Spin correlation in $\Lambda\bar{\Lambda}$ system

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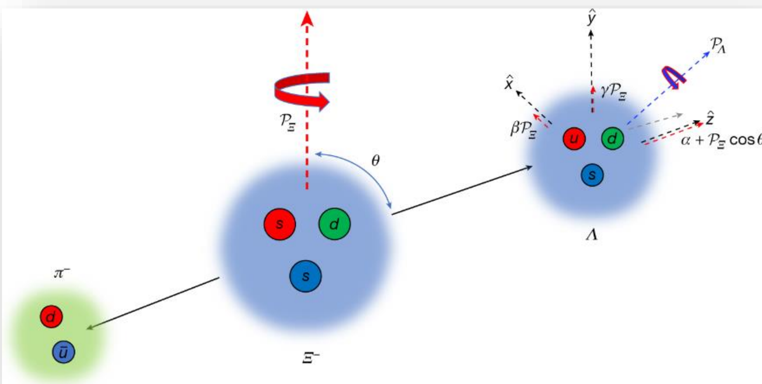
Outline

- **Spin correlation in $e^+e^- \to \gamma^*/\psi \to \Lambda\bar{\Lambda}$**
- **Quantum measurement description of Λ and $\bar{\Lambda}$ decays**
- **Quantum non-locality and entanglement in $\Lambda\bar{\Lambda}$ system in charmonium decays [few body exclusive process]**
- **Quark and hadron spin correlation in coalescence model in high energy HIC**
- **Two examples of spin correlations in HIC: (a) spin correlation in ϕ meson's spin alignment; (b) $\Lambda\Lambda$'s spin correlation as probe to vortical structure of sQGP**
- **Summary**

Particle spin as Qubit: Hyperon decay

Qubit: a two-level system. Spin-1/2 hyperons (Σ, Λ, Ξ) can be good Qubits in particle physics since their spin states can be measured through weak decays.



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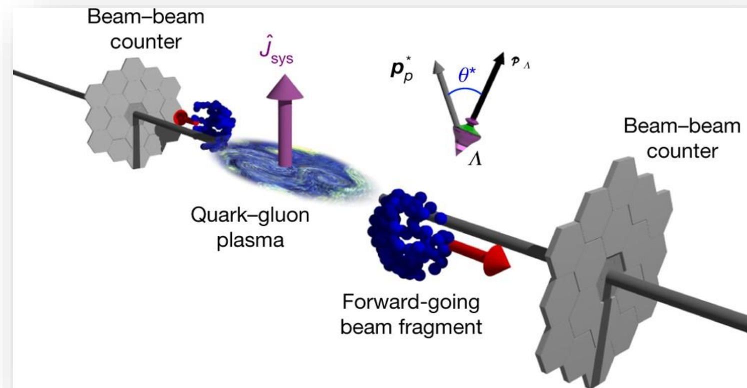

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Global Λ hyperon polarization in nuclear collisions

The STAR Collaboration

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Hyperons' weak decay (e.g. $\Lambda\to p\pi$, $\Xi\to\Lambda\pi$) has being a long history in HEP experiment. The angular distribution server as the polarimeter for itself.

weak decay law

$$\frac{dN}{d\cos\theta} \propto 1 + \alpha\mathcal{P}_B\cos\theta$$

Particle spin as Qubit: Hyperon decay

A typical weak decay process for a spin-1/2 hyperon (anti-hyperon) is $B \to B'M$ ($\bar{B} \to \bar{B}'\bar{M}$). In the rest frame of B, the decay amplitude is

$$
\begin{aligned}
\mathcal{A}_{B\to B'M} =& G_F m_M^2 \bar{u}_{B'}(s', \boldsymbol{p})(C_1 - C_2\gamma_5)u_B(s,0) \\
=& G_F m_M^2 \sqrt{2m_B(E_{B'}+m_{B'})}\chi_{s'}^\dagger \underline{(S + P\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}})}\chi_s
\end{aligned}
$$

S-wave + P-wave

Dirac spinors for particles

$$
u(s,\boldsymbol{p}) = \sqrt{E_p+m}\begin{pmatrix} \chi_s \\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E_p+m}\chi_s \end{pmatrix}
$$

$$
u(s,0) = \sqrt{2m}\begin{pmatrix} \chi_s \\ 0 \end{pmatrix}
$$

spin quantization direction

$$
\boldsymbol{s}_B = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)
$$

$$
\boxed{
\begin{aligned}
\chi_+ &= \begin{pmatrix} e^{-i\phi}\cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} \end{pmatrix} \\
\chi_- &= \begin{pmatrix} -e^{-i\phi}\sin\frac{\theta}{2} \\ \cos\frac{\theta}{2} \end{pmatrix}
\end{aligned}
}
$$

$$
(\boldsymbol{\sigma}\cdot\boldsymbol{s}_B)\chi_s = s\chi_s
$$

Pauli spinors

Hyperon decay as quantum measurement

Squared amplitude: sum over the daughter spin

$$\sum_{s'} |\mathcal{A}_{B\to B'M}|^2 \propto \chi_s^\dagger (S^* + P^* \boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}) \sum_{s'} \chi_{s'}\chi_{s'}^\dagger (S + P\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}) \chi_s$$

$$= \chi_s^\dagger (S^* + P^* \boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}) (S + P\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}) \chi_s$$

$$= \mathrm{Tr}\left[(S + P\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}) \rho_B (S^* + P^*\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}})\right]$$

$$\propto \mathrm{Tr}\left(M_{\boldsymbol{p}}\rho_B M_{\boldsymbol{p}}^\dagger\right)$$

Measurement operator

$$M_{\boldsymbol{p}} = \frac{S + P\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}}{\sqrt{4\pi\left(|S|^2 + |P|^2\right)}}$$

We assume that the hyperon is in the spin up state (100% polarization) along the direction $\vec{s}_B$, then the spin density operator

$$\rho_B = \chi_+\chi_+^\dagger$$

$$= \begin{pmatrix} e^{-i\phi}\cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} \end{pmatrix} \left(e^{i\phi}\cos\frac{\theta}{2}, \sin\frac{\theta}{2}\right)$$

$$= \frac{1}{2}(1 + \boldsymbol{\sigma}\cdot\boldsymbol{s}_B)$$

$$\Longrightarrow$$

$$P(\boldsymbol{p}) = \frac{1}{\Gamma}\frac{d\Gamma}{d\Omega_{\boldsymbol{p}}} = \mathrm{Tr}\left(M_{\boldsymbol{p}}\rho_B M_{\boldsymbol{p}}^\dagger\right)$$

$$= C\,\mathrm{Tr}\left[(S^* + P^*\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}})(S + P\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}})\rho_B\right]$$

$$= \frac{1}{8\pi}\mathrm{Tr}\left[(1 + \alpha\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}})(1 + \boldsymbol{\sigma}\cdot\boldsymbol{s}_B)\right]$$

$$= \frac{1}{4\pi}\left(1 + \underline{\alpha_B}\,\underline{\boldsymbol{s}_B\cdot\hat{\boldsymbol{p}}}\right)$$

$\cos\theta$

Decay constant

$$\alpha_B = \frac{2\mathrm{Re}(S^*P)}{|S|^2 + |P|^2}$$

Quantum measurement

The measurement processes in quantum physics are subject to a collection of measurement operators $\{M_m\}$. When we use the density operator ρ to describe a system being measured, the probability of obtaining a certain outcome m is given by

$$P(m) = Tr\left(M_m \rho M_m^\dagger\right) > 0, \qquad \sum_m P(m) = 1$$

where $\{P(m)\}$ are probability distributions for all possible outcomes.

These two conditions lead to constraints on $\{M_m\}$, the positive semi-deniteness and completeness conditions:

$$M_m^\dagger M_m \geq 0, \qquad \sum_m M_m^\dagger M_m = 1$$

Based on the measurement postulate in quantum mechanics, the initial state ρ instantaneously transforms after the measurement to the state ρ_m

$$\rho \to \rho_m = \frac{M_m \rho M_m^\dagger}{P(m)}, \qquad Tr(\rho_m) = 1$$

The resulting quantum states are described by $\{P(m), \rho_m\}$.

Quantum measurement

The post-measurement state is then

$$\sum_m P(m)\rho_m = \sum_m M_m \rho M_m^\dagger$$

which can be taken as a quantum evolution generated by the measurement. This process is often characterized as a quantum channel

$$\rho \to \mathcal{E}(\rho) = \sum_m M_m \rho M_m^\dagger$$

and the set of $\{M_m\}$ is called Kraus operators.

Hyperon decay as quantum measurement

$$P(\boldsymbol{p}) = \mathrm{Tr}\left(M_{\boldsymbol{p}}\rho_B M_{\boldsymbol{p}}^\dagger\right) = \frac{1}{4\pi}\left(1+\alpha_B \boldsymbol{s}_B\cdot\hat{\boldsymbol{p}}\right), \quad M_{\boldsymbol{p}} = \frac{S+P\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}}{\sqrt{4\pi\left(|S|^2+|P|^2\right)}}$$

$$M_{\boldsymbol{p}}^\dagger M_{\boldsymbol{p}} = \frac{1}{4\pi}\left(1+\alpha_B\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}\right), \quad \int d\Omega_{\boldsymbol{p}} M_{\boldsymbol{p}}^\dagger M_{\boldsymbol{p}} = 1$$

Hyperon decay as quantum measurement

According to the quantum measurement postulate, the spin density operator of the daughter baryon can be obtained via the post-measurement state

$$\rho_{B'}(\hat{\boldsymbol{p}}) = \frac{M_{\boldsymbol{p}}\rho_B M_{\boldsymbol{p}}^{\dagger}}{\mathrm{Tr}\left(M_{\boldsymbol{p}}\rho_B M_{\boldsymbol{p}}^{\dagger}\right)} = \frac{1}{2}\left(1+\boldsymbol{\sigma}\cdot\boldsymbol{s}_{B'}\right)$$

$$\boldsymbol{s}_{B'} = \frac{\alpha_B\hat{\boldsymbol{p}} - \beta_B(\hat{\boldsymbol{p}}\times\boldsymbol{s}_B) + \gamma_B\boldsymbol{s}_B + (1-\gamma_B)(\hat{\boldsymbol{p}}\cdot\boldsymbol{s}_B)\hat{\boldsymbol{p}}}{1+\alpha\boldsymbol{s}_B\cdot\hat{\boldsymbol{p}}}$$

$$\alpha_B = \frac{2\mathrm{Re}(S^*P)}{|S|^2+|P|^2}$$

$$\beta_B = \frac{2\mathrm{Im}(S^*P)}{|S|^2+|P|^2}$$

$$\gamma_B = \frac{|S|^2-|P|^2}{|S|^2+|P|^2}$$

where $\boldsymbol{s}_{B'}$ is the polarization vector of the daughter baryon in the mother baryon's rest frame as a function of $\hat{\boldsymbol{p}}$, $\boldsymbol{\alpha_B}$, $\boldsymbol{\beta_B}$ and $\boldsymbol{\gamma_B}$ are three real parameters in B's weak decay satisfying $\boldsymbol{\alpha_B^2+\beta_B^2+\gamma_B^2=1}$. [T. D. Lee and C. N. Yang, Phys. Rev. 108, 1645 (1957)].

In the hyperon decay process, the daughter baryon may fly in any direction $\hat{\boldsymbol{p}}$ associated with the probability $\boldsymbol{P(\hat{p})}$, which is just the angular distribution of the daughter baryon that can be detected in particle physics experiments.

Hyperon decay as quantum measurement

With the daughter's spin density operator $\rho_{B'}(\hat{\boldsymbol{p}})$, we have an ensemble $\{P(\hat{\boldsymbol{p}}), \rho_{B'}(\hat{\boldsymbol{p}})\}$. According to the generalized measurement postulate, this post-measurement ensemble can be interpreted as a quantum evolution in the channel $\mathcal{E}$ as

$$\begin{aligned}\mathcal{E}(\rho_B) = \int d\Omega_{\boldsymbol{p}} M_{\boldsymbol{p}} \rho_B M_{\boldsymbol{p}}^\dagger = \frac{1}{8\pi} \int d\Omega \left[1 + \alpha_B (\hat{\boldsymbol{p}} \cdot \boldsymbol{s}_B)\right] \\ + \frac{1}{8\pi} \boldsymbol{\sigma} \cdot \int d\Omega \left[\alpha_B \hat{\boldsymbol{p}} - \beta_B (\hat{\boldsymbol{p}} \times \boldsymbol{s}_B) + \gamma_B \boldsymbol{s}_B + (1 - \gamma_B)(\hat{\boldsymbol{p}} \cdot \boldsymbol{s}_B) \hat{\boldsymbol{p}}\right] \\ = \frac{1}{2} \left[1 + \frac{1}{3}(1 + 2\gamma_B) \boldsymbol{\sigma} \cdot \boldsymbol{s}_B\right]\end{aligned}$$

The term $(1 + 2\gamma_B)/3$ just represents the average polarization of the daughter baryon in the rest frame of the mother hyperon.

A chain of decay $B \to B_1 M_1 \to B_2 M_2 M_1$ is called the concatenate quantum measurement with joint probability

$$P(\boldsymbol{p}_1, \boldsymbol{p}_2) = \frac{1}{\Gamma} \frac{d^2\Gamma}{d\Omega_1 d\Omega_2} = \mathrm{Tr}\left[M_{\boldsymbol{p}2}^{1\to 2} M_{\boldsymbol{p}1}^{B\to 1} \rho_B M_{\boldsymbol{p}1}^{\dagger B\to 1} M_{\boldsymbol{p}2}^{\dagger 1\to 2}\right]$$

Joint decay of $B\bar{B}$ and spin-spin correlation

The spin state of two spin-1/2 particles such as $B\bar{B}$ is described by spin density operator

$$\rho_{B\bar{B}} = \frac{1}{4}\left(1 + \boldsymbol{s}_B \cdot \boldsymbol{\sigma} \otimes 1 + 1 \otimes \boldsymbol{s}_{\bar{B}} \cdot \boldsymbol{\sigma} + \sum_{ij} C_{ij}\sigma_i \otimes \sigma_j\right)$$

$$\boldsymbol{s}_B = \langle \boldsymbol{\sigma} \otimes 1 \rangle$$

$$\boldsymbol{s}_{\bar{B}} = \langle 1 \otimes \boldsymbol{\sigma} \rangle$$

$$\underline{C_{ij}} = \langle \sigma_i \otimes \sigma_j \rangle$$

symmetric and real matrix

There are 15 real parameters: $\boldsymbol{s}_B$ (3), $\boldsymbol{s}_{\bar{B}}$ (3), $\boldsymbol{C}_{ij}$ (9).

The joint Hilbert space associated with spin states of $B\bar{B}$ is denoted as $\mathscr{H}_B \otimes \mathscr{H}_{\bar{B}}$ respectively. The one particle density operator can be obtained by taking the partial trace

$$\rho_B = Tr_{\bar{B}}(\rho_{B\bar{B}}) = \frac{1}{2}(1 + \boldsymbol{s}_B \cdot \boldsymbol{\sigma})$$

$$\rho_{\bar{B}} = Tr_B(\rho_{B\bar{B}}) = \frac{1}{2}(1 + \boldsymbol{s}_{\bar{B}} \cdot \boldsymbol{\sigma})$$

According to the quantum measurement postulate, a joint decay process can be regarded as parallel quantum measurement which gives the joint probability

$$P(\boldsymbol{p}, \bar{\boldsymbol{p}}) = Tr\left[(M_{\boldsymbol{p}} \otimes M_{\bar{\boldsymbol{p}}})\,\rho_{B\bar{B}}\left(M_{\boldsymbol{p}}^\dagger \otimes M_{\bar{\boldsymbol{p}}}^\dagger\right)\right]$$

Spin-spin correlation in $e^+e^- \to J/\psi \to Y\bar{Y}$

The spin density operator for $Y\bar{Y}$ can be rewritten as

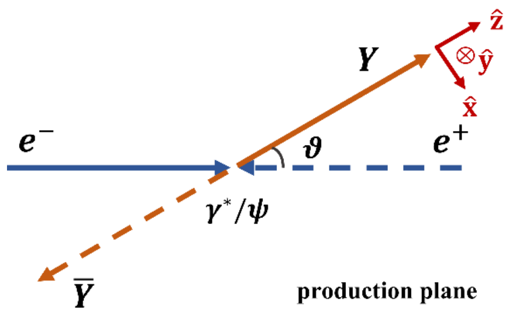
$$\rho_{Y\bar{Y}} = \frac{1}{4}\Theta_{\mu\nu}\sigma_\mu \otimes \sigma_\nu$$

$\mu, \nu = (0, i) = (0, 1, 2, 3)$,
do not distinguish upper or lower indices

$$\sigma_\mu = (1, \boldsymbol{\sigma}),\ \Theta_{i0} = s_{Yi},\ \Theta_{0j} = s_{\bar{Y}j},\ \Theta_{ij} = C_{ij}$$

We choose the helicity rest frame as

$$\hat{\boldsymbol{y}} = -\hat{\boldsymbol{p}}_e \times \hat{\boldsymbol{p}}_Y,\ \ \hat{\boldsymbol{z}} = \hat{\boldsymbol{p}}_Y,\ \ \hat{\boldsymbol{x}} = \hat{\boldsymbol{y}} \times \hat{\boldsymbol{z}}$$



In the rest frames of Y and $\bar{Y}$ we have

$$\Theta_{\mu\nu} = \frac{1}{1+\alpha_\psi\cos^2\theta}\left[\begin{array}{c|ccc} 1+\alpha_\psi\cos^2\theta & 0 & \beta_\psi\sin\theta\cos\theta & 0 \\ \hline 0 & \sin^2\theta & 0 & \gamma_\psi\sin\theta\cos\theta \\ \beta_\psi\sin\theta\cos\theta & 0 & -\alpha_\psi\sin^2\theta & 0 \\ 0 & \gamma_\psi\sin\theta\cos\theta & 0 & \alpha_\psi+\cos^2\theta \end{array}\right]$$

$$\cos\theta = \hat{\mathbf{p}}_e \cdot \hat{\mathbf{p}}_Y$$

$$\alpha_\psi = \frac{s - 4M^2|G_E/G_M|^2}{s + 4M^2|G_E/G_M|^2} \in [-1, 1]$$

$$\Delta\Phi = \arg\{G_E/G_M\} \in (-\pi, \pi]$$

$$\beta_\psi = \sqrt{1-\alpha_\psi^2}\sin(\Delta\Phi), \quad \gamma_\psi = \sqrt{1-\alpha_\psi^2}\cos(\Delta\Phi)$$

Local unitary equivalence and X states

Before our investigation of Bell nonlocality, it is convenient to transform the two-qubit state described by $\Theta_{\mu\nu}$ to the X state by swapping the y and z axes and then diagonalizing C_{ij} in Y and $\bar{Y}$'s rest frame

$$\rho^X_{Y\bar{Y}} = (U_Y \otimes U_{\bar{Y}})\, \rho_{Y\bar{Y}}\, (U_Y \otimes U_{\bar{Y}})^\dagger = \frac{1}{4}\left(1 + a\sigma_z \otimes 1 + 1 \otimes a\sigma_z + \sum_i t_i \sigma_i \otimes \sigma_i\right)$$

Corresponding to

$$\Theta^X_{\mu\nu} = \left[\begin{array}{c|ccc} 1 & 0 & 0 & a \\ \hline 0 & t_1 & 0 & 0 \\ 0 & 0 & t_2 & 0 \\ a & 0 & 0 & t_3 \end{array}\right]$$

$$a = \frac{\beta_\psi \sin\theta \cos\theta}{1 + \alpha_\psi \cos^2\theta}$$

$$t_{1,2} = \frac{1 + \alpha_\psi \pm \sqrt{(1 + \alpha_\psi \cos 2\theta)^2 - \beta_\psi^2 \sin^2 2\theta}}{2(1 + \alpha_\psi \cos^2\theta)}$$

$$t_3 = \frac{-\alpha_\psi \sin^2\theta}{1 + \alpha_\psi \cos^2\theta}$$

The states described by $\rho_{Y\bar{Y}}$ and $\rho^X_{Y\bar{Y}}$ are said to be local unitary equivalent in the sense that they have same quantum correlation properties such as Bell nonlocality and entanglement.

Brief introduction to Bell-CHSH inequality

The Bell inequality is a fundamental concept in quantum mechanics that addresses the nature of correlations predicted by quantum theory compared to those predicted by classical physics (or more specifically, local realism). [John Bell (1964)].

Key Ideas:

Local Realism: This principle assumes two things: (a) Locality: Information cannot travel faster than the speed of light, meaning that an event at one location cannot instantaneously affect another location that is far away. (b) Realism: Physical properties exist with definite values independent of observation.

Quantum Entanglement: In quantum mechanics, entangled particles are in a state where the properties of one particle are correlated with the properties of another, no matter how far apart they are. Measurement on one particle seems to instantaneously affect the state of the other, violating the idea of locality.

Brief introduction to Bell-CHSH inequality

Bell's Theorem:

If the world behaves according to local realism, the correlations between measurements made on separated particles must satisfy the Bell inequality. However, quantum mechanics predicts correlations that can violate this inequality.

Bell-CHSH inequality:

The mathematical expression for Bell's inequality can take various forms depending on the specific scenario, but the most common form is the CHSH (Clauser-Horne-Shimony-Holt, 1969) inequality, which is used in experiments involving measurements of entangled particles.

Consider two particles shared between two distant observers, Alice and Bob. Each observer can perform one of two possible measurements on their particle.

- **Alice measures:** A_1 **or** A_2, $A_{1,2} = \pm 1$
- **Bob measures:** B_1 **or** B_2, $B_{1,2} = \pm 1$
- $E(A_i, B_j)$: **expectation value of the product of the outcomes that Alice measures** A_i **and Bob measures** B_j

Brief introduction to Bell-CHSH inequality

Bell-CHSH inequality:

$$|E(A_1, B_1) + E(A_1, B_2) + E(A_2, B_1) - E(A_2, B_2)| \leq 2$$

Improved Bell-CHSH inequality accounting for detector inefficiencies in experiments (Clauser and Horne, 1974; Fine, 1982)

$$P(A_1, B_1) + P(A_1, B_2) + P(A_2, B_1) - P(A_2, B_2) - P(A_1) - P(B_1) \leq 0$$

where we assume $A_i, B_j \in [0, 1]$, $P(A_i, B_j)$ denotes the probability of the joint outcome of Alice and Bob.

Let us consider an entangled state

$$|\psi(\theta)\rangle = \cos\theta\,|00\rangle + \sin\theta\,|11\rangle \qquad |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix},\ |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

We define two operators $\hat{A}$ and $\hat{B}$ for Alice and Bob for measurement of spins along one specific direction

$$\widehat{A} = \frac{1}{2}(1 + \underbrace{\boldsymbol{a}}_{\text{spin direction}} \cdot \boldsymbol{\sigma}), \quad \widehat{B} = \frac{1}{2}(1 + \underbrace{\boldsymbol{b}}_{\text{spin direction}} \cdot \boldsymbol{\sigma})$$

Bell-CHSH inequality

Calculate the expectation value (probability) for the measurement at spin directions $\vec{a}$ and $\vec{b}$

$$P(A) = \langle\psi(\theta)|\, \widehat{A} \otimes I_B \,|\psi(\theta)\rangle = \frac{1}{2}\left[1 + a_z \cos(2\theta)\right]$$

$$P(B) = \langle\psi(\theta)|\, I_A \otimes \widehat{B} \,|\psi(\theta)\rangle = \frac{1}{2}\left[1 + b_z \cos(2\theta)\right]$$

$$\begin{aligned} P(A,B) &= \langle\psi(\theta)|\, \widehat{A} \otimes \widehat{B} \,|\psi(\theta)\rangle \\ &= \frac{1}{4} + \frac{1}{4}(a_z + b_z)\cos(2\theta) + \frac{1}{4}\left[a_z b_z + (a_x b_x - a_y b_y)\sin(2\theta)\right] \end{aligned}$$

$\theta = \pi/4$
maximally
entangled

$$P(A) = P(B) = \frac{1}{2}$$

$$P(A,B) = \frac{1}{4} + \frac{1}{4}\left[a_z b_z + a_x b_x - a_y b_y\right]$$

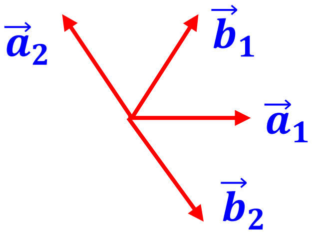
Let us do the measurements at spin directions $\vec{a}_1, \vec{a}_2, \vec{b}_1, \vec{b}_2$ and check CHSH inequality $Q_{CHSH} \leq 0$

$$\begin{aligned} Q_{CHSH} =& P(A_1, B_1) + P(A_1, B_2) + P(A_2, B_1) - P(A_2, B_2) - P(A_1) - P(B_1) \\ =& -\frac{1}{2} + \frac{1}{4}\left[a_{1z}(b_{1z} + b_{2z}) + a_{2z}(b_{1z} - b_{2z})\right] \\ &+ \frac{1}{4}\sin(2\theta)\left[a_{1x}(b_{1x} + b_{2x}) + a_{2x}(b_{1x} - b_{2x}) - a_{1y}(b_{1y} + b_{2y}) - a_{2y}(b_{1y} - b_{2y})\right] \end{aligned}$$

Violation of Bell-CHSH inequality

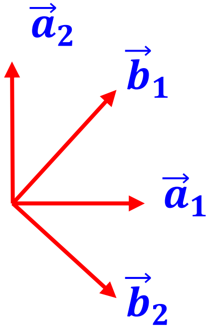
We look at the entangled state with $\theta = \pi/4$. We assume that all spin directions are in the xz-plane with the angles $\phi\left(\vec{a}_1, \vec{a}_2, \vec{b}_1, \vec{b}_2\right) = (0, \frac{2\pi}{3}, \frac{\pi}{3}, -\frac{\pi}{3})$, we obtain

$$
\begin{aligned}
Q_{CHSH} &= -\frac{1}{2} + \frac{1}{4}(\boldsymbol{a}_1 \cdot \boldsymbol{b}_1 + \boldsymbol{a}_1 \cdot \boldsymbol{b}_2 + \boldsymbol{a}_2 \cdot \boldsymbol{b}_1 - \boldsymbol{a}_2 \cdot \boldsymbol{b}_2) \\
&= -\frac{1}{2} + \frac{1}{4}\left(\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + 1\right) = \frac{1}{8} > 0
\end{aligned}
$$



If we choose $\phi\left(\vec{a}_1, \vec{a}_2, \vec{b}_1, \vec{b}_2\right) = (0, \frac{\pi}{2}, \frac{\pi}{4}, -\frac{\pi}{4})$, we obtain the maximum violation of the CHSH inequality (Tsirelson's bound)

$$
\begin{aligned}
Q_{CHSH}^{max} &= -\frac{1}{2} + \frac{1}{4}(\boldsymbol{a}_1 \cdot \boldsymbol{b}_1 + \boldsymbol{a}_1 \cdot \boldsymbol{b}_2 + \boldsymbol{a}_2 \cdot \boldsymbol{b}_1 - \boldsymbol{a}_2 \cdot \boldsymbol{b}_2) \\
&= -\frac{1}{2} + \frac{1}{4}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) = \frac{1}{2}\left(\sqrt{2} - 1\right) > 0
\end{aligned}
$$



Violation of Bell-CHSH inequality

Let us consider another entangled state (the triplet for $\boldsymbol{\theta = \pi/4}$ and singlet for $\boldsymbol{\theta = 7\pi/4}$)

$$|\psi(\theta)\rangle = \cos\theta\,|01\rangle + \sin\theta\,|10\rangle = \begin{cases} \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right), & \theta = \frac{\pi}{4} \\ \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right), & \theta = \frac{7\pi}{4} \end{cases}$$

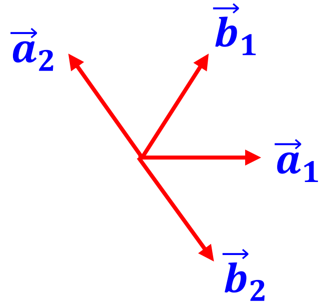
Let us do the measurements at spin directions $\vec{\boldsymbol{a}}_1, \vec{\boldsymbol{a}}_2, \vec{\boldsymbol{b}}_1, \vec{\boldsymbol{b}}_2$ and check CHSH inequality $\boldsymbol{Q_{CHSH} \leq 0}$

$$\begin{aligned} Q_{CHSH} =& P(A_1, B_1) + P(A_1, B_2) + P(A_2, B_1) - P(A_2, B_2) - P(A_1) - P(B_1) \\ =& -\frac{1}{2} - \frac{1}{4}\left[a_{1z}(b_{1z} + b_{2z}) + a_{2z}(b_{1z} - b_{2z})\right] \\ &+ \frac{1}{4}\sin(2\theta)\left[a_{1x}(b_{1x} + b_{2x}) + a_{2x}(b_{1x} - b_{2x}) + a_{1y}(b_{1y} + b_{2y}) + a_{2y}(b_{1y} - b_{2y})\right] \end{aligned}$$

Violation of Bell-CHSH inequality

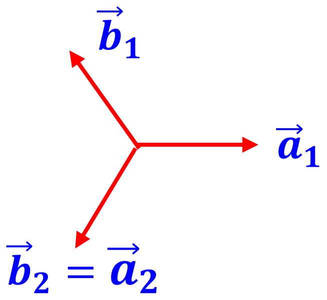
The triplet with $\boldsymbol{\theta = \pi/4}$. We assume that all spin directions are in the xy-plane with the angles $\boldsymbol{\phi\left(\vec{a}_1, \vec{a}_2, \vec{b}_1, \vec{b}_2\right) = (0, \frac{2\pi}{3}, \frac{\pi}{3}, -\frac{\pi}{3})}$, we can check that the CHSH inequality is violated

$$
\begin{aligned}
Q_{CHSH} &= -\frac{1}{2} + \frac{1}{4}\left(\boldsymbol{a}_1 \cdot \boldsymbol{b}_1 + \boldsymbol{a}_1 \cdot \boldsymbol{b}_2 + \boldsymbol{a}_2 \cdot \boldsymbol{b}_1 - \boldsymbol{a}_2 \cdot \boldsymbol{b}_2\right) \\
&= -\frac{1}{2} + \frac{1}{4}\left(\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + 1\right) = \frac{1}{8} > 0
\end{aligned}
$$



The singlet with $\boldsymbol{\theta = 7\pi/4}$. We assume that all spin directions are in the xy-plane with the angles $\boldsymbol{\phi\left(\vec{a}_1, \vec{a}_2, \vec{b}_1, \vec{b}_2\right) = (0, -\frac{2\pi}{3}, \frac{2\pi}{3}, -\frac{2\pi}{3})}$, we can check that the CHSH inequality is violated

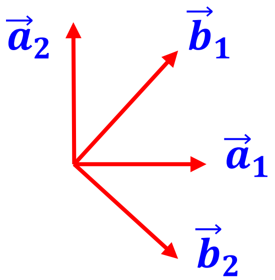
$$
\begin{aligned}
Q_{CHSH} &= -\frac{1}{2} - \frac{1}{4}\left(\boldsymbol{a}_1 \cdot \boldsymbol{b}_1 + \boldsymbol{a}_1 \cdot \boldsymbol{b}_2 + \boldsymbol{a}_2 \cdot \boldsymbol{b}_1 - \boldsymbol{a}_2 \cdot \boldsymbol{b}_2\right) \\
&= -\frac{1}{2} + \frac{1}{4}\left(\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + 1\right) = \frac{1}{8} > 0
\end{aligned}
$$



Maximum violation of Bell-CHSH inequality (Tsirelson's bound)

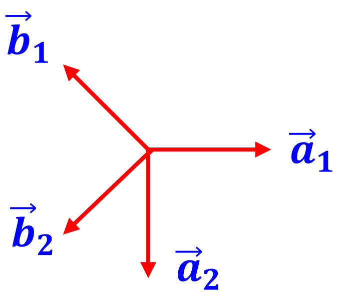
The triplet with $\theta = \pi/4$. **We assume that all spin directions are in the xy-plane with the angles $\phi\left(\vec{a}_1, \vec{a}_2, \vec{b}_1, \vec{b}_2\right) = (0, \frac{\pi}{2}, \frac{\pi}{4}, -\frac{\pi}{4})$, we can check that the CHSH inequality is violated**

$$
\begin{aligned}
Q_{CHSH}^{max} &= -\frac{1}{2} + \frac{1}{4}\left(\boldsymbol{a}_1 \cdot \boldsymbol{b}_1 + \boldsymbol{a}_1 \cdot \boldsymbol{b}_2 + \boldsymbol{a}_2 \cdot \boldsymbol{b}_1 - \boldsymbol{a}_2 \cdot \boldsymbol{b}_2\right) \\
&= -\frac{1}{2} + \frac{1}{4}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) = \frac{1}{2}\left(\sqrt{2} - 1\right) > 0
\end{aligned}
$$



The singlet with $\theta = 7\pi/4$. **We assume that all spin directions are in the xy-plane with the angles $\phi\left(\vec{a}_1, \vec{a}_2, \vec{b}_1, \vec{b}_2\right) = (0, -\frac{\pi}{2}, \frac{3\pi}{4}, -\frac{3\pi}{4})$, we can check that the CHSH inequality is violated**

$$
\begin{aligned}
Q_{CHSH}^{max} &= -\frac{1}{2} - \frac{1}{4}\left(\boldsymbol{a}_1 \cdot \boldsymbol{b}_1 + \boldsymbol{a}_1 \cdot \boldsymbol{b}_2 + \boldsymbol{a}_2 \cdot \boldsymbol{b}_1 - \boldsymbol{a}_2 \cdot \boldsymbol{b}_2\right) \\
&= -\frac{1}{2} + \frac{1}{4}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) = \frac{1}{2}\left(\sqrt{2} - 1\right) > 0
\end{aligned}
$$



Generalized Bell-CHSH inequalities

Generalized Bell-CH inequalities for 3 and 4 measurements

$$
\begin{aligned}
I_3 =& P(A_1,B_1)+P(A_1,B_2)+P(A_1,B_3)+P(A_2,B_1)+P(A_2,B_2) \\
& -P(A_2,B_3)+P(A_3,B_1)-P(A_3,B_2)-P(A_1)-2P(B_1)-P(B_2) \\
& \leq 0 \text{ (for local realistic theory)} \\
I_4 =& P(A_1,B_1)+P(A_1,B_2)+P(A_1,B_3)+P(A_1,B_4)+P(A_2,B_1)+P(A_2,B_2) \\
& +P(A_2,B_3)-P(A_2,B_4)+P(A_3,B_1)+P(A_3,B_2)-P(A_3,B_3) \\
& +P(A_4,B_1)-P(A_4,B_2)-P(A_1)-3P(B_1)-2P(B_2)-P(B_3) \\
& \leq 0 \text{ (for local realistic theory)}
\end{aligned}
$$

For any local realistic theory, these inequalities must hold, but quantum theory can violate them.

Froissart, Nuov Cim B 64, 241 (1981)
Garg and Mermin, Phys. Rev. Lett. 49, 1220 (1982)
Collins and Gisin, J. Phys. A37, 1775 (2004)

Two advantages of Bell-CH inequalities:

- Easily tested in experiments
- Good mathematical structures

New Method of Constructing Bell-CHSH inequalities

New method I: Rearrangement inequalities

C. Qian, Y.-G. Yang, QW and C.-F. Qiao, Phys. Rev. A 103, 062203 (2021)

$$I_2 = x_1(y_1 + y_2) + x_2(y_1 - y_2) - x_1 Y - y_1 X$$

$$I_2^{(0)} = -(x_1 y_1 + x_2 y_2) + x_+ y_- + x_- y_+$$

$$I_2 \leq I_2^{(0)} \leq 0$$

I_2 -- Alice: 2 measurements; Bob: 2 measurements

$$0 \leq x_- \leq x_1, x_2, \cdots, x_m \leq x_+ \leq X$$

$$0 \leq y_- \leq y_1, y_2, \cdots, y_m \leq y_+ \leq Y$$

$$x_- = min(x_1, x_2, \cdots, x_m)$$

$$x_+ = max(x_1, x_2, \cdots, x_m)$$

$$y_- = min(y_1, y_2, \cdots, y_m)$$

$$y_+ = max(y_1, y_2, \cdots, y_m)$$

$$I_3 = x_1(y_2 + y_3) + x_2(y_1 + y_3) + x_3(y_1 + y_2) - x_2 y_2 - x_3 y_3 - (x_1 + x_2)Y - (y_1 + y_2)X$$

$$I_3^{(0)} = -(x_1 y_1 + x_2 y_2 + x_3 y_3) + x_+ y_- + x_- y_+ + (x_1 + x_2 + x_3 - x_+ - x_-)(y_1 + y_2 + y_3 - y_+ - y_-)$$

$$I_3 \leq I_3^{(0)} \leq 0$$

I_3 -- Alice: 3 measurements; Bob: 3 measurements

New Method of Constructing Bell-CHSH inequalities

New method II: Linear inequalities

C. Qian, Y.-G. Yang, QW and C.-F. Qiao, Phys. Rev. A 103, 062203 (2021)

Graphical construction of Bell-CHSH inequalities

$$\begin{array}{c|cc} & -1 & 0 \\ \hline -1 & 1 & 1 \\ 0 & 1 & -1 \end{array} \leqslant \begin{array}{c|c|cc} \theta & & -1 & 0 \\ \hline 1 & 0 & 1 & 0 \\ -1 & -1 & 1 & 0 \end{array} + \begin{array}{c|c|cc} \theta & & -1 & 0 \\ \hline -1 & -1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{array}$$

$$\begin{array}{c|ccc} & -1 & 0 & 0 \\ \hline -2 & 1 & 1 & 1 \\ -1 & 1 & 1 & -1 \\ 0 & 1 & -1 & 0 \end{array} \leqslant \begin{array}{c|c|ccc} \theta & & -1 & 0 & 0 \\ \hline 1 & -1 & 1 & 1 & 0 \\ -1 & -2 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array} + \begin{array}{c|c|ccc} \theta & & -1 & 0 & 0 \\ \hline -1 & -2 & 1 & 1 & 0 \\ 1 & -1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array}$$

$$\begin{array}{c|ccc} \theta & C_{y_1} & \cdots & C_{y_n} \\ \hline \end{array} \Downarrow \theta\left(\textstyle\sum_{j=1}^{n} C_{y_j} y_j\right)$$

$$\begin{array}{c|} \theta \\ \hline C_{x_1} \\ \cdots \\ C_{x_m} \end{array} \Rightarrow \theta\left(\textstyle\sum_{i=1}^{m} C_{x_i} x_i\right)$$

$$\begin{array}{c|ccc} & C_{y_1} & \cdots & C_{y_n} \\ \hline C_{x_1} & C_{x_1 y_1} & \cdots & C_{x_1 y_n} \\ \cdots & \cdots & \cdots & \cdots \\ C_{x_m} & C_{x_m y_1} & \cdots & C_{x_m y_n} \end{array} \Rightarrow Y\sum_{i=1}^{m} C_{x_i} x_i + X\sum_{j=1}^{n} C_{y_j} y_j + \sum_{i=1}^{m}\sum_{j=1}^{n} C_{x_i y_j} x_i y_j$$

More than 100 pages' proofs of 257 Bell-CH inequalities!

New Class of Bell-CH inequalities

New Class of Bell-CH inequalities based on rearrangement inequalities

$$I_{2,CH} \leqslant I_{2,CH}^{(0)} \leqslant -\frac{1}{2}\{[P(x_1,y_1)-P(x_1,y_2)-P(x_2,y_1)+P(x_2,y_2)] + |P(x_1,y_1)-P(x_1,y_2)-P(x_2,y_1)+P(x_2,y_2)|\}$$

Tighter than original CH inequality with LHVT

$$I_{mm}(x_1,\cdots,x_m|y_1,\cdots,y_m) = \sum_{j=1}^{m}\sum_{i=1}^{m+1-j} x_i y_j - \sum_{i=2}^{m} x_i y_{m+2-i} - \sum_{i=1}^{m-1}(m-i)x_i Y - y_1 X$$

$$I_{mm}(x_1,\cdots,x_m|y_1,\cdots,y_m) \leqslant 0$$

C. Qian, Y.-G. Yang, QW and C.-F. Qiao, Phys. Rev. A 103, 062203 (2021)

New Class of Bell-CH inequalities based on linear inequalities

$$\max\left\{I_{k-1,k-1;Q}^{(1)} + \sum_{i=1}^{k-1}[P(x_2,y_i)-P(x_2)],\ I_{k-1,k-1;Q}^{(2)} + \sum_{i=1}^{k-1}[P(x_1,y_i)-P(x_1)]\right\} \leqslant 0,$$

$$I_{k-1,k-1;Q}^{(1)} \equiv \sum_{j=1}^{k-1}\sum_{i=1,i\neq 2}^{k-j} P(x_i,y_j) - \sum_{i=3}^{k-1} P(x_i,y_{k+1-i}) - \sum_{i=1,i\neq 2}^{k-2}(k-1-i)P(x_i) - P(y_1),$$

$$I_{k-1,k-1;Q}^{(2)} \equiv \sum_{j=1}^{k-1}\sum_{i=2}^{k-j} P(x_i,y_j) - \sum_{i=3}^{k-1} P(x_i,y_{k+1-i}) - \sum_{i=2}^{k-2}(k-1-i)P(x_i) - P(y_1).$$

New Bell-CH inequalities have less measurement settings than the original ones

Bell-nonlocality in $\Lambda\bar{\Lambda}$ system

The nonlocal property in a quantum entangled system can be tested by the violation of Bell inequality. The most widely used Bell-type inequality is the CHSH inequality

$$|\langle A_1 \otimes B_1\rangle + \langle A_1 \otimes B_2\rangle + \langle A_2 \otimes B_1\rangle - \langle A_2 \otimes B_2\rangle| \leq 2$$

where $A_i = \boldsymbol{a}_i \cdot \boldsymbol{\sigma},\ B_i = \boldsymbol{b}_i \cdot \boldsymbol{\sigma}\ (i = 1, 2),$ **and** $\langle A \otimes B\rangle = \mathrm{Tr}\left[\rho_{Y\bar{Y}}(\boldsymbol{a} \cdot \boldsymbol{\sigma} \otimes \boldsymbol{b} \cdot \boldsymbol{\sigma})\right]$

Here $\boldsymbol{a}_1,\ \boldsymbol{a}_2,\ \boldsymbol{b}_1,\ \boldsymbol{b}_2$ **are four directions (unit vectors) along which the spin polarization is measured. The inequality can be rewritten in a simpler form**

$$|\boldsymbol{a}_1 \cdot C \cdot (\boldsymbol{b}_1 + \boldsymbol{b}_2) + \boldsymbol{a}_2 \cdot C \cdot (\boldsymbol{b}_1 - \boldsymbol{b}_2)| \leq 2$$

C.Qian, J.-L. Li, A.S. Khan, C.-F. Qiao, PRD(2020)
S. Wu, C. Qian, QW, X.-R. Zhou, PRD(2024)

with C being the correlation matrix C_{ij}. The maximum of the left-hand side can be obtained by tuning $\boldsymbol{a}_1,\ \boldsymbol{a}_2,\ \boldsymbol{b}_1,\ \boldsymbol{b}_2$

$$\mathcal{B}[\rho] \equiv \max_{\boldsymbol{a}_1,\boldsymbol{a}_2,\boldsymbol{b}_1,\boldsymbol{b}_2} |\boldsymbol{a}_1 \cdot C \cdot (\boldsymbol{b}_1 + \boldsymbol{b}_2) + \boldsymbol{a}_2 \cdot C \cdot (\boldsymbol{b}_1 - \boldsymbol{b}_2)|$$

$$= 2\sqrt{m_1 + m_2} \Longrightarrow$$

m_1 and m_2 are are two largest eigenvalues of C^TC

Bell-nonlocality in $\Lambda\bar{\Lambda}$ system

Therefore, the CHSH inequality can be violated iff (if and only if) $m_1 + m_2 > 1$ and the maximum possible violation of the CHSH inequality is the upper bound value $2\sqrt{2}$. We define $m_{12}[\rho] = m_1 + m_2$ to be a measure of the Bell nonlocality.

Since we have put the density operator into the X form, we have

$$C = \mathrm{diag}\left\{t_1, t_2, t_3\right\}, \quad C^{\mathrm{T}}C = \mathrm{diag}\left\{t_1^2, t_2^2, t_3^2\right\}$$

The measure of the Bell nonlocality reads

$$m_{12}\left[\rho^X_{Y\bar{Y}}\right] = \begin{cases} t_1^2 + t_2^2, & \alpha_\psi \geq 0 \\ \max\left\{t_1^2 + t_2^2, t_1^2 + t_3^2\right\}, & \alpha_\psi < 0 \end{cases}$$

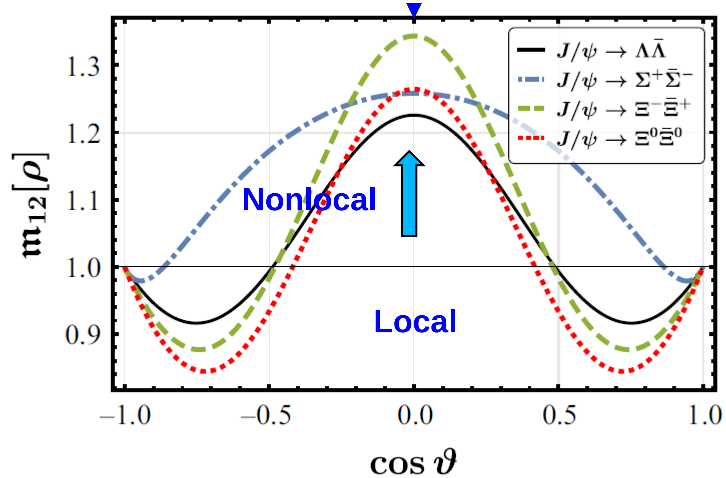
$$\underline{\max_\theta \mathcal{B}\left[\rho^X_{Y\bar{Y}}\right] = 2\sqrt{1+\alpha_\psi^2}}$$

Maximum violation at $\theta = \pi/2$

$$\underline{\theta^*} = \arctan\left|\sqrt{2 - 2\alpha_\psi^2}\frac{\sin\Delta\Phi}{\alpha_\psi}\right|, \quad \text{for } \alpha_\psi \geq 0$$

Non-local range: $(\theta^*, \pi - \theta^*)$

Maximum violation at $\theta = \pi/2$



S. Wu, C. Qian, QW, X.-R. Zhou, PRD(2024)

Quantum entanglement in $\Lambda\bar{\Lambda}$ system

For a bipartite quantum system a state is said to be separable iff the following decomposition holds

$$\rho_{AB} = \sum_k p_k \rho_A^k \otimes \rho_B^k \qquad p_k \geq 0 \text{ and } \sum_k p_k = 1$$

Moreover, the state cannot be decomposed into the above form is called non-separable or entangled.

The concurrence is an entanglement monotone, so it can be regarded as a measure for the entanglement [Wootters, Phys. Rev. Lett. 80, 2245 (1998)]

$$\mathcal{C}[\rho] \equiv \max\{0, \mu_1 - \mu_2 - \mu_3 - \mu_4\} \qquad \begin{aligned} \mathcal{C}[\rho] &= 0, \quad \textit{separable} \\ \mathcal{C}[\rho] &> 0, \quad \textit{entangled} \\ \mathcal{C}[\rho] &= 1, \quad \textit{maximally entangled} \end{aligned}$$

where μ_i $(i = 1, 2, 3, 4)$ are the eigenvalues of the Hermitain matrix $\sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$ and $\tilde{\rho} = \sigma_y \otimes \sigma_y \rho^* \sigma_y \otimes \sigma_y$.

Quantum entanglement in $\Lambda\bar{\Lambda}$ system

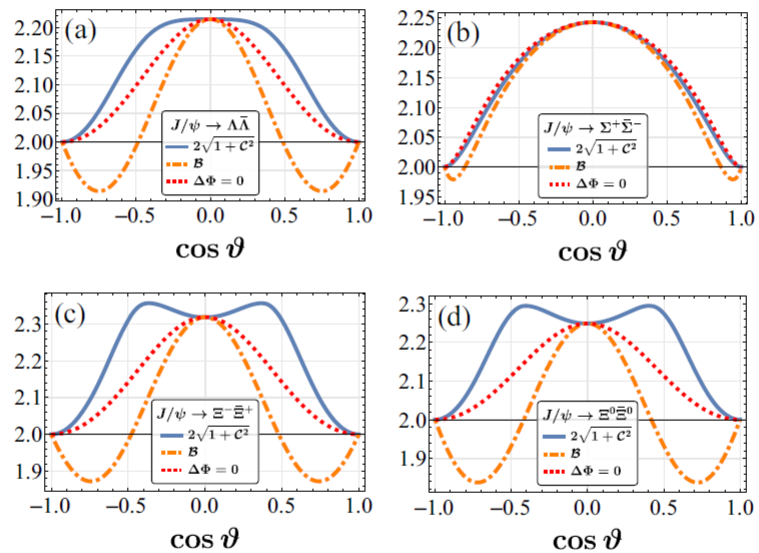
The spin density operator for the hyperon-antihyperon system

$$\rho^{X}_{Y\bar{Y}} = \frac{1}{4}\left(1 + a\sigma_z \otimes 1 + 1 \otimes a\sigma_z + \sum_i t_i \sigma_i \otimes \sigma_i\right)$$

S. Wu, C. Qian, QW, X.-R. Zhou, PRD(2024)

We rewrite $\rho^{X}_{Y\bar{Y}}$ in the σ_z basis

$$\rho^{X}_{Y\bar{Y}} = \frac{1}{4}\begin{bmatrix} 1+2a+t_3 & 0 & 0 & t_1 - t_2 \\ 0 & 1-t_3 & t_1+t_2 & 0 \\ 0 & t_1+t_2 & 1-t_3 & 0 \\ t_1-t_2 & 0 & 0 & 1-2a+t_3 \end{bmatrix}$$



$2\sqrt{m_{12}}$ and $2\sqrt{1+\mathcal{C}^2}$

The concurrence is

$$\mathcal{C}\left[\rho^{X}_{Y\bar{Y}}\right] = \frac{1}{2(1+\alpha_\psi \cos^2\vartheta)} \times \left|1 + \alpha_\psi - \sqrt{(1+\alpha_\psi \cos 2\vartheta)^2 - \beta_\psi^2 \sin^2 2\vartheta}\right|$$

$$\mathcal{B}[\rho] = 2\sqrt{m_{12}} \leq 2\sqrt{1+\mathcal{C}^2[\rho]}$$

Bell nonlocality $\subset$ entanglement

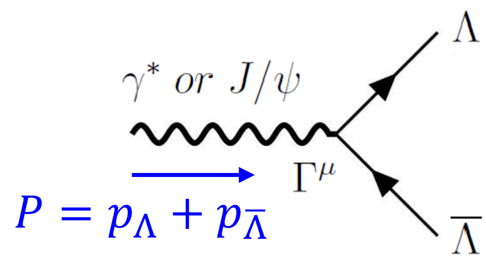
equality holds at $\theta = \pi/2$

Electromagnetic form factors

The role of time-like electromagnetic form factors in quantum nonlocality and entanglement

$$\Gamma^\mu = \gamma^\mu F_1(P^2) + i\frac{\sigma^{\mu\nu}P_\nu}{2M}F_2(P^2)$$

$$G_E(P^2) = F_1 + \frac{P^2}{4M^2}F_2, \quad G_M(P^2) = F_1 + F_2$$



The spin polarization of Λ and $\bar{\Lambda}$ is

$$P_\Lambda = P_{\bar{\Lambda}} = \frac{\sqrt{1-\alpha_\psi^2}\sin\theta\cos\theta}{1+\alpha_\psi\cos^2\theta}\sin(\Delta\Phi)$$

$$\alpha_\psi = \frac{s - 4M^2\left|G_E/G_M\right|^2}{s + 4M^2\left|G_E/G_M\right|^2} \in [-1, 1]$$

$$\Delta\Phi = \arg\left\{G_E/G_M\right\} \in (-\pi, \pi]$$

phase difference between G_E and G_M

If $\Delta\Phi = 0$, $\rho_{Y\bar{Y}}$ is reduced to a BDS form

$$\rho_{Y\bar{Y}}^{\mathrm{BDS}} = \frac{1}{4}\left(1\otimes 1 + \sum_i t_i\sigma_i \otimes \sigma_i\right)$$

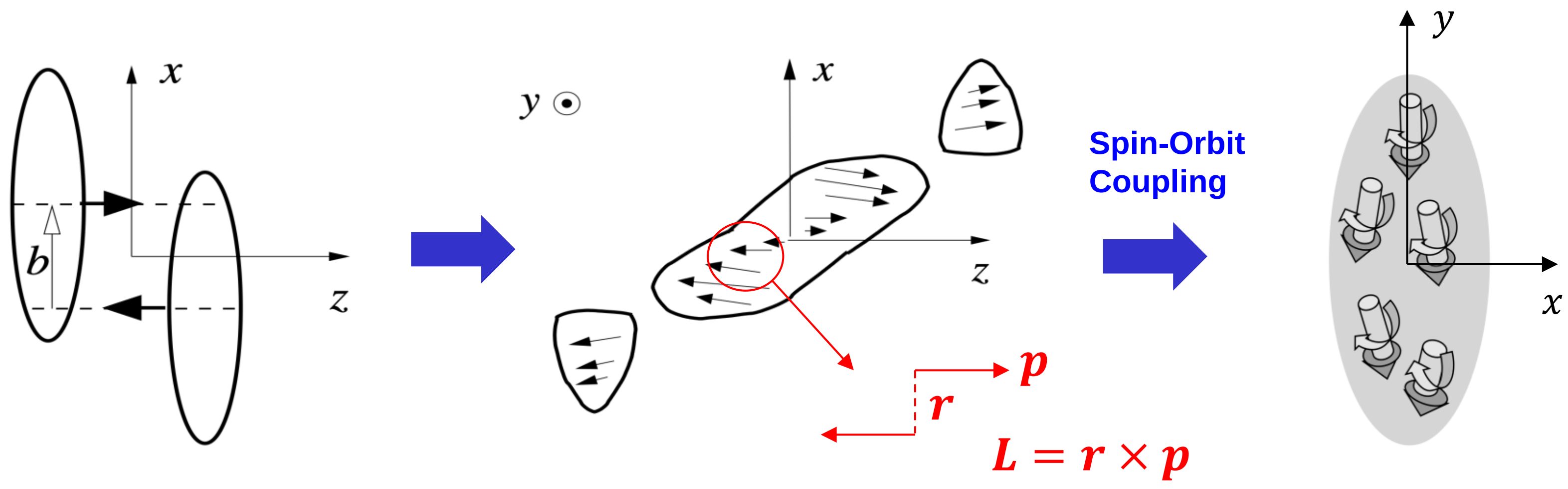
Bell nonlocality == entanglement at all scattering angles

$$\boxed{m_{12}\left[\rho_{Y\bar{Y}}^{\mathrm{BDS}}\right] = 1 + \mathcal{C}^2\left[\rho_{Y\bar{Y}}^{\mathrm{BDS}}\right] = 1 + \left(\frac{\alpha_\psi\sin^2\theta}{1+\alpha_\psi\cos^2\theta}\right)^2 \geq 1}$$

Valid for elementary particles such as $t\bar{t}$ or $\tau^+\tau^-$

Spin correlation in Heavy-Ion collisions

Global OAM and polarization in HIC



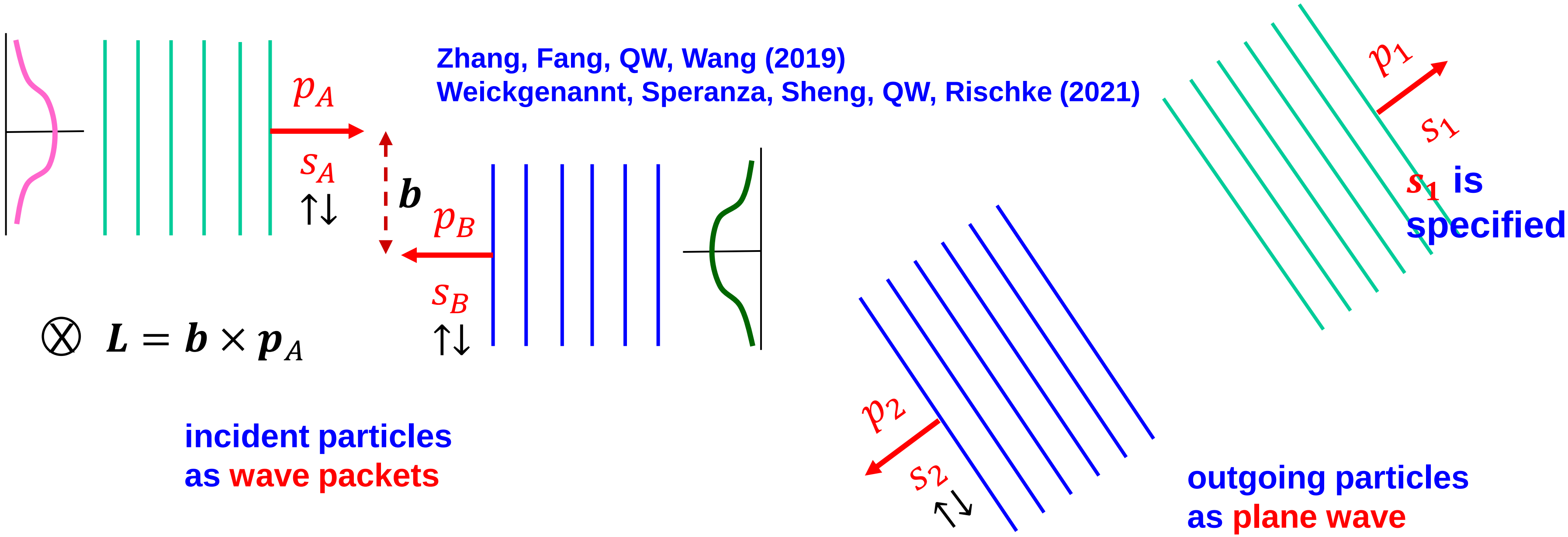
Huge global orbital angular momentum (OAM) is produced in IS of HIC.

Q: How do orbital angular momenta be transferred to the matter in HIC?

A: Part of initial OAM is distributed into the matter in the form of local OAM and then is converted to hadrons' global spin polarization through spin-orbit coupling: e.g. GSP of Λ hyperons **[Liang, Wang (2005)]**

Non-local collisions: collisions of particles as wave packets

Zhang, Fang, QW, Wang (2019)
Weickgenannt, Speranza, Sheng, QW, Rischke (2021)

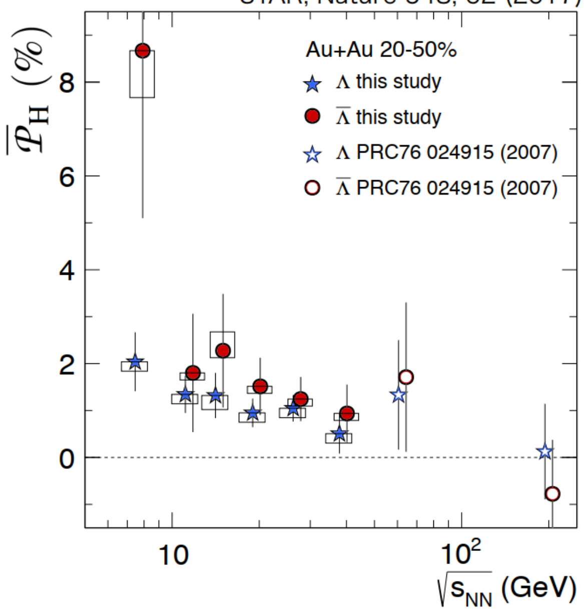


Particle collisions as wave packets: there is a transverse distance between two wave packets (impact parameter) giving non-vanishing OAM and then the polarization of one final particle

$$\boldsymbol{L} = \boldsymbol{b} \times \boldsymbol{p}_A \quad \Longrightarrow \quad \left(\frac{d\sigma}{d\Omega}\right)_{s_1=\uparrow} \neq \left(\frac{d\sigma}{d\Omega}\right)_{s_1=\downarrow}$$

STAR: global polarization of Λ hyperon

STAR, Nature 548, 62 (2017)



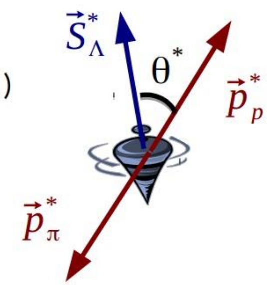
parity-violating decay of hyperons

In case of Λ's decay, daughter proton preferentially decays in the direction of Λ's spin (opposite for anti-Λ)

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha \mathbf{P_\Lambda} \cdot \mathbf{p_p^*})$$

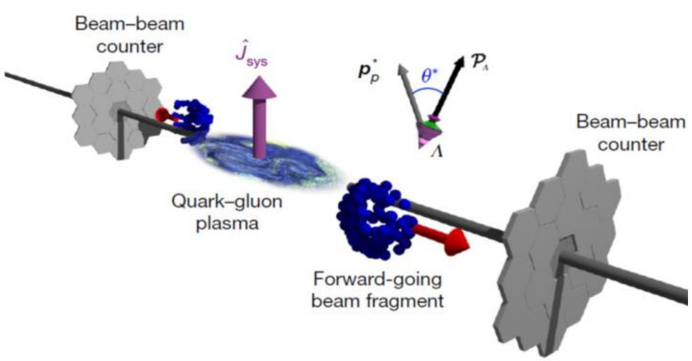
α: Λ decay parameter (=0.642±0.013)
P_Λ: Λ polarization
p_p^*: proton momentum in Λ rest frame

Updated by BES III, PRL129, 131801 (2022)



$$\Lambda \to p + \pi^+$$

(BR: 63.9%, cτ~7.9 cm)

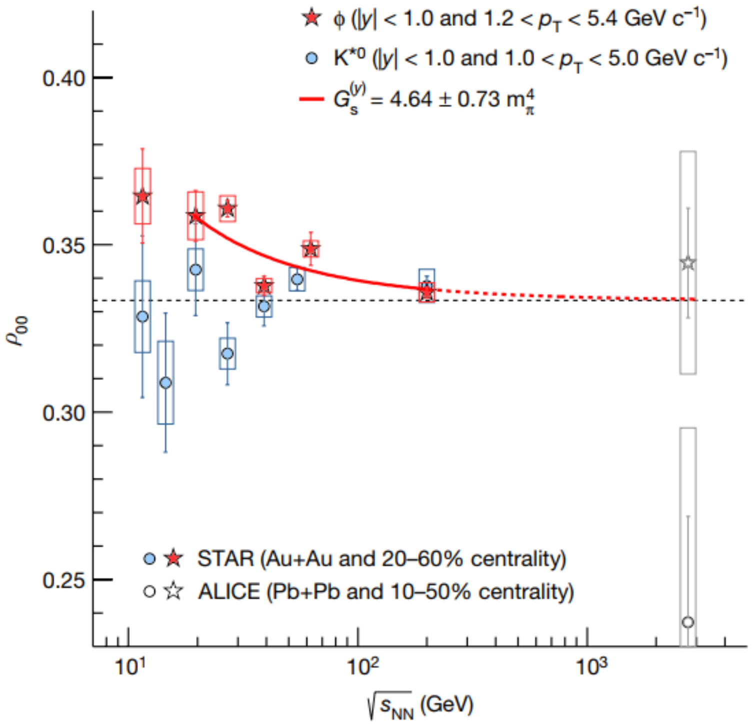


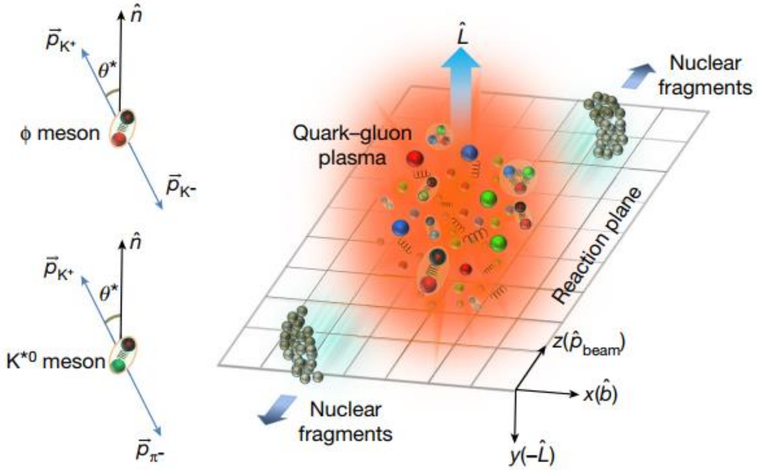
ω = (9 ± 1)x10^{21}/s, the largest angular velocity that has ever been observed in any system

Liang, Wang, PRL (2005)
Betz, Gyulassy, Torrieri, PRC (2007)
Becattini, Piccinini, Rizzo, PRC (2008)
Gao et al., PRC (2008)

STAR: global spin alignments of vector mesons

STAR, Nature 614, 244 (2023);





Implication of correlation or fluctuation of strong force fields

Theory prediction:
Liang, Wang, PLB(2005);
Sheng, Oliva, QW, PRD(2020);
Sheng, Oliva, Liang, et al., PRL(2022).

$$P_\Lambda \sim \langle P_s \rangle, \quad P_{\bar{\Lambda}} \sim \langle P_{\bar{s}} \rangle$$

$$\rho_{00}^{\phi} - \frac{1}{3} \sim \langle P_s P_{\bar{s}} \rangle \neq \langle P_s \rangle \langle P_{\bar{s}} \rangle \sim P_\Lambda P_{\bar{\Lambda}}$$

Global quark and hadron spin correlation in coalescence model in HIC

Global quark spin correlations in relativistic heavy-ion collisions, Lv, Yu, Liang, QW, Wang, PRD(2024)

transition matrices

$$\rho_V = \mathcal{M}\rho_{(q_1\bar{q}_2)}\mathcal{M}^\dagger \Longrightarrow \boldsymbol{q_1\bar{q}_2 \to V}$$

$$\rho_H = \mathcal{M}\rho_{(q_1 q_2 q_3)}\mathcal{M}^\dagger \Longrightarrow \boldsymbol{q_1 q_2 q_3 \to H}$$

$$\rho_{H_1\bar{H}_2} = \mathcal{M}\rho_{(1\cdots 6)}\mathcal{M}^\dagger \Longrightarrow \boldsymbol{q_1 q_2 q_3 \bar{q}_4 \bar{q}_5 \bar{q}_6 \to H_1 \bar{H}_2}$$

Spin density matrix for quarks

The spin density matrices for one quark and two quarks are

$$\rho_{(q)} = \frac{1}{2}\left(1 + \boldsymbol{P}_q \cdot \boldsymbol{\sigma}\right)$$

$$\rho_{(12)} = \frac{1}{4}\left[1 + \boldsymbol{P}_1 \cdot \boldsymbol{\sigma} + \boldsymbol{P}_2 \cdot \boldsymbol{\sigma} + t_{ij}^{(12)} \sigma_{1i} \otimes \sigma_{2j}\right] \Longrightarrow \text{with shortcoming}$$

$$= \rho_{(1)} \otimes \rho_{(2)} + \frac{1}{4} \underline{c_{ij}^{(12)}} \sigma_{1i} \otimes \sigma_{2j} \Longrightarrow \text{improved}$$

2-body genuine correlation

Similarly, the spin density matrix for three quarks has the form

$$\rho_{(123)} = \rho_{(1)} \otimes \rho_{(2)} \otimes \rho_{(3)} + \frac{1}{2^3} \underline{c_{ijk}^{(123)}} \sigma_{1i} \otimes \sigma_{2j} \otimes \sigma_{3k}$$

3-body correlation

$$+ \frac{1}{2^2}\left[\underline{c_{ij}^{(12)}} \sigma_{1i} \otimes \sigma_{2j} \otimes \rho_{(3)} + \underline{c_{jk}^{(23)}} \rho_{(1)} \otimes \sigma_{2j} \otimes \sigma_{3k} \right.$$

$$\left. + \underline{c_{ik}^{(13)}} \sigma_{1i} \otimes \rho_{(2)} \otimes \sigma_{3k}\right]$$

2-body correlations

Spin density matrix for quarks

The spin density matrix for four quarks has the form

$$\rho_{(1234)} = \rho_{(1)} \otimes \rho_{(2)} \otimes \rho_{(3)} \otimes \rho_{(4)} + \frac{1}{2^4} \underline{c_{ijkl}^{(1234)}} \sigma_{1i} \otimes \sigma_{2j} \otimes \sigma_{3k} \otimes \sigma_{4l}$$

4-body correlation

$$+ \frac{1}{2^2} \Big[\underline{c_{ij}^{(12)}} \sigma_{1i} \otimes \sigma_{2j} \otimes \rho_{(3)} \otimes \rho_{(4)} + \underline{c_{kl}^{(34)}} \rho_{(1)} \otimes \rho_{(2)} \otimes \sigma_{3k} \otimes \sigma_{4l}$$
$$+ \underline{c_{ik}^{(13)}} \sigma_{1i} \otimes \rho_{(2)} \otimes \sigma_{3k} \otimes \rho_{(4)} + \underline{c_{jl}^{(24)}} \rho_{(1)} \otimes \sigma_{2j} \otimes \rho_{(3)} \otimes \sigma_{4l}$$
$$+ \underline{c_{il}^{(14)}} \sigma_{1i} \otimes \rho_{(2)} \otimes \rho_{(3)} \otimes \sigma_{4l} + \underline{c_{jk}^{(23)}} \rho_{(1)} \otimes \sigma_{2j} \otimes \sigma_{3k} \otimes \rho_{(4)} \Big]$$

2-body correlations

$$+ \frac{1}{2^3} \Big[\underline{c_{ijk}^{(123)}} \sigma_{1i} \otimes \sigma_{2j} \otimes \sigma_{3k} \otimes \rho_{(4)} + \underline{c_{ijl}^{(124)}} \sigma_{1i} \otimes \sigma_{2j} \otimes \rho_{(3)} \otimes \sigma_{4l}$$
$$+ \underline{c_{ikl}^{(134)}} \sigma_{1i} \otimes \rho_{(2)} \otimes \sigma_{3k} \otimes \sigma_{4l} + \underline{c_{jkl}^{(234)}} \rho_{(1)} \otimes \sigma_{2j} \otimes \sigma_{3k} \otimes \sigma_{4l} \Big]$$

3-body correlations

The polarizations and spin correlations can be extracted by taking expectation values of a direct product of Pauli matrices on spin density matrices.

Spin density matrix for vector mesons

We consider the combination process $q_1\bar{q}_2 \to V$. The spin density matrix of the vector meson is given by

$$\rho_V = \underbrace{\mathcal{M}}\rho_{(q_1\bar{q}_2)}\underbrace{\mathcal{M}^\dagger}$$

transition matrix for $q_1\bar{q}_2 \to V$

The spin density matrix element of vector meson

$$\left|m_{(12)}\right\rangle \equiv |j_1 m_1, j_2 m_2\rangle$$
$$\left|m'_{(12)}\right\rangle \equiv |j_1 m'_1, j_2 m'_2\rangle$$

$$\begin{aligned}
\rho^V_{mm'} &= \langle jm|\,\mathcal{M}\rho_{(q_1\bar{q}_2)}\mathcal{M}^\dagger\,|jm'\rangle \\
&= \sum_{m_{(12)},m'_{(12)}} \langle jm|\,\mathcal{M}\,\left|m_{(12)}\right\rangle \left\langle \underline{m_{(12)}}\right|\rho_{(q_1\bar{q}_2)}\left|\underline{m'_{(12)}}\right\rangle \left\langle m'_{(12)}\right|\mathcal{M}^\dagger\,|jm'\rangle \\
&\Rightarrow N_V \sum_{m_{(12)},m'_{(12)}} \underbrace{\langle jm|m_{(12)}\rangle}_{\text{CG coefficient}}\langle m_{(12)}|\hat{\rho}_{(q_1\bar{q}_2)}|m'_{(12)}\rangle\underbrace{\langle m'_{(12)}|jm'\rangle}_{\text{CG coefficient}}
\end{aligned}$$

Here we assumed $\langle jm|\mathcal{M}|j'm'\rangle = \delta_{jj'}\delta_{mm'}\langle jm|\mathcal{M}|jm\rangle$ due to rotational symmetry of the transition matrix.

Spin density matrix element for vector mesons

Spin density matrix elements of vector mesons are then obtained.
Diagonal elements:

$$\rho_{00}^{V} = \frac{1}{C_V}\left[1 + P_q^x P_{\bar{q}}^x + P_q^y P_{\bar{q}}^y - P_q^z P_{\bar{q}}^z + c_{xx}^{(q\bar{q})} + c_{yy}^{(q\bar{q})} - c_{zz}^{(q\bar{q})}\right]$$

$$\rho_{11}^{V} = \frac{1}{C_V}\left[\left(1 + P_q^z\right)\left(1 + P_{\bar{q}}^z\right) + c_{zz}^{(q\bar{q})}\right]$$

$$\rho_{-1,-1}^{V} = \frac{1}{C_V}\left[\left(1 - P_1^z\right)\left(1 - P_2^z\right) + c_{zz}^{(q\bar{q})}\right]$$

Normalization constant

$$\boxed{C_V = 3 + \boldsymbol{P}_q \cdot \boldsymbol{P}_{\bar{q}} + c_{xx}^{(q\bar{q})} + c_{yy}^{(q\bar{q})} + c_{zz}^{(q\bar{q})}}$$

Off-diagonal elements:

$$\begin{aligned}\rho_{10}^{V} = &\frac{1}{\sqrt{2}C_V}\left\{c_{zx}^{(q\bar{q})} + c_{xz}^{(q\bar{q})} + \left(1 + P_q^z\right)P_{\bar{q}}^x + P_q^x\left(1 + P_{\bar{q}}^z\right)\right.\\ &\left. -i\left[c_{zy}^{(q\bar{q})} + c_{yz}^{(q\bar{q})} + \left(1 + P_q^z\right)P_{\bar{q}}^y + P_q^y\left(1 + P_{\bar{q}}^z\right)\right]\right\}\end{aligned}$$

$$\begin{aligned}\rho_{0,-1}^{V} = &\frac{1}{\sqrt{2}C_V}\left\{-c_{zx}^{(q\bar{q})} - c_{xz}^{(q\bar{q})} + P_q^x\left(1 - P_{\bar{q}}^z\right) + \left(1 - P_q^z\right)P_{\bar{q}}^x\right.\\ &\left. +i\left[c_{zy}^{(q\bar{q})} + c_{yz}^{(q\bar{q})} - P_q^y\left(1 - P_{\bar{q}}^z\right) - \left(1 - P_q^z\right)P_{\bar{q}}^y\right]\right\}\end{aligned}$$

$$\begin{aligned}\rho_{1,-1}^{V} = &\frac{1}{C_V}\left[c_{xx}^{(q\bar{q})} - c_{yy}^{(q\bar{q})} + P_q^x P_{\bar{q}}^x - P_q^y P_{\bar{q}}^y\right.\\ &\left. -i\left(P_q^y P_{\bar{q}}^x + P_q^x P_{\bar{q}}^y + c_{xy}^{(q\bar{q})} + c_{yx}^{(q\bar{q})}\right)\right]\end{aligned}$$

Spin alignment of vector mesons as small effect

If we assume that the polarization and spin correlation are small effects, the average spin alignment is then in the form

$$\begin{aligned}
\langle\rho_{00}^V\rangle \approx & \frac{1}{3}+\frac{2}{9}\left[\langle P_q^x P_{\bar{q}}^x\rangle+\langle P_q^y P_{\bar{q}}^y\rangle-2\langle P_q^z P_{\bar{q}}^z\rangle\right] \\
& +\frac{2}{9}\left[\left\langle c_{xx}^{(q\bar{q})}\right\rangle+\left\langle c_{yy}^{(q\bar{q})}\right\rangle-2\left\langle c_{zz}^{(q\bar{q})}\right\rangle\right] \\
\langle\rho_{11}^V\rangle \approx & \frac{1}{3}+\langle P_q^z\rangle+\langle P_{\bar{q}}^z\rangle-\frac{1}{9}\left[\langle P_q^x P_{\bar{q}}^x\rangle+\langle P_q^y P_{\bar{q}}^y\rangle-2\langle P_q^z P_{\bar{q}}^z\rangle\right] \\
& -\frac{1}{9}\left[\left\langle c_{xx}^{(q\bar{q})}\right\rangle+\left\langle c_{yy}^{(q\bar{q})}\right\rangle-2\left\langle c_{zz}^{(q\bar{q})}\right\rangle\right] \\
\langle\rho_{-1,-1}^V\rangle \approx & \frac{1}{3}-\langle P_q^z\rangle-\langle P_{\bar{q}}^z\rangle-\frac{1}{9}\left[\langle P_q^x P_{\bar{q}}^x\rangle+\langle P_q^y P_{\bar{q}}^y\rangle-2\langle P_q^z P_{\bar{q}}^z\rangle\right] \\
& -\frac{1}{9}\left[\left\langle c_{xx}^{(q\bar{q})}\right\rangle+\left\langle c_{yy}^{(q\bar{q})}\right\rangle-2\left\langle c_{zz}^{(q\bar{q})}\right\rangle\right]
\end{aligned}$$

Correlation in spin polarization

Genuine correlation from dynamical processes

quadrupole structure

$$\boxed{\mathrm{Tr}\rho^V=1}$$

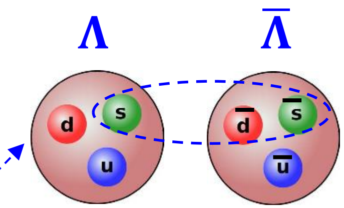
Here the spin quantization direction is along +z direction. The three polarization vectors (direction of the vector field) for the vector meson in the rest frame are

$$\epsilon_0=\boldsymbol{e}_z,\quad \epsilon_1=-\frac{1}{\sqrt{2}}(\boldsymbol{e}_x+i\boldsymbol{e}_y),\quad \epsilon_{-1}=\frac{1}{\sqrt{2}}(\boldsymbol{e}_x-i\boldsymbol{e}_y)$$

Spin correlation for $\Lambda\bar{\Lambda}$

When all two-particle spin correlations ($c_{ij}^{(ab)} \neq 0$, all other $c_{i_1\cdots i_n}^{(q_1\cdots q_n)} = 0$ with $3 \leq n \leq 6$) are considered, the result for the spin correlation of $\Lambda\bar{\Lambda}$ is

$$
\begin{aligned}
c_{zz}^{\Lambda\bar{\Lambda}} =& P_s^z P_{\bar{s}}^z + \frac{1}{B_{\Lambda\bar{\Lambda}}}\left(c_{zz}^{(s\bar{s})} + \text{four and six body correlation}\right) \\
\approx& P_s^z P_{\bar{s}}^z + c_{zz}^{(s\bar{s})} + \text{four or more body correlation}
\end{aligned}
$$

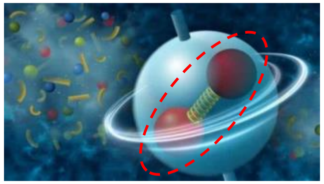


where $B_{\Lambda\bar{\Lambda}}$ is defined as

$$
B_{\Lambda\bar{\Lambda}} = 1 - \boldsymbol{P}_u \cdot \boldsymbol{P}_d - \boldsymbol{P}_{\bar{u}} \cdot \boldsymbol{P}_{\bar{d}} - c_{ii}^{(ud)} - c_{ii}^{(\bar{u}\bar{d})} + \text{four body correlation}
$$

We can take average of $c_{zz}^{\Lambda\bar{\Lambda}}$ over all $\Lambda\bar{\Lambda}$ events

$$
\left\langle c_{zz}^{\Lambda\bar{\Lambda}} \right\rangle \approx \langle P_s^z P_{\bar{s}}^z \rangle + \left\langle c_{zz}^{(s\bar{s})} \right\rangle + \text{four or more body correlation}
$$



In $\Lambda\bar{\Lambda}$ correlation, s is in Λ and $\bar{s}$ is in $\bar{\Lambda}$, i.e. the correlation of constituent quarks in different particles → long range correlation

In spin alignment of ϕ meson $\left\langle \rho_{00}^{\phi} \right\rangle$, there are also $\langle P_s^z P_{\bar{s}}^z \rangle$ and $\left\langle c_{zz}^{(s\bar{s})} \right\rangle$, but the average here is taken inside the ϕ meson → **short range correlation**

Spin correlation of ΛΛ as probe to vortical structure of QGP fluid

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Vortical Fluid and Λ Spin Correlations in High-Energy Heavy-Ion Collisions

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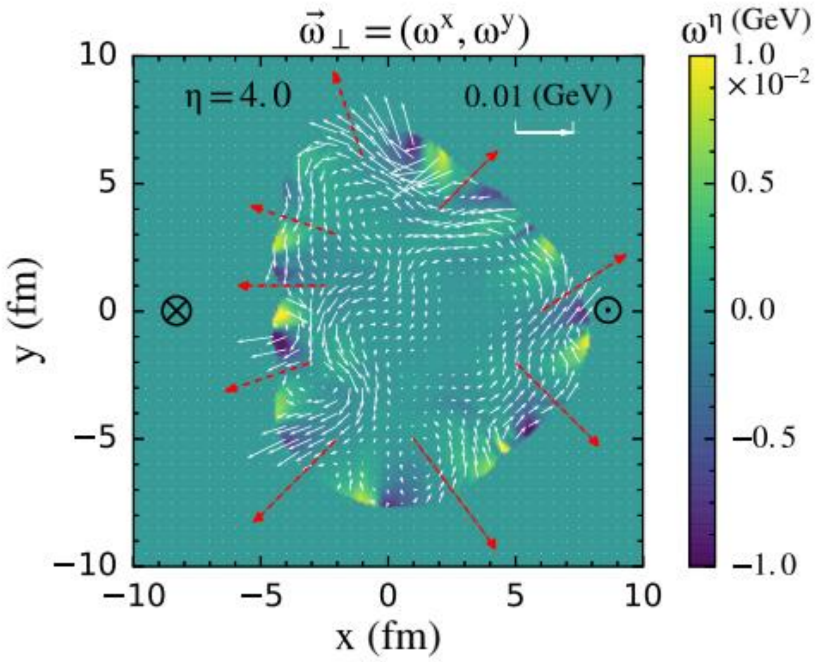
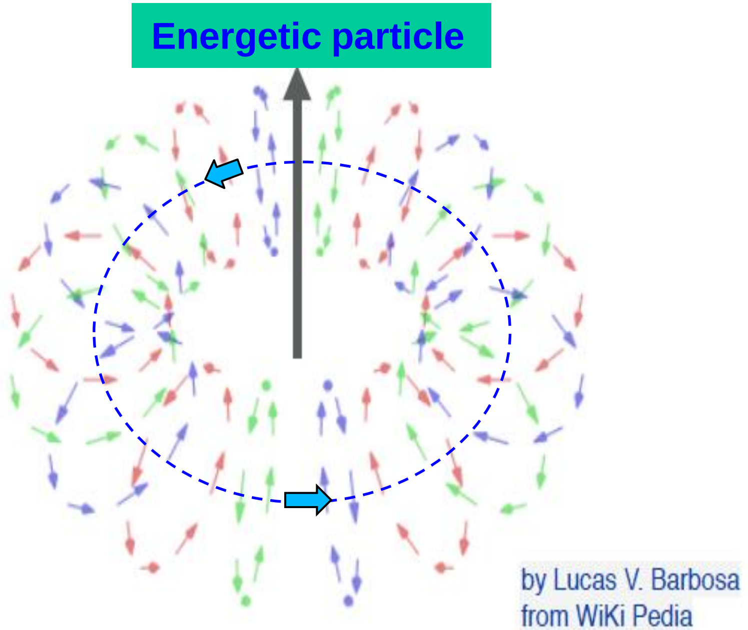
Turbulence and vortex rings in high energy HIC





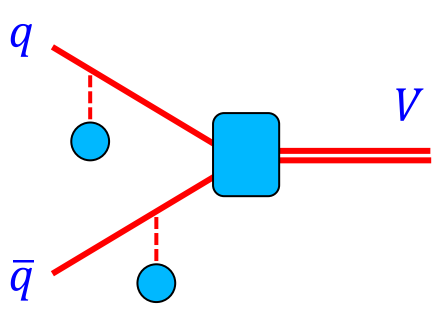
Pang, Petersen, QW, et al. PRL(2016);
Xia, Li, Tang, QW, PRC(2018);
Lisa, Barbon, Serenone, Shen, PRC (2021)

Spin correlation of Λ Λ can probe the vortical structure in sQGP

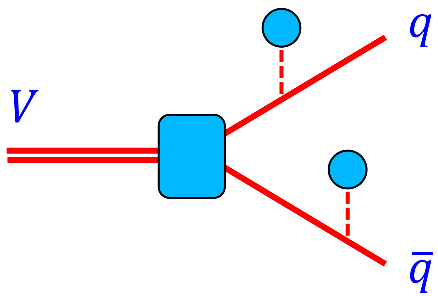
Relativistic Spin Boltzmann (Kinetic) Equation for vector mesons in quark coalescence model

Sheng, Oliva, Liang, QW, et al., PRL (2023), PRD(2024)

Review on QKE and SKE based on Wigner functions:
Hidaka, Pu, QW, Yang, Prog. Part. Nucl. Phys. 127 (2022) 103989



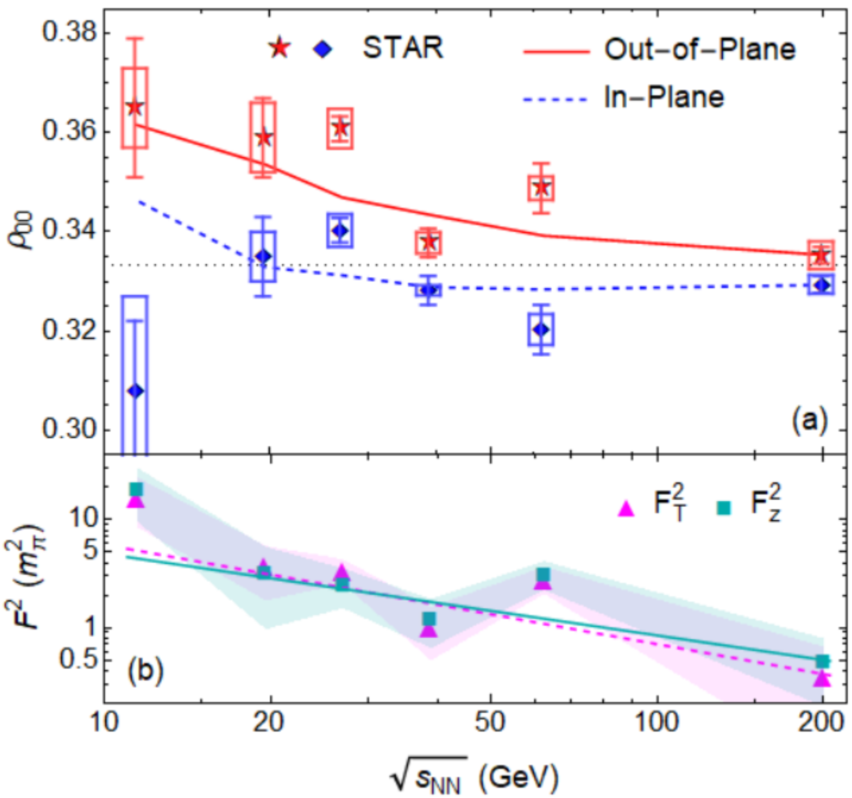
Quark coalescence to V-meson



V-meson dissociation to quarks

Quark coalescence model:
Greco, Ko, Levai (2003);
Fries, Mueller et al (2003);
Yang, Hwa (2003).

Spin density matrix element for vector mesons

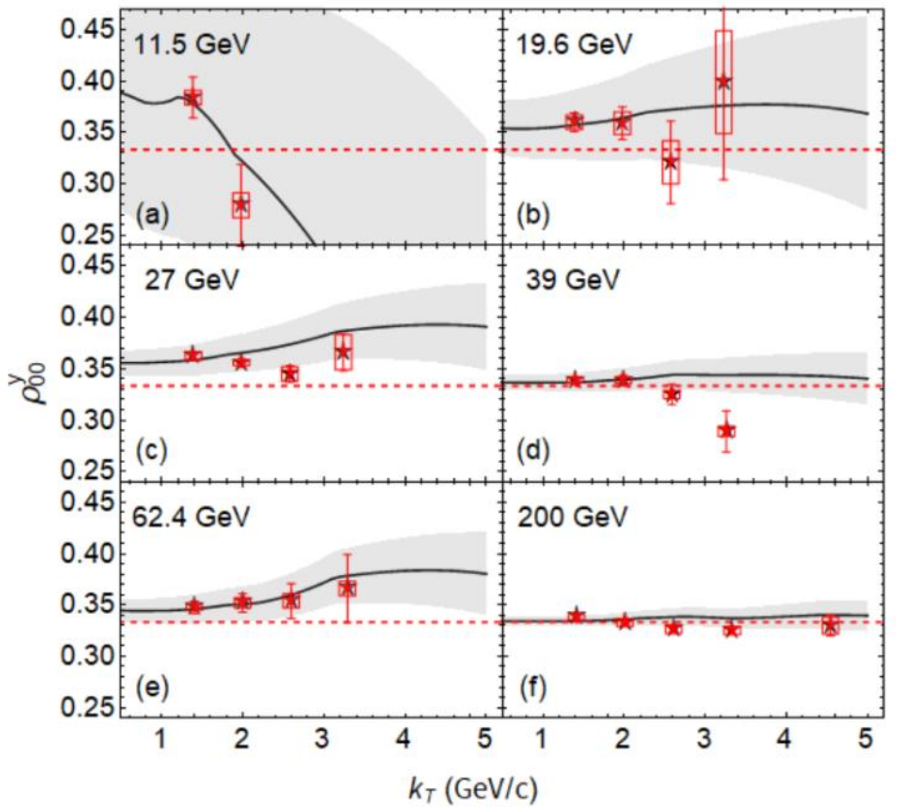


Sheng, Lucia, Liang, QW, et al, PRL (2023), PRD(2024)

(a) The STAR's data on phi meson's ρ_{00}^{y} (out-of-plane, red stars) and ρ_{00}^{x} (in-plane, blue diamonds) in 0-80% Au+Au collisions as functions of collision energies. The red-solid line and blue-dashed line are calculated with values of F_T^2 and F_z^2 from fitted curves in (b).

(b) Values of F_T^2 (magenta triangles) and F_z^2 (cyan squares) with shaded error bands extracted from the STAR's data on the phi meson's ρ_{00}^{y} and ρ_{00}^{x} in (a). The magenta-dashed line (cyan-solid line) is a fit to the extracted F_T^2 (F_z^2) as a function of $\sqrt{s_{NN}}$.

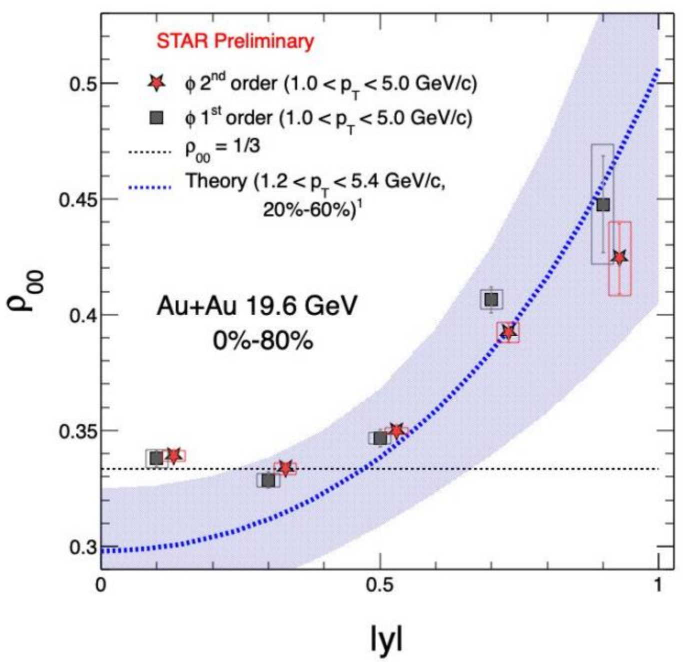
Spin density matrix element for vector mesons



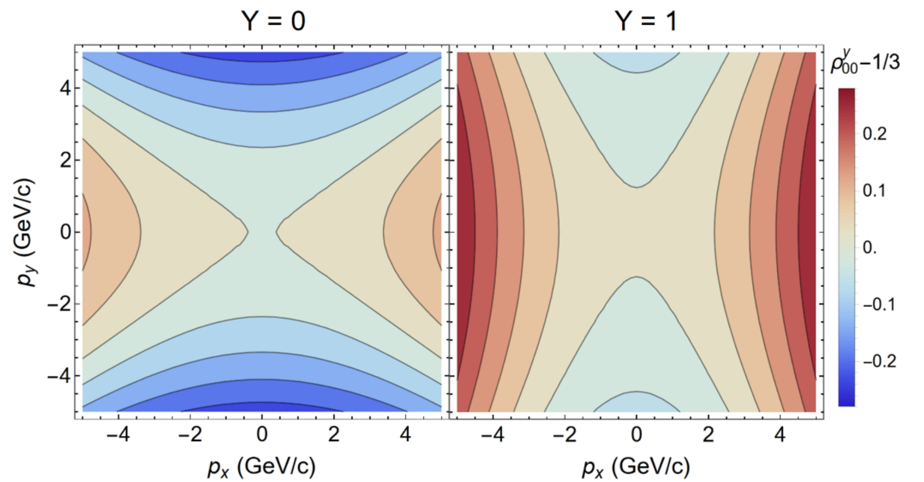
Calculated ρ_{00}^{y} (solid line) of ϕ mesons as functions of transverse momenta in 0-80% Au+Au collisions at different colliding energies in comparison with STAR data. Shaded error bands are from the extracted parameters F_T^2 and F_z^2 .

Sheng, Lucia, Liang, QW, et al, PRL (2023), PRD(2024)

Our prediction on rapidity dependence of ρ_{00}^{y}



Sheng, Pu, QW, PRC(2023)



If B^2 and E^2 is isotropic in all directions in lab frame, we have simple formula with clear physics

$$\langle \delta\rho_{00}^{y}\rangle(\mathbf{p}) = \frac{8}{3m_\phi^4}(C_1+C_2)F^2\left(\frac{p_x^2+p_z^2}{2}-p_y^2\right)$$

$$\propto \frac{1}{2}p_T^2\left[3\cos(2\varphi)-1\right]+\left(m_\phi^2+p_T^2\right)\sinh^2 Y$$

Summary

Take-home message:

Spin correlation is a new tool to study quantum properties of sQGP.