2109.04956

Dispersive analysis of glueball properties

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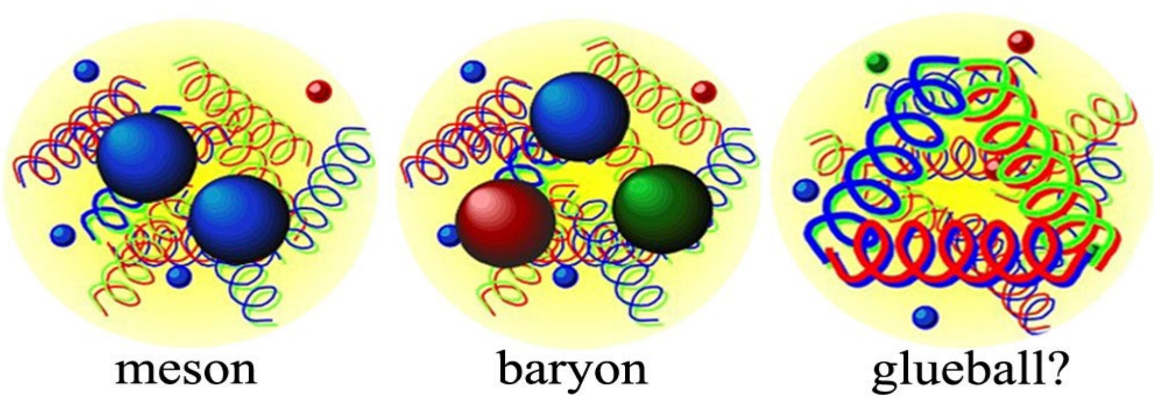
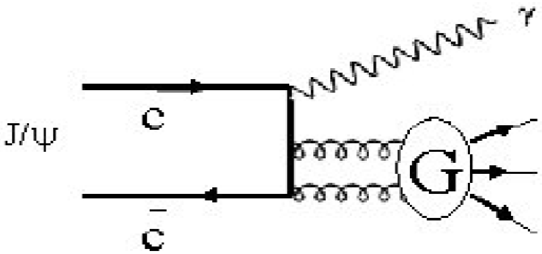


Glueballs

- Quest for glueballs lasted for decades
- Quenched Lattice QCD (LQCD), sum rules (SR) gave scalar glueball mass 1.5-1.7 GeV (Chen et al. 06, Narison 98)
- Large $B(J/\psi \to \gamma f_0(1710)) \approx 10^{-3}$ supports f0(1710) as a candidate



- Quenched LQCD, SR gave pseudoscalar glueball mass > 2 GeV (Morningstar, Peardon 99; Narison 98)
- No strong candidate with mass > 2 GeV from J/psi radiative decays; X(2370) ~ 10E-5, quantum nonumber?

Motivation

- Conventional QCD sum rules: resonances assumed to exist and parametrized into spectral density

Shifman et al, 1979

$$\mathrm{Im}\Pi(q^2) = \frac{\pi}{2} f_V^2 \delta(q^2 - m_V^2) + \pi \rho^h(q^2)\theta(q^2 - s_0^h)$$

spectral density | decay constant | vector mass

- fine for well-established states, but not for uncertain states like glueballs
- New technique needed
- Will analyze glueball properties in dispersive approach developed recently

Formalism

Contour integration

- Two-current correlator $J_\mu = (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)/\sqrt{2}$

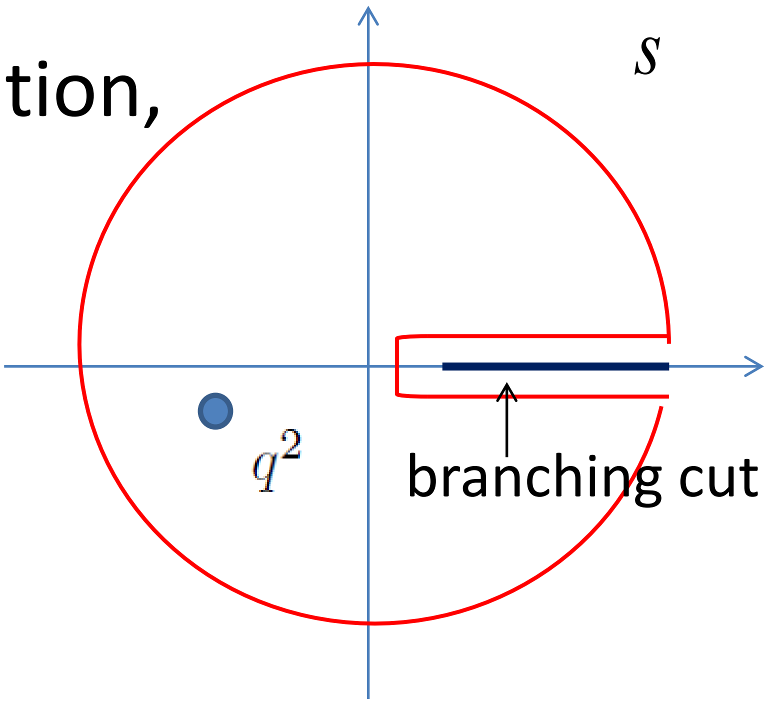
$$\Pi_{\mu\nu}(q^2) = i\int d^4x e^{iq\cdot x}\langle 0|T[J_\mu(x)J_\nu(0)]|0\rangle$$

$$= (q_\mu q_\nu - g_{\mu\nu}q^2)\Pi(q^2)$$

← vacuum polarization function

- Identity from contour integration, because $\Pi(s)$ (photon self-energy) has no pole

$$\Pi(q^2) = \frac{1}{2\pi i}\oint ds \frac{\Pi(s)}{s-q^2}$$



Quark side

- Work on correlator at large q^2 (deep Euclidean region)
- Operator product expansion reliable

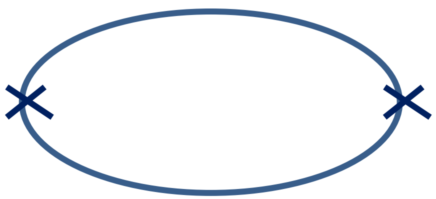
higher order

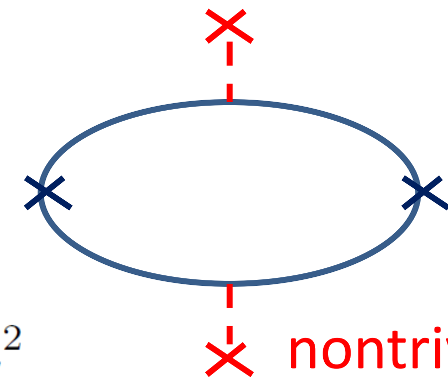
higher powers

$$\Pi^{\mathrm{OPE}}(q^2) = \frac{1}{4\pi^2}\left(1+\frac{\alpha_s}{\pi}\right)\ln\frac{\mu^2}{-q^2} + \frac{1}{12\pi}\frac{\langle\alpha_s G^2\rangle}{(q^2)^2} + 2\frac{\langle m_q\bar{q}q\rangle}{(q^2)^2} + \frac{224\pi}{81}\frac{\kappa\alpha_s\langle\bar{q}q\rangle^2}{(q^2)^3}$$

perturbative piece

4-quark condensate





$$\frac{1}{4\pi^2}\left(1+\frac{\alpha_s}{\pi}\right)\ln\frac{\mu^2}{-q^2} \equiv c\ln\frac{\mu^2}{-q^2}$$

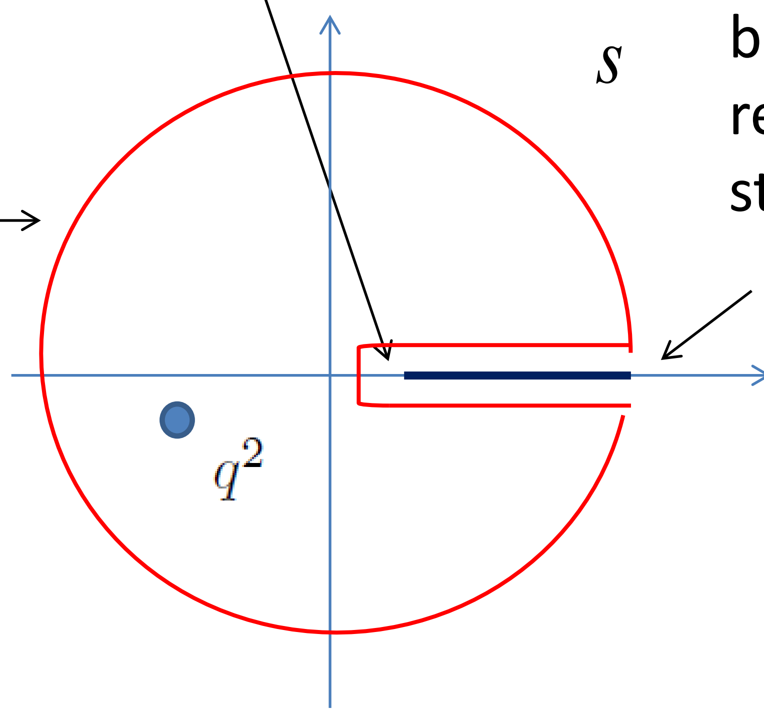
Hadron side

Contour integral

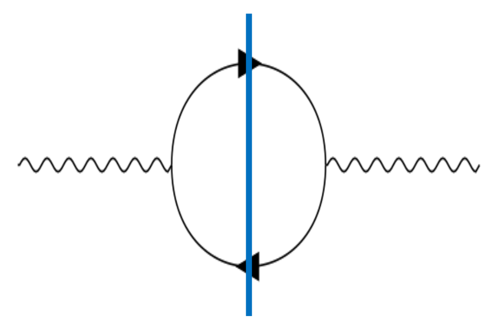
nonperturbative spectral function

$$\frac{1}{2\pi i}\oint ds\frac{\Pi(s)}{s-q^2} = \frac{1}{\pi}\int_0^R ds\frac{\mathrm{Im}\Pi(s)}{s-q^2} + \frac{1}{2\pi i}\int_C ds\frac{\Pi^{\mathrm{pert}}(s)}{s-q^2}$$

contribution from large circle C of radius R will be cancelled



branch cut caused by real intermediate states due to time-like $q^2 > 0$ (log term)



Dispersion relation

- Rewrite pert piece as contour integral

$$\Pi^{\rm OPE}(q^2) = \frac{1}{2\pi i}\oint ds \frac{\Pi^{\rm pert}(s)}{s-q^2} + \frac{1}{12\pi}\frac{\langle \alpha_s G^2\rangle}{(q^2)^2} + 2\frac{\langle m_q \bar{q}q\rangle}{(q^2)^2} + \frac{224\pi}{81}\frac{\kappa\alpha_s\langle\bar{q}q\rangle^2}{(q^2)^3}$$

due to analyticity of perturbation theory

- Equality of two sides gives dispersion relation
- Contributions from big circles cancel, and spectral functions from branch cuts remain

$$\frac{1}{\pi}\int_0^R ds \frac{{\rm Im}\Pi(s)}{s-q^2} = \frac{1}{\pi}\int_0^R ds \frac{{\rm Im}\Pi^{\rm pert}(s)}{s-q^2} + \frac{1}{12\pi}\frac{\langle \alpha_s G^2\rangle}{(q^2)^2} + 2\frac{\langle m_q \bar{q}q\rangle}{(q^2)^2} + \frac{224\pi}{81}\frac{\kappa\alpha_s\langle\bar{q}q\rangle^2}{(q^2)^3}$$

Weakness of sum rules

- How to handle excited-state contribution?
- Rely on parametrization, quark-hadron duality

$$\mathrm{Im}\Pi(q^2) = \pi f_V^2 \delta(q^2 - m_V^2) + \mathrm{Im}\Pi^{\mathrm{pert}}(q^2)\theta(q^2 - s_0)$$

observables: decay constant, mass

continuum threshold

- Duality may fail
- Stability in unphysical Borel mass? (equivalent to q)
- Usually not; rely on discretionary prescription; tune s0 to make 70% (30%) perturbative (nonperturbative) contribution

Idea

- Start with analyticity like sum rules
- OPE in Euclidean region calculable to high orders and powers with universal condensates
- Handle dispersion relation as inverse problem
- Solve for spectral density from inputs directly
- No presumption of resonances, no continuum threshold (free parameter), no duality
- Systematic framework; high predictive power
- Limitation: observables must be formulated as correlators (inapplicable to jet observables)

UV subtraction

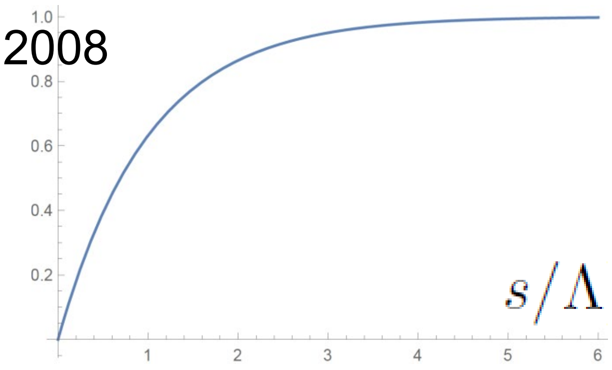
arbitrary R switched into arbitrary scale

- Subtracted spectral function

$$\Delta\rho(s,\Lambda) = \rho(s) - \frac{1}{\pi}\mathrm{Im}\Pi^{\mathrm{pert}}(s)[1-\exp(-s/\Lambda)]$$

- Maintain low-energy behavior $\rho(s) \sim s$ as $s \to 0$
- Bear resonance structure the same as $\rho(s)$

Kwon et al 2008



- Circle radius R can be pushed to infinity

$$\int_0^\infty ds \frac{\Delta\rho(s,\Lambda)}{s-q^2} = \int_0^\infty ds \frac{ce^{-s/\Lambda}}{s-q^2} + \frac{1}{12\pi}\frac{\langle\alpha_s G^2\rangle}{(q^2)^2} + 2\frac{\langle m_q\bar{q}q\rangle}{(q^2)^2} + \frac{224\pi}{81}\frac{\kappa\alpha_s\langle\bar{q}q\rangle^2}{(q^2)^3}$$

- No duality assumed at finite s

Solving integral equation

Fredholm integral equation

- Goal is to solve ill-posed integral equation

unknown spectral density
to be solved

$$\int_0^\infty dy \frac{\rho(y)}{x-y} = \omega(x)$$

OPE input

1st kind of Fredholm integral equation

- How to solve it? Notoriously difficult
- Discretization does not work

ill-posedness

- Discretizing integral equation fails

$$\sum_i M_{ij}\rho_j = \omega_i \qquad M_{ij} = \begin{cases} 1/(i-j), & i \neq j \\ 0, & i = j \end{cases}$$

unknowns input

- Rows M_{ij} and $M_{(i+1)j}$ become almost identical for fine meshes, det(M) ~ 0
- Matrix M becomes singular; M^{-1} diverges quickly
- Solution diverges and sensitive to variation of inputs

Strategy

- Suppose $\rho(y)$ decreases quickly enough
- Expansion into powers of 1/x justified

$$\frac{1}{x-y} = \sum_{m=1}^{N} \frac{y^{m-1}}{x^m} \qquad \omega(x) = \sum_{n=1}^{N} \frac{b_n}{x^n}$$

true for OPE

- Suppose $\omega(x)$ can be expanded
- Decompose

$$\rho(y) = \sum_{n=1}^{N} a_n y^{\alpha} e^{-y} L_{n-1}^{(\alpha)}(y)$$

generalized Laguerre polynomials

depend on $\rho(y)$ at $y \to 0$

- Orthogonality

$$\int_0^{\infty} y^{\alpha} e^{-y} L_m^{(\alpha)}(y) L_n^{(\alpha)}(y) dy = \frac{\Gamma(n+\alpha+1)}{n!} \delta_{mn}$$

Solution

- Equating coefficients of $1/x^n$

$$Ma = b \qquad M_{mn} = \int_0^\infty dy y^{m-1+\alpha} e^{-y} L_{n-1}^{(\alpha)}(y)$$

matrix (→ M)

input $b = (b_1, b_2, \cdots, b_N)$

unknown $a = (a_1, a_2, \cdots, a_N)$

- Solution $a = M^{-1} b$
- True solution can be approached by increasing N, but M^{-1} diverges with N
- Additional polynomial gives $1/x^{N+1}$ correction, beyond considered precision

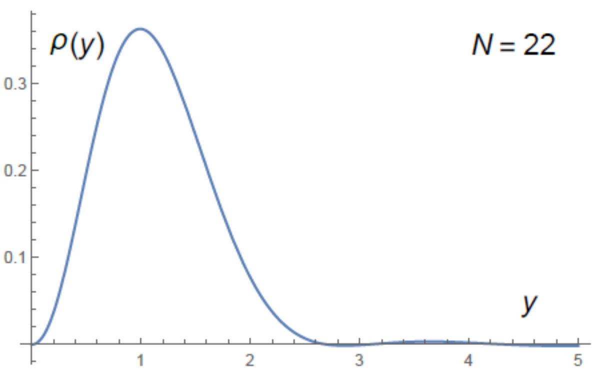
due to orthogonality

Test examples

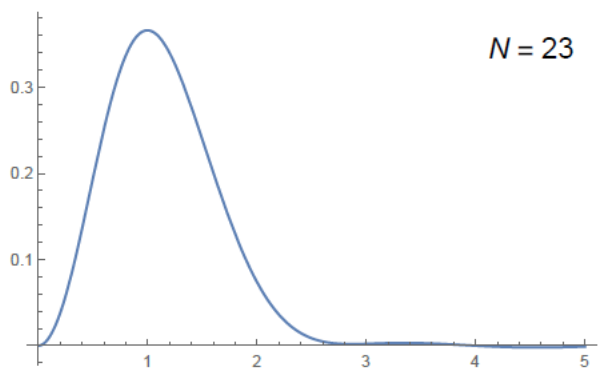
- Generate mock data from $\rho(y) = y^2 e^{-y^2}$

$$b_n = \int_0^\infty dy y^{n-1} y^2 e^{-y^2} \Leftarrow \int_0^\infty dy \frac{\rho(y)}{x-y} = \omega(x)$$

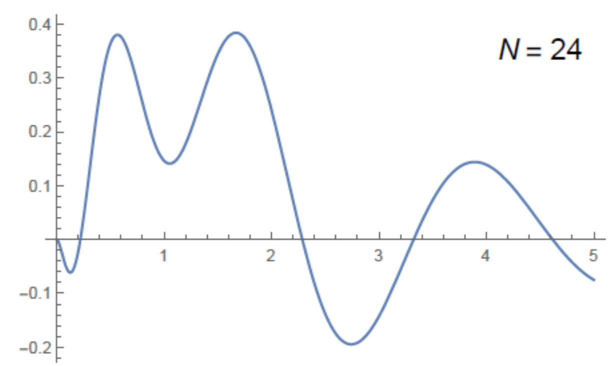
- Compute matrix M with $\alpha = 2$
- Solution stable for N > 20, becomes oscillatory as N=24 due to divergent M^{-1}



$a_{22}/a_{21} \approx 1$



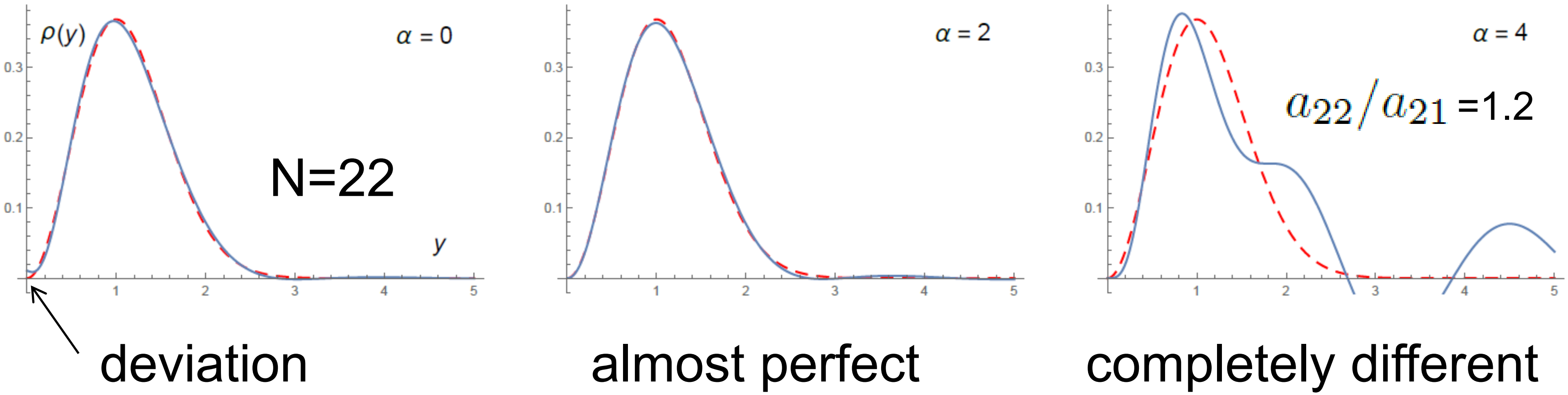
$a_{23}/a_{22} \approx 2$



$a_{24}/a_{23} \approx 58$

Boundary conditions

- Test choices of α (red: true solution)



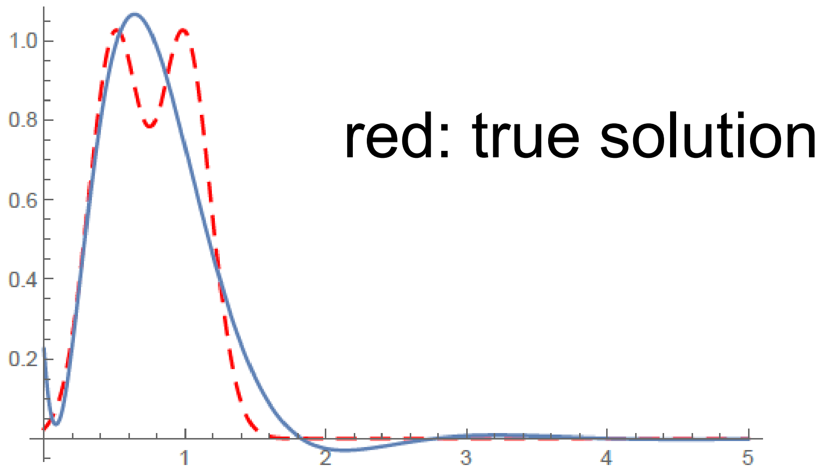
- Parameter α determined by boundary conditions of solution
- Boundary conditions help getting correct solutions

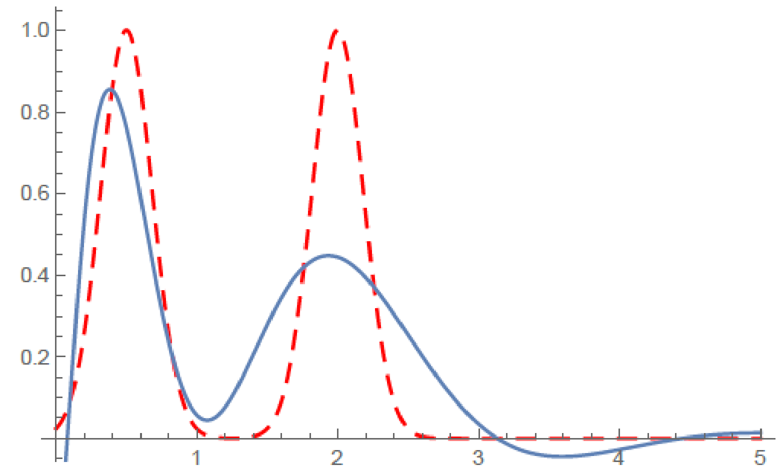
Resolution

- e^{-y} implies resolution power $\Delta y \sim 1$
- Test double peak functions

$$\rho_1(y) = e^{-20(y-0.5)^2} + e^{-20(y-1.0)^2} \qquad \Delta y \sim 0.5$$

$$\rho_2(y) = e^{-20(y-0.5)^2} + e^{-20(y-2.0)^2} \qquad \Delta y \sim 1.5$$





- Fine structure can't be resolved (ill-posed) rho, glueball mass

Features of solution

- Dimensionless $\Delta\rho(s,\Lambda) \to \Delta\rho(s/\Lambda)$
- Variable changes $x = q^2/\Lambda$, $y = s/\Lambda$

$$\int_0^\infty dy \frac{\Delta\rho(y)}{x-y} = \int_0^\infty dy \frac{ce^{-y}}{x-y} - \frac{1}{12\pi}\frac{\langle \alpha_s G^2\rangle}{x^2\Lambda^2} - 2\frac{\langle m_q \bar{q}q\rangle}{x^2\Lambda^2} - \frac{224\pi}{81}\frac{\kappa\alpha_s\langle\bar{q}q\rangle^2}{x^3\Lambda^3}$$

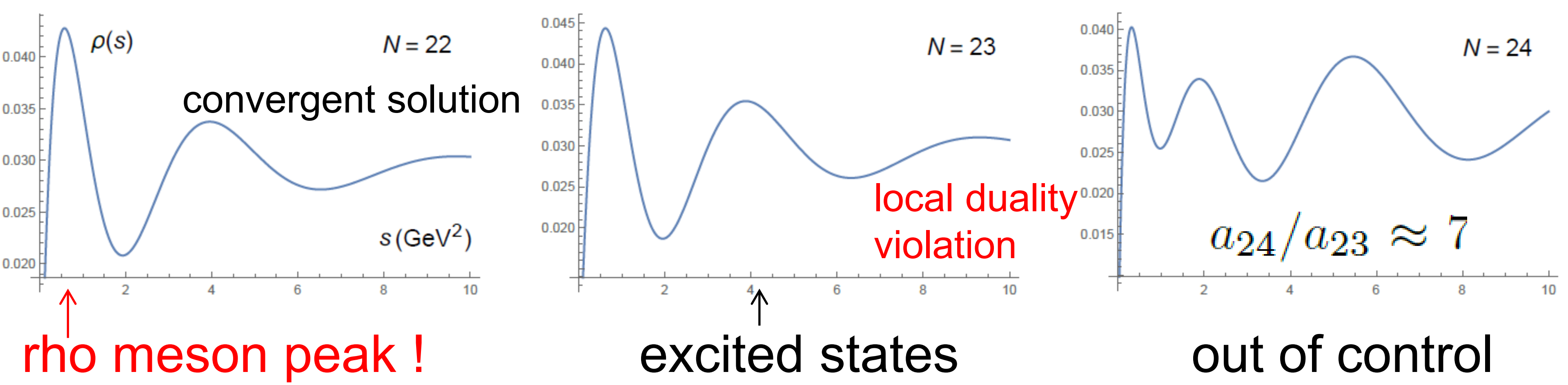
- Λ sets resolution power, ~ O(1) GeV
- Physical solution is independent of Λ
- As it is large enough, condensate effects disappear, scaling of solution appears
- A solution $\Delta\rho(y)$ implies $\Delta\rho(s/\Lambda)$ is solution for any Λ, with which mass grows

rho meson spectral density

- Compute M with $\alpha = 1$
- OPE input known in the literature

$$\langle m_q \bar{q} q \rangle = 0.007 \times (-0.246)^3 \ \mathrm{GeV}^4, \quad \langle \alpha_s G^2 \rangle = 0.08 \ \mathrm{GeV}^4$$

$$\alpha_s \langle \bar{q} q \rangle^2 = 1.49 \times 10^{-4} \ \mathrm{GeV}^6, \quad \alpha_s = 0.5, \quad \kappa = 2,$$

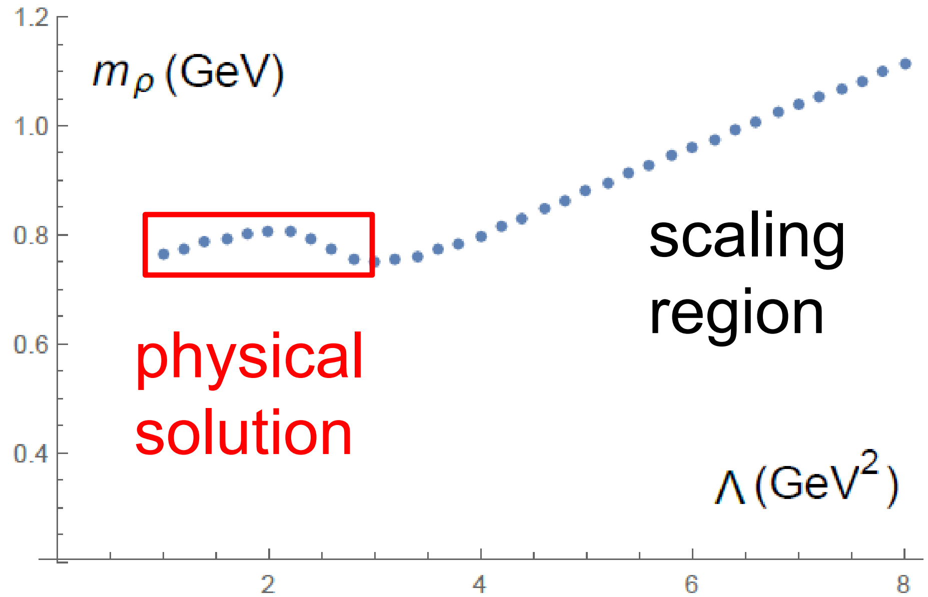


- Positivity satisfied automatically

rho meson mass

- Vary Λ, find peak location
- Tiny error, stable solution

$$m_\rho = (0.78 \pm 0.03)\ \text{GeV}$$



- Including RG effect, condensate variation

$$m_\rho = (0.78 \pm 0.07)\ \text{GeV}$$

- Decay constant, area under peak w/o $\text{Im}\Pi^{\text{pert}}$

$$f_\rho^2 \approx \int_0^\infty ds \Delta\rho(s, \Lambda) \qquad f_\rho \approx 0.20\ \text{GeV}$$

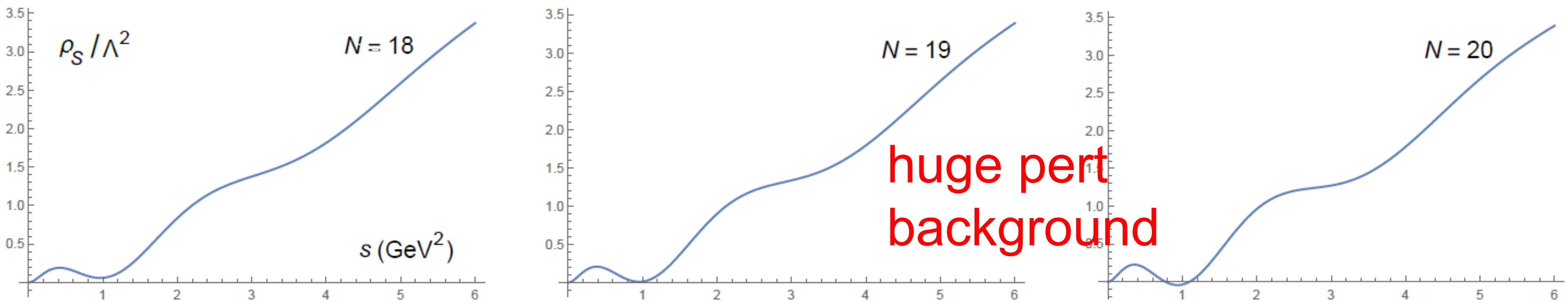
consistent with PDG

Glueball correlators

- Correlator $\Pi_G(q^2) = i\int d^4x e^{iq\cdot x}\langle 0|TO_G(x)O_G(0)|0\rangle$

$$
\begin{aligned}
O_S(x) &= \alpha_s G^a_{\mu\nu}(x)G^{a\mu\nu}(x) \\
O_P(x) &= \alpha_s G^a_{\mu\nu}(x)\tilde{G}^{a\mu\nu}(x) \qquad \tilde{G}_{\mu\nu} \equiv i\epsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}/2
\end{aligned}
$$

- Input $\Pi_G^{\rm OPE}(q^2) = \cdots$ see 2109.04956
- Spectral density for scalar glueball



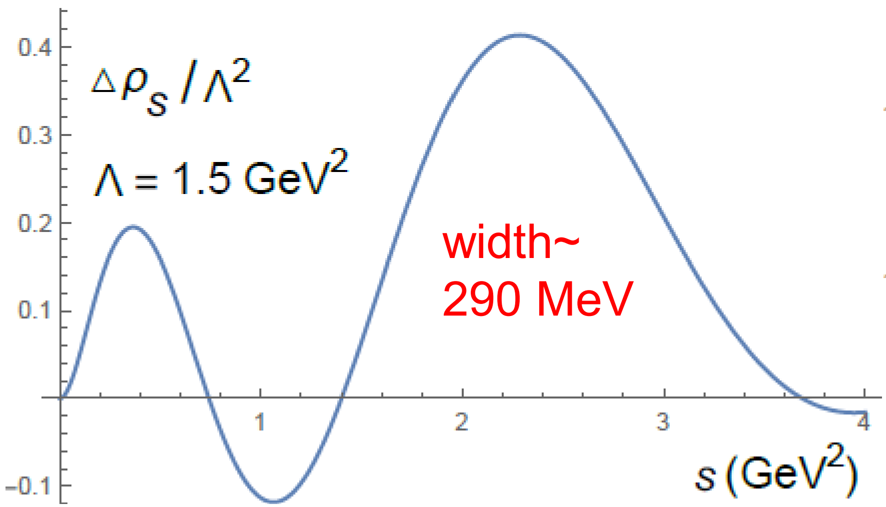
huge pert background

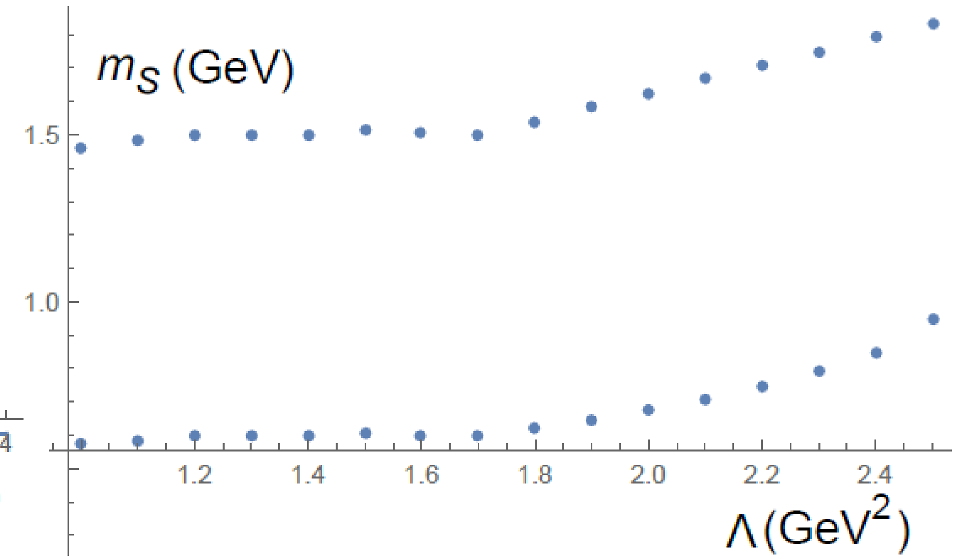
two peaks appear !

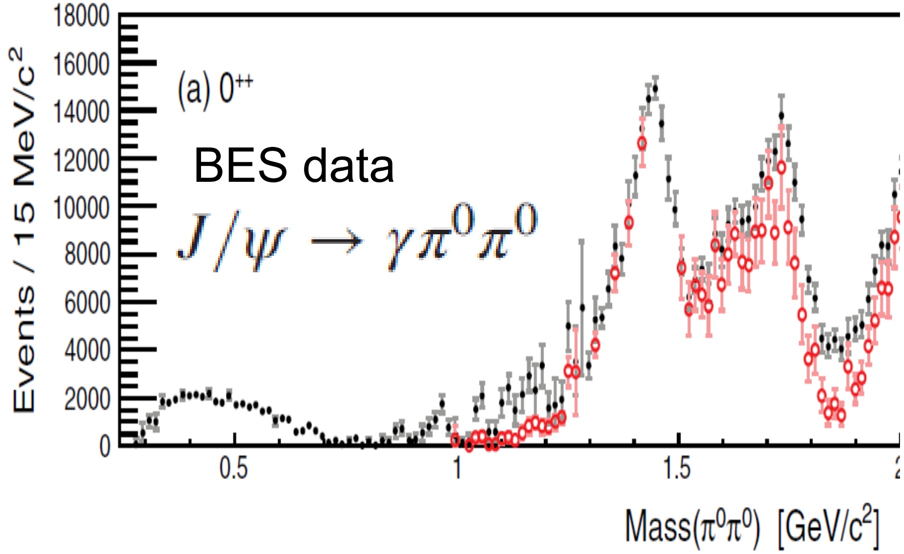
divergent M^{-1}
positivity lost

Scalar glueball mass

- Subtracted spectral density and mass







$m_{S_1} = (0.60 \pm 0.01)$ GeV $\rightarrow f_0(500)$

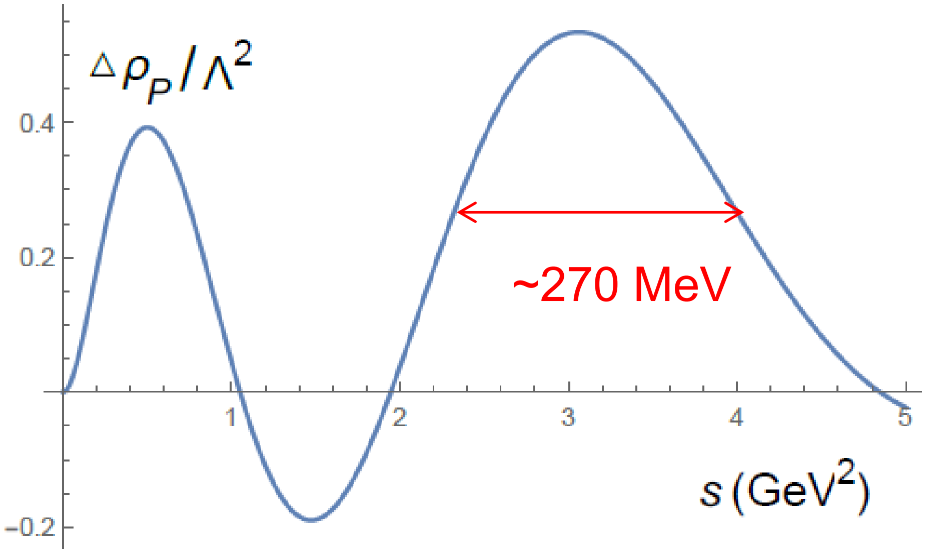
little gluonium in $f_0(980)$?

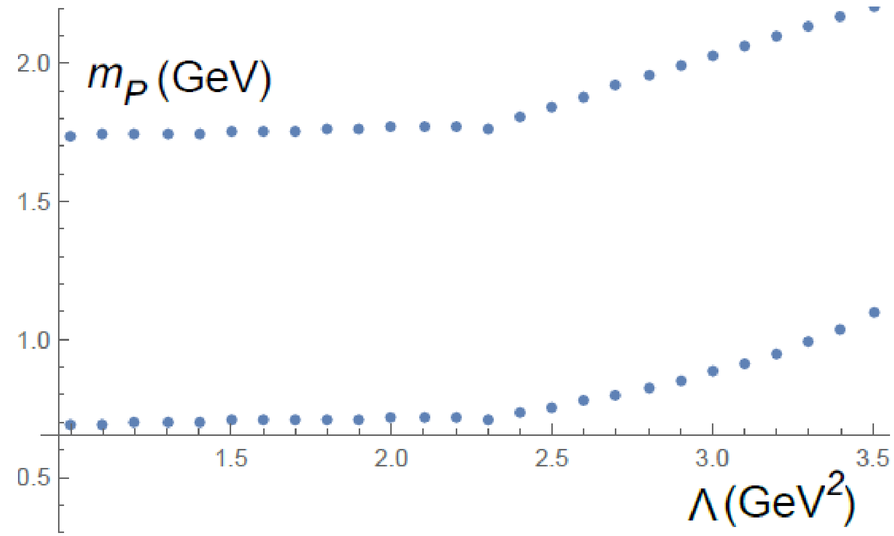
$m_{S_2} = (1.50 \pm 0.01)$ GeV

$f_0(1370),\ f_0(1500)\ \ f_0(1710)$

width~
112 MeV

Pseudoscalar glueball mass





$m_{P_1} = (0.71 \pm 0.02)$ GeV $\rightarrow \eta, \eta'$ $\quad m_{P_2} = (1.75 \pm 0.02)$ GeV $\rightarrow \eta(1760)$

- $\eta(1760)$ proposed by Page, XQ Li in 1998 width ~ 240 MeV
- Quenched LQCD gave 2.6 GeV in 1999
- $J/\psi \rightarrow \gamma X(2370)$ unknown parity, BR~10E(-5)
- $X(1835)$ but seen in $J/\psi \rightarrow \gamma\gamma\phi$, unlikely

BR ~ 10E(-4)

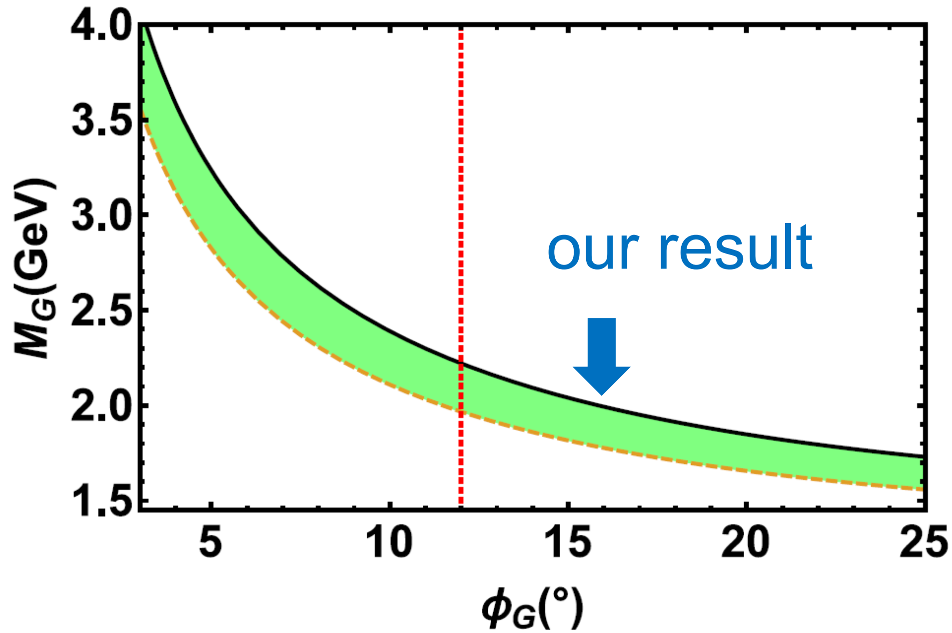
Comparison to earlier work

Tetramixing + anomalous Ward identity

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \\ |G\rangle \\ |\eta_c\rangle \end{pmatrix} = U_{34}(\theta)U_{14}(\phi_G)U_{12}(\phi_Q) \begin{pmatrix} |\eta_8\rangle \\ |\eta_1\rangle \\ |g\rangle \\ |\eta_Q\rangle \end{pmatrix}$$

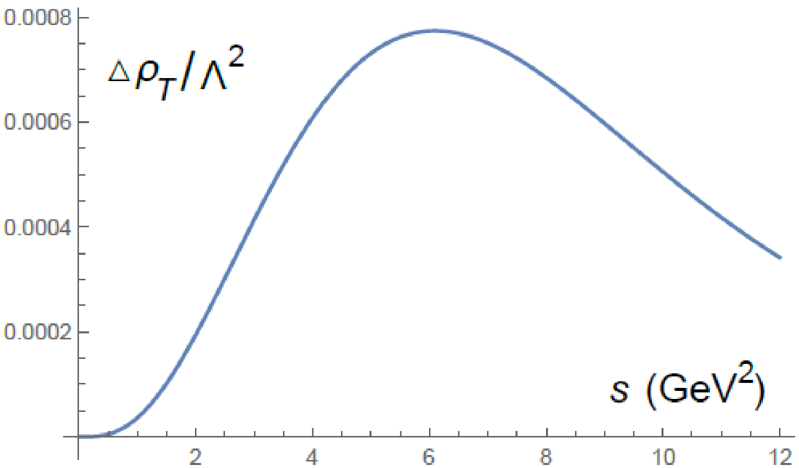
$$U_{14}(\phi_G) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi_G & \sin\phi_G & 0 \\ 0 & -\sin\phi_G & \cos\phi_G & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

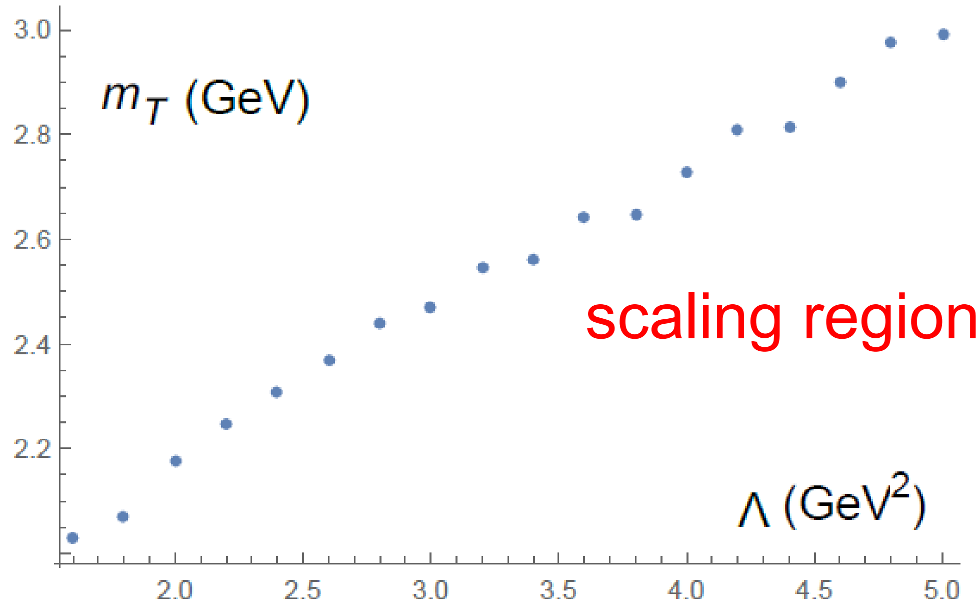
``not lower than 1.8 GeV”,
W. Qin, Q. Zhao, X. H. Zhong,
1712.02550



Tensor glueball mass

- No solution!





- Single condensate $\langle \alpha_s^2 G^4 \rangle$ in OPE, insufficient nonpert information to establish resonance
- Narison found $m_T \approx 2.0\ \mathrm{GeV}$ in 1998, but no stability as varying continuum threshold

  instead, chose an inflection point

Correlator at zero momentum

- Once $\Delta\rho_G$ is obtained, can calculate

$$\chi_t = i\int d^4x \langle 0|Q(x)Q(0)|0\rangle$$

local operator,
UV divergence

- Can be made well-define in our formalism

perturbative piece

$$\Pi_G(q^2) \;=\; \int_0^R \frac{ds}{s-q^2}\left\{\underline{\Delta\rho_G(s,\Lambda)} + s^2\left[A_0^{(G)} + 2A_1^{(G)}\ln\frac{s}{\mu^2} + A_2^{(G)}\left(3\ln^2\frac{s}{\mu^2} - \pi^2\right)\right]\exp(-s/\Lambda)\right.$$
$$\left. + B_1^{(G)}\langle\alpha_s G^2\rangle\exp(-s^2/\Lambda^2)\right\} + \frac{1}{2\pi i}\oint ds\frac{\Pi_G^{\rm perp}(s)}{s-q^2},$$

condensates

UV contribution from large circle R

known and finite!

$$\Pi_G(0) \;=\; \lim_{\epsilon\to 0}\Pi_G(-\epsilon\Lambda)$$

Topological susceptibility

- Low-energy theorem

$\beta_0 = 11N_c/3 - 2N_f/3$

$$\Pi_S(q^2=0) = \frac{32\pi}{\beta_0}\langle\alpha_s G^2\rangle,$$

$$\Pi_P(q^2=0) = (32\pi^2\alpha_s)^2\chi_t$$

gluon condensate fixed by rho mass

this relation then fixes triple-gluon condensate (quite uncertain)

triple-gluon condensate used to predict χ_t

$\langle gG^3\rangle = 0.27\ \mathrm{GeV}^2\ \langle\alpha_s G^2\rangle$ instanton

$\langle gG^3\rangle = -1.5\langle\alpha_s G^2\rangle^{3/2},$ lattice

- Prediction $\chi_t^{1/4} = 75\text{-}78\ \mathrm{MeV}$ almost independent of Λ

Lattice: $\chi_t^{1/4} = 66\text{-}120\ \mathrm{MeV}$

chiral perturbation: $\chi^{1/4} \approx 75\ \mathrm{MeV}$

Summary

- Solve dispersion relation as inverse problem, no duality assumption, no pole parametrization, no discretionary prescription
- Resonances appear naturally, rho mass produced with acceptable OPE inputs
- Same inputs predict $f_0(1370)$, $f_0(1500)$ $f_0(1710)$ and $\eta(1760)$ as glue-rich states
- $f_0(500)$ and η , η' have small amount of gluonium
- Current OPE inputs insufficient to establish tensor glueball