Gauge Gravity Duality 2024

Holographic spin alignment for vector mesons

Yan-Qing Zhao

Hainan Normal University

In collaborate with: Xin-Li Sheng, Defu Hou, Si-Wen Li, Francesco Becattini

Based on: JHEP 08 (2024) 070 ; PRD 110 (2024) 5, 056047

What spin alignment ?

- The spin state of particles exhibits a certain non-random arrangement in specific directions.

Why spin alignment ?

- Different hadronization mechanisms:

STAR, Nature 614, 244 (2023).

Spin alignments

Global spin alignment in heavy − ion collisions

DELPHI, Phys. Lett. B 406, 271 (1997) OPAL, Phys. Lett. B 412, 210 (1997); Z. fur Physik C Particles Fields 74, 437 (1997); Eur. Phys. J. C 16, 61 (2000).

Helicity − basis spin alignments in e^+e^- *collisions*

Measurements of the global spin alignment in this case provide a good opportunity to study the polarization correlation in the sQGP produced in heavy ion collisions.

How spin alignment ?

- Spin alignment is described by 00-element of meson's normalized spin density matrix.

Theoretical approaches

➢Coalescence model with spin

- Quark/antiquark polarized by external field
- Non-equilibrium process described by kinetic theory

- Z.-T. Liang, X.-N. Wang, PLB 629, 20 (2005).
- XLS, Q. Wang, X.-N. Wang PRD 102, 056013 (2020).
- A. Kumar, B. Mueller, D.-L. Yang, PRD 108, 016020 (2023).
- XLS, L.Oliva, Z.-T.Liang, Q.Wang, X.-N.Wang, PRL 131, 042304 (2023);

➢Spin kinetic equation

- D. Wagner, N. Weickgenannt, E. Speranza, PRR 5, 013187 (2023)
- S. Fang, S. Pu, D.-L. Yang, PRD 109, 034034 (2024)
- Y.-L. Yin, W.-B. Dong, J.-Y. Pang, S. Pu, Q. Wang, arXiv:2402.03672

➢Linear response theory

- F. Li, S. Liu, arXiv: 2206.11890
- W.-B. Dong, Y.-L. Yin, XLS, S.-Z. Yang, Q. Wang,
- arXiv:2311.18400.

➢Spectral function method

In this talk ! ! !

- Splitting between spectral functions of longitudinal and transverse modes due to external fields or motion relative to a thermal background, calculated by QFT, NJL model, holographic model…
- Meson at thermodynamical equilibrium

- XLS, S.-Y. Yang, Y.-L. Zou, D. Hou, arXiv: 2209.01872.
- A. Kumar, B. Mueller, D.-L. Yang, PRD 108, 016020 (2023). M. Wei, M. Huang, CPC 47, 104105 (2023).
- W.-B. Dong, Y.-L. Yin, XLS, S.-Z. Yang, Q. Wang, arXiv:2311.18400.
- XLS, Y.-Q. Zhao, S.-W. Li, F. Becattini, D. Hou, arXiv:2403.07522
- Y.-Q. Zhao, XLS, S.-W. Li, D. Hou, arXiv:2403.07468

A talk by Shu Lin in first day ! ! !

Theoretical approaches

1. Z.-T. Liang, X.-N. Wang, PLB 629, 20 (2005)
2. F. Becattini, L. Csernai, D.-J. Wang, PRC 88, 034905 (2013)
3. Y.-G. Yang, R.-H. Fang, Q. Wang, X.-N. Wang, PRC 97, 034917 (2018)
4. X.-L. S, L. Oliva, Q. Wang, PRD 101, 096005 (2020)
5. F. Li, S. Liu, arXiv: 2206.11890
6. M. Wei, M. Huang, CPC 47, 104105 (2023)
7. S. Fang, S. Pu, D.-L. Yang, arXiv:2311.15197.
8. P. H. D. Moura, K. J. Goncalves, G. Torrieri, PRD 108, 034032 (2023)
9. X.-L. Xia, H. Li, X.-G. Huang, H.-Z. Huang, PLB 817, 136325 (2021)
10. A. Kumar, P. Gubler, D.-L. Yang, arXiv:2312.16900
11. D. Wagner, N. Weickgenannt, E. Speranza, PRR 5, 013187 (2023)
12. W.-B. Dong, Y.-L. Yin, XLS, S.-Z. Yang, Q. Wang, arXiv:2311.18400.
13. F. Sun, J. Shao, R. Wen, K. Xu, M. Huang, arXiv: 2402.16595.
14. X.-L. S, S.-Y. Yang, Y.-L. Zou, D. Hou, arXiv:2209.01872
15. Y.-Q. Zhao, X.-L. S, S.-W. Li, D. Hou, JHEP 08 (2024) 070
16. B. Muller, D.-L. Yang, PRD 105, 1 (2022).
17. J.-H. Gao, PRD 104, 076016 (2021)
18. A. Kumar, B. Muller, D.-L. Yang, PRD 108, 016020 (2023)
19. X.-L. S, S. Pu, Q. Wang, PRC 108, 054902 (2023).
20. X.-L. S, Y.-Q. Zhao, S.-W. Li, F. Becattini, D. Hou, PRD 110 (2024) 5, 056047
21. X.-L. S, L. Oliva, Z.-T. Liang, Q. Wang, X.-N. Wang, PRL 131, 042304 (2023); PRD 109, 036004 (2024)

Smaller deviation from 1/3 (c_{hadro}, c_{EM}, c_F, c_A, c_h)

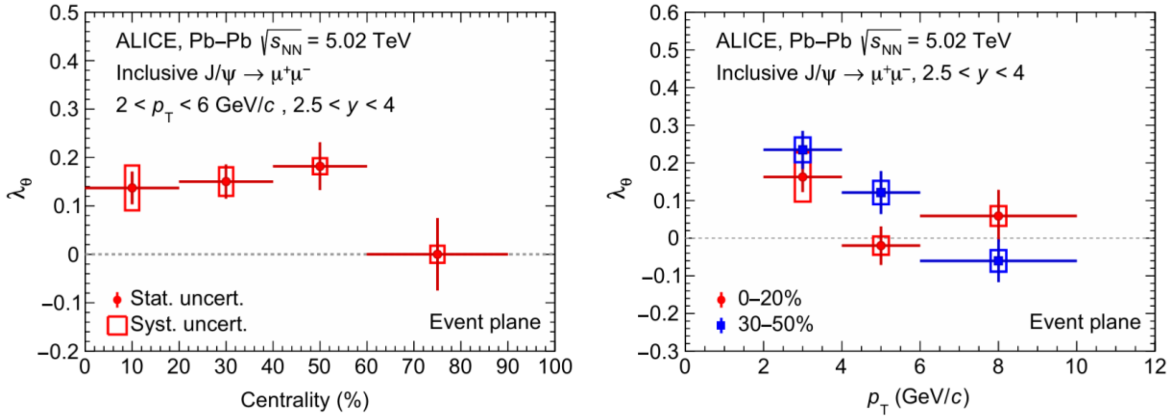
Larger deviation from 1/3 ? (c_{strong})

$$\rho_{00} \approx \frac{1}{3} + c_{hadro} + c_{EM} + c_F + c_A + c_h + c_{strong} + \cdots$$

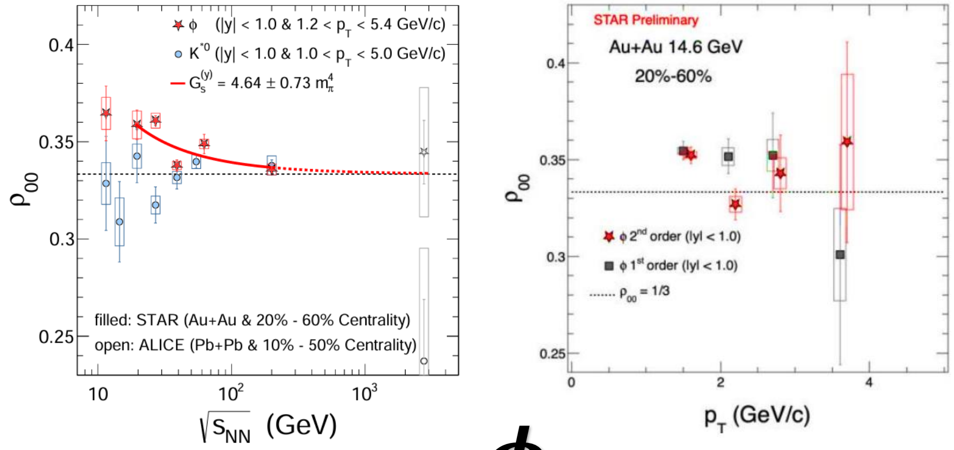
- c_{strong} → Anisotropic strong force [4,15,18-21]
- c_h → Helicity Polarization[17]
- c_A → Anomalous color field [16]
- c_F → Fragmentation [1]
- c_{EM} → Electromagnetic fields [3,4,14,15]
- c_{hadro} → Hydrodynamic gradient (vorticity; Acceleraction; shear tensor; second order;)[1-13]

Experiment results

ALICE, Phys. Rev. Lett. 131 (4) (2023) 042303.



STAR, Nature 614 (7947) (2023) 244–248.



J/ψ

$$\lambda_\theta = \frac{1-3\rho_{00}}{1+\rho_{00}},$$

$\rho_{00} < 1/3$

$\rho_{00} > 1/3$

ϕ

$$\lambda_\theta = \frac{3\rho_{00}-1}{1-\rho_{00}}$$

J/ψ are preferably to be transversely polarized.

ϕ are preferably to be longitudinally polarized.

$\rho_{00} = 1/3$ no spin alignment

Outline

- Theory
  - Spin alignment
  - Correlation function
- Momentum induced spin alignment
- Magnetic field induced spin alignment
- Summary and Outlook

- Theory
  - Spin alignment
  - Correlation function
- Momentum induced spin alignment
- Magnetic field induced spin alignment
- Summary and Outlook

Spin alignment

- S-matrix element($J/\psi \to l + \bar{l}$ and $\phi \to l + \bar{l}$.):

C. Gale and J. I. Kapusta, Nucl. Phys. B 357 (1991) 65–89

$$S_{fi} = \int d^4x d^4y \langle f, l\bar{l} | J_\mu(y) G_R^{\mu\nu}(x-y) J_\nu^l(x) | i \rangle$$

where

J_μ is the current that couples to V.M. ;

J_ν^l is the leptonic current

$$J_\nu^l(x) = g_{Ml\bar{l}} \bar{\psi}_l(x) \Gamma_\nu \psi_l(x)$$

- Propagators (vacuum):

$$G_{R/A}^{\mu\nu} = -\frac{\eta^{\mu\nu} + p^\mu p^\nu / p^2}{p^2 + m_V^2 \pm i m_V \Gamma}$$

- Retarded current-current correlation:

$$D^{\mu\nu}(x,p) \equiv \int d^4y \theta(y^0) \langle [J^\mu(y), J^\nu(0)] \rangle_{T(x)} e^{-ip\cdot y}$$

- Spectral function:

$$\varrho_{\alpha\beta}(x,p) \equiv -\mathrm{Im} D_{\alpha\beta}(x,p)$$

- Differential production rate:

$$n(x,p) = -\frac{2g_{Ml\bar{l}}^2}{3(2\pi)^5}\left(1 - \frac{2m_l^2}{p^2}\right)\sqrt{1 + \frac{4m_l^2}{p^2}}\, p^2 n_B(x,\omega) \times \left(\eta_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right) G_A^{\mu\alpha}(p) \varrho_{\alpha\beta}(x,p) G_R^{\beta\nu}(p),$$

Coordinate space

Y. Burnier, M. Laine, M. Vepsalainen,, JHEP 02 (2009) 008.

L. D. McLerran, T. Toimela, Phys. Rev. D 31 (1985) 545.

H. A. Weldon, Phys. Rev. D 42 (1990) 2384–2387.

Spin alignment

➢ Spectral function:

$$\varrho^{\mu\nu}(x,p)=\sum_{\lambda,\lambda'=0,\pm1} v^{\mu}(\lambda,p)v^{*\nu}(\lambda',p)\tilde{\varrho}_{\lambda\lambda'}(x,p)$$

➢ Polarization vectors:

$$v^{\mu}(\lambda,p)=\left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}_{\lambda}}{M},\boldsymbol{\epsilon}_{\lambda}+\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}_{\lambda}}{M(\omega+M)}\mathbf{p}\right)$$

Orthonormality conditions:

$$\eta_{\mu\nu}v^{\mu}(\lambda,p)v^{*\nu}(\lambda',p)=\delta_{\lambda\lambda'}$$

Completeness condition :

$$\sum_{\lambda} v^{\mu}(\lambda,p)v^{*\nu}(\lambda,p)=(\eta^{\mu\nu}+p^{\mu}p^{\nu}/p^{2})$$

➢ Dilepton production rate:

$$n_{\lambda}(x,p)=-\frac{2g^{2}_{Ml\bar{l}}}{3(2\pi)^{5}}\left(1-\frac{2m_{l}^{2}}{p^{2}}\right)\times\sqrt{1+\frac{4m_{l}^{2}}{p^{2}}}\,\frac{p^{2}n_{B}(x,\omega)\tilde{\varrho}_{\lambda\lambda}(x,p)}{(p^{2}+m_{V}^{2})^{2}+m_{V}^{2}\Gamma^{2}}.$$

➢ Spin alignment:

$$\rho_{00}(x,\mathbf{p})\equiv\frac{\int d\omega\, n_{0}(x,p)}{\sum_{\lambda=0,\pm1}\int d\omega\, n_{\lambda}(x,p)}$$

Spin space

- Theory
  - Spin alignment
  - Correlation function
- Momentum induced spin alignment
- Magnetic field induced spin alignment
- Summary and Outlook

Correlation function

- **AdS/CFT dictionary:**

$$Z_{\mathrm{QFT}}[A_\mu^{(0)}] = Z_{\mathrm{gravity}}[A_\mu]$$

where

$$Z_{\mathrm{QFT}}[A_\mu^{(0)}] = \left\langle \exp\left\{ \int_{\partial\mathcal{M}} J^\mu A_\mu^{(0)} d^4x \right\} \right\rangle,$$

$$Z_{\mathrm{gravity}}[A_\mu] = \exp\{-S_{\mathrm{bulk}}[A_\mu]\}$$

- **Bulk geometry:** $0 \leq \zeta \leq \zeta_h$

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu + g_{\zeta\zeta}d\zeta^2$$

- **Two-point correlators**

$$\langle [J^\mu, J^\nu] \rangle = D^{\mu\nu} \propto \frac{\delta^2 Z_{\mathrm{QFT}}}{\delta A_\mu^{(0)} \delta A_\nu^{(0)}} = \frac{\delta^2 Z_{\mathrm{gravity}}}{\delta A_\mu^{(0)} \delta A_\nu^{(0)}}$$

- **Bulk mesonic action:**

T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113 (2005) 843882.
T. Sakai, S. Sugimoto, Prog. Theor. Phys. 114 (2005) 1083–1118.
A. Karch, E. Katz, D. T. Son, M. A. Stephanov, Phys. Rev. D 74(2006) 015005.

$$S_{\mathrm{bulk}} = -\int d^4x d\zeta Q(\zeta) F_{MN} F^{MN}$$

- **Equation of motion:** $\partial_M[Q(\zeta)F^{MN}] = 0$ **Radial gauge condition:** $A_\zeta = 0$
- **Fourier transformation:** $A_\mu(x,\zeta) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x} A_\mu(p,\zeta)$
- **Electric fields:** $E_i(p,\zeta) \equiv -p_0 A_i(p,\zeta) + p_i A_0(p,\zeta)$
- **Boundary condition:** $\lim_{\zeta\to 0} \tilde{E}_i(j,p,\zeta) = \delta_{ij}$
- **Equation of motion :** $\partial_\zeta^2 E_i(p,\zeta) + \frac{[\partial_\zeta Q(\zeta) g^{\zeta\zeta}]}{Q(\zeta) g^{\zeta\zeta}} [\partial_\zeta E_i(p,\zeta)] - \frac{p^2}{g^{\zeta\zeta}} E_i(p,\zeta)$

$$+ (-p_0 g_{i\mu} + p_i g_{0\mu})(\partial_\zeta g^{\mu\nu})[\partial_\zeta A_\nu(p,\zeta)] = 0,$$

Correlation function

- Correlation function:

D. T. Son and A. O. Starinets, JHEP 09 (2002) 042

$$D^{\mu\nu}(p)=\lim_{\zeta\to 0} g^{\zeta\zeta}g^{\mu\alpha}Q(\zeta)\frac{\delta[\partial_\zeta A_\alpha(p,\zeta)]}{\delta A_\nu(p,\zeta)}\Big|_{A_\mu(p,0)=0}$$

$$D^{\mu 0}(p)=-\lim_{\zeta\to 0}\frac{p_j}{p_0}\left(g^{\mu k}-\frac{p^\mu p^k}{p^2}\right)\frac{1}{\zeta}\partial_\zeta \tilde{E}_k(j,p,\zeta),$$

$$D^{\mu i}(p)=\lim_{\zeta\to 0}\left(g^{\mu k}-\frac{p^\mu p^k}{p^2}\right)\frac{1}{\zeta}\partial_\zeta \tilde{E}_k(i,p,\zeta).$$

- Ward identity: $p_\mu D^{\mu\nu}=p_\mu D^{\nu\mu}=0$

- Spectral function:

$$\tilde{\varrho}_{\lambda\lambda}(p)=v^*_\mu(\lambda,p)v_\nu(\lambda,p)\mathrm{Im}D^{\mu\nu}(p)$$

- Theory
  - Spin alignment
  - Correlation function
- **Momentum induced spin alignment**
- Magnetic field induced spin alignment
- Summary and Outlook

Momentum induced spin alignment

- Holographic background metric:

$$ds^2 = \frac{L^2}{\zeta^2}\left(-f(\zeta)dt^2 + dx^2 + dy^2 + dz^2 + \frac{d\zeta^2}{f(\zeta)}\right)$$

$$f(\zeta) = 1 - \zeta^4/\zeta_h^4$$

- Function:

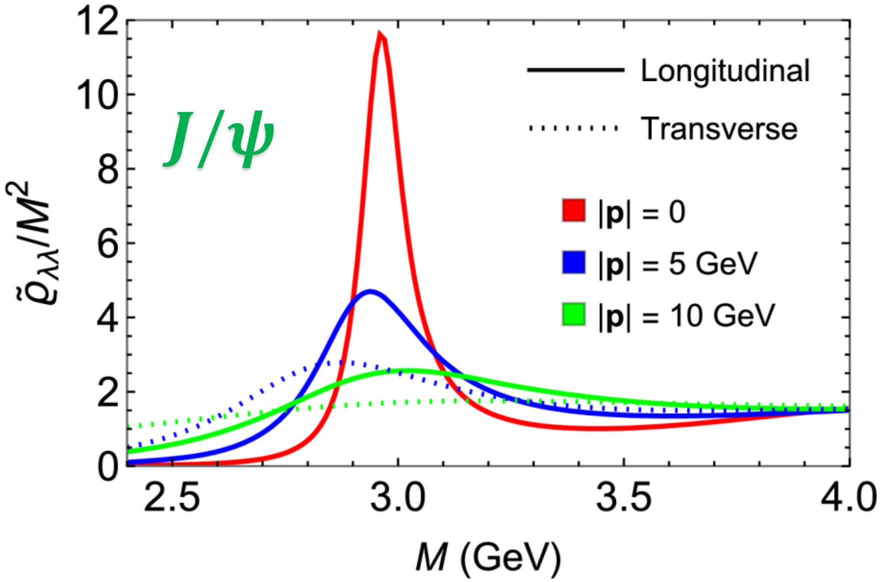
$$Q(\zeta) = e^{-\Phi(\zeta)}\sqrt{-g}/(4g_5^2)$$

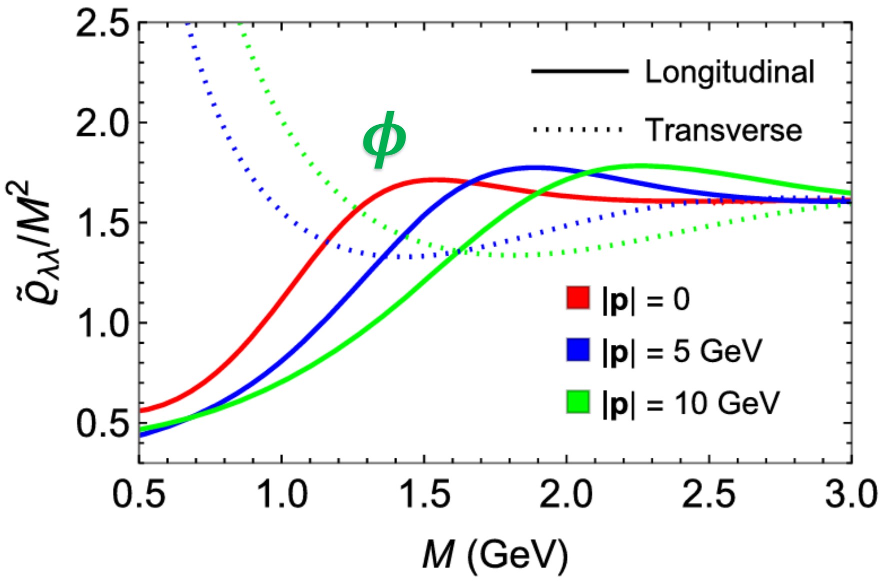
- Dilaton field:

$$\Phi(\zeta) = c\zeta^2$$

Spectral function

$T = 150\ MeV$





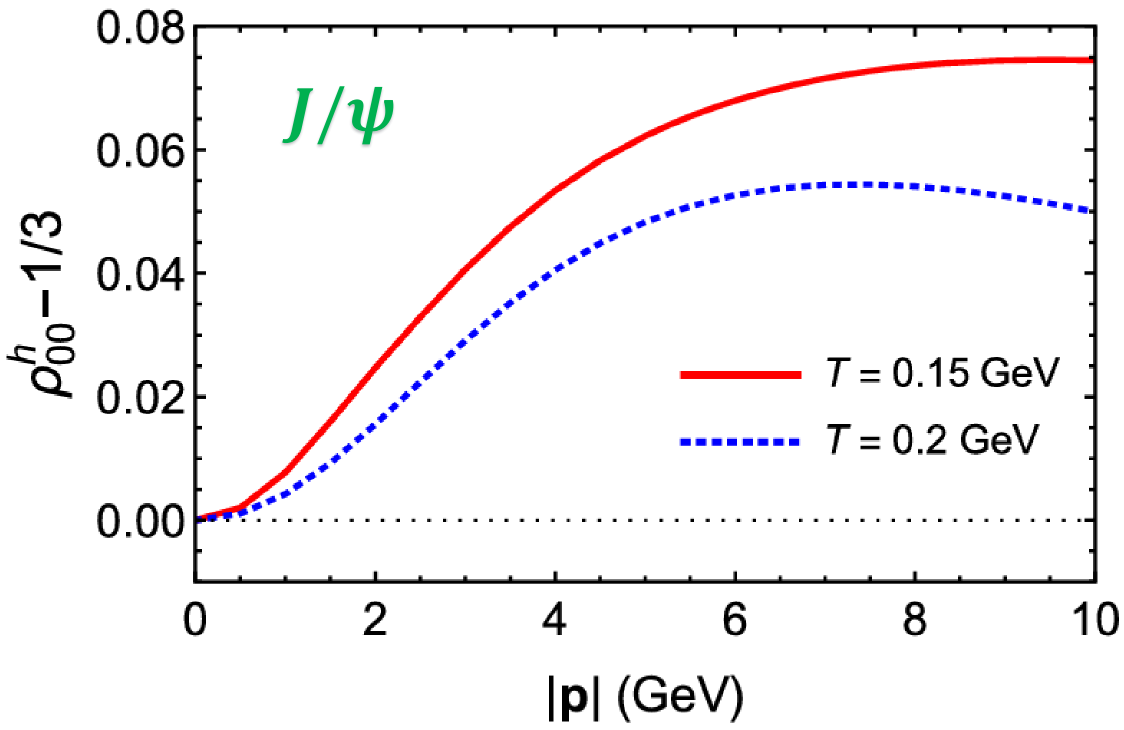
- At zero momentum, spectral functions for all spin states are degenerate because of the rotation symmetry.
- High-momentum resonances are harder to be produced than low-momentum resonances.

- $s - \bar{s}$ pairs cannot exist as quasi-stable particles.
- Spectral functions should be interpreted as probabilities of $s - \bar{s}$ pairs.

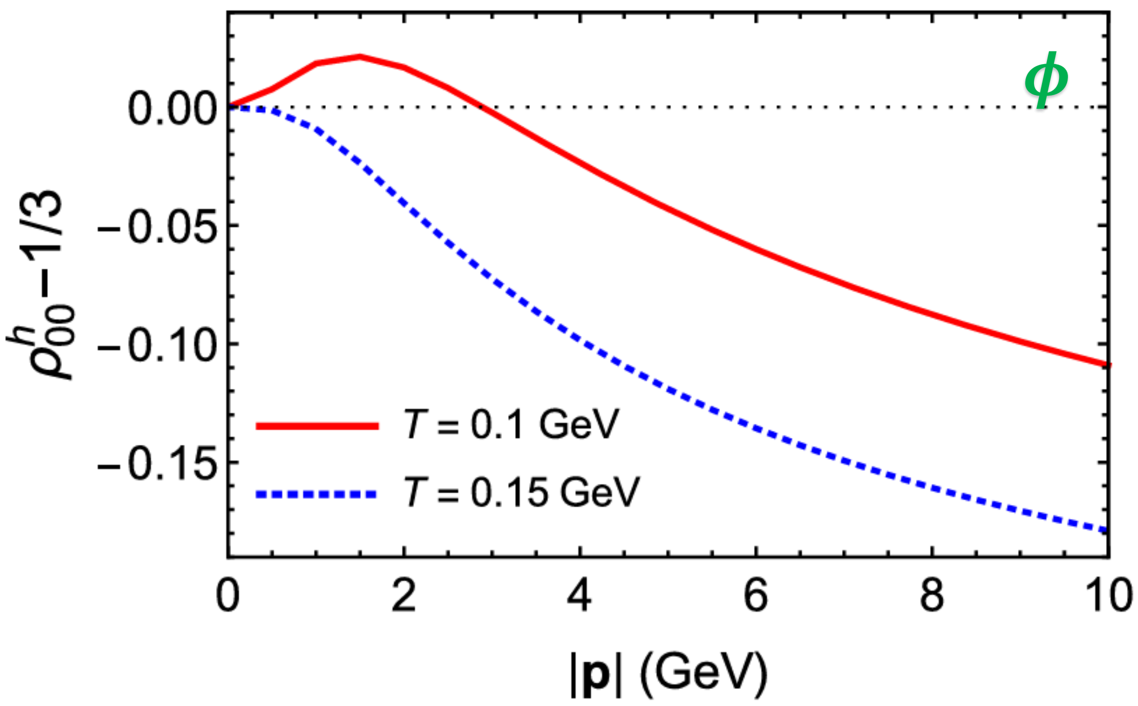
Momentum induced spin alignment

Spin alignment in the helicity frame

$$\lambda_\theta = \frac{1-3\rho_{00}}{1+\rho_{00}},$$



$$\lambda_\theta = \frac{3\rho_{00}-1}{1-\rho_{00}}$$

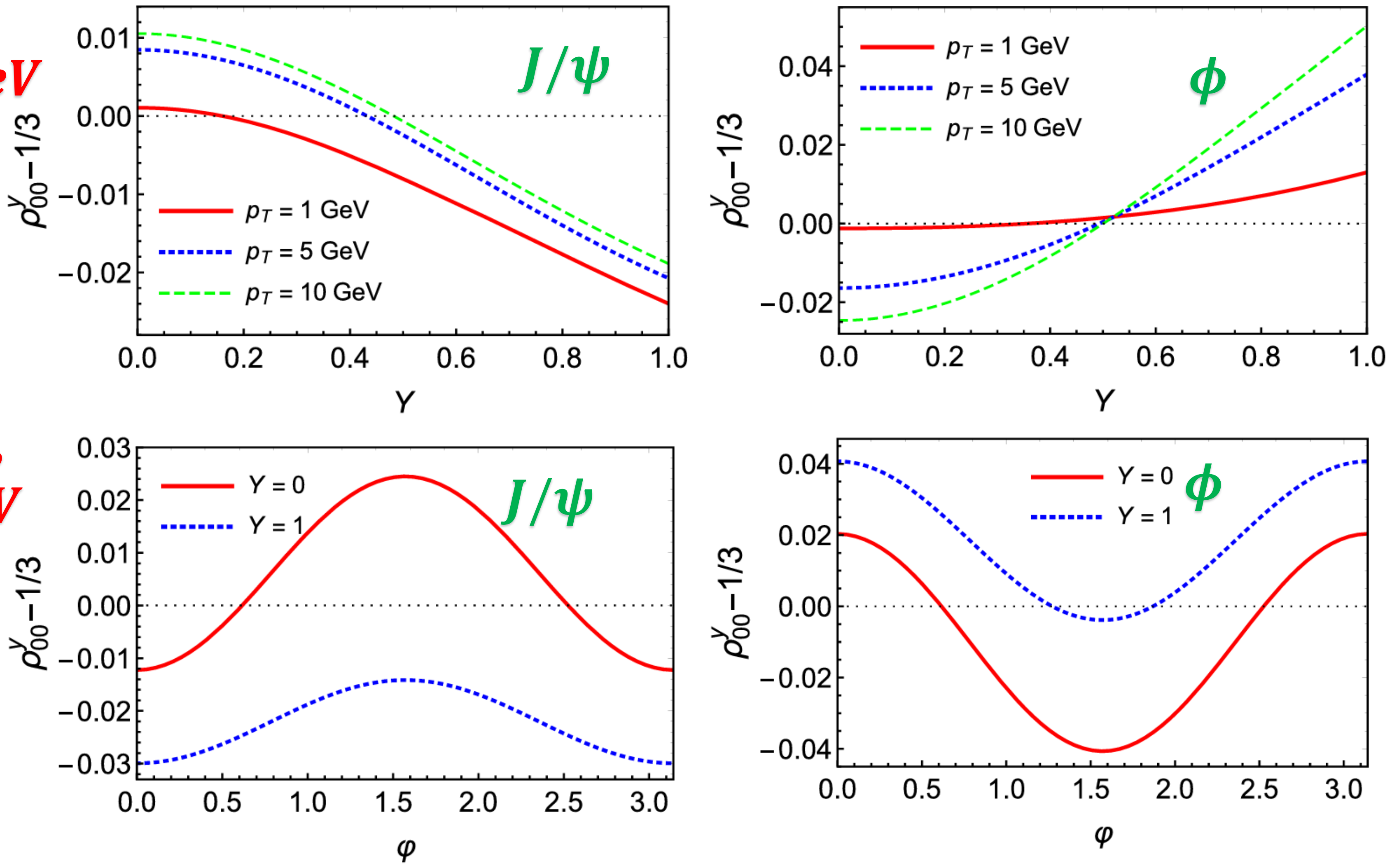


Momentum induced spin alignment

Global Spin alignment

$T = 150\ MeV$

$p_T = 2\ GeV,$
$T = 150\ MeV$



- Theory
  - Spin alignment
  - Correlation function
- Momentum induced spin alignment
- **Magnetic field induced spin alignment**
- Summary and Outlook

Magnetic field induced spin alignment

J/ψ

- **Action:**

$$S=\frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left(R+\frac{12}{L^2}\right)+\frac{1}{8\pi G_5}\int d^4x\sqrt{-\gamma}\left(K-\frac{3}{L}\right)+S_{\mathrm{f}},$$

$$S_{\mathrm{f}}=-\frac{N_c}{16\pi^2}\int d^4x\int_0^{\zeta_h}d\zeta Q(\zeta)\mathrm{Tr}\left(F_L^2+F_R^2\right),$$

- **Holographic background metric:**

$$ds^2=\frac{L^2}{\zeta^2}\left(-f(\zeta)dt^2+h_T(\zeta)(dx^2+dy^2)+h_P(\zeta)dz^2+\frac{d\zeta^2}{f(\zeta)}\right).$$

$$f(\zeta)=1-\frac{\zeta^4}{\zeta_h^4}+\frac{2}{3}\frac{e^2B^2}{1.6^2}\zeta^4\ln\frac{\zeta}{\zeta_h}+\mathcal{O}(e^4B^4),$$

$$h_T(\zeta)=1-\frac{4}{3}\frac{e^2B^2}{1.6^2}\zeta_h^4\int_0^{\zeta/\zeta_h}\frac{y^3\ln y}{1-y^4}dy+\mathcal{O}(e^4B^4),$$

$$h_P(\zeta)=1+\frac{8}{3}\frac{e^2B^2}{1.6^2}\zeta_h^4\int_0^{\zeta/\zeta_h}\frac{y^3\ln y}{1-y^4}dy+\mathcal{O}(e^4B^4),$$

- **Hawking temperature:**

$$T=\frac{1}{4\pi}\left|\frac{4}{\zeta_h}-\frac{2}{3}\frac{e^2B^2}{1.6^2}\zeta_h^3\right|.$$

- **Magnetic field constraint conditions**

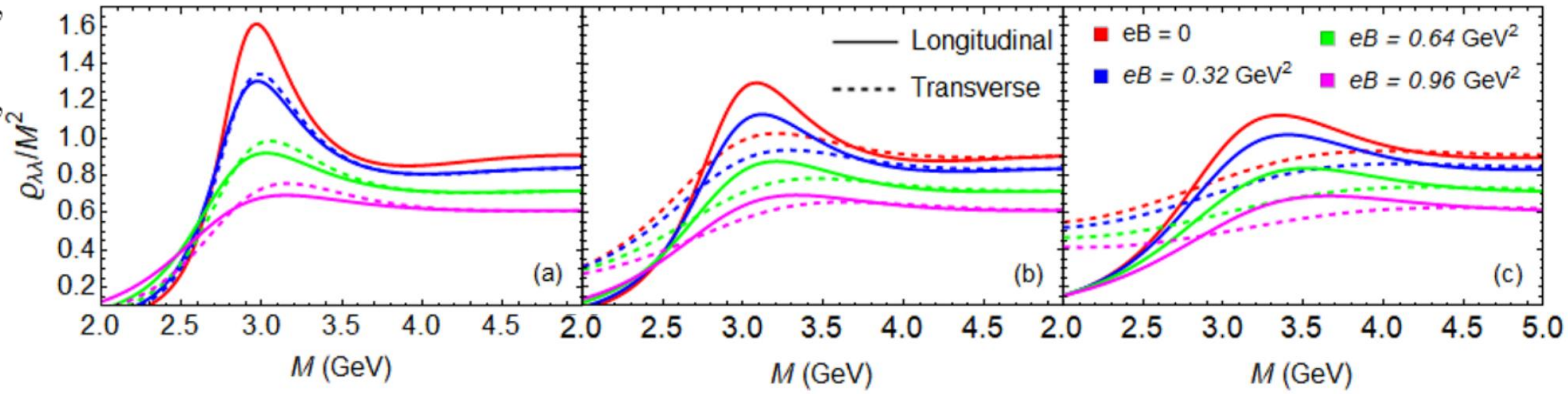
$$eB<\sqrt{\frac{3}{2}}\frac{1.6}{\zeta_h^2}\approx\frac{1.96}{\zeta_h^2}$$

- **Magnetic field parallel to momentum**

Spectral Function:

$T = 0.2\ \mathrm{GeV}$ | $\mathbf{p}=(0,0,p)$

$P = 0$ | $P = 5\ GeV$ | $P = 10\ GeV$

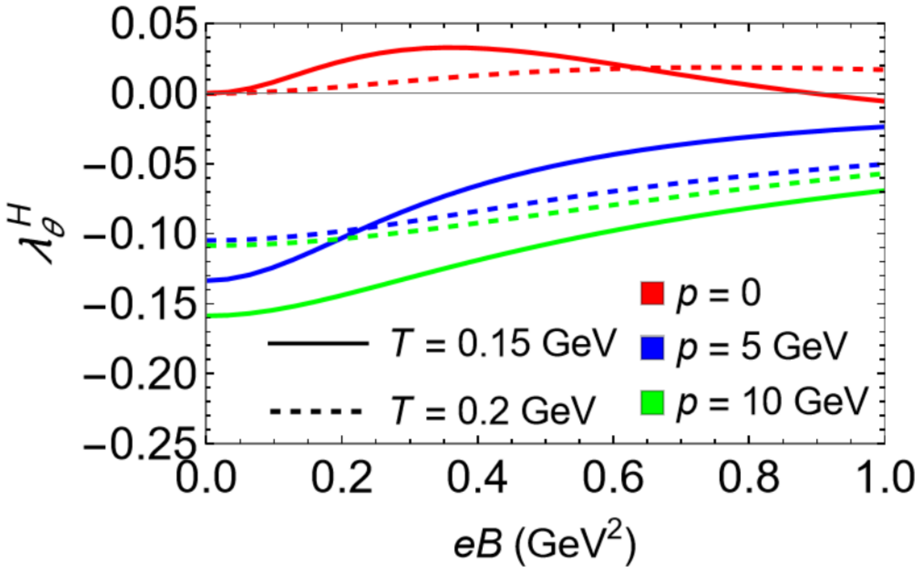


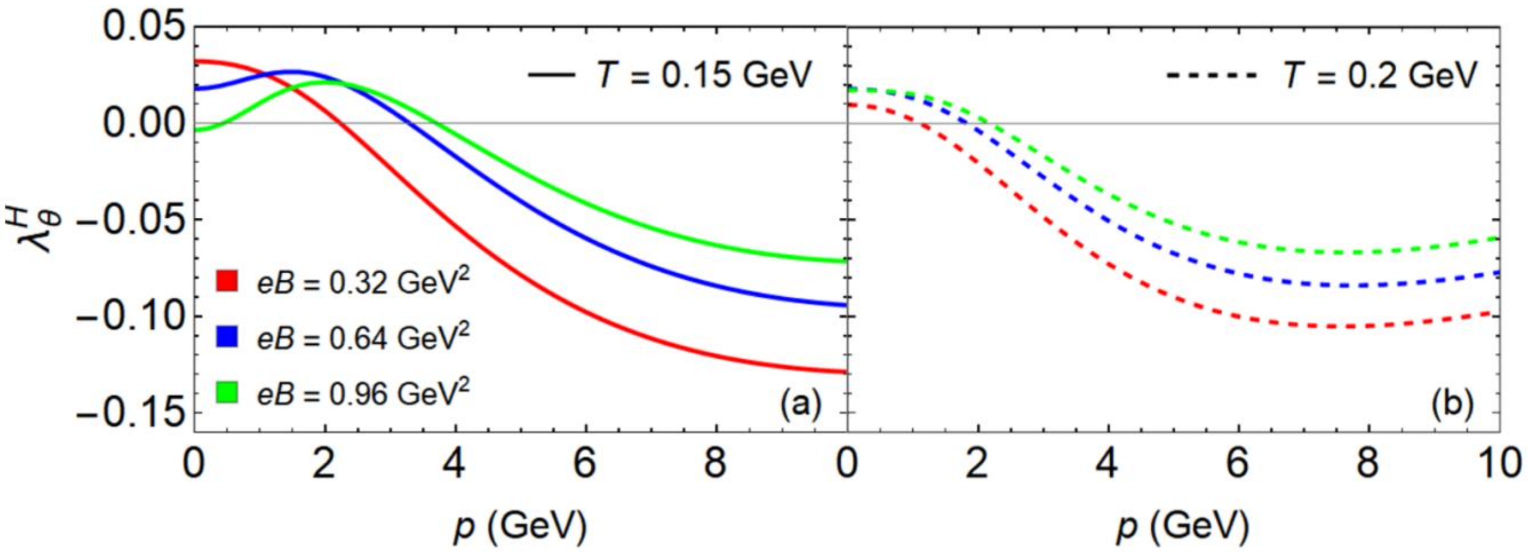
Magnetic field induced spin alignment

➢ **Magnetic field parallel to momentum** $\mathbf{p} = (0, 0, p)$

Magnetic Field:

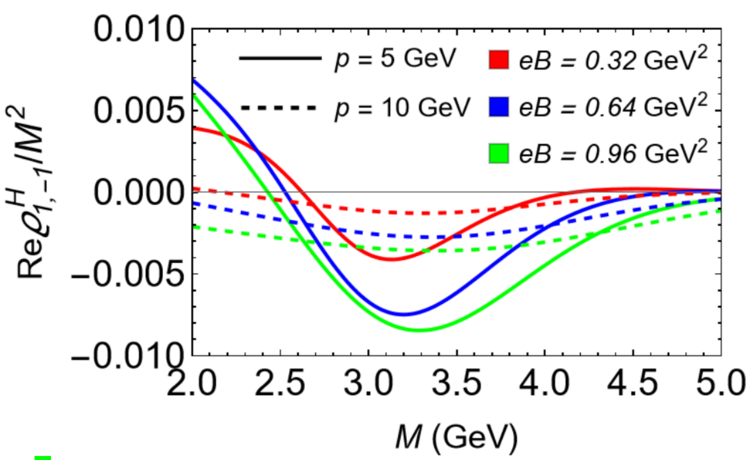
Momentum:



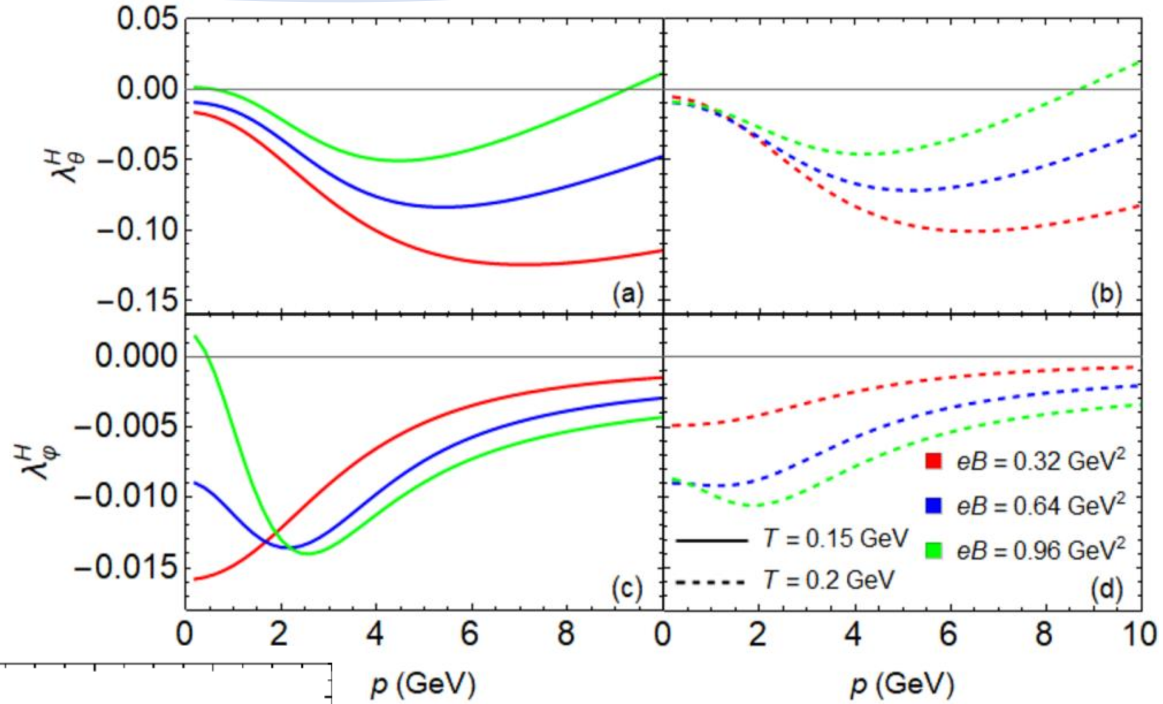


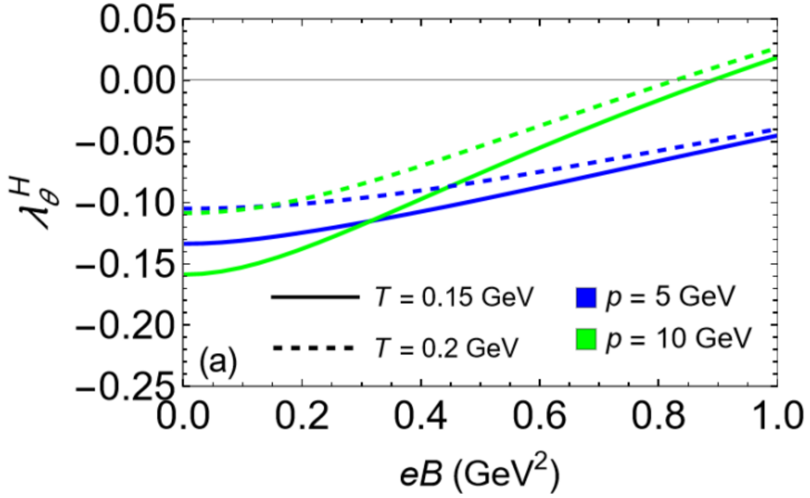
Magnetic field induced spin alignment

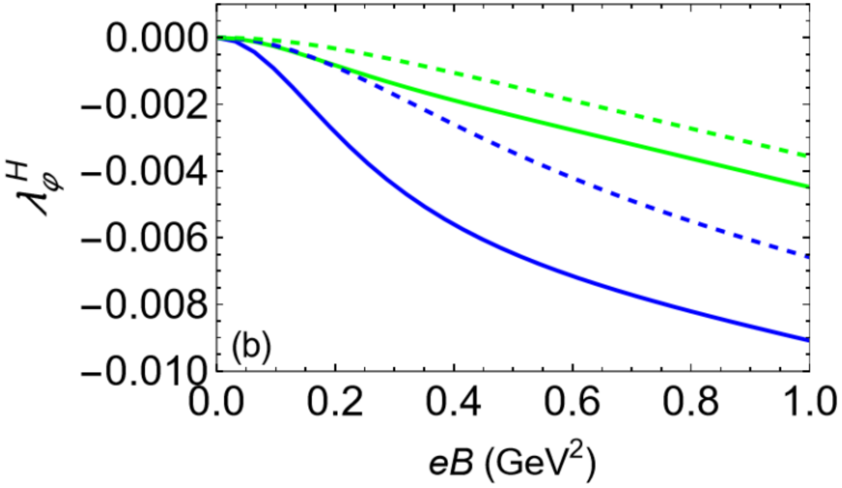
➢ Magnetic field perpendicular to momentum



$$\mathbf{p} = (p, 0, 0)$$







$$\lambda_\theta = \frac{1 - 3\rho_{00}}{1 + \rho_{00}},$$

$$\lambda_\varphi = \frac{2\mathrm{Re}\rho_{1,-1}}{1 + \rho_{00}},$$

Magnetic field induced spin alignment

➢ Application to heavy-ion collisions(the direction of magnetic field along the y-direction)

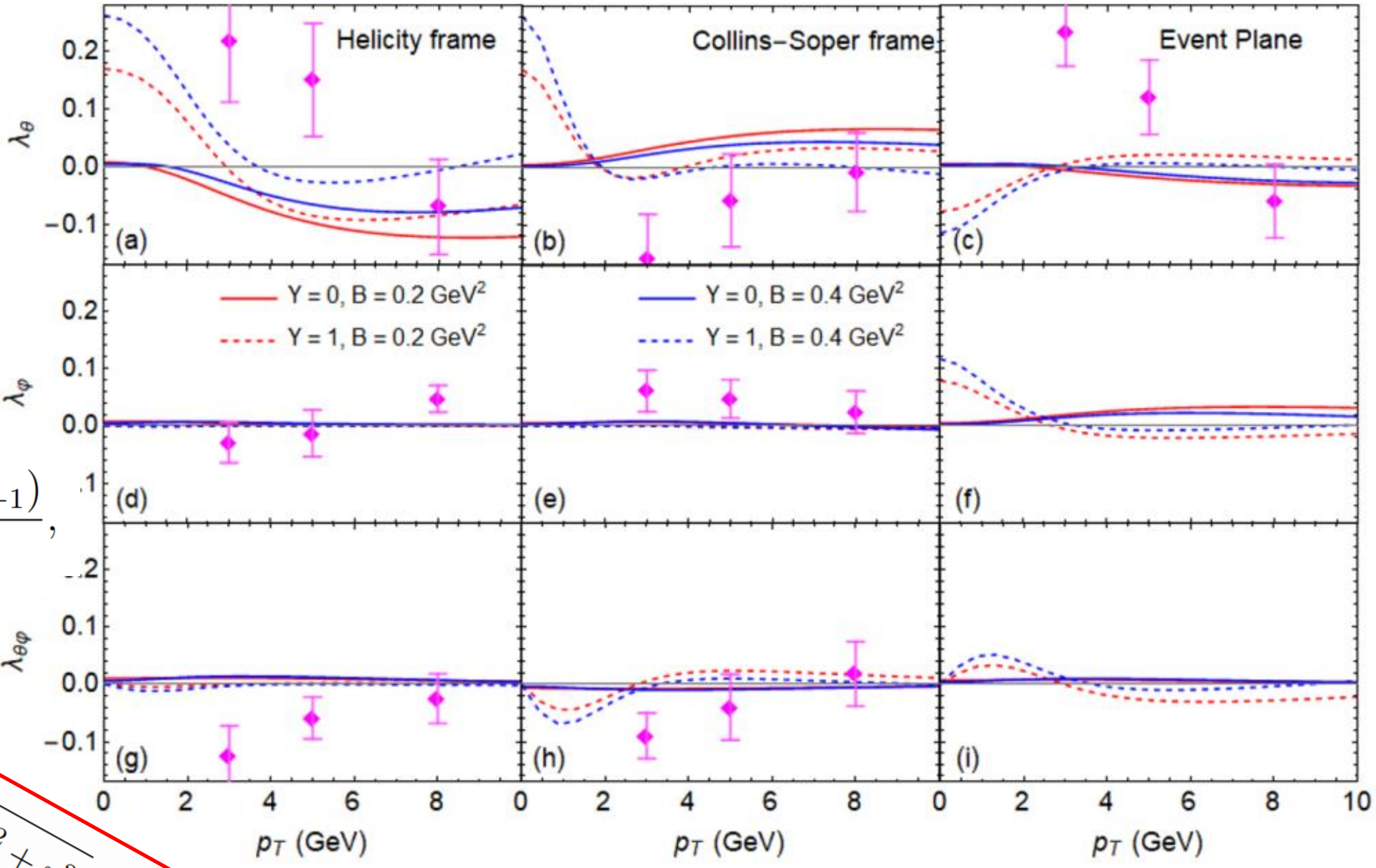
$T = 0.15\ \text{GeV}$

$$\lambda_\theta = \frac{1-3\rho_{00}}{1+\rho_{00}},$$

$$\lambda_\varphi = \frac{2\text{Re}\rho_{1,-1}}{1+\rho_{00}},$$

$$\lambda_{\theta\varphi} = \frac{\sqrt{2}\text{Re}(\rho_{01}-\rho_{0,-1})}{1+\rho_{00}},$$

$$\mathbf{p} = \left(p_T\cos\varphi, p_T\sin\varphi, \sqrt{M^2+p_T^2}\sinh(Y)\right)$$



ALICE Collaboration, S. Acharya et al., Phys. Rev. Lett. 131 no. 4,(2023) 042303

ALICE Collaboration, S. Acharya et al., Phys. Lett. B 815 (2021) 136146

- Theory
  - Spin alignment
  - Correlation function
- Momentum induced spin alignment
- Magnetic field induced spin alignment
- **Summary and Outlook**

Summary and Outlook

- The spin alignment can be purely induced by the motion of vector meson relative to the background.
- The holographic prediction shows that J/ψ and ϕ have opposite behaviours. $J/\psi(\phi)$ are preferably to be transversely(longitudinally) polarized.
- The meson's spin alignment is a non-perturbative property in the strongly interacting matter.
- Magnetic field induces $\lambda_\theta^H > 0$ when the meson's momentum p is very small, while $\lambda_\theta^H < 0$ when p is large enough.
- Comparisons with experimental data show qualitative agreement for spin parameters λ_θ and λ_φ in the helicity and Collins-Soper frames.
- We also find significant differences between our results for $\lambda_{\theta\varphi}^H$ and λ_θ^{EP} with experiments .

Thanks!