Effective Field Theory (EFT)

- why EFT ?
- examples of EFT ⟵ there are many EFTs....
- the Standard Model Effective Field Theory (SMEFT)
  most relevant for BSM
- phenomenology (if there's time)

note his opinion on Hierarchy problem...
(see my lecture for my opinion)

useful refs.

★ Manohar's lectures on EFT 1804.05863 or TASI 2022
2025 Sakurai Prize (with Jenkins)

old ones: 1006.2142 Skiba , hep-ph/0308266 Rothstein
nucl-th/0510023 Kaplan

book : Introduction to EFT . Burgess

SMEFT :

Lectures on SMEFT , Falkowski (no arxiv)
warsaw basis 1008.4884
Higgs+EW SMEFT 1308.1879 (Barcelona) Elias-Miro, Espinosa, Masso, Pomarol
SMEFTsim 3.0 Ilaria Brivio 2012.11343
well maintained ....

Why EFT?

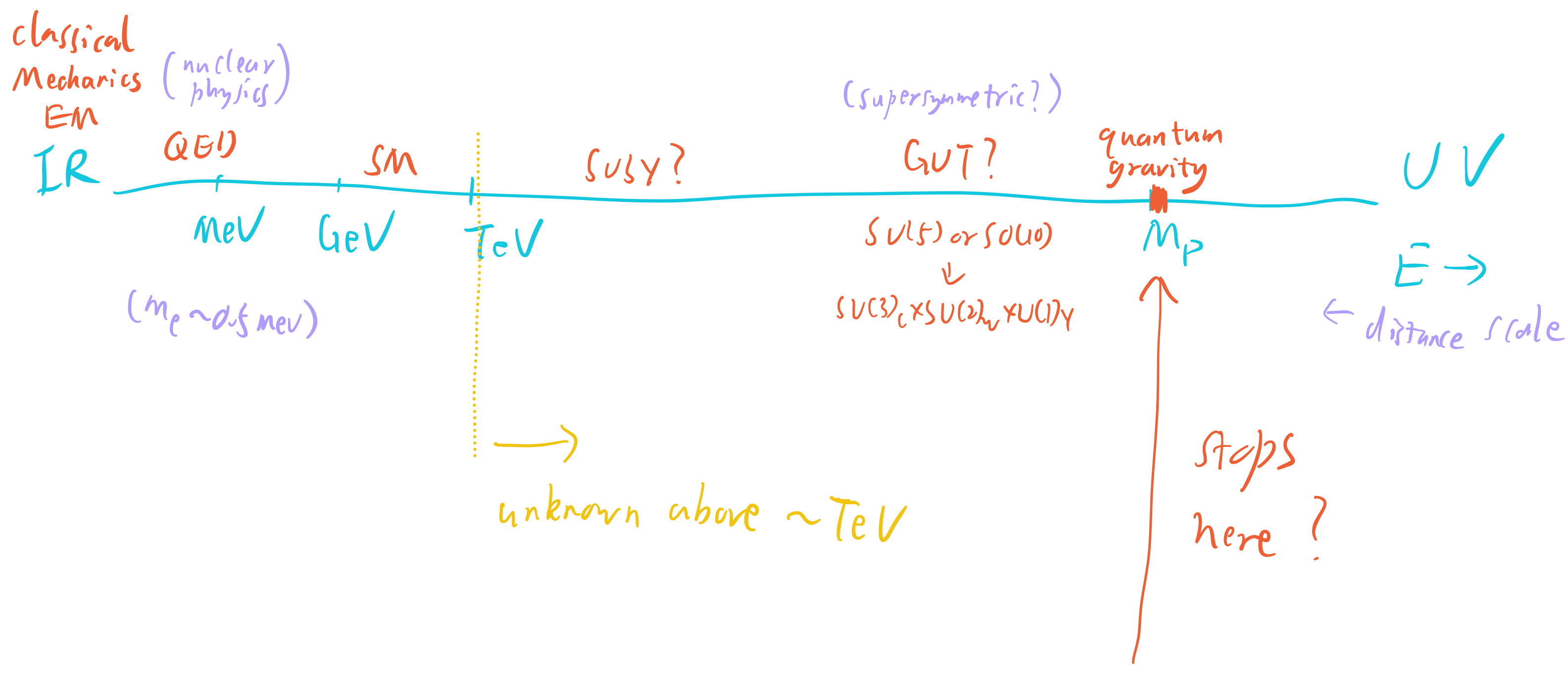
(We think) Every theory is an effective theory!

Reductionism: (还原论)

A more fundamental theory will appear at a higher energy / smaller length scale.

The more correct way to ask "物质是否无限可分"

(natural units: 1 unit $[E] \sim [L]^{-1}$)



(or maybe there is an ultimate theory?)

Quantum gravity ⇔ spacetime quantized?
notion of energy / distance breaks down?

Not everyone believes Reductionism...

Key ingredient: locality (many definitions, here it means)

Measurements at large distance / low energy should not be sensitive to the physics at small distance / high energy.

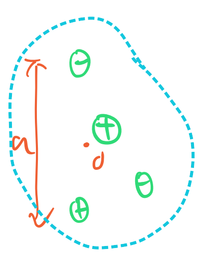
Engineers don't need to learn QFT to build bridges! car

Classical Mechanics is replaced by QFT / QM / Relativity at small scale / high energy

does not mean it's wrong, it's an effective theory at large scale / low energy.

- Even if we know the more fundamental theory (e.g. QFT), it can be more convenient to use an EFT (e.g. Classical Mech.) at low energy!

example 1 Multipole Expansion in Electrostatics



$a \sim$ scale of the charge distribution

$\vec{r}$. $V(\vec{r})$ electric potential

locality: For $r \gg a$, this looks like a point charge!

$$V(\vec{r}) = \frac{1}{r}\sum_{l,m} b_{lm}\frac{1}{r^l}\underbrace{Y_{lm}(\theta,\varphi)}_{\text{spherical harmonics}}$$

(kind of like operators in field theory)

spherical harmonics (球谐函数)

$$= \frac{1}{r}\sum_{\substack{l,m \\ =0,1,\ldots}} c_{lm}\left(\frac{a}{r}\right)^l Y_{lm} \qquad b_{lm} \equiv c_{lm}\, a^l$$

c_{lm} s are dimensionless parameters (usually of order 1).

separation of scales ($\frac{a}{r} \ll 1$) ⇒ The expansion is useful,

i.e. we only need to keep a few terms in the l expansion to get a good approximation

(more terms ⇒ better accuracy)

	distance	energy	
IR	r	$E \sim 1/r$	collider energy or EW scale v
UV	a	$\Lambda \sim 1/a$	scale of new physics
expansion parameter	$\frac{a}{r}$	$\frac{E}{\Lambda}$	(or $\frac{v}{\Lambda}$)
coupling (wilson coefficients)	c_{lm}	$c_i^{(n)}$ — dim-n operator	

- There is no precise definition of a. One could only measure the combination $b_{lm} = c_{lm}\, a^l$. $\left(\frac{c_i^{(n)}}{\Lambda^{n-4}}\right)$

- If we know the charge distribution (UV theory), we can do the expansion to find out all the b_{lm} (c_{lm}). This is called matching. (top down)

- If we don't know ......, we can treat all C_{lm} s as free parameters and try to measure them experimentally. (bottom up)

After truncating the series (throwing away terms with $l > l_{max}$) there are a finite number of parameters.
If we make enough measurements we can constrain all parameters (C_{lm}).

- To precisely determine the values of C_{lm} we can either
  - make very precise measurement at large r (low energy)
  - make measurements at small r (high energy)

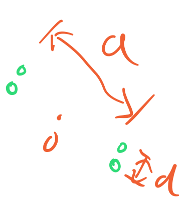
energy vs. precision (or both!)

Important aspects for colliders.

If $r \simeq a$, the expansion breaks down!

High energy is always good, but EFT may not be valid!

multiple scales



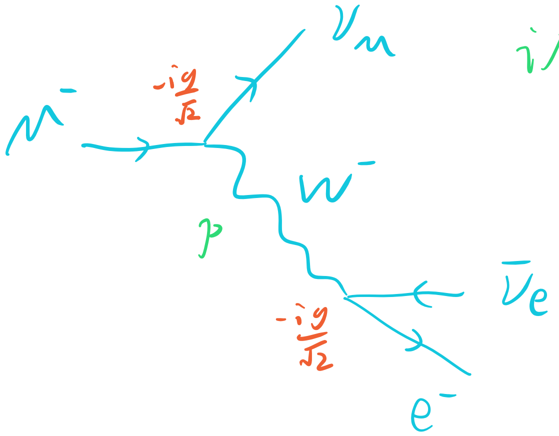
$\vec{r}\,.$

$$r \gg a \gg d$$

$$(\Lambda_{EW} \ll \Lambda_{SUSY} \ll \Lambda_{GUT})$$

example 2 Fermi's theory

muon decay



$$iM = \left(\frac{-ig}{\sqrt{2}}\right)^2 (\bar{\nu}_\mu \gamma^\mu \mu_L)(\bar{e}_L \gamma^\nu \nu_e) \cdot \frac{-ig_{\mu\nu}}{p^2 - M_W^2}$$

(ignore W width since $p^2 \ll M_W^2$)

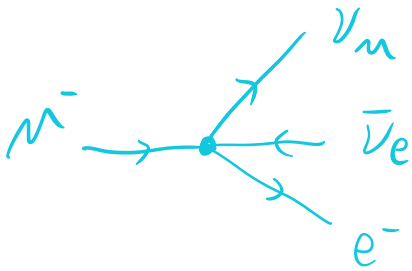
For $p^2 \ll M_W^2$, we can expand the operator

$$\frac{1}{p^2 - M_W^2} = -\frac{1}{M_W^2}\left(1 + \frac{p^2}{M_W^2} + \frac{p^4}{M_W^4} + \cdots\right)$$

keeping only the 1st term we have

$$iM = \frac{-ig^2}{2M_W^2} (\bar{\nu}_\mu \gamma^\mu \mu_L)(\bar{e}_L \gamma_\mu \nu_e) + O\left(\frac{1}{M_W^4}\right)$$

4-fermion contact interaction



which can be produced by the local Lagrangian

often written as $\frac{c}{\Lambda^2}$ where $\Lambda \sim M_W^2$, if the UV theory is unknown.

$$\mathcal{L} = \overbrace{-\frac{g^2}{2M_W^2}}\underbrace{(\bar{\nu}_\mu \gamma^\mu \mu_L)(\bar{e}_L \gamma_\mu \nu_e)}_{\text{dimension-6 operator}} + O\left(\frac{1}{M_W^4}\right)$$

dimension-6 operator, what Fermi wrote down.

If we keep more terms in the Lagrangian we'll generate higher dimensional operators, e.g. the $\frac{1}{M_W^4}$ term corresponds to dim-8 operators.

(Higher dimensional operators may look very complicated, it's actually much easier to use on-shell amplitudes!)

This is the simplest example of the matching between the full model (SM) and the low-energy effective field theory (Fermi's theory).

- This way of matching is called amplitude matching.
  (Finding a EFT that gives the same low energy amplitudes as the full theory.)
- Matching can also be understood from the path integral picture:

Consider light field(s) ϕ
heavy field(s) Φ

generating functional (or the 1 light PI effective action)

$$Z_{uv}[J_\phi, J_\Phi] = \int D\phi\, D\Phi\, \exp\left\{ i\int d^4x \left[\mathcal{L}_{uv}(\phi, \Phi) + J_\phi \phi + J_\Phi \Phi \right] \right\}$$

at low energy, Φ cannot be excited, set $J_\Phi = 0$
"integrating out the heavy fields Φ"
literally

$$Z_{EFT}[J_\phi] = \int D\phi\, \exp\left\{ i\int d^4x \left[\mathcal{L}_{EFT}(\phi) + J_\phi \phi \right] \right\}$$

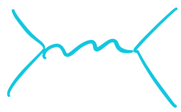
many literature on this ...... (Xiaochuan Lu, Zhengkang Zhang ...)

Note: The two pictures of matching are equivalent.

back to $\frac{1}{p^2 - M_W^2} = -\frac{1}{M_W^2}\left(1 + \frac{p^2}{M_W^2} + \frac{p^4}{M_W^4} + \cdots\right)$

- For $p^2 \ll M_W^2$, the 4F operator gives a very good approximation of the full theory. This is the case for muon decay. ($p^2 < m_\mu^2$ $\quad \frac{m_\mu^2}{M_W^2} \sim 10^{-6}$)

- For $p^2 \gtrsim M_W^2$, the expansion breaks down!

  consider $2 \to 2$ scattering 

  for $E_{cm}^2 \sim p^2$ we'll see the resonance!

- The coefficient of the 4F operator is $-\frac{g^2}{2M_W^2} = \frac{-2}{v^2}$.

  Measuring muon decay only tells us the value of v (or $G_F \equiv \frac{1}{\sqrt{2}v^2}$), but not M_W, which depends on g.

- But M_W is the scale at which the EFT breaks down!

In our world, $g \simeq 0.65$.

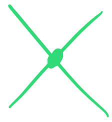
Suppose we've measured G_F but don't know the value of g.

The scale at which the EFT breaks down depends on the value of the coupling!

If g is $\begin{cases} \text{very small}, & W, Z \text{ would be much lighter} \\ \text{very large}, & \cdots\cdots \text{heavier}. \end{cases}$ cutoff Λ 没有下限 有上限

but if $g \gtrsim 4\pi$, the theory becomes non-perturbative! $\Lambda_{max} \sim \frac{1}{2} 4\pi \cdot 246\,\text{GeV} \simeq 1.5\,\text{TeV}$

- perturbative unitarity bound

2→2 scattering   dim-6 4f operator  $\frac{-g^2}{2M_W^2}$ $\mathcal{M} \sim E^2$

unitarity bound is violated for $\sqrt{s} \gtrsim 1.5\,\text{TeV}$

corresponds to the value of M_W with $g \sim 4\pi$!

This is the scale at which the EFT has to break down.

(but for weakly coupled theory the EFT would break down at a lower scale.)

- unitarity is violated ✗
- perturbative (EFT) expansion breaks down ✓

each term in the expansion is much larger than the sum.

- In this simple example, if we also measure the dim-8 coefficient $\left(\sim \frac{g^2}{M_W^4}\right)$ we can derive the W mass.

  In more complicated cases (with multiple heavy particles) it is in general not possible.

- global fcn ....

example 3 a complex scalar EFT

(with a U(1) global symmetry)

- bottom up approach, writing down all operators in the EFT approach.

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 |\phi|^2 - \frac{\lambda}{4} |\phi|^4 \qquad [O] \leq 4$$

$$+ \sum_i \frac{C_i^{(5)}}{\Lambda} O_i^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \sum_i \frac{C_i^{(7)}}{\Lambda^3} O_i^{(7)} + \cdots\cdots$$

has dimension $4-n$

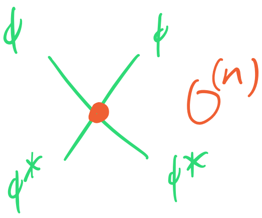
$[\mathcal{L}] = 4$, $\quad [O^{(n)}] = n$, $\quad [C] = 0$, $\quad [\Lambda] = 1$

$\Lambda \sim$ scale of the new physics (mass of the new particles)

Large $\Lambda \Rightarrow$ The EFT expansion is good.

Operators in $\mathcal{L}$ are divided into 3 classes based on their mass dimensions

		coupling dimension	
$[O]<4$	relevant	>0	$\lvert\phi\rvert^2$
$[O]=4$	marginal	$=0$	$\lvert\phi\rvert^4$
$[O]>4$	irrelevant	<0	



$$M \sim \left(\frac{E}{\Lambda}\right)^{n-4}$$

Small effects at low energy

EFT breaks down at high energy! With $[O]>4$, the theory cannot be a complete theory $\Rightarrow$ why it's called an EFT.

This classification is also related to renormalizability — we'll come back to it later.

Now let's try to write down the higher dim. operators...

Each $O_i^{(n)}$ need to be invariant under Lorentz, U(1) symmetry.

No odd dimension operator!

$\phi^*\phi\phi^*\partial_\mu\phi$ not Lorentz invariant

$|\phi|^4\phi$ not U(1) invariant

Large $\Lambda \Rightarrow$ leading contribution: $O_i^{(6)}$!

Bottom-up approach: write down all possible $O_i^{(6)}$!

Not all of them are independent!

<u>Operator redundancy</u>

Operators are related by

- Integration by parts (IBP)
- Equation of Motion (EOM) why EOM works beyond the classical level
- Fierz identity ... (no use for scalar theory)

(more generally, field redefinition)

Let's try to write down all possible d6 operators:

0∂: $|\phi|^6$

2∂:

$$(\partial^\mu \phi^* \partial_\mu \phi)\, \phi^* \phi, \quad \underbrace{(\partial^\mu \partial_\mu \phi^*)\phi\, \phi^* \phi, \quad \left((\partial^\mu \phi^*)\phi\right)^2}_{+h.c.}, \dots$$

4∂:

$$\underbrace{(\partial^\mu \partial_\mu \phi^*)(\partial^\nu \partial_\nu \phi), \quad \text{------}}_{\text{only 1 indep operator}}$$

IBP:

total derivative

$$(\partial^\mu \partial_\mu \phi^*)(\partial^\nu \partial_\nu \phi) = \partial^\mu\left[(\partial_\mu \phi^*)(\partial^\nu \partial_\nu \phi)\right]$$

$$- \underbrace{\partial_\mu \phi^* \partial^\mu}_{\text{moved } \partial^\mu \text{ here, equivalent operator}} \partial^\nu \partial_\nu \phi$$

$$\ldots\ldots$$

EOM:

$$\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = 0 \qquad (\phi \leftrightarrow \phi^*)$$

$$-m^2 \phi^* - \frac{\lambda}{2} \phi^* \phi^* \phi - \partial_\mu \partial^\mu \phi^* + \overbrace{\ldots\ldots}^{d \geq 5} = 0$$

$$\partial_\mu \partial^\mu \phi^* = -m^2 \phi^* - \frac{\lambda}{2} \phi^* \phi^* \phi + \underbrace{\ldots\ldots}_{d \geq 5}$$

$$\partial_\mu \partial^\mu \phi = -m^2 \phi - \frac{\lambda}{2} \phi \phi \phi^* + \overbrace{\ldots\ldots}^{d \geq 5}$$

---

things get more complicated if we go beyond d6!

$$\frac{c}{\Lambda^2} (\partial^\mu \partial_\mu \phi^*) \phi \phi^* \phi$$

$$= \overbrace{-\frac{c}{\Lambda^2} m^2 |\phi|^4}^{\text{absorb in a redefinition of } \lambda} - \frac{c}{\Lambda^2} \frac{\lambda}{2} |\phi|^6 + \overbrace{\ldots\ldots}^{d \geq 8 \text{, ignore}}$$

We can eliminate $(\partial^\mu \partial_\mu \phi^*) \phi \phi^* \phi$ in favor of $|\phi|^6$

$$(\partial^\mu \partial_\mu \phi^*)(\partial^\nu \partial_\nu \phi)$$

total derivative

$$\partial^\mu\left((\partial_\mu\phi^*)\phi\,\phi^*\phi\right)$$

$$= (\partial^\mu\partial_\mu\phi^*)\phi\phi^*\phi + 2(\partial_\mu\phi^*\partial^\mu\phi)\phi^*\phi$$

$$+ \underbrace{\left((\partial_\mu\phi^*)\phi\right)^2}_{\text{can eliminate}} \qquad \cancel{+ (\partial_\mu\phi^*)\phi\phi^*(\partial^\mu\phi)}$$

2 independent d6 operators!

We can chose $|\phi|^6$, $|\phi|^2\partial_\mu\phi^*\partial^\mu\phi$.

This is called choosing a basis. (chose which redundant operators to eliminate)

Of course we can chose a different basis.

Why not just keep redundant operators? < convenience, global fit

(important to know how many indep. physical parameters we have)

☆ Physics are basis-independent.

Physicists are basis-dependent!

HEFT workshop 吵架……

Instead of e.o.m. we can perform field redefinition ....

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 |\phi|^2 - \frac{\lambda}{4} |\phi|^4$$

$$+ \frac{C_1}{\Lambda^2} |\phi|^6 + \frac{C_2}{\Lambda^2} |\phi|^2 \partial_\mu \phi^* \partial^\mu \phi$$

$$\underbrace{+\left(\frac{C_3}{\Lambda^2} (\partial^\mu \partial_\mu \phi^*) \phi \phi^* \phi + h.c.\right) + \frac{C_4}{\Lambda^2} (\partial^\mu \partial_\mu \phi^*)(\partial^\nu \partial_\nu \phi)}$$

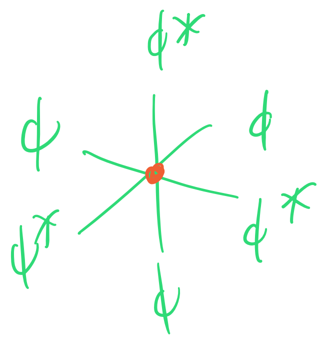
Field redefinition   chose α_1, α_2 to cancel

$$\phi \to \phi + \alpha_1 \frac{\phi^* \phi \phi}{\Lambda^2} + \alpha_2 \frac{\partial_\nu \partial^\nu \phi}{\Lambda^2}$$

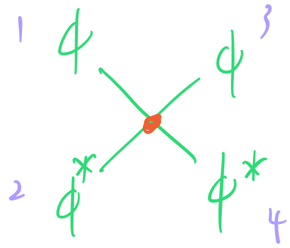
does not change kinetic, mass terms

indep. operators $\Longleftrightarrow$ (massless) on-shell amplitude

$$[A] = 4 - n$$



$\sim \frac{C_1}{\Lambda^2}$



$\sim \frac{C_2}{\Lambda^2}(s+u)$

general:

$$\alpha s + \cancel{\beta t} + \gamma u \qquad (s+t+u=0)$$

$s \Longleftrightarrow u$ symmetric $\Rightarrow \alpha = \gamma$

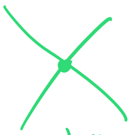
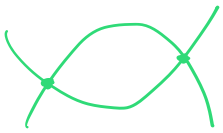
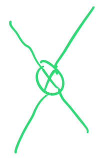
$(\partial^2\phi)^2$ cancel with propagator

$(\partial^2 \phi^4)$

$$\frac{1}{p^2\left(1+\frac{\phi^2}{\Lambda^2}\right)} = \frac{1}{p^2}\left(1 - \frac{c p^2}{\Lambda^2} + \dots\right)$$

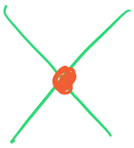
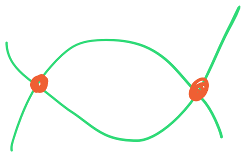
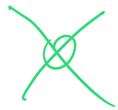
What about renormalizability?

Operators with $d > 4$ are non-renormalizable?

 $\sim \lambda$  $\sim \lambda^2$  counter term $\sim |\phi|^4$

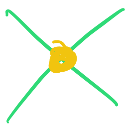
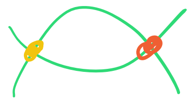
$|\phi|^4$

no need to add extra terms in $\mathcal{L}$ ⇒ renormalizable

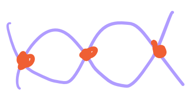
 $\sim \frac{C_{(6)}}{\Lambda^2}$  $\sim \frac{C_{(6)}^2}{\Lambda^4}$  counter term d8 $|\partial_\mu \phi|^4$

$|\phi|^2 (\partial_\mu \phi^* \partial^\mu \phi)$

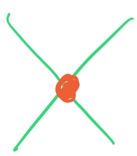
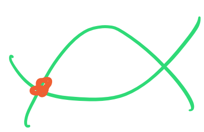
need to add ∞ many terms in $\mathcal{L}$ ⇒ non-re..!

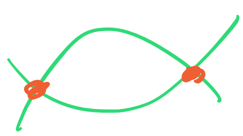
need to add  $\sim \frac{C_{(8)}}{\Lambda^4}$ ⇒  $\sim \frac{C_{(6)} C_{(8)}}{\Lambda^6}$

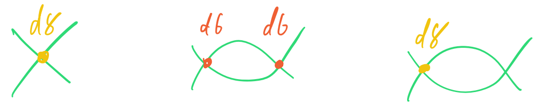
⇒ need d10 counter term! ....

or  $\sim \frac{C_{(6)}^3}{\Lambda^6}$ ⇒ need d10 counter term!

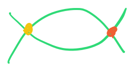
Correct argument: We only work up to a fixed order in the EFT expansion and discard all higher order terms

up to d6 (& 1 loop): keep   $\sim \frac{1}{\Lambda^2}$ counter term also d6

discard  , ... $\sim \frac{1}{\Lambda^4}$

up to $\frac{1}{\Lambda^4}$: add 

$\sim \frac{1}{\Lambda^4}$

discard  .... $\frac{1}{\Lambda^6}$

☆ EFT is renormalizable order by order in the EFT expansion.

why SMEFT ?

SM is incomplete (gravity, dark matter, matter anti-matter asymmetry ......)

There must be BSM New physics but we don't know what it is. (some people think they know ....)

light particle
very weak coupling
~~SMEFT~~

heavy particle ($M \gg v$)
SMEFT ✓

- strictly speaking, graviton makes SMEFT invalid.
  (But that's ok ....)

since we don't know what it is

- bottom-up approach: Be agnostic about the UV physics and try to systematically parameterize its effects at low energies. by writing down higer dimension operators. (top-down: model building ....)
- useful even if we know the UV model for calculation ....
  resumming large logs with RG ...

SMEFT

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \sum_i \frac{C_i^{(5)}}{\Lambda} \mathcal{O}_i^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \sum_i \frac{C_i^{(7)}}{\Lambda^3} \mathcal{O}_i^{(7)} + \dots$$

$$\cancel{[\mathcal{L}] = 4, \quad [\mathcal{O}^{(n)}] = n, \quad [\Lambda] = 1}$$

Note: each $\mathcal{O}_i$ needs to be invariant under Lorentz and gauge transformations $\underline{SU(3) \times SU(2) \times U(1)}$

non-decoupling physics

HEFT: only $SU(3) \times U(1)$. (linear vs. non-linear ...)

bottom up approach: write down all possible operators!

(not all operators are independent ....)

dim 5: only 1 type of operators $\sim LLHH$ (Weinberg operator)

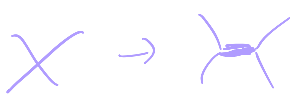
HW: write down the exact form of the Weinberg operator

neutrino majorana mass

$$\mathcal{L} \sim \frac{c}{\Lambda} LLHH \to c\frac{v^2}{\Lambda} \nu\nu \qquad \left.\begin{matrix} c \sim 1 \\ \Lambda \sim \Lambda_{GUT} \end{matrix}\right\} \Rightarrow m_\nu \sim 10^{-2}\,\text{eV}$$

Seesaw mechanism: large Λ $\Rightarrow$ naturally explains why m_ν is small!

several possible UV completions: type I, II, III, ...



Furthermore, all odd dimension operators violate Baryon (B) or Lepton (L) numbers.

B, L, charge of $U(1)_B$, $U(1)_L$ global symmetry

	B	L
q	$\frac{1}{3}$	0
$\bar{q}$	$-\frac{1}{3}$	0
l	0	1
$\bar{l}$	0	-1

$\not{B}$ $\not{L}$ effects are usually strongly constrained (e.g. proton decay).

Assuming B, L are conserved around the TeV scale

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \sum_i \frac{C_i^{(8)}}{\Lambda^2} O_i^{(8)} + \cdots$$

How many independent parameters do we have? (B & L conserved)

	1 generation		3 generations	
dim-6	76	1008.4884	2499	Manohar et al. 1312.2014
dim-8	895		36971	2005.00008 理论所组 2005.00059 Murphy

Hilbert series: Murayama et al. 1512.03433

Warsaw basis 1008.4884

- first to write down a complete d6 basis
- try to eliminate operators with more derivatives in favor of operators with more fields.

Buchmüller & Wyler almost did it in 1986.... (why no one completed it in 24 years!)

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

4+3+3

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde\varphi\, W^I_{\mu\nu}$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p \gamma^\mu d_r)$

8 × 3

briefly explain each type of operators ...

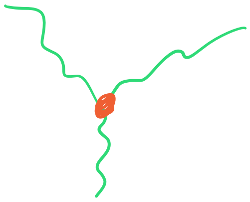
$(\bar LL)(\bar LL)$		$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p\gamma_\mu l_r)(\bar l_s\gamma^\mu l_t)$	Q_{ee}	$(\bar e_p\gamma_\mu e_r)(\bar e_s\gamma^\mu e_t)$	Q_{le}	$(\bar l_p\gamma_\mu l_r)(\bar e_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar q_s\gamma^\mu q_t)$	Q_{uu}	$(\bar u_p\gamma_\mu u_r)(\bar u_s\gamma^\mu u_t)$	Q_{lu}	$(\bar l_p\gamma_\mu l_r)(\bar u_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p\gamma_\mu \tau^I q_r)(\bar q_s\gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar d_p\gamma_\mu d_r)(\bar d_s\gamma^\mu d_t)$	Q_{ld}	$(\bar l_p\gamma_\mu l_r)(\bar d_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p\gamma_\mu l_r)(\bar q_s\gamma^\mu q_t)$	Q_{eu}	$(\bar e_p\gamma_\mu e_r)(\bar u_s\gamma^\mu u_t)$	Q_{qe}	$(\bar q_p\gamma_\mu q_r)(\bar e_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p\gamma_\mu \tau^I l_r)(\bar q_s\gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar e_p\gamma_\mu e_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar u_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p\gamma_\mu u_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar u_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p\gamma_\mu T^A u_r)(\bar d_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar d_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar d_s\gamma^\mu T^A d_t)$

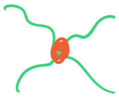
5+7+8

$(\bar LR)(\bar RL)$ and $(\bar LR)(\bar LR)$		B-violating	
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\varepsilon_{jk}(\bar q_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\varepsilon_{jk}(\bar q_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\varepsilon_{jk}(\bar q_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar l_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar q_s^k \sigma^{\mu\nu} u_t)$		

5

= 59 operators

1)  (transverse) anomalous triple gauge coupling aTGC

 & (qartic GC) aQGC

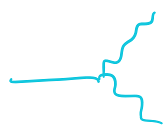
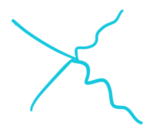
2) $\varphi \hookrightarrow H$ $\quad |H|^6 \to$  h^3 modifies h^3 & h^4 couplings

$(\partial |H|^2)^2$ modify $\partial_\mu h \, \partial^\mu h$, h wave function renormalization

shift Higgs couplings

3) $\psi^2 \varphi^3 \rightarrow$ modify Yukawa coupling (relation between m & y) $\cancel{y = \frac{v}{\sqrt{2}}}$

4) $|H|^2 V_{\mu\nu} V^{\mu\nu}$

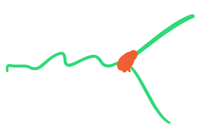
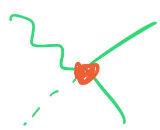
- $h V_{\mu\nu} V^{\mu\nu}$ 
- $h h V_{\mu\nu} V^{\mu\nu}$ 

different from $h Z^\mu Z_\mu \quad h W^\mu W_\mu$

5) $H \to V$ dipole 

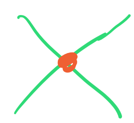
real magnetic

imaginary electric

6) $H \to V$  $\Longleftrightarrow$ 

modifies SM $Vf\bar{f}$ coupling $\qquad$ contact interaction

7) 4f interaction



1 generation:

59 operators $\begin{cases} 17 \text{ non hermitian} \Rightarrow \text{complex coefficient} \\ 42 \text{ hermitian} \Rightarrow \text{real coefficient} \end{cases}$

$42 + 17 \times 2 = 76$ parameters

3 generations: 2499 parameters!

(many of them are 4f operators)

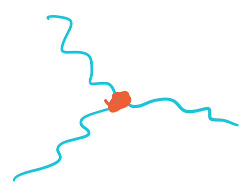
In other bases, we sometimes keep operators with more derivatives.

e.g. $\mathcal{O}_{2W} = -\frac{1}{2}(D^\mu W^a_{\mu\nu})^2 \quad \mathcal{O}_{2B} = -\frac{1}{2}(\partial^\mu B_{\mu\nu})^2$

useful in describing universal contributions to 4f interactions.

$\mathcal{O}_{HW} = ig(D^\mu H)^\dagger \sigma^a (D^\nu H) W^a_{\mu\nu}$

$\mathcal{O}_{HB} = ig'(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}$



useful for describing (longitudinal) anomalous triple gauge couplings (aTGCs)

(may skip)

RG running

1308.2627
1310.4838 Alonso, Jenkins, Manohar, Trott
1312.2014

running couplings $\Longleftrightarrow$ resum large logs

tree level calculation + running couplings give a reasonably accurate prediction.

suppose vev $\to 0$, $[g_{SM}] = 0$

let $C_i^{(6)} = \frac{c_i^{(6)}}{\Lambda^2}$ $\quad [C_i] = -2$

d6 RGE $\beta_{C_i} \equiv \mu \frac{d}{d\mu} C_i = \gamma_{ij} C_j$ ← still holds!

anomalous dimension matrix depends on g_{SM}

This is the only term allowed by dimensional analysis!

How about vev $\neq 0$?

C_i contribute to $\beta_{g_{SM}} = \mu \frac{d}{d\mu} g_{SM} = \cdots + \gamma_i v^2 C_i g_{SM} + \cdots$

$C_i^2 \quad C_i^{(8)}$ contribute to β_{C_i}

d6^2 d8 ← can ignore since they are higher order

(d8 RGEs are more complicated, has both d8 & d6^2 contributions)

Solve RGE. If expand to 1st loop order (not resumed)

$$C_i(\mu) = C_i(\mu_0) + \log\frac{\mu_1}{\mu_0}\,\gamma_{ij}\,C_j$$

note: important to resum!

generally solve numerically.

RGE implies:
- running
- mixing (off-diagonal terms in γ_{ij})

Matching & Running

UV

Λ new physics

close

μ matching scale

RG running

v electroweak scale (where we do measurements)

IR

mixing: Operators not generated at matching scale can be generated at a lower scale via running!

running effects can be significant if $v \ll \Lambda$!!

more conveniently calculated in the EFT.

See examples in Manohar / Skiba 's lecture notes,

Schwartz 31.3

RGE $\Leftrightarrow$ on-shell amplitude   see e.g. 1505.01844 Cheung & Shen

Phenomenology

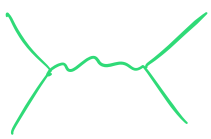
$$\mathcal{L}_{SMEFT} \longrightarrow \underset{\text{amplitude}}{\mathcal{M}} \longrightarrow \underset{\text{observable}}{\frac{d\sigma}{d\Omega}, \ \frac{dP}{d\Omega}}$$

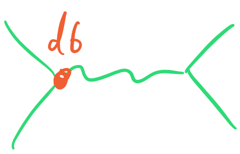
expand in terms of $\frac{1}{\Lambda}$

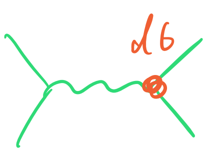
$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_6 + \mathcal{L}_8 + \cdots$$

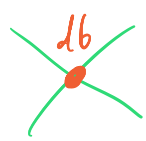
$\mathcal{M}$ $\quad q\bar{q} \to l^+ l^-$

SM

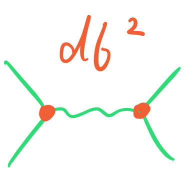


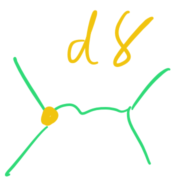
$+$ 

$+$ 

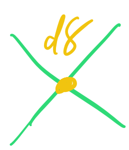
$+$ 

$+ \cdots \qquad \frac{1}{\Lambda^2}$

$+$ 

$+$ 

$+$ 

$+$ 

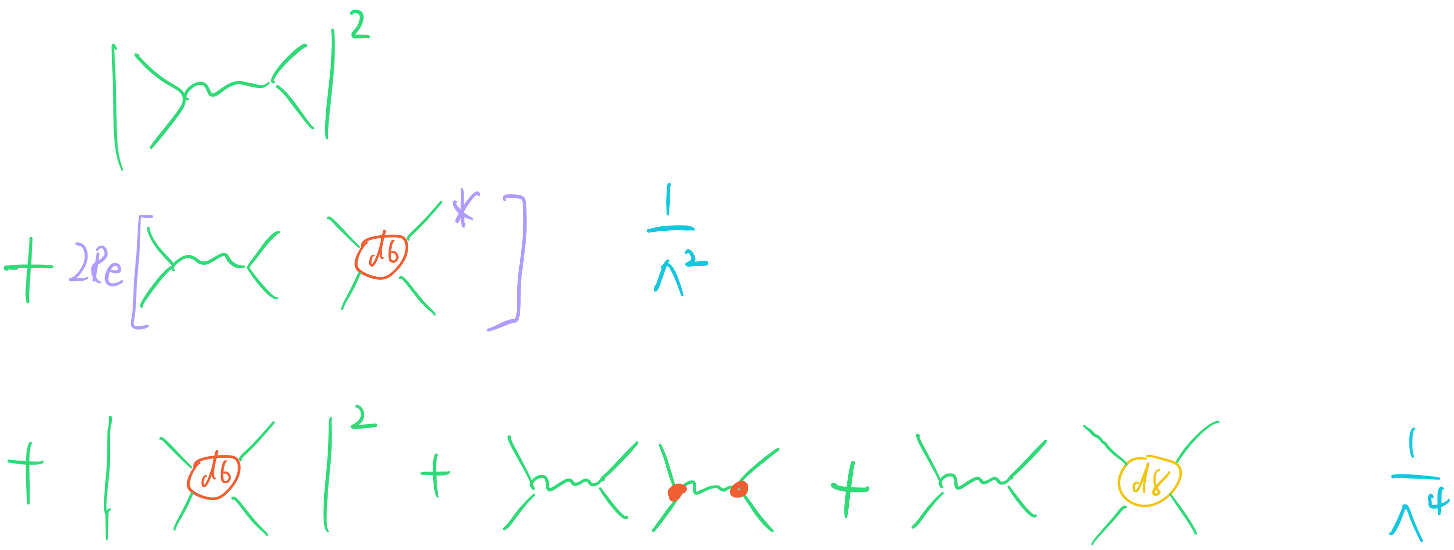
$+ \cdots \qquad \frac{1}{\Lambda^4}$

$+ \cdots$

($d6^2$ & $d8$ are formally indistinguishable)

Higher dimensional operators can contribute to m, Γ, which appears in dinominators $\frac{1}{p^2-m^2-im\Gamma}$. we don't consider it here (or just expand?)

$\sigma \sim |\mathcal{M}|^2$



$+ \; \cdots\cdots$

We can truncate σ at

collider energy

$\frac{1}{\Lambda^2}$ is a very good approximation if $E \ll \Lambda$, $v \ll \Lambda$

$(1+x)^2 \simeq 1+2x$ if x is very small!

$\frac{1}{\Lambda^4}$ strictly speaking, need to calculate d8

~~what if Λ is not that large, shall we keep $d6^2$?~~

Typically:

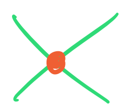
If measurement is very precise $\Rightarrow$ can constrain $\Lambda \gg E$ ✓

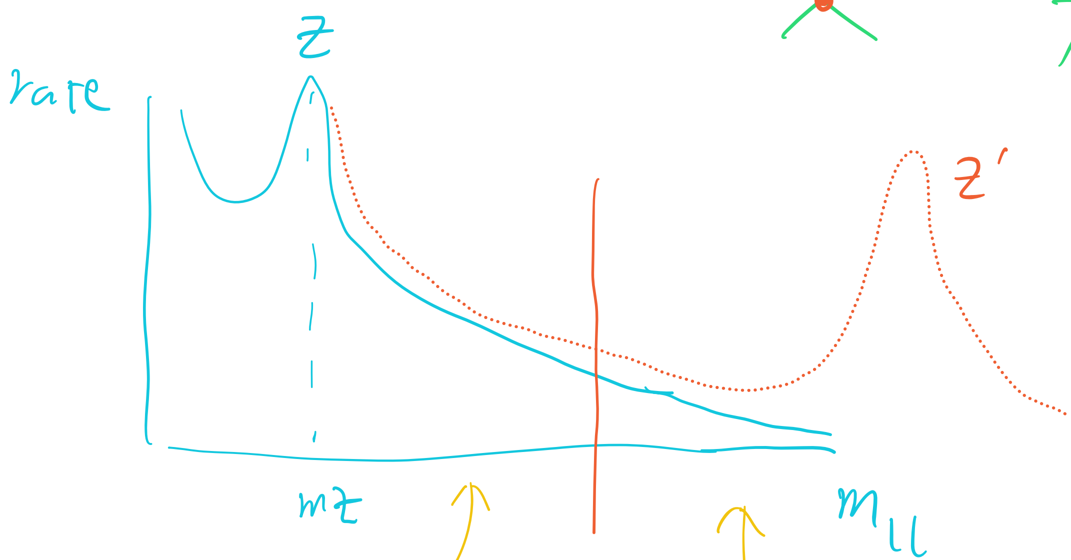
$\Rightarrow$ ok to just keep $\frac{1}{\Lambda^2}$ (ideal case!)

What if it's not the case?

typical LHC measurement

Drell-Yan $pp \to l^+ l^-$


$\sim \frac{E^2}{\Lambda^2}$



small E, large statistics

easier to constrain $\Lambda \gg E$

ok to truncate at $\frac{1}{\Lambda^2}$

cut here and throw away $\Rightarrow$?

(or do something else)

large E, small statistics

can't really tell if a Z' is near by.

EFT valid?

(Lepton colliders usually don't have this problem.)

Important exceptions of the $\frac{1}{\Lambda^2}$ power counting:

when SM contribution is absent or strongly suppressed

leading order: $d6^2 \sim \frac{1}{\Lambda^4}$

$SM \cdot d6,\ SM \cdot d8 \ll d6^2 \ \ldots$

- rare process

  flavor violation ...

  proton decay $\quad \Gamma_p \sim \frac{m_p^5}{\Lambda^4} \quad \Lambda \gtrsim 10^{15}\ \mathrm{GeV}$

- Fermi's theory: Weak interaction

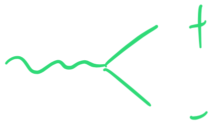
$\Gamma_\mu \sim \frac{m_\mu^5}{\Lambda_{EW}^4}$!

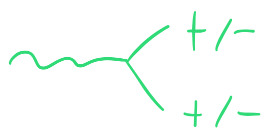
no interference with QED!

- The interference term with SM is suppressed.

  Different helicity amplitudes ...

  e.g. $Z_\mu \bar{f}_{L/R} \gamma^\mu f_{L/R}$ $\qquad$ $Z_{\mu\nu} \bar{f}_L \sigma^{\mu\nu} f_R$





no interference in the $m_f \to 0$ limit!

How SMEFT modifies SM parameters

The SM has a set of free parameters to be fixed by experiments.

$$g \quad g' \quad v \quad \lambda \quad y_t \quad \cdots\cdots$$

which are related to

$$m_Z, \quad m_W, \quad \underset{\text{muon decay}}{G_F}, \quad \underset{\text{electron magnetic moment}}{\overset{\text{fine structure constant}}{\alpha}}, \quad m_h, \quad m_t, \quad \cdots\cdots$$

These relations can be modified by higher dimensional operators!

3rd generation

$$\text{e.g.} \quad \mathcal{L} \supset y_t \bar{Q}_L^{(3)} t_R \tilde{H} + \frac{C_t}{\Lambda^2}|H|^2 \bar{Q}_L^{(3)} t_R \tilde{H} + h.c.$$

$$= \frac{y_t}{\sqrt{2}}(v+h)\bar{t}_L t_R + \frac{C_t}{\Lambda^2}\frac{(v+h)^3}{2\sqrt{2}}\bar{t}_L t_R + h.c.$$

SM $C_t = 0$: $\mathcal{L} = \underbrace{\frac{y_t v}{\sqrt{2}}}_{m_t} \bar{t}_L t_R + \underbrace{\frac{y_t}{\sqrt{2}}}_{g_{htt}} h \bar{t}_L t_R + h.c.$

with C_t : $\mathcal{L} = \underbrace{\left(\frac{y_t v}{\sqrt{2}} + \frac{C_t v^3}{2\sqrt{2}\Lambda^2}\right)}_{m_t} \bar{t}_L t_R + \underbrace{\left(\frac{y_t}{\sqrt{2}} + \frac{C_t 3v^2}{2\sqrt{2}\Lambda^2}\right)}_{g_{htt}} h \bar{t}_L t_R + h.c. + \ldots$ more h

Question: Does the measurement of m_t gives us a constraint on C_t?

If $C_t \neq 0$, is m_t not 173 GeV anymore?

No! Because a nonzero C_t only changes the "inferred value" of y_t.

We need 2 measurements to fix 2 parameters.

In other words, C_t changes the relation between m_t & g_{htt}

$$m_t = \frac{y_t v}{\sqrt{2}} + \frac{C_t v^3}{2\sqrt{2}\Lambda^2} \qquad \frac{y_t}{\sqrt{2}} = \frac{m_t}{v} - \frac{C_t v^2}{2\sqrt{2}\Lambda^2}$$

$$g_{htt} = \frac{m_t}{v} + \frac{C_t v^2}{\sqrt{2}\Lambda^2}$$

More generally, any operator of the form $|H|^2 O_{SM}$ can only be probed with the "Higgs particle"!

$g_{SM} O_{SM}$ vs. $g_{SM} O_{SM} + \frac{c}{\Lambda^2} |H|^2 O_{SM}$

$$= g_{SM} O_{SM} + \frac{c}{\Lambda^2} \frac{v^2}{2} O_{SM} + \text{terms with } h$$

$$= \left(g_{SM} + \frac{c v^2}{2 \Lambda^2} \right) O_{SM} + \text{terms with } h$$

redefine $\bar{g} = g_{SM} + \frac{c v^2}{2 \Lambda^2}$

$$= \bar{g}\, O_{SM} + \text{terms with } h$$

(can also be h in the loop)

Similarly, $O_6 \equiv (H^\dagger H)^3$ can only be probed by measuring the Higgs self coupling!

HW