Holographic boundary conformal field theory with TTbar deformation

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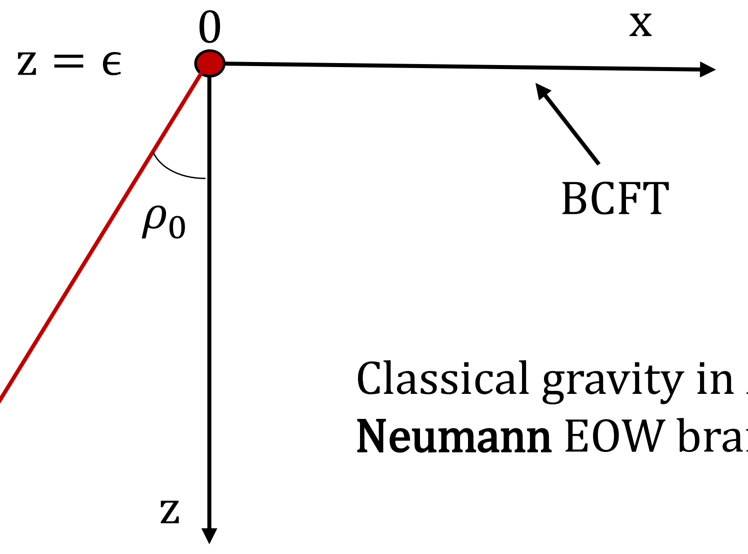
Based on [2411.06345](2411.06345)

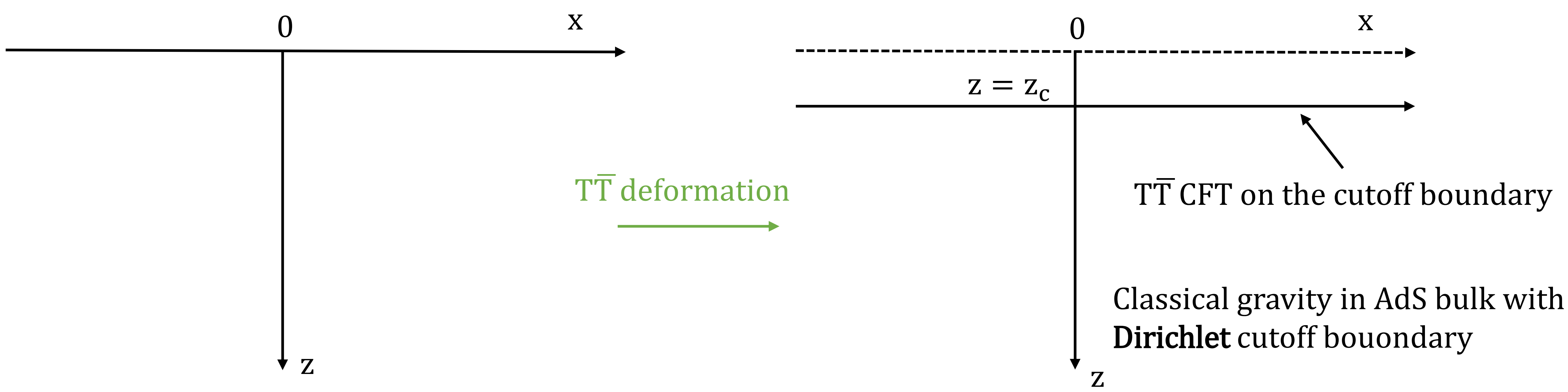
▶ AdS/BCFT



Classical gravity in AdS bulk with
Neumann EOW brane

EOW brane with Neumann
boundary condition

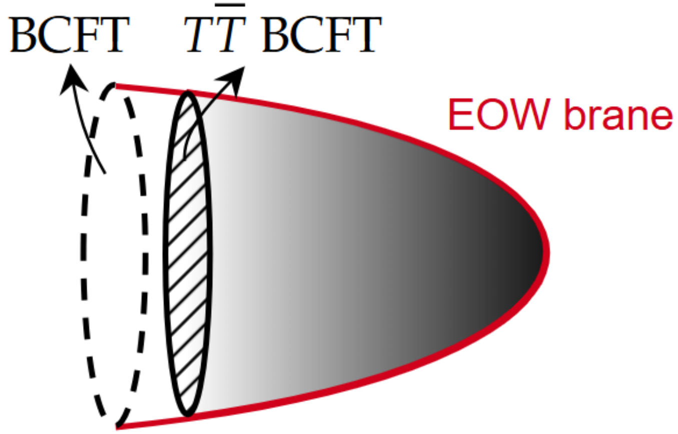
Holographic TTbar deformation



AdS/TTbar BCFT

Gravity side: Neumann EOW brane + Dirichlet cutoff bouondary

Field theory side: boundary + TTbar deformation

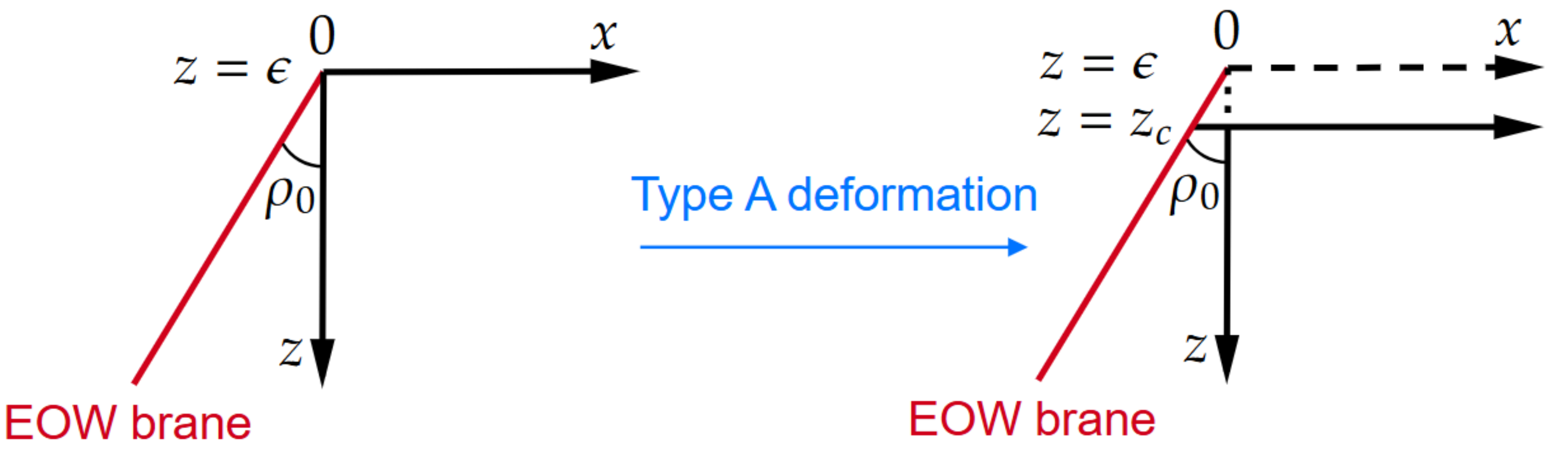


Two kinds of possibilities:

Type A: boundary is deformed by TTbar

Type B: boundary is unaffected by TTbar

▶ Type A

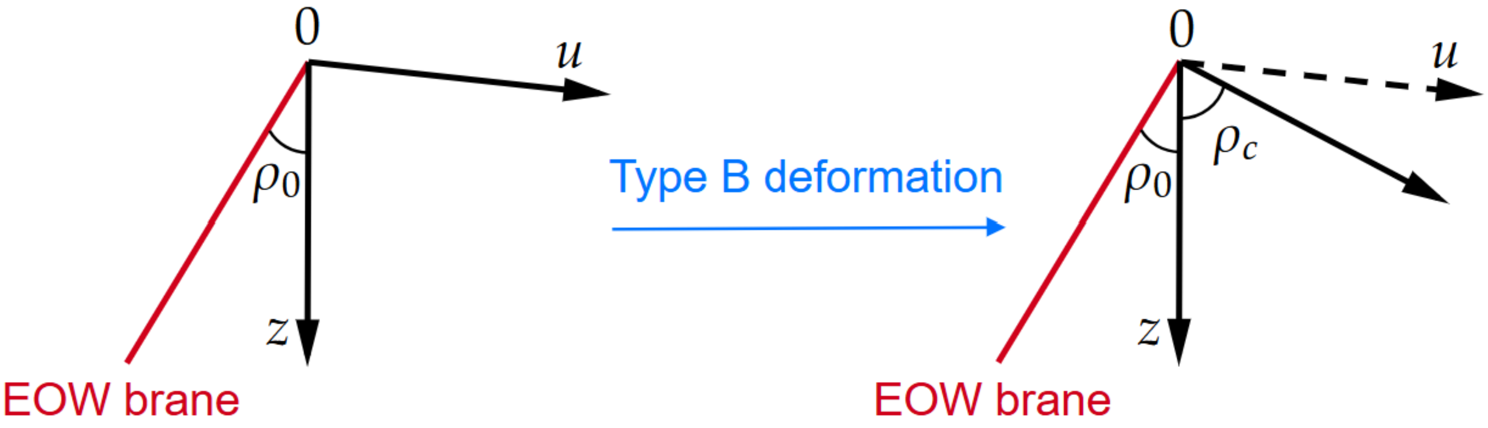


Bulk dual:

$$\begin{aligned} ds^2 &= \frac{l^2}{z^2}(-dt^2 + dz^2 + dx^2) \\ &= d\rho^2 + l^2 \cosh^2 \frac{\rho}{l} \left(\frac{-dt^2 + du^2}{u^2} \right) , \quad \rho \leq \rho_0 , \quad z \geq z_c . \end{aligned}$$

$$\lambda = \frac{8G_N}{l} z_c^2$$

Type B



Bulk dual: $$ds^2 = d\rho^2 + l^2 \cosh^2 \frac{\rho}{l} \left(\frac{-dt^2 + du^2}{u^2} \right) , \quad -\rho_c \leq \rho \leq \rho_0$$

Holographic dictionary in Type B

Extrinsic curvature on Dirichlet boundary:

$$K_{ab}=\frac{\tanh\frac{\rho_c}{l}}{l}h_{ab}=\frac{\tanh\frac{\rho_c}{l}}{l}\begin{bmatrix}-\frac{l^2\cosh^2\frac{\rho_c}{l}}{u^2} & 0\\ 0 & \frac{l^2\cosh^2\frac{\rho_c}{l}}{u^2}\end{bmatrix}$$

Brown-York tensor:

$$T_{ab}=\frac{1}{8\pi G_N}\left(K_{ab}-Kh_{ab}+\frac{1}{l}h_{ab}\right)$$

Trace flow equation from gravity side:

$$T^a_a=\frac{1}{8\pi G_N l}\frac{1}{\cosh^2\frac{\rho_c}{l}}-32\pi G_N l T\bar{T}$$

$\updownarrow$

Trace flow equation from field th side:

$$T^a_a=-\frac{c}{24\pi}\mathcal{R}[h]-4\pi\lambda T\bar{T}$$

$$c=\frac{3l}{2G_N}\,,\qquad \lambda=8G_N l$$

Key insight

Yes, **Type A**

Boundary of BCFT moves holographically

No, **Type B**

Boundary entropy in Type A

Consider TTbar BCFT in a disk.

Boundary entropy in field th side: $S_{\text{bdy}}^{\text{disk}} = \log g = \log\langle 0|B\rangle$ or $S_{\text{bdy}} = S^{\text{BCFT}}([0,L]) - \frac{1}{2}S^{\text{CFT}}([-L,L])$

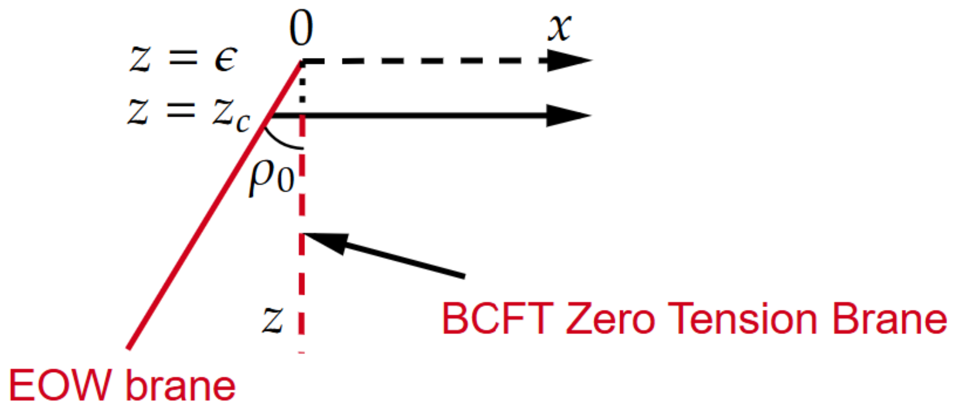
$\updownarrow$

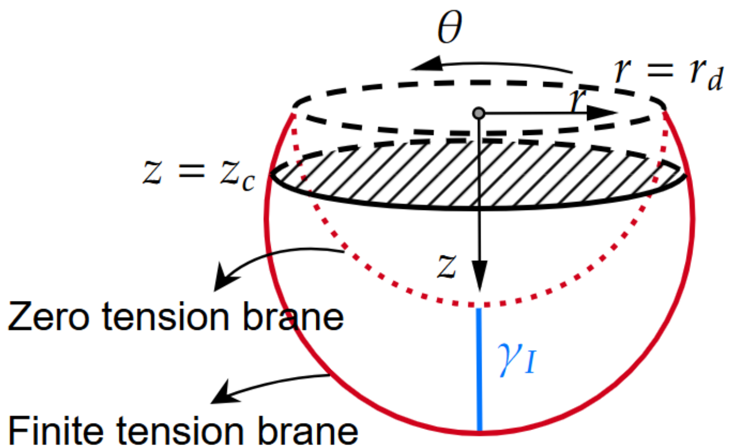
Boundary entropy in gravity side: **Bulk on-shell action between zero tension and finite tension brane** or **Minimal surface anchored on zero tension and finite tension brane**

Qusetion: Where is zero tension brane?

Boundary entropy in Type A

Choice 1: BCFT zero tension brane





This choice implies that we treat the field theory degrees of freedom on the cutoff disk, located between the BCFT zero tension and finite tension brane, as boundary degrees of freedom.

Boundary entropy in Type A

Boundary entropy from Bulk on-shell action:

The on-shell action bounded by a finite tension brane is:

$$
\begin{aligned}
I_E =& I_{\text{bulk}} + I_{\text{brane}} + I_{\text{ct}} \\
=& -\frac{1}{16\pi G}\int dx^3\sqrt{g}\left(R+\frac{2}{l^2}\right) - \frac{1}{8\pi G}\int d^2x\sqrt{h}(K-T) - \frac{1}{8\pi G}\int d^2x\sqrt{h}(K-\frac{1}{l}) \\
=& -\frac{1}{16\pi G}\int_{z_c}^{r_d(\cosh\frac{\rho_0}{l}+\sinh\frac{\rho_0}{l})} dz \int_0^{\sqrt{r_d^2\cosh^2\frac{\rho_0}{l}-(z-r_d\sinh\frac{\rho_0}{l})^2}} dr\frac{-4}{l^2}\frac{l^3}{z^3}2\pi r \\
& -\frac{1}{8\pi G}\int_{z_c}^{r_d(\cosh\frac{\rho_0}{l}+\sinh\frac{\rho_0}{l})} dz\frac{1}{l}\tanh\frac{\rho_0}{l}2\pi\frac{l^2}{z^2}r_d\cosh\frac{\rho_0}{l} \\
& -\frac{1}{8\pi G}\int_0^{\sqrt{r_d^2\cosh^2\frac{\rho_0}{l}-(z_c-r_d\sinh\frac{\rho_0}{l})^2}} dr\frac{1}{l}2\pi r\frac{l^2}{z_c^2} \\
=& -\frac{l}{4G}\left(\frac{\rho_0}{l}+\log\frac{r_d}{z_c}\right) .
\end{aligned}
$$

For **choice 1**, the on-shell action within BCFT zero tension brane is: $I_E|_{\rho_0\to 0} = -\frac{l}{4G}\log\frac{r_d}{z_c}$

The boundary entropy is given by: $S_{\text{bdy}}^{\text{disk}} = -(I_E - I_E|_{\rho_0\to 0}) = \frac{\rho_0}{4G}$.

Boundary entropy in Type A

Boundary entropy from minimal surface:

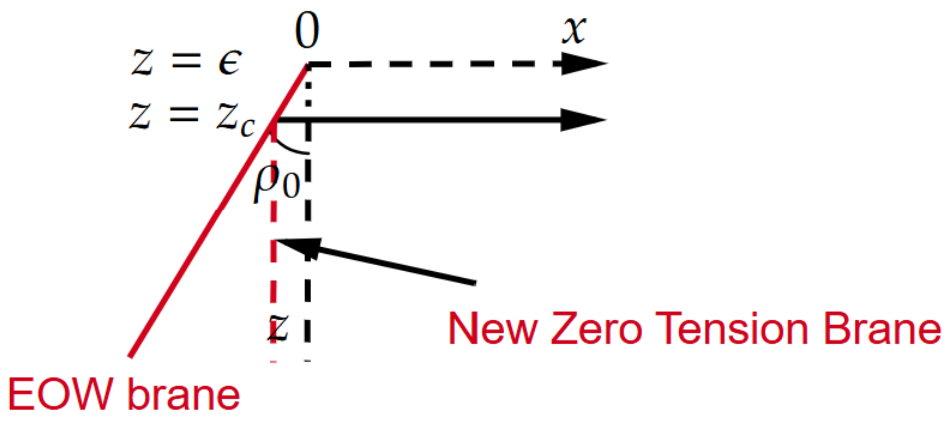
$$S_{\text{bdy}}^{\text{RT}} = \frac{\text{Area}(\gamma_I)}{4G}$$

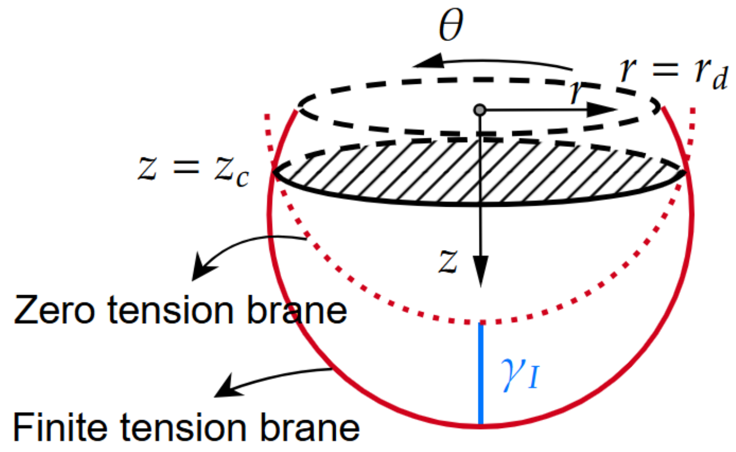
$$S_{\text{bdy}}^{\text{RT}} = \frac{1}{4G}\int_{r_d}^{r_d(\cosh\frac{\rho_0}{l}+\sinh\frac{\rho_0}{l})} dz\frac{l}{z} = \frac{\rho_0}{4G}$$

The two results agree with each other and is the same as BCFT boundary entropy. This indicates that if we regard the degree of freedom in the extended interval between BCFT zero tension brane and finite tension brane as boundary degrees of freedom, then the total boundary entropy will be conserved along TTbar deformation.

Boundary entropy in Type A

Choice 2: New zero tension brane





This choice implies that we treat the field theory degrees of freedom on the cutoff disk, located between the BCFT zero tension and finite tension brane, as bulk degrees of freedom.

Boundary entropy in Type A

Boundary entropy from Bulk on-shell action:

For **choice 2**, the on-shell action within new zero tension brane is:

$$I_E\big|_{\rho_0\to 0, r_d\to\sqrt{r_d(r_d+2z_c\sinh\frac{\rho_0}{l})}} = -\frac{l}{8G}\log\frac{r_d(r_d+2z_c\sinh\frac{\rho_0}{l})}{z_c^2} \ .$$

The boundary entropy is given by:

$$\begin{aligned} S_{\text{bdy}}^{\text{disk}} &= -(I_E - I_E\big|_{\rho_0\to 0, r_d\to\sqrt{r_d(r_d+2z_c\sinh\frac{\rho_0}{l})}}) \\ &= \frac{\rho_0}{4G} - \frac{l}{8G}\log(1+\frac{2z_c}{r_d}\sinh\frac{\rho_0}{l}) \ . \end{aligned}$$

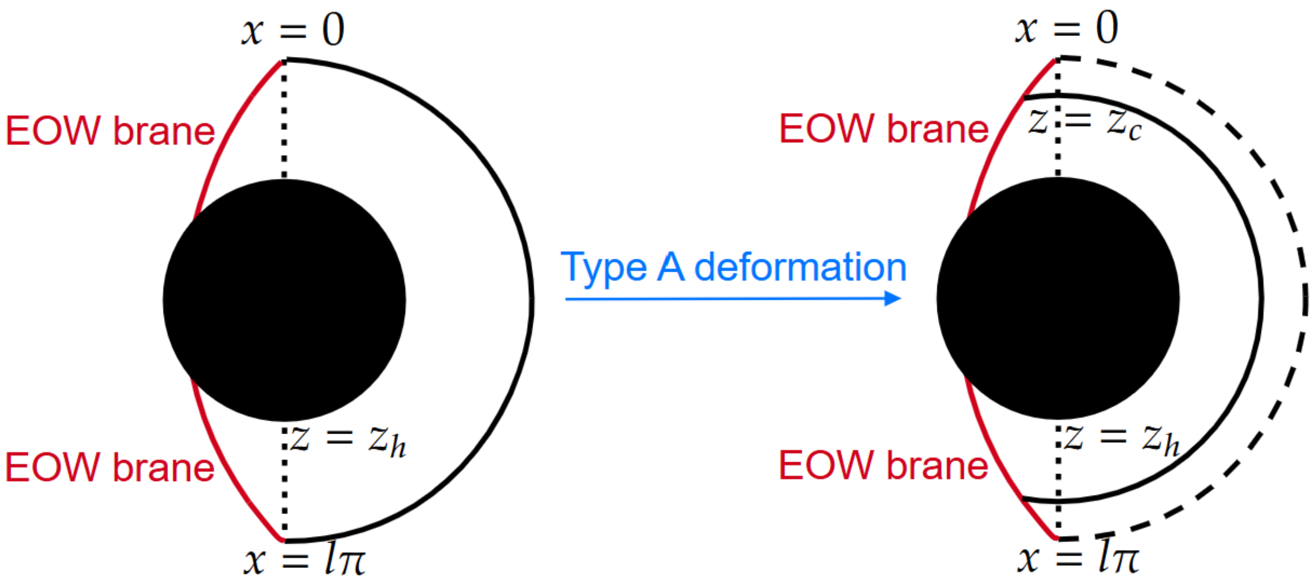
Boundary entropy in Type A

Boundary entropy from minimal surface:

$$S_{\mathrm{bdy}}^{\mathrm{RT}} = \frac{1}{4G}\int^{r_d(\cosh\frac{\rho_0}{l}+\sinh\frac{\rho_0}{l})}_{\sqrt{r_d(r_d+2z_c\sinh\frac{\rho_0}{l})}} dz\frac{l}{z} = \frac{\rho_0}{4G} - \frac{l}{8G}\log(1+\frac{2z_c}{r_d}\sinh\frac{\rho_0}{l})\ .$$

The two results agree with each other. In this case, we observe that the boundary entropy is no longer constant; it depends on the TTbar deformation parameter. This result clearly indicates that the boundary of the BCFT is deformed, and we can use boundary entropy to quantify the amount of boundary deformation.

Energy spectrum in Type A



Energy spectrum in Type A

Energy spectrum from gravity side:

Energy density from Brown-York tensor:

$$e = u^i u^j T_{ij} = \frac{u^i u^j}{8\pi G}\left(K_{ij} - K h_{ij} + h_{ij}\right)$$

$$e = \frac{1}{8\pi G l}\left(1 - \sqrt{f(z_c)}\right) = \frac{1}{8\pi G l}\left(1 - \sqrt{1 - \frac{z_c^2}{z_H^2}}\right) = \frac{1}{8\pi G l}\left(1 - \sqrt{1 - \frac{8 G_N M z_c^2}{l^2}}\right)$$

Energy obtained by integrating over the spatial region:

$$\begin{aligned} E &= \int_{-z_H \cdot \mathrm{arcsinh}\left(\frac{lTz}{z_H\sqrt{1-l^2T^2}}\right)}^{\pi + z_H/l \cdot \mathrm{arcsinh}\left(\frac{lTz}{z_H\sqrt{1-l^2T^2}}\right)} d\theta \frac{l^2}{z_c} e \\ &= \int_{-z_H \cdot \mathrm{arcsinh}\left(\frac{lTz}{z_H\sqrt{1-l^2T^2}}\right)}^{\pi + z_H/l \cdot \mathrm{arcsinh}\left(\frac{lTz}{z_H\sqrt{1-l^2T^2}}\right)} \frac{l}{8\pi G_N z_c}\left[1 - \sqrt{1 - \frac{8 G_N M z_c^2}{l^2}}\right] d\theta \\ &= \frac{l}{8\pi G_N z_c}\left[\pi + 2 z_H/l \cdot \mathrm{arcsinh}\left(\frac{lTz}{z_H\sqrt{1-l^2T^2}}\right)\right] \cdot \left[1 - \sqrt{1 - \frac{8 G_N M z_c^2}{l^2}}\right] \end{aligned}$$

Dimensionless quantity obtained by multiplying energy with proper length:

$$\begin{aligned} \mathcal{E} &= E \cdot \int_{-z_H \cdot \mathrm{arcsinh}\left(\frac{lTz}{z_H\sqrt{1-l^2T^2}}\right)}^{\pi + z_H/l \cdot \mathrm{arcsinh}\left(\frac{lTz}{z_H\sqrt{1-l^2T^2}}\right)} d\theta \frac{l^2}{z_c} \\ &= \frac{l^3}{8\pi G_N z_c^2}\left[\pi + 2 z_H/l \cdot \mathrm{arcsinh}\left(\frac{lTz}{z_H\sqrt{1-l^2T^2}}\right)\right]^2 \cdot \left[1 - \sqrt{1 - \frac{8 G_N M z_c^2}{l^2}}\right] \end{aligned}$$

Energy spectrum in Type A

The bulk picture tells us that the boundaries only truncate the spatial circle while preserving the energy density. This suggests we can quantify their impact by recognizing that the effect is simply to rescale the spatial length. The rate of rescaling is

$$f(\lambda) \equiv \frac{1}{2\pi}\left[\pi + \frac{2z_H}{l}\operatorname{arcsinh}\left(\frac{lTz_c}{z_H\sqrt{1-l^2T^2}}\right)\right] = \frac{1}{2} + \frac{z_H}{\pi l}\operatorname{arcsinh}\left(\frac{lT\sqrt{\frac{\lambda l}{8G_N}}}{z_H\sqrt{1-l^2T^2}}\right)$$

Energy spectrum in Type A

Energy spectrum from field th side:

Without boundaries, the spatial length of field theory is $L = 2\pi l$

After adding boundaries, the spatial length of deformed theory is $L_b = 2\pi l f(\lambda) = L f(\lambda)$

The stress energy tensor of TTbar BCFT in finite interval have zero momentum, and we can separately compute the expectation value of TTbar operator as

$$\begin{aligned}\langle n|T\bar{T}|n\rangle &= \frac{1}{8}\langle n|T|n\rangle\langle n|\bar{T}|n\rangle - \frac{1}{8}\langle n|\Theta|n\rangle\langle n|\Theta|n\rangle \\ &= -\frac{1}{4}\left(\langle n\left|T_{\tau\tau}\right|n\rangle\langle n\left|T_{xx}\right|n\rangle\right),\end{aligned}$$

$$\begin{aligned}\langle n\left|T_{\tau\tau}\right|n\rangle &= \frac{E_n}{L_b} = \frac{E_n}{Lf(\lambda)}, \\ \langle n\left|T_{xx}\right|n\rangle &= \frac{\partial E_n}{\partial L_b} = \frac{\partial E_n}{\partial L}\cdot\frac{1}{f(\lambda)}.\end{aligned}$$

Energy spectrum in Type A

Energy spectrum from field th side:

From the definition of TTbar deformation

$$S_{T\bar{T}} = 2\pi\lambda \int d^2x T\bar{T}$$

We have

$$H_{T\bar{T}} = 2\pi\lambda \int_0^{Lf(\lambda)} d\theta \; T\bar{T}$$

Thus

$$\begin{aligned}\frac{\partial}{\partial\lambda}\langle H_{T\bar{T}}\rangle &= 2\pi Lf(\lambda)\langle T\bar{T}\rangle + 2\pi L\lambda f'(\lambda)\langle T\bar{T}\rangle|_{Lf(\lambda)} \\ &= 2\pi Lf(\lambda)\langle T\bar{T}\rangle + \frac{f'(\lambda)}{f(\lambda)}\langle H_{T\bar{T}}\rangle,\end{aligned}$$

Final equation is :

$$2\frac{\partial E_n}{\partial\lambda} - \frac{2f'(\lambda)}{f(\lambda)}E_n + \frac{\pi E_n}{f(\lambda)}\frac{\partial E_n}{\partial L} = 0$$

Agree with gravity result under

$$M_n = M \cdot l$$

$$\lambda = \frac{8G}{l} z_c^2$$

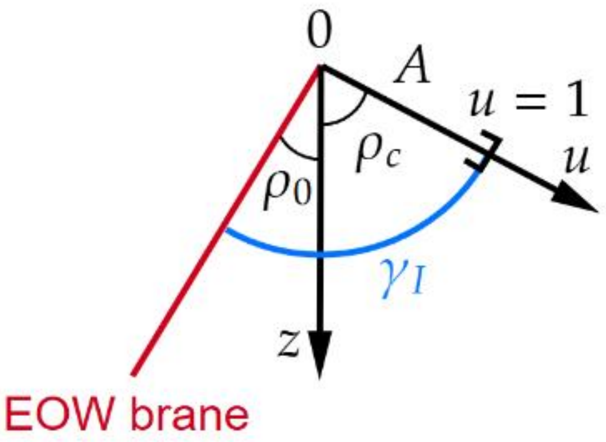
Final solution is :

$$E_n = \frac{Lf(\lambda)}{\pi\lambda}\left[1 - \sqrt{1 - \frac{4\pi^2\lambda M_n}{L^2}}\right]$$

Dimensionless quantity:

$$\mathcal{E}_N = \frac{L^2 f(\lambda)^2}{\pi\lambda}\left[1 - \sqrt{1 - \frac{4\pi^2\lambda M_n}{L^2}}\right]$$

Entanglement entropy in Type B



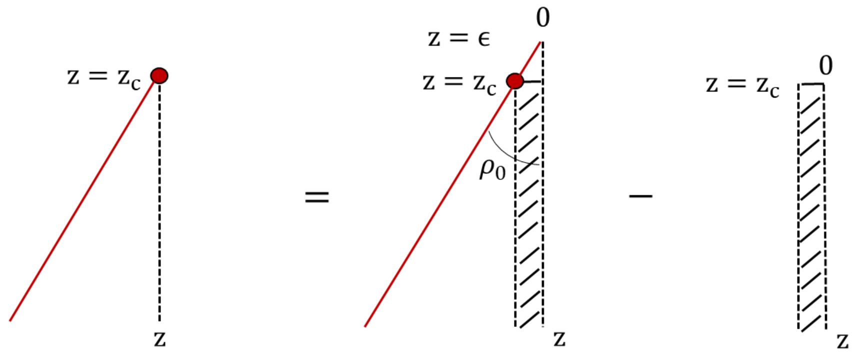
$$S(A) = \frac{\mathrm{Area}(\gamma_I)}{4G_N} = \frac{1}{4G_N}\int_{-\rho_c}^{\rho_0} d\rho = \frac{\rho_c}{4G_N} + \frac{\rho_0}{4G_N}$$

Field theory interpretation of boundary deformation

Shift of boundary position

Small ⟷ Displacement operator $\mathcal{D} = T_{xx}$

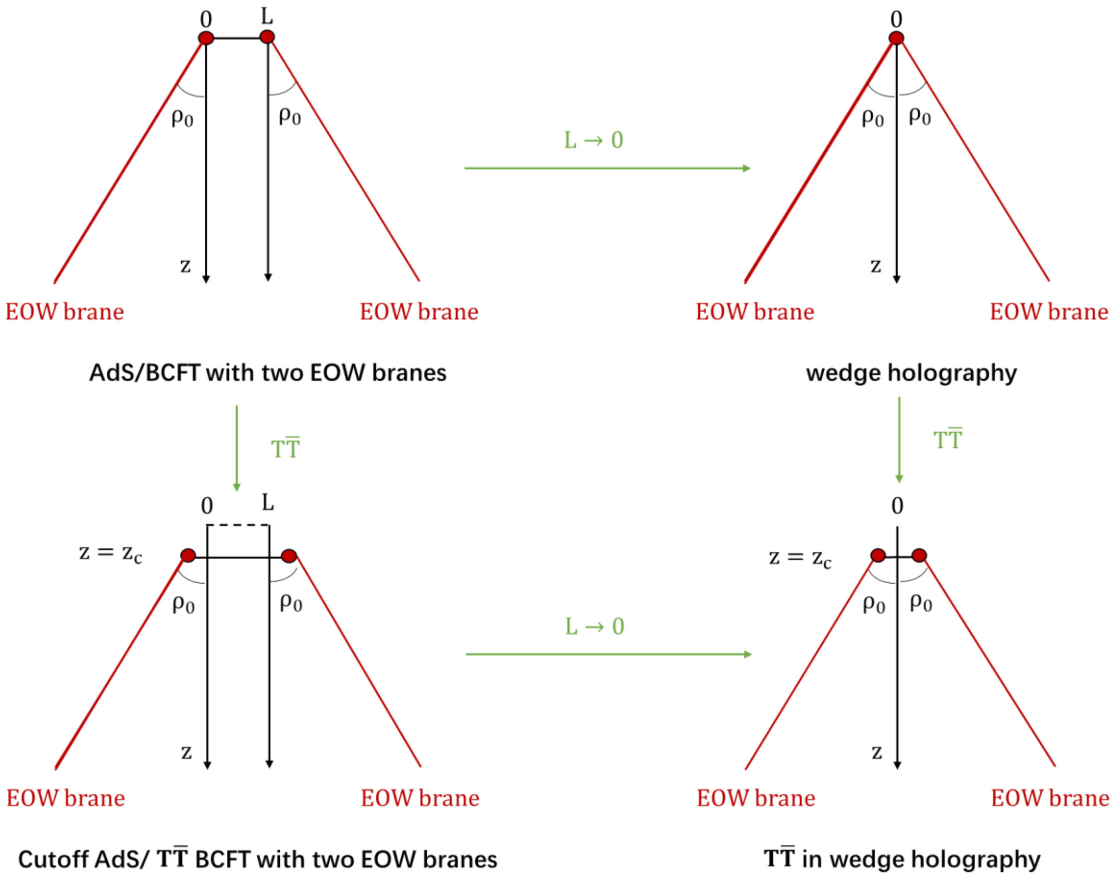
Finite ⟷



$$\mathbf{W} = \mathbf{W_0} - \Delta\mathbf{W}$$

$$S_{1\mathrm{d}} = S_0 - 2\pi\lambda z_c \sinh\frac{\rho_0}{l}\int_{x=-z_c\sinh\frac{\rho_0}{l}} dt\,\frac{\mathcal{D}^2}{\pi - 16\lambda\mathcal{D}}$$

TTbar deformation in wedge holography



Thank you for your attention！