Is holographic quark-gluon plasma homogeneous?

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Gauge/gravity duality 2024

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in collaboration with

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Niko Jokela, Aleksi Piispa (Helsinki) [2405.02392, 2405.02394, 2405.02399]

Outline

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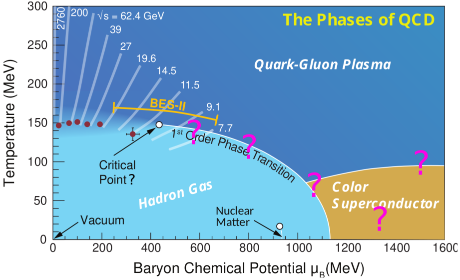
QCD phase diagram and the critical point

Search for the critical point: ongoing effort at RHIC

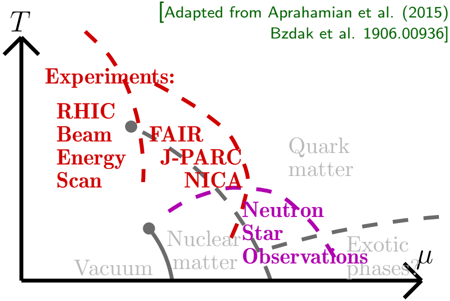
- Beam Energy Scan stage I results available
- Stage II finished, results being analyzed

Will be extended by future experiments at FAIR, J-PARC, NICA

Neutron star observations give complementary information at high density



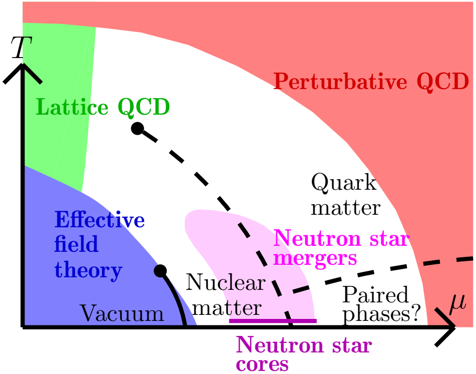
[Adapted from Aprahamian et al. (2015)
Bzdak et al. 1906.00936]



QCD phase diagram and the critical point

Theoretical approaches

- First-principles methods do not work in the region relevant for critical point
- Phase diagram or even relevant phases not known
- May include spatially modulated phases
- Can be accessed via the gauge/gravity duality?



- Basic idea (bottom-up): use the gauge/gravity duality to extrapolate lattice (and other) data to higher density

[DeWolfe et al. 1012.1864; Knaute et al. 1702.06731; Critelli et al. 1706.00455 Jokela, MJ, Remes 1809.07770; Demircik, Ecker, MJ 2112.12157 Cai, He, Li, Wang 2201.02004; Li, Liang, He, Li 2305.13874 . . .]

[See also the talk by Mei Huang]

Outline

1. Introduction
2. Holographic models
3. Spatial Instability
4. Conclusion

Generic holographic approach: fields

We want to describe holographically (chirally symmetric) QCD plasma (N_f massless flavors)

Most important (relevant and marginal) operators

- $T_{\mu\nu}$, dual to the metric $g_{\mu\nu}$
- Gluon operator $G^2_{\mu\nu}$, dual to a scalar (the dilaton) ϕ
- Flavor currents $\bar{\psi}_i\gamma_\mu(1\pm\gamma_5)\psi_j$, dual to the gauge fields $(A_\mu^{L/R})_{ij}$ (with $i,j=1\ldots N_f$) – global $\mathrm{U}(N_f)_L\times\mathrm{U}(N_f)_R$ of QCD promoted to gauge symmetry
- ~~Flavor bilinears $\bar{\psi}_i\psi_j$ dual to a complex scalar T_{ij}~~ – irrelevant in chirally symmetric phase

What are our options for the choice of 5D action?

Chern-Simons (CS) terms in holography

Chiral anomalies in QCD: consider the chiral $\mathrm{U}(N_f)_L \times \mathrm{U}(N_f)_R$ coupled to external fields A_L, A_R

- Under transformation with parameters $\Lambda_{L/R}$

$$S_{\mathrm{QCD}} \mapsto S_{\mathrm{QCD}} + \frac{iN_c}{24\pi^2}\int \mathrm{Tr}\left[\Lambda_L F_L \wedge F_L - \Lambda_R F_R \wedge F_R + \cdots\right]$$

Holographic counterpart

- External fields promoted to 5D gauge fields
- Gauge variation at the boundary must agree with the anomaly
- 5D CS term – unique when chiral symmetry intact

[Witten hep-th/9802150]

$$S_{\mathrm{CS}} = \frac{iN_c}{24\pi^2}\int \mathrm{Tr}\Big[-iA_L \wedge F_L \wedge F_L + \frac{1}{2}A_L \wedge A_L \wedge A_L \wedge F_L + \frac{i}{10}A_L \wedge A_L \wedge A_L \wedge A_L \wedge A_L - (L \leftrightarrow R)\Big]$$

- Generalizations (e.g. chirally broken) worked out

[Casero, Paredes, Kiritsis hep-th/0702155; Lau, Sugimoto 1612.09503; MJ, Kiritsis, Nitti, Préau 2209.05868]

- Note: $U(1)_A$ anomaly is a separate issue – not needed here

Generic holographic approach: actions

We write down expected (two-derivative) terms

$$S = S_{\rm gr} + S_{\rm matter} + S_{\rm CS}$$

where $S_{\rm CS}$ is fixed by anomalies, and

$$S_{\rm gr} = M_{\rm p}^3 N_c^2 \int d^5x\, \sqrt{-\det g}\left[R - \frac{4}{3}(\partial\phi)^2 + V_{\rm g}(\phi)\right]$$

Choice of $S_{\rm matter}$ less obvious. Options: $S_{\rm matter} = S_{\rm DBI}$ or $S_{\rm matter} = S_{\rm YM}$, with

1. $$S_{\rm DBI} = M_{\rm p}^3 N_c \int V_{\rm f}(\phi)\,{\rm Tr}\left[\sqrt{-\det\left[g_{\mu\nu} + w(\phi)(F_L)_{\mu\nu}\right]} + (L \leftrightarrow R)\right]$$
2. $$S_{\rm YM} = M_{\rm p}^3 N_c \int Z(\phi)\,{\rm Tr}\left[F_L^2 + F_R^2\right]$$

- Background gauge fields sourced by $\mu_B \Rightarrow$ at small density, $F_{L/R}$ small $\Rightarrow$ DBI and YM reduce to the same choice
- Potentials (V_g, V_f, w or V_g, Z) to be fixed by QCD data

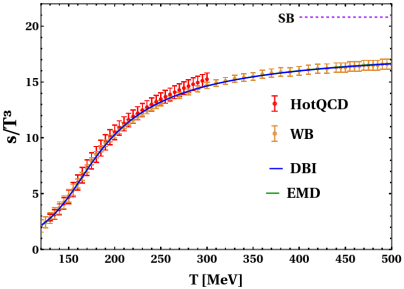
Fitting the potentials to data

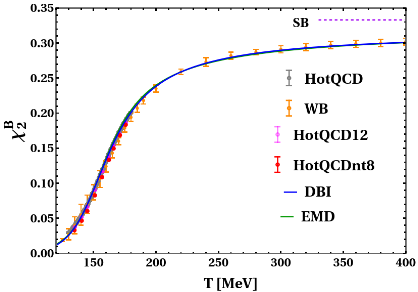
Potentials determined by comparison to lattice data

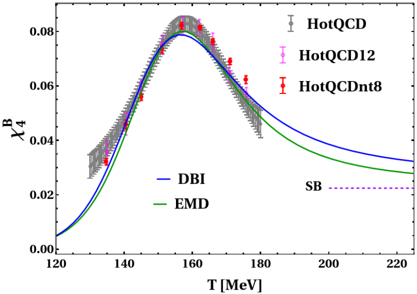
- Data for Yang-Mills (V_g)
- Data for full QCD (other potentials):
  equation of state, $\chi_2^B = \frac{d^2p}{d\mu_B^2}\Big|_{\mu_B=0}$ ...

In case of DBI action we use two approaches

1. With confinement and phase transition (V-QCD)
2. Without confinement, direct fit to data





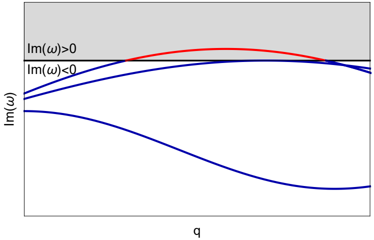


Outline

Inhomogeneity in holographic plasma?

Spatially modulated instability

[Nakamura, Ooguri, Park 0911.0679;
Ooguri, Park 1011.4144]

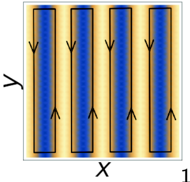


- Exponentially growing perturbation at $q \neq 0$: a quasi-normal mode with $\mathrm{Im}\,\omega > 0$
- The Chern-Simons term can drive such a modulated instability at finite density

Schematic fluctuation equation

$$\psi''(r) + \left(A' + \frac{f'}{f}\right)\psi'(r) + \underbrace{\frac{qn}{M_p^3 f e^{2A} Z(\phi)^2}\psi(r)}_{\text{From CS term}} + \left(\frac{\omega^2}{f^2} - \frac{q^2}{f}\right)\psi(r) = 0$$

$\psi = \delta A^x_{L/R} \pm i\delta A^y_{L/R}$ $\qquad$ $r =$ holographic coord.

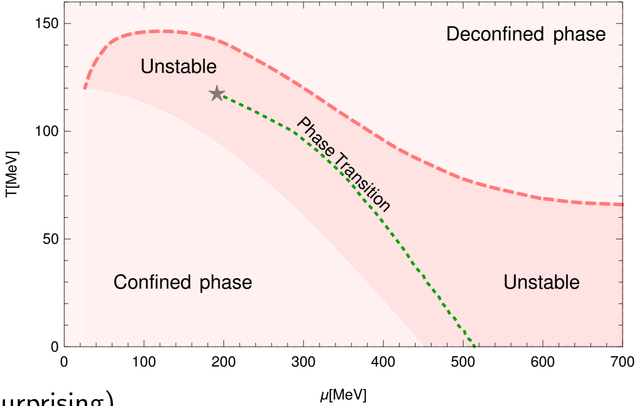


- Ground state: Modulated 5D gauge fields dual to modulated persistent chiral currents in field theory

Modulated instability in V-QCD

The region where instability exists in V-QCD

[Cruz Rojas, Demircik, MJ 2405.02399]



- The Chern-Simons term is strong enough to create an instability in V-QCD (unsurprising)
- Instability is found at low T and large density (expected)
- Instability is also found at higher T, near the regime with critical point?! (a big surprise)
- Estimate for transition and critical point from earlier work

[Demircik, Ecker, MJ 2112.12157]

Model dependence: fitting uncertainty

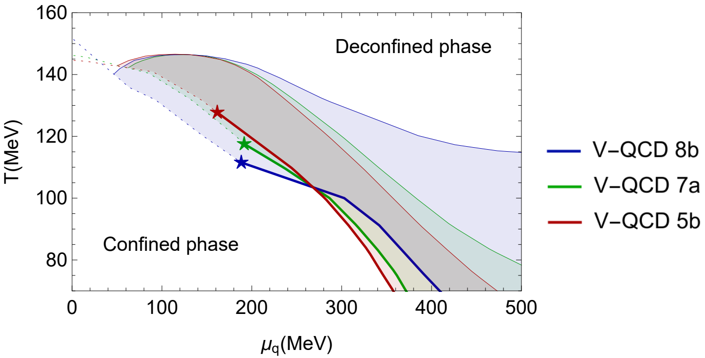
Low-density instability would be phenomenologically highly interesting and potentially testable

- There may be caveats and uncertainties (choices in fitting the data, model dependence and reliability. . .)
- However, at low densities, expect that models strictly fixed by lattice data
- Important to check this!

[Demircik, Jokela, MJ, Piispa 2405.02392]

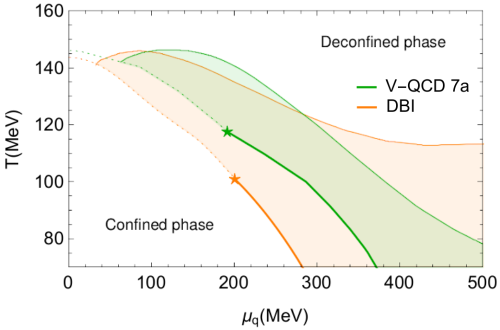
Parameter depedence in V-QCD: rather weak

- Onset of instability solidly determined by lattice fit

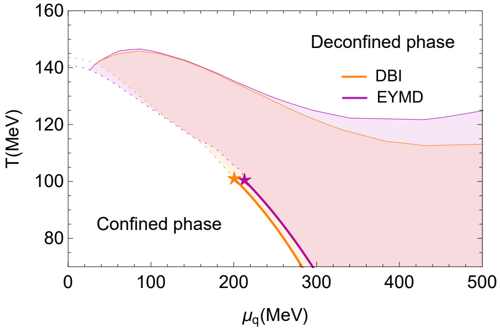


Model dependence: other checks

Vary fitting strategy



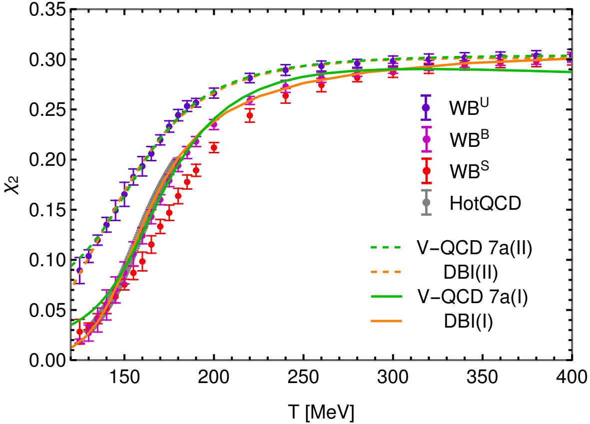
Vary matter action



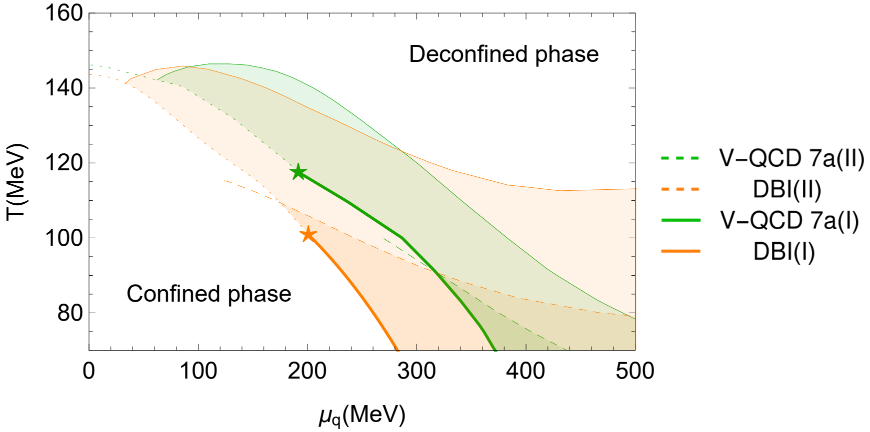
- Only minor changes – in particular, DBI and Yang-Mills actions give essentially identical results
- This means that the instability appears in a wide class of models in the literature

Model dependence: strange quark mass

- Instability potentially sensitive to fit to $\chi_2 = \frac{d^2p}{d\mu^2}\Big|_{\mu=0}$
- Lattice data shows mild flavor dependence [Borsanyi et al. 1112.4416]
- Naive test: fit instead of the full χ_2 the light quark χ_2 (dashed curves) of the $N_f = 2 + 1$ lattice result $\Rightarrow$ isolate the instability in the light quark sector



Model dependence: strange quark mass



- Rather strong suppression of the instability!
- However, not a consistent check due to strange quark effects in lattice data
- Moreover, fit to strange quark χ_2 would instead enhance instability
- Therefore further careful study is required

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Conclusion

- Holographic bottom-up QCD models anchored to lattice data suffer from strong Nakamura-Ooguri-Park instability
  - Model dependence weak, so perhaps also a feature of real QCD?
- Appears at high density, region potentially reached in neutron star cores and neutron star mergers
- A surprise: also found at low density and high temperature, region reachable by lattice or experiments
- Dependence on fitting procedure and choice of flavor action small at low density – affects ALL models fitted to equation of state and χ_2^B
- Flavor effects, in particular dependence on strange quark mass, expected to be significant
- Next step, therefore: add separate flavors and strange quark mass – in progress with Toshali Mitra – fitting already done

Thank you!

Generic holographic approach: fitting strategies

Potentials are determined by comparing with lattice results for QCD thermodynamics. Two main strategies:

Strategy I: Include confined phase, with $S_{\text{on-shell}} = \mathcal{O}(N_c^0)$, and the transition to a deconfined phase, with $S_{\text{on-shell}} = \mathcal{O}(N_c^2)$

- Used in Improved Holographic QCD and V-QCD models
  [Gürsoy, Kiritsis 0707.1324; Gürsoy, Kiritsis, Nitti 0707.1349; MJ, Kiritsis 1112.1261]
- Fit lattice data above $T = T_c$
  [Gürsoy, Kiritsis, Mazzanti, Nitti 0903.2859; Jokela, MJ, Remes 1809.07770]
- Faithful to the behavior in the limit of large N_c

Strategy II: Only deconfined black holes: no phase transition at low density

- Fit lattice data at all temperatures
  [Gubser, Nellore, Pufu, Rocha 0804.1950; Gubser, Nellore 0804.0434; DeWolfe, Gubser, Rosen 1012.1864; . . .]
- Follows the behavior in the phase diagram of QCD (crossover at low density)

We study both approaches

Fitting the models: setup

Solve numerically black hole geometries

$$ds^2 = e^{2A(r)}\left(\frac{1}{f(r)}dr^2 - f(r)dt^2 + d\vec{x}^2\right)$$

with a horizon $f(r = r_h) = 0$ and a background gauge field

$$A_L^t(r) = A_R^t(r) = \Phi(r)\mathbb{I}$$

Black hole thermodynamics $\Rightarrow$ equation of state

$$T = \frac{1}{4\pi}|f'(r_h)| \qquad s = 4\pi M_p^2 N_c^2 e^{3A(r_h)}$$

Relation between quark number n and chemical potential (for YM action)

$$\mu = \Phi(r = 0) = n\int_0^{r_h} \frac{1}{e^A Z(\phi)}$$

Numerical expansion $\Rightarrow$ susceptibilities

$$\chi_k(T, \mu) = \frac{\partial^k p(T, \mu)}{\partial\mu^k} = \frac{\partial^{k-1} n(T, \mu)}{\partial\mu^{k-1}}$$

Constraining the potentials

In the UV ($\lambda \to 0$):

- UV expansions of potentials matched with perturbative QCD beta functions $\Rightarrow$ asymptotic freedom and logarithmic flow of the coupling and quark mass, as in QCD

[Gürsoy, Kiritsis 0707.1324; MJ, Kiritsis 1112.1261]

In the IR ($\lambda \to \infty$): various qualitative constraints

- Linear confinement, discrete glueball & meson spectrum, linear radial trajectories
- Existence of a "good" IR singularity
- Correct behavior at large quark masses
- Working potentials often string-inspired power-laws, multiplied by logarithmic corrections (i.e, first guesses usually work!)

[Gürsoy, Kiritsis, Nitti 0707.1349; MJ, Kiritsis 1112.1261; Arean, Iatrakis, MJ, Kiritsis 1309.2286, 1609.08922; MJ 1501.07272]

Final task: determine the potentials in the middle, $\lambda = \mathcal{O}(1)$

- Qualitative comparison to lattice/experimental data

Ansatz for potentials, $(x = 1)$

$$V_g(\lambda) = 12\left[1 + V_1\lambda + \frac{V_2\lambda^2}{1+\lambda/\lambda_0} + V_{\mathrm{IR}}\, e^{-\lambda_0/\lambda}(\lambda/\lambda_0)^{4/3}\sqrt{\log(1+\lambda/\lambda_0)}\right]$$

$$V_{f0}(\lambda) = W_0 + W_1\lambda + \frac{W_2\lambda^2}{1+\lambda/\lambda_0} + W_{\mathrm{IR}}\, e^{-\lambda_0/\lambda}(\lambda/\lambda_0)^2$$

$$\frac{1}{w(\lambda)} = w_0\left[1 + \frac{w_1\lambda/\lambda_0}{1+\lambda/\lambda_0} + \bar{w}_0\, e^{-\lambda_0/\lambda w_s}\frac{(w_s\lambda/\lambda_0)^{4/3}}{\log(1+w_s\lambda/\lambda_0)}\right]$$

$$V_1 = \frac{11}{27\pi^2}\ , \qquad V_2 = \frac{4619}{46656\pi^4}$$

$$W_1 = \frac{8+3\,W_0}{9\pi^2}\ ; \qquad W_2 = \frac{6488+999\,W_0}{15552\pi^4}$$

Fixed UV/IR asymptotics $\Rightarrow$ fit parameters only affect details in the middle

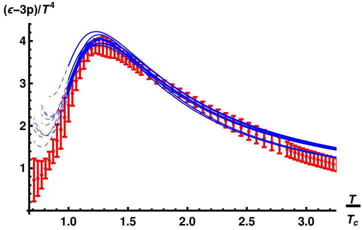
Fitting example: V-QCD (strategy I)

Fit to lattice data near $\mu = 0$ with DBI action and fitting strategy I (with transition): the V-QCD model (in the chirally symmetric phase) [MJ, Jokela, Remes, 1809.07770]

- Choose suitable Ansätze for the potentials, many parameters
- Parameters adjusted "by hand"
- Good description of lattice data – nontrivial result!
- Flat direction in the fit $\Rightarrow$ a one-parameter family of models

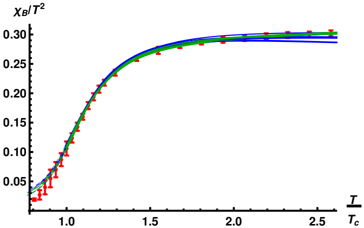
Interaction measure $\frac{\epsilon - 3p}{T^4}$, 2+1 flavors

[Data: Borsanyi et al. 1309.5258]



Baryon number susceptibility $\chi_2 = \left.\frac{d^2 p}{d\mu^2}\right|_{\mu=0}$

[Data: Borsanyi et al. 1112.4416]



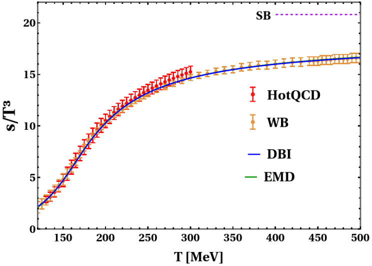
Fitting example: direct fit (strategy II)

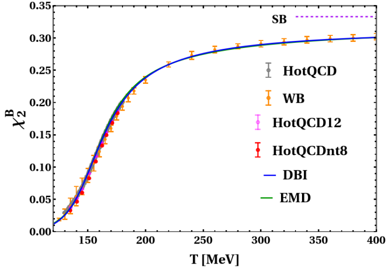
Use strategy II (no phase transition) with both DBI and YM [Jokela, MJ, Piispa 2405.02394]

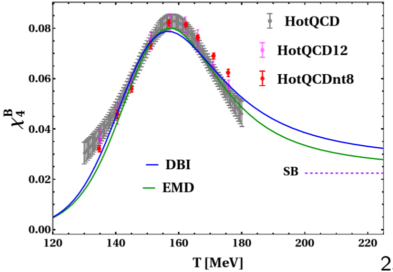
Systematic statistical fit to

1. Equation of state (through entropy density)
2. Cumulants χ_2 and χ_4

- (Here YM $\to$ EMD: for Abelian background, Yang-Mills=Maxwell)

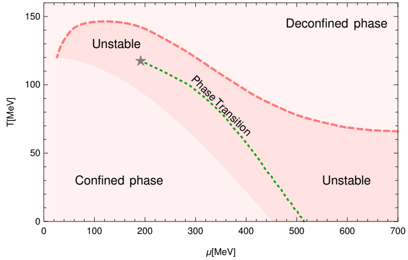






How does the instability arise?

Looks quite different from Nakamura-Ooguri-Park, where the onset was at fixed μ/T...what is going on?



- Also differs from result in Witten-Sakai-Sugimoto

[Ooguri, Park 1011.4144]

- Look at the fluctuation equation

$$\psi'' + \left(A' + \frac{f'}{f}\right)\psi' + \frac{qn}{M_p^3 f e^{2A} Z(\phi)^2}\psi + \left(\frac{\omega^2}{f^2} - \frac{q^2}{f}\right)\psi = 0$$

- Values of ϕ largest near horizon, and grow for smaller black holes
- Smallest black holes found near the deconfinement transition

[Alho, MJ, Kajantie, Kiritsis, Rosen, Tuominen 1312.5199]

- $Z(\phi)$ determined by fit to χ_2: fast increase of χ_2 with T
  $\Rightarrow$ fast decrease of Z with ϕ
- Enhances instability strongly for small black holes