EFT Homework WINP2025

(Doing this homework is completely optional.)

Consider the Higgs sector in the SM, which we write as

$$\mathcal{L}_{\text{Higgs}} = (D_{\mu}H)^{\dagger}(D^{\mu}H) - V(H), \qquad V(H) = -\mu^2 H^{\dagger}H + \lambda (H^{\dagger}H)^2.$$
(1)

With $\mu^2 > 0$ and $\lambda > 0$, the Higgs field H develops a vacuum expectation value (vev). Without loss of generality, and in the unitary gauge, we can write $H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix}$, where v is the vev and h is the "Higgs particle". We can also write V in terms of h in the following form:

$$V(h) = \frac{1}{2}m_h^2 h^2 + \lambda_3 h^3 + \lambda_4 h^4 + \text{constant}.$$
 (2)

- (a) Find the vev v by writing $\phi = v + h$ and solving $\frac{dV}{d\phi}|_{\phi=v} = 0$, and show that V is at its minimum at the vacuum. Express v and m_h in terms of μ^2 and λ . From measurements we know that v = 246 GeV and $m_h = 125 \text{ GeV}$. For now, we can ignore their measurement uncertainties. Work out the values of μ^2 and λ . Then express λ_3 and λ_4 in terms of v and m_h and work out their numerical values.
- (b) Let's now add a dimension-6 operator $\mathcal{O}_6 = (H^{\dagger}H)^3$ to the Lagrangian. We can write the potential as

$$V(H) = -\mu^2 H^{\dagger} H + \lambda (H^{\dagger} H)^2 + \frac{c_6}{\Lambda^2} (H^{\dagger} H)^3, \qquad (3)$$

where c_6 is a dimensionless parameter and Λ is the scale of some new physics. Let's define $\frac{\bar{c}_6}{v^2} \equiv \frac{c_6}{\Lambda^2}$. Working with \bar{c}_6 will turn out to be much simpler. If we require the potential to have a minimum, what does it imply on the sign of \bar{c}_6 ? Assuming we still have $\mu^2 > 0$ and $\lambda > 0$, find out the new vev v and write it in terms of μ^2 , λ and \bar{c}_6 .

- (c) Express μ^2 and λ in terms of v, m_h and \bar{c}_6 . For λ , you may find it helpful to first express m_h^2 in terms of v, λ and \bar{c}_6 . Note that v = 246 GeV and $m_h = 125 \text{ GeV}$ are physical parameters which are fixed by experiments. However, the presence of \bar{c}_6 changes the relation between μ^2 , λ and v, m_h , so μ^2 and λ will take different values from the SM ones if \bar{c}_6 is non-zero.
- (d) With 3 parameters we need at least 3 measurements to determine their values. In addition to v and m_h , we can measure the so-called "triple Higgs coupling", which is λ_3 in Eq. (2). It is a very difficult measurement. To see how the triple Higgs coupling is related to c_6 , it is most convenient to define a quantity $\kappa_3 \equiv \frac{\lambda_3}{\lambda_{3,SM}}$, where $\lambda_{3,SM}$ is the value of λ_3 in the SM without \mathcal{O}_6 , which you have worked out in part a). Write κ_3 in terms of v, m_h and \bar{c}_6 . You don't need to plugging in the numerical values of v and m_h .

(e) The quartic Higgs coupling, λ_4 , is even more difficult to measure. However, with just the \mathcal{O}_6 operator, λ_4 is not an independent parameter. Work out the relation between $\kappa_4 \equiv \frac{\lambda_4}{\lambda_{4,\text{SM}}}$ and κ_3 . Does this relation still hold if we add an additional dimension-8 operator $\mathcal{O}_8 = (H^{\dagger}H)^4$?