# Quark models: What can they teach us?

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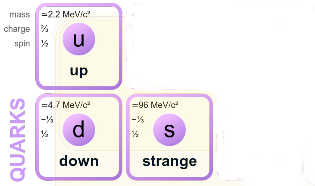
# Introduction

# Quark model: The structure of hadrons

1964 — Quark model by Gell-Mann & Zweig $\Longrightarrow$ $SU(3)$ multiplets

"Ordinary" hadrons*:

- Meson consists of quark and antiquark
- Baryon consists of 3 quarks

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**All hadrons understood $\Longrightarrow$ No "misterious" states**

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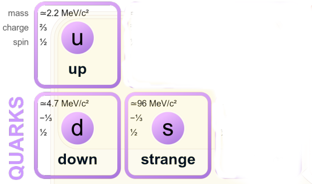


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Contemporary status — input for the quark model

- 6 quarks belonging to 3 generations
- 3 light quarks ($u$, $d$, $s$ with $m_u \approx 2$ MeV, $m_d \approx 5$ MeV, $m_s \approx 94$ MeV)
- 2 heavy quarks ($c$, $b$ with $m_c \approx 1.3$ GeV, $m_b \approx 4.2$ GeV) that form hadrons
- $t$ quark with the mass $m_t \approx 170$ GeV that decays too fast to form hadrons

# Quark model: The structure of hadrons

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Setting up the language

- Quark-antiquark mesons and 3-quark baryons are ordinary hadrons
- Further multiquark states (tetraquarks, pentaquarks, hydrids, etc) are exotic hadrons
- Many exotic candidates are found experimentally
- Various theoretical approaches are suggested to exotic hadrons

Disclaimer:

In this lecture, only ordinary quark-antiquark mesons will be discussed in the framework of Quantum mechanical potential quark model and chiral quark model inspired by Quantum field theory

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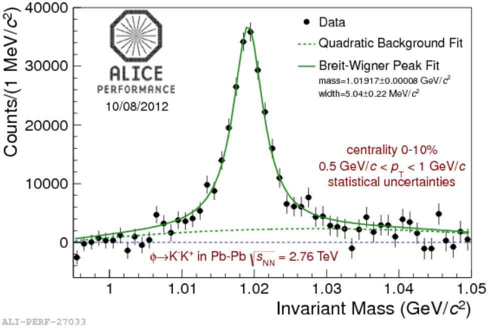
# Experiment: Mass & Width



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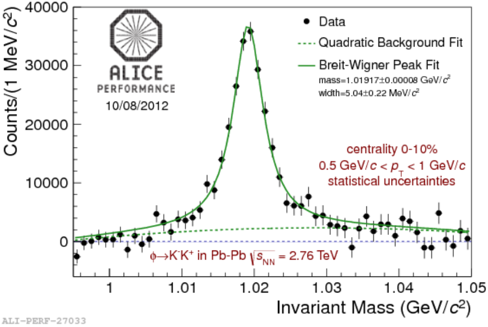
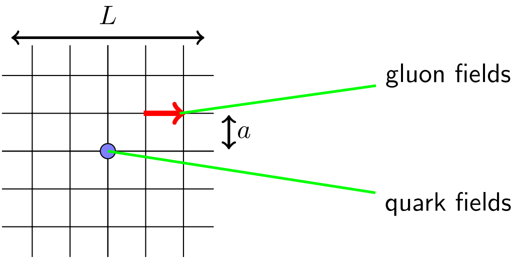



$$\mathcal{A} = \mathcal{A}_{\mathrm{bg}} + \mathcal{A}_{\mathrm{BW}}$$

$$\mathcal{A}_{\mathrm{BW}} \propto \frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

# Lattice simulations

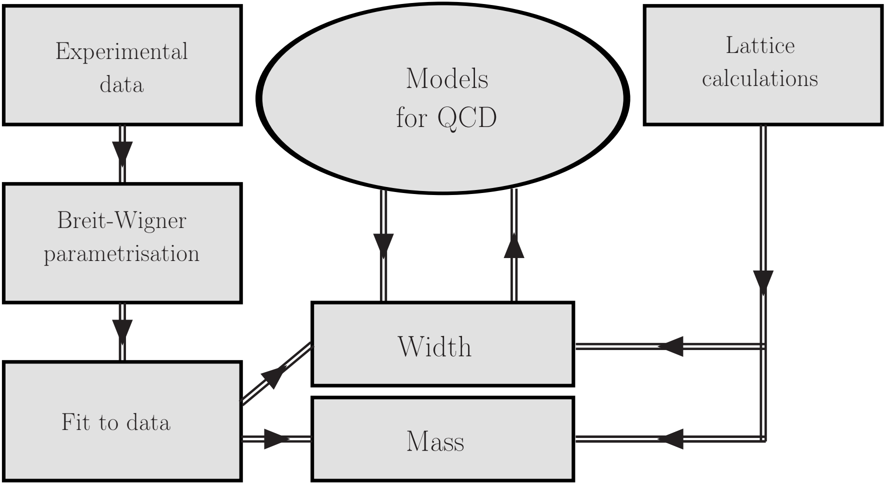



$$C_{ij}(t) = \langle 0|O_i(t)O_j(0)|0\rangle = \sum_n \frac{e^{-E_n t}}{2E_n}\langle 0|O_i(0)|n\rangle\langle n|O_j^\dagger(0)|0\rangle$$

Approaching the real world:

- Continuum limit $\Longrightarrow a \to 0$
- Infinite box $\Longrightarrow L \to \infty$
- Unphysical light quark mass $\Longrightarrow$ Chiral extrapolation

# Approach to ordinary hadrons



# Potential quark model

# Particle propagator in external field

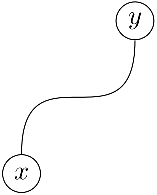
Scalar particle in external field ($D_\mu = \partial_\mu - ieA_\mu$)

$$G(x,y|A) = (m^2 - D^2)^{-1}_{xy} = \langle y| \int_0^\infty ds e^{-s(m^2-D^2)} |x\rangle$$

Fock-Feynman-Schwinger representation

$$G(x,y|A) = \int_0^\infty ds \int (Dz)_{xy}\, e^{-K} \exp\left( ie \int_x^y A_\mu(z) dz_\mu \right)$$

$$K = m^2 s + \frac{1}{4}\int_0^s d\tau \left(\frac{dz_\mu}{d\tau}\right)^2$$



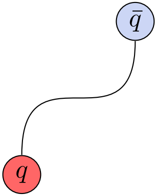
# Meson operator

Gauge invariant nonlocal object

$$\Psi(x,y) = \bar{\psi}_\alpha(x)\Phi^\alpha_\beta(x,y)\psi^\beta(y)$$

with non-abelien parallel transporter (Schwinger line)

$$\Phi = P\exp\left(ig\int_x^y A^a_\mu \frac{\lambda^a}{2} dz_\mu\right)$$

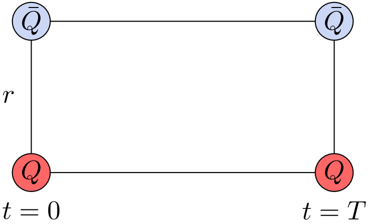



$$A^a_\mu = \underbrace{B^a_\mu\text{(background field)}}_{\text{Confinement}} + \underbrace{a^a_\mu\text{(perturbative excitation)}}_{\text{colour Coulomb interaction}}$$

# Wilson loop, area law, confinement

Propagation of $\bar{Q}Q$ meson

$$G = \langle \Psi(x', y') \Psi^\dagger(x, y) \rangle = \mathrm{Tr}\left[ S_Q(x, x'|B) \Phi(x', y') S_{\bar{Q}}(y', y|B) \Phi(y, x) \right]$$




Wilson loop and area law

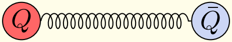
$$\langle W(C) \rangle = \langle \mathrm{Tr} P \exp(ig \oint_C B_\mu^a \frac{\lambda^a}{2} dz_\mu) \rangle_B \sim e^{-\sigma S_{\min}} \sim e^{-\int_0^T V_{\mathrm{conf}}(r) dt}$$

$$V_{\mathrm{conf}}(r) = \sigma r$$

# Wilson loop, area law, confinement

Propagation of $\bar{Q}Q$ meson

Phenomenological static potential between quarks




$$V(r) = \sigma r - \frac{C_F \alpha_s}{r} \qquad \text{(Cornell potential)}$$

$$\left(\frac{\lambda^a}{2} \times \frac{\lambda^a}{2}\right)_\alpha^\beta = C_F \delta_\alpha^\beta \qquad C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}$$

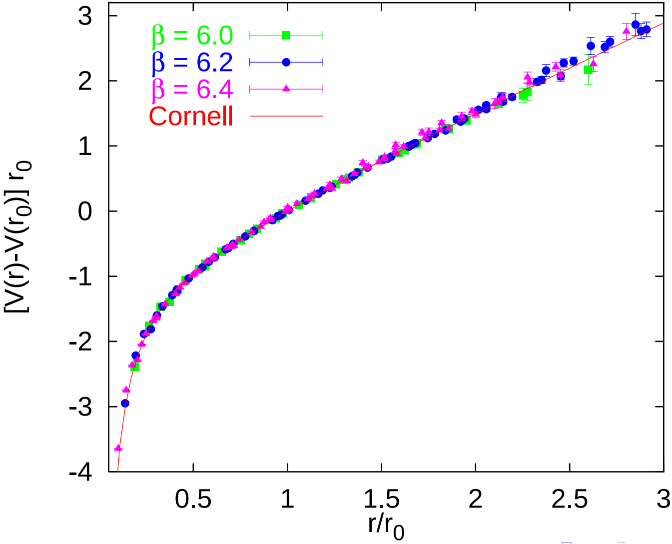
$$\sigma = \sigma_F = \frac{4}{3}\sigma_0$$

$$\sigma \simeq 0.16..0.2 \text{ GeV}^2 \qquad \alpha_s \simeq 0.3..0.6$$

$V_{\text{conf}}(r) = \sigma r$

# Static $Q\bar{Q}$ potential from lattice

(Bali'2001)



# Radial Regge trajectory for light mesons

Salpeter equation for light-light meson

$$\left(2\sqrt{p_r^2+m_q^2}+\sigma r\right)\psi = M\psi$$

WKB spectrum for $m_q \ll \sqrt{\sigma} \quad \Longrightarrow \quad M_n^2 = 4\pi\sigma n + \ldots$

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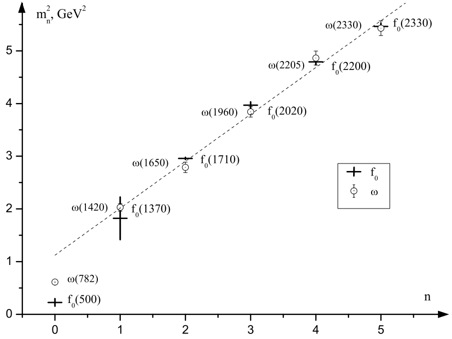
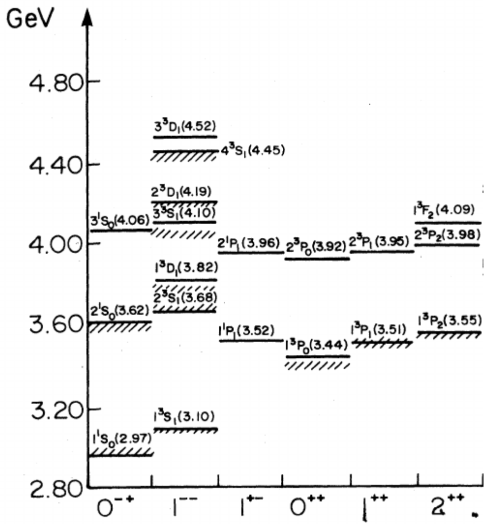



(Adapted from Afonin, Solomko, Eur.Phys.J.C 82 (2022) 3, 195)

# Meson spectrum in relativistic potential quark model

(Godfrey,Isgur'1985)




Masses[a]

$$\tfrac{1}{2}(m_u + m_d) = 220 \ \text{MeV}$$
$$m_s = 419 \ \text{MeV}$$
$$m_c = 1628 \ \text{MeV}$$
$$m_b = 4977 \ \text{MeV}$$

Potentials

$$b = 0.18 \ \text{GeV}^2$$
$$\alpha_s^{\text{critical}} = 0.60$$

# Useful trick: Einbein field

(Brink,Di Vecchia,Howe'1977)

- Simple idea (einbein field)

$$\sqrt{ab} = \left(\frac{a\nu}{2} + \frac{b}{2\nu}\right)\Big|_{\nu=\nu_{\rm ext}} \qquad \nu_{\rm ext} = \sqrt{\frac{b}{a}}$$

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$$\sigma r \quad \longrightarrow \quad \frac{\nu}{2} + \frac{\sigma^2 r^2}{2\nu}$$

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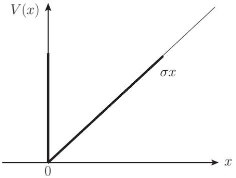
- Simplified kinetic term ($\mu_{\rm ext} \simeq \langle \sqrt{\boldsymbol{p}^2 + m^2} \rangle$ — effective quark mass)

$$\sqrt{\boldsymbol{p}^2 + m^2} \quad \longrightarrow \quad \frac{\boldsymbol{p}^2 + m^2}{2\mu} + \frac{\mu}{2}$$

# Useful trick at work

One-dimensional linear potential

$$V(x) = \begin{cases} \sigma x, & x \geqslant 0 \\ \infty, & x < 0 \end{cases}$$




Schrödinger equation and wave function

$$\left(\frac{p^2}{2m} + V(x)\right)\psi = E\psi \qquad \psi_n(x) = \mathcal{N}_n Ai(y) \qquad y = (2m\sigma)^{1/3}(x - E_n/\sigma)$$

Quantisation condition

$$\psi_n(x=0) = 0 \quad \Longrightarrow \quad E_n = -\frac{\sigma^{2/3}}{(2m)^{1/3}} a_n \ \text{ with } \ Ai(a_n) = 0$$

Approximate formula for zeros of Airy function ($(3\pi/2)^{2/3} \approx 2.81$)

$$a_n \approx -\left(\frac{3\pi}{2}\right)^{2/3}\left(n - \frac{1}{4}\right)^{2/3} \qquad n = 1, 2, \ldots$$

# Useful trick at work

One-dimensional linear potential


In einbein field formalism:

$$\sigma x \to \frac{\nu}{2} + \frac{1}{2} m\omega^2 x^2 \qquad \omega = \frac{\sigma}{\sqrt{m\nu}}$$

$$E_n = \frac{\nu}{2} + \omega \left[ \{2(n-1)+1\} + \frac{1}{2} \right] \quad \Longrightarrow \quad -\frac{\sigma^{2/3}}{(2m)^{1/3}} a_n^{\rm ein}$$

$$a_n^{\rm ein} = -3\left(n - \frac{1}{4}\right)^{2/3} \qquad n = 1, 2, \ldots$$

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# Behind the trick: Free relativistic particle

- Lagrange function & reparametrisation (gauge) invariance

$$S = \int_{\tau_i}^{\tau_f} \mathcal{L} d\tau \qquad \mathcal{L} = -m\sqrt{\dot{x}^2} \qquad \tau \to f(\tau) \quad \Longrightarrow \quad S = \text{inv}$$

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- Hamilton function

$$p_\mu = \frac{\partial \mathcal{L}}{\partial \dot{x}^\mu} = -m\frac{\dot{x}_\mu}{\sqrt{\dot{x}^2}} \quad \Longrightarrow \quad \mathcal{H} = p_\mu \dot{x}^\mu - \mathcal{L} = 0$$

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$$p^2 = m^2$$

- Quantisation $\Longrightarrow$ Klein-Gordon equation

$$(\Box + m^2)\phi = 0$$

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- Different forms of dinamics (different "gauges")

instant form: $\tau = x_0 \Longrightarrow \mathcal{L} = -m\sqrt{1 - \dot{\boldsymbol{x}}^2} \Longrightarrow H = \sqrt{\boldsymbol{p}^2 + m^2}$

light cone form: $\tau = x_+ = (x_0 + x_3)/\sqrt{2} \Longrightarrow H = (\boldsymbol{p}_\perp^2 + m^2)/(2p_+)$

# Behi ... rticle

- Lagrange

$S =$ ... $S = $ inv

- Hamilton

... $= 0$

- Contraine

- Quantisat

- Different

instant fo ... $\sqrt{\boldsymbol{p}^2 + m^2}$

light cone form: $\tau = x_+ = (x_0 + x_3)/\sqrt{2} \implies H = (\boldsymbol{p}_\perp^2 + m^2)/(2p_+)$

In einbein field formalism:

$$\mathcal{L} = -\frac{m^2}{2\mu} - \frac{\mu \dot{x}^2}{2}$$

$$p_\mu = \frac{\partial \mathcal{L}}{\partial \dot{x}^\mu} = -\mu \dot{x}_\mu$$

$$\pi = \frac{\partial \mathcal{L}}{\partial \dot{\mu}} = 0 \quad \implies \quad \varphi_1 = \pi$$

$$\mathcal{H} = p_\mu \dot{x}^\mu + \pi \dot{\mu} - \mathcal{L} = -\frac{1}{2\mu}(p^2 - m^2)$$

$$\varphi_2 = \{\varphi_1 \mathcal{H}\} = \frac{\partial \mathcal{H}}{\partial \mu} = \frac{1}{2\mu^2}(p^2 - m^2)$$

$$\{\varphi_1 \varphi_2\} \approx 0 \qquad \mathcal{H} \approx 0$$

Quantisation:

$$\frac{\partial \phi}{\partial \mu} = 0 \qquad (\Box + m^2)\phi = 0$$

# Two free relativistic particles

(Kalashnikova,AN'1997)

- Two einbein fields

$$\mathcal{L} = -m_1\sqrt{\dot{x}_1^2} - m_2\sqrt{\dot{x}_2^2} \quad \to \quad -\frac{\mu_1}{2} - \frac{\mu_1\dot{x}_1^2}{2} - \frac{\mu_2}{2} - \frac{\mu_2\dot{x}_2^2}{2}$$

- Centre-of-mass motion separation

$$x = x_1 - x_2 \qquad X = \zeta x_1 + (1-\zeta)x_2 \qquad \zeta = \frac{\mu_1}{\mu_1+\mu_2} \qquad M = \mu_1 + \mu_2$$

- Primary constraints

$$\varphi_1 = \Pi \qquad \varphi_2 = \pi + (Px) = 0$$

- Conclusion: auxiliary variables may mix with physical degrees of freedom
- Proper time gauge $(Px) = 0$ (co-moving frame)

$$\left[\Box_X + \left(\sqrt{p_i^2+m_1^2} + \sqrt{p_i^2+m_2^2}\right)^2\right]\Psi(X,x) = 0 \qquad p_i = e_\mu^{(i)}p^\mu$$

# Two free relativistic particles

(Kalashnikova,AN'1997)

- Two einbein fields
- Ce

$x$ ... $\mu_1 + \mu_2$

- Pr
- Co ... dom

$$H = \sqrt{\boldsymbol{P}^2 + \mathcal{M}^2}$$

$$\mathcal{M} = \sqrt{\boldsymbol{k}^2 + m_1^2} + \sqrt{\boldsymbol{k}^2 + m_2^2}$$

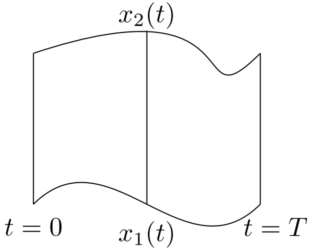
$$\boldsymbol{k} = \boldsymbol{p}_1 - \frac{\boldsymbol{P}\sqrt{\boldsymbol{p}_1^2 + m_1^2}}{\mathcal{M}} + \frac{(\boldsymbol{P}\boldsymbol{p}_1)\boldsymbol{P}}{\mathcal{M}(\mathcal{M}+H)} \qquad \boldsymbol{P} = \boldsymbol{p}_1 + \boldsymbol{p}_2$$

- Proper time gauge $(Px) = 0$ (co-moving frame)

$$\left[\Box_X + \left(\sqrt{p_i^2 + m_1^2} + \sqrt{p_i^2 + m_2^2}\right)^2\right]\Psi(X, x) = 0 \qquad p_i = e_\mu^{(i)} p^\mu$$

# QCD string

(Dubin,Kaidalov,Simonov'1993)




$$S_{\min} = \int_0^T dt \int_0^1 d\beta \sqrt{(\dot{w}w')^2 - \dot{w}^2 w'^2}$$

$$w(\beta,t) = \beta x_1(t) + (1-\beta)x_2(t)$$

$$x_{10} = x_{20} = t$$

# QCD string

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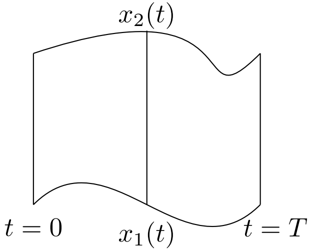



$$S_{\min}=\int_0^T dt\int_0^1 d\beta\sqrt{(\dot{w}w')^2-\dot{w}^2w'^2}$$

$$w(\beta,t)=\beta x_1(t)+(1-\beta)x_2(t)$$

$$x_{10}=x_{20}=t$$

$$H=\sum_{i=1}^{2}\left(\frac{p_r^2+m_i^2}{2\mu_i}+\frac{\mu_i}{2}\right)+\int_0^1 d\beta\left(\frac{\sigma^2r^2}{2\nu}+\frac{\nu}{2}\right)$$

$$+\frac{\boldsymbol{L}^2}{2r^2[\mu_1(1-\zeta)^2+\mu_2\zeta^2+\int_0^1 d\beta\nu(\beta-\zeta)^2]}$$

$$\zeta=\frac{\mu_1+\int_0^1 d\beta\nu\beta}{\mu_1+\mu_2+\int_0^1 d\beta\nu}$$

# QCD string

(Dubin,Kaidalov,Simonov'1993)

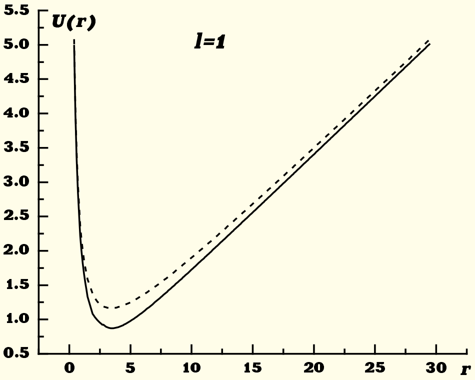
$\dot{w}^2 w'^2$

$_2(t)$

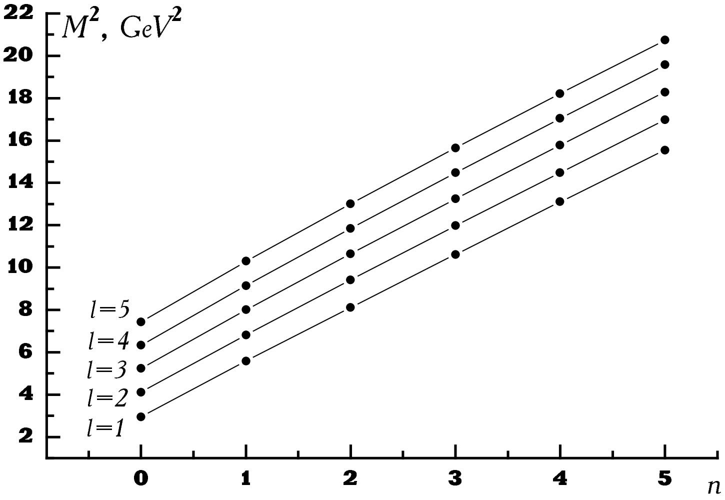
$t = 0$

$H = \sum_{i}$

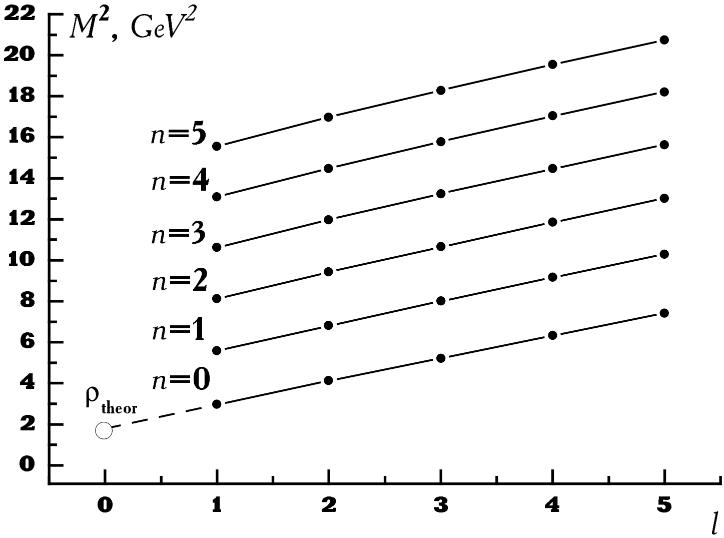
$-\zeta)^2]$



# Radial Regge trajectories



# Angular momentum Regge trajectories



# Foldy-Wouthuysen transformation in QED

- Free Dirac equation

$$H = \boldsymbol{\alpha}\boldsymbol{p} + \beta m \quad \Longrightarrow \quad \tilde{H} = U_F H U_F^\dagger = \begin{pmatrix} \sqrt{\boldsymbol{p}^2 + m^2} & 0 \\ 0 & -\sqrt{\boldsymbol{p}^2 + m^2} \end{pmatrix}$$

$$U_F = \cos\theta + (\boldsymbol{\gamma}\boldsymbol{n})\sin\theta \qquad \boldsymbol{n} = \frac{\boldsymbol{p}}{|\boldsymbol{p}|} \qquad \tan 2\theta = \frac{|\boldsymbol{p}|}{m}$$

- Dirac equation in external field to order $\mathcal{O}(1/m)$ (Pauli equation)

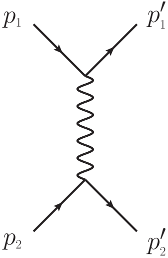
$$H = \boldsymbol{\alpha}(\boldsymbol{p} - e\boldsymbol{A}) + \beta m + eA_0 \quad \Longrightarrow \quad \tilde{H} = \frac{(\boldsymbol{p} - e\boldsymbol{A})^2}{2m} - \frac{e}{2m}\boldsymbol{\sigma}\boldsymbol{H} + eA_0$$

- Spin-orbital interaction to order $\mathcal{O}(1/m^2)$ ($\boldsymbol{A} = 0$, $eA_0 = V(r)$)

$$V_{LS} = -\frac{e}{4m^2}\boldsymbol{\sigma}[\boldsymbol{E} \times \boldsymbol{p}] \underset{\text{central field}}{=} \frac{\boldsymbol{S}\boldsymbol{L}}{2m^2 r} V'(r)$$

# Breit equation in QED

Interaction potential is Fourier transform of the Born amplitude




$$\hat{U}(\hat{\mathbf{p}}_1,\ \hat{\mathbf{p}}_2,\ \mathbf{r}) =$$

$$= \frac{e^2}{r} - \frac{\pi e^2 \hbar^2}{2c^2}\left(\frac{1}{m_1^2} + \frac{1}{m_2^2}\right)\delta(\mathbf{r}) - \frac{e^2}{2m_1 m_2 c^2 r}\left[\hat{\mathbf{p}}_1\hat{\mathbf{p}}_2 + \frac{\mathbf{r}(\mathbf{r}\hat{\mathbf{p}}_1)\hat{\mathbf{p}}_2}{r^2}\right] -$$

$$- \frac{e^2\hbar}{4m_1^2c^2r^3}[\mathbf{r}\hat{\mathbf{p}}_1]\sigma_1 + \frac{e^2\hbar}{4m_2^2c^2r^3}[\mathbf{r}\hat{\mathbf{p}}_2]\sigma_2 - \frac{e^2\hbar}{2m_1m_2c^2r^3}\{[\mathbf{r}\hat{\mathbf{p}}_1]\sigma_2 - [\mathbf{r}\hat{\mathbf{p}}_2]\sigma_1\} +$$

$$+ \frac{e^2\hbar^2}{4m_1m_2c^2}\left\{\frac{\sigma_1\sigma_2}{r^3} - 3\frac{(\sigma_1\mathbf{r})(\sigma_2\mathbf{r})}{r^5} - \frac{8\pi}{3}\sigma_1\sigma_2\delta(\mathbf{r})\right\}. \qquad (83,15)$$

# Spin-dependent interactions in mesons

(Eichten,Feinberg'1981;Gromes'1984)

- Spin-dependent potentials to order $\mathcal{O}(1/m^2)$

$$
\begin{aligned}
V_{\rm SD}(r) &= \left(\frac{\boldsymbol{S}_1\boldsymbol{L}}{2m_1^2 r}+\frac{\boldsymbol{S}_2\boldsymbol{L}}{2m_2^2 r}\right)\left[V_0'(r)+2V_1'(r)\right]+\frac{(\boldsymbol{S}_1+\boldsymbol{S}_2)\boldsymbol{L}}{m_1 m_2 r}V_2'(r) \\
&+ \frac{(3(\boldsymbol{S}_1\boldsymbol{n})(\boldsymbol{S}_2\boldsymbol{n})-\boldsymbol{S}_1\boldsymbol{S}_2)}{3m_1m_2}V_3(r)+\frac{\boldsymbol{S}_1\boldsymbol{S}_2}{3m_1m_2}V_4(r)
\end{aligned}
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$$V_0'(r)+V_1'(r)-V_2'(r)=0$$

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- Gromes relation

$$V_0'(r)+V_1'(r)-V_2'(r)=0$$

- Spin-dependent interaction from Cornell potential

$$V_0'(r)=\sigma+\frac{4}{3}\frac{\alpha_s}{r^2} \qquad V_1'(r)=-\sigma \qquad V_2'(r)=\frac{4}{3}\frac{\alpha_s}{r^2}$$

$$V_3(r)=\frac{4\alpha_s}{r^3} \qquad V_4(r)=\frac{32}{3}\pi\alpha_s\delta^{(3)}(\boldsymbol{r})$$

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\end{aligned}
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- Gror

- Spin-

- In einbein field formalism $\Longrightarrow$ $m \to \mu$
- For heavy quarks $\Longrightarrow$ $\mu \approx m$
- For light quarks $\Longrightarrow$ $\mu \gg m$

$$V_0'(r)=\sigma+\frac{4}{3}\frac{\alpha_s}{r^2} \qquad V_1'(r)=-\sigma \qquad V_2'(r)=\frac{4}{3}\frac{\alpha_s}{r^2}$$

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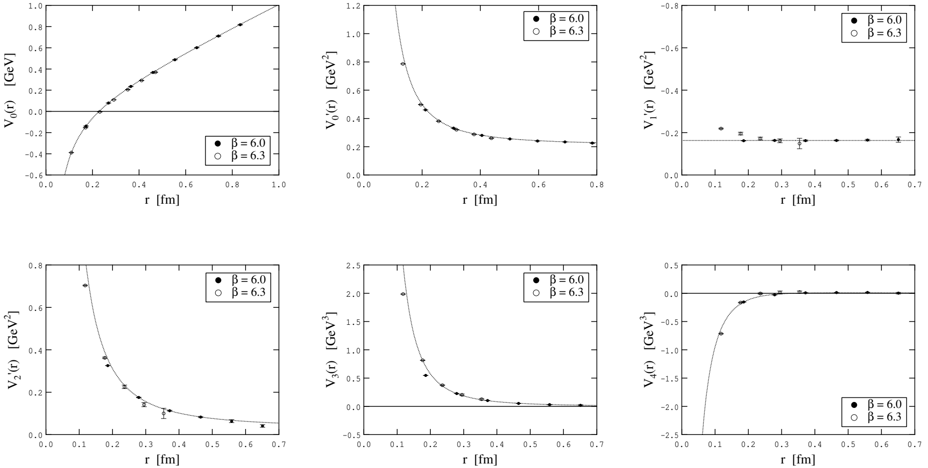
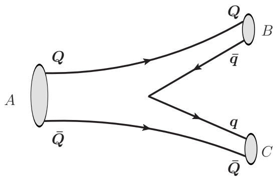
# Spin-dependent potentials from lattice

(Koma & Koma'2007)



# Decays




Pair creation Hamiltonian ($^3P_0$ model)

$$H_{\text{int}} = g \int d^3x \, \bar{\psi}\psi$$

Leading contribution to the mass

$$\Delta M_{\text{A}} = \int \frac{d^3p}{(2\pi)^3} \frac{|\langle \text{BC}|H_{\text{int}}|\text{A}\rangle|^2}{M_{\text{A}} - E_{\text{BC}} + i0} \qquad E_{\text{BC}} = \sqrt{\boldsymbol{p}^2 + M_{\text{B}}^2} + \sqrt{\boldsymbol{p}^2 + M_{\text{C}}^2}$$

Mass shift and decay width

$$\delta M_{\text{A}} = \text{Re}(\Delta M_{\text{A}}) \qquad \Gamma(\text{A} \to \text{BC}) = -2\text{Im}(\Delta M_{\text{A}})$$

# Problem with the pion

- Typical mass of not excited hadron

  $$M \simeq 2E_q + \sigma \langle r \rangle \simeq 2 \times 300 \text{ MeV} + (0.16 \text{ GeV}^2) \times (0.6 \text{ fm}) \sim 1 \text{ GeV}$$

- Typical size of spin-dependent interactions

  $$\Delta M_{\rm SD} \simeq M_{\rho(1450)} - M_{\pi(1300)} \simeq 150 \text{ MeV}$$

- Physical mass of the $\rho$ meson

  $$M_\rho \simeq 770 \text{ MeV} \ \sim 1 \text{ GeV}$$

- Naively expected mass of the pion

  $$M_\pi \simeq M_\rho - 150 \text{ MeV} \simeq 600 \text{ MeV}$$

- Physical mass of the pion

  $$M_\pi \simeq 140 \text{ MeV} \ \ll 1 \text{ GeV}$$

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Why is the pion $\pi(140)$ so light?!

# Spontaneous breaking of chiral symmerty

# Chiral symmetry

- Free Dirac Lagrangian

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi$$

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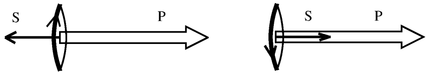
$$\psi_R = \frac{1}{2}(1+\gamma^5)\psi \quad \psi_L = \frac{1}{2}(1-\gamma^5)\psi \quad \psi = \psi_R + \psi_L$$



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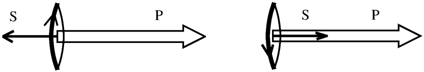


- Simple but instructive decomposition

$$\bar{\psi}\gamma^\mu\psi = \bar{\psi}_R\gamma^\mu\psi_R + \bar{\psi}_L\gamma^\mu\psi_L \qquad \bar{\psi}\psi = \bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R$$

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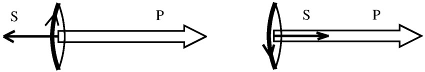


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- In the strict chiral limit ($m = 0$)
  - $\psi_L$ and $\psi_R$ are decoupled and transform independently
  - Axial current $j^5_\mu = \bar{\psi}\gamma^5\gamma_\mu\psi$ is conserved $\implies$ $[Q_5 H] = 0$
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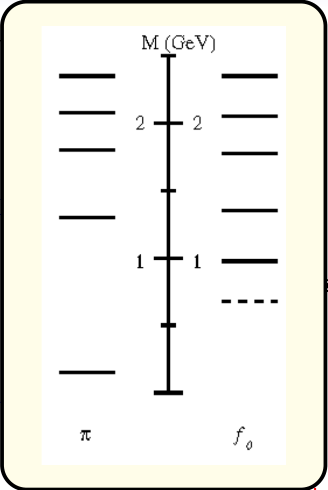


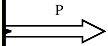


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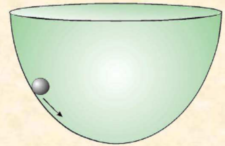
# Spontaneous breaking of symmetry



- Symmetric vacuum is stable
- Massive excitations over vacuum

$$V(\phi) = \underbrace{V(0)}_{\text{Energy shift}} + \frac{1}{2}\underbrace{V''(0)}_{m^2>0}\phi^2 + \ldots$$

- No tachyon in the spectrum
- Spectrum of excitations inherits symmetry of the vacuum

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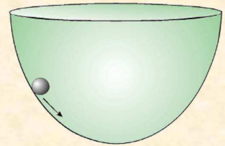

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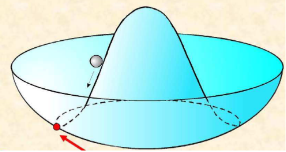
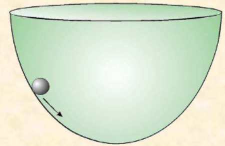
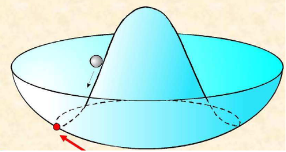


- Symmetric vacuum is unstable
- Tachyon in the spectrum built over symmetric vacuum: $V''(0) = m^2 < 0$
- True vacuum is selected among many possibilities $\Longrightarrow$ Symmetry is spontaneously broken
- Physical vacuum and excitations over it are NOT symmetric
- Massless mode — Goldstone boson

# Spontaneous breaking of symmetry





- Symm
- Massi

$V(\phi)$ … $\underbrace{(0)}_{\text{Energy shift}}$ … $2$ … $\underbrace{(0)\phi}_{m^2>0}$ … $+\dots$

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over

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How can we have the pion as a Goldstone boson related to spontaneous breaking of chiral symmetry and a quark-antiquark meson at the same time?

# From full QCD to chiral quark model

- Stationary point approach

$$\int dA e^{-f(A)} \sim e^{-f(A_0)} \qquad f'(A_0) = 0$$

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- Integrating out "gluons" ($A^3$ and $A^4$ terms neglected)

$$\int DA\, e^{\int(-A^2+2AJ)d^4x} \sim e^{\int J^2 d^4x} \qquad J \sim \bar{\psi}\mathcal{O}\psi$$

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- Spontaneous chiral symmetry breaking in simple (NJL) model for QCD

(Nambu,Jona-Lasinio'1961)

$$\mathcal{L}_{\text{int}} = \frac{\lambda}{4}\int d^3x \left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma^5\psi\right)^2\right] = \lambda \int d^3x \left(\bar{\psi}_R\psi_L\right)\left(\bar{\psi}_L\psi_R\right)$$

$$S = S_0 + S_0\Sigma S_0 + \dots \quad \Longrightarrow \quad S^{-1} = S_0^{-1} - \Sigma$$

$$\Sigma = m = 2$$

$$= \frac{i}{2}\lambda \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr} S(p) = \lambda \int \frac{d^3p}{(2\pi)^3} \frac{m}{\sqrt{\boldsymbol{p}^2 + m^2}}$$

# From full QCD to chiral quark model

Gap (mass-gap equation):

$$m\left(1-\lambda\int^{\Lambda}\frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{\boldsymbol{p}^2+m^2}}\right)=0$$

- Weak coupling regime $\lambda < \lambda_{\rm crit} = \left(\frac{2\pi}{\Lambda}\right)^2$

$$m=0$$

- Strong coupling regime $\lambda > \lambda_{\rm crit}$

$$m \neq 0 \quad \Longrightarrow \quad \text{Gap in the spectrum of excitations}$$

$$\langle\bar{\psi}\psi\rangle \neq 0 \quad \Longrightarrow \quad \text{Chiral symmetry is broken spontaneously}$$

1961)

$$S=S_0+S_0\Sigma S_0+\ldots \quad \Longrightarrow \quad S^{-1}=S_0^{-1}-\Sigma$$

$\Sigma = m = 2$  $= \dfrac{i}{2}\lambda\displaystyle\int\frac{d^4p}{(2\pi)^4}{\rm Tr}S(p) = \lambda\int\frac{d^3p}{(2\pi)^3}\frac{m}{\sqrt{\boldsymbol{p}^2+m^2}}$

# Model for QCD in two dimensions

# QCD$_2$ in axial gauge

('t Hooft'1974;Bars,Green'1978)

- Lagrangian of QCD$_2$ ('t Hooft model)

$$L(x) = -\frac{1}{4}F^a_{\mu\nu}(x)F^a_{\mu\nu}(x) + \bar{\psi}(x)\Big[i(\partial_\mu - igA^a_\mu t^a)\gamma_\mu - m\Big]\psi(x)$$

- "Dirac" matrices

$$\gamma_0 \equiv \beta = \sigma_3 \quad \gamma_1 = i\sigma_2 \quad \gamma_5 \equiv \alpha = \gamma_0\gamma_1 = \sigma_1$$

- Axial (Coulomb) gauge

$$A_1(x_0, x) = 0 \qquad D^{ab}_{00}(x_0 - y_0, x - y) = -\frac{i}{2}\delta^{ab}|x-y|\delta(x_0 - y_0)$$

- Interaction Hamiltonian

$$H_{\rm int} = -\frac{g^2}{2}\int dxdy \left(\psi^\dagger(t,x)\frac{\lambda^a}{2}\psi(t,x)\right)|x-y|\left(\psi^\dagger(t,y)\frac{\lambda^a}{2}\psi(t,y)\right)$$

- Large-$N_c$ limit

$$\gamma = \frac{g^2 N_c}{4\pi} \underset{N_c\to\infty}{\to} \text{const}$$

# $\mathrm{QCD}_2$ in axial gauge

('t Hooft'1974;Bars,Green'1978)

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$$\psi(t,x) = \int \frac{dk}{2\pi} e^{ikx}[b(k,t)u(k) + d^\dagger(-k,t)v(-k)]$$

$$u(k) = T(k)\begin{pmatrix}1\\0\end{pmatrix} \quad v(-k) = T(k)\begin{pmatrix}0\\1\end{pmatrix} \quad T(k) = e^{-\frac{1}{2}\theta(k)\gamma_1}$$

- Interaction Hamiltonian

$$H_{\rm int} = -\frac{g^2}{2}\int dxdy\left(\psi^\dagger(t,x)\frac{\lambda^a}{2}\psi(t,x)\right)|x-y|\left(\psi^\dagger(t,y)\frac{\lambda^a}{2}\psi(t,y)\right)$$

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# Bogoliubov transformation: From "bare" to "dressed" particles

- Hamiltonian in terms of "bare" particles (bosons)

$$H = h_1 a^\dagger a + \frac{1}{2} h_2 (a^\dagger a^\dagger + aa)$$

- "Dressed" particles (quasiparticles)

$$a = ub + vb^\dagger \qquad a^\dagger = ub^\dagger + vb \qquad u^2 - v^2 = 1 \implies [bb^\dagger] = [aa^\dagger] = 1$$

with convenient parametrisation: $u = \cosh\theta$ and $v = \sinh\theta$

- Hamiltonian in terms of dressed operators ($H = H_0 + :H_2:$)

$$H_0 = -\frac{1}{2} h_1 + \frac{1}{2}\left(h_1 \cosh 2\theta + h_2 \sinh 2\theta\right)$$

$$:H_2: = (h_1 \cosh 2\theta + h_2 \sinh 2\theta) b^\dagger b + \frac{1}{2}(h_1 \sinh 2\theta + h_2 \cosh 2\theta)(b^\dagger b^\dagger + bb)$$

# Bogoliubov transformation: From "bare" to "dressed" particles

- Hami
- "Dres

$a$ ... $] = 1$

with

- Hami

$: H_2$ ... $b^\dagger + bb)$ ... $2$

---

- Condition (equation for $\theta$)

$$h_1 \sinh 2\theta + h_2 \cosh 2\theta = 0$$

ensures both $E_{\rm vac}$=min and $: H_2 :$ is diagonal

- Physical versus trivial vacuum

$$a\,|0\rangle_0 = 0 \qquad b\,|0\rangle = 0 \qquad |0\rangle \neq |0\rangle_0$$

- Diagonalised Hamiltonian in terms of dressed operators

$$H = \frac{1}{2}\left(\sqrt{h_1^2 - h_2^2} - h_1\right) + \sqrt{h_1^2 - h_2^2}\, b^\dagger b$$

- Trivial vacuum (with $E_{\rm vac}^{(0)} = 0$) is unstable

$$E_{\rm vac} = \langle 0|H|0\rangle < 0$$

# Physical vacuum versus trivial vacuum

Before Bogoliubov-Valatin transformation

$$a\,|0\rangle_0 = 0$$

After Bogoliubov-Valatin transformation

$$b\,|0\rangle = 0 \qquad b = ua - va^\dagger \qquad |0\rangle = \sum_{n=0}^{\infty} C_n\,|n\rangle_0$$

$$(ua - va^\dagger)\sum_{n=0}^{\infty}|0\rangle_0 = 0 \quad \Longrightarrow \quad C_{2n+1} = 0 \quad C_{2n} = \left(\frac{v}{u}\right)^n \sqrt{\frac{(2n-1)!!}{(2n)!!}}$$

$$|0\rangle = \sum_{n=0}^{\infty} (\tanh\theta)^n \sqrt{\frac{(2n-1)!!}{(2n)!!}}\,|2n\rangle$$

# Hamiltonian approach to 't Hooft model

- Normally ordered Hamiltonian ($\psi \sim b + d^\dagger$)

$$H = H_0 + :H_2: + :H_4:$$

- The vacuum energy is a minimum

$$E_{\rm vac} = \langle 0|H|0\rangle = H_0 = \text{min}$$

- Quadratic part $:H_2:$ (describes dressing of quarks) is diagonal
- Quartic part $:H_4:$ (describes interaction of dressed quarks) is suppressed by $N_c$
- Mass-gap equation

$$p\cos\theta(p) - m\sin\theta(p) = \frac{\gamma}{2}\int\frac{dk}{(p-k)^2}\sin[\theta(p)-\theta(k)]$$

- Dressed quark dispersion law

$$E_p = m\cos\theta(p) + p\sin\theta(p) + \frac{\gamma}{2}\int\frac{dk}{(p-k)^2}\cos[\theta(p)-\theta(k)]$$

# Solutions of mass-gap equation in 't Hooft model

- Free solution

$$\theta = \arctan \frac{p}{m} \qquad E_p = \sqrt{p^2 + m^2}$$

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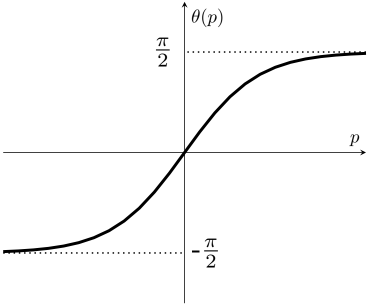
- Physical chirally nonsymmetric solution




$$\langle\bar\psi\psi\rangle = -\frac{N_c}{\pi}\int_0^\infty dp\cos\theta(p) \neq 0$$

$$\langle\bar\psi\psi\rangle_{m=0} = -\frac{1}{\sqrt{6}}N_c\sqrt{\gamma}$$

# Bound state equation

Operators creating and annihilating quark-antiquark pairs

$$M^\dagger(p,p') = \frac{1}{\sqrt{N_c}}\sum_\alpha b^\dagger_\alpha(p')d^\dagger_\alpha(-p) \qquad M(p,p') = \frac{1}{\sqrt{N_c}}\sum_\alpha d_\alpha(-p)b_\alpha(p')$$

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Hamiltonian in terms of such compound operators

$$H \sim H_0 + M^\dagger M + \frac{1}{2}\left(M^\dagger M^\dagger + MM\right)$$

# Bound state equation

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$$m_n^\dagger(Q) = \int \frac{dq}{2\pi} \left\{ M^\dagger(q-Q,q)\varphi_+^n(q,Q) + M(q,q-Q)\varphi_-^n(q,Q) \right\}$$

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Orthogonality & completeness

$$\int \frac{dp}{2\pi} \left( \varphi_+^n(p,Q)\varphi_+^{n'}(p,Q) - \varphi_-^n(p,Q)\varphi_-^{n'}(p,Q) \right) = \delta_{nn'}$$

$$\sum_{n=0}^{\infty} (\varphi_+^n(p,Q)\varphi_+^n(k,Q) - \varphi_-^n(p,Q)\varphi_-^n(k,Q)) = 2\pi\delta(p-k)$$

# Bound state equation

Operators creating and annihilating quark-antiquark pairs

$$M^\dagger(p,p') = \frac{1}{\sqrt{N}} \sum b^\dagger_\alpha(p') d^\dagger_\alpha(-p) \qquad M(p,p') = \frac{1}{\sqrt{N}} \sum d_\alpha(-p) b_\alpha(p')$$

Hamilton

Meson cr

$- v^2 = 1)$

Orthogon

- Each meson is described by two wave functions simultaneously, $\varphi^\pm$
- $\varphi^+$ describes forward in time motion of $q\bar{q}$ pair
- $\varphi^-$ describes backward in time motion of $q\bar{q}$ pair
- Quark and antiquark simultaneously proceed from forward to backward in time motion and vice versa because of instantaneous interaction
- $\varphi^+$ and $\varphi^-$ obey system of coupled equations (bound state equation)
- For excited mesons, $|\varphi^-| \ll |\varphi^+|$

$$\sum_{n=0}^{\infty} (\varphi^n_+(p,Q)\varphi^n_+(k,Q) - \varphi^n_-(p,Q)\varphi^n_-(k,Q)) = 2\pi\delta(p-k)$$

# The chiral pion

Solution of bound state equation for the chiral pion

$$\varphi_{\pm}^{\pi}(p,Q) = \sqrt{\frac{\pi}{2Q}}\left(\cos\frac{\theta(Q-p)-\theta(p)}{2} \pm \sin\frac{\theta(Q-p)+\theta(p)}{2}\right)$$

The pion decay constant $f_\pi$

$$\langle\Omega\,|\,J_\mu^5(x)\,|\,\pi(Q)\,\rangle = f_\pi Q_\mu \frac{e^{-iQx}}{\sqrt{2Q_0}} \qquad f_\pi = \sqrt{\frac{N_c}{\pi}}$$

Pion mass

$$M_\pi^2 = 2m\int_0^\infty dp\cos\theta(p)$$

Gell-Mann-Oakes-Renner relation

$$f_\pi^2 M_\pi^2 = -2m\langle\bar{\psi}\psi\rangle$$

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Solution of bound state equation for the chiral pion

$$\varphi^{\pi}_{\pm}(p,Q)=\sqrt{\frac{\pi}{2Q}}\left(\cos\frac{\theta(Q-p)-\theta(p)}{2}\pm\sin\frac{\theta(Q-p)+\theta(p)}{2}\right)$$

- Pion as Goldstone boson for SBCS appears from $:H_2:$
- Pion as a quark-antiquark meson appears from $:H_2:+:H_4:$
- Dualism of the pion: bound state equation for the pion is identical to the mass-gap equation for the chiral angle
- Both wave functions of the pion ($\varphi^{\pi}_{\pm}$) are equally important
- If $\varphi^{\pi}_{-}$ is not retained, the pion becomes unphysically heavy

Gell-Mann-Oakes-Renner relation

$$f_\pi^2 M_\pi^2=-2m\langle\bar{\psi}\psi\rangle$$

# Planar and non-planar diagrams

$$\langle (A_\mu)^\alpha_\beta (A_\nu)^\gamma_\delta \rangle \quad \propto \quad \left(\frac{\lambda^a}{2}\right)^\alpha_\beta \left(\frac{\lambda^a}{2}\right)^\gamma_\delta \quad = \quad \frac{1}{2}\left(\delta^\alpha_\delta \delta^\gamma_\beta - \frac{1}{N_c}\delta^\alpha_\beta \delta^\gamma_\delta\right) \quad \xrightarrow[N_c \to \infty]{} \quad \frac{1}{2}\delta^\alpha_\delta \delta^\gamma_\beta$$

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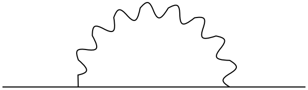

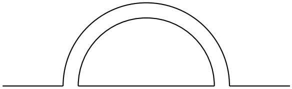

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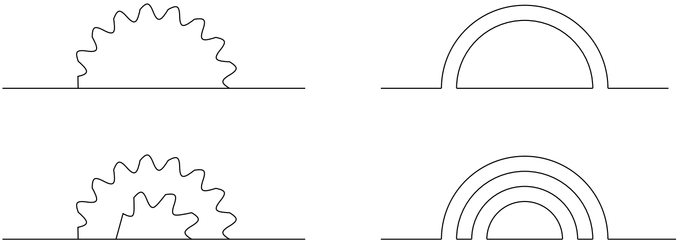

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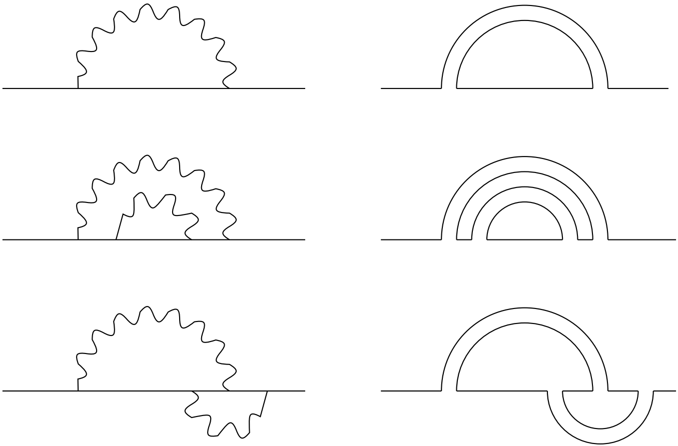
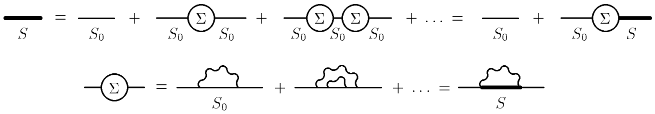
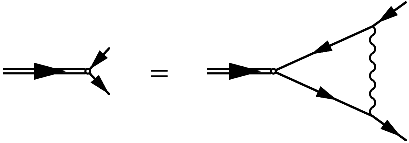


# Diagrammatic approach to 't Hooft model

- Dyson series (in rainbow approximation) for dressed quark propagator




- Bethe-Salpeter equation (in ladder approximation) for quark-antiquark meson



- Mass-gap equation and bound-state equation derived using diagrams and Hamiltonian approach coincide

# Infrared divergent and finite quantities: An instructive lesson

- Interquark potential in principal value prescription

$$V(x) = -P\int_{-\infty}^{+\infty}\frac{dp}{2\pi}\frac{e^{ipx}}{p^2} = -|x|\int_0^{+\infty}\frac{d\xi}{\pi}\frac{\cos\xi - 1}{\xi^2} = \frac{1}{2}|x|$$

- Interquark potential with finite infrared regulator

$$V(x) = -\int_{-\infty}^{+\infty}\frac{dp}{2\pi}\frac{e^{ipx}}{p^2+\mu_{\rm IR}^2} = -\frac{1}{2\mu_{\rm IR}}e^{-\mu_{\rm IR}|x|} \underset{\mu_{\rm IR}\to 0}{=} -\frac{1}{2\mu_{\rm IR}} + \frac{1}{2}|x| + \ldots$$

- Infrared divergent piece appears in not observable quantities (potential, $E_p$, etc) but cancels in physical ones (chiral angle, bound state equation, etc)
- Infrared divergence shows up in not gauge invariant objects (e.g. single quark) indicating that they are not observable
- Only gauge invariant objects are Poincare invariant (Bars & Green'1978)

# Confining chiral quark model for QCD in four dimensions

# Confining chiral quark model for QCD in 3+1

(Orsay group'1980s;Bicudo,Ribeiro'1990s)

- Interacting colour charge densities

$$H_{\rm int}=\frac12\int d^3xd^3y\left(\psi^\dagger(t,\boldsymbol{x})\frac{\lambda^a}{2}\psi(t,\boldsymbol{x})\right)V(|\boldsymbol{x}-\boldsymbol{y}|)\left(\psi^\dagger(t,\boldsymbol{y})\frac{\lambda^a}{2}\psi(t,\boldsymbol{y})\right)$$

$$\psi(t,\boldsymbol{x})=\sum_{s=\uparrow,\downarrow}\int\frac{d^3p}{(2\pi)^3}e^{i\boldsymbol{px}}\left(b_{\boldsymbol{p}s}u_{\boldsymbol{p}s}[\varphi_p]+d^\dagger_{-\boldsymbol{p}-s}v_{-\boldsymbol{p}-s}[\varphi_p]\right)$$

- Normally ordered Hamiltonian

$$H=E_{\rm vac}[\varphi_p]+:H_2:+:H_4:$$

- The energy of the vacuum is a minimum $\Longrightarrow$ mass-gap equation for $\varphi_p$
- Quadratic part $:H_2:$ describes dressed quarks
- Quartic part $:H_4:$ describes mesons
- Employ large-$N_c$ logic $\Longrightarrow$ nonplanar diagrams neglected & only leading-order contributions in $N_c$ retained

# Mass-gap equation

- Define auxiliary functions

$$A_p = m + \frac{1}{2}\int \frac{d^3k}{(2\pi)^3} V(\boldsymbol{p}-\boldsymbol{k})\sin\varphi_k$$

$$B_p = p + \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\,(\hat{\boldsymbol{p}}\hat{\boldsymbol{k}})V(\boldsymbol{p}-\boldsymbol{k})\cos\varphi_k$$

- Vacuum energy

$$E_{\rm vac}[\varphi_p] = -N_c V\int \frac{d^3p}{(2\pi)^3}\Big(A_p\sin\varphi_p + B_p\cos\varphi_p\Big)$$

- Dressed quark dispersion law

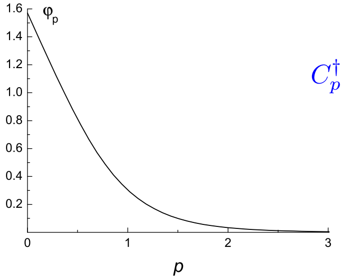
$$E_p = A_p\sin\varphi_p + B_p\cos\varphi_p$$

- Mass-gap equation for the chiral angle

$$A_p\cos\varphi_p - B_p\sin\varphi_p = 0$$

# Chiral symmetry breaking in GNJL

(Bicudo,Ribeiro'1990s)




$$C_p^\dagger = \sum_{\alpha=1}^{N_c} \sum_{s,s'=\uparrow,\downarrow} b_{\alpha s}^\dagger(\boldsymbol{p}) \underbrace{[(\boldsymbol{\sigma}\hat{\boldsymbol{p}})i\sigma_2]_{ss'}}_{^3P_0 \text{ operator}} d_{\alpha s'}^\dagger(\boldsymbol{p})$$

$$Q^\dagger = \frac{1}{2}\sum_{\boldsymbol{p}} \varphi_p C_p^\dagger$$

Broken vacuum

$$|0\rangle = e^{Q-Q^\dagger}|0\rangle_0 = \prod_p \left[\cos^2\frac{\varphi_p}{2} + \sin\frac{\varphi_p}{2}\cos\frac{\varphi_p}{2}C_p^\dagger + \frac{1}{2}\sin^2\frac{\varphi_p}{2}C_p^{\dagger 2}\right]|0\rangle_0$$

Chiral condensate

$$\langle\bar{\psi}\psi\rangle = -\frac{N_C}{\pi^2}\int_0^\infty dp\, p^2 \sin\varphi_p$$

# The chiral pion

Bound state equation for the pion

$$\begin{cases} [2E_p - M_\pi]\varphi_\pi^+(p) = \int \frac{q^2 dq}{(2\pi)^3}\left[T_\pi^{++}(p,q)\varphi_\pi^+(q) + T_\pi^{+-}(p,q)\varphi_\pi^-(q)\right] \\ [2E_p + M_\pi]\varphi_\pi^-(p) = \int \frac{q^2 dq}{(2\pi)^3}\left[T_\pi^{-+}(p,q)\varphi_\pi^+(q) + T_\pi^{--}(p,q)\varphi_\pi^-(q)\right] \end{cases}$$

$$T_\pi^{++}(p,q) = T_\pi^{--}(p,q) = -\int d\Omega_q V(\boldsymbol{p}-\boldsymbol{q})\left[\cos^2\frac{\varphi_p-\varphi_q}{2} - \frac{1-(\hat{\boldsymbol{p}}\hat{\boldsymbol{q}})}{2}\cos\varphi_p\cos\varphi_q\right]$$

$$T_\pi^{+-}(p,q) = T_\pi^{-+}(p,q) = -\int d\Omega_q V(\boldsymbol{p}-\boldsymbol{q})\left[\sin^2\frac{\varphi_p-\varphi_q}{2} + \frac{1-(\hat{\boldsymbol{p}}\hat{\boldsymbol{q}})}{2}\cos\varphi_p\cos\varphi_q\right]$$

Chiral pion wave function in centre-of-mass frame (chiral limit of $M_\pi = 0$)

$$\varphi_+^\pi(p) = \varphi_-^\pi(p) = \mathcal{N}_\pi \sin\varphi_p$$

$$2E_p\varphi_\pi(p) = \int \frac{q^2 dq}{(2\pi)^3}[T_\pi^{++}(p,q) + T_\pi^{+-}(p,q)]\varphi_\pi(q) = -\int \frac{d^3q}{(2\pi)^3}V(\boldsymbol{p}-\boldsymbol{q})\varphi_\pi(q)$$

Bound state equation for the pion as Salpeter-like equation

$$[2E_p + V(r)]\varphi_\pi = 0$$

# Chiral symmetry in heavy-light mesons

- Bound state equation for opposite-parity heavy-light mesons

$$\psi'(\boldsymbol{p}) = (\boldsymbol{\sigma}\hat{\boldsymbol{p}})\psi(\boldsymbol{p})$$

$$E_p\psi(\boldsymbol{p}) + \int \frac{d^3k}{(2\pi)^3} V(\boldsymbol{p}-\boldsymbol{k}) \left[C_pC_k + (\boldsymbol{\sigma}\hat{\boldsymbol{p}})(\boldsymbol{\sigma}\hat{\boldsymbol{k}})S_pS_k\right]\psi(\boldsymbol{k}) = E\psi(\boldsymbol{p})$$

$$E_p\psi'(\boldsymbol{p}) + \int \frac{d^3k}{(2\pi)^3} V(\boldsymbol{p}-\boldsymbol{k}) \left[S_pS_k + (\boldsymbol{\sigma}\hat{\boldsymbol{p}})(\boldsymbol{\sigma}\hat{\boldsymbol{k}})C_pC_k\right]\psi'(\boldsymbol{k}) = E\psi'(\boldsymbol{p})$$

$$C_p = \sqrt{\frac{1+\sin\varphi_p}{2}} \qquad S_p = \sqrt{\frac{1-\sin\varphi_p}{2}}$$

- If $\varphi_p \to 0$ mesons with opposite parity become degenerate

$$C_p^2 - S_p^2 = \sin\varphi_p$$

- Interpretation: SBCS, as intrinsically quantum effect, must fade out in the quasiclassical part of the spectrum (Glozman'2000s)

# Infrared-finite quark energy

- Linear confining potential

$$V(r) = -\int \frac{d^3p}{(2\pi)^3} \frac{8\pi\sigma}{(p^2+\mu_{\rm IR}^2)^2} e^{i\vec{p}\vec{r}} = \frac{\sigma}{\mu_{\rm IR}} e^{-\mu_{\rm IR} r} \underset{\mu_{\rm IR}\to 0}{=} -\frac{\sigma}{\mu_{\rm IR}} + \sigma r + \ldots$$

- Auxiliary functions $A_p$ and $B_p$

$$A_p = \frac{\sigma}{2\mu_{\rm IR}} \sin\varphi_p + A_p^{\rm fin} \qquad B_p = \frac{\sigma}{2\mu_{\rm IR}} \cos\varphi_p + B_p^{\rm fin}$$

- Mass-gap equation

$$A_p^{\rm fin} \cos\varphi_p - B_p^{\rm fin} \sin\varphi_p = 0$$

- Dispersion law

$$E_p = \frac{\sigma}{2\mu_{\rm IR}} + \ldots$$

- Infrared-finite quark energy and dynamical mass

$$\omega_p = p \lim_{\mu_{\rm IR}\to 0} \frac{E_p}{B_p} = \frac{p}{\cos\varphi_p} = \left(p^2 + (\underbrace{p\tan\varphi_p}_{M_p})^2\right)^{1/2}$$

# GNJL at finite temperatures

- Fermi-Dirac distributions at $T \neq 0$

$$\langle b^\dagger_{\boldsymbol{p}s} b_{\boldsymbol{p}s} \rangle = n_p = \left(1 + e^{(\sqrt{p^2+M_p^2}-\mu)/T}\right)^{-1} \underset{T\to\infty}{\longrightarrow} \frac{1}{2}$$

$$\langle d^\dagger_{\boldsymbol{p}s} d_{\boldsymbol{p}s} \rangle = \bar{n}_p = \left(1 + e^{(\sqrt{p^2+M_p^2}+\mu)/T}\right)^{-1} \underset{T\to\infty}{\longrightarrow} \frac{1}{2}$$

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- For vanishing chemical potential $\mu = 0$

$$n_p = \bar{n}_p = \left(1 + e^{(\sqrt{p^2+M_p^2})/T}\right)^{-1}$$

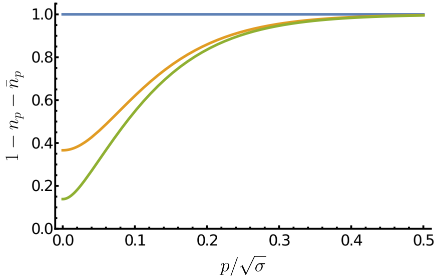
- Modified auxiliary functions at finite $T$

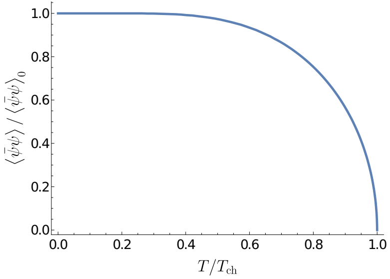
$$\tilde{A}_p = m + \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}(1-2n_k)V(\boldsymbol{p}-\boldsymbol{k})\sin\varphi_k$$

$$\tilde{B}_p = p + \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}(1-2n_k)(\hat{\boldsymbol{p}}\hat{\boldsymbol{k}})V(\boldsymbol{p}-\boldsymbol{k})\cos\varphi_k$$

# Critical temperature and chiral restoration

(Glozman,AN,Wagenbrunn'2024)




Result of numerical calculation

$$\langle\bar{\psi}\psi\rangle_0 \equiv \langle\bar{\psi}\psi\rangle_{T=0} \approx -0.0123(\sqrt{\sigma})^3 \qquad T_{\rm ch} \approx 0.084\sqrt{\sigma}$$

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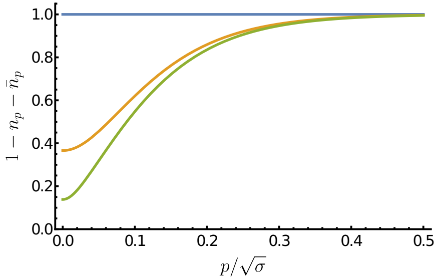


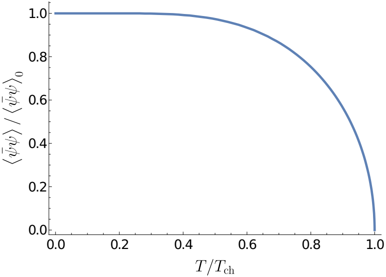


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$$T_{\rm ch} \approx 0.37|\langle\bar{\psi}\psi\rangle_0|^{1/3} \approx 90\ \text{MeV} \quad (cf\ T_{\rm ch}^{\rm lat} \approx 130\ \text{MeV})$$

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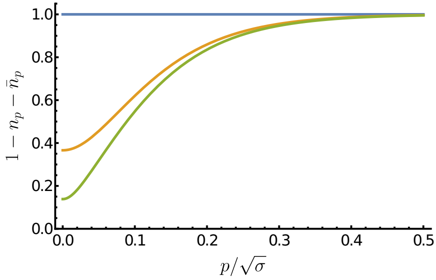


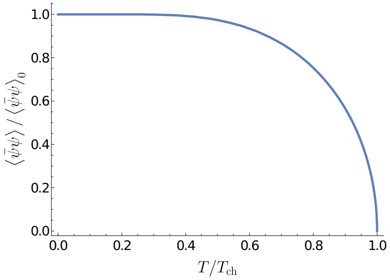


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Chiral condensate behaviour near $T_{\rm ch}$

$$\langle\bar{\psi}\psi\rangle_{T\to T_{\rm ch}} / \langle\bar{\psi}\psi\rangle_0 = 2.39(1-T/T_{\rm ch})^{0.54} \quad (\text{compatible with } \sqrt{1-T/T_{\rm ch}})$$

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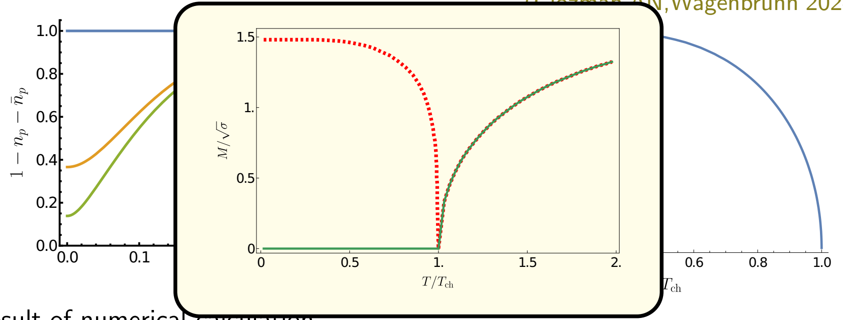


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# Hadrons in chirally unbroken phase

(Banks,Casher'1980)

$$\langle\bar{\psi}\psi\rangle = -\pi\rho(0) = -\lim_{m\to 0}\int_{-\infty}^{+\infty} d\lambda \frac{\rho(\lambda)}{m - i\lambda}$$

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(Denissenya,Glozman,Lang'2015)

$$\tilde{S} = S - \sum_{n=1}^{k} \frac{1}{\lambda_n}|\lambda_n\rangle\langle\lambda_n| \qquad i\not{D}\psi_n(x) = \lambda_n\psi_n(x)$$

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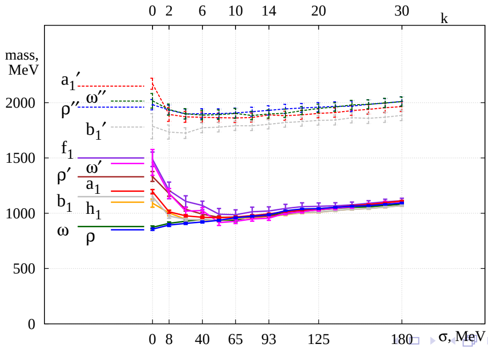
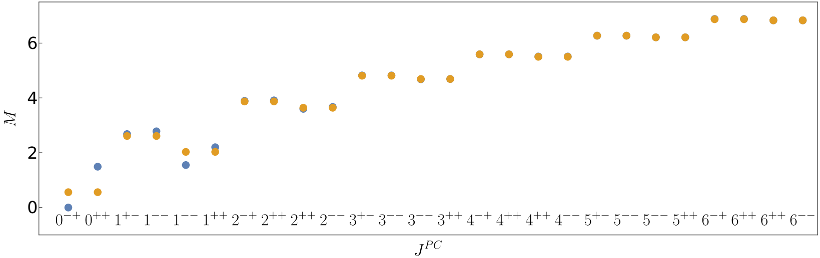
$$\langle\bar{\psi}\psi\rangle = -\pi\rho(0) = -\lim_{m\to 0}\int_{-\infty}^{+\infty} d\lambda \frac{\rho(\lambda)}{m - i\lambda}$$

(Denissenya,Glozman,Lang'2015)

$$\tilde{S} = S - \sum_{n=1}^{k}\frac{1}{\lambda_n}|\lambda_n\rangle\langle\lambda_n| \qquad i\not{D}\psi_n(x) = \lambda_n\psi_n(x)$$



# Hadrons above $T_{\mathrm{ch}}$




- Chiral symmetry is restored
- Confinement persists $\Longrightarrow$ hadrons survive as bound states of quarks
- What emergent symmetries are observed in the spectrum of $\bar{q}q$ mesons?

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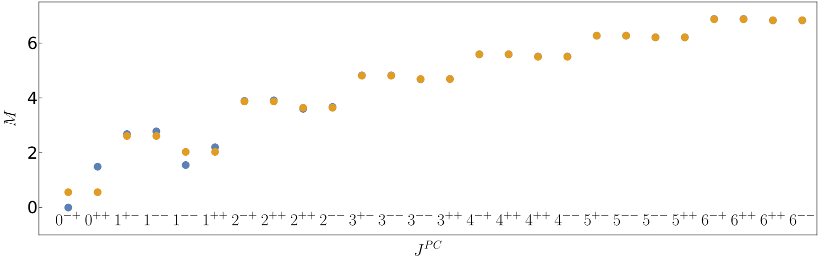


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## Work in progress...

## Stay tuned!

# Conclusions

- Quark models have a long history and provide strong and physically transparent theoretical tool
- Many phenomena inherent in QCD can be studied and understood employing quark models
- Predictions of quark models for many hadrons comply very well with the experimental data
- There are certain limitations on use of quark models in hadronic physics
- Still quark models have strong potential in studies of various phenomena in strong interactions
- Beyond applicability domain, quark models are used as classification basis