Holography from Conformal Field Theory

Gauge/Gravity Duality Conference 2024
November 30 - December 4, 2024

Robert de Mello Koch

School of Science
Huzhou University

RdMK, G. Kemp and J. Van Zyl, JHEP **04** (2024), 079, arXiv:2403.07606
RdMK and J. Van Zyl, JHEP **09** (2024), 022, arXiv:2406.18248
and work in progress.

December 3, 2024

Quantum Channels

A quantum channel: completely positive trace preserving linear map $\mathcal{E} : L(\mathcal{H}_1) \to L(\mathcal{H}_2)$. $L(\mathcal{H}_i)$ set of linear operators acting on Hilbert space $\mathcal{H}_i$. Quantum channel maps density matrices to density matrices. General transformation of state in an open quantum system.

A quantum channel $\mathcal{E}$ is reversible if one can find a recovery channel $\mathcal{R}$ such that

$$\mathcal{R} \circ \mathcal{E}(\rho) = \rho \qquad \forall \rho \in S(\mathcal{H})$$

$\mathcal{E}$ produces a (discrete) evolution of a state. After evolution, the result of measurement of an observable O is $\mathrm{tr}(\mathcal{E}(\rho)O)$. Describe the evolution of observables by channel $\mathcal{E}^*$ called Hilbert-Schmidt dual map defined as

$$\mathrm{tr}(\rho\mathcal{E}^*(O)) = \mathrm{tr}(\mathcal{E}(\rho)O) \qquad \forall \rho, O$$

Every quantum channel $\mathcal{E}$ has an operator sum rep in terms of non-unique *Kraus operators* $\{A_i\}$ such that,

$$\mathcal{E}(\rho) = \sum_i A_i \rho A_i^\dagger \qquad\qquad \sum_i A_i^\dagger A_i = I$$

$\mathcal{E}^*$ has operator sum representation. The Kraus operators for $\mathcal{E}^*$ are $\{A_a^\dagger\}$ instead of $\{A_a\}$.

Relative Entropy

Relative entropy, defined for a pair of density matrices ρ and σ

$$S(\rho||\sigma) = \mathrm{tr}(\rho \log \rho - \rho \log \sigma)$$

is an important observable:

- Its UV behaviour is better than that of entanglement entropy.
- It is not negative and is only zero when $\rho = \sigma$. It is a measure of how well we can distinguish two states.
- It is non-increasing under the action of any quantum channel $\mathcal{E}$ i.e.

$$S(\rho||\sigma) \geqslant S(\mathcal{E}(\rho)||\mathcal{E}(\sigma))$$

- There exists a quantum channel $\mathcal{R}$ such that for all states $\rho \in S(\mathcal{H})$, $\mathcal{R} \circ \mathcal{E}(\rho) = \rho$ iff $S(\rho||\sigma) = S(\mathcal{E}(\rho)||\mathcal{E}(\sigma))$ for all $\rho, \sigma \in S(\mathcal{H})$.

Quantum Channels and AdS/CFT

According to the AdS/CFT duality every question about bulk physics can be answered in the boundary CFT. We don't know how to translate all questions about bulk physics to questions in the boundary CFT. Completing this bulk-boundary dictionary is the aim of the bulk reconstruction program.

The AdS/CFT correspondence can be modelled as a quantum channel between the Hilbert space of the bulk and the Hilbert space of CFT. We can refine this to a quantum channel between the Hilbert space of a subregion of the bulk and the Hilbert space of a subregion of the CFT.

It is known that AdS/CFT defines a reversible quantum channel, at large N, for the subregion case.

The recovery channel provides the CFT representation for a bulk field, so it provides a bulk reconstruction.

The recovery channel is known as the Petz recovery channel.

Universal recovery channel and the Petz map

There exists a quantum channel $\mathcal{R}$ such that for all states $\rho \in S(\mathcal{H})$, $\mathcal{R} \circ \mathcal{E}(\rho) = \rho$ iff $S(\rho||\sigma) = S(\mathcal{E}(\rho)||\mathcal{E}(\sigma))$ for all $\rho, \sigma \in S(\mathcal{H})$.

Partition the boundary into a region A and its complement $\bar{A}$. The entanglement wedge of boundary region A is the bulk domain of dependence of any bulk spacelike surface whose boundary is the union of A and the codimension two extremal surface whose boundary is ∂A.

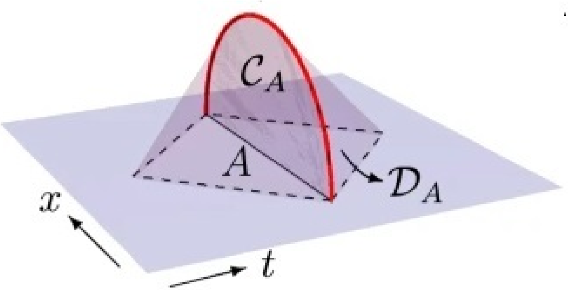
Jafferis, Lewkowycz, Maldacena, Suh, arXiv:1512.06431 show relative entropy of two states in boundary region A equals relative entropy of corresponding bulk states in $\mathcal{E}_A$ up to subleading corrections. Using this Dong, Harlow, Wall, arXiv:1601.05416 proved any bulk operator ϕ_a acting in $\mathcal{H}_a$ is represented in CFT as operator O_A with support only on $\mathcal{H}_A$.

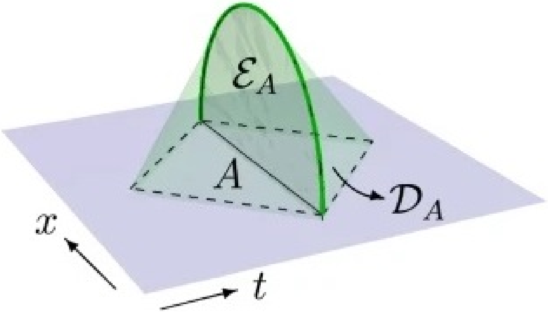
The explicit form of the quantum channel $\mathcal{R}$ for the set of states $\{\mathcal{E}(\rho)|\forall\rho \in S(\mathcal{H})\}$ is given as a function of a reference quantum state $\sigma \in S(\mathcal{H}_A)$ and the channel $\mathcal{E}$ itself as

$$\mathcal{R}(\cdot) = \mathcal{P}_{\sigma,\mathcal{E}}(\cdot) = \sigma^{1/2}\mathcal{E}^*\left(\mathcal{E}(\sigma)^{-1/2}(\cdot)\mathcal{E}(\sigma)^{-1/2}\right)\sigma^{1/2}$$

where $\mathcal{E}^*$ is the dual channel of $\mathcal{E}$. $\mathcal{P}_{\sigma,\mathcal{E}}$ is known as *Petz recovery channel*.

Entanglement Wedge Reconstruction





Goal of this talk

Collective field theory provides a constructive approach to the AdS/CFT correspondence. Using it we can obtain a completely explicit formula for any bulk operator ϕ_a acting within $\mathcal{H}_a$ as a CFT operator O_A with support only on $\mathcal{H}_A$.

The Petz recovery channel provides a completely explicit formula for any bulk operator ϕ_a acting within $\mathcal{H}_a$ as a CFT operator O_A with support only on $\mathcal{H}_A$.

Do these two reconstructions agree?

Constructive Holography

Constructive holography is accomplished by a change to gauge invariant field variables in the CFT. A change of spacetime coordinates is needed to give the bulk interpretation of the collective field theory. Das and Jevicki, Phys. Rev. D **68** (2003), 044011

Starting from the CFT and carrying out these two steps, one obtains the higher dimensional gravitational theory.

Holography is demonstrated by giving a holographic map constructed by:

1. Reducing gravity to physical and independent degrees of freedom.
2. Reducing CFT to its independent degrees of freedom.
3. Identifying the complete set of degrees of freedom of CFT with those of gravity.

We work entirely at the leading order at large N. Thus we reproduce linearized gravity. We work entirely around empty AdS.

Higher Spin Gravity/$O(N)$ Vector Model Duality

Polyakov, Klebanov hep-th/0210114 conjectured a duality between Vasiliev's theory of higher spin gravity and critical $O(N)$ vector model.

M. A. Vasiliev, Phys. Lett. B **243** (1990), 378-382.

Higher gravity has a scalar field and spinning gauge fields of every even integer spin.

Vector model single trace primaries include a scalar primary and a conserved current of every even integer spin.

Giombi, Yin arXiv:0912.3462 proved that leading three point correlators computed on the two sides match.

$$\mathrm{AdS}_4 \leftrightarrow \mathrm{CFT}_3$$

Counting degrees of freedom

Counting in gravity: Work in lightcone gauge (AdS_4) $A^{+\mu_2\cdots\mu_{2s}} = 0$. All components $A^{-\mu_2\cdots\mu_{2s}}$ are determined by constraints. Dynamical fields are X, Z polarizations: $A^{XZXZ\cdots ZZ}$. Gauge field is symmetric and traceless, $\Rightarrow$ **two independent** physical degrees of freedom at each spin.

Metsaev **NPB** 563 (1999) 295 hep-th/9906217

Counting in CFT: The spinning currents (CFT_3) are symmetric, traceless, conserved rank $2s$ tensors $J_{\mu_1\cdots\mu_{2s}}$.

There are $\frac{(2s+1)(2s+2)}{2}$ symmetric rank $2s$ tensors.

There are $4s+1$ symmetric, traceless rank $2s$ tensors.

There are **two independent** symmetric, traceless, conserved rank $2s$ tensors.

$\Rightarrow$ **number of independent components of the spinning primary match the number of physical and independent components of the gauge field.**

Reduce gravity to independent and physical fields. Reduce CFT to its independent fields. To construct holographic map, identify physical degrees of freedom.

Equal time bilocal fields

From OPE: bilocal packages the complete set of single trace primary operators

$$\begin{aligned}\sigma(t_1,\vec{x}_1,t_2,\vec{x}_2) &= \sum_{a=1}^{N}\phi^a(t_1,\vec{x}_1)\phi^a(t_2,\vec{x}_2)\\ &= \sum_{s=0}^{\infty}\sum_{d=0}^{\infty} c_{sd}\left(y^\mu\frac{\partial}{\partial x^\mu}\right)^d y_{\mu_1}\cdots y_{\mu_{2s}} j_{(2s)}^{\mu_1\cdots\mu_{2s}}(x)\end{aligned}$$

where $x^\mu = \frac{1}{2}(x_1^\mu + x_2^\mu)$ and $y^\mu = \frac{1}{2}(x_1^\mu - x_2^\mu)$.

Equal x^+ bilocal $\sigma(x^+,x_1^-,x_1,x_2^-,x_2)=\sum_a\phi^a(x^+,x_1^-,x_1)\phi^a(x^+,x_2^-,x_2)$ has $y^+=0$

$\Rightarrow$ packages only currents with $-$ and x polarizations.

Using known transformation rule of scalar field and the OPE, we verify symmetries are implemented as in the reduced theory obtained by eliminating $+$ polarizations.

The equal x^+ bilocal theory provides reduction of CFT to independent fields.

RdMK, JHEP **08** (2023), 056 2307.05032

Conformal symmetry

The equal x^+ bilocal theory provides reduction of CFT to independent fields.
Obtain a representation of symmetry on $j^{xx\cdots x}$ and $j^{-x\cdots x}$.

RdMK, JHEP **08** (2023), 056 2307.05032

To work out the so(2,3) AdS isometry (= conformal) generators after reducing to physical degrees of freedom we need to:

• Fix a gauge and solve the associated gauge constraint. Isometries are generated using the Killing vectors as usual.

• Since conformal transformations move out of lightcone gauge, each conformal transformation must be supplemented with a compensating gauge transformation, that restores the gauge.

• Reduce to independent degrees of freedom by solving the symmetric and traceless constraints.

Result is a set of transformation defined on $A^{XX\cdots X}$ and $A^{ZX\cdots X}$ fields.

Metsaev **NPB** 563 (1999) 295 hep-th/9906217

Repackaging Higher Spin Gravity

Many fields on AdS$_4$:

Co-ordinates: $X^+ \equiv X^2 + X^0 \qquad X^- \equiv X^2 - X^0, \qquad X \equiv X^1 \qquad Z$

Metric: $ds^2 = \frac{dX^+ dX^- + dX^2 + dZ^2}{Z^2}$

Fields: $A^{XX\cdots X}(X^+, X^-, X, Z)$, $A^{ZX\cdots X}(X^+, X^-, X, Z)$, $\Phi(X^+, X^-, X, Z)$

One field on AdS$_4 \times$S^1:

Co-ordinates: $X^+ \equiv X^2 + X^0 \qquad X^- \equiv X^2 - X^0, \qquad X \equiv X^1 \qquad Z \qquad \theta$

Metric: $ds^2 = \frac{dX^+ dX^- + dX^2 + dZ^2}{Z^2}$

Field: $\Phi(X^+, X^-, X, Z, \theta) = \sum_{s=0}^{\infty} \left(\cos(2s\theta) \frac{A^{XX\cdots XX}}{Z} + \sin(2s\theta) \frac{A^{XX\cdots XZ}}{Z} \right)$

Matching Fields

$$\sigma(x^+, x_1^-, x_1, x_2^-, x_2) = \sigma_0(x^+, x_1^-, x_1, x_2^-, x_2) + \frac{1}{\sqrt{N}}\eta(x^+, x_1^-, x_1, x_2^-, x_2)$$

$\sigma_0(x^+, x_1^-, x_1, x_2^-, x_2)$ is the large N two point function.

$$\eta(x^+, x_1^-, x_1, x_2^-, x_2) \leftrightarrow \Phi(X^+, X^-, X, Z, \theta)$$

RdMK, Jevicki, Jin and Rodrigues, Phys. Rev. D **83** (2011) 025006.

The bilocal transforms in a tensor product. The complete collection of higher spin fields transform in a direct sum. The natural change of basis

$$V_{[\frac{1}{2},0]} \otimes V_{[\frac{1}{2},0]} \longrightarrow V_{[1,0]} \oplus \bigoplus_{s=2,4,\cdots} V_{[s+1,s]}$$

determines a map between CFT and bulk coordinates.

(Think of addition of angular momentum: $\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$.)

Change of Spacetime Co-ordinates

The bilocal transforms in $V_{\frac{1}{2},0} \otimes V_{\frac{1}{2},0}$ $\qquad$ $(L^A \in \mathrm{so}(2,3))$

$$L^A_{\otimes}\sigma \quad = \quad \Big(L^A\phi^a(x^+,x_1^-,x_1)\phi^a(x^+,x_2^-,x_2) + \phi^a(x^+,x_1^-,x_1)L^A\phi^a(x^+,x_2^-,x_2)\Big)$$

$$V_{\frac{1}{2},0} \otimes V_{\frac{1}{2},0} = V_{1,0} \oplus \bigoplus_{s=2,4,6,\cdots} V_{s+1,s}$$

The complete collection of higher spin fields fill out the reducible representation $V_{1,0} \oplus \bigoplus_{s=2,4,6,\cdots} V_{s+1,s}$

$$L^A_{\oplus}\Phi(X^+,X^-,X,Z,\theta) = \sum_{s=0}^{\infty}\left(\cos(2s\theta)L^A_{2s}\frac{A^{XX\cdots XX}}{Z} + \sin(2s\theta)L^A_{2s}\frac{A^{XX\cdots XZ}}{Z}\right)$$

We want to change from the natural representation $(L^A_{\otimes})$ of the CFT to the representation that is natural for the bulk gravity $(L^A_{\oplus})$.

Change of Spacetime Coordinates

Bilocal field $\eta(x^+, x_1^-, x_1, x_2^-, x_2)$. 5 coordinates in CFT: $x^+, x_1^-, x_1, x_2^-, x_2$.

Higher spin gravity field $\Phi(X^+, X^-, X, Z, \theta)$. 5 coordinates in gravity: X^+, X^-, X, Z, θ.

Symmetry: $X^- \to X^- + a$ in gravity and $x^- \to x^- + b$ in CFT motivates the Fourier transform:

$$\eta(x^+, p_1^+, x_1, p_2^+, x_2) \quad = \quad \int \frac{dx_1^-}{2\pi} \int \frac{dx_2^-}{2\pi} \, \eta(x^+, x_1^-, x_1, x_2^-, x_2) \; e^{-ip_1^+ x_1^- - ip_2^+ x_2^-}$$

5 coordinates in CFT: $x^+, p_1^+, x_1, p_2^+, x_2$.

$$\Phi(X^+, P^+, X, Z, \theta) \quad = \quad \int \frac{dX^-}{2\pi} \, \Phi(X^+, X^-, X, Z, \theta) \; e^{-iP^+ X^-}$$

5 coordinates in gravity: X^+, P^+, X, Z, θ.

Change of Spacetime Coordinates

Bilocal field $\eta(x^+, p_1^+, x_1, p_2^+, x_2)$. 5 coordinates in CFT: $x^+, p_1^+, x_1, p_2^+, x_2$.
Higher spin gravity field $\Phi(X^+, P^+, X, Z, \theta)$. 5 coordinates in gravity: X^+, P^+, X, Z, θ.

$$x_1 = X + Z \tan\left(\frac{\theta}{2}\right) \qquad x_2 = X - Z\cot\left(\frac{\theta}{2}\right) \qquad x^+ = X^+$$

$$p_1^+ = P^+ \cos^2\left(\frac{\theta}{2}\right) \qquad p_2^+ = P^+ \sin^2\left(\frac{\theta}{2}\right)$$

$$X = \frac{p_1^+ x_1 + p_2^+ x_2}{p_1^+ + p_2^+} \qquad Z = \frac{\sqrt{p_1^+ p_2^+}}{p_1^+ + p_2^+} |x_1 - x_2|$$

$$P^+ = p_1^+ + p_2^+ \qquad \theta = 2\tan^{-1}\left(\sqrt{\frac{p_2^+}{p_1^+}}\right)$$

$$L_\oplus^A \Phi = 2\pi P^+ \sin\theta \; L_\otimes^A \eta$$

RdMK, Jevicki, Jin and Rodrigues, Phys. Rev. D **83** (2011) 025006.
RdMK, G. Kemp and J. Van Zyl, JHEP **04** (2024), 079, arXiv:2403.07606

Summary: Constructive Holography

$$\begin{aligned}\sigma(x^+,x_1^-,x_1,x_2^-,x_2) &= \sum_{a=1}^{N}\phi^a(x^+,x_1^-,x_1)\phi^a(x^+,x_2^-,x_2)\\ &= \sigma_0(x^+,x_1^-,x_1,x_2^-,x_2)+\frac{1}{\sqrt{N}}\eta(x^+,x_1^-,x_1,x_2^-,x_2)\end{aligned}$$

$$X = \frac{p_1^+x_1+p_2^+x_2}{p_1^++p_2^+} \qquad Z = \frac{\sqrt{p_1^+p_2^+}}{p_1^++p_2^+}|x_1-x_2| \qquad X^+ = x^+$$

$$P^+ = p_1^++p_2^+ \qquad \theta = 2\tan^{-1}\left(\sqrt{\frac{p_2^+}{p_1^+}}\right)$$

The collective field $\eta(x^+,x_1^-,x_1,x_2^-,x_2)$ is defined on $\mathrm{AdS}_4\times\mathrm{S}^1$. The gravity fields are coefficients of a harmonic expansion on S^1. Note non-trivial field redfinition!

$$\begin{aligned}\Phi &= \sum_{s=0}^{\infty}\left(\cos(2s\theta)\frac{A^{XX\cdots XX}(X^+,P^+,X,Z)}{Z}+\sin(2s\theta)\frac{A^{XX\cdots XZ}(X^+,P^+,X,Z)}{Z}\right)\\ &= 2\pi P^+\sin\theta\;\eta(X^+,P^+\cos^2\frac{\theta}{2},X+Z\tan\frac{\theta}{2},P^+\sin^2\frac{\theta}{2},X-Z\cot\frac{\theta}{2})\end{aligned}$$

Bulk Reconstruction

Reconstruction $\Phi(X^+, P^+, X, Z, \theta)$ obeys correct eqn of motion with correct boundary condition.

GKPW dictionary is local and Z dependent.

$$A_{M_1\cdots M_{2s}} \sim Z^{2-2s} A^{\mathrm{non-norm}}_{M_1\cdots M_{2s}}(X^+, X^-, X) + Z^{1+2s} A^{\mathrm{norm}}_{M_1\cdots M_{2s}}(X^+, X^-, X)$$

GKPW says: $j_{M_1,M_2\cdots M_{2s}} \propto A^{\mathrm{norm}}_{M_1,M_2\cdots M_{2s}}$

Field from constructive holography obeys: dMK, Jevicki, Rodrigues, Yoon, arXiv:1408.4800

$$\frac{\partial^s}{\partial X^{-s}} \Phi_s(X^+; X^-, X, 0) = 16\pi\mathcal{N} \sum_{k=0}^{s} \frac{(-1)^k \partial_-^{s-k} \phi^a(X^+, X^-, X) \partial_-^k \phi^a(X^+, X^-, X)}{\Gamma\left(s-k+\frac{1}{2}\right) \Gamma\left(k+\frac{1}{2}\right) k!(s-k)!}$$

where $\Phi_s \equiv \frac{A^{xx\cdots xx}}{Z}$. Recall that

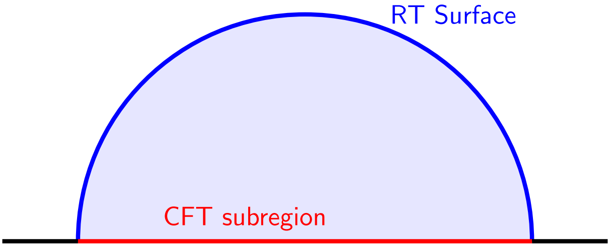
$$J_{\mu_1\mu_2\cdots\mu_{2s}}(t, \vec{x}) \alpha^{\mu_1}\alpha^{\mu_2}\cdots\alpha^{\mu_{2s}} = \sum_{a=1}^{N}\sum_{k=0}^{2s} \frac{(-1)^k \ :(\alpha\cdot\partial)^{2s-k}\phi^a \ (\alpha\cdot\partial)^k \phi^a :}{k!(2s-k)!\Gamma(k+\frac{1}{2})\Gamma(2s-k+\frac{1}{2})}$$

(Mintun, Polchinski, arXiv:1411.3151) showed usual GKPW rule of de Donder gauge maps to contructive holography result in lightcone gauge.

Subregion Duality

Which subregion of the CFT (if any) is dual to a given subregion of the bulk?

Entanglement wedge reconstruction claims that everything from the boundary up to the RT surface can be reconstructed.



Subregion Duality

Which subregion of the CFT (if any) is dual to a given subregion of the bulk?

Consider localized CFT excitations, at time x^+, with first at (x_1, p_1^+) and second at (x_2, p_2^+), described as wavepackets, tightly peaked at x_1 and x_2 along direction x transverse to the light cone, and smeared along x^-.

$$\left(X - \frac{x_1 + x_2}{2}\right)^2 + Z^2 = \left(\frac{x_1 - x_2}{2}\right)^2$$

The bulk excitation is on a semicircle in the X, Z plane, with radius $(x_1 - x_2)/2$ and center $X = (x_1 + x_2)/2$ and $Z = 0$. To locate the excitation specify angle θ

$$\tan\theta = \frac{Z}{X - \frac{x_1+x_2}{2}} = \frac{2\sqrt{p_1^+ p_2^+}}{p_1^+ - p_2^+}$$

This angle θ is the angle θ appearing in the map.

Subregion Duality

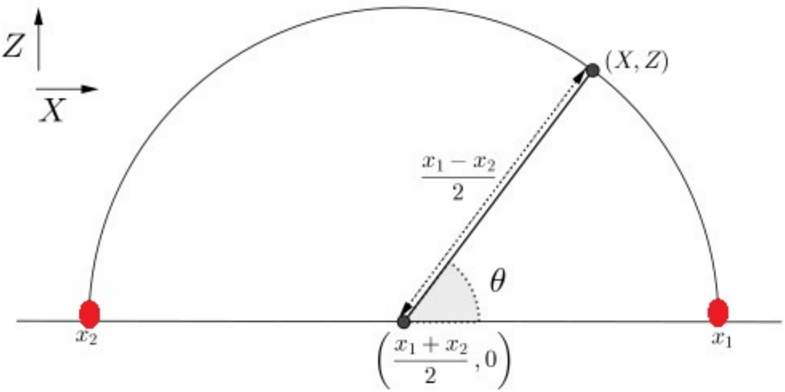


Figure: The bilocal describing excitations localized at (x_1, p_1^+) and (x_2, p_2^+) corresponds to a bulk excitation localized at (X, Z) as shown. This figure lives on a constant $x^+ = X^+$ slice. The angle θ is specified by $\tan\theta = \frac{Z}{X - \frac{x_1 + x_2}{2}}$.

Bulk Reconstruction

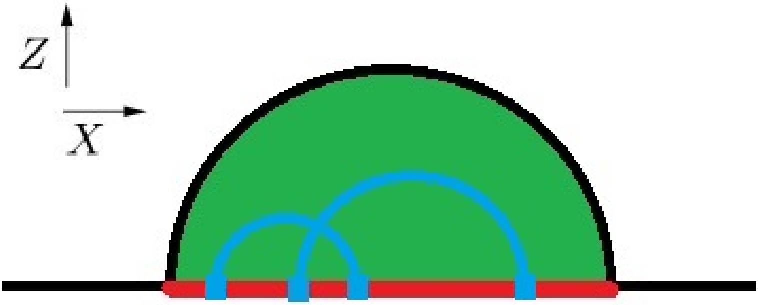


Figure: Using bilocals restricted to the red region of the CFT we are able to reconstruct bulk field living in the area shaded in green. This figure is for fixed time $x^+ = X^+$.

What is the interpretation of the boundary of the green region? The boundary is a geodesic so that the green region is the entanglement wedge.

This completes our discussion of entanglement wedge reconstruction using constructive holography.

We want to compare this answer with that obtained from the Petz recovery channel.

HKLL Reconstruction

Hamilton, Kabat, Lishytz, Lowe hep-th/0606141

Mode expansion of bulk field: $\phi(X) = \sum_n (f_n(X) a_n + h.c.)$

Using the extrapolate dictionary

$$O(t,\Omega) = \lim_{\rho\to\pi/2} \frac{1}{\cos^\Delta \rho}\, \phi(t,\rho,\Omega) \qquad \Rightarrow \qquad O(x) = \sum_n (\tilde{g}_n(x) a_n + h.c.)$$

$$\Rightarrow \quad a_n = \frac{1}{M_n} \int dx O(x) g_n^*(x)$$

This gives the reconstruction

$$\Rightarrow \qquad \phi(X) = \sum_n \left(\frac{1}{M_n} f_n(X) \int dx\ O(x) \tilde{g}_n^*(x) + h.c. \right)$$

Exchanging summation and integration: $\phi(X) = \int dx\ K(X|x) O(x)$.
$K(X|x) = \sum_n M_n^{-1} f_n(X)\ \tilde{g}_n^*(x) + h.c.$ is the smearing function.

Reconstruction from constructive holography can also be written using a smearing function. It does not agree with HKLL.

Code subspace

A low-energy subspace of the full CFT Hilbert space that corresponds to semi-classical bulk geometries in the anti-de Sitter (AdS) spacetime.

Starting with the global AdS vacuum state $|\Omega\rangle$ we define

$$\mathcal{H}_{\mathcal{C}} = span\{|\Omega\rangle, \phi_i(x)|\Omega\rangle, ..., \phi_i(x_1)\phi_j(x_2)|\Omega\rangle, ...\}, \tag{1}$$

where range of i and the number of ϕ insertions are finite. Bulk reconstruction is within a given code subspace. We can define the code subspace around any semi-classical state.

Construct the code subspace using the Reeh-Schlieder theorem. Act on the global vacuum with the operator algebra of region A. A choice of basis for this operator algebra is the Rindler modes $O_{\omega\lambda;A}$ and $O^{\dagger}_{\omega\lambda;A}$. A basis for code subspace at large N is

$$|\{j_{\omega,\lambda}, \Delta_{\omega,\lambda}\}\rangle = \prod_{\omega,\lambda} (O_{\omega\lambda;A})^{j_{\omega,\lambda}} (O^{\dagger}_{\omega\lambda;A})^{j_{\omega,\lambda}+\Delta_{\omega,\lambda}} |\Omega\rangle \tag{2}$$

where $j \in N$ and $\Delta \in Z$.

Bahiru, Vardian, arXiv:2210.00602.

Quantum Channel for AdS/CFT

Global HKLL is a map from bulk states to CFT states $V : \mathcal{H}_{\mathrm{bulk}} \to \mathcal{H}_{CFT}$

Assume $\mathcal{H}_{bulk} = \mathcal{H}_a \otimes \mathcal{H}_{\bar{a}}$ and $\mathcal{H}_{CFT} = \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}}$.

Define $\mathcal{E} : S(\mathcal{H}_a) \to S(\mathcal{H}_A)$. $S(\mathcal{H}_a)$ are density matrices in bulk region a and $S(\mathcal{H}_A)$ are density matrices in boundary region A. The complementary bulk region $\bar{a}$ is in state $\sigma_{\bar{a}}$.

The quantum channel $\mathcal{E}$ is

$$\mathcal{E}(*) = \mathrm{tr}_{\bar{A}}\big(V(* \otimes \sigma_{\bar{a}})V^\dagger\big)$$

Assume both σ_a and $\sigma_{\bar{a}}$ are in maximally mixed states. Then Petz recovery channel gives

$$O_A = \tau_A^{-1/2} \mathrm{tr}_{\bar{A}}\big(V(\phi_a)V^\dagger\big)\tau_A^{-1/2} \qquad \tau_A = \mathrm{tr}_{\bar{A}} P_{\mathrm{code}}$$

$$\langle \phi_a \rangle_{\rho_{bulk}} = \langle \Phi_{a,HKLL} \rangle_{\rho_{CFT}}$$

implies that $V(\phi_a)V^\dagger = P_{code}\Phi_{a,HKLL}P_{code}$, so bulk operator ϕ in entanglement wedge maps to boundary operator with support only in region A

$$O_A = \tau_A^{-1/2} \mathrm{tr}_{\bar{A}}\big(P_{code}\Phi_{HKLL}P_{code}\big)\tau_A^{-1/2}$$

Petz Reconstruction

HKLL reconstruction: $\Phi_{HKLL}(X) = \int_{bdy} dtdx\ K^g(X|t,x)O(t,x)$
A Bogoloiubov transformation relates the global oscillators to Rindler oscillator.

$$\mathrm{tr}_{\bar{A}}\big(P_{code}O_{\omega,\lambda;A}P_{code}\big) = O_{\omega,\lambda;A}\tau_A \qquad \mathrm{tr}_{\bar{A}}\big(P_{code}O^\dagger_{\omega,\lambda;A}P_{code}\big) = \tau_A O^\dagger_{\omega,\lambda;A}$$

The Petz bulk reconstruction

$$\Phi_A(X) = \int d\omega d\lambda\ \Big(\mathcal{F}_{\omega,\lambda;A}(X)\ O_{\omega\lambda,A} + \mathcal{F}^*_{\omega,\lambda;A}(X)\ O^\dagger_{\omega\lambda,A}\Big)$$

can be rewritten as

$$\Phi_A(X) = \int d\tau dx\ K_{Petz,A}(X|\tau,x)\ O(\tau,x)$$

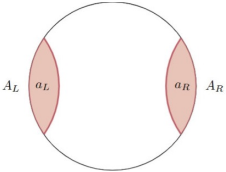
$$K_{Petz,A}(X|\tau,x) = \int d\omega d\lambda\ \mathcal{F}_{\omega,\lambda;A}(X)e^{i\omega\tau}Y^*_\lambda(x)$$

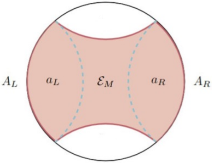
Reconstruction from constructive holography can also be written using a smearing function. For spin-0 is an exact match to $K_{Petz,A}(X|\tau,x)$. Higher spins are in progress.

Discussion and Future Directions

Constructive Holography builds a higher dimensional gravitational theory start from the CFT. We have seen that scalar fields reconstructed with constructive holography agree with those reconstructed using the Petz map.

It is interesting to explore fields that belong to the entanglement wedge but not to the causal wedge.





Generalize away from an equal x^+ description?

Generalize to two time descriptions.

RdMK, Jevicki, Suzuki, Yoon, arXiv:1810.02332,
Aharony, Chester, Urbach, arXiv:2011.06328.

Thanks for your attention!