Computation of the HVP contribution to the muon anomaly with precise lattice spacing determination

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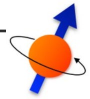
Sep 20 2024

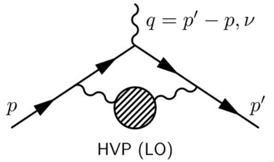
Unblinding, Jul 18 2024

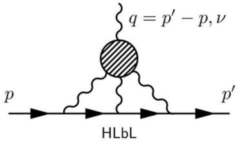
- ***Muon anomaly***
- Lattice QCD and physical point
- Window observables and results

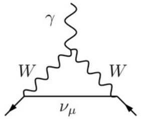
Muon anomaly

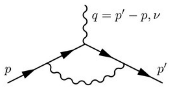
Contribution	Section	Equation	Value $\times 10^{11}$	References
Experiment (E821)		Eq. (8.13)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	Sec. 2.3.7	Eq. (2.33)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	Sec. 2.3.8	Eq. (2.34)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	Sec. 2.3.8	Eq. (2.35)	12.4(1)	Ref. [8]
HVP LO (lattice, $udsc$)	Sec. 3.5.1	Eq. (3.49)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)	Ref. [31]
HLbL (lattice, uds)	Sec. 5.7	Eq. (5.49)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)	Refs. [18–30, 32]
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)	Refs. [33, 34]
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	Sec. 8	Eq. (8.5)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)	Refs. [18–32]
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	Sec. 8	Eq. (8.14)	279(76)	





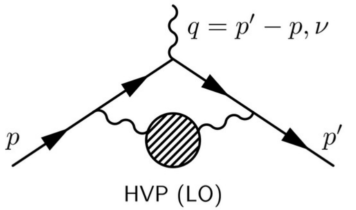






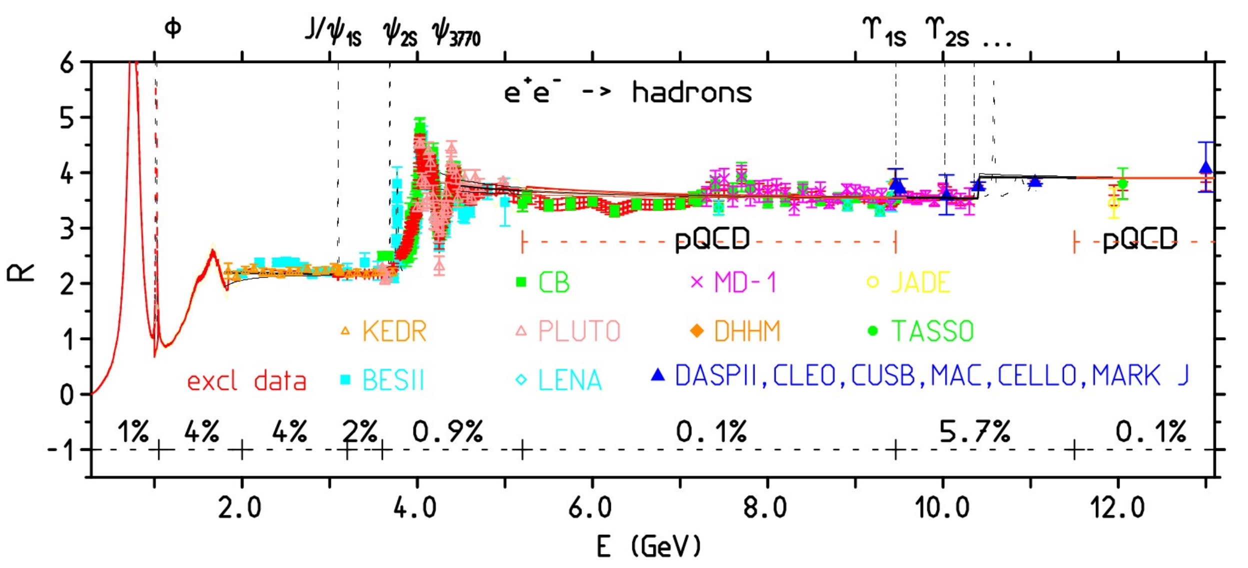
T. Aoyama, et al., Phys.Rept. 887 (2020) 1-166

Muon anomaly



- Hadronic R-ratio from the experimentally measured cross section

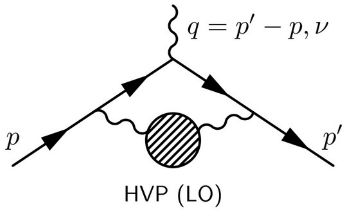
$$R(s) \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{4\pi\alpha^2/(3s)} \qquad a_\mu^{\text{LO,HVP}} = \int_{m_\pi^2}^{\infty} \frac{ds}{s} R(s) K(s)$$



- Heavily dominated by lowest energy regions
- Muon has a factor of $m_\mu^2/(3s)$ to enhance higher energy regions compared to electron
- Region between 0.4 and 1.0 GeV dominant
- Tensions between experiments in this region

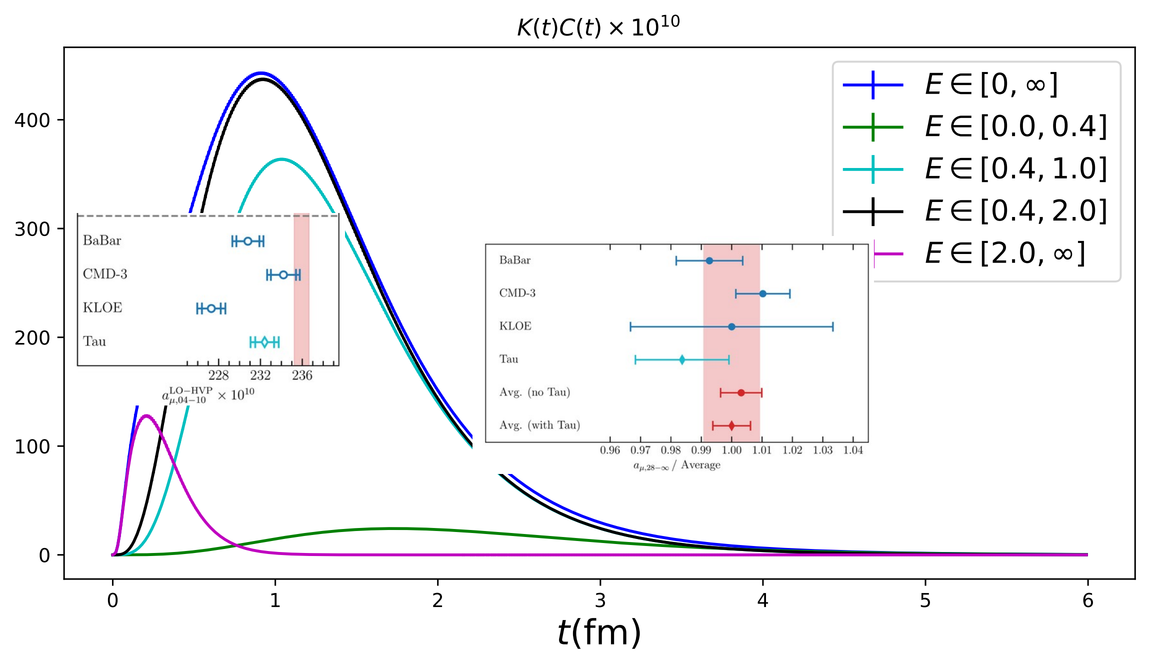
[1] T. Aoyama, et al., Phys.Rept. 887 (2020) 1-166

Muon anomaly



- The Euclidean correlator C(t) is related R-ratio [1]

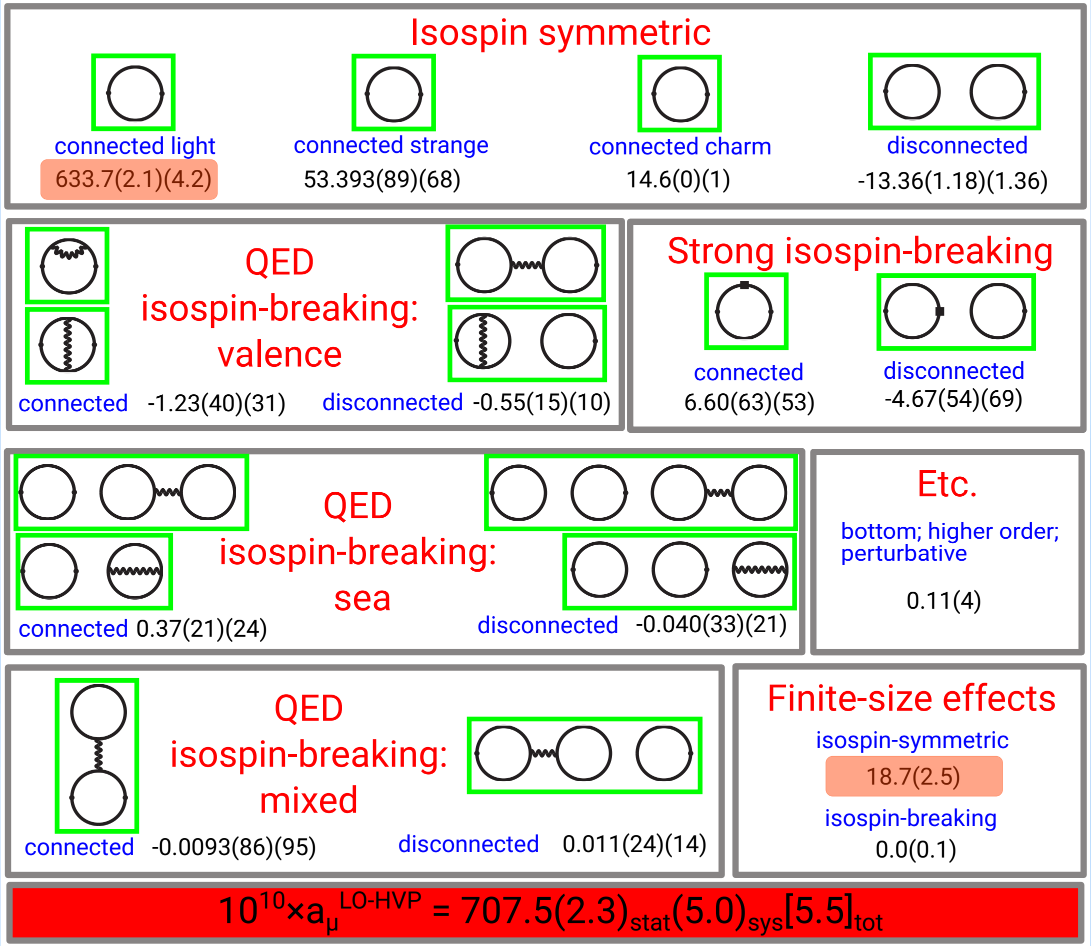
$$C(t) = \frac{1}{(12\pi^2)} \int_0^\infty d(\sqrt{s}) R(s) s e^{-\sqrt{s}t} \qquad a_\mu^{\mathrm{LO-HVP}} = \int \mathrm{d}t\, \tilde{K}(t) C(t)$$



- Around rho peak contributes most
- Tensions clearly observed between 0.4 and 1.0 fm
- Long distance > 2.8 fm : no obvious tensions

[1] D. Bernecker and H. B. Meyer, Eur. Phys. J. A47 (2011)
[2] A. Keshavarzi, D. Nomura, T. Teubner, KNT19, Phys. Rev. D 101, 014029 (2020)

Muon anomaly



$$C(t) \equiv \frac{1}{3}\sum_i \langle \int d^3x\, j_i(\vec{x},t) j_i(\vec{0},0)\rangle$$

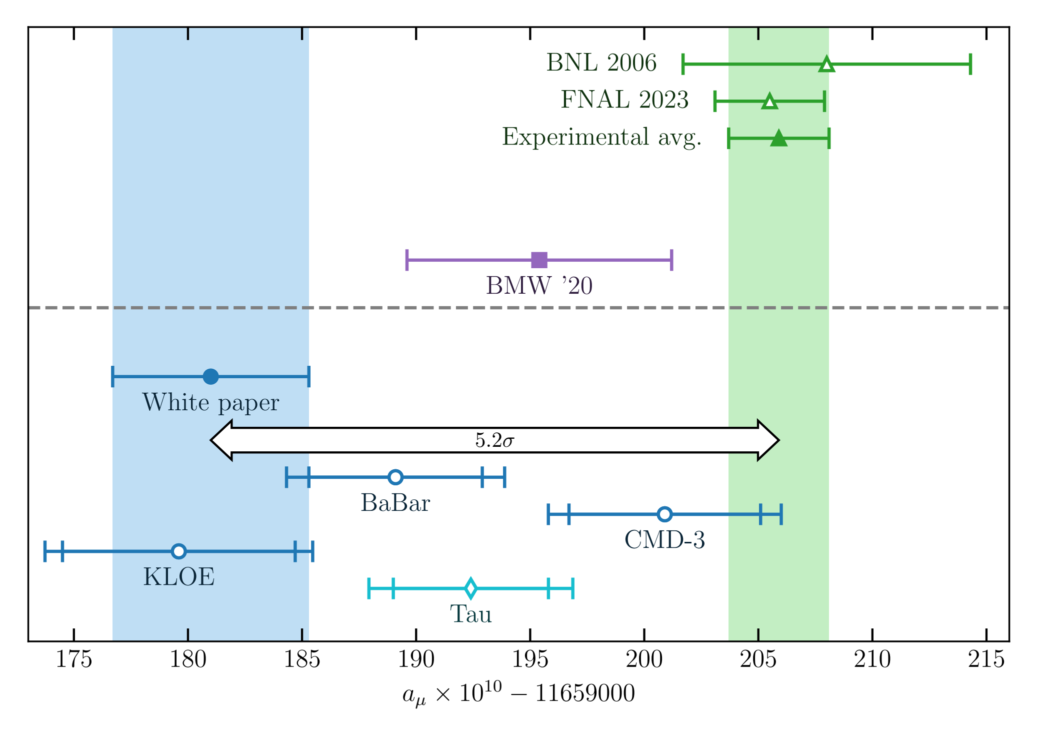
- Connected light dominates HVP contribution and error
- Systematic errors dominant:

$$1) a \to 0\ ;\ 2) L \to \infty$$

- Direct calculation of all QED and strong-iso-spin breaking effects

Sz. Borsanyi, et al., BMWc, Nature 593 (2021) 7857, 51-55

Muon anomaly



- Experimental result is 4.2σ SM predictions (WP Aoyama et al., 2020)
- BMW20 result is 2.1σ higher than R-ratio and consistent with experiment value at 1.5σ level
- ***Improve LO HVP***
  - Calculation with new 0.048 fm lattice to reduce $a \to 0$ error
  - Euclidean-time "tail" [2] with experimental data to reduce statistical and finite-volume errors

[1] Sz. Borsanyi, et al., BMWc, Nature 593 (2021) 7857, 51-55
[2] T. Blum, et al., RBC/UKQCD, Phys.Rev.Lett. 121 (2018) 2, 022003

- Muon anomaly
- ***Lattice QCD and physical point***
- Window observables and results

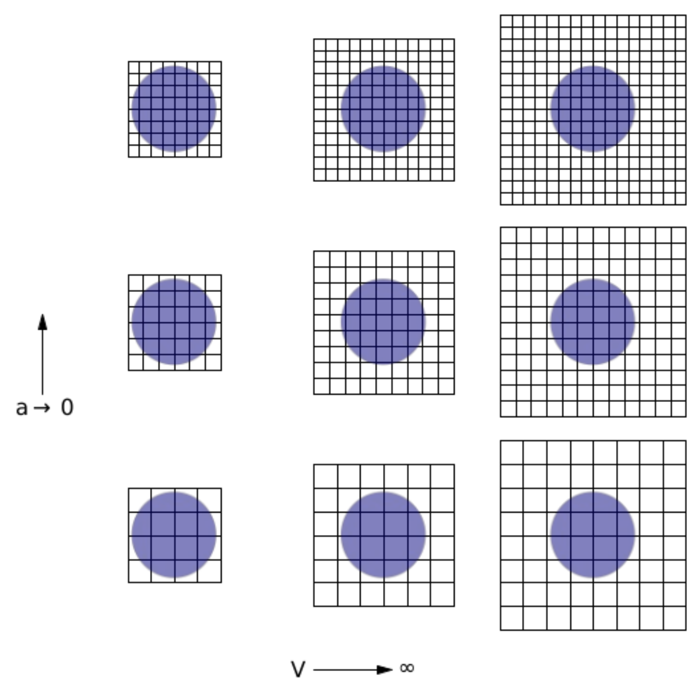
Lattice QCD path integral

- Euclidean Lagrangian with strong and electromagnetic interactions

$$\mathcal{L} = -\frac{1}{2g^2}\mathrm{Tr}G_{\mu\nu}G_{\mu\nu} - \frac{1}{4e^2}F_{\mu\nu}F_{\mu\nu} + \bar{\psi}\left[\gamma_\mu\left(\partial_\mu + G_\mu + qA_\mu\right)\right]\psi$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu + [G_\mu, G_\nu]$$



- Bare free parameters

$$\beta = \frac{6}{g_0^2}, \quad m_u = m_d, m_s, m_c = m_s * 11.85$$

- Match to physical world

$$M^2_{\pi_\chi} = M_{\pi_0} \equiv \frac{1}{2}(M^2_{uu} + M^2_{dd})$$

$$M^2_{K_\chi} \equiv \frac{1}{2}(M^2_{K_0} + M^2_{K_+} - M^2_{\pi_+})$$

$$M^2_\Omega$$

$$\Delta M^2_K \equiv M^2_{K_0} - M^2_{K_+}$$

$$M^2_{\pi_\chi}$$

$$M^2_{ss}$$

$$w_0$$

$$\Delta M^2 \equiv M^2_{dd} - M^2_{uu}$$

Staggered fermions

- A ``naive" discretization of the fermion on the lattice

$$\partial_\mu \psi(x) \to \frac{1}{2a}[\psi(n+\hat{\mu}) - \psi(n-\hat{\mu})],$$

- Fermion doubling problem

$$S^{-1}(p) = \sum_\mu \frac{i}{a}\gamma_\mu \sin(ap_\mu)$$

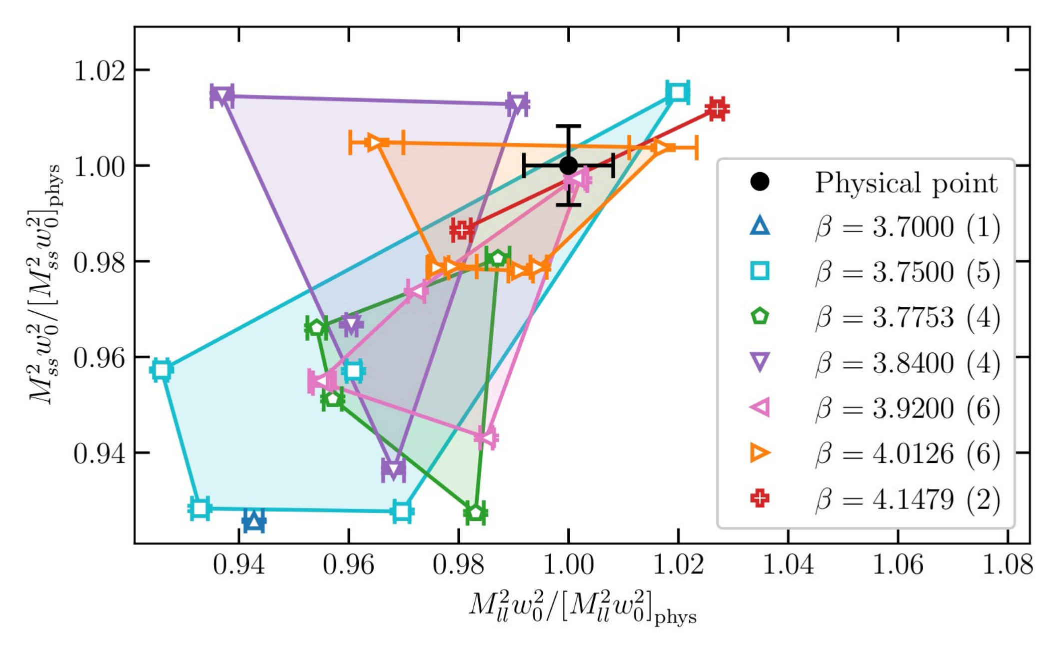
- Nielsen-Ninomiya theorem cannot have satisfy all : locality, single flavor in the continuum, chiral symmetry

- Staggered fermions with 4th root (taste breaking effects)

$$\sum_n \bar{\chi}_n \left[\sum_\mu \frac{\eta_{n,\mu}}{2} \left(U_{n,\mu}\chi_{n+\mu} - U^\dagger_{n-\mu,\mu}\chi_{n-\mu} \right) \right]$$

[1] H.B. Nielsen, M. Ninomiya, Nuclear Physics B. 185 (1): 20–40

Physical point



- $N_f = 2 + 1 + 1$ staggered fermions
- Stout smearing n = 4, ρ = 0.125

➢ M_π and M_{ss} around physical point with the finest lattice spacing 0.048 fm

➢ The lattice scale is set by Ω baryon mass
  - moderate quark mass dependence
  - precisely determined in a lattice simulation
  - known experimental value to an accuracy better than a permil level

➢ Precise determination of w_0 from M_Ω

Ω correlation function

- For the Ω baryon, two staggered baryon operators in [1] and an operator which only couples to a single taste [2]

$$\Omega_{\mathrm{VI}}(t) = \sum_{x_k even} \epsilon_{abc} \left[S_1\chi_a S_{12}\chi_b S_{13}\chi_c - S_2\chi_a S_{21}\chi_b S_{23}\chi_c + S_3\chi_a S_{31}\chi_b S_{32}\chi_c\right](x)$$

$$\Omega_{\mathrm{XI}}(t) = \sum_{x_k even} \epsilon_{abc} \left[S_1\chi_a S_2\chi_b S_3\chi_c\right](x)$$

$$\Omega_{\mathrm{Ba}}(t) = \left[2\delta_{\alpha 1}\delta_{\beta 2}\delta_{\gamma 3} - \delta_{\alpha 3}\delta_{\beta 1}\delta_{\gamma 2} - \delta_{\alpha 2}\delta_{\beta 3}\delta_{\gamma 1} + (\cdots \beta \leftrightarrow \gamma \cdots)\right]$$

$$\sum_{x_k even} \epsilon_{abc} \left[S_1\chi_{a\alpha} S_{12}\chi_{b\beta} S_{13}\chi_{c\gamma} - S_2\chi_{a\alpha} S_{21}\chi_{b\beta} S_{23}\chi_{c\gamma} + S_3\chi_{a\alpha} S_{31}\chi_{b\beta} S_{32}\chi_{c\gamma}\right](x)$$

- Wuppertal smearing connects 2a lattice spacing

$$\left[\hat{W}v\right]_x = (1-\sigma)v_x + \frac{\sigma}{6}\sum_{\mu=1,2,3}\left(U^{3d}_{\mu,x}U^{3d,\dagger}_{\mu,x+\mu}v_{x+2\mu} + U^{3d}_{u,x-\mu}U^{3d,\dagger}_{\mu,x-2\mu}v_{x-2\mu}\right)$$

[1] M. F. L. Golterman and J. Smit, Nucl. Phys. B 255 (1985), 328-340
[2] J. A. Bailey, Phys. Rev. D 75 (2007), 114505

GEVP method

- Staggered Ω correlators with positive and oscillating negative parity states:

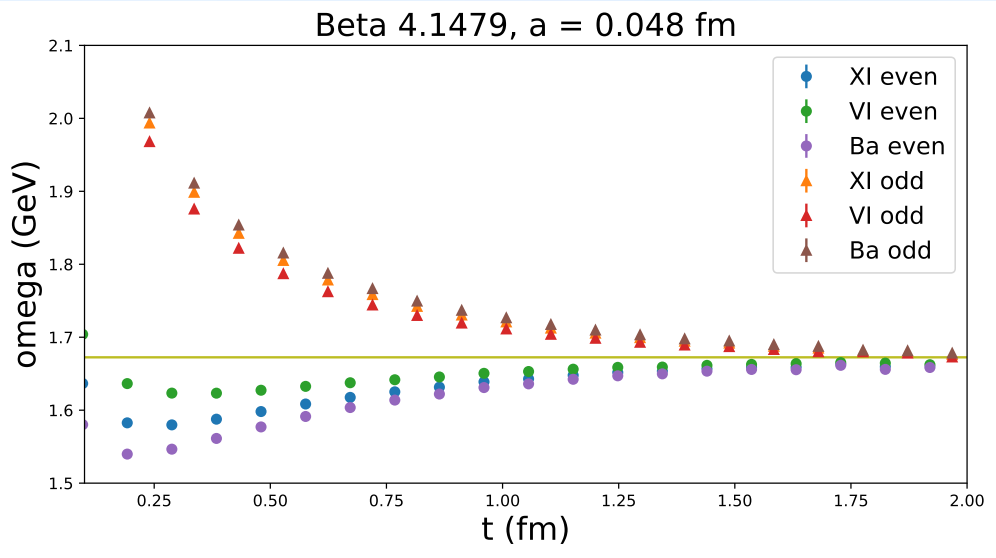
$$H(t,A,M) = A_0 e^{-M_0 t} + (-1)^{t+1} A_1(M_1,t) e^{-M_1 t} + A_2 e^{-M_2 t} + (-1)^{t+1} A_3 e^{-M_3 t} + \cdots$$

- Use time shift to create an “new” operator [1]

$$H(t+2t_s) = \sum_i \left[A'_i e^{-2M_i t_s}\right] e^{-M_i t}$$

- Combine smeared-point, smeared-smeared, point-point correlators into one matrix for GEVP

- Presence of oscillations makes the time shifted GEVP procedure more efficient



$$C(t \in \mathrm{even/odd}) = \frac{1}{2} log(\frac{C(t+2)}{C(t)})$$

[1] C. Aubin, and K. Orginos, AIP Conf.Proc. 1374 (2011) 1, 621-624

GEVP method

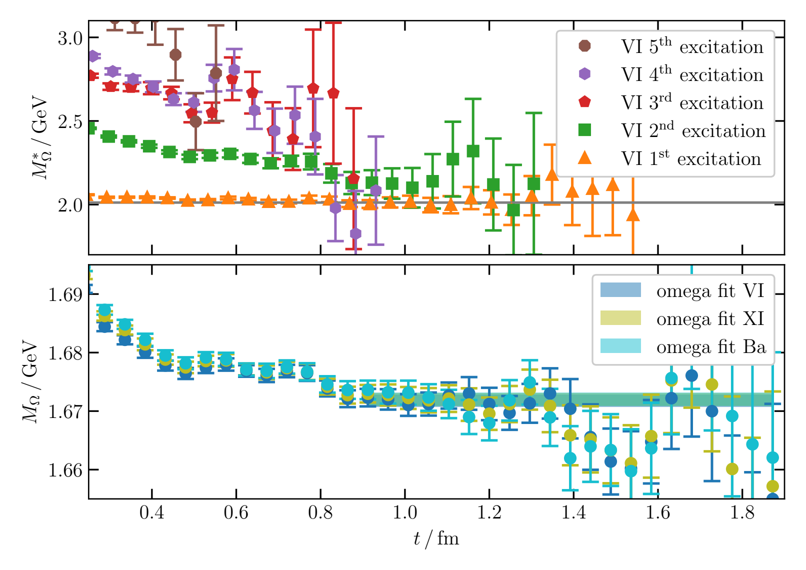
$$\mathbf{H}(t) = \left(\begin{array}{cc|cccc} H^{pp}_{t+2t_p+0} & H^{pp}_{t+2t_p+1} & H^{ps}_{t+t_p+0} & H^{ps}_{t+t_p+1} & H^{ps}_{t+t_p+2} & H^{ps}_{t+t_p+3} \\ H^{pp}_{t+2t_p+1} & H^{pp}_{t+2t_p+2} & H^{ps}_{t+t_p+1} & H^{ps}_{t+t_p+2} & H^{ps}_{t+t_p+3} & H^{ps}_{t+t_p+4} \\ \hline H^{sp}_{t+t_p+0} & H^{sp}_{t+t_p+1} & H^{ss}_{t+0} & H^{ss}_{t+1} & H^{ss}_{t+2} & H^{ss}_{t+3} \\ H^{sp}_{t+t_p+1} & H^{sp}_{t+t_p+2} & H^{ss}_{t+1} & H^{ss}_{t+2} & H^{ss}_{t+3} & H^{ss}_{t+4} \\ H^{sp}_{t+t_p+2} & H^{sp}_{t+t_p+3} & H^{ss}_{t+2} & H^{ss}_{t+3} & H^{ss}_{t+4} & H^{ss}_{t+5} \\ H^{sp}_{t+t_p+3} & H^{sp}_{t+t_p+4} & H^{ss}_{t+3} & H^{ss}_{t+4} & H^{ss}_{t+5} & H^{ss}_{t+6} \end{array}\right)$$

- Point source correlators get additional time shift t_p to suppress its large excited state effects

- Solve the Generalized Eigenvalue Problem (GEVP)

$$\mathbf{H}(t_a)v_i(t_a,t_b) = \lambda_i(t_a,t_b)\mathbf{H}(t_b)v_i(t_a,t_b)$$

➢ Ground state is extracted wth 0.1% precision



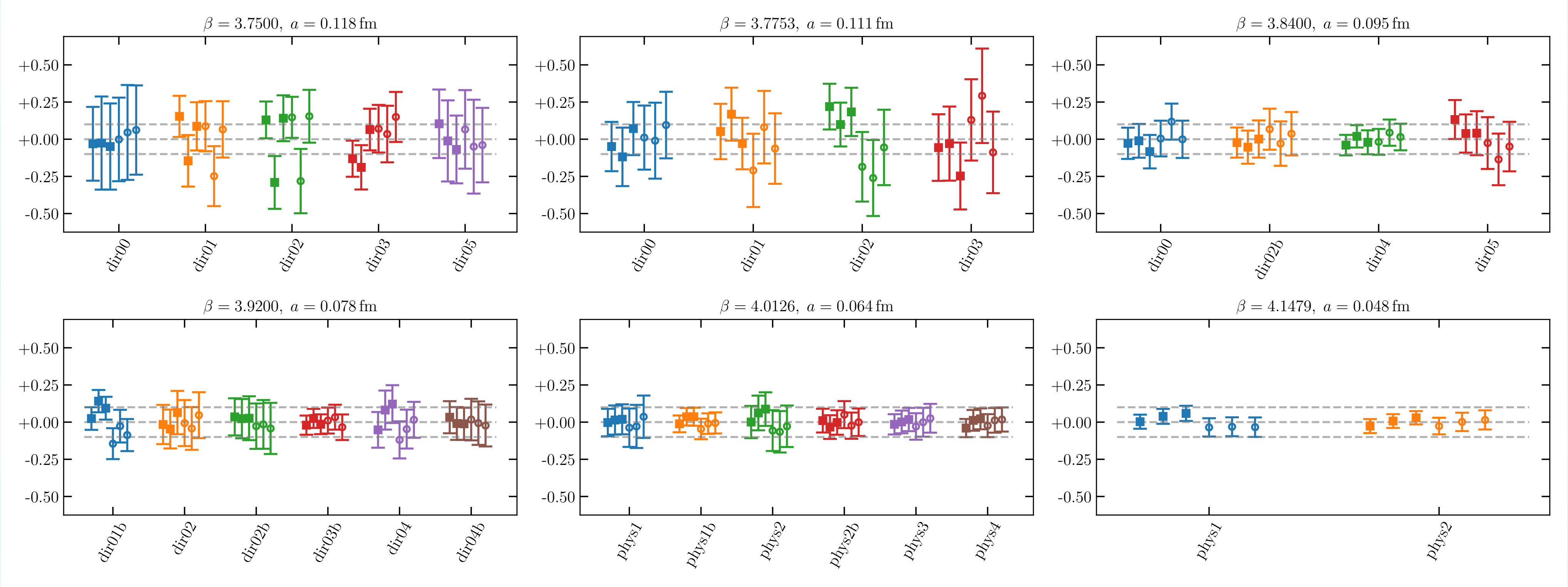
Ω measurements

β	#conf	$N_{\rm Wptl}$	$N_{\rm 3d}$	t_p	t_a	t_b	range #1	range #2	# pt, sm sources
3.7000	904	24	32	1	4	7	7...15	8...15	28928, 229376
3.7500	2072	30	40	1	4	7	8...18	9...18	66208, 530176
3.7553	1907	34	46	1	4	7	9...19	10...19	61024, 488192
3.8400	2949	46	62	2	4	9	10...20	11...20	125440, 2807552
3.9200	4296	67	90	2	6	9	12...25	13...25	137472, 3038720
4.0126	6980	101	135	3	6	9	15...30	16...30	223360, 4235520
4.1479	5017	178	238	5	6	11	19...40	21...40	160544, 2068736

- **Over 30,000 gauge configurations**
- **10’s of millions measurements**
- **Measurements on GPUs based on Quda [1] and Qlattice [2]**

[1] https://github.com/lattice/quda
[2] https://github.com/jinluchang/Qlattice

Ω masses



- Correlated (first 3 points) and uncorrelated (second 3 points) are consistent
- Taste breaking effects not observed between different Omega operators
- Reached less than 0.1% error on 0.048 fm lattices

Continuum extrapolation formula

- The logarithmic derivative of the gauge-action density along the gradient flow time

$$W_\tau[U] \equiv \frac{d(\tau^2 E[U,\tau])}{dlog\tau}, \langle W_{\tau=w_0^2} \rangle = 0.3$$

- Continuum extrapolations as a Taylor expansion of a^2

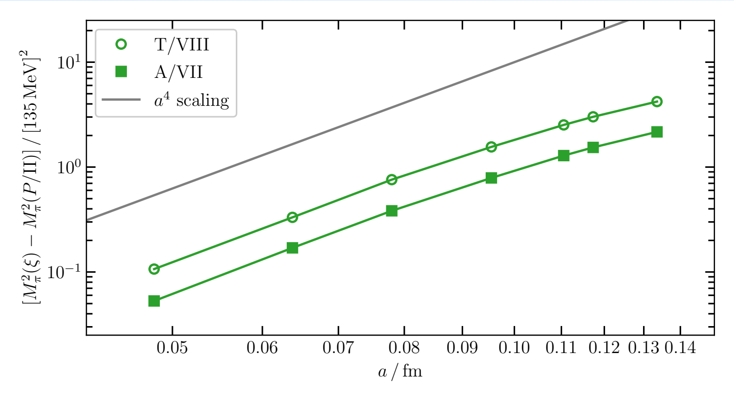
$$Y = Y_0 + A_1 a^2 + A_2 a^4 + A_3 a^6 + \cdots$$

- Or non-analytic from the Symanzik effective theory [1] with \`n\` unknown

$$a^2 \to \alpha_s(a)^n a^2$$

- Major staggered artifact (taste violation) scales with a power of n ≈ 3

$$\Delta_{KS}(\xi) \equiv M_\pi^2(\xi) - M_{ll}^2$$



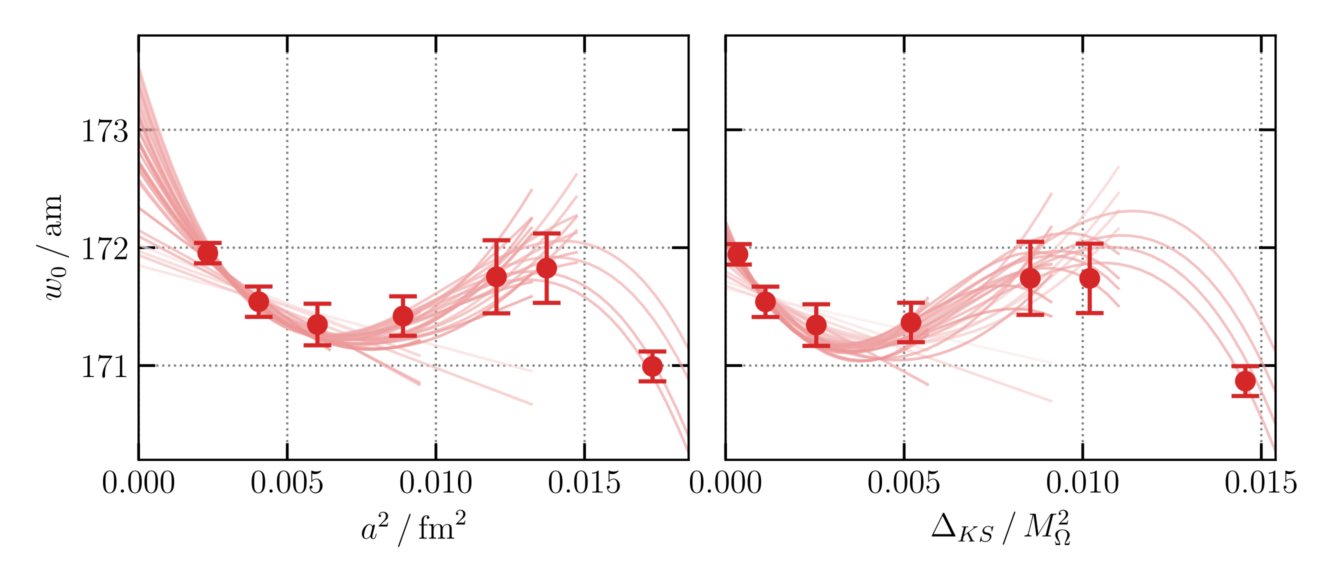
[1] N. Husung, P. Marquard and R. Sommer, Eur.Phys.J.C 80 (2020) 3, 200

Continuum extrapolation

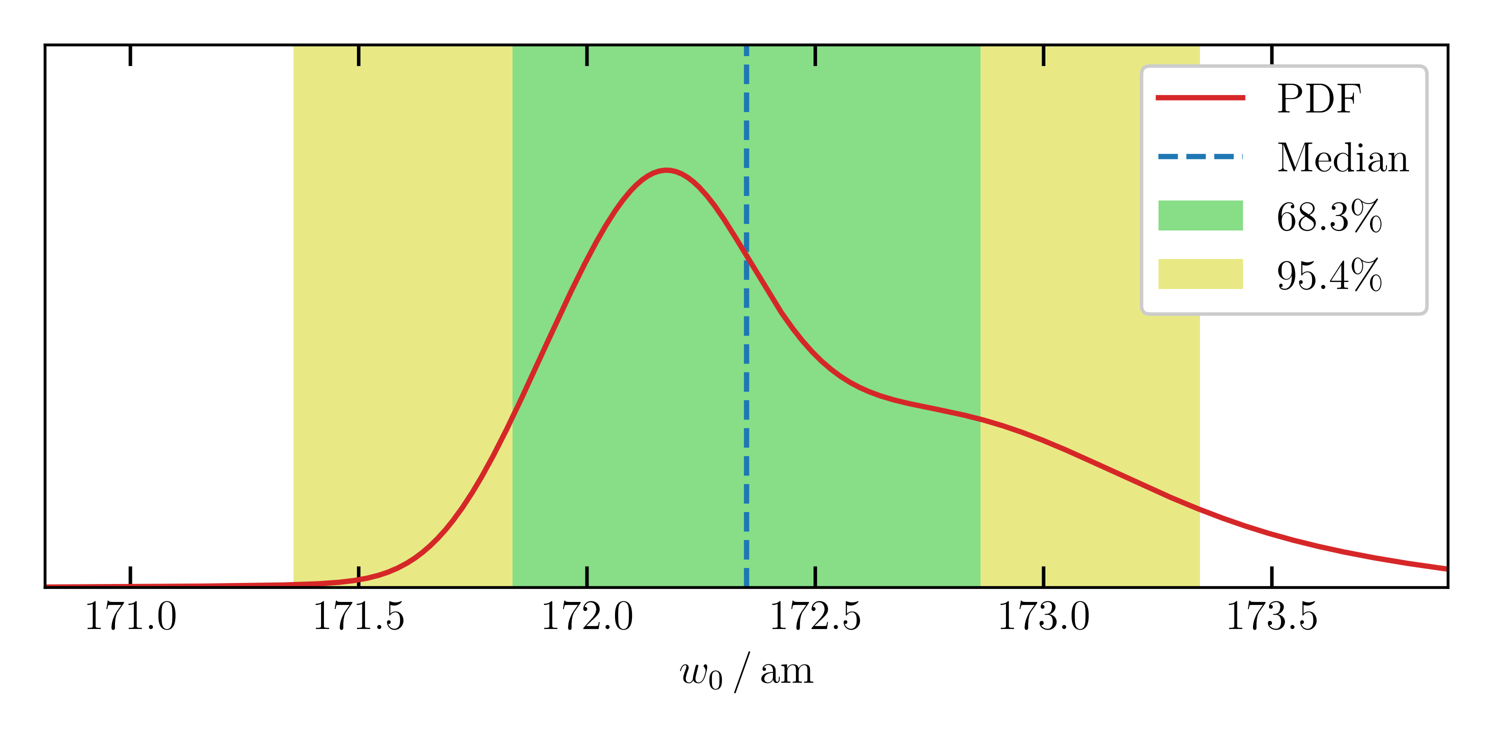
- Observable $Y = w_0 M_\Omega$

$$Y = A(a^2) + A'(\Delta_{KS}) + (B_0 + B_1 a^2)X_l + (C_0 + C_1 a^2)X_s$$

- ➢ $A(a^2)$ or $A'(\Delta_{KS})$
- ➢ beta cuts 0, 1, 2, 3, 4
- ➢ B1 or C1 included or not
- ➢ different Omega fits
- ➢ different meson fits



Fits distribution



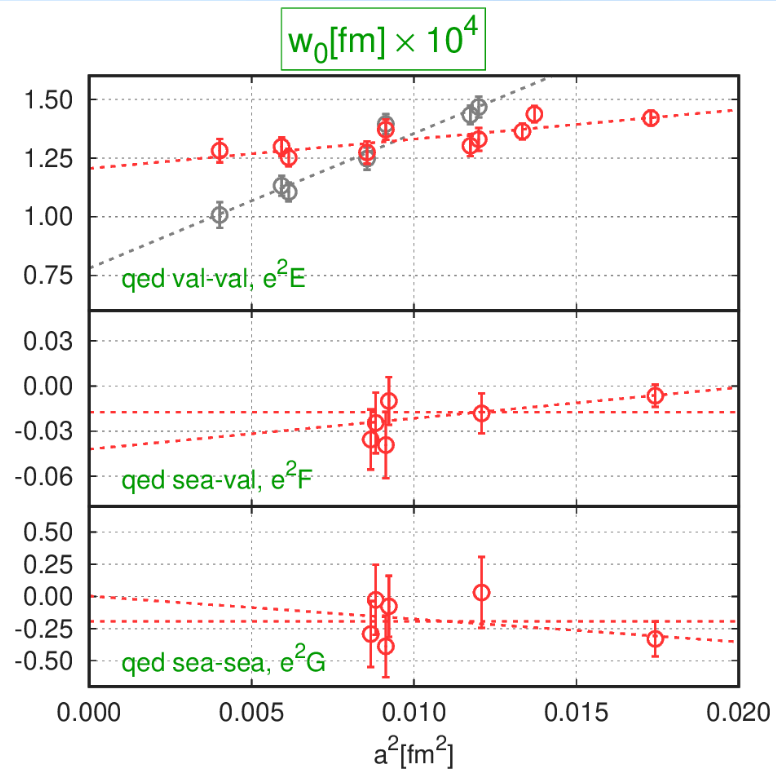
Electromagnetic effects

- Observable $Y = w_0 M_\Omega$

$$Y = A + BX_l + CX_s + DX_{\delta m} + Ee_v^2 + Fe_v e_s + Ge_s^2$$

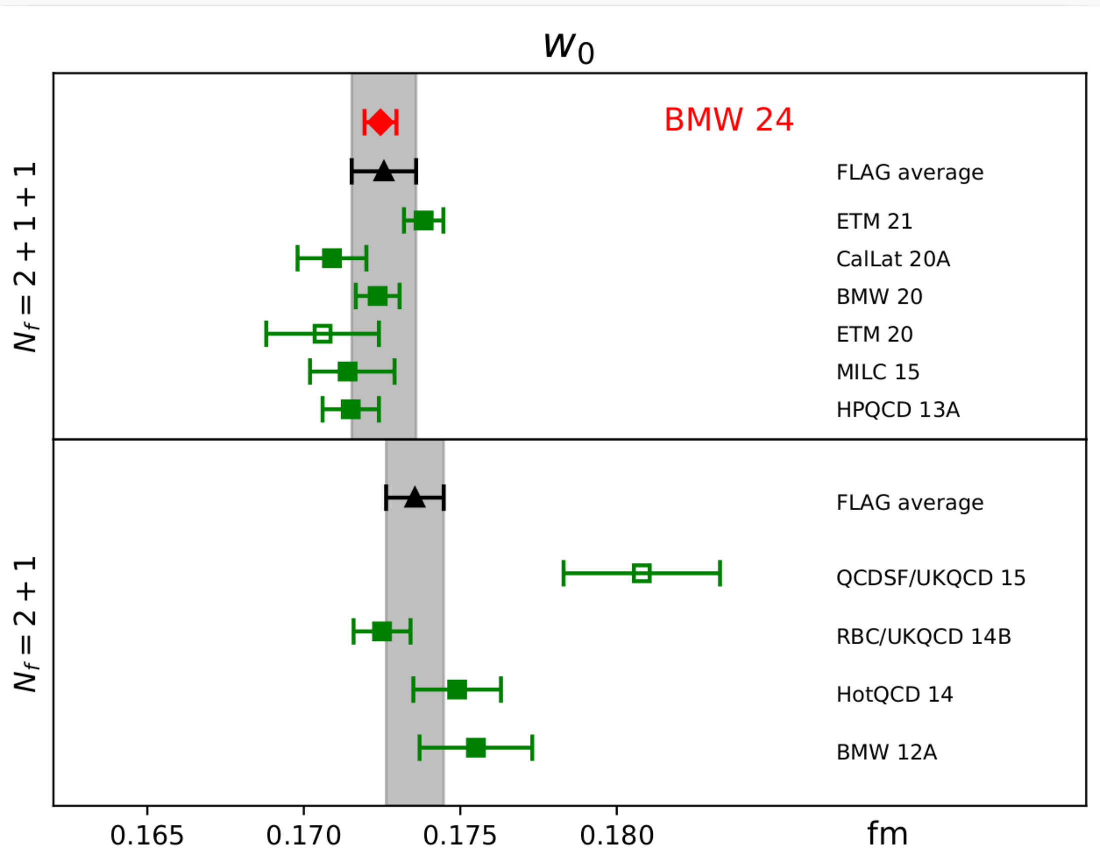
- Fit to a system of equations [1]

$$[Y]_0 = [A + BX_l + CX_s]_0$$
$$[Y]'_m = [DX_{\delta m}]'_m$$
$$[Y]''_{20} = [A + BX_l + CX_s + DX_{\delta m}]''_{20} + [E]_0$$
$$[Y]''_{11} = [A + BX_l + CX_s + DX_{\delta m}]''_{11} + [F]_0$$
$$[Y]''_{02} = [A + BX_l + CX_s + DX_{\delta m}]''_{02} + [G]_0$$



[1] Sz. Borsanyi, et al., BMWc, Nature 593 (2021) 7857, 51-55

Final results



- 7 lattice spacings all at physical pion mass
- Omega baryon statistical errors well under control
- Electromagnetic effects included

$$[w_0]_{\text{phys}} = 0.17245(22)(46)[51]\ \text{fm}$$

$$[M_{ss}]_{\text{phys}} = 689.89(28)(40)[49]\ \text{MeV}$$

$$[\Delta M^2]_{\text{phys}} = M_{uu}^2 - M_{dd}^2$$
$$= 13170(320)(270)[420]\ \text{MeV}^2$$

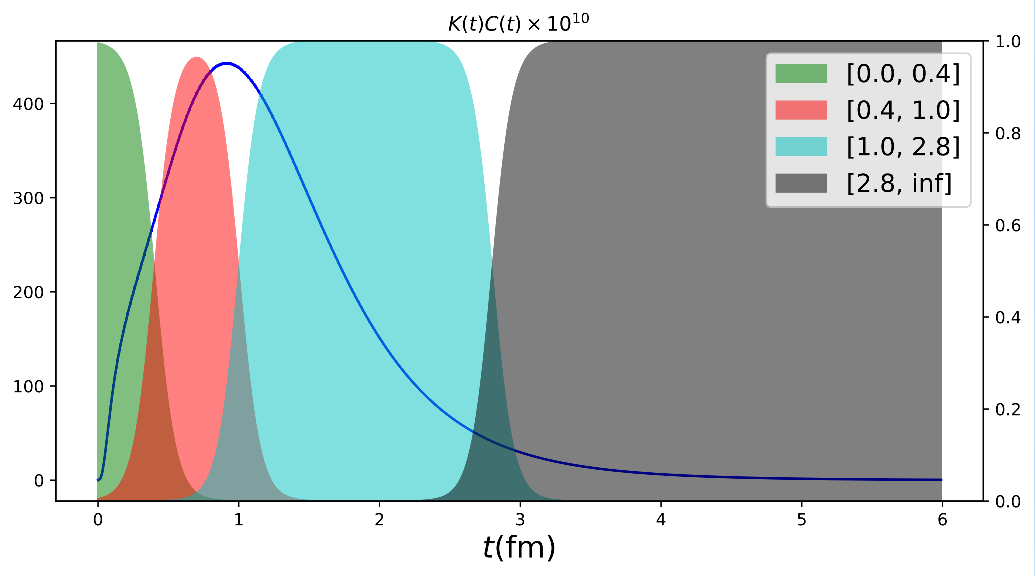
$$[M_{\pi_\chi}]^{exp}_{\text{phys}} = 134.9768(5)\ \text{MeV}$$

- Muon anomaly
- Lattice QCD and physical point
- ***Window observables and results***

Window quantities

- Time-momentum representation :

$$a_\mu^{\mathrm{LO-HVP}} = \int \mathrm{d}t\, \tilde{K}(t) C(t)$$



- Window quantities[1]:

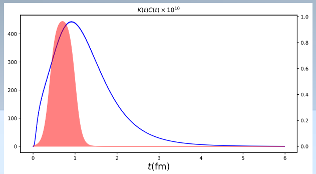
$$a_\mu^{\mathrm{win}} = \sum_t K(t)C(t) \times [\theta(t, t_0, \Delta) - \theta(t, t_1, \Delta)]$$

$$\theta(t, t_0, \Delta) = \frac{1}{2}(1 + \tanh(\frac{t - t'}{\Delta}))$$

- Comparison among lattice groups for intermediate window between t = 0.4 to 1 fm
  - No signal-to-noise problem
  - Small-t cutoff effects suppressed
  - Long-distance volume effects suppressed

[1] T. Blum, et al., (RBC/UKQCD), Phys. Rev. Lett. 121, 022003 (2018)

Light connected intermediate window



- Connected light window contributions (LMA)

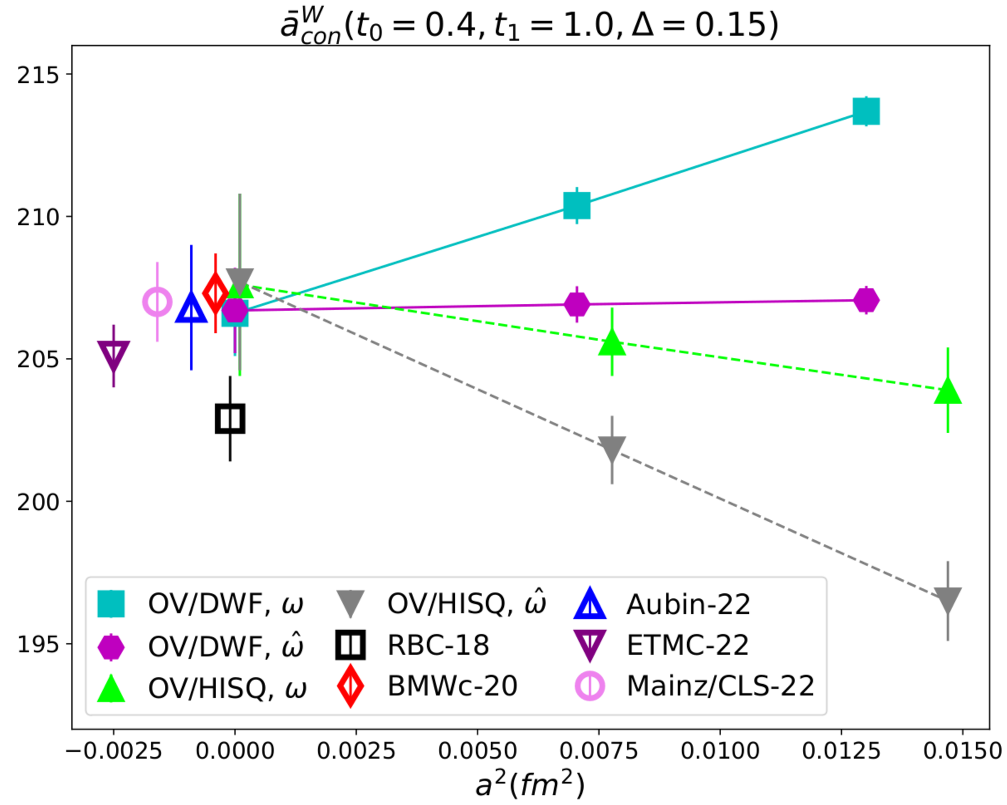
$$a_\mu^{\text{win}} = \sum_t K_t C(t) \times [\theta(t, t_0, \Delta) - \theta(t, t_1, \Delta)]$$

- Two difference weighting functions

$$\omega(t) = 4\alpha^2 \int_0^\infty \frac{\mathrm{d}q^2}{m_\mu^2} f\left(\frac{q^2}{m_\mu^2}\right) \left[\frac{\cos(tq) - 1}{q^2} + \frac{1}{2}t^2\right]$$

$$\hat{\omega}(t) = 4\alpha^2 \int_0^\infty \frac{\mathrm{d}q^2}{m_\mu^2} f\left(\frac{q^2}{m_\mu^2}\right) \left[\frac{\cos(tq) - 1}{[\frac{2}{a}\sin(\frac{qa}{2})]^2} + \frac{1}{2}t^2\right]$$

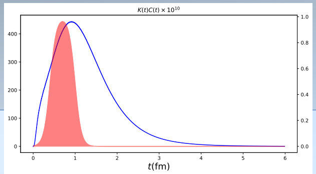
- OV/DWF and OV/HISQ are consistent at continuum limit
- OV/DWF result is higher than unitary DWF[1], consistent with latest DWF [2]



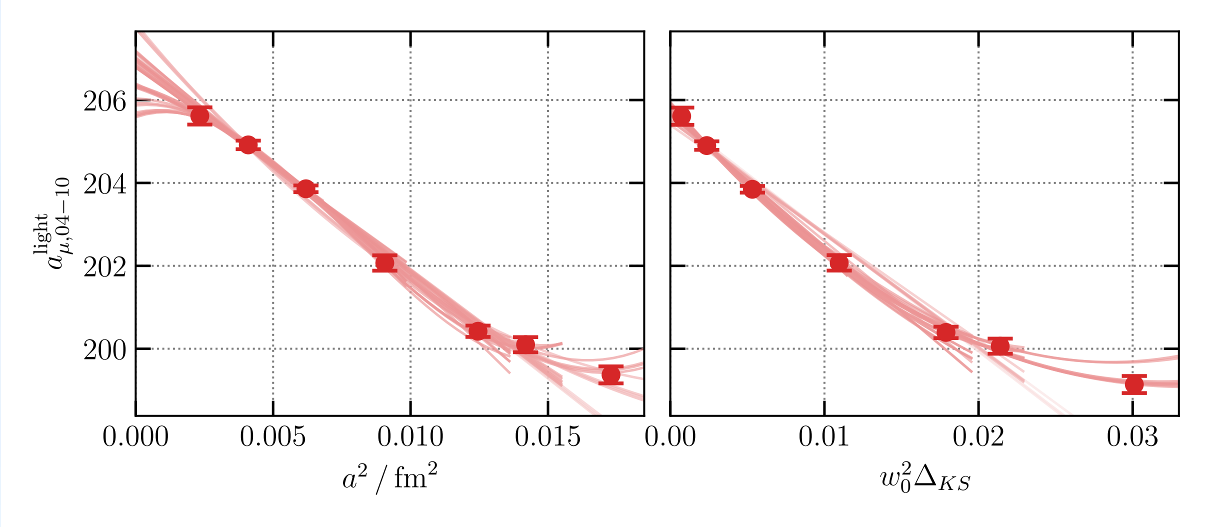
[1] T. Blum, et al., RBC/UKQCD, Phys.Rev.Lett. 121 (2018) 2, 022003
[2] T. Blum, et al., RBC/UKQCD, Phys.Rev.D 108 (2023) 5, 054507
[3] G. Wang, T. Draper, K.-F. Liu, Y.-B. Yang, Phys.Rev.D 107 (2023) 3, 03451

64I on the ORISE Supercomputer of Chinese Academy of Sciences

Light connected intermediate window



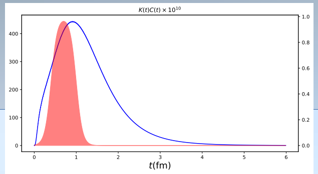
- Updates from BMW 2024 with 0.048 fm lattice
  - 2880 fits with different continuum extrapolations and mass fit ranges
  - Dominant uncertainty is the $a \to 0$ error
  - Results correspond to the reference box-size 6.272 fm

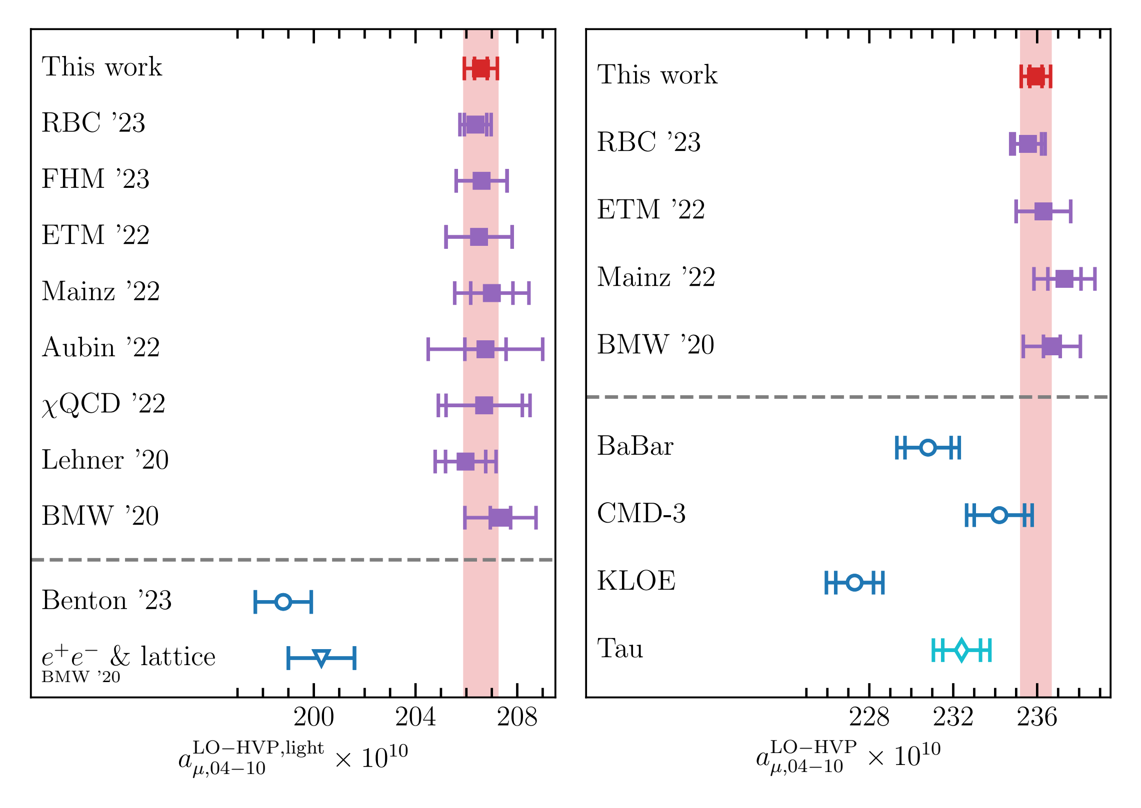


- ***Error budget***

Median	206.03	
Total error	0.65	0.31 %
Statistical error	0.25	0.12 %
Systematic error	0.60	0.29 %
Pseudoscalar fit range	0.01	$<$ 0.01 %
Physical value of M_{ss}	0.01	$<$ 0.01 %
w_0 scale setting	0.21	0.10 %
Lattice spacing cuts	0.14	0.07 %
Order of fit polynomials	0.20	0.10 %
Continuum parameter (Δ_{KS} or a^2)	0.40	0.20 %

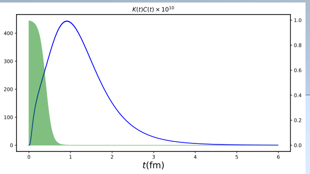
Light connected intermediate window





- Latest lattice results are consistent within errors
- Have large tensions between experimental results
- Recent CMD-3 consistent with lattice

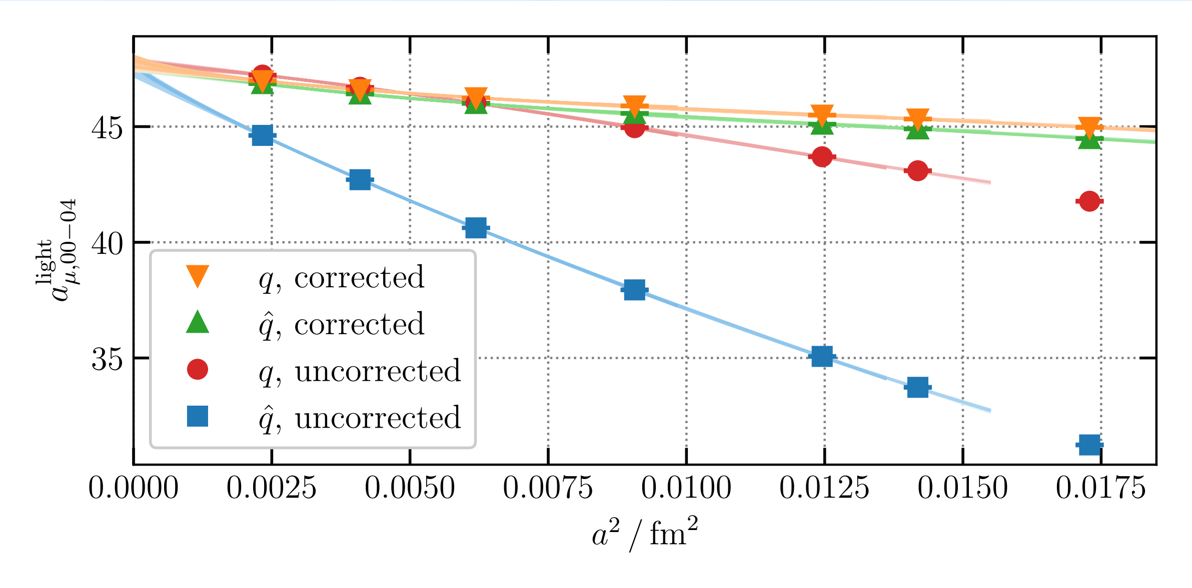
Short distance light connected window



- Lattice artifacts are logarithmically enhanced
- Leading-order infinite-volume massless staggered perturbation theory

$$a^{\text{light}}_{\mu,00-04} \to a^{\text{light}}_{\mu,00-04} + a^{\text{tree}}_{\mu,00-04}(0) - a^{\text{tree}}_{\mu,00-04}(a)$$

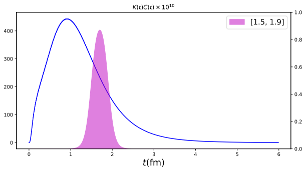
- The logarithmically enhanced cutoff effect has a different sign for the two kernels
- Logarithmic terms included in fits



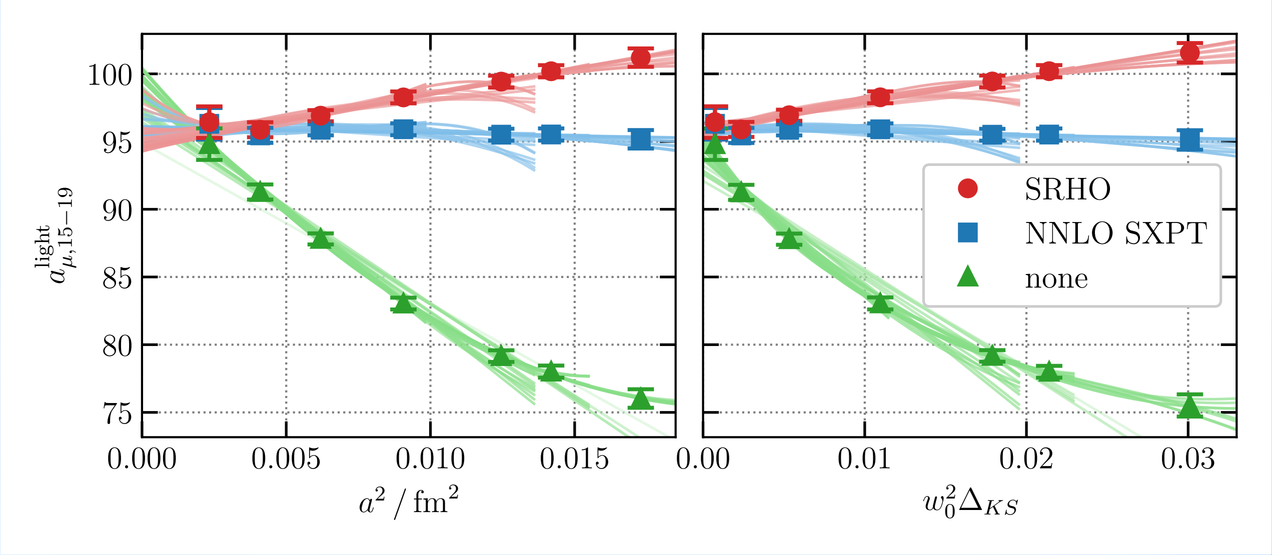
- ***Error budget***

Median	47.821	
Total error	0.153	0.32 %
Statistical error	0.040	0.08 %
Systematic error	0.148	0.31 %
Pseudoscalar fit range	0.002	< 0.01 %
Physical value of M_{ss}	< 0.001	< 0.01 %
w_0 scale setting	0.010	0.02 %
Tree-level corrections & $\hat{q}$	0.103	0.22 %
Lattice spacing cuts	0.077	0.16 %
Order of fit polynomials	0.085	0.18 %

Long-distance window



- Large taste-breaking effects
  - staggered version of the rho-pion-gamma model (SRHO)
  - NNLO staggered chiral perturbation theory (NNLO SXPT)
- Dominant uncertainties are the scale setting and taste breaking correction error
- Instead of `a` NNLO SXPT have mass shift for each taste as input

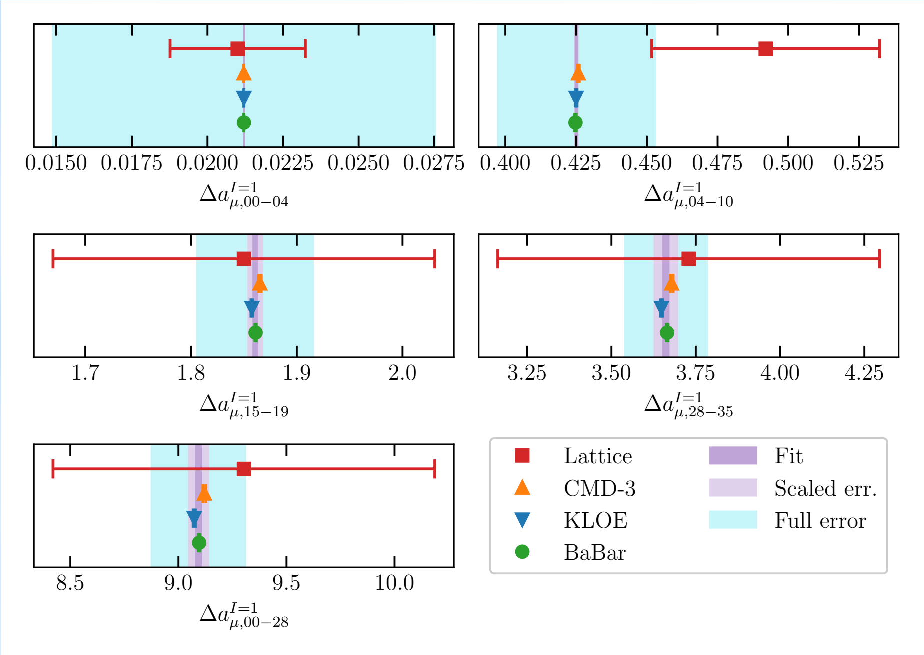


- ***Error budget***

Median	95.61	
Total error	1.60	1.68 %
Statistical error	1.14	1.19 %
Systematic error	1.13	1.18 %
Pseudoscalar fit range	0.03	0.03 %
Physical value of M_{ss}	0.01	0.01 %
w_0 scale setting	0.67	0.70 %
Taste breaking correction	0.40	0.42 %
Lattice spacing cuts	0.11	0.12 %
Order of fit polynomials	0.21	0.22 %
Continuum parameter (Δ_{KS} or a^2)	0.34	0.36 %

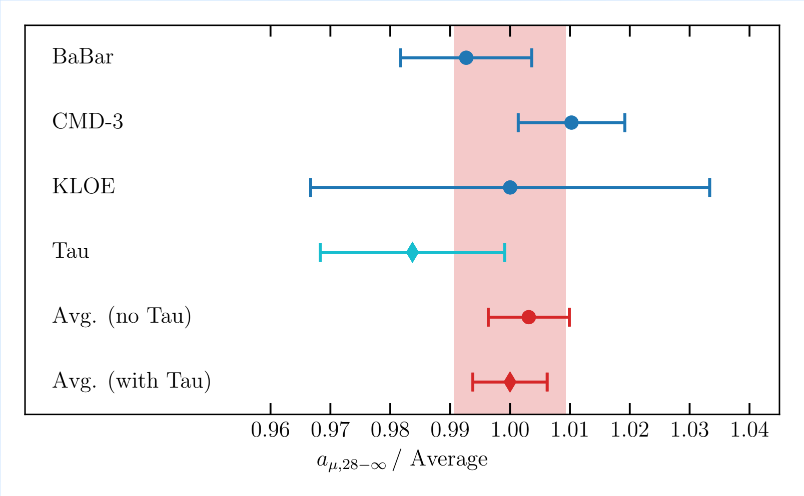
Finite volume corrections

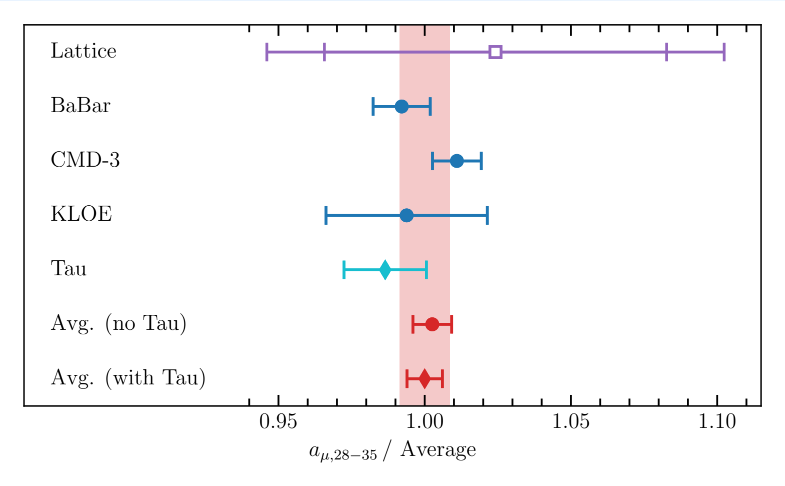
- Direct Lattice calculation and chiral perturbation theory (ChPT)
  - Difference between box size 6.272 fm and a large box size 10.752 fm
  - NLO and NNLO staggered ChPT for 10.752 fm to infinity



- Descriptions based on ππ states should reproduce FV effects very well
  - Use R-ratio data with a combination of the Meyer-Lellouch-Lüscher [47–49] (MLL) and the Hansen-Patella [50,51] (HP) methods to predict Finite-volume corrections from 6.272 fm to infinity
  - Confirms direct lattice calculation

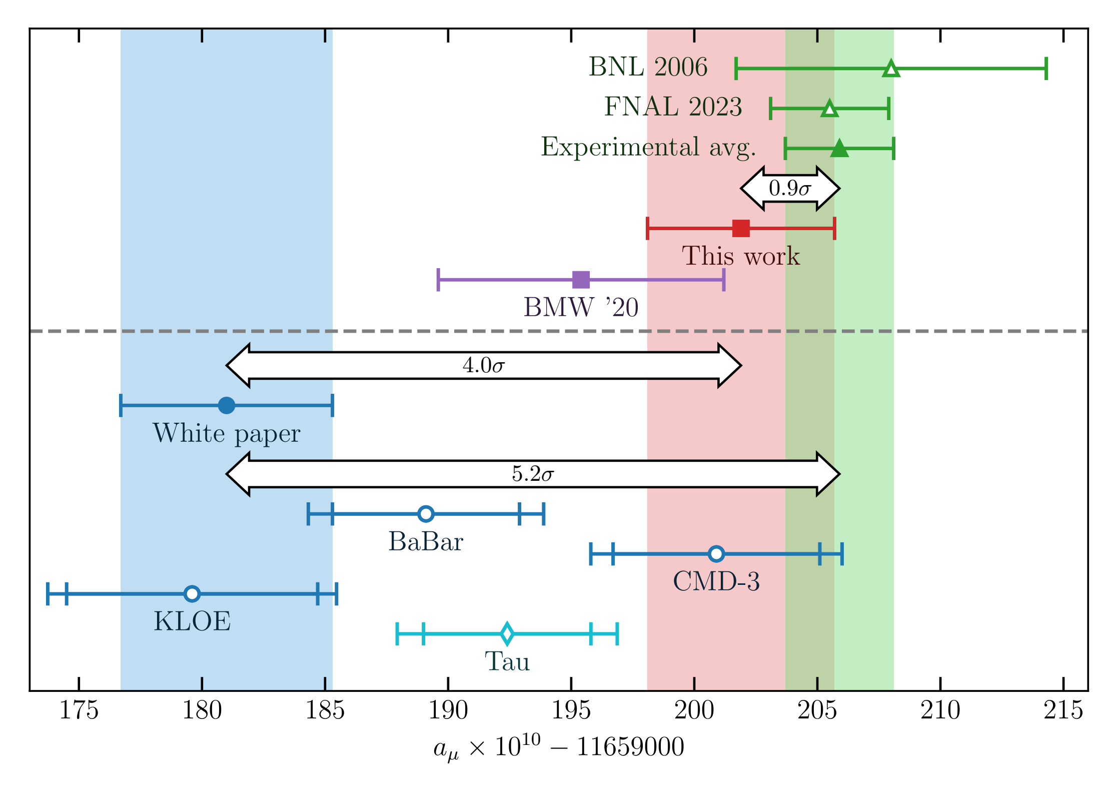
Data-driven tail





- With 2.8 fm, final result is still dominated by the lattice contribution
- The data-driven “tail” reduces the finite-volume correction and statistical error
- Four experiments are consistent for the “tail”
- The experimental results are compatible to lattice at long distance
- Experimental results have negligible errors for “tail”

Results for HVP contribution



- Reduce total uncertainties by 40%
  - Additional lattice at 0.048 fm
  - Use data-driven "tail" after 2.8 fm
  - Finite L and T corrections reduced by a factor of 2
  - Determine the physical point with very precise computation of omega baryon mass

BMW24 result diff with experiment by only 0.9σ

Thank You