QCD phase diagram from a 5-D holographic hardwall model.

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based on arXiv: 2408.10986, 2412.xxxxx

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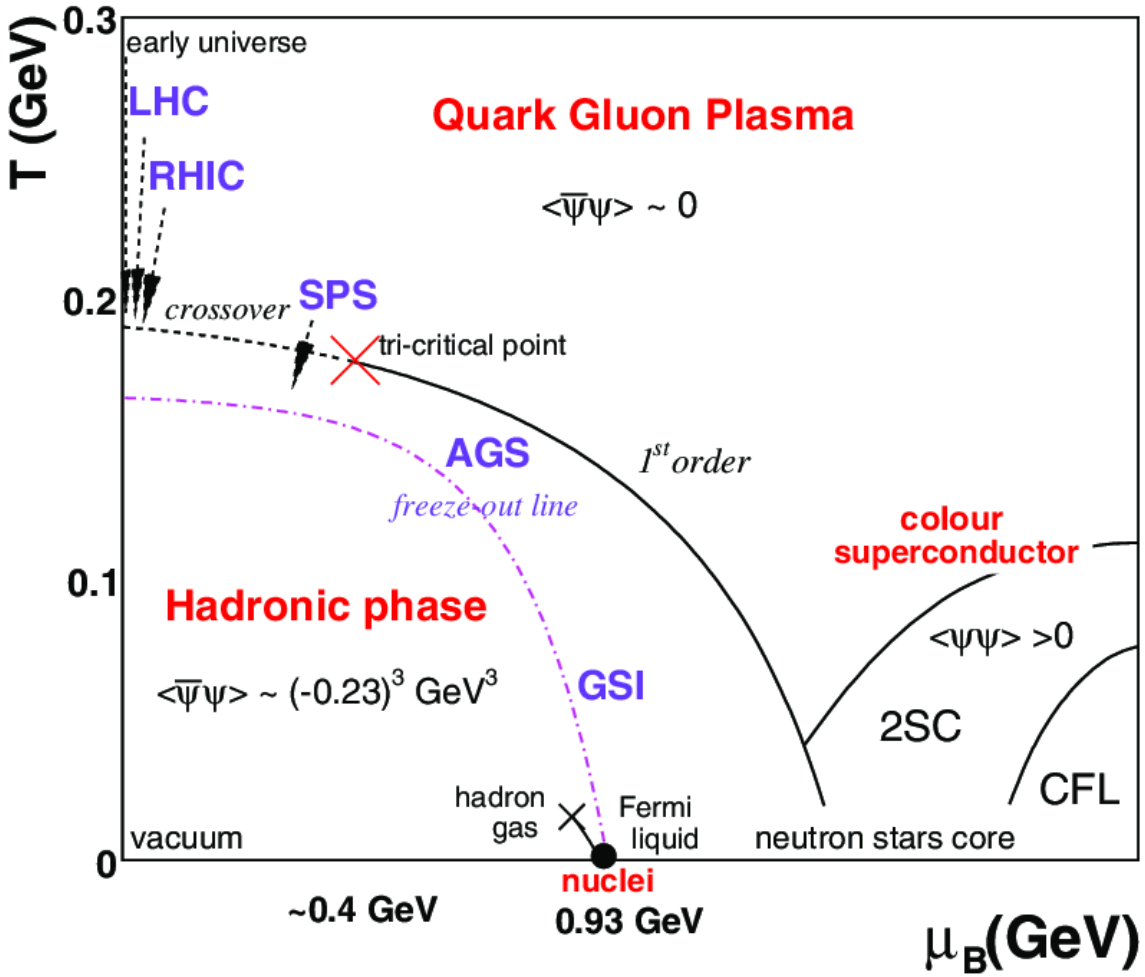
QCD Phase Diagram

CMS Report Volume II: 2007



Presentation Outline

Introduction

$$Z_{\rm QCD} = \int \mathcal{D}[A_\mu, \psi, \overline{\psi}]\, e^{-\int^\beta dt \int d^3x \left[\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \overline{\psi}^a_{i,\alpha} (\not{D} + \gamma^0 \mu_B - m) \psi^a_{i,\alpha}\right]}$$

Properties of QCD:
Confinement, Asymptotic freedom, Axial Anomaly, Chiral symmetry breaking, Color Superconductivity
Methods:

1. Lattice QCD: Suitable for vanishing densities.
2. EFTs: Valid at low densities and low temperatures.
3. Holographic QCD Approaches
   - **Top-down Models**: String theory (e.g., D3/D7, D4/D8).
   - **Bottom-up Models**: Phenomenological, e.g., Hardwall, Softwall, VQCD.

Holographic Hardwall Model

$$S_0 = \frac{N_c^2}{8\pi^2 L^3}\int^{z_0} d^5x\sqrt{g}\,(R-2\Lambda) - \frac{\theta N_f N_c}{24\pi^2 L}\int^{z_0} d^5x\sqrt{g}\frac{F^2}{4} + S_{bdry}$$

$$S_{bdry} = -\frac{N_c^2}{4\pi^2 L^3}\int d^4x\sqrt{\gamma}\left(\Theta - \frac{3}{L}\right)\Big|_{z=\epsilon}$$

with N_f, N_c, and L are the number of flavors, colors and AdS radius, respectively.

Normalization parameter θ: **Dilaton coupling, Compactification of Dp-brane**

Solutions and Phases

The diagonal metric ansatz:

$$ds^2 = -\frac{g(z)}{h(z)}L^2dt^2 + \frac{L^2}{z^2}d\vec{x}^2 + \frac{L^2}{g(z)}dz^2$$

and the gauge field ansatz $A_0 = \phi(z)$.
Four boundary conditions: g, h, ϕ and ϕ'.

There are three possible geometries:

1. **Thermal AdS:** zero density, confined phase.
2. **Charged Black Hole:** finite density, deconfined phase.
3. **Charged AdS:** finite density, confined phase. **B. H. Lee, C. Park, S. J. Sin : 2009**

Charged AdS

The charged AdS is a **horizonless geometry** in the bulk that represents a finite-density confined phase of the boundary theory.

$$g = z^2\left(1 - cz^4 + \frac{(2\pi)^4}{\theta N_f N_c}\rho_B^2 z^6\right); \quad h = z^4; \quad \phi = \mu - \frac{12\pi^2}{\theta N_f}\rho_B z^2;$$

The pressure

$$p = \frac{N_c^2}{8\pi^2}\left(\frac{z_0^2 + g(z_0)}{z_0^6} + \frac{(2\pi)^4 z_0^2}{\theta N_f N_c}\rho_B^2\right)$$

$g(z_0) = \bar{g}_0 + \frac{\tilde{g}_0(\mu, T)}{N_c}$
ρ-meson mass: $\implies \bar{g}_0 \simeq 6.$

Physically motivated boundary condition

Sources are behind the cutoff:

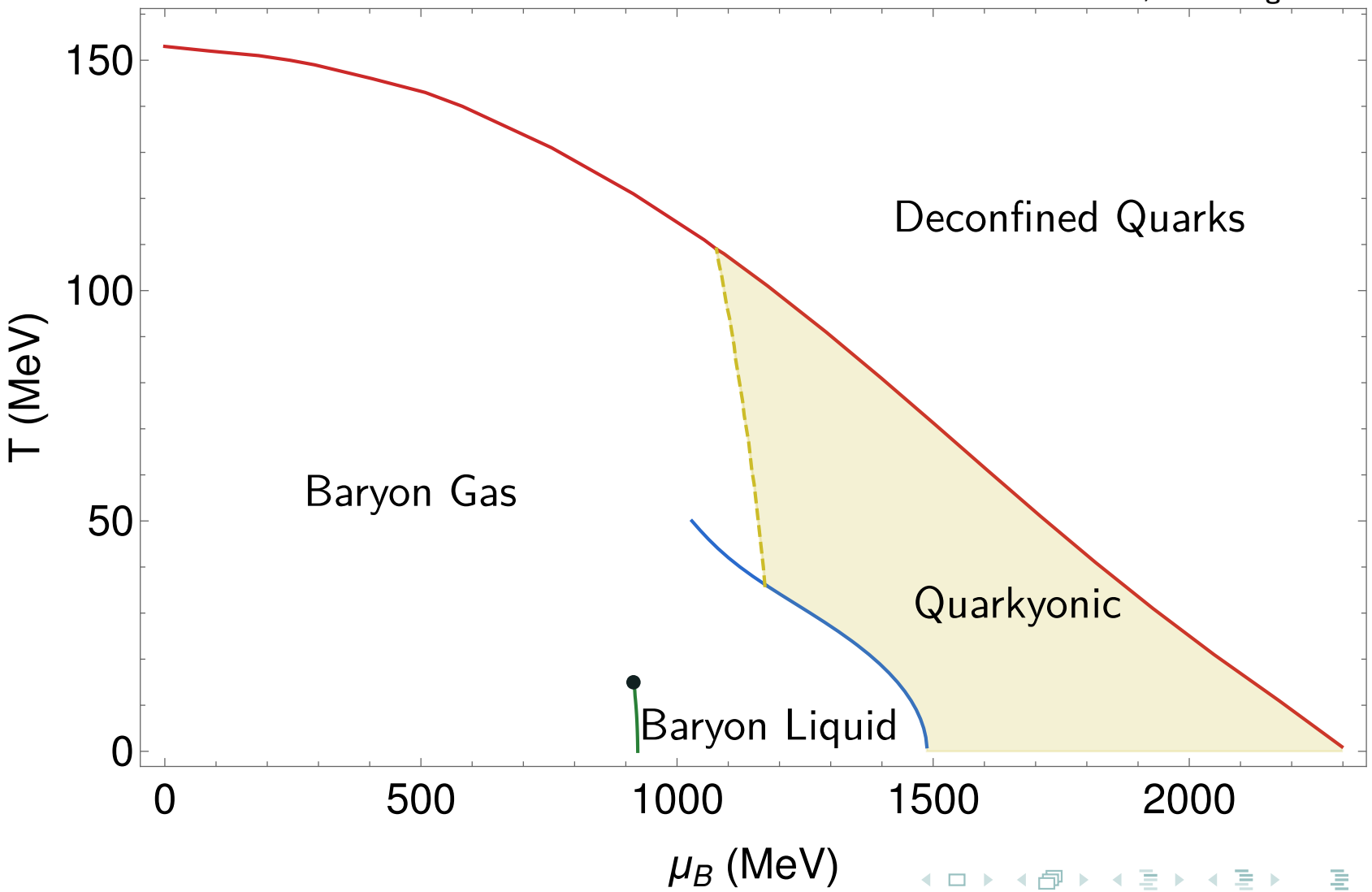
1. **Gauss' Law:** ρ_B
2. **Path Integral:** $p = p_B = \frac{N_c}{8\pi^2}\left(\frac{\tilde{g}_0(\mu,T)}{z_0^6} + \frac{2\pi^2 z_0^2}{\theta N_f}\rho_B^2\right)$

QCD Phenomenology

- **Nambu–Jona-Lasinio:** Temperature and chemical potential dependent constituent quark mass. **M. Asakawa, K. Yazaki: 1989**
- **Van der Waals:** Isospin-symmetric nuclear matter (in-medium ChPT). **S. Fiorilla, N. Kaiser, and W. Weise: 2012**

Phase Diagram for $\theta \sim 10$

AS, K. P. Yogendran: 2024



Condensates in holographic model

A complex scalar field ψ breaks $U(1)_B$ symmetry spontaneously.

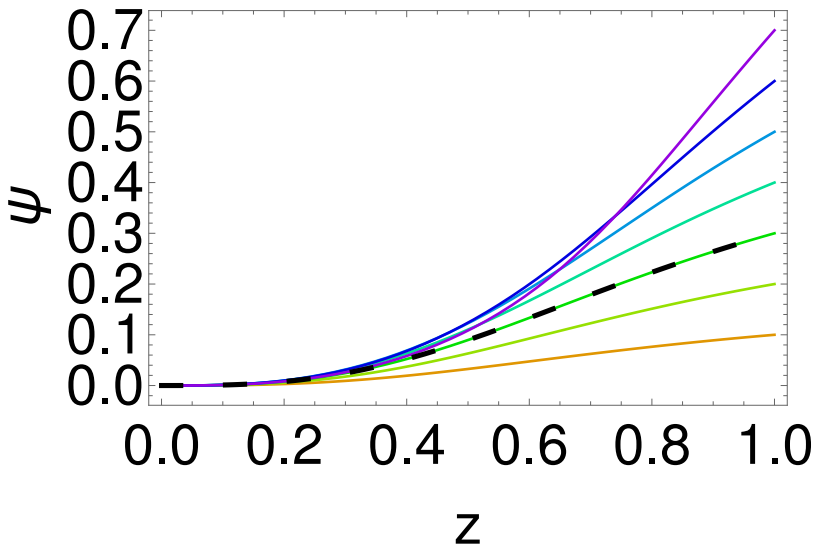
Gubser:2008

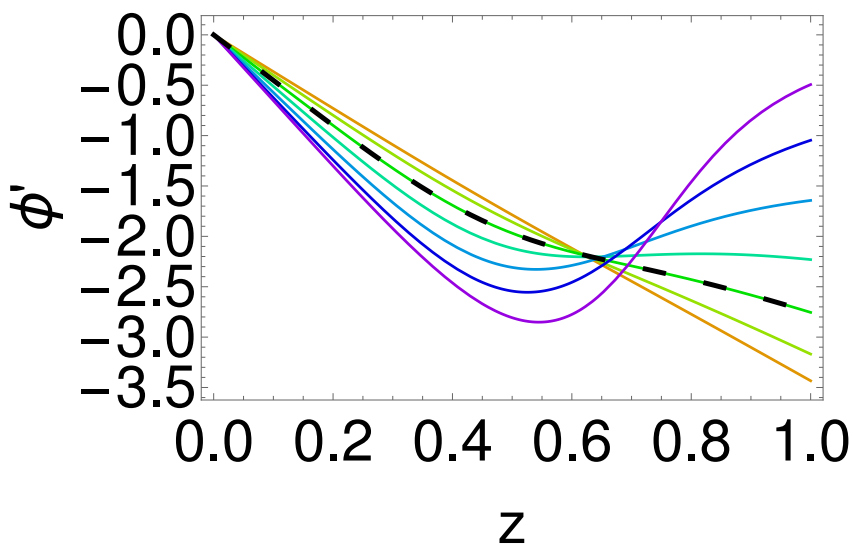
$$S = S_0 - \int d^5x\sqrt{g}\left(|D\psi|^2 + m^2|\psi|^2\right) - \int d^4x\sqrt{\gamma}\frac{d-\Delta}{2L}|\psi|^2$$

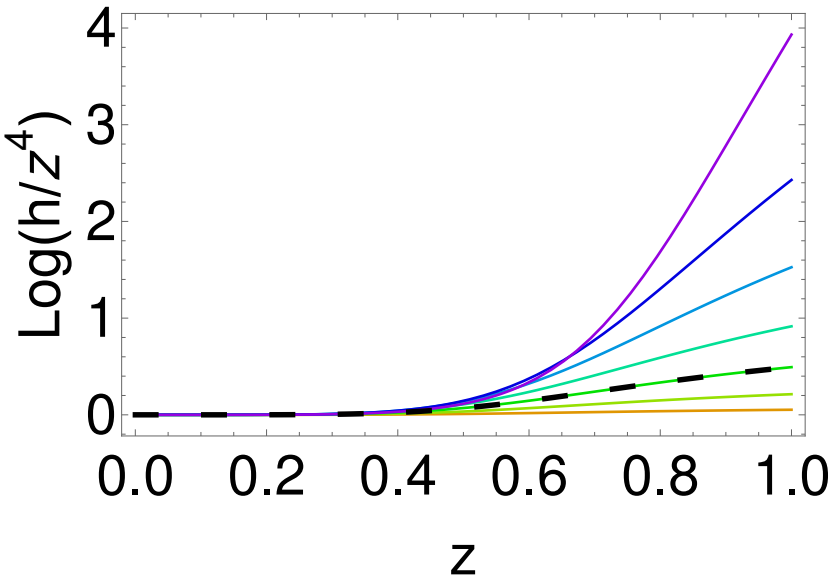
- Scaling dimension: $m^2L^2 = \Delta(\Delta - d)$
- Charge: $D_\mu = \partial_\mu - \iota q A_\mu$
- Asymptotic analysis: $\psi \sim \psi_+ z^{\Delta_+} + \psi_- z^{\Delta_-} + \ldots$

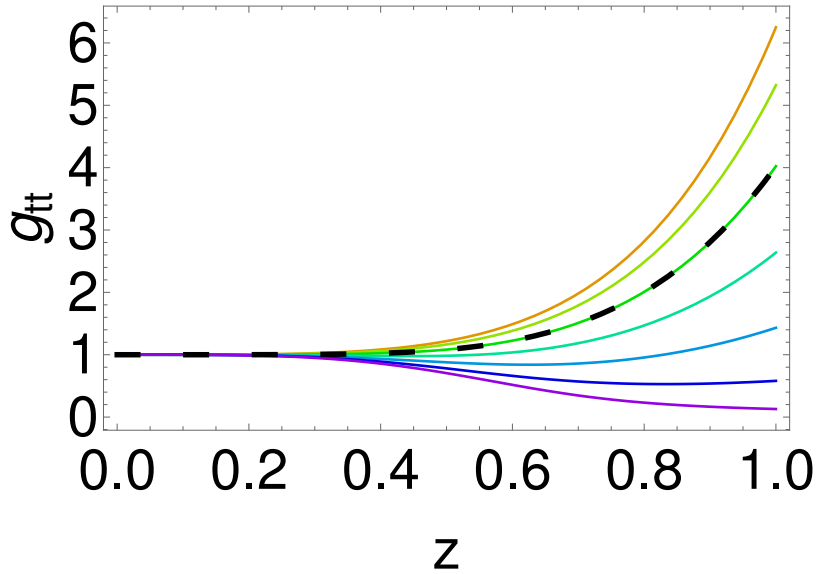
Condensate Solutions:

1. Charged black hole: Regularity condition at the horizon.
2. Charged AdS: IR boundary condition?


ψ
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.0 0.2 0.4 0.6 0.8 1.0
z


ϕ'
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
−3.5
0.0 0.2 0.4 0.6 0.8 1.0
z


Log(h/z^4)
0
1
2
3
4
0.0 0.2 0.4 0.6 0.8 1.0
z


g_tt
0
1
2
3
4
5
6
0.0 0.2 0.4 0.6 0.8 1.0
z

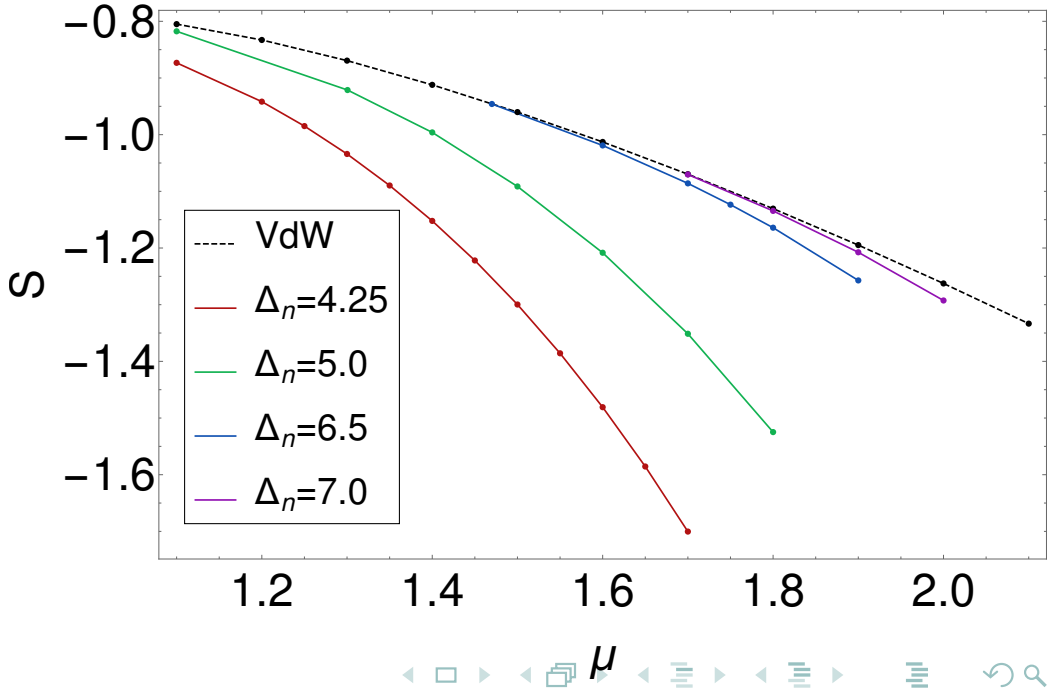
Which condensate?

NJL

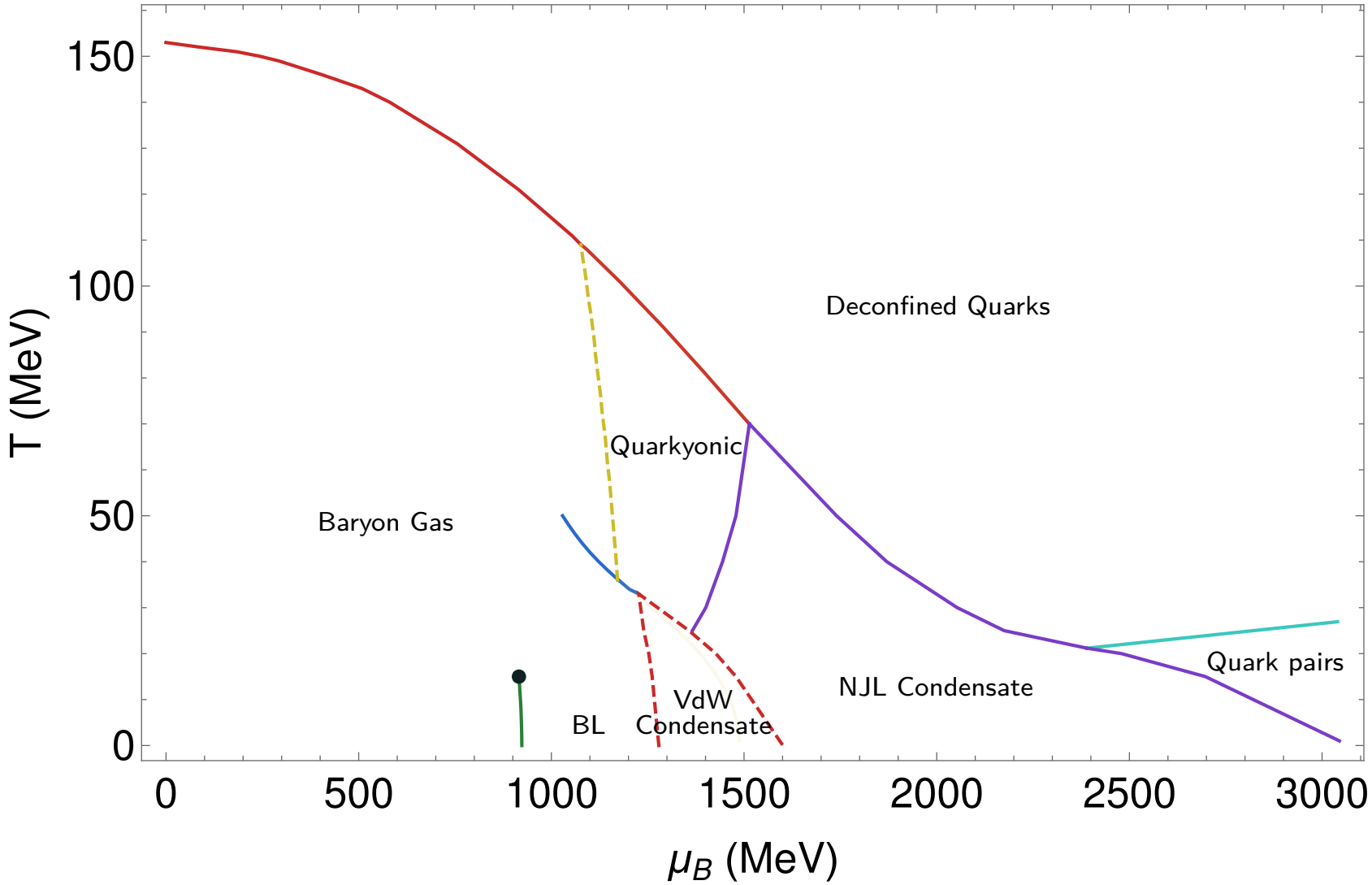
1. Charge of the Operator $q = 2$
2. Scaling dimension $\Delta = 3$

VdW

1. Charge of the Operator $q = 2N_c$
2. Scaling dimension $\Delta = 3N_c$?



A Phase Diagram



Conclusions and Discussion

- **5-D Hardwall Model**:
  1. Effective in capturing low-density confined phases.
  2. Allows for baryonic condensates with different charges and scaling dimensions
- **Future Directions**:
  1. Incorporate Isospin.
  2. Compact star structure.
  3. Baryons in deep IR.
  4. Top down completion

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Thank You