrm{GeV}}\right) & \text{long-baseline experiments} & \Delta m^2 \gtrsim 10^{-3}\ \mathrm{eV}^2 \\ 10^4\ \frac{\mathrm{km}}{\mathrm{GeV}} & \text{atmospheric neutrino experiments} & \Delta m^2 \gtrsim 10^{-4}\ \mathrm{eV}^2 \\ 10^{11}\ \frac{\mathrm{m}}{\mathrm{MeV}} & \text{solar neutrino experiments} & \Delta m^2 \gtrsim 10^{-11}\ \mathrm{eV}^2 \end{array} \right.$$

ν Oscillation: A QM phenomenon



ν_e

ν_μ

ν_τ

ν Oscillations: CP, T & CPT properties

Right-handed antineutrinos are described by CP-conjugated fields:

$$\nu_{\alpha L}^{\mathrm{CP}} = \gamma^0 \, \mathcal{C} \, \overline{\nu_{\alpha L}}^T$$

C $\Longrightarrow$ Particle $\leftrightarrows$ Antiparticle

P $\Longrightarrow$ Left-Handed $\leftrightarrows$ Right-Handed

ν Oscillations: CP, T & CPT properties

$$\text{Fields:} \quad \nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{kL} \quad \xrightarrow{\text{CP}} \quad \nu_{\alpha L}^{\text{CP}} = \sum_k U_{\alpha k}^* \nu_{kL}^{\text{CP}}$$

$$\text{States:} \quad |\nu_\alpha\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle \quad \xrightarrow{\text{CP}} \quad |\bar{\nu}_\alpha\rangle = \sum_k U_{\alpha k} |\bar{\nu}_k\rangle$$

$$\underline{\text{NEUTRINOS}} \qquad U \quad \leftrightarrows \quad U^* \qquad \underline{\text{ANTINEUTRINOS}}$$

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sum_{k,j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E} \right)$$

$$P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}(L, E) = \sum_{k,j} U_{\alpha k} U_{\beta k}^* U_{\alpha j}^* U_{\beta j} \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E} \right)$$

ν Oscillations: CP, T & CPT properties

$$P_{\nu_\alpha \to \nu_\beta} \quad \xrightarrow{\text{CPT}} \quad P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$$

CPT Asymmetries: $A^{\text{CPT}}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}$

Local Quantum Field Theory $\Longrightarrow$ $A^{\text{CPT}}_{\alpha\beta} = 0$ CPT Symmetry

$$P_{\nu_\alpha \to \nu_\beta}(L, E) = \sum_{k,j} U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j} \exp\left(-i \frac{\Delta m^2_{kj} L}{2E}\right)$$

is invariant under CPT: $U \leftrightarrows U^* \qquad \alpha \leftrightarrows \beta$

$$\boxed{P_{\nu_\alpha \to \nu_\beta} = P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha}}$$

$\boxed{P_{\nu_\alpha \to \nu_\alpha} = P_{\bar{\nu}_\alpha \to \bar{\nu}_\alpha}}$ (solar ν_e, reactor $\bar{\nu}_e$, accelerator ν_μ)

ν Oscillations: CP, T & CPT properties

$$P_{\nu_\alpha \to \nu_\beta} \quad \xrightarrow{\text{CP}} \quad P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$$

CP Asymmetries: $A^{\text{CP}}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta}$

$$\boxed{A^{\text{CP}}_{\alpha\beta}(L, E) = 4 \sum_{k>j} \text{Im}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] \sin\left(\frac{\Delta m^2_{kj} L}{2E}\right)}$$

Jarlskog rephasing invariant: $\text{Im}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] = \pm J$

$$J = c_{12} s_{12} c_{23} s_{23} c^2_{13} s_{13} \sin\delta_{13}$$

$$J \neq 0 \quad \Longleftrightarrow \quad \vartheta_{12}, \vartheta_{23}, \vartheta_{13} \neq 0, \pi/2 \quad \delta_{13} \neq 0, \pi$$

ν Oscillations: CP, T & CPT properties

$$
\begin{aligned}
\text{CPT} \quad \Longrightarrow \quad 0 &= A^{\text{CPT}}_{\alpha\beta} \\
&= P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha} \\
&= P_{\nu_\alpha \to \nu_\beta} - P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta} \leftarrow A^{\text{CP}}_{\alpha\beta} \\
&+ P_{\bar{\nu}_\alpha \to \bar{\nu}_\beta} - P_{\nu_\beta \to \nu_\alpha} \leftarrow -A^{\text{CPT}}_{\beta\alpha} = 0 \\
&+ P_{\nu_\beta \to \nu_\alpha} - P_{\bar{\nu}_\beta \to \bar{\nu}_\alpha} \leftarrow A^{\text{CP}}_{\beta\alpha} \\
&= A^{\text{CP}}_{\alpha\beta} + A^{\text{CP}}_{\beta\alpha} \qquad \Longrightarrow \qquad \boxed{A^{\text{CP}}_{\alpha\beta} = -A^{\text{CP}}_{\beta\alpha}}
\end{aligned}
$$

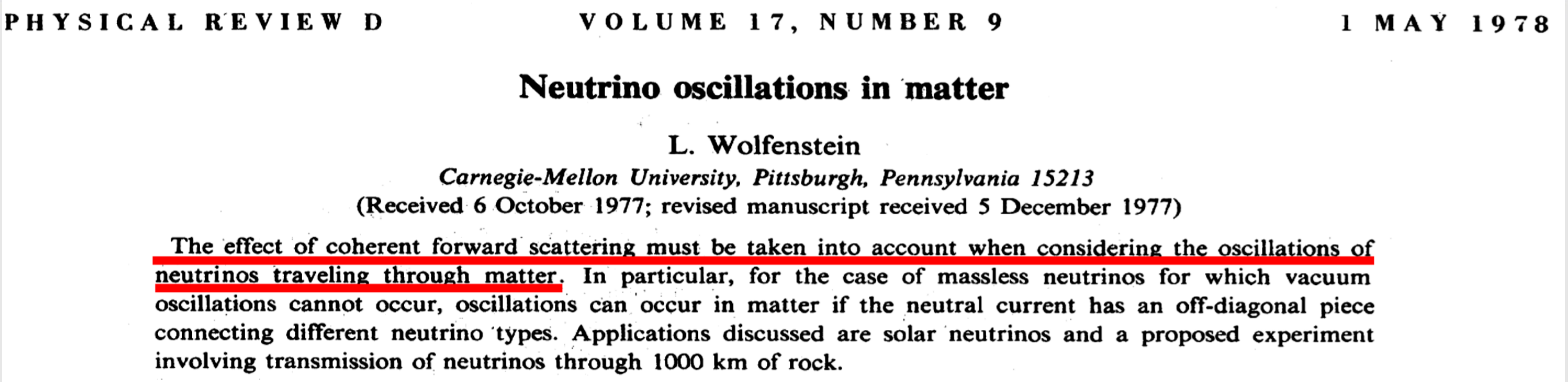
ν Oscillations: CP, T & CPT properties

$$P_{\nu_\alpha \to \nu_\beta} \quad \xrightarrow{\text{T}} \quad P_{\nu_\beta \to \nu_\alpha}$$

T Asymmetries: $A^{\text{T}}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\nu_\beta \to \nu_\alpha}$

$$\begin{aligned} \text{CPT} \quad \Longrightarrow \quad 0 &= A^{\text{CPT}}_{\alpha\beta} \\ &= P_{\nu_\alpha \to \nu_\beta} - P_{\bar\nu_\beta \to \bar\nu_\alpha} \\ &= P_{\nu_\alpha \to \nu_\beta} - P_{\nu_\beta \to \nu_\alpha} \leftarrow A^{\text{T}}_{\alpha\beta} \\ &+ P_{\nu_\beta \to \nu_\alpha} - P_{\bar\nu_\beta \to \bar\nu_\alpha} \leftarrow A^{\text{CP}}_{\beta\alpha} \\ &= A^{\text{T}}_{\alpha\beta} + A^{\text{CP}}_{\beta\alpha} \\ &= A^{\text{T}}_{\alpha\beta} - A^{\text{CP}}_{\alpha\beta} \qquad \Longrightarrow \quad \boxed{A^{\text{T}}_{\alpha\beta} = A^{\text{CP}}_{\alpha\beta}} \end{aligned}$$

ν Oscillations: Matter Effect



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Neutrino oscillations in matter

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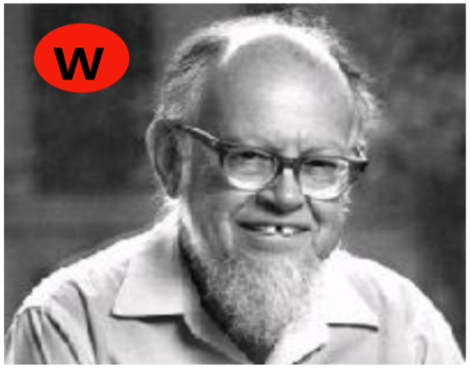
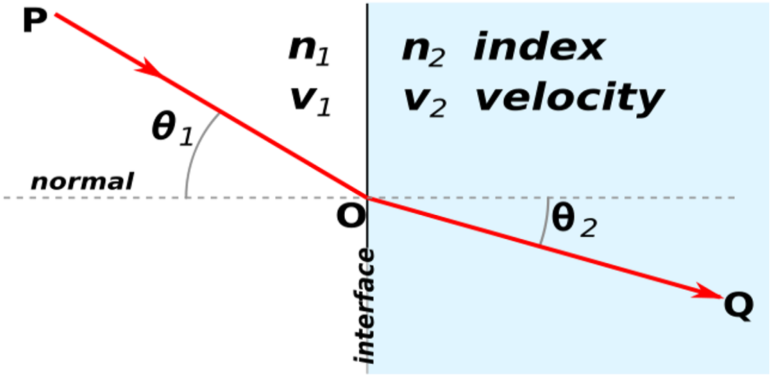
The effect of coherent forward scattering must be taken into account when considering the oscillations of neutrinos traveling through matter. In particular, for the case of massless neutrinos for which vacuum oscillations cannot occur, oscillations can occur in matter if the neutral current has an off-diagonal piece connecting different neutrino types. Applications discussed are solar neutrinos and a proposed experiment involving transmission of neutrinos through 1000 km of rock.



Lincoln Wolfenstein (1923-2015)



Refraction of light in media

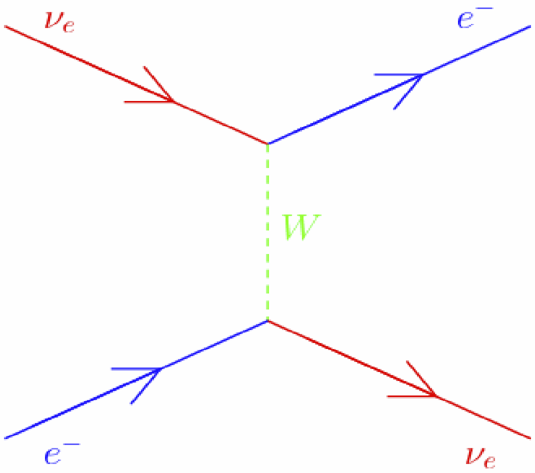
$$\nu_e : \exp[ipx(n_{\rm nc} + n_{\rm cc} - 1)]$$
$$\nu_\mu : \exp[ipx(n_{\rm nc} - 1)]$$
$$\nu_\tau : \exp[ipx(n_{\rm nc} - 1)]$$

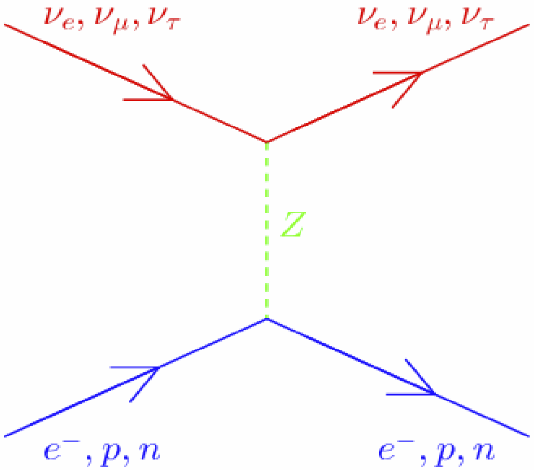
Refraction of neutrinos in media, where both CC and NC interactions contribute to refractive indices (not far from 1)

When neutrinos are traveling in matter, the effect of coherent forward scattering with background particles leads to a modification of their energies. Such a modification can be described by an potential energy. The difference between the potentials of distinct neutrino flavors is relevant for neutrino oscillations.

ν Oscillations: Matter Effect

coherent interactions with medium: forward elastic CC and NC scattering





$$\boxed{V_{\text{CC}} = \sqrt{2} G_{\text{F}} N_e} \qquad V_{\text{NC}}^{(e^-)} = -V_{\text{NC}}^{(p)} \quad \Rightarrow \quad \boxed{V_{\text{NC}} = V_{\text{NC}}^{(n)} = -\frac{\sqrt{2}}{2} G_{\text{F}} N_n}$$

$$V_e = V_{\text{CC}} + V_{\text{NC}} \qquad V_\mu = V_\tau = V_{\text{NC}}$$

only $V_{\text{CC}} = V_e - V_\mu = V_e - V_\tau$ is important for flavor transitions

antineutrinos: $\overline{V}_{\text{CC}} = -V_{\text{CC}}$ $\qquad \overline{V}_{\text{NC}} = -V_{\text{NC}}$

ν Oscillations: Matter Effect

- Flavor neutrino ν_α with momentum p: $|\nu_\alpha(p)\rangle = \sum_k U^*_{\alpha k}\,|\nu_k(p)\rangle$
- Evolution is determined by Hamiltonian
- Hamiltonian in vacuum: $\mathcal{H} = \mathcal{H}_0$

$$\mathcal{H}_0\,|\nu_k(p)\rangle = E_k\,|\nu_k(p)\rangle \qquad E_k = \sqrt{p^2 + m_k^2}$$

- Hamiltonian in matter: $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I$ $\qquad \mathcal{H}_I\,|\nu_\alpha(p)\rangle = V_\alpha\,|\nu_\alpha(p)\rangle$
- Schrödinger evolution equation: $i\,\frac{\mathrm{d}}{\mathrm{d}t}\,|\nu(p,t)\rangle = \mathcal{H}|\nu(p,t)\rangle$
- Initial condition: $|\nu(p,0)\rangle = |\nu_\alpha(p)\rangle$
- For $t > 0$ the state $|\nu(p,t)\rangle$ is a superposition of all flavors:

$$|\nu(p,t)\rangle = \sum_\beta \varphi_\beta(p,t)|\nu_\beta(p)\rangle$$

- Transition probability: $P_{\nu_\alpha\to\nu_\beta} = |\varphi_\beta|^2$

ν Oscillations: Matter Effect

evolution equation of states

$$i \frac{\mathrm{d}}{\mathrm{d}t} |\nu(p,t)\rangle = \mathcal{H} |\nu(p,t)\rangle \,, \qquad |\nu(p,0)\rangle = |\nu_\alpha(p)\rangle$$

flavor transition amplitudes

$$\varphi_\beta(p,t) = \langle \nu_\beta(p) | \nu(p,t) \rangle \,, \qquad \varphi_\beta(p,0) = \delta_{\alpha\beta}$$

evolution of flavor transition amplitudes

$$i \frac{\mathrm{d}}{\mathrm{d}t} \varphi_\beta(p,t) = \langle \nu_\beta(p) | \mathcal{H} | \nu(p,t) \rangle$$

$$i \frac{\mathrm{d}}{\mathrm{d}t} \varphi_\beta(p,t) = \langle \nu_\beta(p) | \mathcal{H}_0 | \nu(p,t) \rangle + \langle \nu_\beta(p) | \mathcal{H}_I | \nu(p,t) \rangle$$

ν Oscillations: Matter Effect

$$i\,\frac{\mathrm{d}}{\mathrm{d}t}\,\varphi_\beta(p,t) = \langle \nu_\beta(p)|\mathcal{H}_0|\nu(p,t)\rangle + \langle \nu_\beta(p)|\mathcal{H}_I|\nu(p,t)\rangle$$

$$\langle \nu_\beta(p)|\mathcal{H}_0|\nu(p,t)\rangle =$$

$$\sum_\rho \sum_{k,j} \underbrace{\langle \nu_\beta(p)|\nu_k(p)\rangle}_{U_{\beta k}} \underbrace{\langle \nu_k(p)|\mathcal{H}_0|\nu_j(p)\rangle}_{\delta_{kj}E_k} \underbrace{\langle \nu_j(p)|\nu_\rho(p)\rangle}_{U^*_{\rho j}} \underbrace{\langle \nu_\rho(p)|\nu(p,t)\rangle}_{\varphi_\rho(p,t)}$$

$$= \sum_\rho \sum_k U_{\beta k} E_k U^*_{\rho k}\,\varphi_\rho(p,t)$$

$$\langle \nu_\beta(p)|\mathcal{H}_I|\nu(p,t)\rangle = \sum_\rho \underbrace{\langle \nu_\beta(p)|\mathcal{H}_I|\nu_\rho(p)\rangle}_{\delta_{\beta\rho}V_\beta} \underbrace{\langle \nu_\rho(p)|\nu(p,t)\rangle}_{\varphi_\rho(p,t)}$$

$$= \sum_\rho \delta_{\beta\rho} V_\beta\,\varphi_\rho(p,t)$$

$$\boxed{i\,\frac{\mathrm{d}}{\mathrm{d}t}\,\varphi_\beta = \sum_\rho \left(\sum_k U_{\beta k}\,E_k\,U^*_{\rho k} + \delta_{\beta\rho}\,V_\beta \right) \varphi_\rho}$$

ν Oscillations: Matter Effect

ultrarelativistic neutrinos: $E_k = p + \frac{m_k^2}{2E}$ $\quad E = p \quad t = x$

$$V_e = V_{\text{CC}} + V_{\text{NC}} \qquad V_\mu = V_\tau = V_{\text{NC}}$$

$$i \frac{\mathrm{d}}{\mathrm{d}x} \varphi_\beta(p,x) = (p + V_{\text{NC}})\, \varphi_\beta(p,x) + \sum_\rho \left(\sum_k U_{\beta k} \frac{m_k^2}{2E} U_{\rho k}^* + \delta_{\beta e}\, \delta_{\rho e}\, V_{\text{CC}} \right) \varphi_\rho(p,x)$$

$$\psi_\beta(p,x) = \varphi_\beta(p,x)\, e^{ipx + i \int_0^x V_{\text{NC}}(x')\, \mathrm{d}x'}$$

$$\Downarrow$$

$$i \frac{\mathrm{d}}{\mathrm{d}x} \psi_\beta = e^{ipx + i \int_0^x V_{\text{NC}}(x')\, \mathrm{d}x'} \left(-p - V_{\text{NC}} + i \frac{\mathrm{d}}{\mathrm{d}x} \right) \varphi_\beta$$

$$\boxed{i \frac{\mathrm{d}}{\mathrm{d}x} \psi_\beta = \sum_\rho \left(\sum_k U_{\beta k} \frac{m_k^2}{2E} U_{\rho k}^* + \delta_{\beta e}\, \delta_{\rho e}\, V_{\text{CC}} \right) \psi_\rho}$$

$$\boxed{P_{\nu_\alpha \to \nu_\beta} = |\varphi_\beta|^2 = |\psi_\beta|^2}$$

ν Oscillations: Matter Effect

evolution of flavor transition amplitudes in matrix form

$$\boxed{i \frac{\mathrm{d}}{\mathrm{d}x} \Psi_\alpha = \frac{1}{2E} \left(U \, \mathbb{M}^2 \, U^\dagger + \mathbb{A} \right) \Psi_\alpha}$$

$$\Psi_\alpha = \begin{pmatrix} \psi_e \\ \psi_\mu \\ \psi_\tau \end{pmatrix} \qquad \mathbb{M}^2 = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} \qquad \mathbb{A} = \begin{pmatrix} A_{\mathrm{CC}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A_{\mathrm{CC}} = 2EV_{\mathrm{CC}} = 2\sqrt{2}EG_{\mathrm{F}}N_e$$

effective mass-squared matrix in vacuum

$$\mathbb{M}^2_{\mathrm{VAC}} = U \, \mathbb{M}^2 \, U^\dagger \xrightarrow{\text{matter}} U \, \mathbb{M}^2 \, U^\dagger + 2 \, E \, \underset{\substack{\uparrow \\ \text{potential due to coherent} \\ \text{forward elastic scattering}}}{\mathbb{V}} = \mathbb{M}^2_{\mathrm{MAT}}$$

effective mass-squared matrix in matter

ν Oscillations: Matter Effect for 2 flavors

$\nu_e \to \nu_\mu$ transitions with $U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$

$$U\,\mathbb{M}^2\,U^\dagger = \begin{pmatrix} \cos^2\vartheta\, m_1^2 + \sin^2\vartheta\, m_2^2 & \cos\vartheta\sin\vartheta\,(m_2^2 - m_1^2) \\ \cos\vartheta\sin\vartheta\,(m_2^2 - m_1^2) & \sin^2\vartheta\, m_1^2 + \cos^2\vartheta\, m_2^2 \end{pmatrix}$$

$$= \frac{1}{2}\underbrace{\begin{pmatrix} m_1^2 + m_2^2 & 0 \\ 0 & m_1^2 + m_2^2 \end{pmatrix}}_{\uparrow\ \text{irrelevant common phase}} + \frac{1}{2}\begin{pmatrix} -\Delta m^2\cos 2\vartheta & \Delta m^2\sin 2\vartheta \\ \Delta m^2\sin 2\vartheta & \Delta m^2\cos 2\vartheta \end{pmatrix}$$

$$\Delta m^2 \equiv m_2^2 - m_1^2$$

ν Oscillations: Matter Effect for 2 flavors

$$\boxed{i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix} = \frac{1}{4E}\begin{pmatrix}-\Delta m^2\cos 2\vartheta + 2A_{\mathrm{CC}} & \Delta m^2\sin 2\vartheta \\ \Delta m^2\sin 2\vartheta & \Delta m^2\cos 2\vartheta\end{pmatrix}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix}}$$

initial ν_e $\Longrightarrow$ $\begin{pmatrix}\psi_e(0)\\ \psi_\mu(0)\end{pmatrix} = \begin{pmatrix}1\\ 0\end{pmatrix}$

$$\boxed{\begin{array}{l} P_{\nu_e\to\nu_\mu}(x) = |\psi_\mu(x)|^2 \\ P_{\nu_e\to\nu_e}(x) = |\psi_e(x)|^2 = 1 - P_{\nu_e\to\nu_\mu}(x) \end{array}}$$

ν Oscillations: Constant Matter Density

$$\boxed{i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix}=\frac{1}{4E}\begin{pmatrix}-\Delta m^2\cos 2\vartheta+2A_{\mathrm{CC}} & \Delta m^2\sin 2\vartheta\\ \Delta m^2\sin 2\vartheta & \Delta m^2\cos 2\vartheta\end{pmatrix}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix}}$$

$$\frac{\mathrm{d}A_{\mathrm{CC}}}{\mathrm{d}x}=0$$

diagonalization of effective Hamiltonian: $\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix}=\begin{pmatrix}\cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}}\\ -\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix}$

$$\begin{pmatrix}\cos\vartheta_{\mathrm{M}} & -\sin\vartheta_{\mathrm{M}}\\ \sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}-\Delta m^2\cos 2\vartheta+2A_{\mathrm{CC}} & \Delta m^2\sin 2\vartheta\\ \Delta m^2\sin 2\vartheta & \Delta m^2\cos 2\vartheta\end{pmatrix}\begin{pmatrix}\cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}}\\ -\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}=$$

$$=\begin{pmatrix}A_{\mathrm{CC}}-\Delta m^2_{\mathrm{M}} & 0\\ 0 & A_{\mathrm{CC}}+\Delta m^2_{\mathrm{M}}\end{pmatrix}$$

$$i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix}=\frac{1}{4E}\left[\underset{\substack{\uparrow\\ \text{irrelevant common phase}}}{\begin{pmatrix}A_{\mathrm{CC}} & 0\\ 0 & A_{\mathrm{CC}}\end{pmatrix}}+\begin{pmatrix}-\Delta m^2_{\mathrm{M}} & 0\\ 0 & \Delta m^2_{\mathrm{M}}\end{pmatrix}\right]\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix}$$

ν Oscillations: Constant Matter Density

Effective Mixing Angle in Matter

$$\tan 2\vartheta_{\mathrm{M}} = \frac{\tan 2\vartheta}{1 - \dfrac{A_{\mathrm{CC}}}{\Delta m^2 \cos 2\vartheta}}$$

Effective Squared-Mass Difference

$$\Delta m^2_{\mathrm{M}} = \sqrt{\left(\Delta m^2 \cos 2\vartheta - A_{\mathrm{CC}}\right)^2 + \left(\Delta m^2 \sin 2\vartheta\right)^2}$$

Resonance ($\vartheta_{\mathrm{M}} = \pi/4$)

$$A^{\mathrm{R}}_{\mathrm{CC}} = \Delta m^2 \cos 2\vartheta \quad \Longrightarrow \quad N^{\mathrm{R}}_e = \frac{\Delta m^2 \cos 2\vartheta}{2\sqrt{2} E G_{\mathrm{F}}}$$

ν Oscillations: Constant Matter Density

$$i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_1^{\mathrm{M}}\\\psi_2^{\mathrm{M}}\end{pmatrix}=\frac{1}{4E}\begin{pmatrix}-\Delta m_{\mathrm{M}}^2 & 0\\0 & \Delta m_{\mathrm{M}}^2\end{pmatrix}\begin{pmatrix}\psi_1^{\mathrm{M}}\\\psi_2^{\mathrm{M}}\end{pmatrix}$$

$$\begin{pmatrix}\psi_e\\\psi_\mu\end{pmatrix}=\begin{pmatrix}\cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}}\\-\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_1^{\mathrm{M}}\\\psi_2^{\mathrm{M}}\end{pmatrix}\quad\Rightarrow\quad\begin{pmatrix}\psi_1^{\mathrm{M}}\\\psi_2^{\mathrm{M}}\end{pmatrix}=\begin{pmatrix}\cos\vartheta_{\mathrm{M}} & -\sin\vartheta_{\mathrm{M}}\\\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_e\\\psi_\mu\end{pmatrix}$$

$$\nu_e\to\nu_\mu\quad\Longrightarrow\quad\begin{pmatrix}\psi_e(0)\\\psi_\mu(0)\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix}\quad\Longrightarrow\quad\begin{pmatrix}\psi_1^{\mathrm{M}}(0)\\\psi_2^{\mathrm{M}}(0)\end{pmatrix}=\begin{pmatrix}\cos\vartheta_{\mathrm{M}}\\\sin\vartheta_{\mathrm{M}}\end{pmatrix}$$

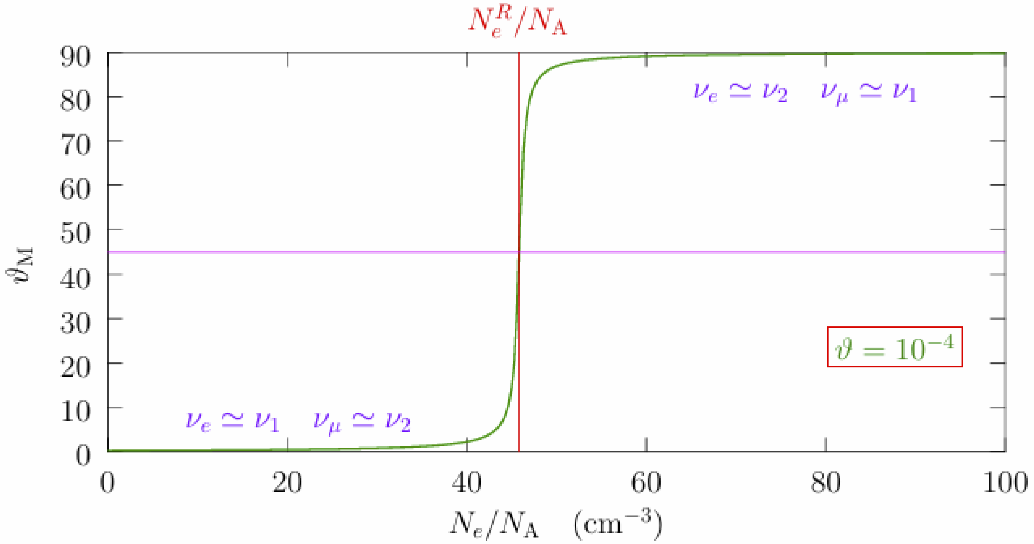
$$\psi_1^{\mathrm{M}}(x)=\cos\vartheta_{\mathrm{M}}\exp\left(i\frac{\Delta m_{\mathrm{M}}^2x}{4E}\right)$$

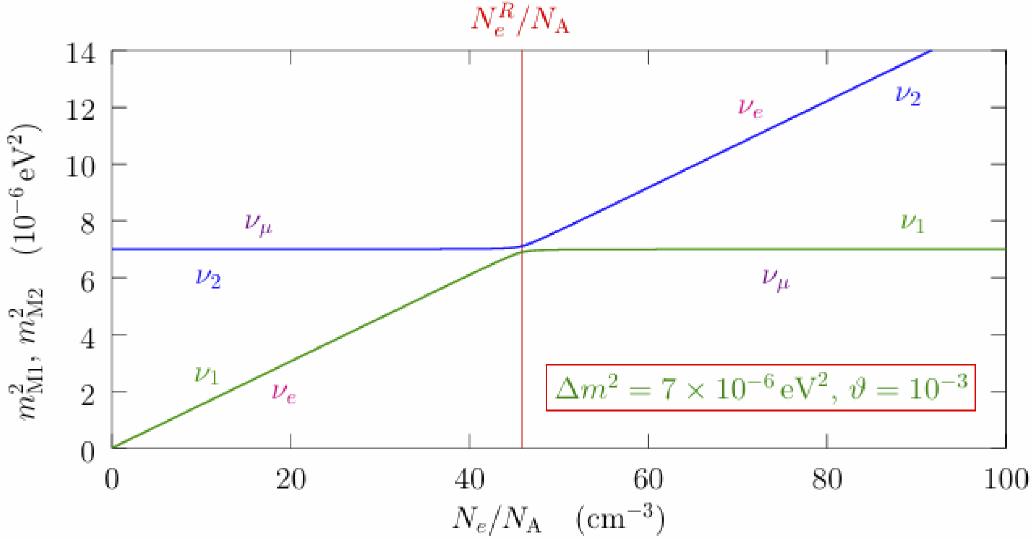
$$\psi_2^{\mathrm{M}}(x)=\sin\vartheta_{\mathrm{M}}\exp\left(-i\frac{\Delta m_{\mathrm{M}}^2x}{4E}\right)$$

$$P_{\nu_e\to\nu_\mu}(x)=|\psi_\mu(x)|^2=\left|-\sin\vartheta_{\mathrm{M}}\psi_1^{\mathrm{M}}(x)+\cos\vartheta_{\mathrm{M}}\psi_2^{\mathrm{M}}(x)\right|^2$$

$$\boxed{P_{\nu_e\to\nu_\mu}(x)=\sin^2 2\vartheta_{\mathrm{M}}\ \sin^2\left(\frac{\Delta m_{\mathrm{M}}^2x}{4E}\right)}$$

ν Oscillations: Constant Matter Density





$$\nu_e = \cos\vartheta_M\,\nu_1 + \sin\vartheta_M\,\nu_2$$
$$\nu_\mu = -\sin\vartheta_M\,\nu_1 + \cos\vartheta_M\,\nu_2$$

$$\tan 2\vartheta_M = \frac{\tan 2\vartheta}{1 - \dfrac{A_{CC}}{\Delta m^2 \cos 2\vartheta}}$$

$$\Delta m_M^2 = \left[\left(\Delta m^2 \cos 2\vartheta - A_{CC}\right)^2 + \left(\Delta m^2 \sin 2\vartheta\right)^2 \right]^{1/2}$$

MSW Effect !

ν Oscillations: Non-constant Matter Density

$$\boxed{i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix} = \frac{1}{4E}\begin{pmatrix}-\Delta m^2\cos 2\vartheta + 2A_{\mathrm{CC}} & \Delta m^2 \sin 2\vartheta\\ \Delta m^2 \sin 2\vartheta & \Delta m^2 \cos 2\vartheta\end{pmatrix}\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix}}$$

tentative diagonalization: $\begin{pmatrix}\psi_e\\ \psi_\mu\end{pmatrix} = \begin{pmatrix}\cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}}\\ -\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix}$

$$i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}}\\ -\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix} =$$

$$= \frac{1}{4E}\begin{pmatrix}-\Delta m^2\cos 2\vartheta + 2A_{\mathrm{CC}} & \Delta m^2 \sin 2\vartheta\\ \Delta m^2 \sin 2\vartheta & \Delta m^2 \cos 2\vartheta\end{pmatrix}\begin{pmatrix}\cos\vartheta_{\mathrm{M}} & \sin\vartheta_{\mathrm{M}}\\ -\sin\vartheta_{\mathrm{M}} & \cos\vartheta_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix}$$

if matter density is not constant $\mathrm{d}\vartheta_{\mathrm{M}}/\mathrm{d}x \neq 0$

$$i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix} = \left[\underset{\uparrow}{\frac{A_{\mathrm{CC}}}{4E}} + \frac{1}{4E}\begin{pmatrix}-\Delta m^2_{\mathrm{M}} & 0\\ 0 & \Delta m^2_{\mathrm{M}}\end{pmatrix} + \begin{pmatrix}0 & -i\frac{\mathrm{d}\vartheta_{\mathrm{M}}}{\mathrm{d}x}\\ i\frac{\mathrm{d}\vartheta_{\mathrm{M}}}{\mathrm{d}x} & 0\end{pmatrix}\right]\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix}$$

irrelevant common phase

ν Oscillations: Non-constant Matter Density

$$i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix} = \left[\frac{1}{4E}\underset{\substack{\uparrow\\ \text{adiabatic}}}{\begin{pmatrix}-\Delta m^2_{\mathrm{M}} & 0\\ 0 & \Delta m^2_{\mathrm{M}}\end{pmatrix}} + \underset{\substack{\uparrow\\ \text{non-adiabatic}\\ \text{maximum at resonance}}}{\begin{pmatrix}0 & -i\dfrac{\mathrm{d}\vartheta_{\mathrm{M}}}{\mathrm{d}x}\\ i\dfrac{\mathrm{d}\vartheta_{\mathrm{M}}}{\mathrm{d}x} & 0\end{pmatrix}}\right]\begin{pmatrix}\psi_1^{\mathrm{M}}\\ \psi_2^{\mathrm{M}}\end{pmatrix}$$

initial conditions:

$$\begin{pmatrix}\psi_1^{\mathrm{M}}(0)\\ \psi_2^{\mathrm{M}}(0)\end{pmatrix} = \begin{pmatrix}\cos\vartheta^0_{\mathrm{M}} & -\sin\vartheta^0_{\mathrm{M}}\\ \sin\vartheta^0_{\mathrm{M}} & \cos\vartheta^0_{\mathrm{M}}\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix} = \begin{pmatrix}\cos\vartheta^0_{\mathrm{M}}\\ \sin\vartheta^0_{\mathrm{M}}\end{pmatrix}$$

solution approximating all non-adiabatic $\nu_1^{\mathrm{M}} \leftrightarrows \nu_2^{\mathrm{M}}$ transitions in resonance

$$\begin{aligned}
\psi_1^{\mathrm{M}}(x) \;\simeq\; & \left[\cos\vartheta^0_{\mathrm{M}}\exp\left(i\int_0^{x_{\mathrm{R}}}\frac{\Delta m^2_{\mathrm{M}}(x')}{4E}\,\mathrm{d}x'\right)\mathcal{A}^{\mathrm{R}}_{11} + \sin\vartheta^0_{\mathrm{M}}\exp\left(-i\int_0^{x_{\mathrm{R}}}\frac{\Delta m^2_{\mathrm{M}}(x')}{4E}\,\mathrm{d}x'\right)\mathcal{A}^{\mathrm{R}}_{21}\right]\\
& \times\exp\left(i\int_{x_{\mathrm{R}}}^{x}\frac{\Delta m^2_{\mathrm{M}}(x')}{4E}\,\mathrm{d}x'\right)\\
\psi_2^{\mathrm{M}}(x) \;\simeq\; & \left[\cos\vartheta^0_{\mathrm{M}}\exp\left(i\int_0^{x_{\mathrm{R}}}\frac{\Delta m^2_{\mathrm{M}}(x')}{4E}\,\mathrm{d}x'\right)\mathcal{A}^{\mathrm{R}}_{12} + \sin\vartheta^0_{\mathrm{M}}\exp\left(-i\int_0^{x_{\mathrm{R}}}\frac{\Delta m^2_{\mathrm{M}}(x')}{4E}\,\mathrm{d}x'\right)\mathcal{A}^{\mathrm{R}}_{22}\right]\\
& \times\exp\left(-i\int_{x_{\mathrm{R}}}^{x}\frac{\Delta m^2_{\mathrm{M}}(x')}{4E}\,\mathrm{d}x'\right)
\end{aligned}$$

ν Oscillations: Non-constant Matter Density

$$\psi_e(x) = \cos\vartheta\,\psi_1^{\mathrm{M}}(x) + \sin\vartheta\,\psi_2^{\mathrm{M}}(x)$$

Applied to Solar & Supernova neutrinos

neglect interference (averaged over energy spectrum)

$$\overline{P}_{\nu_e\to\nu_e}(x) = |\langle\psi_e(x)\rangle|^2 = \cos^2\vartheta\ \cos^2\vartheta_{\mathrm{M}}^0\,|\mathcal{A}_{11}^{\mathrm{R}}|^2 + \cos^2\vartheta\ \sin^2\vartheta_{\mathrm{M}}^0\,|\mathcal{A}_{21}^{\mathrm{R}}|^2$$
$$+ \sin^2\vartheta\ \cos^2\vartheta_{\mathrm{M}}^0\,|\mathcal{A}_{12}^{\mathrm{R}}|^2 + \sin^2\vartheta\ \sin^2\vartheta_{\mathrm{M}}^0\,|\mathcal{A}_{22}^{\mathrm{R}}|^2$$

conservation of probability (unitarity)

$$|\mathcal{A}_{12}^{\mathrm{R}}|^2 = |\mathcal{A}_{21}^{\mathrm{R}}|^2 = P_c \qquad |\mathcal{A}_{11}^{\mathrm{R}}|^2 = |\mathcal{A}_{22}^{\mathrm{R}}|^2 = 1 - P_c$$

$P_c \equiv$ crossing probability

$$\boxed{\overline{P}_{\nu_e\to\nu_e}(x) = \frac{1}{2} + \left(\frac{1}{2} - P_c\right)\cos 2\vartheta_{\mathrm{M}}^0\ \cos 2\vartheta}$$

[Parke, PRL 57 (1986) 1275]

ν Oscillations: Non-constant Matter Density

$$P_c = \frac{\exp\left(-\frac{\pi}{2}\gamma F\right) - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}{1 - \exp\left(-\frac{\pi}{2}\gamma \frac{F}{\sin^2\vartheta}\right)}$$

[Kuo, Pantaleone, PRD 39 (1989) 1930]

adiabaticity parameter: $$\gamma = \left.\frac{\Delta m^2_M/2E}{2|d\vartheta_M/dx|}\right|_R = \frac{\Delta m^2 \sin^2 2\vartheta}{2E\cos 2\vartheta \left|\frac{d\ln A_{CC}}{dx}\right|_R}$$

$A \propto x$ $F = 1$ (Landau-Zener approximation) [Parke, PRL 57 (1986) 1275]

$A \propto 1/x$ $F = \left(1 - \tan^2\vartheta\right)^2 / \left(1 + \tan^2\vartheta\right)$ [Kuo, Pantaleone, PRD 39 (1989) 1930]

$A \propto \exp(-x)$ $F = 1 - \tan^2\vartheta$

[Pizzochero, PRD 36 (1987) 2293]

[Toshev, PLB 196 (1987) 170]

[Petcov, PLB 200 (1988) 373]

Review: [Kuo, Pantaleone, RMP 61 (1989) 937]

In ν Oscillations Dirac = Majorana

Evolution of Amplitudes: $$i\frac{\mathrm{d}\psi_\alpha}{\mathrm{d}x} = \frac{1}{2E}\sum_\beta \left(UM^2U^\dagger + 2EV\right)_{\alpha\beta}\psi_\beta$$

difference: $$\begin{cases} \text{Dirac:} & U^{(\mathrm{D})} \\ \text{Majorana:} & U^{(\mathrm{M})} = U^{(\mathrm{D})}D(\lambda) \end{cases}$$

$$D(\lambda) = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & e^{i\lambda_{21}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{i\lambda_{N1}} \end{pmatrix} \quad \Rightarrow \quad D^\dagger = D^{-1}$$

$$M^2 = \begin{pmatrix} m_1^2 & 0 & \cdots & 0 \\ 0 & m_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_N^2 \end{pmatrix} \quad \Longrightarrow \quad DM^2 = M^2D \quad \Longrightarrow \quad DM^2D^\dagger = M^2$$

$$U^{(\mathrm{M})}M^2(U^{(\mathrm{M})})^\dagger = U^{(\mathrm{D})}DM^2D^\dagger(U^{(\mathrm{D})})^\dagger = U^{(\mathrm{D})}M^2(U^{(\mathrm{D})})^\dagger$$

Common Question: Do Charged Leptons Oscillate?

- Mass is the only property which distinguishes e, μ, τ.
- The flavor of a charged lepton is defined by its mass!
- By definition, the flavor of a charged lepton cannot change.

THE FLAVOR OF CHARGED LEPTONS DOES NOT OSCILLATE

[CG, Kim, FPL 14 (2001) 213] [CG, hep-ph/0409230] [Akhmedov, JHEP 09 (2007) 116]

参考书

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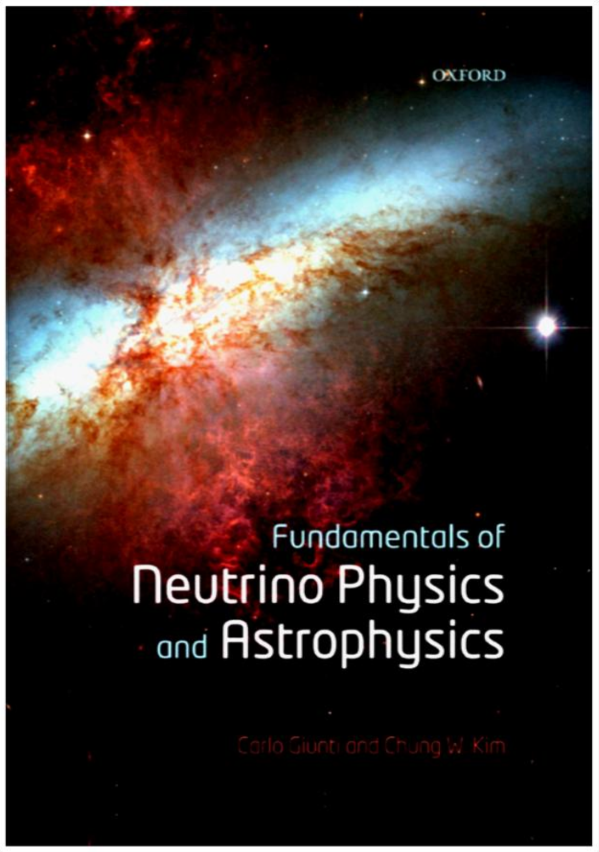
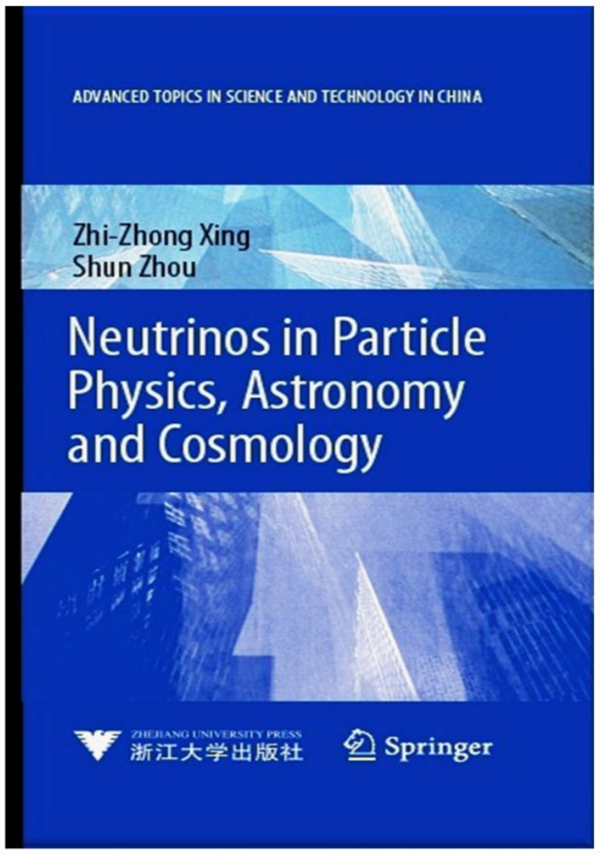
A. Strumia & F. Vissani

hep-ph/0606054





- **文献搜索**
- http://inspirehep.net/
- https://arxiv.org/
- https://ui.adsabs.harvard.edu/

Thanks!

谢谢!

弱相互作用: 费米理论 v.s. 标准模型

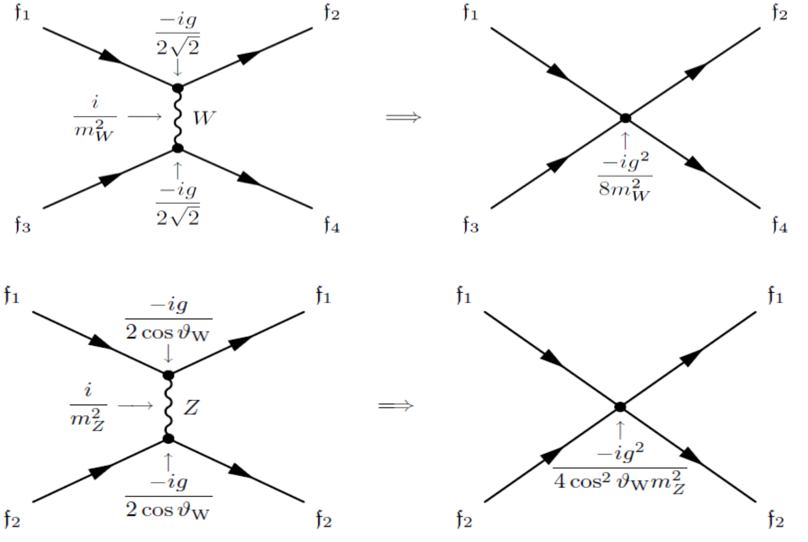
费米相互作用：Current-current interaction *Fermi (1934)*

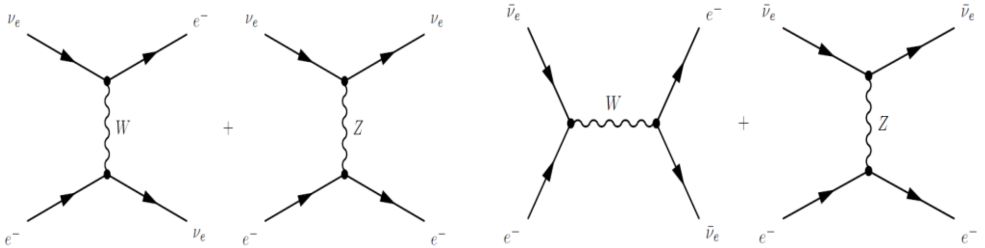
$$\frac{G_\mathrm{F}}{\sqrt{2}} = \frac{g^2}{8\, m_W^2}$$

$$j^\rho_{W,\mathrm{L}} = 2 \sum_{\alpha=e,\mu,\tau} \overline{\nu_{\alpha L}}\, \gamma^\rho\, \ell_{\alpha L} = \sum_{\alpha=e,\mu,\tau} \overline{\nu_\alpha}\, \gamma^\rho \left(1-\gamma^5\right) \ell_\alpha$$

$$j^\rho_{Z,\nu} = \sum_{\alpha=e,\mu,\tau} \overline{\nu_{\alpha L}}\, \gamma^\rho\, \nu_{\alpha L} = \frac{1}{2} \sum_{\alpha=e,\mu,\tau} \overline{\nu_\alpha}\, \gamma^\rho \left(1-\gamma^5\right) \nu_\alpha$$

$$\mathscr{L}^{(\mathrm{CC})}_{\mathrm{I,L}} = -\frac{g}{2\sqrt{2}} \left(j^\rho_{W,\mathrm{L}} W_\rho + j^\rho_{W,\mathrm{L}}{}^\dagger W^\dagger_\rho \right) \qquad \mathscr{L}^{(\mathrm{NC})}_{\mathrm{I},\nu} = -\frac{g}{2\cos\vartheta_\mathrm{W}}\, j^\rho_{Z,\nu}\, Z_\rho$$





$$\mathscr{L}_\mathrm{eff}\big(\overset{(-)}{\nu}_e e^- \to \overset{(-)}{\nu}_e e^-\big) = -\frac{G_\mathrm{F}}{\sqrt{2}} \Big\{ \left[\overline{\nu_e}\, \gamma^\rho \left(1-\gamma^5\right) e\right] \left[\overline{e}\, \gamma_\rho \left(1-\gamma^5\right) \nu_e\right]$$
$$+ \left[\overline{\nu_e}\, \gamma^\rho \left(1-\gamma^5\right) \nu_e\right] \left[\overline{e}\, \gamma_\rho \left(g_V^l - g_A^l \gamma^5\right) e\right] \Big\}$$

$$\mathscr{L}_\mathrm{eff}\big(\overset{(-)}{\nu}_e e^- \to \overset{(-)}{\nu}_e e^-\big) = -\frac{G_\mathrm{F}}{\sqrt{2}} \left[\overline{\nu_e}\, \gamma^\rho \left(1-\gamma^5\right) \nu_e\right] \left[\overline{e}\, \gamma_\rho \left(\left(1+g_V^l\right) - \left(1+g_A^l\right) \gamma^5\right) e\right]$$

Z 耦合

Fermions	g_L	g_R	g_V	g_A
ν_e, ν_μ, ν_τ	$g_L^\nu = \frac{1}{2}$	$g_R^\nu = 0$	$g_V^\nu = \frac{1}{2}$	$g_A^\nu = \frac{1}{2}$
e, μ, τ	$g_L^l = -\frac{1}{2} + s_W^2$	$g_R^l = s_W^2$	$g_V^l = -\frac{1}{2} + 2s_W^2$	$g_A^l = -\frac{1}{2}$
u, c, t	$g_L^U = \frac{1}{2} - \frac{2}{3} s_W^2$	$g_R^U = -\frac{2}{3} s_W^2$	$g_V^U = \frac{1}{2} - \frac{4}{3} s_W^2$	$g_A^U = \frac{1}{2}$
d, s, b	$g_L^D = -\frac{1}{2} + \frac{1}{3} s_W^2$	$g_R^D = \frac{1}{3} s_W^2$	$g_V^D = -\frac{1}{2} + \frac{2}{3} s_W^2$	$g_A^D = -\frac{1}{2}$

	Natural units	**Realistic units**
Phase factors	$\exp\left(-iE_{1,2}t\right)$	$\exp\left(-i\frac{E_{1,2}}{\hbar}t\right)$
Energies and momentum	$E_{1,2}=\sqrt{p^2+m_{1,2}^2}$	$E_{1,2}=\sqrt{p^2c^2+m_{1,2}^2c^4}$
Energy difference	$\Delta E=\frac{\Delta m^2}{2E}$	$\Delta E=\frac{\Delta m^2c^3}{2p}=\frac{\Delta m^2c^4}{2E}$
Time and distance	$t=L$	$t=\frac{L}{c}$
Oscillation argument	$\frac{1}{2}\Delta Et=\frac{\Delta m^2L}{4E}$	$\frac{1}{2}\frac{\Delta E}{\hbar}t=\frac{c^3}{\hbar}\cdot\frac{\Delta m^2L}{4E}$