The effect of charm quark on chiral phase transition in holographic QCD

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Introduction

- Quantum Chromodynamics (QCD) is the theory of the strong nuclear interactions.

$$\mathcal{L} = -\frac{1}{2g^2} \operatorname{Tr} F_{\mu\nu} F^{\mu\nu} + \sum_{i=1}^{N_f} i\bar{\psi}_i \not{D} \psi_i$$

- Decompose the fermionic kinetic terms into into left-handed and right-handed parts:

$$G_F = U(N_f)_L \times U(N_f)_R \Longrightarrow SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A \,.$$

- The quark condensate – also known as a chiral condensate – is a vacuum expectation value of the composite operators

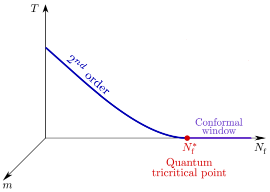
$$\langle \bar{\psi}_{-i} \psi_{+j} \rangle = -\sigma \delta_{ij} \longrightarrow \langle \bar{\psi}_{-i} \psi_{+j} \rangle \mapsto \sigma \left(L^\dagger R \right)_{ij}$$

- At low temperatures chiral condensate spontaneously breaks chiral symmetry while at high temperatures the interactions among the quarks and anti-quarks are weak, no such condensate exists, and chiral symmetry is manifest (Collins, Perry PRL 75).

Introduction

- Investigating the thermal QCD phase transition is an important subject for understanding the hot environment relevant to the initial stage of heavy-ion collisions.
- The order of the chiral phase transition depends on the number of quark flavors (N_f) and the current quark masses (m_f).
- Universality, 3D sigma models: 1st order for $N_f \geq 3$ with massless quarks (Pisarski, Wilczek PRD 84).

- According to the recent lattice simulation, the chiral phase transition is estimated to be of the second order with $N_f \leq 6$ after extrapolation to the massless limit (JHEP11(2021)141).



- We address the thermal chiral phase transition in such a multi-flavor system, specifically focusing on the four-quark flavor system, by using the holographic QCD approach.

AdS/CFT Correspondence

- AdS/CFT Correspondence (Adv.Theor.Math.Phys.2,231 (1998))

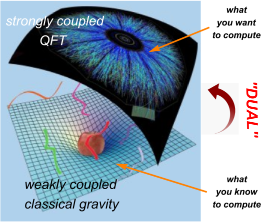
Type IIB string theory on $AdS_5 \times S^5$ in low-energy approximation $\Longleftrightarrow$ $\mathcal{N} = 4$ Super Yang-Mills theory on AdS boundary in the limit $\lambda = g_{YM}^2 N_c >> 1$

- The parameters on the field theory side, i.e., g_{YM} and N_c, are mapped to the parameters g_s and l_s on the string theory side by

$$g_{YM}^2 = 2\pi g_s \quad \text{and} \quad 2\lambda = 2g_{YM}^2 N_c = L^4/l_s$$

- GKP-Witten relation (hep-th/9802109, hepth/ 9802150):
  The partition function of a gauge theory with scale invariance (conformal invariance) is equivalent to the partition function of string theory on the AdS_5 spacetime.

$$Z_{CFT} = Z_{AdS_5}$$



1908.02667v2 [hep-th]

Soft-wall model with multi-quark flavors

- The temperature is introduced through the geometries with black holes. We take the solution of the simple AdS-Schwarzschild (AdS-SW) black hole

$$ds^2 = \frac{L^2}{z^2}\left(-f(z)dt^2 + \frac{1}{f(z)}dz^2 + dx_i dx^i\right),$$

$$f(z) = 1 - \frac{z^4}{z_h^4}, \quad \text{with} \quad f(zh) = 0$$

.

- The temperature of the hot QCD system defined from the Hawking temperature:

$$T = \frac{1}{4\pi}\left|\frac{df(z)}{dz}\right|_{z\to z_h} = \frac{1}{\pi z_h}$$

- Conformal invariance broken by a background dilaton field in the bulk: $\phi(z)$

- At IR region $\phi(z \to \infty) \approx +z^2$ is necessary to get the linear confinement (Karch et al. 2006)

- At UV $\phi(z \to 0) \approx -z^2$ is required to describe the spontaneous symmetry breaking in the chiral limit (Chelabi et al. 2015)

$$\Phi(z) = -\mu_1^2 z^2 + (\mu_1^2 + \mu_0^2) z^2 \tanh(\mu_2^2 z^2)$$

Soft-wall model with multi-quark flavors

- Operators/fields correspondence:

4D : $\mathcal{O}(x)$	5D : $\phi(x,z)$	p	Δ	$(M_5)^2$
$\bar{q}_L\gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R\gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3

- The 5D action:

$$S_M = -\int d^5x\sqrt{-g}e^{-\Phi}\left\{\mathrm{Tr}\left[\left(D^M X\right)^\dagger(D_M X) + \frac{1}{4g_5^2}\left(L^{MN}L_{MN} + R^{MN}R_{MN}\right)\right] + V(X)\right\}$$

- $V(X)$ represents the scalar potential which takes the polynomial form of X:

$$V(X) = V_0(X) + V_{\mathrm{det}} = M_5^2\mathrm{Tr}\left[X^\dagger X\right] + \lambda\mathrm{Tr}\left[|X|^4\right] + \gamma\mathrm{Re}\left[\det(X)\right]$$

$M_5^2 = (\Delta - p)(\Delta + p - 4) = -3$ ($p = 0$ and $\Delta = 3$), $X = e^{i\pi}X_0e^{i\pi}$, $g_5^2 = 12\pi^2/N_c$,
$X_0 = \mathrm{diag}\left(\frac{\chi_u(z)}{\sqrt{2}}, \frac{\chi_d(z)}{\sqrt{2}}, \frac{\chi_s(z)}{\sqrt{2}}, \frac{\chi_c(z)}{\sqrt{2}}\right)$

Soft-wall model with multi-quark flavors

- Since we are concerned with the chiral phase transition, we set the bulk gauge fields L_M and R_M to zero.

$$S\left[\chi_l, \chi_s, \chi_c\right] = -\int d^5x\sqrt{-g}e^{-\Phi}\left\{g^{zz}\left(\chi_l'^2 + \frac{1}{2}\chi_s'^2 + \frac{1}{2}\chi_c'^2\right)\right.$$
$$\left. + \left[-3\left(\chi_l^2 + \frac{1}{2}\chi_s^2 + \frac{1}{2}\chi_c^2\right) + v_4\left(2\chi_l^4 + \chi_s^4 + \chi_c^4\right) + 3v_{\text{det}}\chi_l^2\chi_s\chi_c\right]\right\}$$

- The equations of motion for χ_f $(f = l, s, c)$ read

$$\chi_l'' + \left(-\frac{3}{z} - \Phi' + \frac{f'}{f}\right)\chi_l' + \frac{1}{z^2 f}\left(3\chi_l - 3v_{\text{det}}\chi_l\chi_s\chi_c - 4v_4\chi_l^3\right) = 0,$$
$$\chi_s'' + \left(-\frac{3}{z} - \Phi' + \frac{f'}{f}\right)\chi_s' + \frac{1}{z^2 f}\left(3\chi_s - 3v_{\text{det}}\chi_l^2\chi_c - 4v_4\chi_s^3\right) = 0,$$
$$\chi_c'' + \left(-\frac{3}{z} - \Phi' + \frac{f'}{f}\right)\chi_c' + \frac{e^1}{z^2 f}\left(3\chi_c - 3v_{\text{det}}\chi_l^2\chi_s - 4v_4\chi_c^3\right) = 0,$$

Soft-wall model with multi-quark flavors

- The UV solutions of χ_f near the boundary $z = 0$:

$$\chi_l = a_l z - \left(\mu_1^2 - a_l^2 v_4 - \frac{3}{2} a_s a_c v_{det}\right) a_l z^3 \log(z) + b_l z^3 + \ldots..,$$
$$\chi_s = a_s z - \left(\mu_1^2 a_s - a_s^3 v_4 - \frac{3}{2} a_l^2 a_c v_{det}\right) z^3 \log(z) + b_s z^3 + \ldots..,$$
$$\chi_c = a_c z - \left(\mu_1^2 a_c - a_c^3 v_4 - \frac{3}{2} a_l^2 a_s v_{det}\right) z^3 \log(z) + b_c z^3 + \ldots..,$$

a_f correspond to the current quark masses:

$$a_l = m_l \zeta, \quad a_s = m_s \zeta, \quad a_c = m_c \zeta.$$

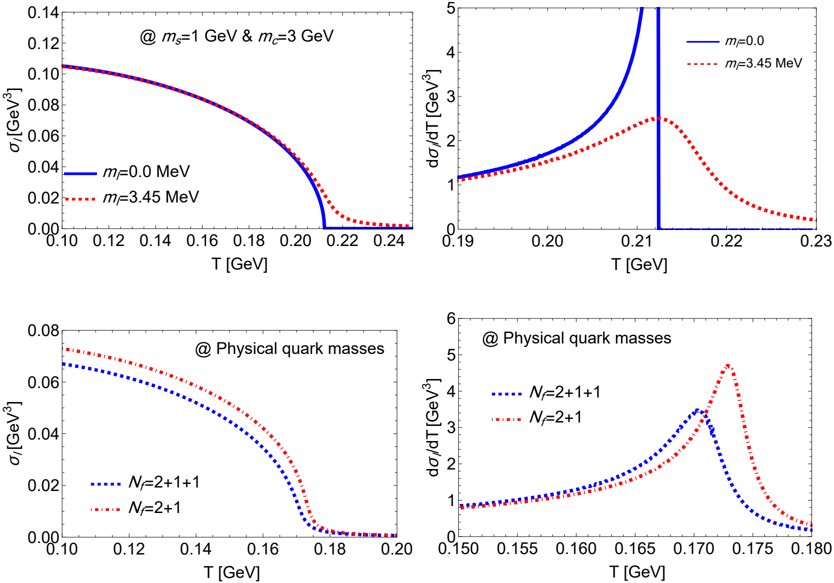
b_f correspond to the quark condensates:

$$b_l = \frac{\sigma_l}{\zeta}, \quad b_s = \frac{\sigma_s}{\zeta}, \quad b_c = \frac{\sigma_c}{\zeta}. \tag{1}$$

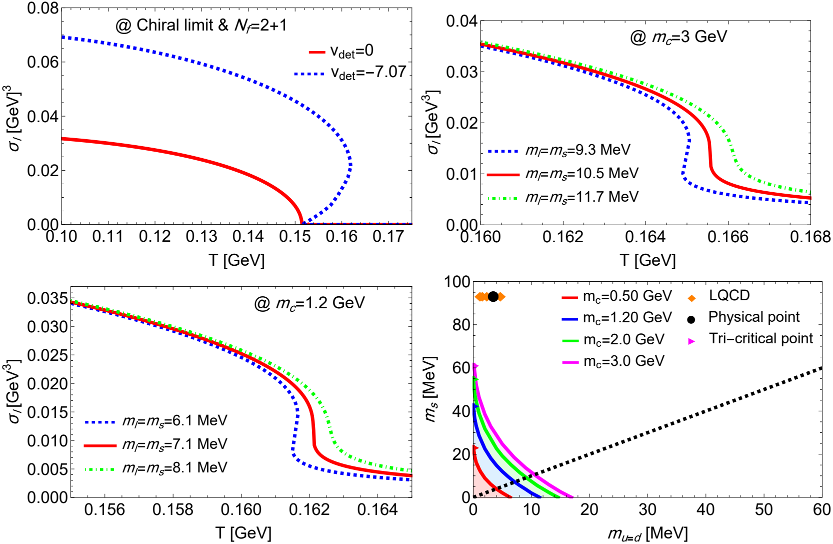
- Avoid the singularity at the horizon $z = z_h$:

$$f' \chi_l' + \frac{1}{z^2}\left(3\chi_l - 3v_{\mathrm{det}} \chi_l \chi_s \chi_c - 4v_4 \chi_l^3\right)|_{z=z_h-\epsilon} = 0,$$
$$f' \chi_s' + \frac{1}{z^2}\left(3\chi_s - 3v_{\mathrm{det}} \chi_l^2 \chi_c - 4v_4 \chi_s^3\right)|_{z=z_h-\epsilon} = 0,$$
$$f' \chi_c' + \frac{1}{z^2}\left(3\chi_c - 3v_{\mathrm{det}} \chi_l^2 \chi_s - 4v_4 \chi_c^3\right)|_{z=z_h-\epsilon} = 0.$$

Chiral Phase Transition

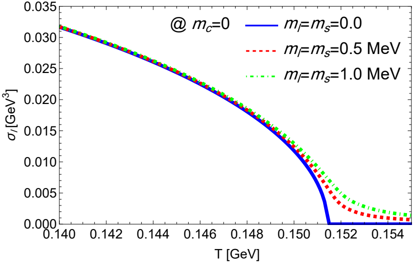


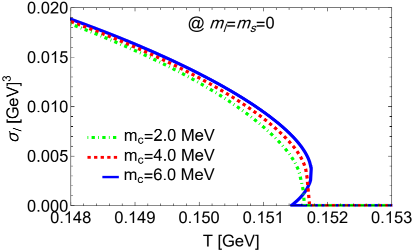
Three-flavor first order phase transition

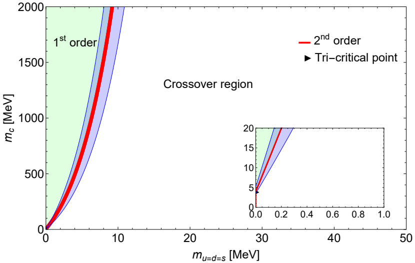


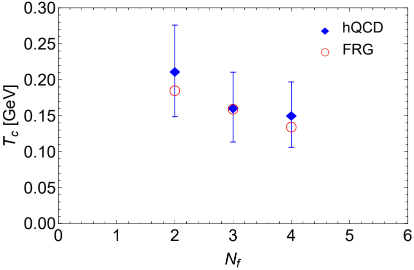
HotQCD:Phys. Rev. Lett. 123, 062002 (2019)

Four-flavor Phase Diagram





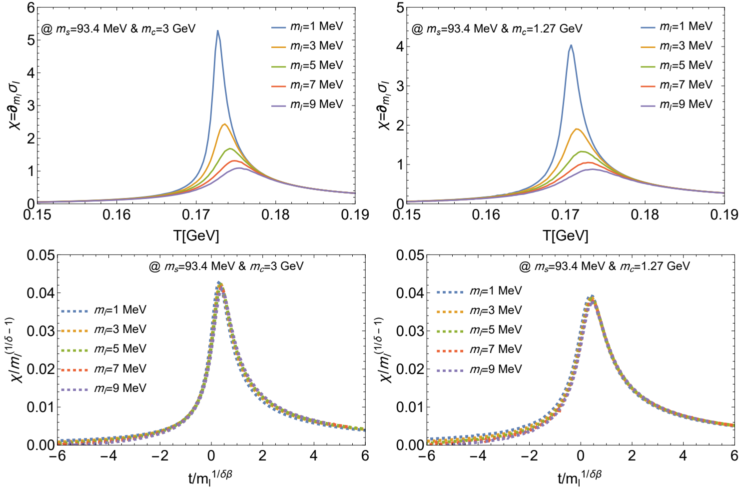




FRG:JHEP 06, 024 (2006), JHEP 05, 060 (2010)

Critical Exponents

$$\chi(t, m_l) = m_l^{1/\delta - 1} f_\chi \left(t/m_l^{1/(\delta\beta)} \right), \text{ for } m_l \sim m_l^{\mathrm{cri}}$$



The value of the critical exponents in the case of $m_l^{\mathrm{cri}} = 0,\ m_s^{\mathrm{cri}} = 93.4\ \mathrm{MeV},\ m_c = 3\ \mathrm{GeV}$ are $\beta = 0.47$ and $\delta = 3.3$. For the case of four quark flavor, $m_l^{\mathrm{cri}} = 0,\ m_s^{\mathrm{cri}} = 93.4\ \mathrm{MeV},\ m_c = 1.27\ \mathrm{GeV}$, the critical exponents are evaluated as $\beta = 0.50$ and $\delta = 3.0$.

Summary

- In the massless limit of the two-quark (three-quark) flavor system, the order phase transition is second (first) order phase transition.
- Applying our framework to the three-quark flavor system for various values of quark masses, we have illustrated the chiral phase diagram on the plane of the light quark mass m_l and the strange quark mass m_s, which is a part of the Columbia plot.
- Given the phase transition order observed in the massless four-quark flavor system, we have proposed the new phase diagram with the four-quark flavors, which is described on the $m_{l=s}$-m_c plane.
- Our estimated critical temperatures are comparable to those of FRG method in the massless systems within the systematic uncertainty.
- Finally, we estimate the critical exponents for two-, three-, and four-quark flavor system.

谢谢大家！

Thanks for your attention!