Gauge Gravity Duality 2024

30 November 2024 to 4 December 2024
Sanya, Hainan, China

# Deep Learning Bulk Spacetime from Boundary quantum data

2024.12. 01


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Gwangju Institute of Science and Technology


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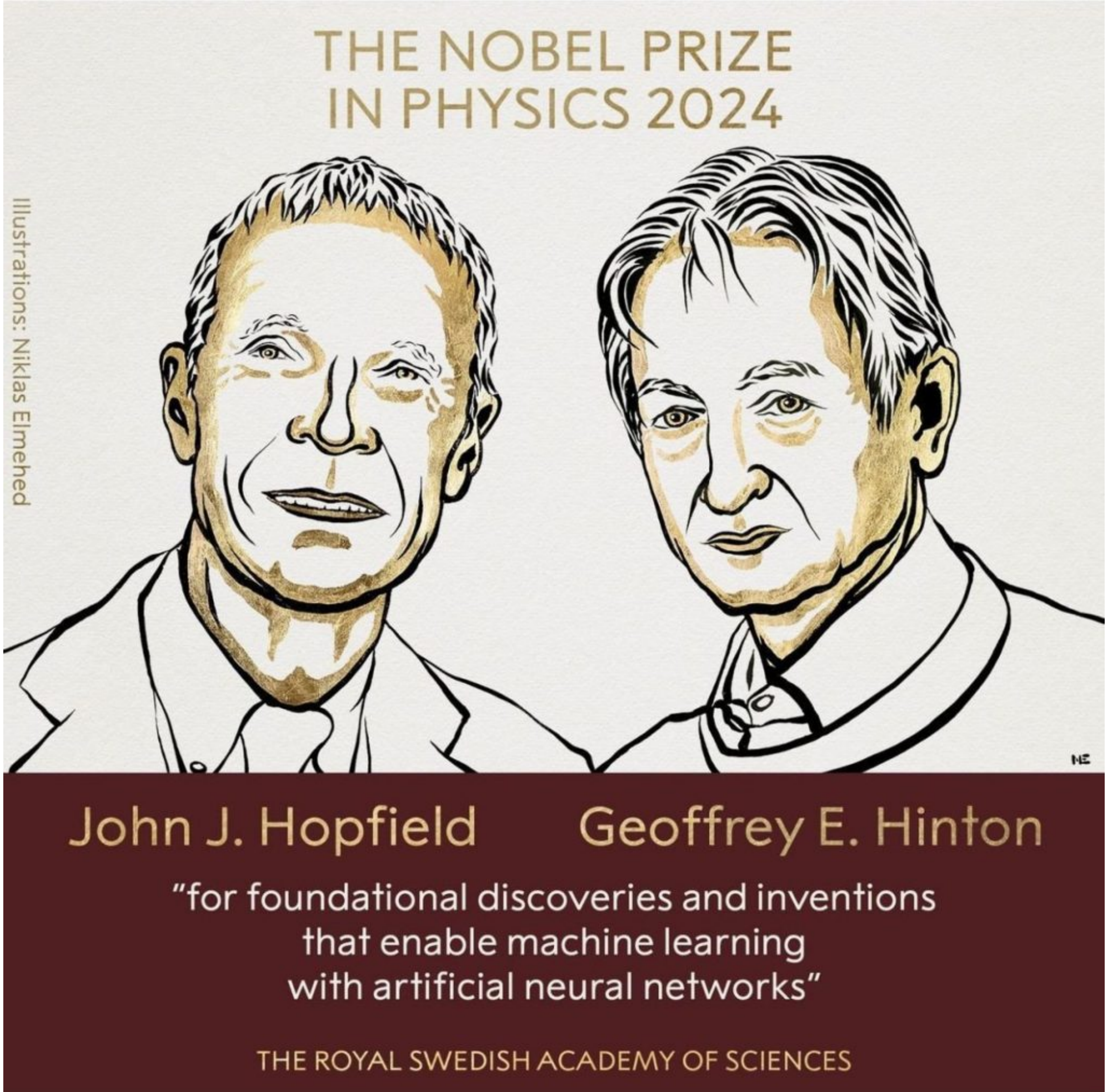
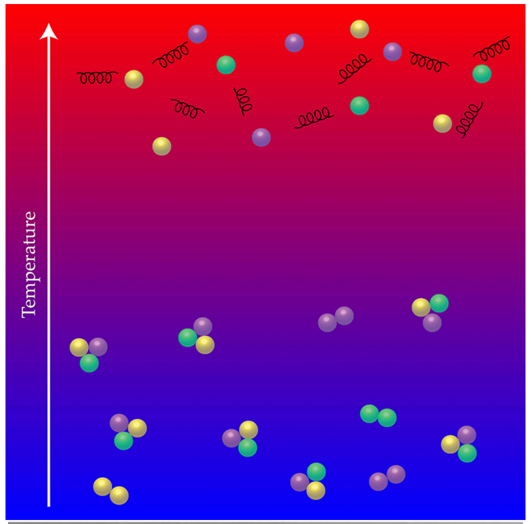
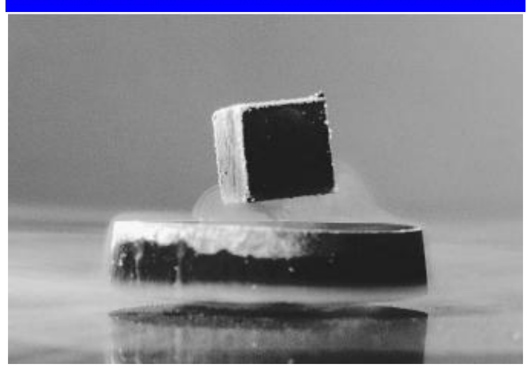

THE NOBEL PRIZE
IN PHYSICS 2024
Illustrations: Niklas Elmehed
John J. Hopfield
Geoffrey E. Hinton
“for foundational discoveries and inventions
that enable machine learning
with artificial neural networks”
THE ROYAL SWEDISH ACADEMY OF SCIENCES

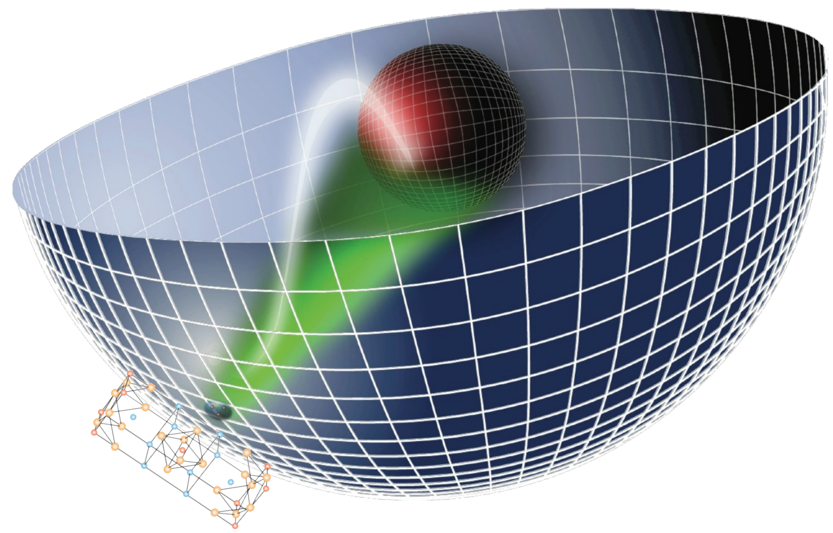
Holography

# Deep Learning Bulk Spacetime from Boundary data



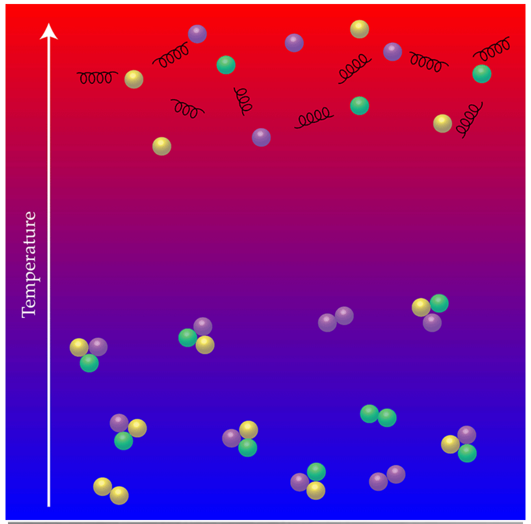



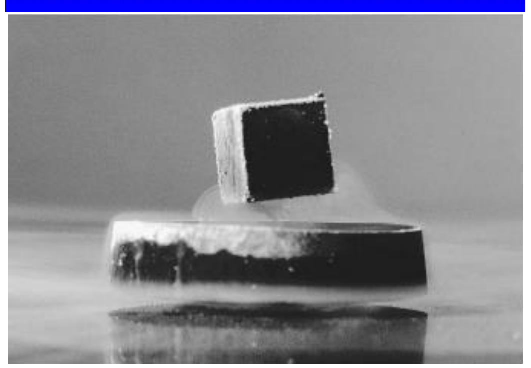
Quantum physics in 4D = Gravity in 5D



Holography

# Deep Learning Bulk Spacetime from Boundary data



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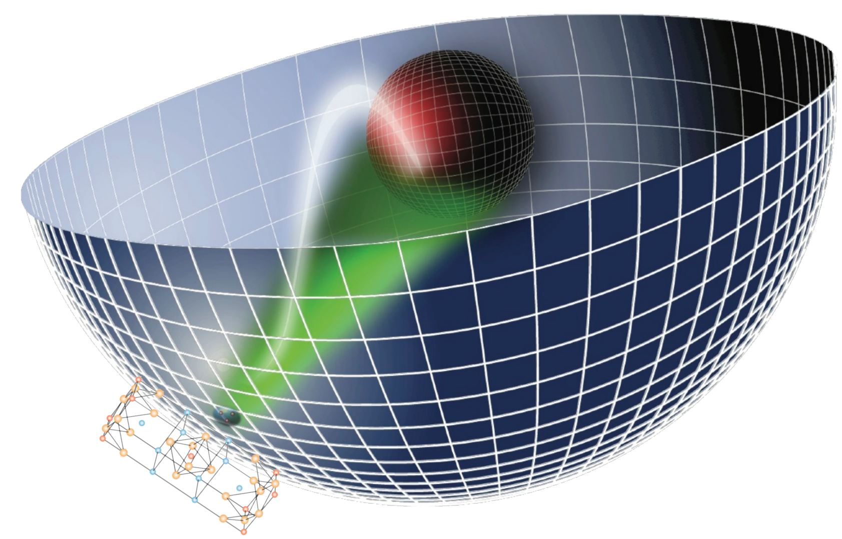
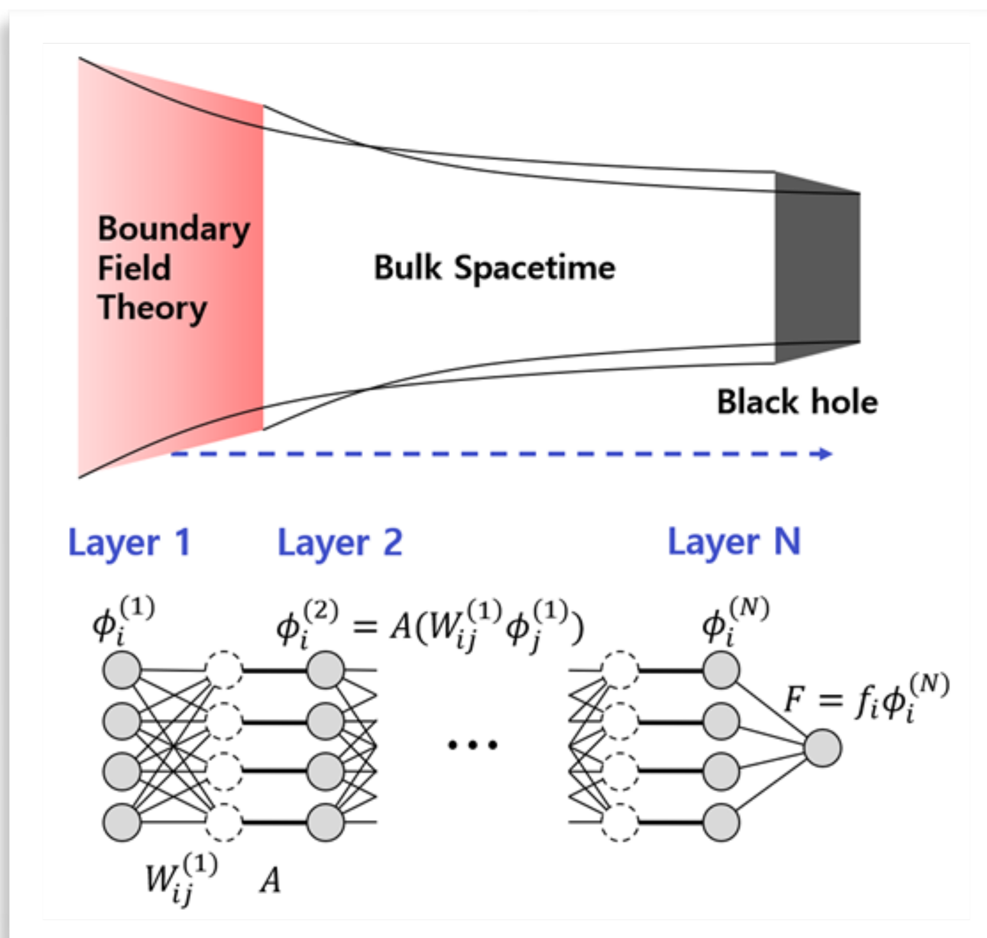




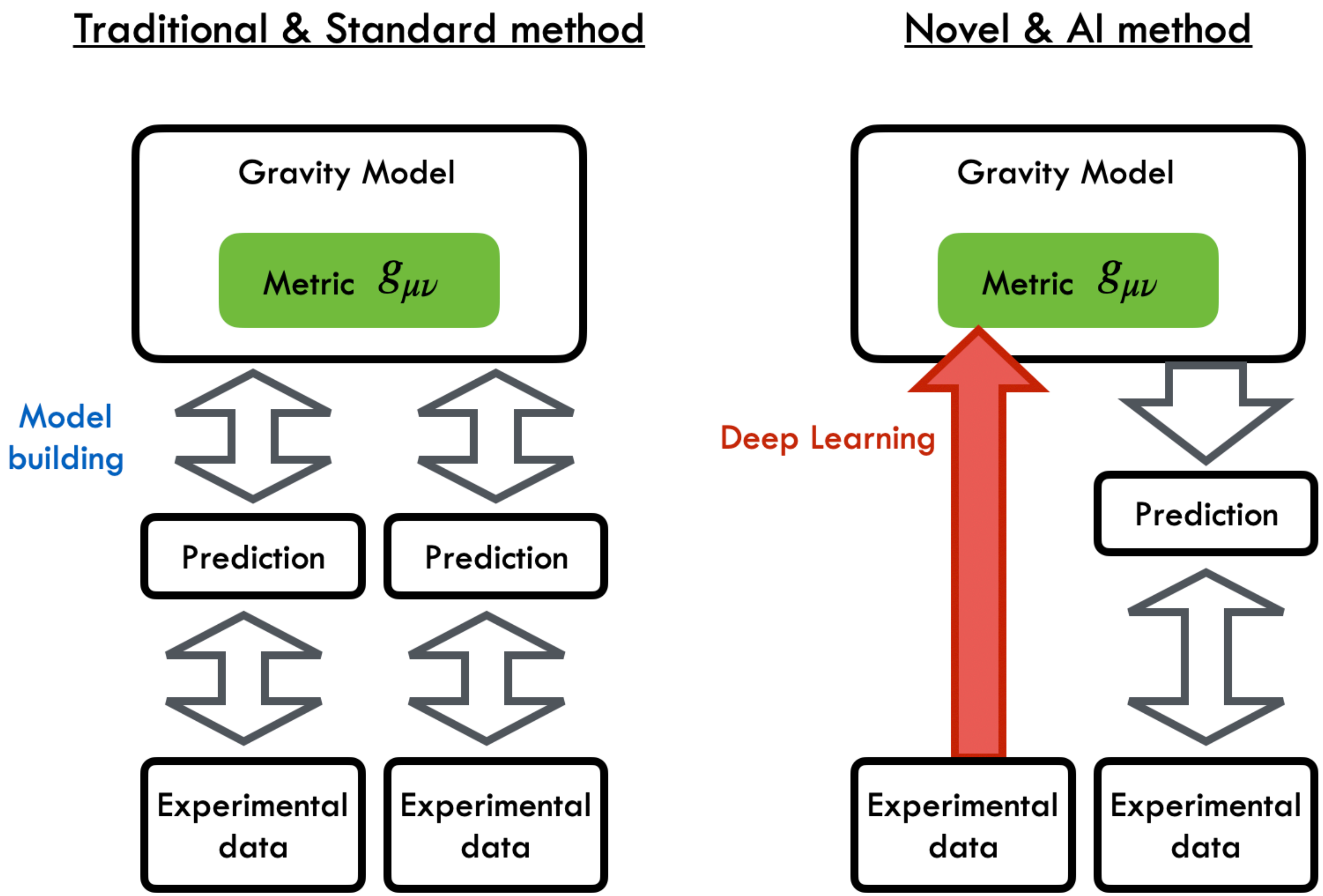

ML Questions:

Can we understand
the extra (holographic) dimension
as a deep neural network?

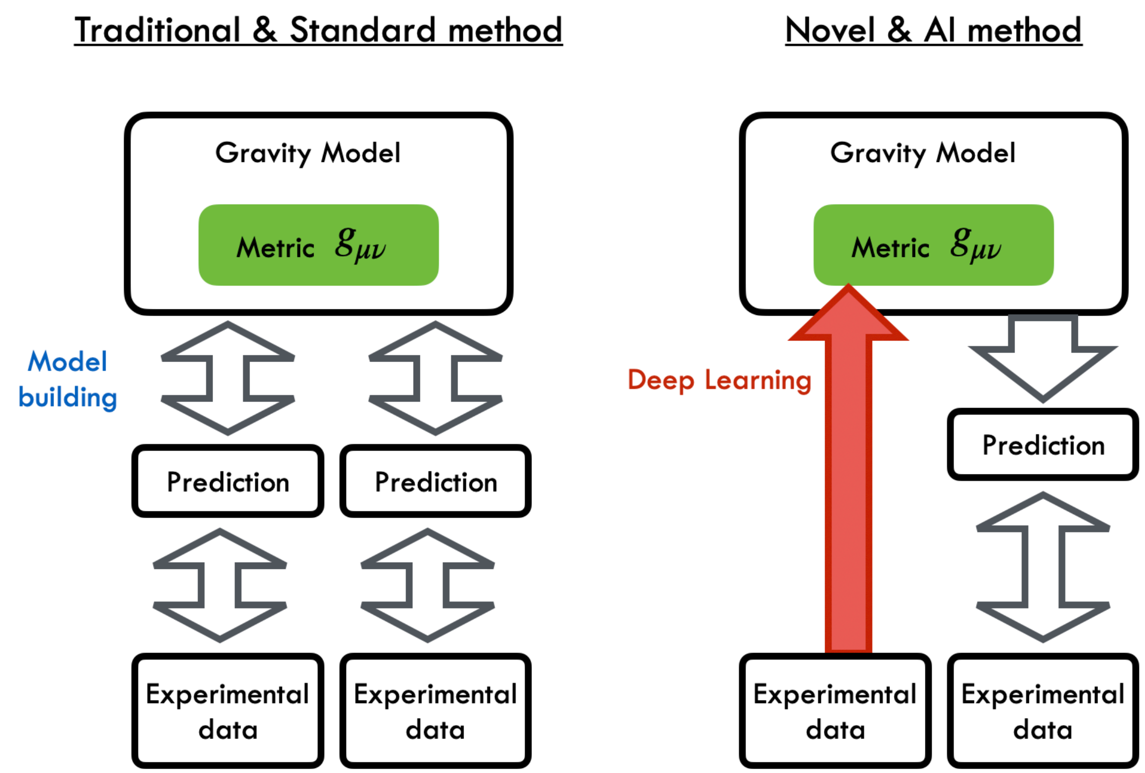
Can we use a deep neural network
as a useful tool for holography?

Answer:
Positive for both

# Deep Learning as a methodology



# Deep Learning as a methodology




Traditional method: from Bulk to Boundary

- Intuition, principle (ex: symmetry), "genius" etc required to make a model
- From a model, data are produced

AI method: from boundary to bulk

- Big data required
- Model yielding the answer is given by machine without any understanding
- Intuition, principle (ex: symmetry), etc implied by the model will be discovered by human

For a difficult problem,
once we are given a qualitative answer we can understand it more easily.
(for example, model of "T-linear resistivity + $T^2$ -Hall angle together" )

# My own three motivations to study machine learning

- Surprised by Machine
- Physics motivation
- Quantum computing, brain and human, etc

2016 March


Why ML?

Surprisingly, there are still many new ways to play Go!

Likewise, machines may reveal unexpected new ways of understanding nature.

2016 March


Why ML?

Surprisingly, there are still many new ways to play Go!

Likewise, machines may reveal unexpected new ways of understanding nature.

Amazing

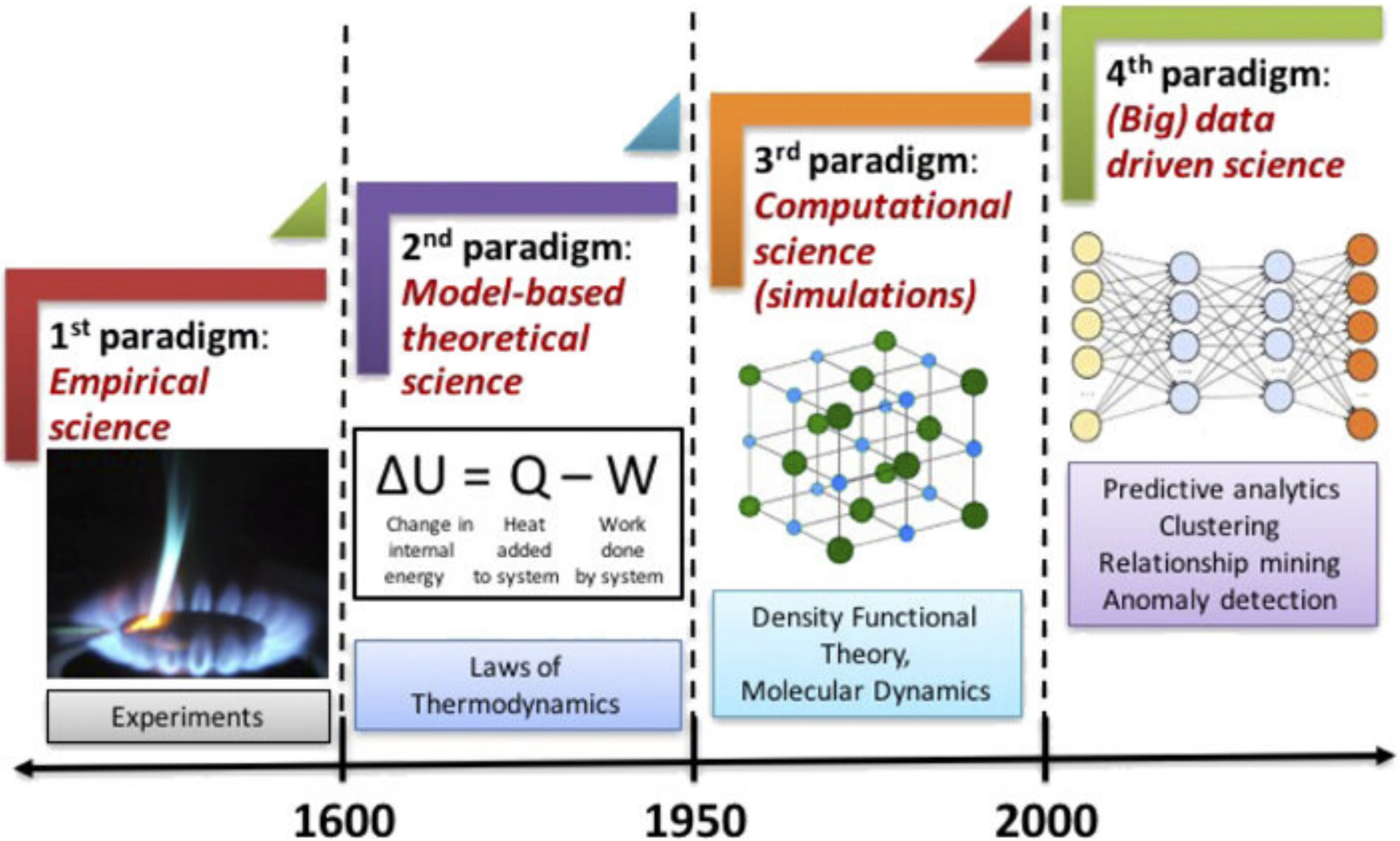
ChatGPT

## The status of ML as a “general” research tool

It's time to use machine learning as a toolbox.
We use Mathematica without fully understanding how it works.
We don't feel guilty using Mathematica,
so using machine learning isn't cheating either.

Knowing the answer (assisted by machines)
will be helpful for real understanding.

Time to collaborate with Machine.




Stolen from Xun Chen's talk yesterday

Physics Reports 839 (2020) 1–117

Contents lists available at ScienceDirect

Physics Reports

journal homepage: www.elsevier.com/locate/physrep

# Data science applications to string theory

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# Machine learning and the physical sciences

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## CONTENTS

MLPhys 学術変革領域研究(A) 学習物理学の創成
Foundation of "Machine Learning Physics"


Announcement | Registration | Program | Info

## Past conferences

String Data 2023

String Data 2022

String Data 2021

String Data 2020

String Data 2019

String Data 2018

String Data 2017

## Invited spearkers

Gabriel Lopes Cardoso (Lisbon, IST)

François Charton (META AI)

Sergei Gukov (Caltech)

James Halverson (Northeastern University)

Song He (Jilin University / Max Planck Institute Potsdam)

Edward Hirst (Queen Mary, University of London)

Vishunu Jejjala (University of the Witwatersrand in Johannesburg)

Hyun-Sik Jeong (Institute for Theoretical Physics UAM-CSIC in Madrid)

Keun-Young Kim (GIST)

Sven Krippendorf (Arnold Sommerfeld Center for Theoretical Physics, LMU Munich)

Anindita Maiti (Perimeter Institute)

Fabian Ruehle (Northeastern University)

Matthew Schwartz (Harvard University)

Rak-Kyeong Seong (UNIST)

Eva Silverstein (Stanford)

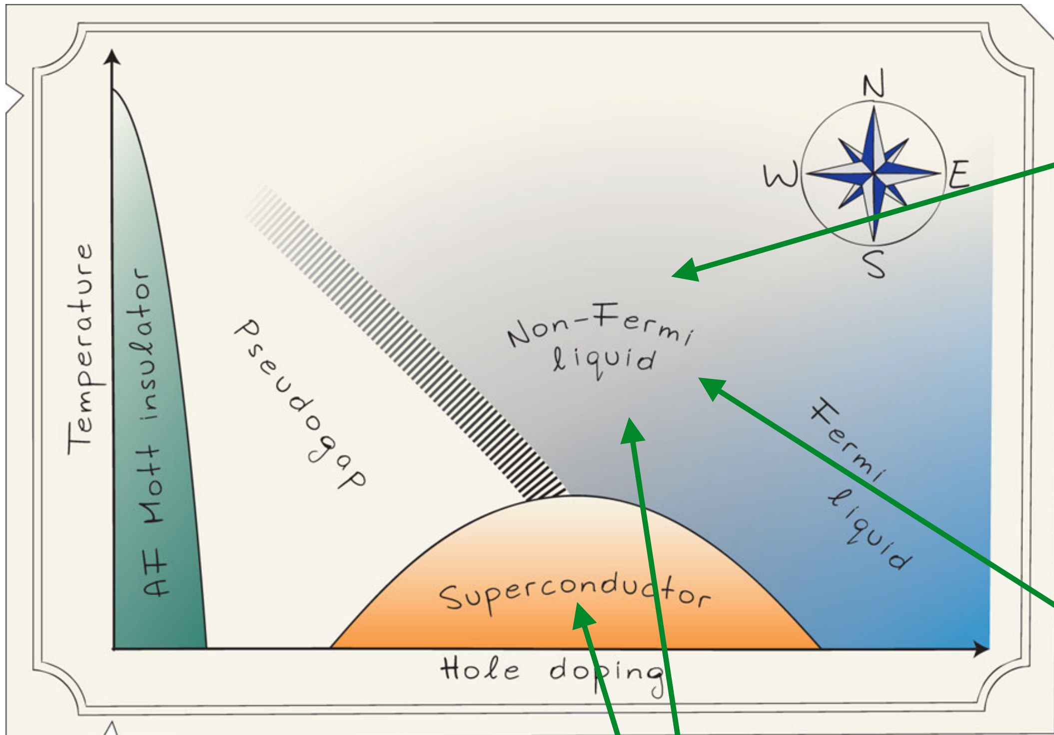
(More to be confirmed)

# My own three motivations to study machine learning

- Surprised by Machine
- Physics motivation
- Quantum computing, brain and human, etc

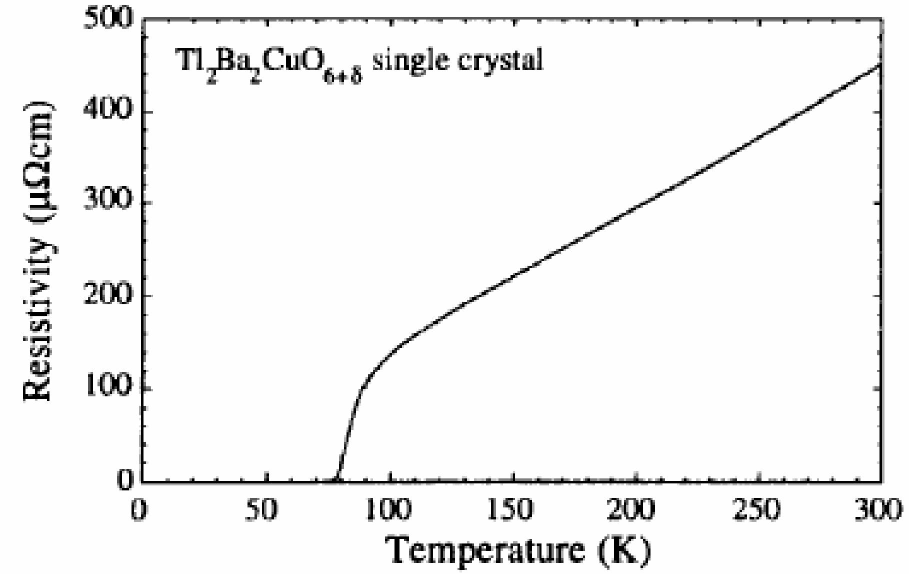
# Some universal properties in CMT

Cuprate phase diagram

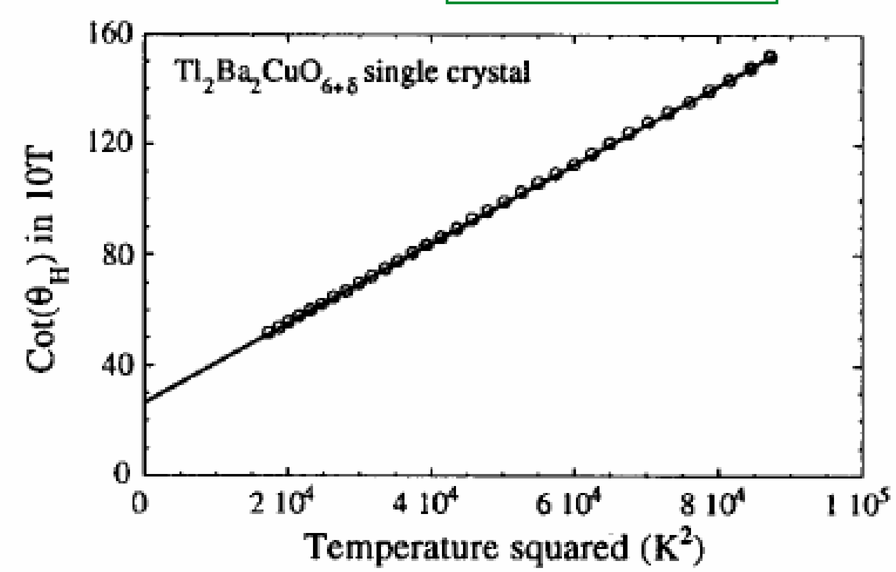



[Peter Wahl: 2012, Nature Physics]

- DC resistivity $\boxed{\rho \sim T}$

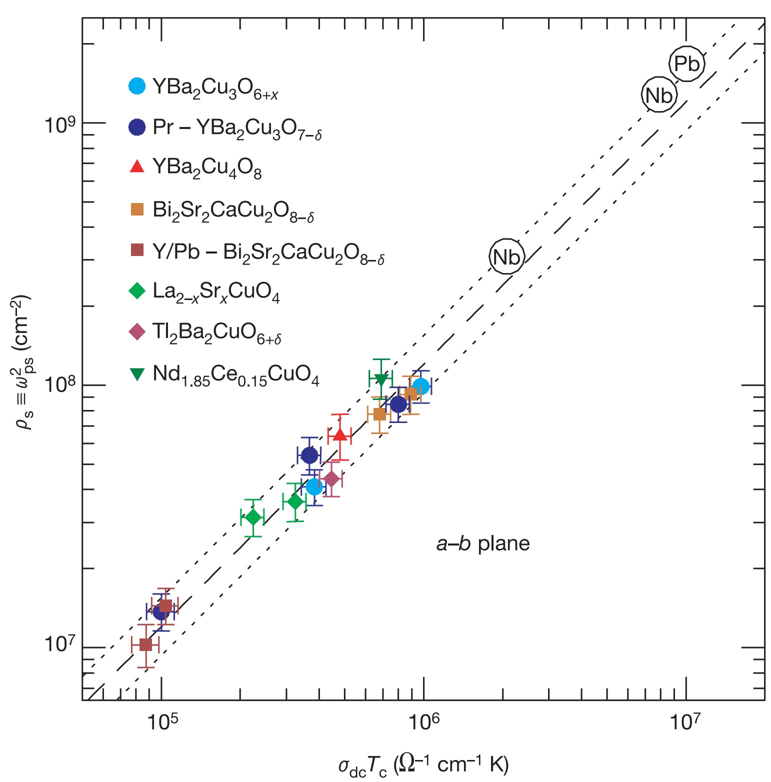



- Hall angle $\boxed{\sigma_{xx}/\sigma_{xy} \sim T^2}$




Mackenzie, 1997

- Homes law $\boxed{\rho_s(T=0) = C\sigma_{DC}(T_c)T_c}$



# Some universal properties in CMT

Science

Current Issue

HOME > SCIENCE > VOL. 377, NO. 6602 > STRANGER THAN METALS

REVIEW | PHYSICS

## Stranger than metals

PHILIP W. PHILLIPS, NIGEL E. HUSSEY, AND PETER ABBAMONTE Authors Info & Affiliations

*SCIENCE* • 8 Jul 2022 • Vol 377, Issue 6602 • DOI: 10.1126/science.abh4273

# Some universal properties in CMT

| | $\rho \propto T$ as $T \to \infty$ | $\rho \propto T$ as $T \to 0$ | Extended criticality | $\cot\Theta_H \propto T^2$ (at low $H$) | Modified Kohler's (at low $H$) | $H$-linear MR (at high $H$) | Quadrature MR |
|---|---|---|---|---|---|---|---|
| UD $p$-cuprates | ✓ [6] | × [20] | × [21] | ✓ [22] | ✓ [23] | - | - |
| OP $p$-cuprates | ✓ [4] | - | - | ✓ [24] | ✓ [25] | ✓ [26] | × [27] |
| OD $p$-cuprates | ✓ [6] | ✓ [8] | ✓ [8] | ✓ [28] | × [29] | ✓ [29] | ✓ [29] |
| $La_{2-x}Ce_xCuO_4$ | × [30] | ✓ [31] | ✓ [31] | × [32] | × [33] | ✓ [34] | × [34] |
| $Sr_2RuO_4$ | ✓ [35] | × [36] | × [37] | × [38] | × [36] | × [36] | × [36] |
| $Sr_3Ru_2O_7$ | ✓ [10] | ✓ [10] | × [10] | × | - | - | - |
| $FeSe_{1-x}S_x$ | × [39] | ✓ [40] | × [40] | ✓ [41] | ✓ [41] | ✓* [42] | ✓* [42] |
| $BaFe_2(As_{1-x}P_x)_2$ | × [43] | ✓ [44] | × [44] | - | ✓ [45] | ✓ [46] | ✓ [46] |
| $Ba(Fe_{1/3}Co_{1/3}Ni_{1/3})_2As_2$ | - | ✓ [47] | × [47] | - | - | ✓ [47] | ✓ [47] |
| $YbRh_2Si_2$ | × [48] | ✓ [49] | ✓ [50] | ✓ [51] | - | - | - |
| $YbBAl_4$ | × [52] | ✓** [52] | ✓** [52] | - | - | - | - |
| $CeCoIn_5$ | × [53] | ✓ [54] | × [54] | ✓ [53] | ✓ [53] | - | - |
| $CeRh_6Ge_4$ | × [55] | ✓ [55] | × [55] | - | - | - | - |
| $(TMTSF)_2PF_6$ | - | ✓ [56] | ✓ [56] | - | - | - | - |
| MATBG | ✓ [57] | ✓ [58] | ✓ [58] | ✓ [59] | - | - | - |

a

| | $\rho \propto T$ as $T \to 0$ | $\rho \propto T$ as $T \to \infty$ | $\sigma \propto \omega^{-2/3}$ | Quadrature MR | Extended criticality | Experimental Prediction |
|---|---|---|---|---|---|---|
| Phenomenological | | | | | | |
| MFL | ✓ [65] | × [65] | × | × | × | loop currents [104] |
| EFL | - [b] | - | - | × | × | loop currents [105] |
| Numerical | | | | | | |
| ECFL | × | ✓ [106] | - | - | × | × |
| HM (QMC/ED/CA) | - [107] | ✓ [107–111] | × | - | - | - |
| DMFT/EDMFT | ✓ [112] | ✓ [113, 114] | × | - | ✓ [114] | - |
| QCP | ✓ [115] | - | - | - | × | - |
| Gravity-based | | | | | | |
| SYK | ✓ [116, 117] | ✓[c] [117] | × | ✓[d] [118] | - | × |
| AdS/CFT | ✓ [119] | ✓ [119] | ✓[e] [88, 123] | × | × | × |
| AD/EMD | ✓ [124–126] | ✓ [88, 123, 124, 126, 127] | ✓ [88, 123, 127] | × | ✓ [123] | Fractional A-B [126] |

No concrete holography model of "T-linear resistivity + $T^2$ -Hall angle together" yet, even though there are many interesting holography models partly successful?

# ML Example: Holographic model

EMD(Einstein Maxwell Dilaton) model [ArXiv:1005.4690][hep-th], [ArXiv:1401.5436][hep-th]

$$S = \int \mathrm{d}^{p+1}x \sqrt{-g}\left[R - \frac{1}{2}\partial\phi^2 - \frac{1}{4}Z(\phi)F^2 + V(\phi) - \frac{1}{2}Y(\phi)\sum_{i=1}^{p-1}\partial\psi_i^2\right].$$

$\longrightarrow$ Many variations

$$Z(\phi) \sim e^{\gamma\phi}, \qquad V(\phi) \sim V_0 e^{-\delta\phi}, \qquad Y(\phi) \sim e^{\lambda\phi}$$

$$\mathrm{d}s^2 = r^{\frac{2\theta}{p-1}}\left[-f(r)\frac{\mathrm{d}t^2}{r^{2z}} + \frac{L^2\mathrm{d}r^2}{r^2 f(r)} + \frac{\mathrm{d}\vec{x}^2}{r^2}\right], \qquad A = Qr^{\zeta-z}\mathrm{d}t\,, \qquad \phi = \kappa \ln r$$

**Scientific Organizers**

Matteo Baggioli (SJTU), Rong-Gen Cai (ITP, CAS and NBU), Johanna Erdmenger (JMU), Xian-Hui Ge (SHU), Elias Kiritsis (APC, Paris and Crete U.), Li Li (ITP, CAS), Wei-Jia Li (DUT), Ya-Wen Sun (UCAS)

+ many colleagues here

# ML Example: Holographic model

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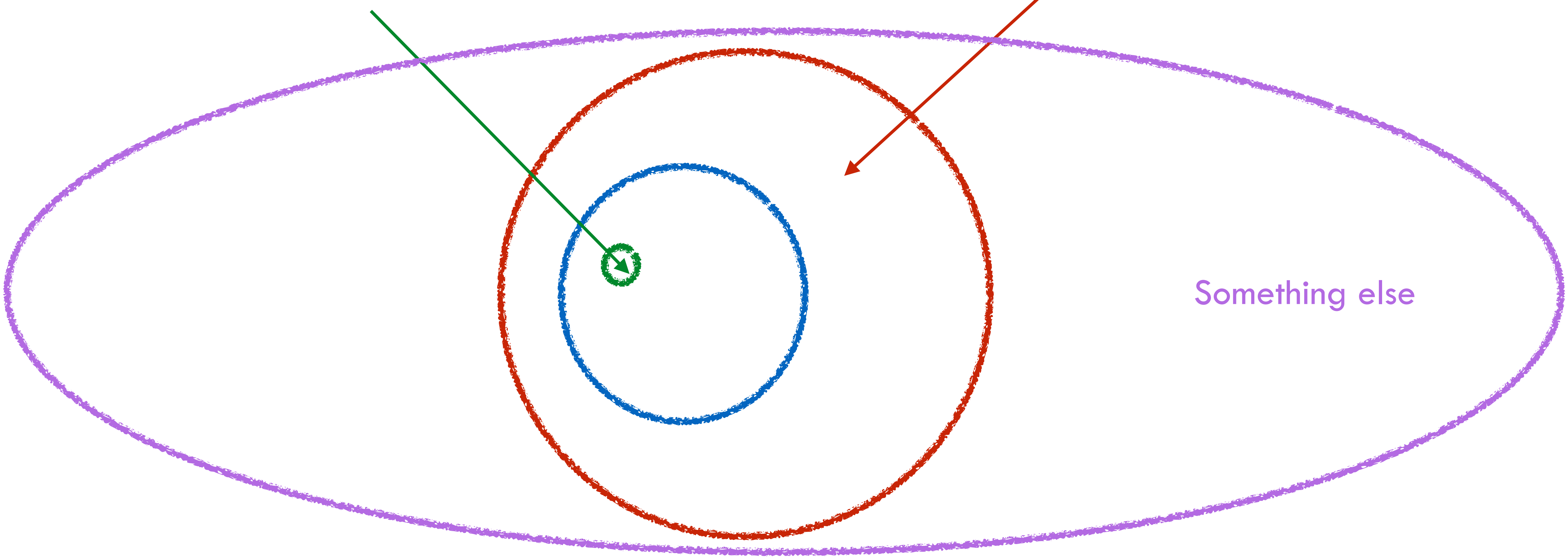

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Many variations

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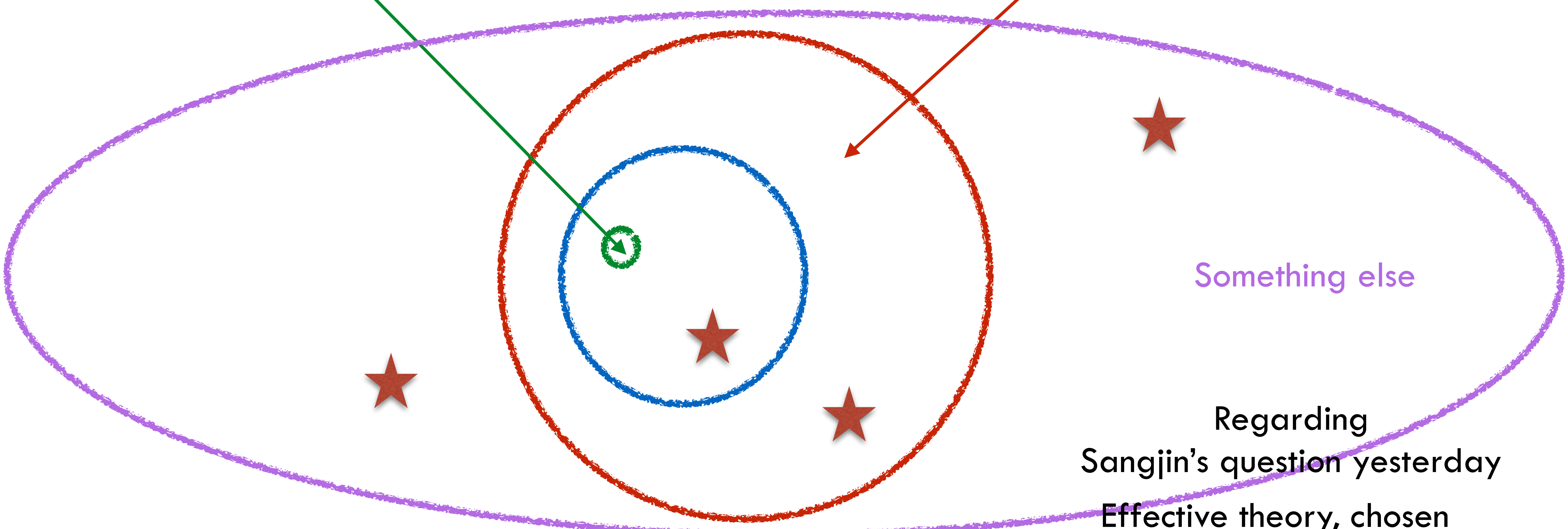
$$\mathrm{d}s^2 = r^{\frac{2\theta}{p-1}} \left[ -f(r)\frac{\mathrm{d}t^2}{r^{2z}} + \frac{L^2\mathrm{d}r^2}{r^2 f(r)} + \frac{\mathrm{d}\vec{x}^2}{r^2} \right], \quad A = Q r^{\zeta - z}\mathrm{d}t\,, \quad \phi = \kappa \ln r$$




Regarding
Sangjin's question yesterday

Effective theory, chosen
by simplicity and other observables

# ML Example: Holographic model

EMD(Einstein Maxwell Dilaton) model

$$S = \int \mathrm{d}^{p+1}x\,\sqrt{-g}\left[R - \frac{1}{2}\partial\phi^2 - \frac{1}{4}Z(\phi)F^2 + V(\phi) - \frac{1}{2}Y(\phi)\sum_{i=1}^{p-1}\partial\psi_i^2\right].$$

Many variations

$$Z(\phi) \sim e^{\gamma\phi}, \qquad V(\phi) \sim V_0 e^{-\delta\phi}, \qquad Y(\phi) \sim e^{\lambda\phi}$$

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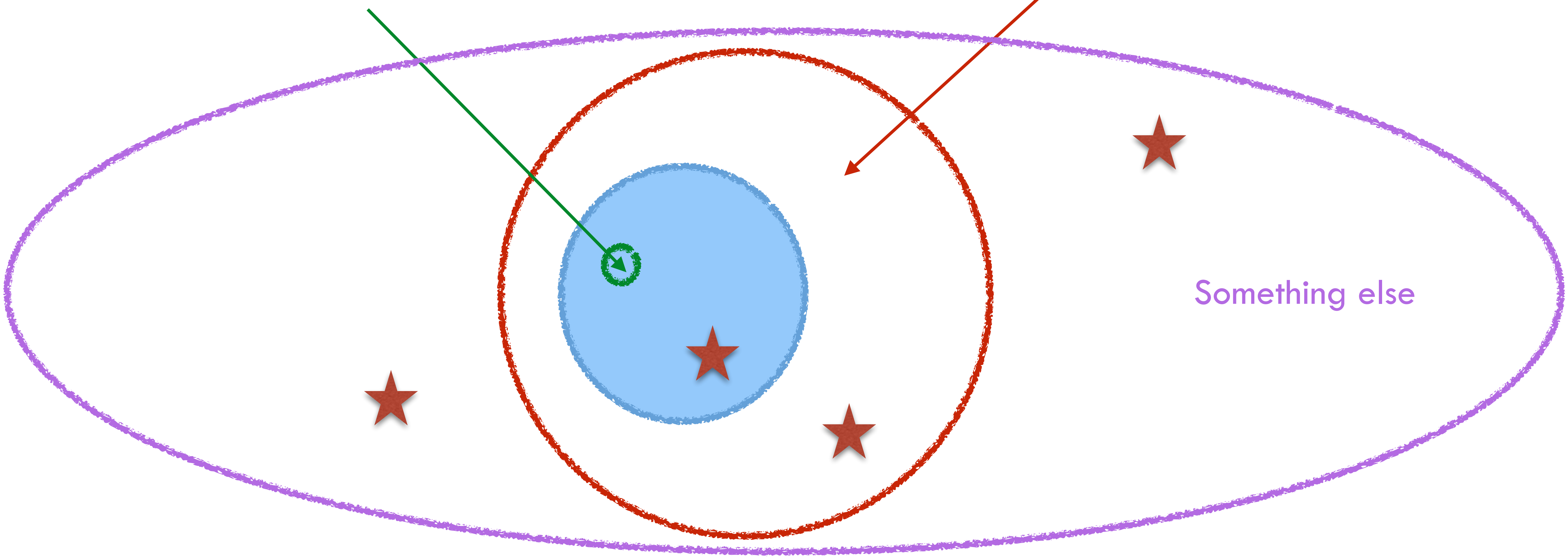

# My own three motivations to study machine learning

- Surprised by Machine
- Physics motivation
- Quantum computing, brain and human, etc

# What I have done for my goal

- Exercise 1: Metric from optical conductivity
- Exercise 2: Metric from entanglement entropy
- Problem classification and technique development in general

# Reference I

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## Deep learning bulk spacetime from boundary optical conductivity

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# AdS/Deep learning: optical conductivity

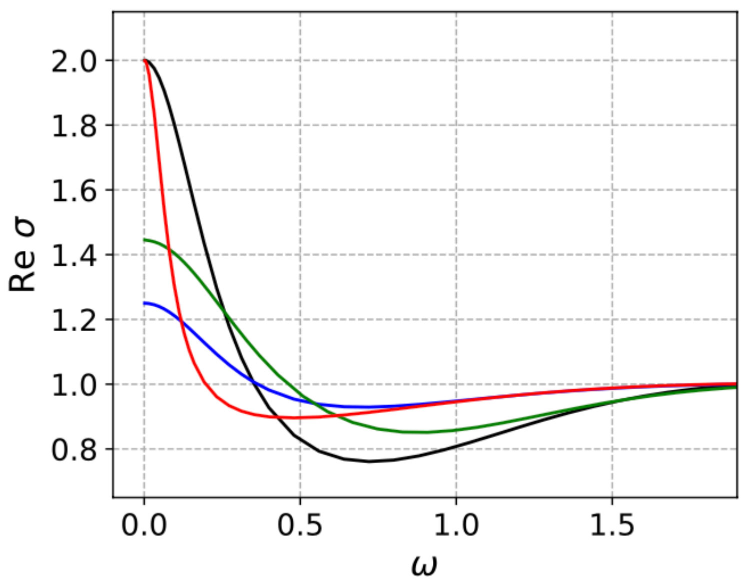
**Setup**

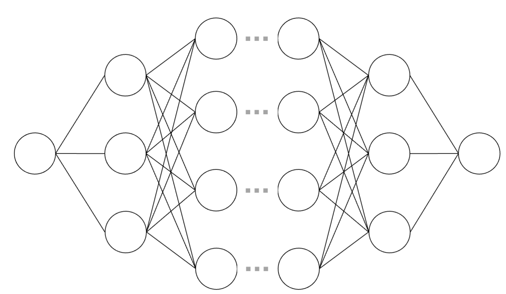
$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left(R + 6 - \frac{1}{4}F_{ab}F^{ab} - \frac{1}{2}\sum_{I=1}^{2}(\partial X_I)^2\right)$$

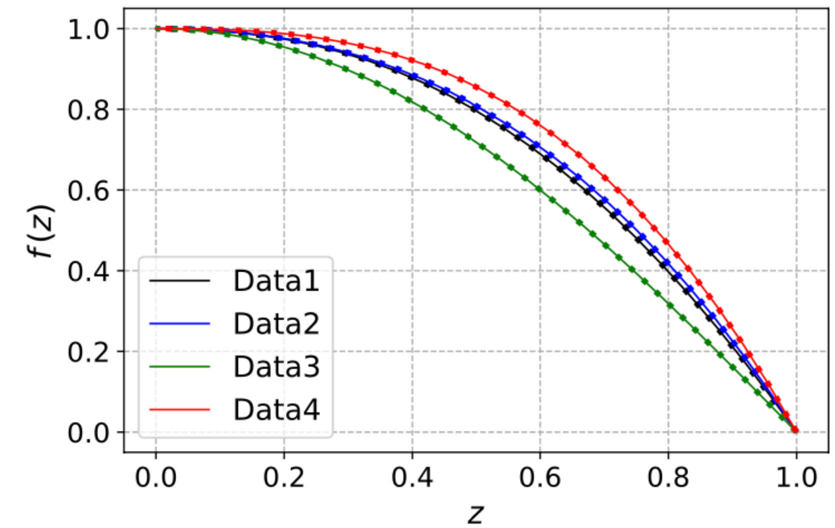
$$\mathrm{d}s^2 = \frac{1}{z^2}\left[-f(z)\mathrm{d}t^2 + \frac{\mathrm{d}z^2}{f(z)} + \mathrm{d}x^2 + \mathrm{d}y^2\right],$$

$$A = \mu\,(1-z)\,\mathrm{d}t\,, \qquad X_1 = \alpha\,x\,, \quad X_2 = \alpha\,y$$

What is the bulk metric giving the conductivity at boundary





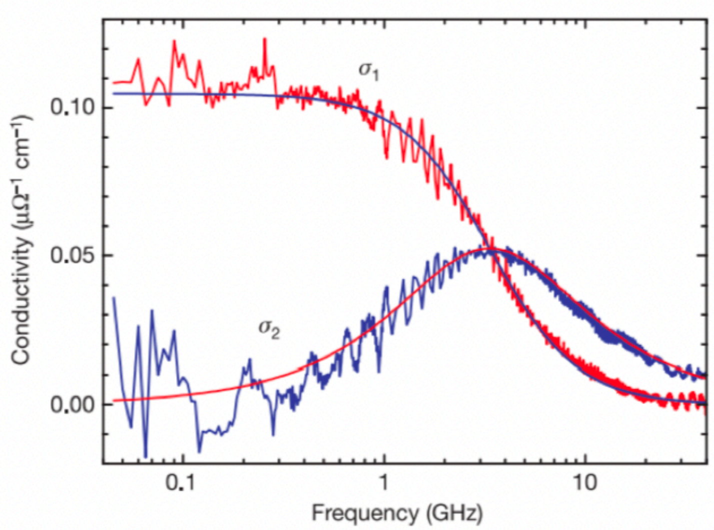
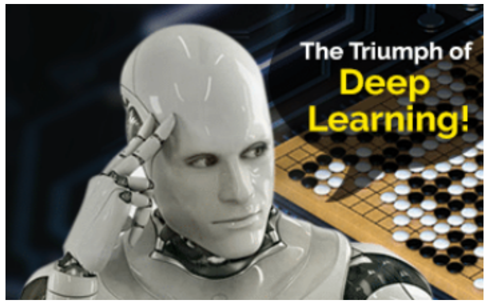
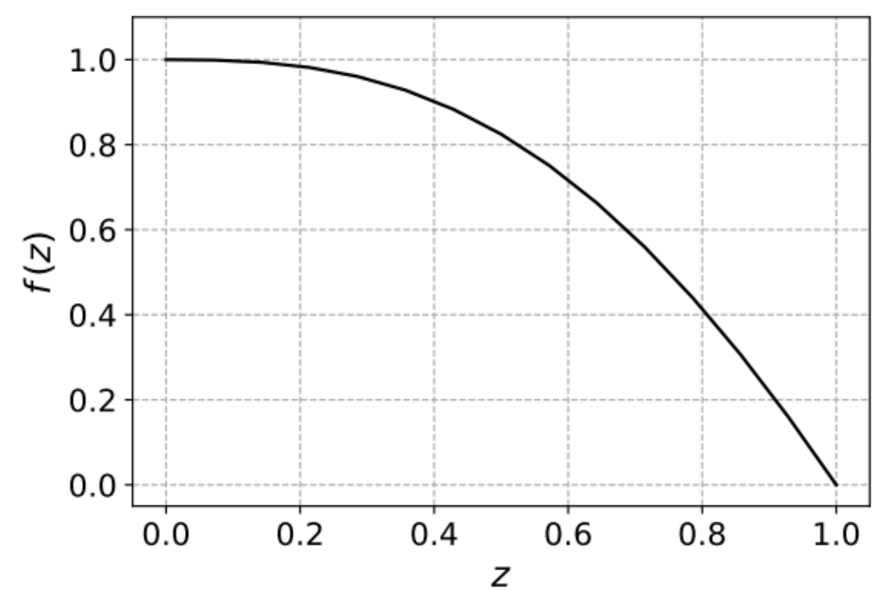




**Figure 2 | Conductivity spectrum of $UPd_2Al_3$ at temperature 2.75 K; both real and imaginary parts ($\sigma_1$ and $\sigma_2$, respectively) are shown.** The fit ($\sigma_1 + i\sigma_2 = \sigma_0(1 - i\omega\tau)^{-1}$, $\sigma_0 = 0.105\,\mu\Omega^{-1}\,\mathrm{cm}^{-1}$, $\tau = 4.8 \times 10^{-11}$ s) documents the excellent agreement of experimental data and the Drude prediction. The characteristic relaxation rate $1/\tau$ is marked by the decrease in $\sigma_1$ and the maximum in $\sigma_2$ around 3 GHz.



# What I have done for my goal

- Exercise 1: Metric from optical conductivity
- Exercise 2: Metric from entanglement entropy
- Problem classification and technique development in general

# Holographic reconstruction of black hole spacetime: machine learning and entanglement entropy

**Byoungjoon Ahn,**[a] **Hyun-Sik Jeong,**[b,c] **Keun-Young Kim**[a,d] **and Kwan Yun**[a]

[a] *Department of Physics and Photon Science, Gwangju Institute of Science and Technology, 123 Cheomdan-gwagiro, Gwangju 61005, Korea*

[b] *Instituto de Física Teórica UAM/CSIC, Calle Nicolás Cabrera 13-15, 28049 Madrid, Spain*

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[d] *Research Center for Photon Science Technology, Gwangju Institute of Science and Technology, 123 Cheomdan-gwagiro, Gwangju 61005, Korea*

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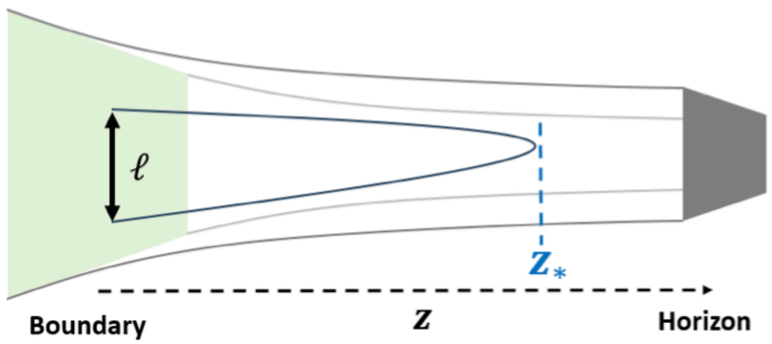
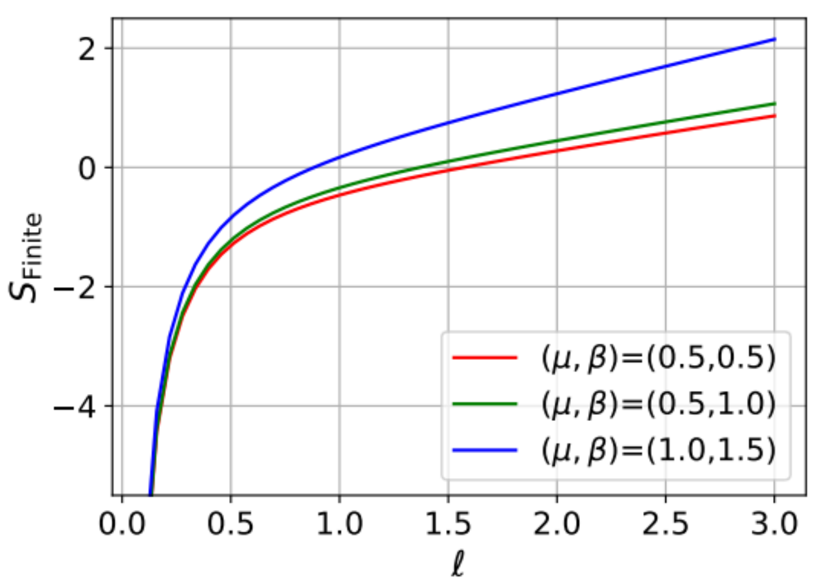
ArXiv:2404:07395 Accepted in JHEP

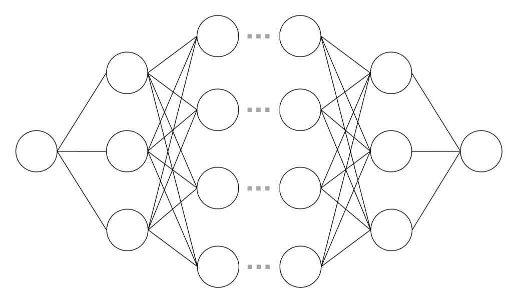
# AdS/Deep learning: entanglement entropy

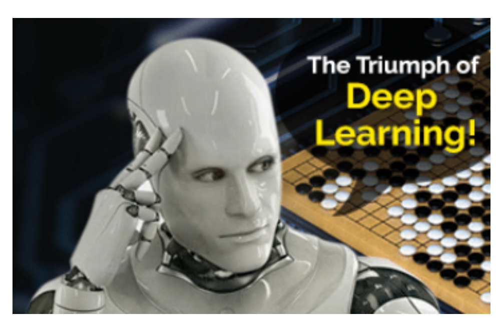
**Setup** Gubser-Rocca case

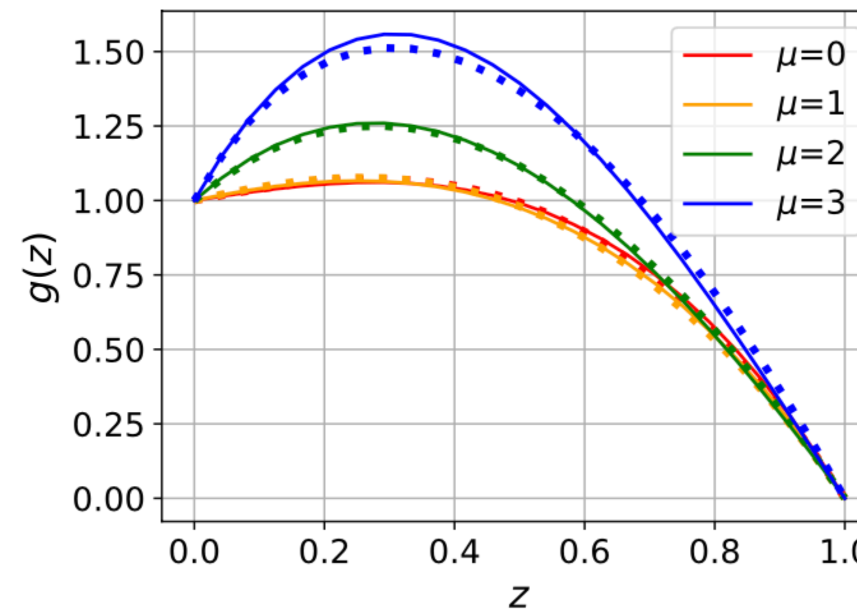
$$S=\int \mathrm{d}^4x\sqrt{-g}\left(R+6\cosh\phi-\frac{1}{4}e^{\phi}F^2-\frac{3}{2}(\partial\phi)^2-\frac{1}{2}\sum_{I=1}^{2}(\partial\psi_I)^2\right)\qquad ds^2=\frac{L^2}{z^2}\left[-g(z)dt^2+\frac{dz^2}{g(z)}+h(z)(dx^2+dy^2)\right]$$

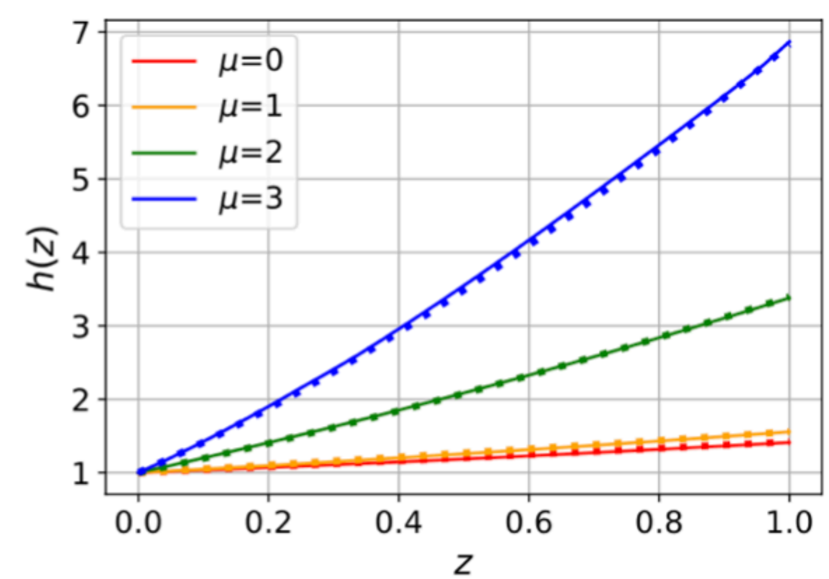




# What I have done for my goal

- Exercise 1: Metric from optical conductivity
- Exercise 2: Metric from entanglement entropy
- Problem classification and technique development in general

# Original Idea

PHYSICAL REVIEW D **98,** 046019 (2018)

## Deep learning and the AdS/CFT correspondence

Koji Hashimoto,[1] Sotaro Sugishita,[1] Akinori Tanaka,[2,3,4] and Akio Tomiya[5]
[1]*Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan*
[2]*Mathematical Science Team, RIKEN Center for Advanced Intelligence Project (AIP), 1-4-1 Nihonbashi, Chuo-ku, Tokyo 103-0027, Japan*
[3]*Department of Mathematics, Faculty of Science and Technology, Keio University, 3-14-1 Hiyoshi, Kouhoku-ku, Yokohama 223-8522, Japan*
[4]*Interdisciplinary Theoretical & Mathematical Sciences Program (iTHEMS) RIKEN 2-1, Hirosawa, Wako, Saitama 351-0198, Japan*
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(Received 18 March 2018; published 27 August 2018)

We present a deep neural network representation of the AdS/CFT correspondence, and demonstrate the emergence of the bulk metric function via the learning process for given data sets of response in boundary quantum field theories. The emergent radial direction of the bulk is identified with the depth of the layers, and the network itself is interpreted as a bulk geometry. Our network provides a data-driven holographic modeling of strongly coupled systems. With a scalar $\phi^4$ theory with unknown mass and coupling, in unknown curved spacetime with a black hole horizon, we demonstrate that our deep learning (DL) framework can determine the systems that fit given response data. First, we show that, from boundary data generated by the anti–de Sitter (AdS) Schwarzschild spacetime, our network can reproduce the metric. Second, we demonstrate that our network with experimental data as an input can determine the bulk metric, the mass and the quadratic coupling of the holographic model. As an example we use the experimental data of the magnetic response of the strongly correlated material $Sm_{0.6}Sr_{0.4}MnO_3$. This AdS/DL correspondence not only enables gravitational modeling of strongly correlated systems, but also sheds light on a hidden mechanism of the emerging space in both AdS and DL.

Chinese Physics C Vol. 45, No. 7 (2021) 073111

# AdS/Deep-Learning made easy: simple examples*

Mugeon Song[1,#†] Maverick S. H. Oh[1,2,#‡] Yongjun Ahn[1§] Keun-Young Kima[1♮]

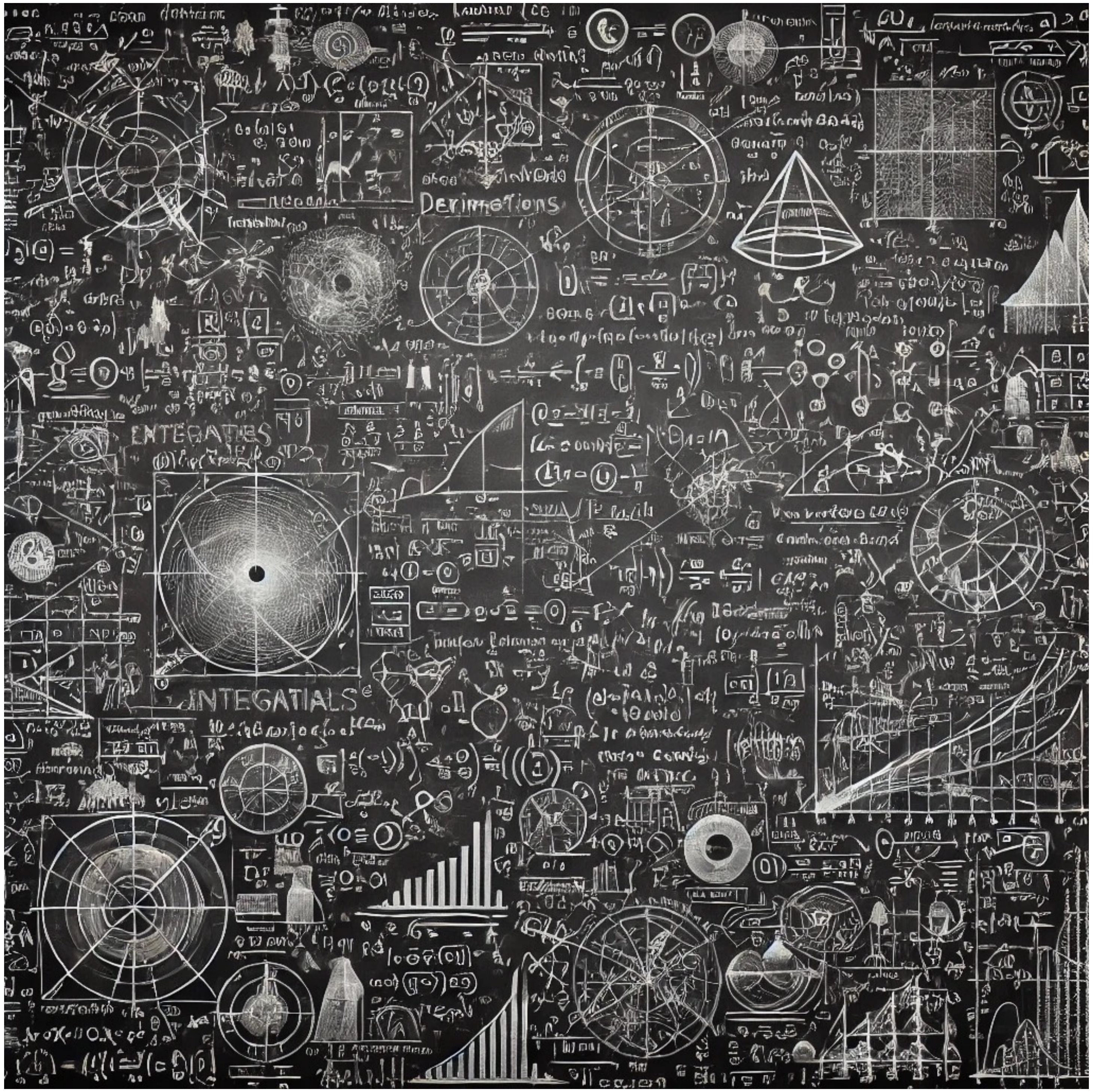
[1]Gwangju Institute of Science and Technology (GIST), Department of Physics and Photon Science, Gwangju, South Korea
[2]University of California–Merced, Department of Physics, Merced, CA, USA

**Abstract:** Deep learning has been widely and actively used in various research areas. Recently, in gauge/gravity duality, a new deep learning technique called AdS/DL (Deep Learning) has been proposed. The goal of this paper is to explain the essence of AdS/DL in the simplest possible setups, without resorting to knowledge of gauge/gravity duality. This perspective will be useful for various physics problems: from the emergent spacetime as a neural network to classical mechanics problems. For prototypical examples, we choose simple classical mechanics problems. This method is slightly different from standard deep learning techniques in the sense that we not only have the right final answers but also obtain physical understanding of learning parameters.

**Keywords:** gauge/gravity duality, holographic principle, machine learning

**DOI:** 10.1088/1674-1137/abfc36

# Problem classification



Complicated equations by "ChatGPT"

**Differential equation**

$$A(z)f''(z)+B(z)f'(z)+C(z)f(z) = D(z)g(z)$$

$$E(z)g''(z)+F(z)g'(z)+G(z)g(z) = H(z)f(z)$$

Ex) boundary transport
by bulk fluctuation equations

**Integral equation**

$$A(\alpha) = \int F[\, f_1(r;\alpha), f_2(r;\alpha), \cdots ]\, dr$$

$$B(\alpha) = \int G[\, f_1(r;\alpha), f_2(r;\alpha), \cdots ]\, dr$$

Ex) boundary entanglement entropy
by bulk metric

Inverse Problem
Optimization problem

Holography

# Deep Learning $\boxed{\text{Bulk Spacetime from Boundary data}}$

- What is "Machine Learning" & "Deep Learning"?

  - Deep Learning 101: standard story

- Physics equation related Deep Learning I

  - Deep Learning for ODE: classical mechanics

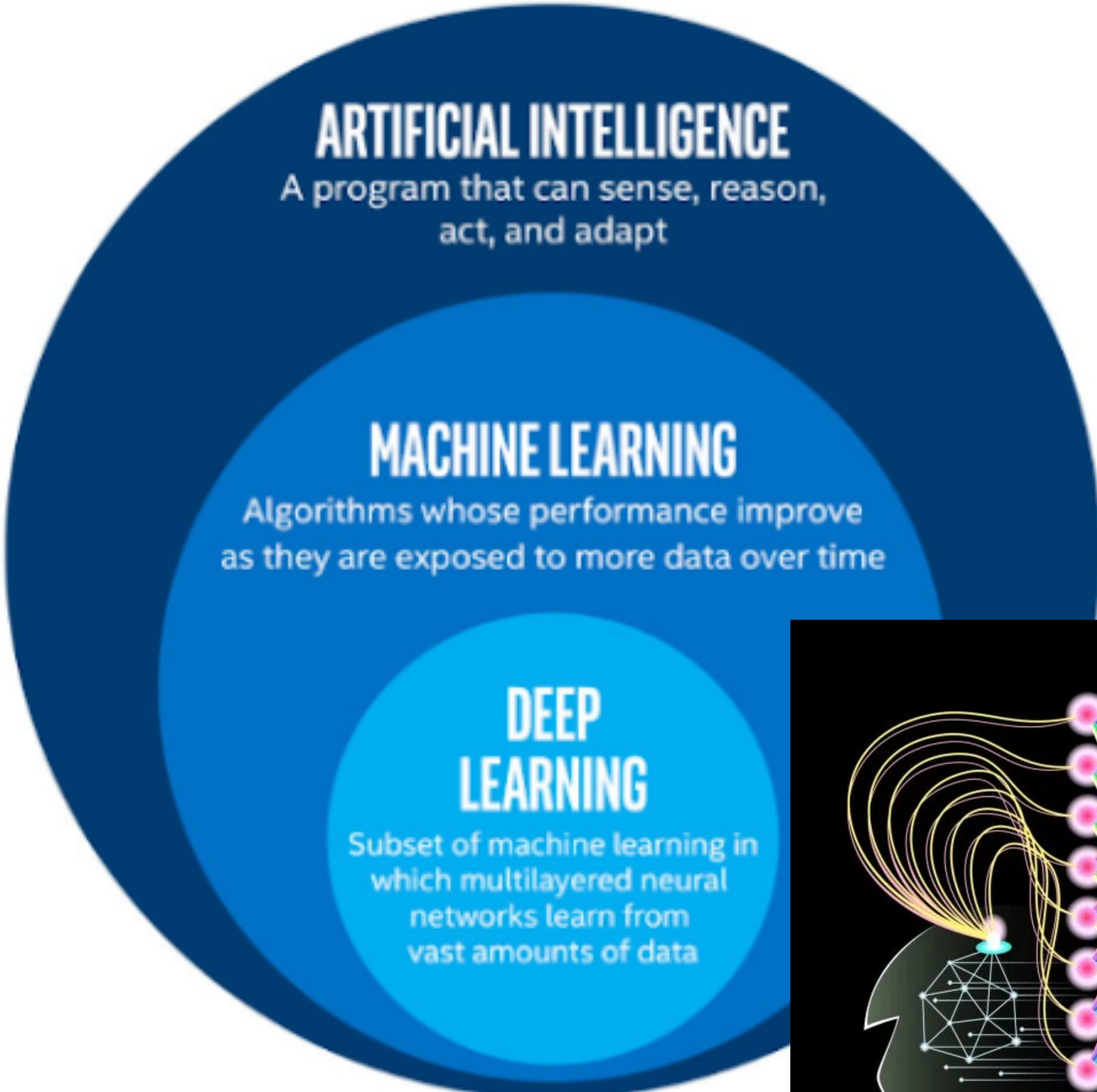
  - AdS/Deep Learning: optical conductivity

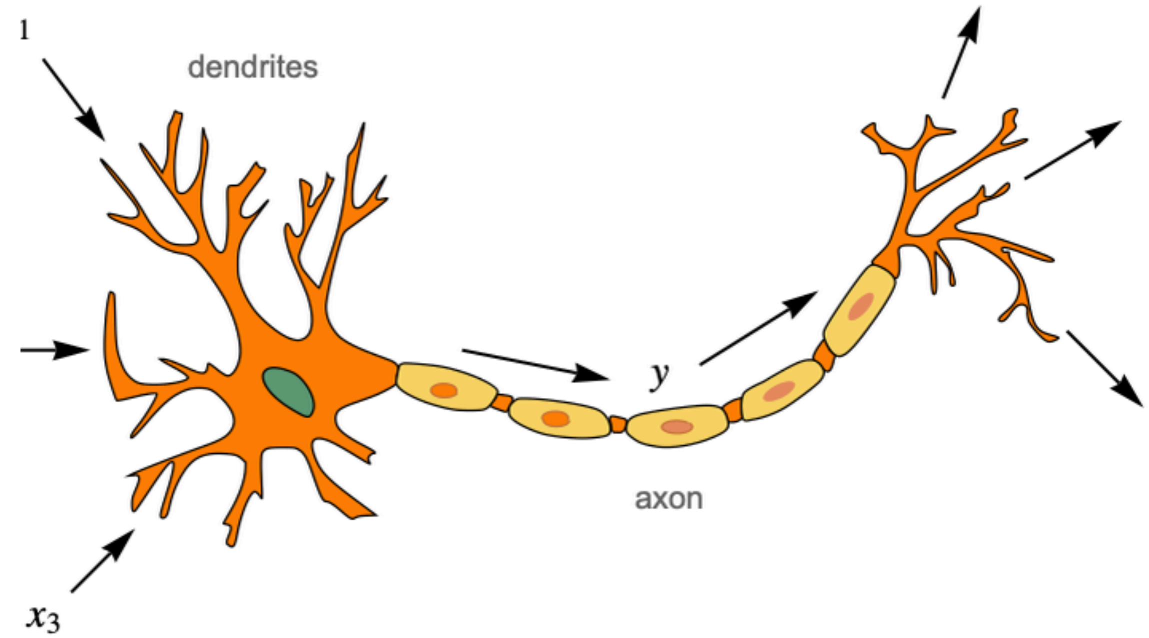
- Physics equation related Deep Learning II

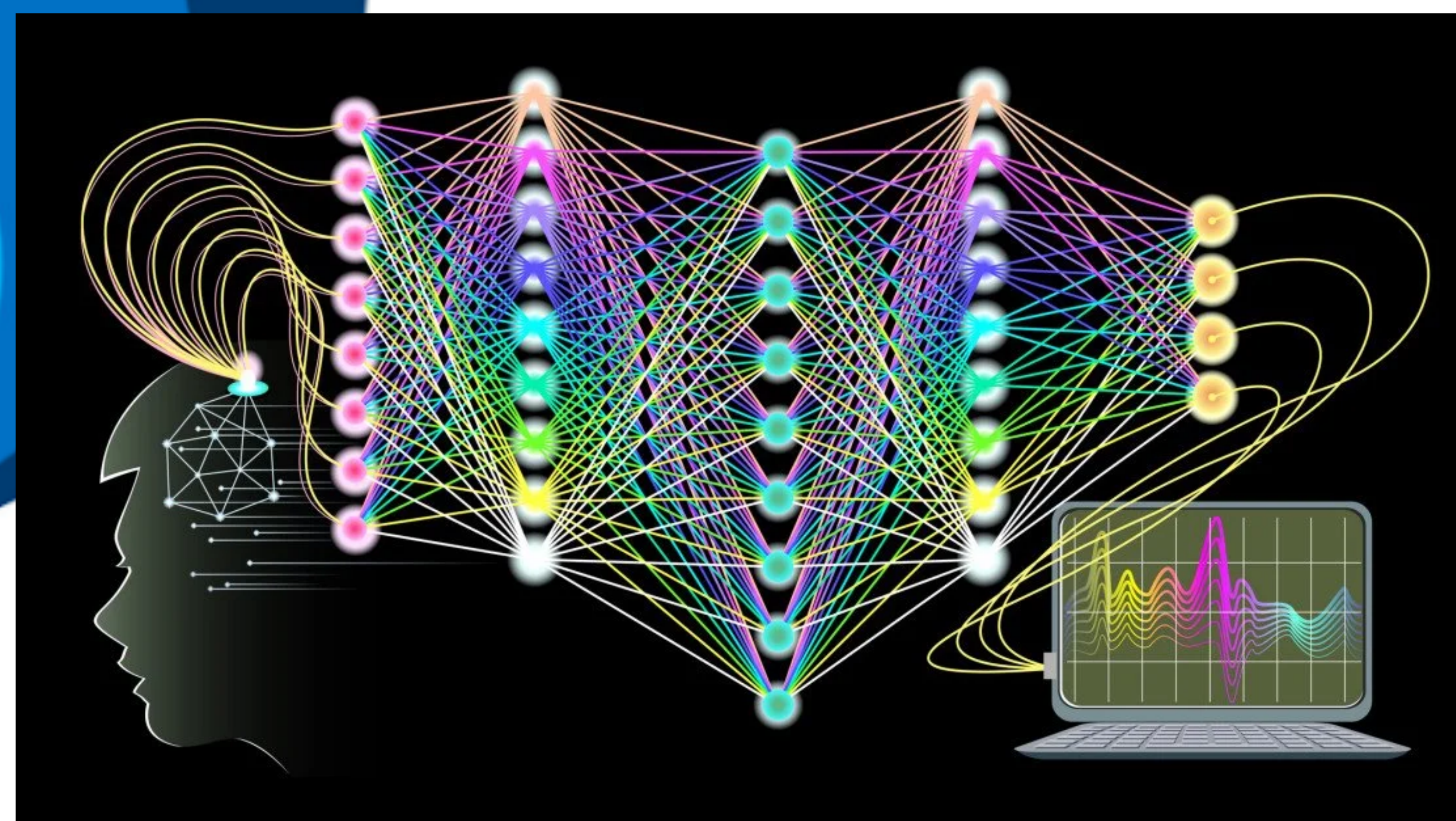
  - Deep Learning for Integral: classical electrostatics

  - AdS/Deep Learning: Entanglement entropy

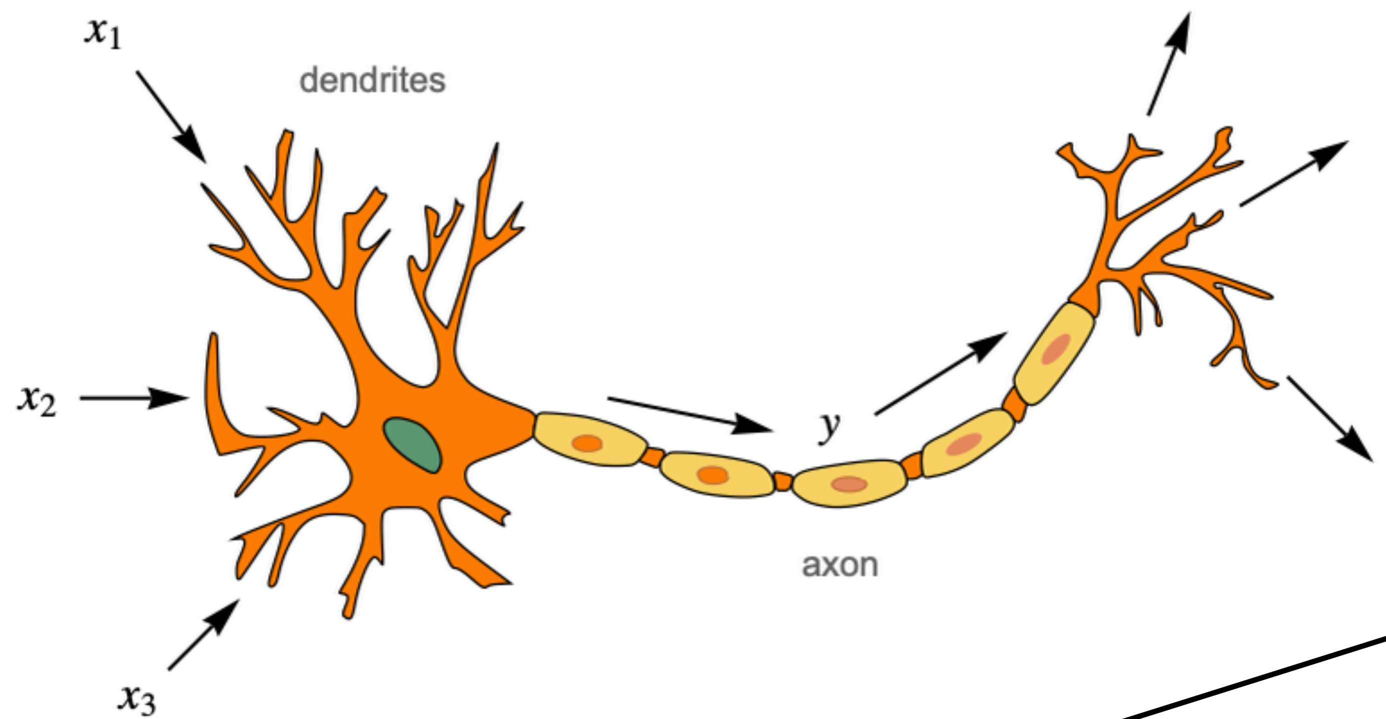
# Deep Learning 101

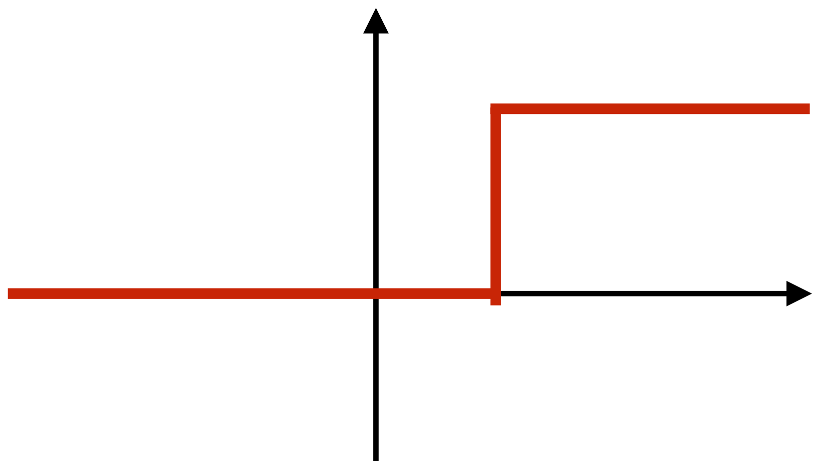


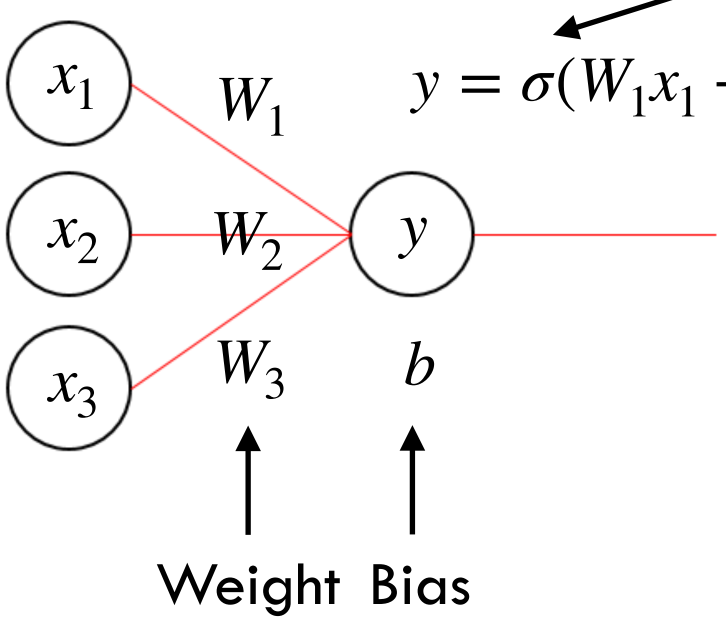
# Deep Learning 101

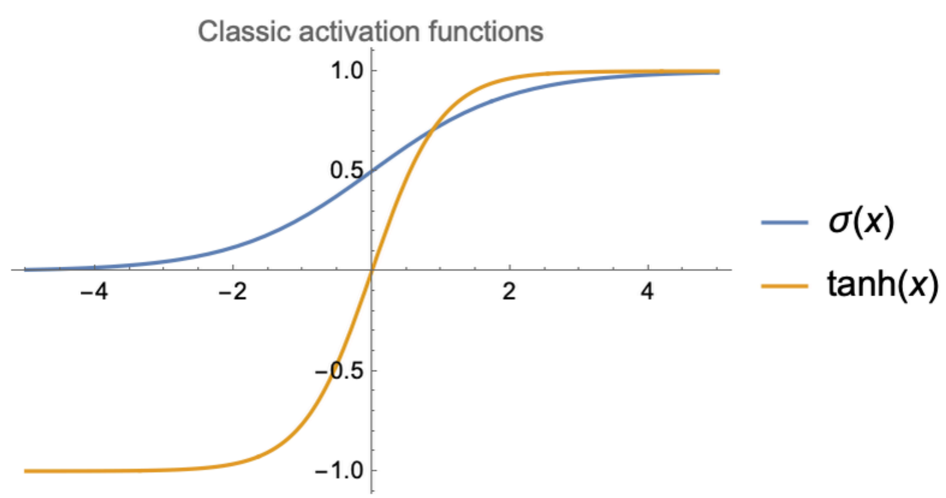


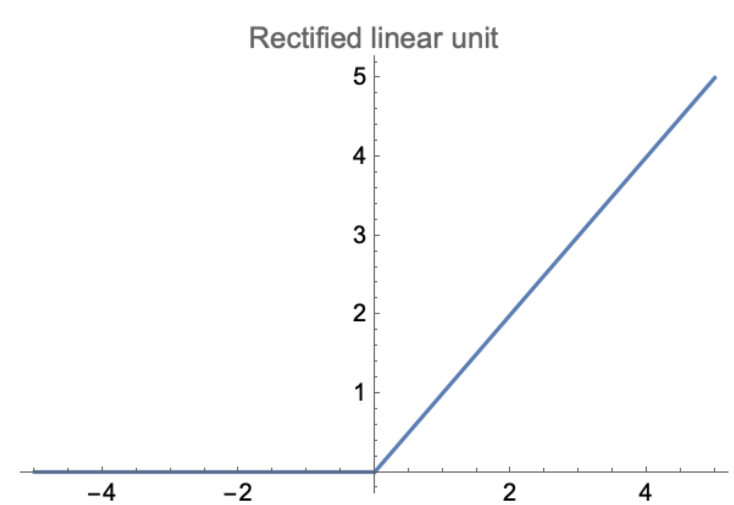



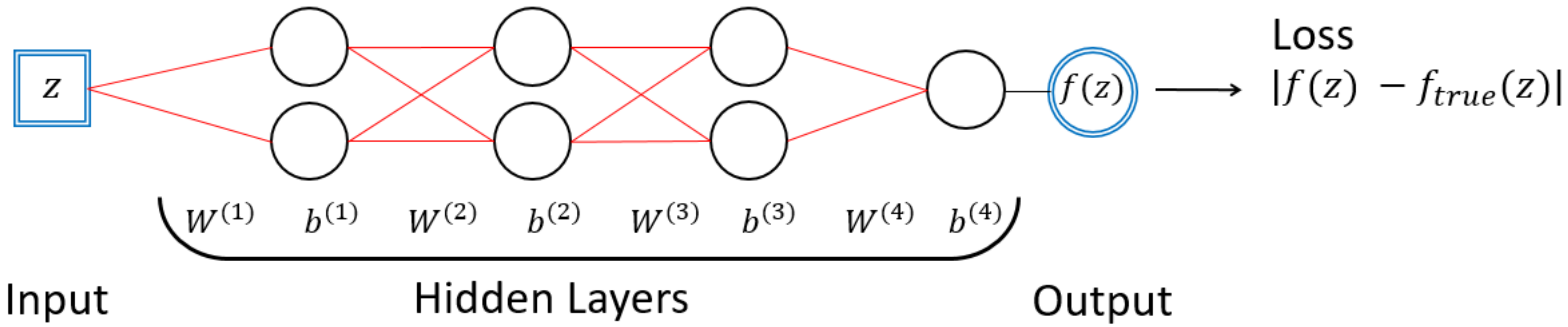
Activation function

$$y = \sigma(W_1 x_1 + W_2 x_2 + W_3 x_3 + b)$$



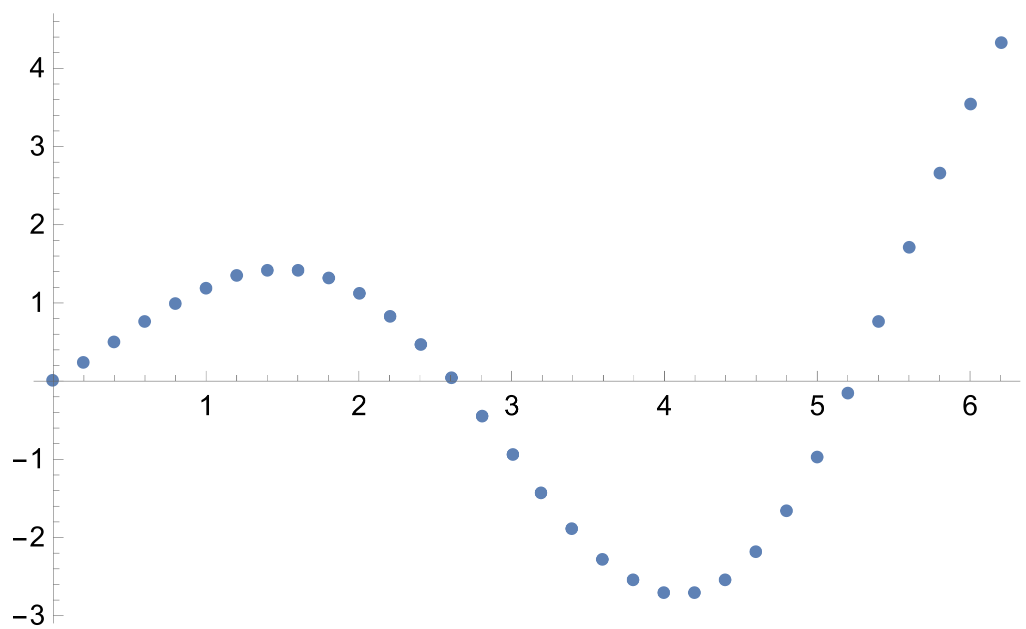



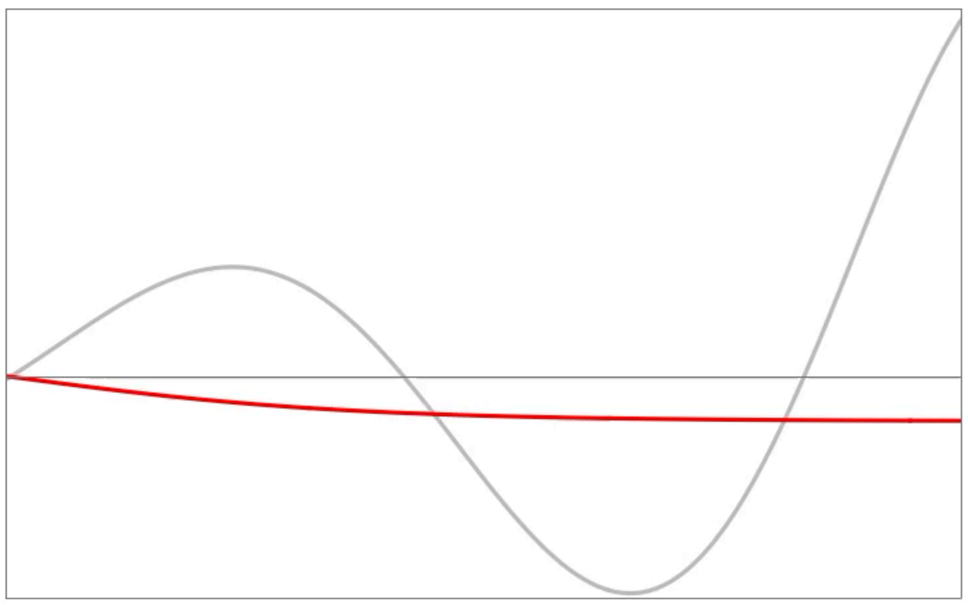





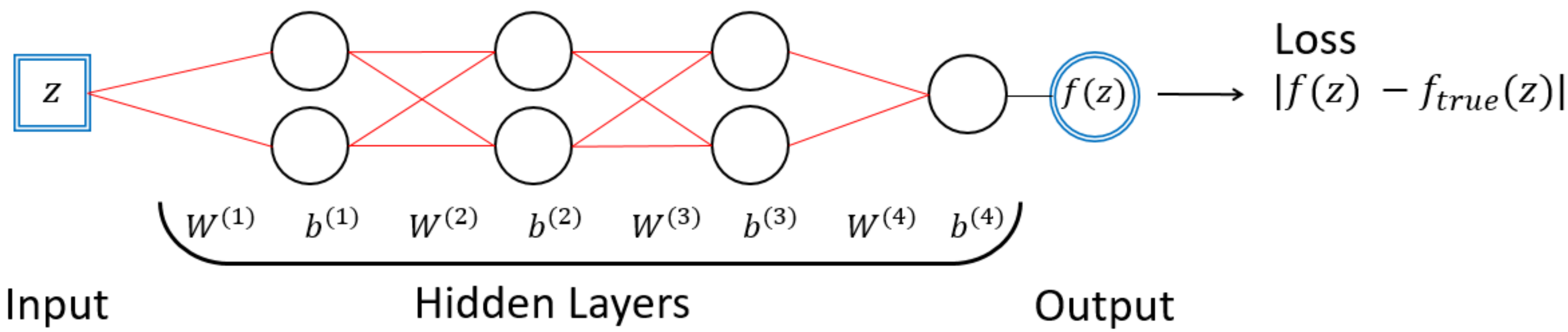

$$z_j^{(i+1)} = \sigma(W_{jk}^{(i)} \cdot z_k^{(i)} + b_j^{(i)})$$ ~2000 epoch

```
net = NetChain[{LinearLayer[2], Tanh, LinearLayer[2], Tanh, LinearLayer[2], Tanh, LinearLayer[]}]
      |네트워크 체인  |선형층            |쌍곡…  |선형층            |쌍곡…  |선형층            |쌍곡…  |선형층
solutions = NetTrain[net, data, <|"Property" → Function[#Net[Range[0, 2 π, 0.2]]], "Form" → "List",
            |네트워크 훈련                      |함수
   "Interval" → Quantity[30, "Rounds"]|>,  (*LearningRate→0.001,*) TimeGoal → 600];
                |수량                                              |작업 시간 지정
```



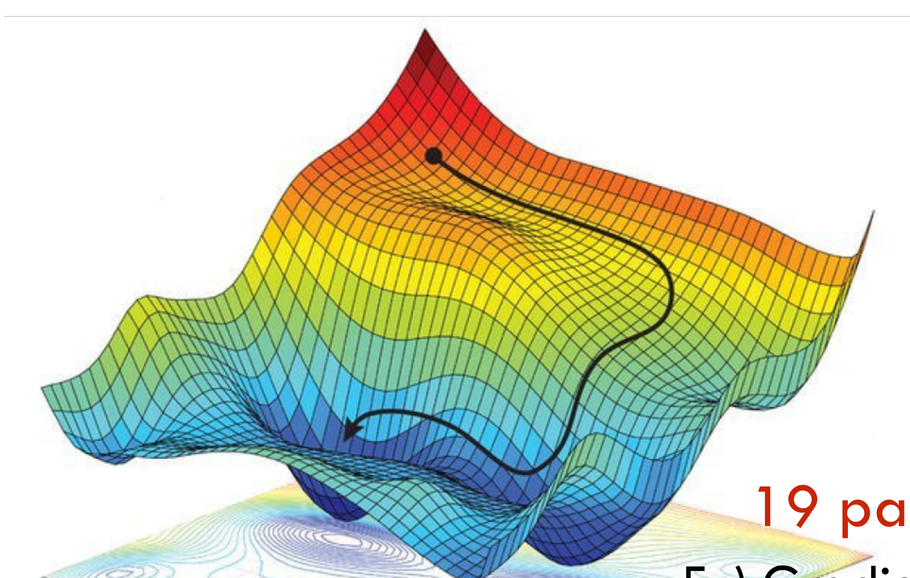
# Standard Deep Learning




$$z_j^{(i+1)} = \sigma(W_{jk}^{(i)} \cdot z_k^{(i)} + b_j^{(i)})$$ ~2000 epoch 19 parameters

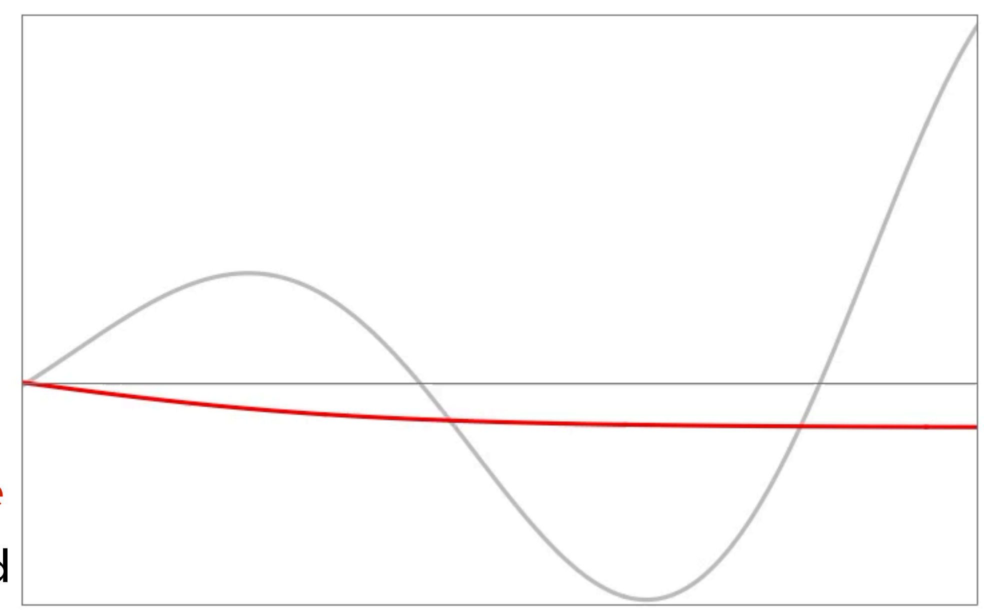
$$\tanh\left[\begin{pmatrix}\square & \square\end{pmatrix} \tanh\left[\begin{pmatrix}\square&\square\\\square&\square\end{pmatrix}\left(\tanh\left[\begin{pmatrix}\square&\square\\\square&\square\end{pmatrix}\left(\tanh\left[\begin{pmatrix}\square\\\square\end{pmatrix} z + \begin{pmatrix}\square\\\square\end{pmatrix}\right]\right) + \begin{pmatrix}\square\\\square\end{pmatrix}\right] + \begin{pmatrix}\square\\\square\end{pmatrix}\right)\right] + (\square)\right]$$

$$\begin{pmatrix}0.878421 & 6.21182\end{pmatrix}\begin{pmatrix}1.55725 & -2.14132\\-2.49736 & -1.0517\end{pmatrix}\begin{pmatrix}-1.21558 & -0.693247\\2.86862 & 3.01908\end{pmatrix}\begin{pmatrix}0.317991\\0.521212\end{pmatrix}\begin{pmatrix}-1.96854\\-0.020086\end{pmatrix}\begin{pmatrix}0.320358\\-0.614063\end{pmatrix}\begin{pmatrix}-0.442947\\0.854547\end{pmatrix}\begin{pmatrix}1.33224\end{pmatrix}$$

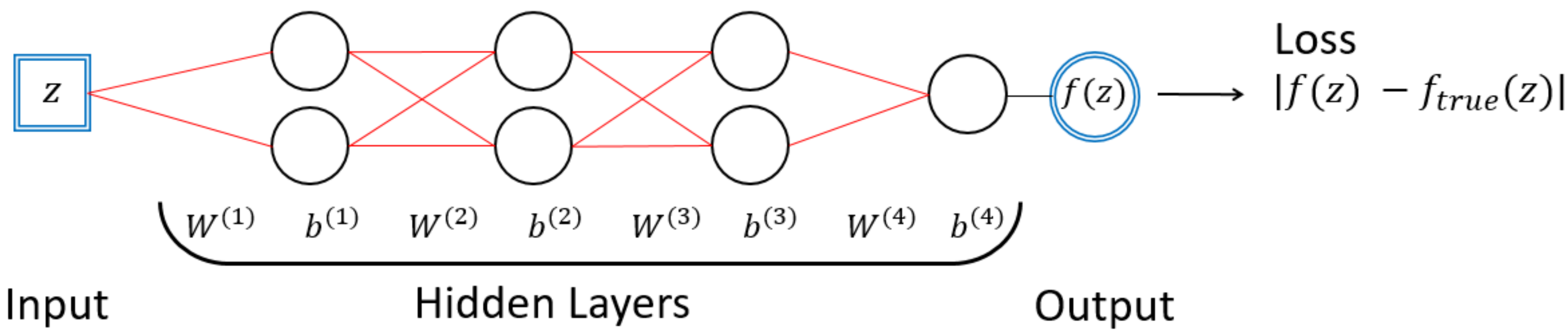


19 parameter space

Ex) Gradient decent method



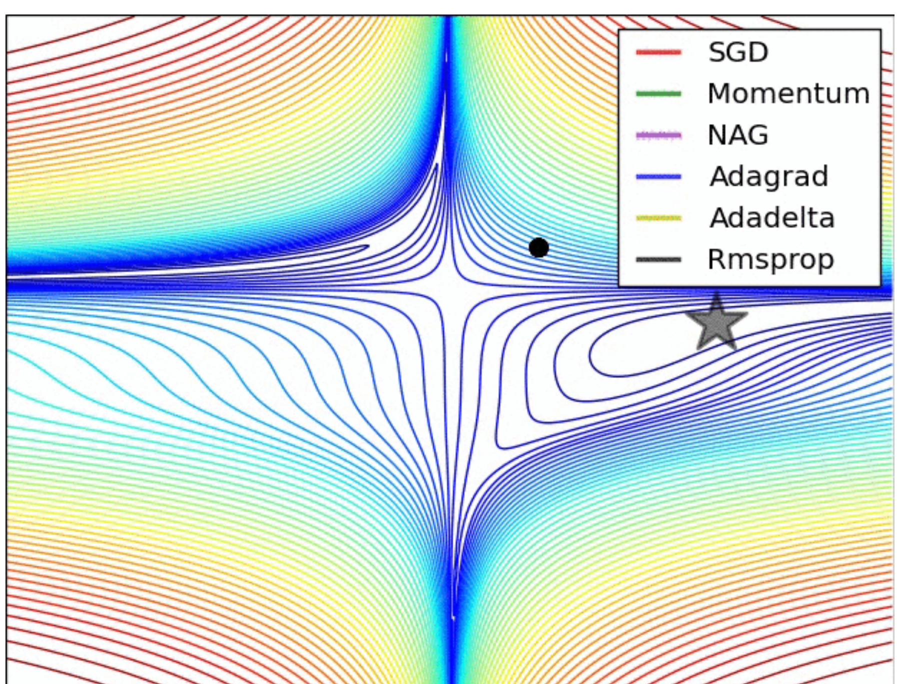
# Standard Deep Learning

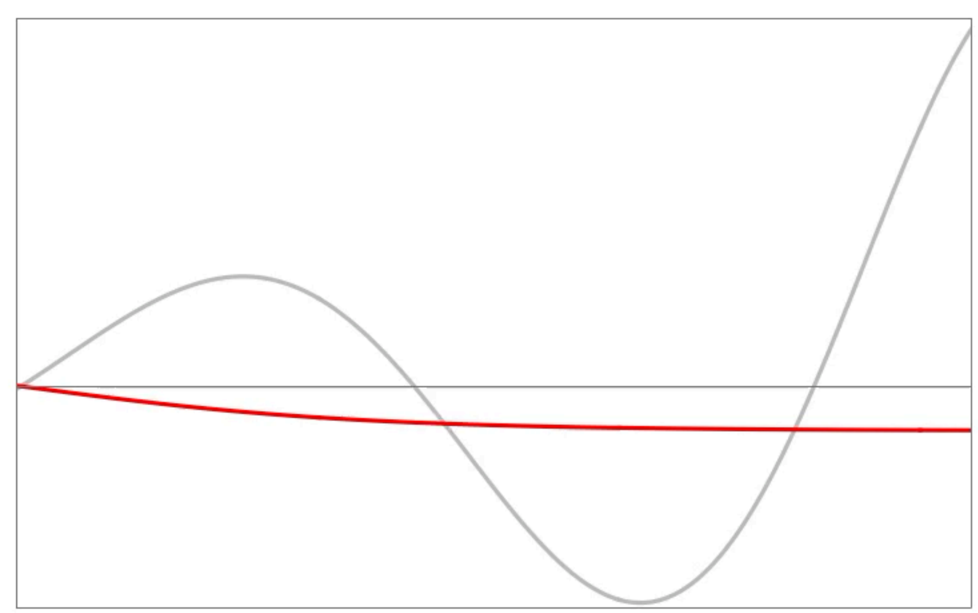



$$z_j^{(i+1)} = \sigma(W_{jk}^{(i)} \cdot z_k^{(i)} + b_j^{(i)})$$ ~2000 epoch 19 parameters

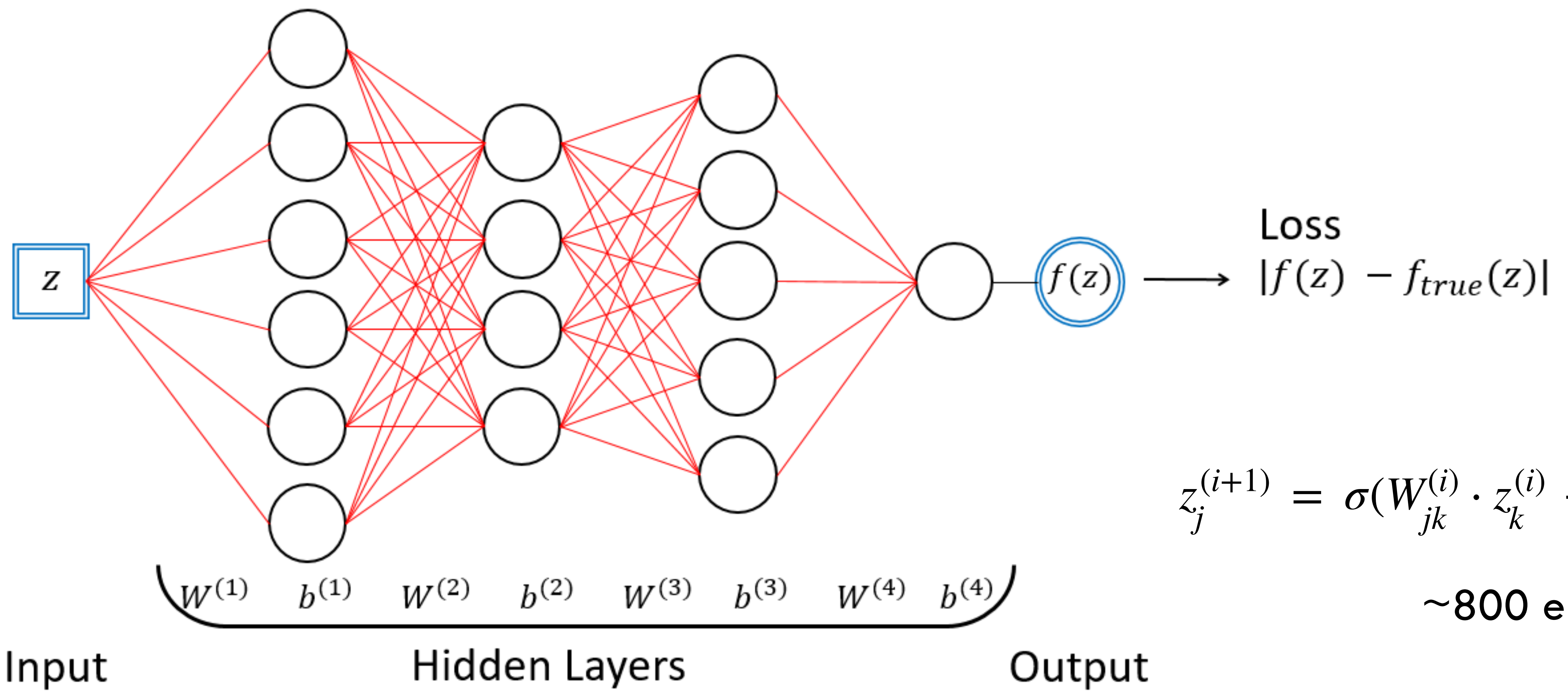
$$\tanh\left[\begin{pmatrix}\square & \square\end{pmatrix} \tanh\left[\begin{pmatrix}\square & \square \\ \square & \square\end{pmatrix}\left(\tanh\left[\begin{pmatrix}\square & \square \\ \square & \square\end{pmatrix}\left(\tanh\left[\begin{pmatrix}\square \\ \square\end{pmatrix} z + \begin{pmatrix}\square \\ \square\end{pmatrix}\right]\right) + \begin{pmatrix}\square \\ \square\end{pmatrix}\right] + \begin{pmatrix}\square \\ \square\end{pmatrix}\right)\right] + \begin{pmatrix}\square\end{pmatrix}\right]$$

$$\begin{pmatrix}0.878421 & 6.21182\end{pmatrix} \begin{pmatrix}1.55725 & -2.14132 \\ -2.49736 & -1.0517\end{pmatrix} \begin{pmatrix}-1.21558 & -0.693247 \\ 2.86862 & 3.01908\end{pmatrix} \begin{pmatrix}0.317991 \\ 0.521212\end{pmatrix} \begin{pmatrix}-1.96854 \\ -0.020086\end{pmatrix} \begin{pmatrix}0.320358 \\ -0.614063\end{pmatrix} \begin{pmatrix}-0.442947 \\ 0.854547\end{pmatrix} \begin{pmatrix}1.33224\end{pmatrix}$$



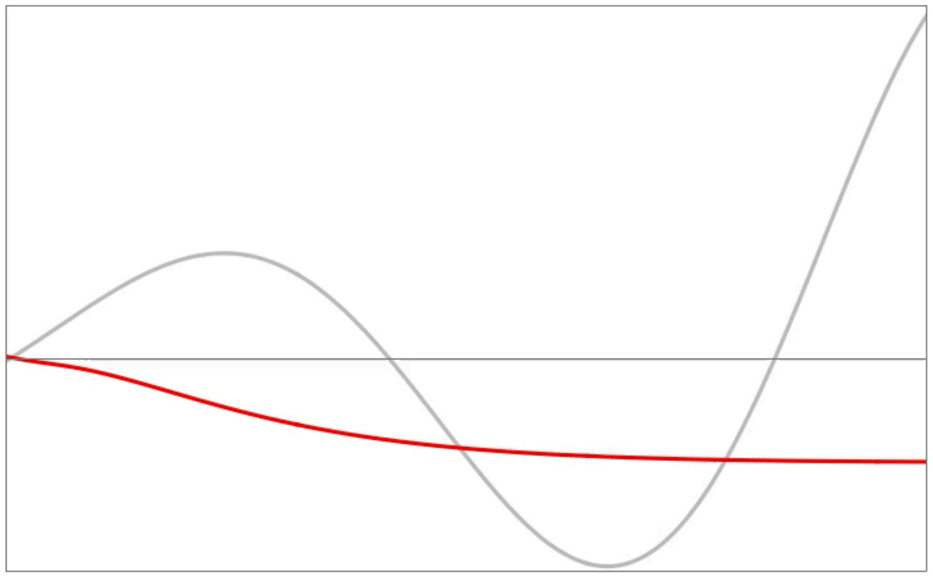



# Standard Deep Learning

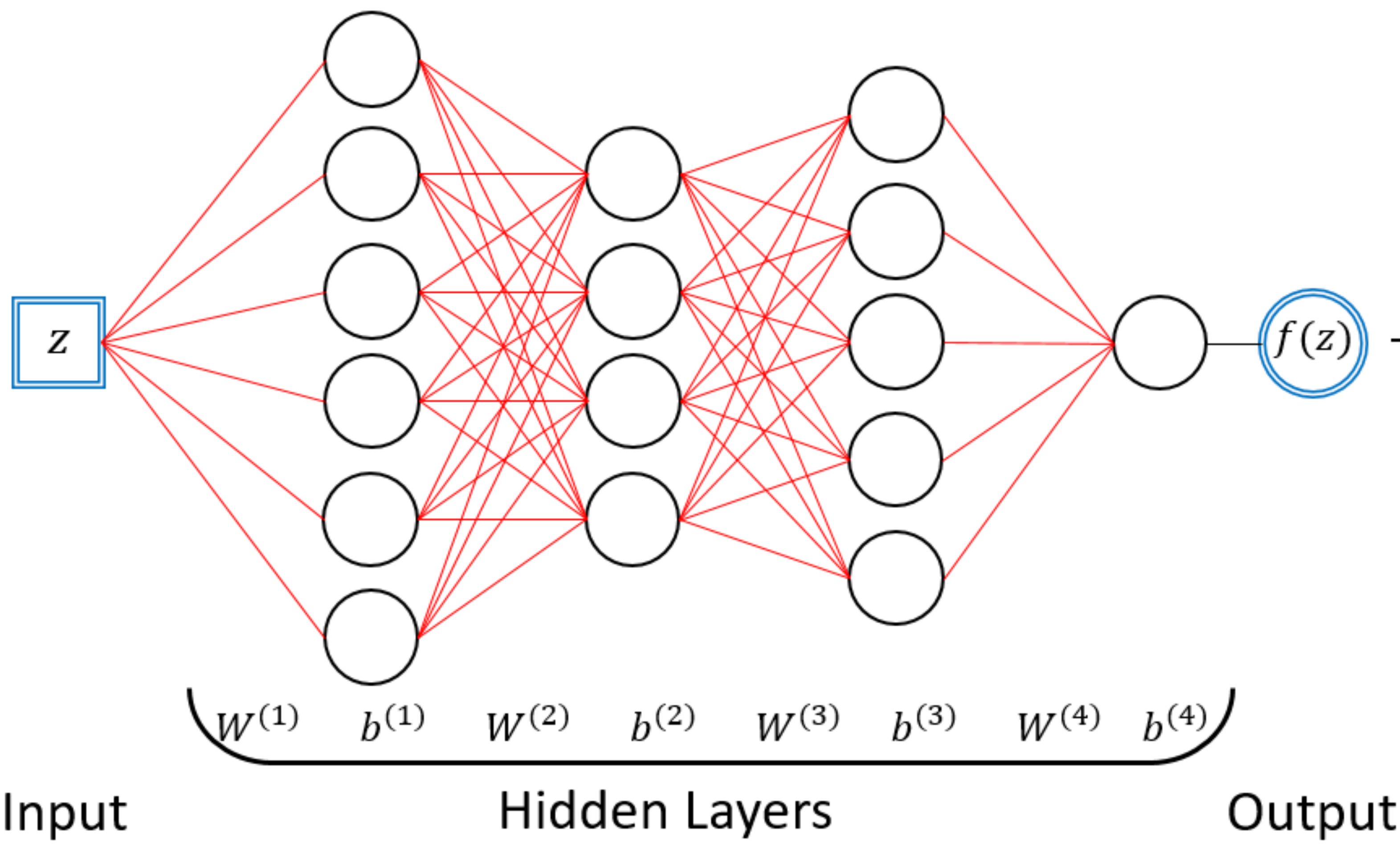



$$z_j^{(i+1)} = \sigma(W_{jk}^{(i)} \cdot z_k^{(i)} + b_j^{(i)})$$

~800 epoch



# Standard Deep Learning




Loss
$|f(z) - f_{true}(z)|$

$$z_j^{(i+1)} = \sigma(W_{jk}^{(i)} \cdot z_k^{(i)} + b_j^{(i)})$$

~800 epoch

71 parameters

$$\text{Show}\Big[\Big\{\begin{pmatrix} 0.155467 \\ 0.503477 \\ -0.19021 \\ -0.573821 \\ 0.238013 \\ 0.93589 \end{pmatrix}, \begin{pmatrix} -0.421564 \\ -0.664526 \\ 0.768171 \\ 1.02467 \\ -1.01427 \\ -0.94677 \end{pmatrix}, \begin{pmatrix} 0.584066 & 0.331801 & -0.29404 & -0.586447 & 0.820878 & 1.04534 \\ 0.208755 & 0.465765 & -1.21082 & -0.63667 & 1.73886 & -0.53486 \\ 0.918991 & 0.328462 & -1.08441 & 0.474161 & -0.0382366 & -0.502777 \\ 0.600695 & -0.32061 & -0.914458 & 0.73871 & 0.124301 & 0.402687 \end{pmatrix},$$

$$\begin{pmatrix} 0.172725 \\ -0.0946543 \\ 0.0526899 \\ 0.201293 \end{pmatrix}, \begin{pmatrix} 0.308126 & -1.0333 & 1.21593 & 1.62945 \\ -0.217696 & 0.696293 & -0.338222 & -2.68779 \\ 0.0759481 & -0.664867 & 0.86521 & 1.7725 \\ 0.445185 & 0.0762033 & -1.15522 & -2.01414 \\ -0.595479 & 0.916218 & -0.400813 & -2.12668 \end{pmatrix}, \begin{pmatrix} 0.621626 \\ -0.192633 \\ 0.402367 \\ -0.14795 \\ -0.331163 \end{pmatrix},$$

$$\begin{pmatrix} 0.903621 & -2.09077 & 1.97153 & -1.85892 & -1.05424 \end{pmatrix}, \begin{pmatrix} 0.319326 \end{pmatrix}\Big\}\Big]$$

Holography

# Deep Learning $\boxed{\text{Bulk Spacetime from Boundary data}}$

- What is “Machine Learning” & “Deep Learning”?

  - Deep Learning 101: standard story

- Physics equation related Deep Learning I

  - Deep Learning for ODE: classical mechanics

  - AdS/Deep Learning: optical conductivity

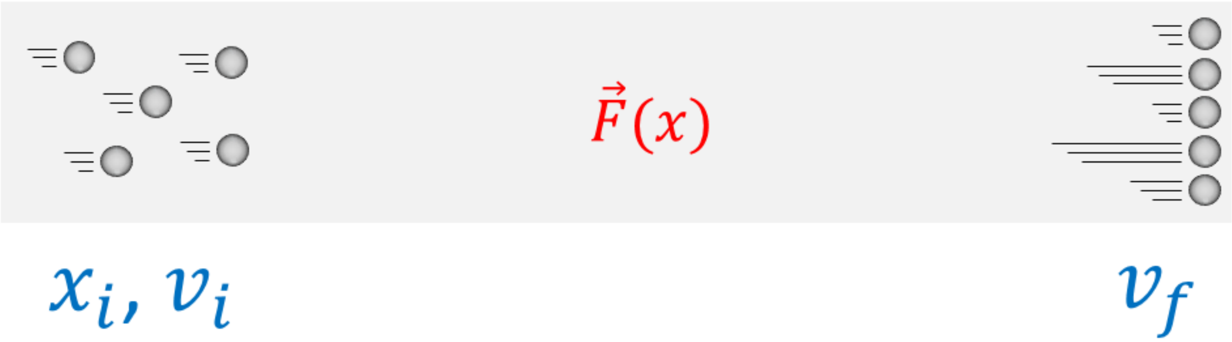
- Physics equation related Deep Learning II

  - Deep Learning for Integral: classical electrostatics

  - AdS/Deep Learning: Entanglement entropy

$$m\ddot{x} = F(x)$$

$$\dot{x} = v\,, \quad \dot{v} = \frac{1}{m}F$$



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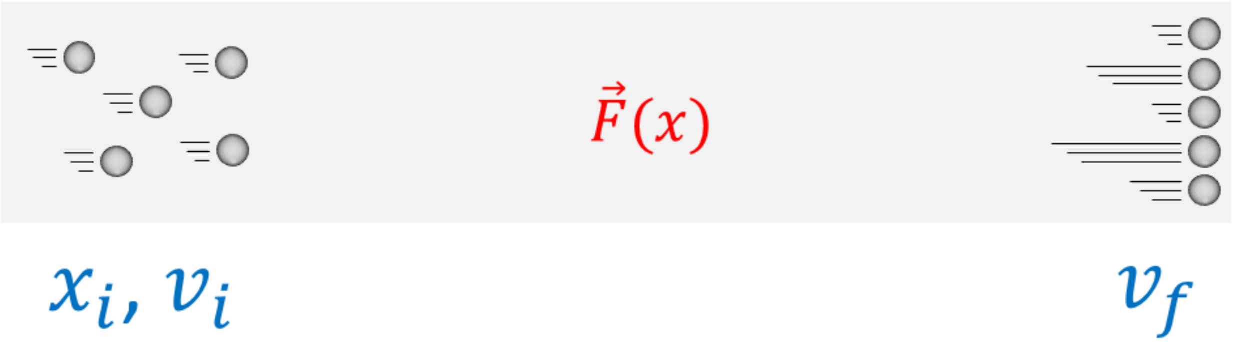
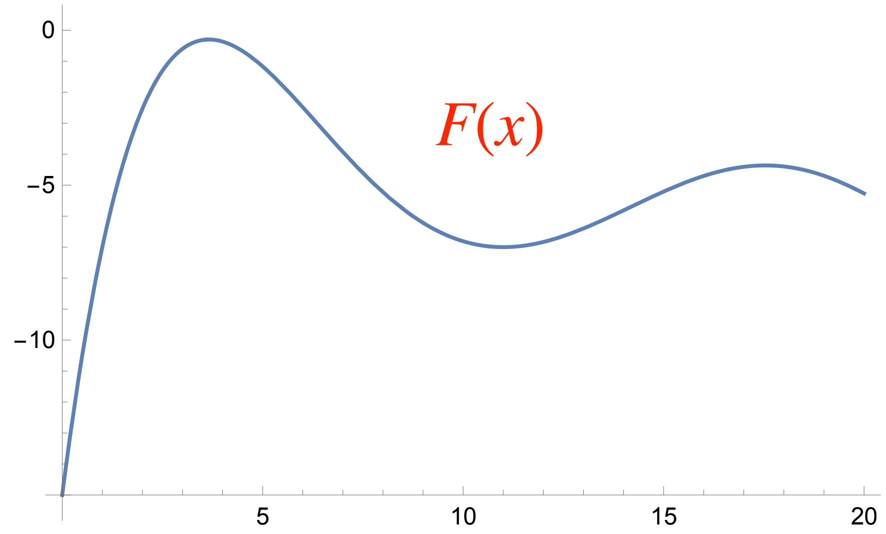


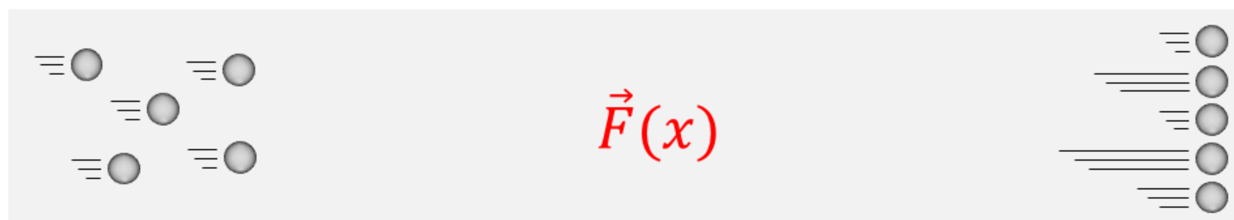




$$F_{2,\mathrm{True}}(x) = \frac{1}{8000}(x-1)(x-11)^2(x-23)^2 - 0.7$$

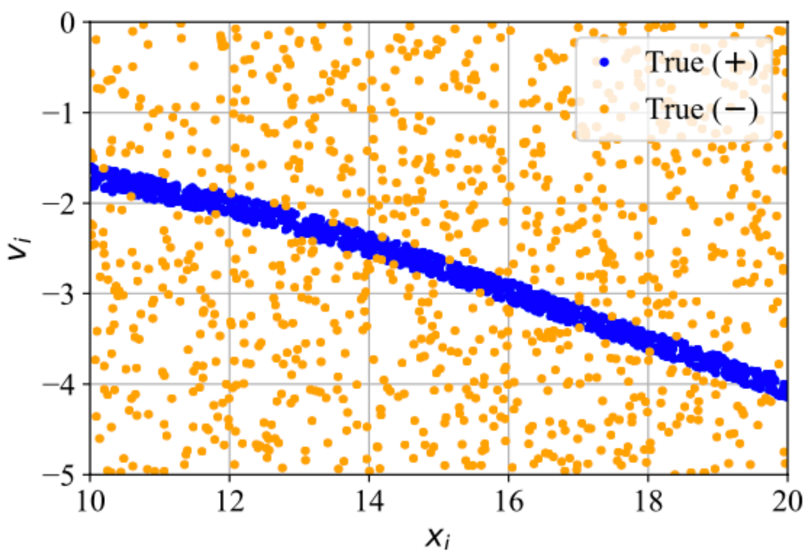
# Deep Learning for ODE: classical mechanics

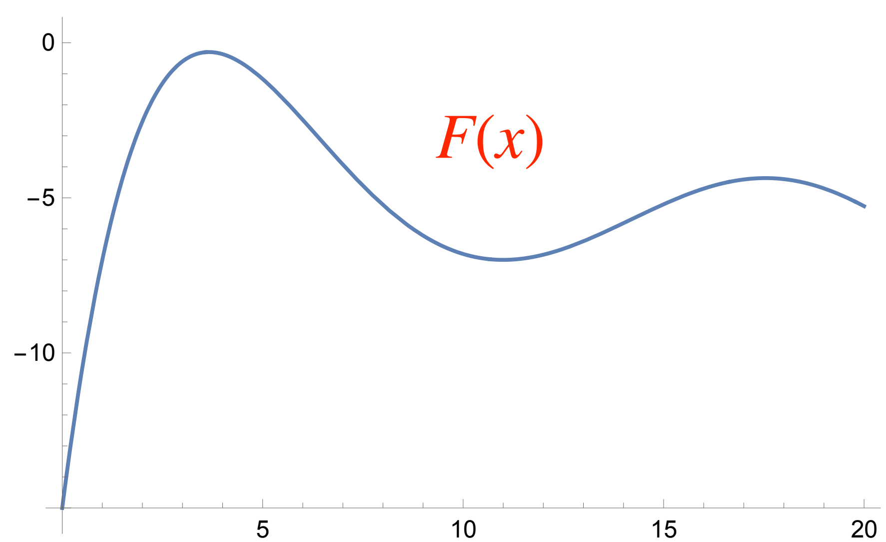
$$m\ddot{x} = F(x)$$

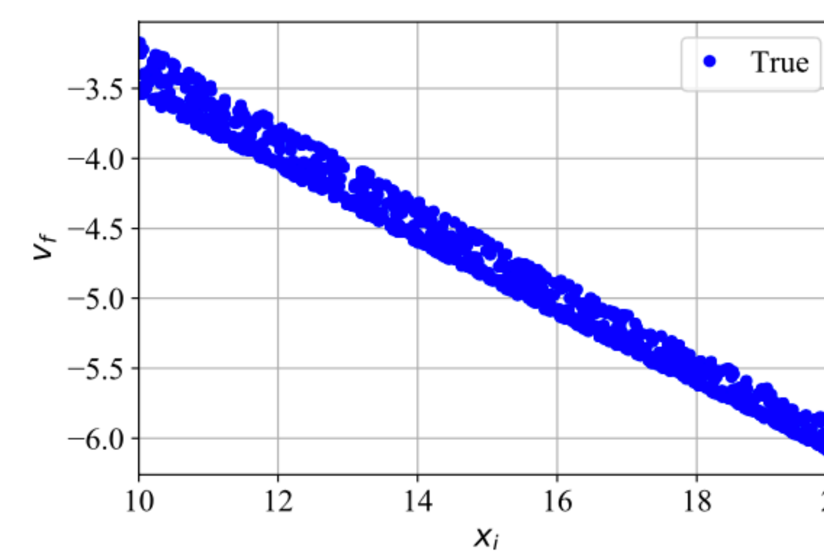
$$\dot{x} = v\,, \quad \dot{v} = \frac{1}{m}F$$




$x_i, v_i$ $\qquad$ $v_f$




$$F_{2,\mathrm{True}}(x) = \frac{1}{8000}(x-1)(x-11)^2(x-23)^2 - 0.7$$

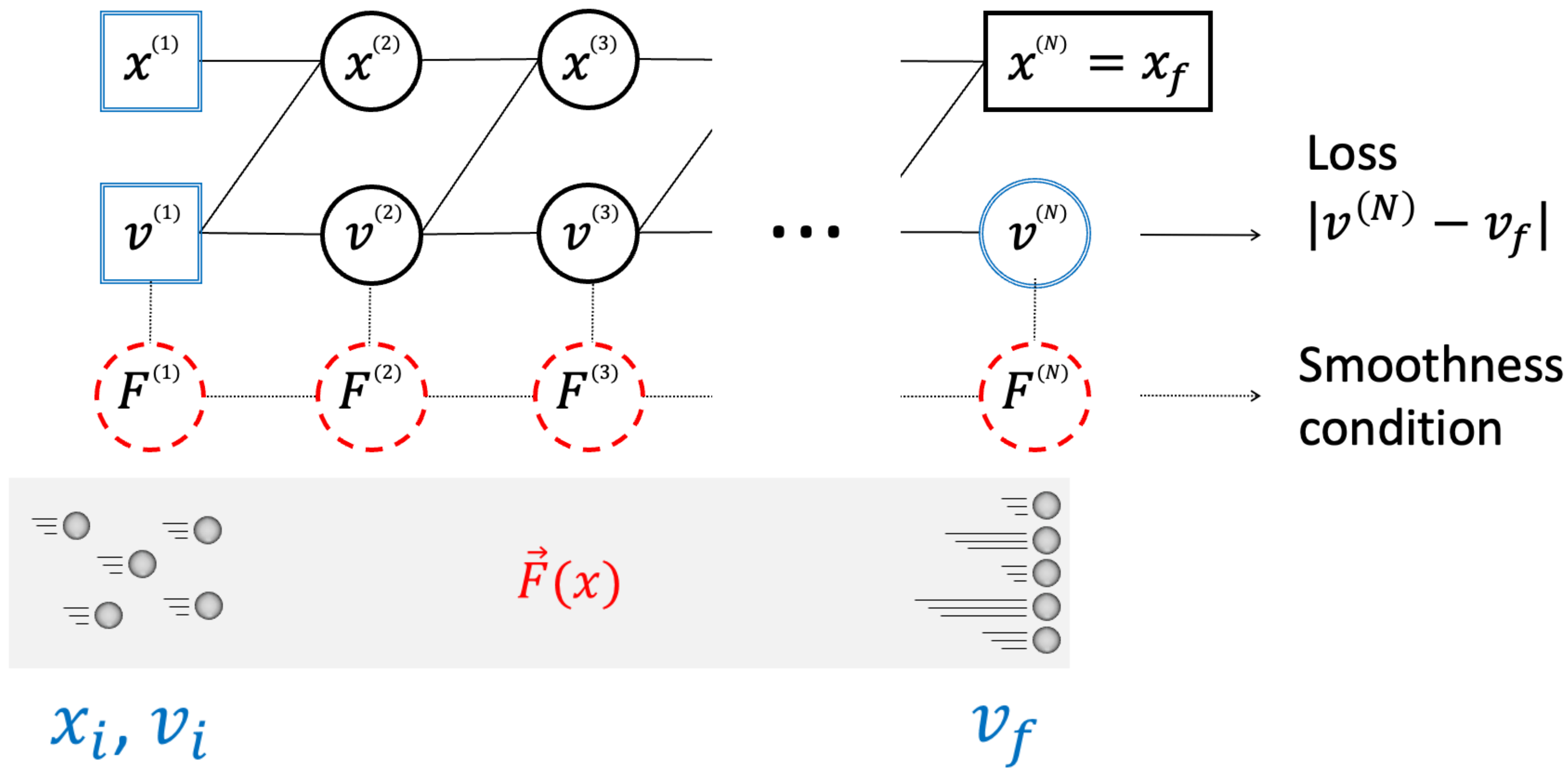
# Deep Learning for ODE: classical mechanics

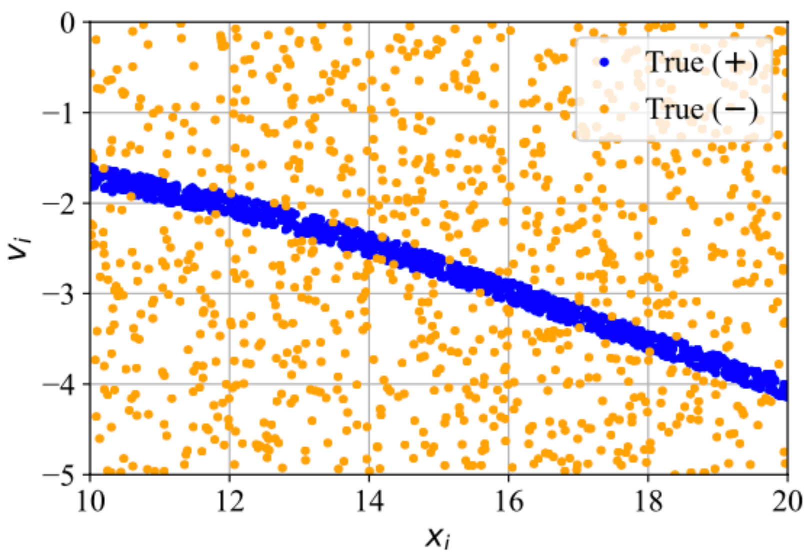
$$x^{(i+1)} = x^{(i)} + \Delta t \cdot v^{(i)}$$

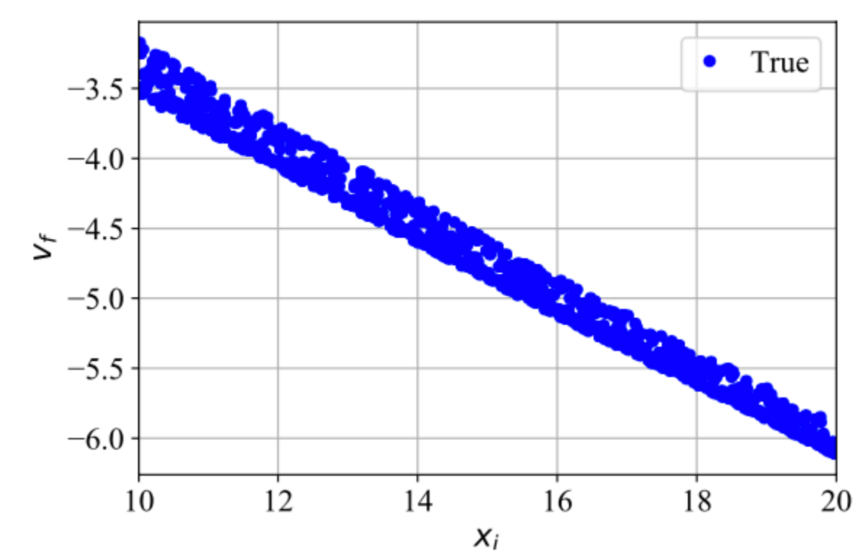
$$v^{(i+1)} = v^{(i)} + \Delta t \cdot F(x^{(i)})$$

$$m\ddot{x} = F(x)$$

$$\dot{x} = v\,, \quad \dot{v} = \frac{1}{m}F$$



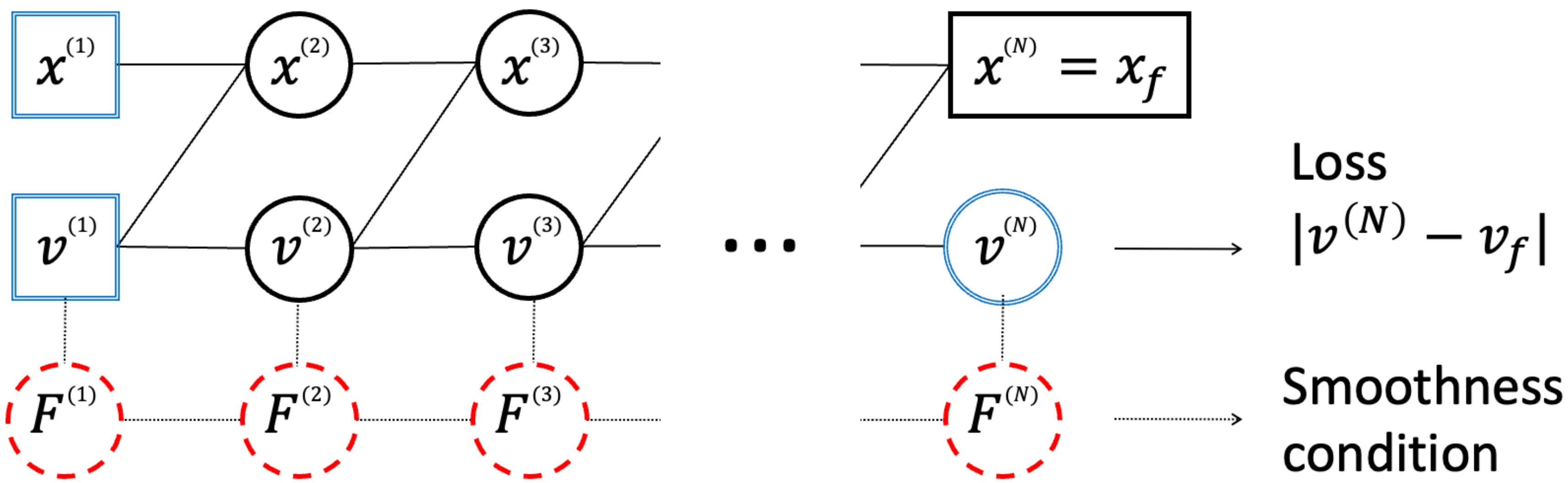



# Deep Learning for ODE: classical mechanics

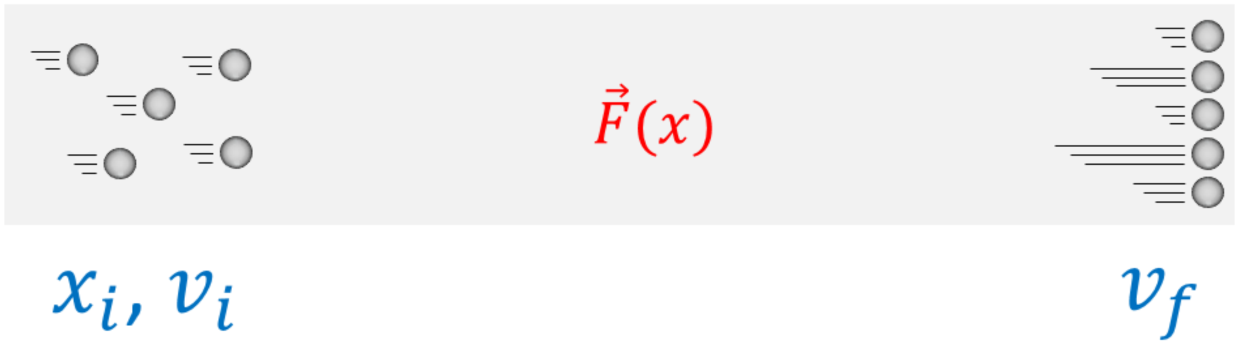
$$x^{(i+1)} = x^{(i)} + \Delta t \cdot v^{(i)}$$

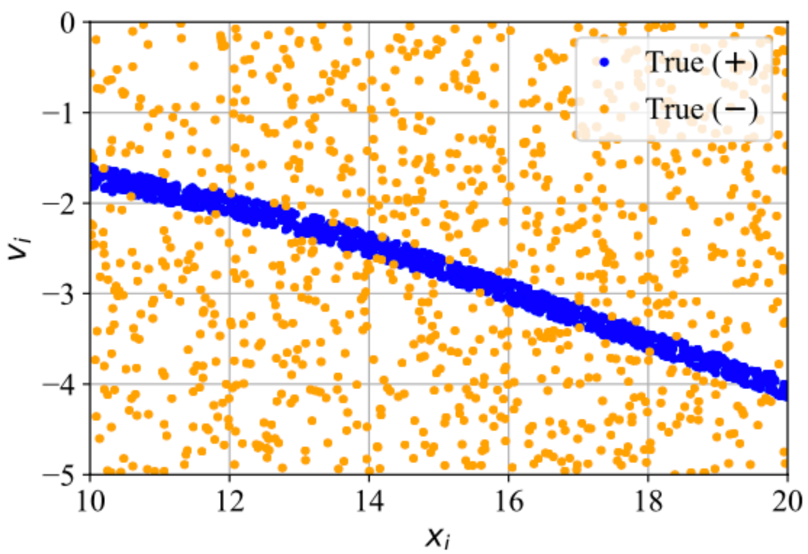
$$v^{(i+1)} = v^{(i)} + \Delta t \cdot F(x^{(i)})$$

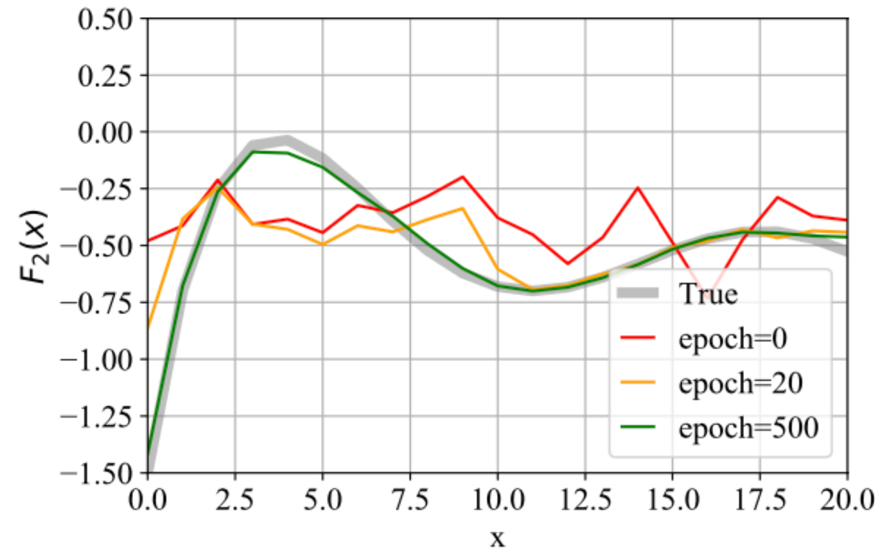


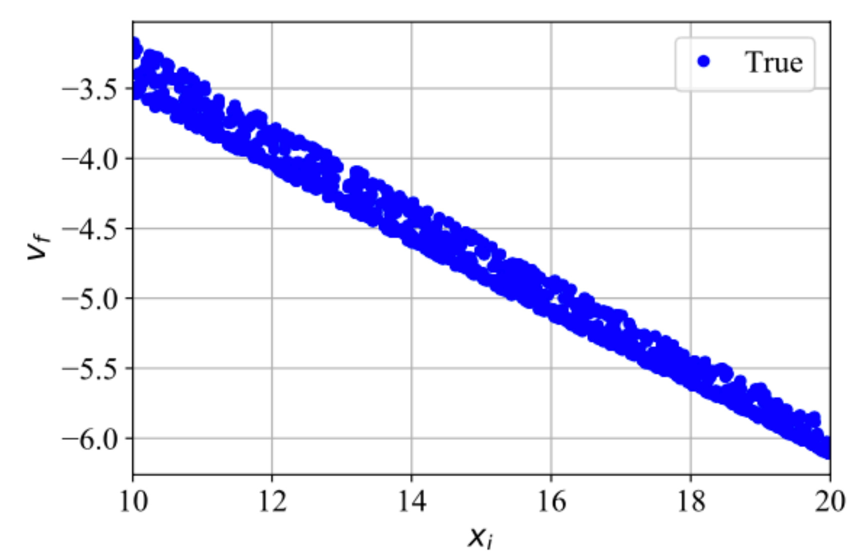

$$m\ddot{x} = F(x)$$

$$\dot{x} = v\,, \quad \dot{v} = \frac{1}{m}F$$




$$F_{2,\mathrm{True}}(x) = \frac{1}{8000}(x-1)(x-11)^2(x-23)^2 - 0.7$$

- What is “Machine Learning” & “Deep Learning”?

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- Physics equation related Deep Learning II

  - Deep Learning for Integral: classical electrostatics

  - AdS/Deep Learning: Entanglement entropy

$$m\ddot{x} = F$$

$$A(z)f''(z)+B(z)f'(z)+C(z)f(z) = F(z)$$

$$A(z)f''(z)+B(z)f'(z)+C(z)f(z) = D(z)g(z)$$

$$E(z)g''(z)+F(z)g'(z)+G(z)g(z) = H(z)f(z)$$

$$\partial_z^2 A_x = \zeta\,\partial_z A_x + \left(\frac{z^2\mu^2}{f} - \xi\right) A_x + \frac{iz\mu}{f}\Phi\,,$$

$$\partial_z^2 \Phi = \zeta\,\partial_z \Phi + \left(\frac{\alpha^2}{f} + \frac{f'}{zf} - \xi\right) \Phi - \frac{iz\alpha^2\mu}{f} A_x\,,$$

$$\zeta := \frac{2i\omega}{(1-z)f'(1)} - \frac{f'(z)}{f(z)}\,, \qquad \xi := \frac{\omega^2}{f(z)^2} + \frac{i\omega}{(1-z)f'(1)}\left(\frac{i\omega}{(1-z)f'(1)} - \frac{1}{1-z} - \frac{f'(z)}{f(z)}\right)$$

# AdS/Deep learning: optical conductivity

**Action**

$$S = \int d^4x\sqrt{-g}\left(R + 6 - \frac{1}{4}F_{ab}F^{ab} - \frac{1}{2}\sum_{I=1}^{2}(\partial X_I)^2\right)$$

**EOM**

$$R_{ab} - \frac{1}{2}g_{ab}\left(R + 6 - \frac{1}{4}F_{ab}F^{ab} - \frac{1}{2}\sum_{I=1}^{2}(\partial X_I)^2\right) - F_{ac}F_b^{\ c} - \frac{1}{2}\sum_{I=1}^{2}\partial_a X_I \partial_b X_I = 0\,,$$

$$\nabla^a F_{ab} = 0\,, \qquad \nabla_a \nabla^a X_I = 0\,,$$

**Background**

$$ds^2 = \frac{1}{z^2}\left[-f(z)dt^2 + \frac{dz^2}{f(z)} + dx^2 + dy^2\right], \qquad f(z) = 1 - \frac{\alpha^2}{2}z^2 - \left(1 - \frac{\alpha^2}{2} + \frac{\mu^2}{4}\right)z^3 + \frac{\mu^2}{4}z^4$$

$$A = \mu\,(1-z)\,dt\,, \qquad X_1 = \alpha\, x\,, \quad X_2 = \alpha\, y$$

**Flucutation EOM I**

$$\delta g_{tx} = e^{-i\omega t}\frac{h_{tx}(z)}{z^2}\,, \qquad \delta A_x = e^{-i\omega t}a_x(z)\,, \qquad \delta X_1 = e^{-i\omega t}\frac{\psi_x(z)}{\alpha}$$

$$a_x''(z) + \frac{f'(z)}{f(z)}a_x'(z) + \left(\frac{\omega^2}{f(z)^2} - \frac{\mu^2 z^2}{f(z)}\right)a_x(z) - \frac{i\mu z}{f(z)}\phi(z) = 0\,, \qquad \phi(z) := -\frac{f(z)\psi_x'(z)}{\omega\, z}$$

$$\phi''(z) + \frac{f'(z)}{f(z)}\phi'(z) + \left(\frac{\omega^2}{f(z)^2} - \frac{\alpha^2}{f(z)} - \frac{f'(z)}{zf(z)}\right)\phi(z) + \frac{i\alpha^2\mu z}{f(z)}a_x(z) = 0\,,$$

$$\boxed{\begin{aligned}\sigma(\omega) &= \frac{1}{i\omega}G^R_{j^x j^x}(\omega) = \frac{1}{i\omega}\frac{a_x^{(R)}}{a_x^{(S)}}\\ &= \frac{A_x'(z_{\text{fin}})}{i\omega A_x(z_{\text{fin}})} - \frac{1}{f'(1)}\end{aligned}}$$

**Flucutation EOM II**

$$A_x(z) := (1-z)^{-\frac{i\omega}{f'(1)}}a_x(z)\,, \qquad \Phi(z) := (1-z)^{-\frac{i\omega}{f'(1)}}\phi(z)\,,$$

$$\boxed{\begin{aligned}\partial_z^2 A_x &= \zeta\,\partial_z A_x + \left(\frac{z^2\mu^2}{f} - \xi\right)A_x + \frac{iz\mu}{f}\Phi\,,\\ \partial_z^2 \Phi &= \zeta\,\partial_z \Phi + \left(\frac{\alpha^2}{f} + \frac{f'}{zf} - \xi\right)\Phi - \frac{iz\alpha^2\mu}{f}A_x\,,\\ \zeta &:= \frac{2i\omega}{(1-z)f'(1)} - \frac{f'(z)}{f(z)}\,, \quad \xi := \frac{\omega^2}{f(z)^2} + \frac{i\omega}{(1-z)f'(1)}\left(\frac{i\omega}{(1-z)f'(1)} - \frac{1}{1-z} - \frac{f'(z)}{f(z)}\right)\end{aligned}}$$

# AdS/Deep learning: optical conductivity

$$m\ddot{x} = F(x) \qquad \dot{x} = v\,, \quad \dot{v} = \frac{1}{m}F \qquad \begin{aligned} x^{(i+1)} &= x^{(i)} + \Delta t \cdot v^{(i)} \\ v^{(i+1)} &= v^{(i)} + \Delta t \cdot F(x^{(i)}) \end{aligned}$$

$$\partial_z^2 A_x = \zeta\,\partial_z A_x + \left(\frac{z^2\mu^2}{f} - \xi\right) A_x + \frac{iz\mu}{f}\Phi\,,$$

$$\partial_z^2 \Phi = \zeta\,\partial_z \Phi + \left(\frac{\alpha^2}{f} + \frac{f'}{zf} - \xi\right)\Phi - \frac{iz\alpha^2\mu}{f}A_x\,,$$

$$\zeta := \frac{2i\omega}{(1-z)f'(1)} - \frac{f'(z)}{f(z)}\,, \qquad \xi := \frac{\omega^2}{f(z)^2} + \frac{i\omega}{(1-z)f'(1)}\left(\frac{i\omega}{(1-z)f'(1)} - \frac{1}{1-z} - \frac{f'(z)}{f(z)}\right)$$

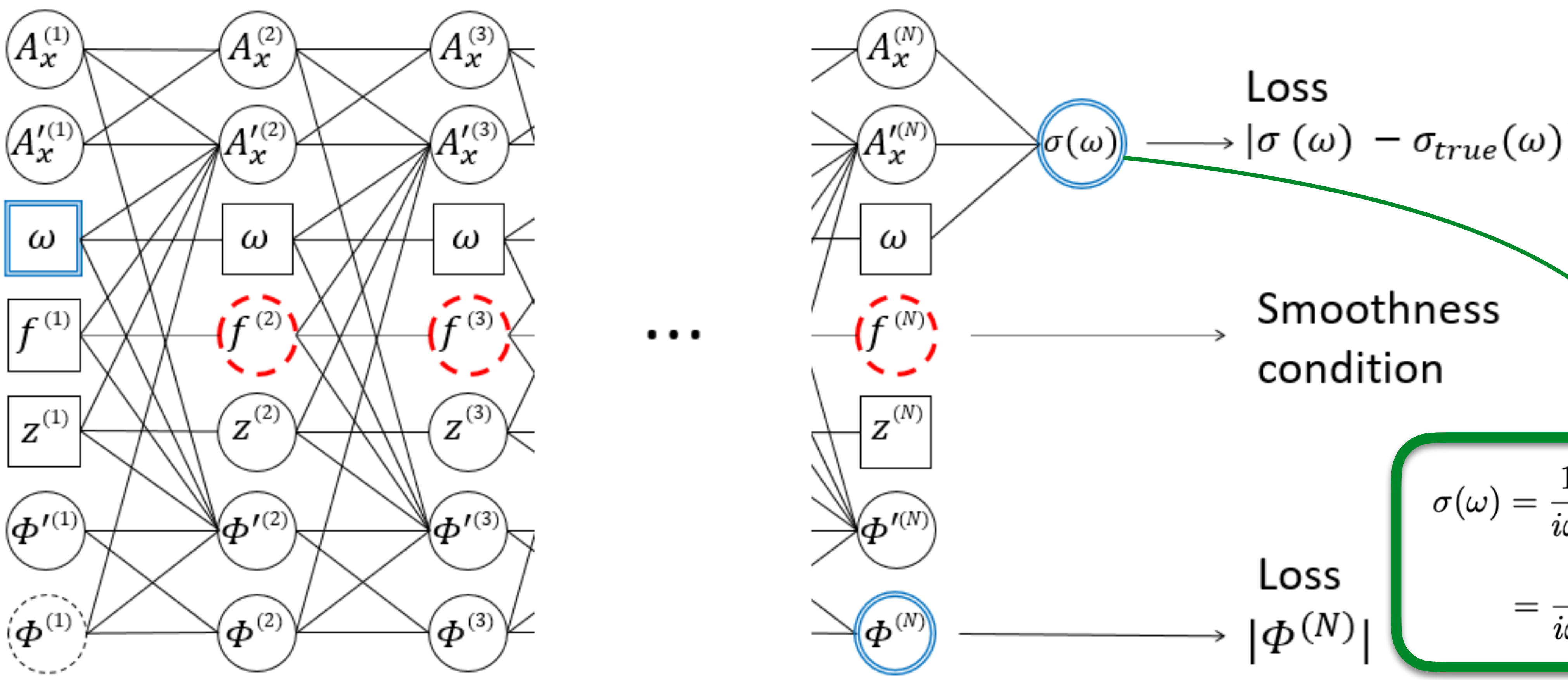
$$A_x^{(i+1)} = A_x^{(i)} + \Delta z \cdot A_x'^{(i)}$$

$$\Phi^{(i+1)} = \Phi^{(i)} + \Delta z \cdot \Phi'^{(i)}$$

$$A_x'^{(i+1)} = \left(\frac{z^2\mu^2}{f} - \xi\right)\Delta z A_x^{(i)} + \frac{iz\mu}{f}\Delta z \Phi^{(i)} + (1 + \zeta\Delta z)\,A_x'^{(i)}$$

$$\Phi'^{(i+1)} = -\frac{iz\alpha^2\mu}{f}\Delta z A_x^{(i)} + \left(\frac{\alpha^2}{f} + \frac{f'}{zf} - \xi\right)\Delta z \Phi^{(i)} + (1 + \zeta\Delta z)\,\Phi'^{(i)}$$

# AdS/Deep learning: optical conductivity




$$\sigma(\omega) = \frac{1}{i\omega} G^R_{j^x j^x}(\omega) = \frac{1}{i\omega}\frac{a_x^{(R)}}{a_x^{(S)}} = \frac{A_x'(z_{\text{fin}})}{i\omega A_x(z_{\text{fin}})} - \frac{1}{f'(1)}$$

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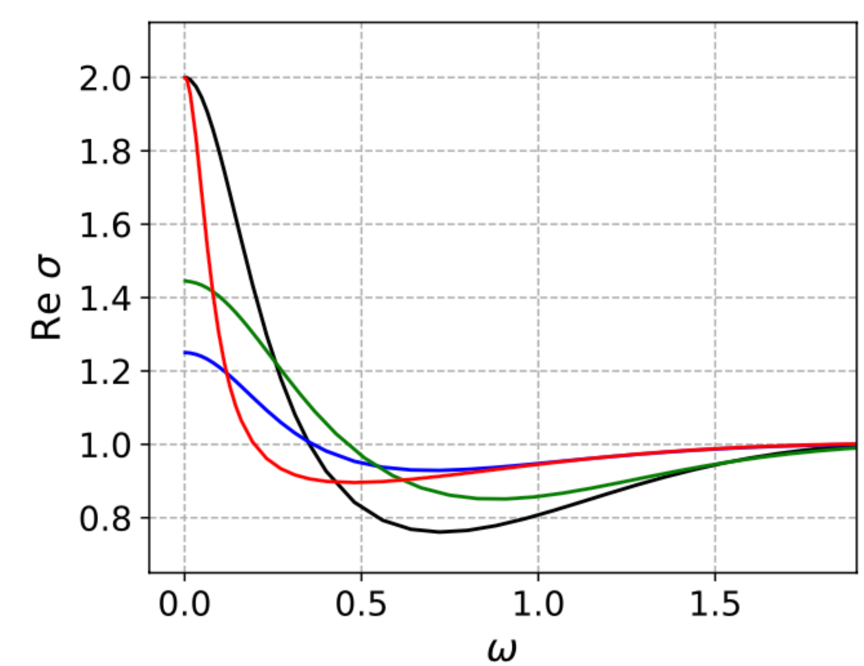
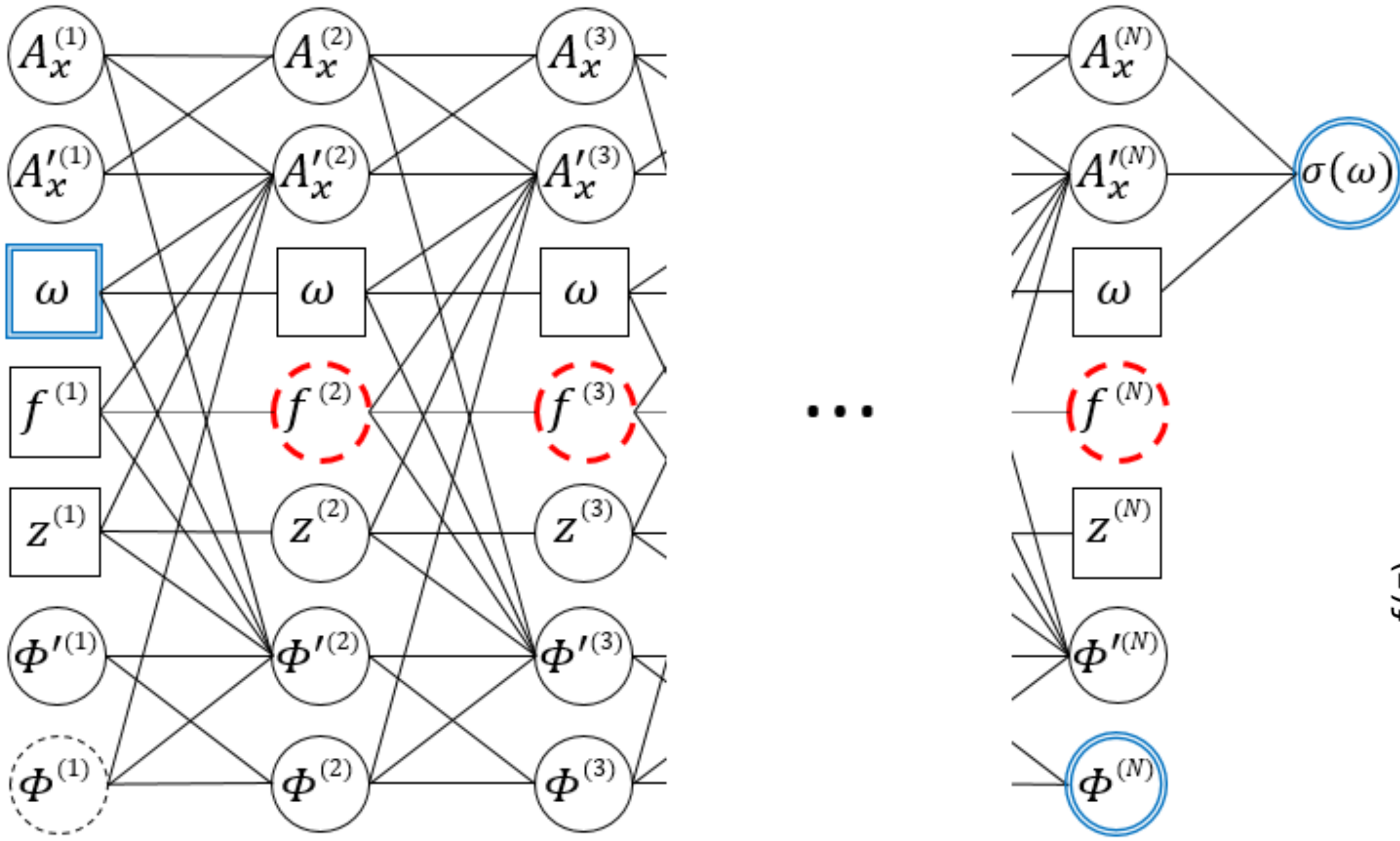
$$\Phi'^{(i+1)} = -\frac{iz\alpha^2\mu}{f}\Delta z A_x^{(i)} + \left(\frac{\alpha^2}{f} + \frac{f'}{zf} - \xi\right)\Delta z \Phi^{(i)} + (1+\zeta\Delta z)\,\Phi'^{(i)}$$

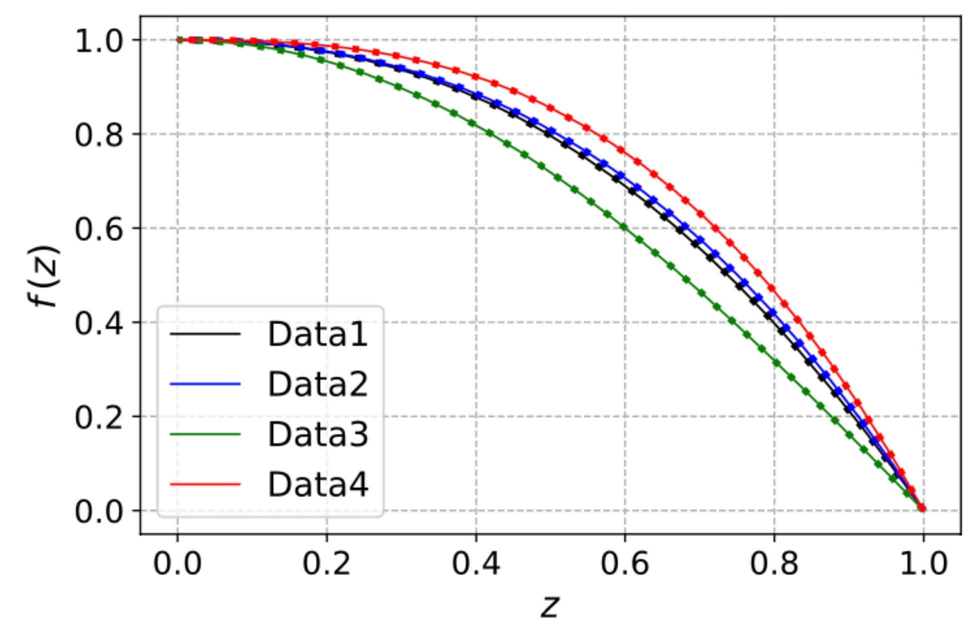


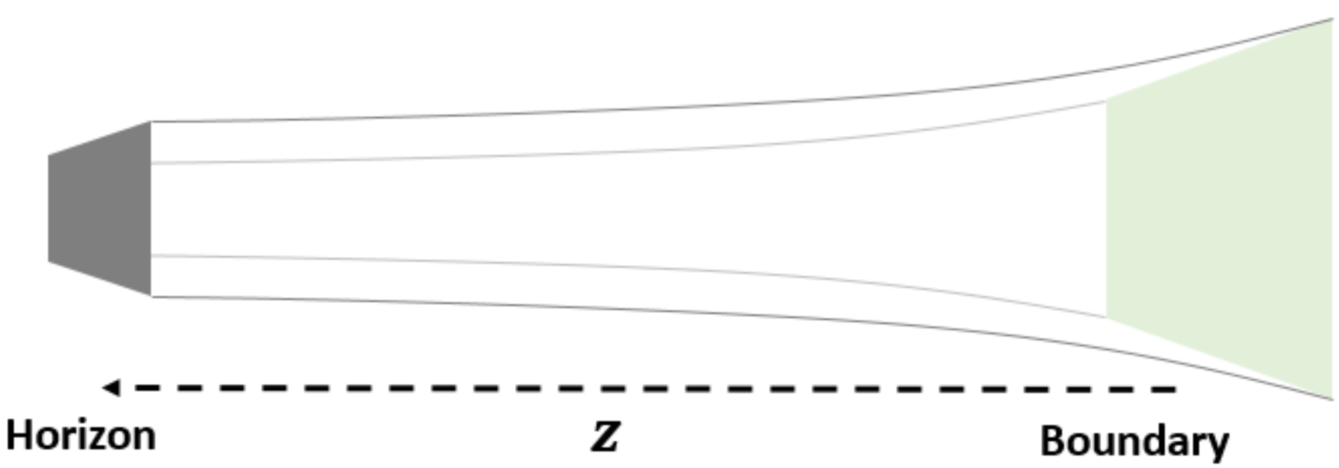
# AdS/Deep learning: optical conductivity

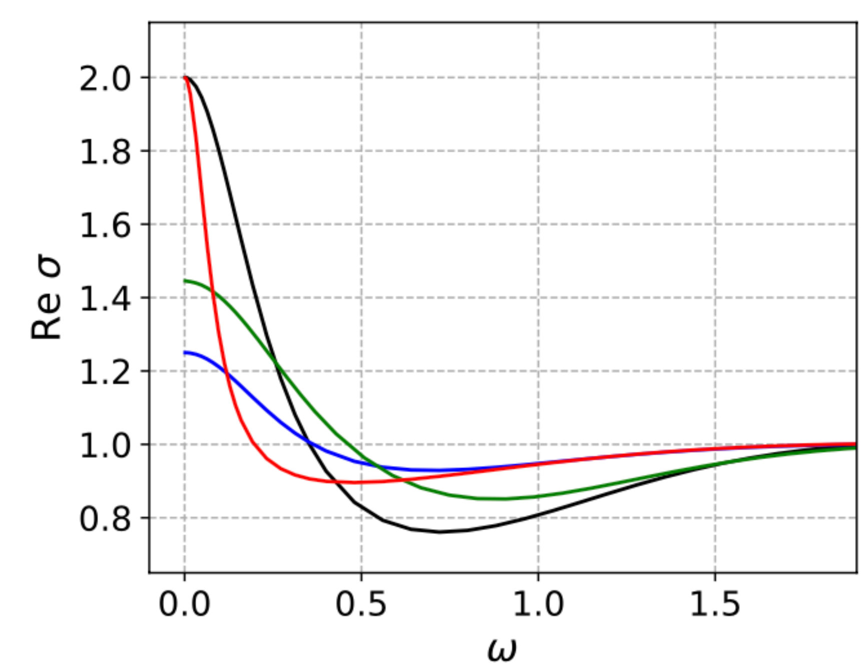



Loss

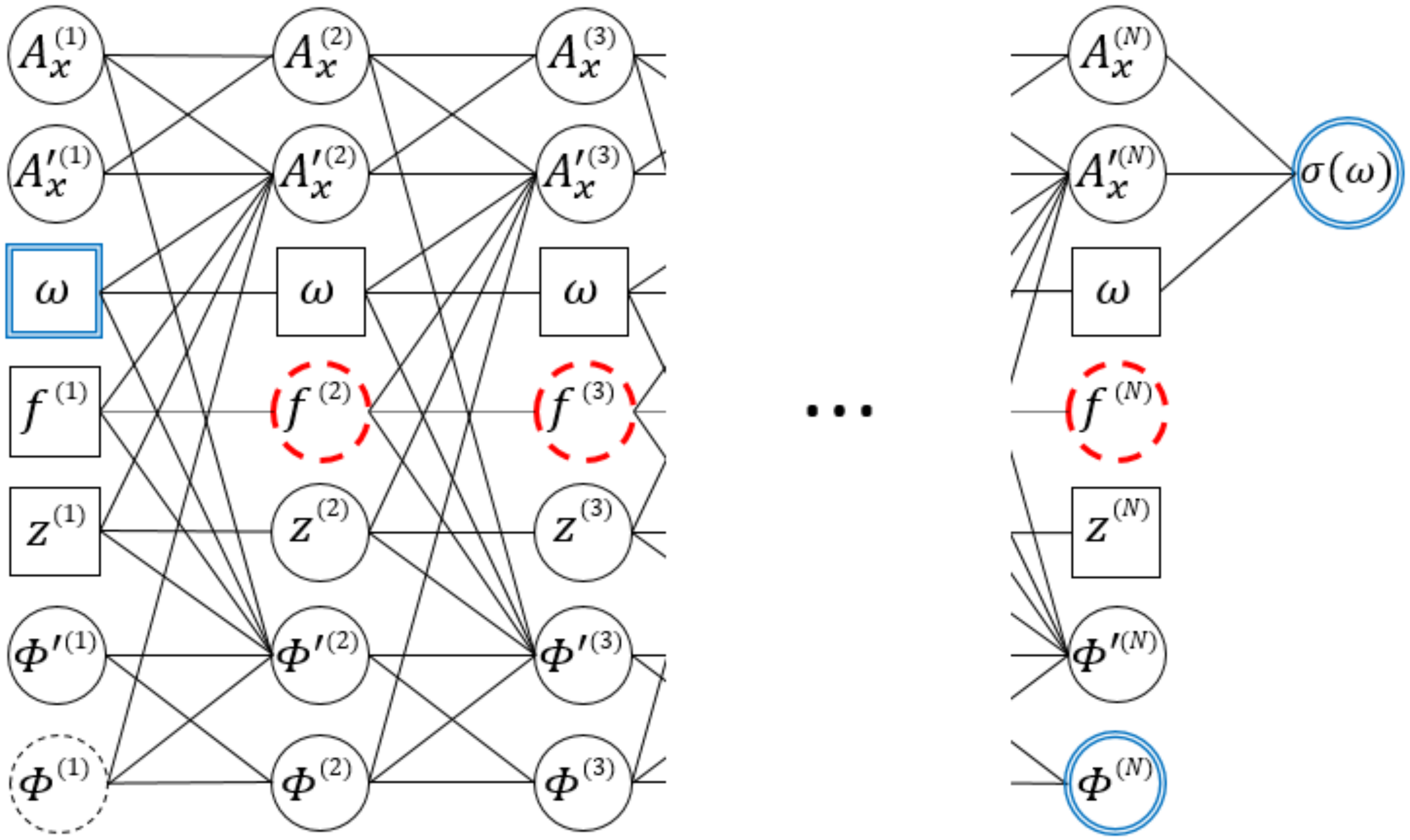
$\longrightarrow |\sigma(\omega) - \sigma_{true}(\omega)|$



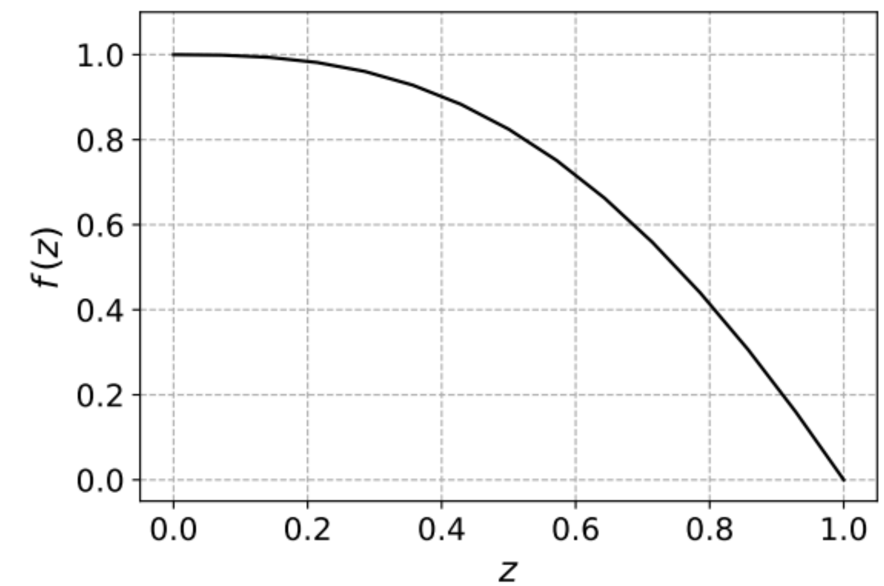



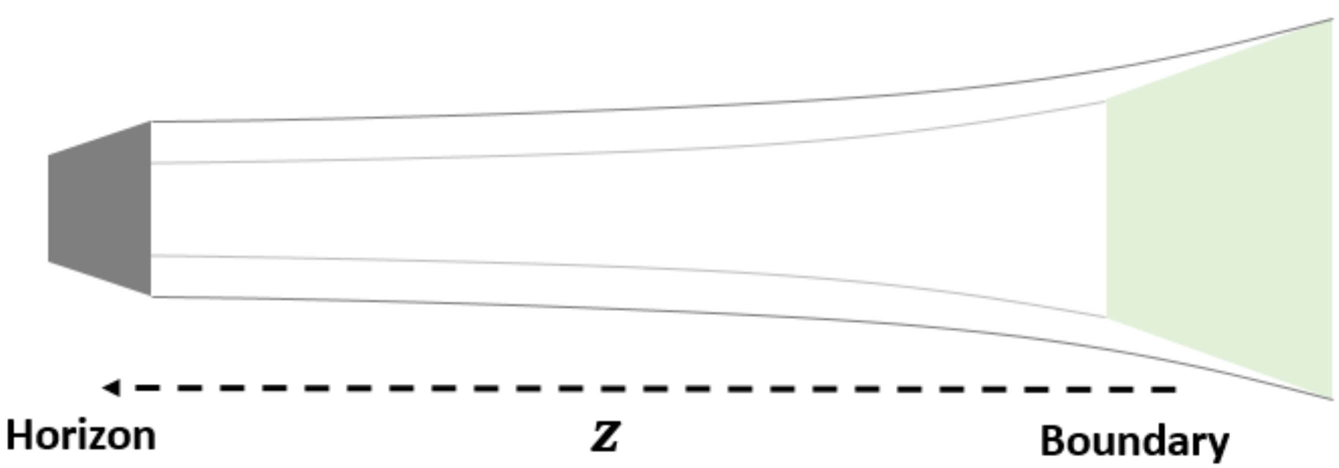
# AdS/Deep learning: optical conductivity



Loss

$\longrightarrow |\sigma(\omega) - \sigma_{true}(\omega)|$





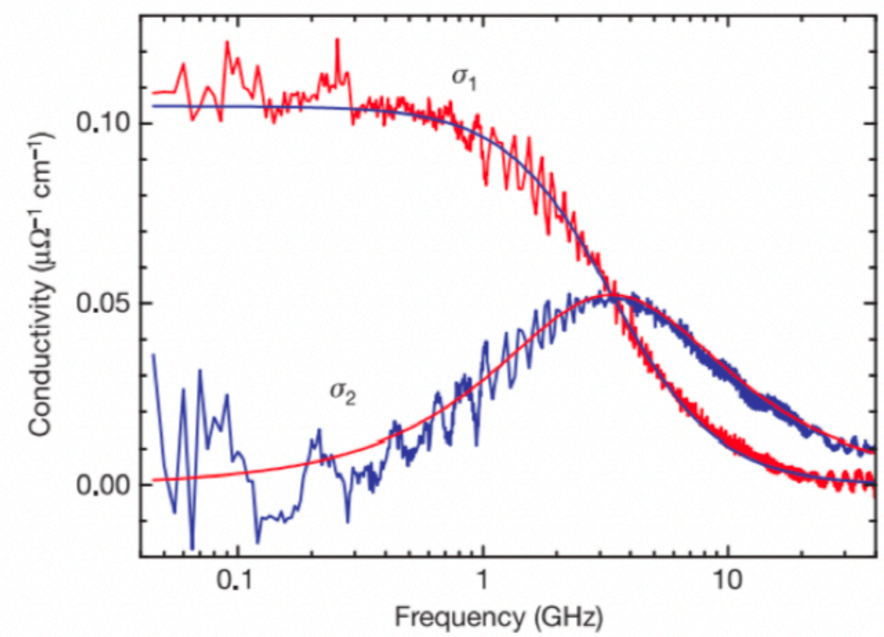



**Figure 2 | Conductivity spectrum of $UPd_2Al_3$ at temperature 2.75 K**

# AdS/Deep learning: optical conductivity

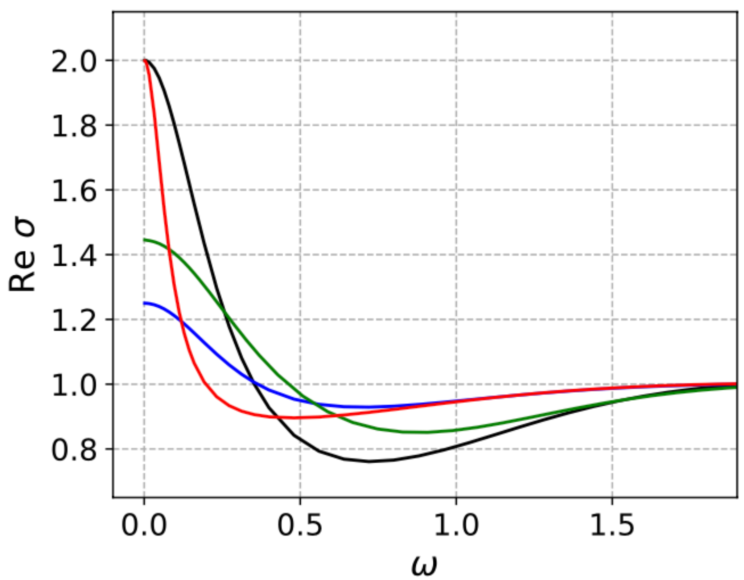
**Setup**

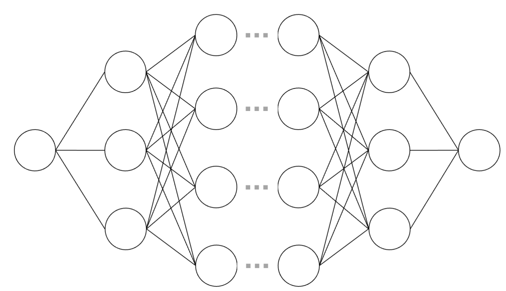
$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left(R + 6 - \frac{1}{4}F_{ab}F^{ab} - \frac{1}{2}\sum_{I=1}^{2}(\partial X_I)^2\right)$$

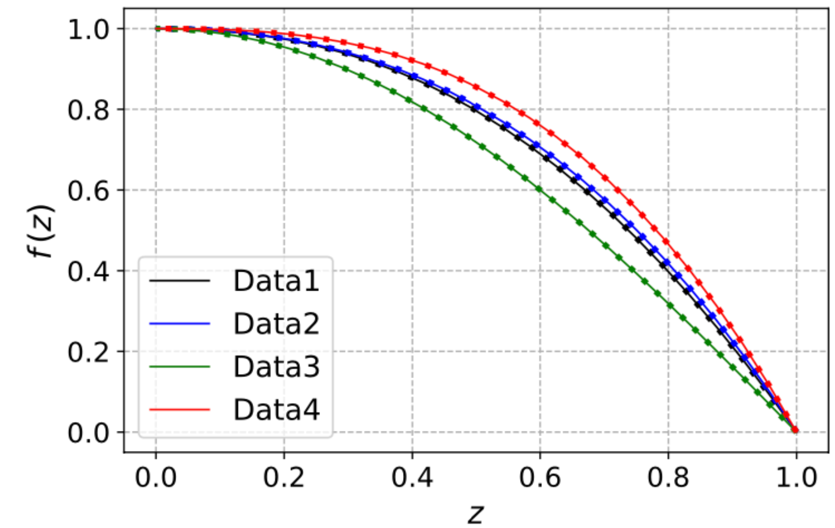
$$\mathrm{d}s^2 = \frac{1}{z^2}\left[-f(z)\mathrm{d}t^2 + \frac{\mathrm{d}z^2}{f(z)} + \mathrm{d}x^2 + \mathrm{d}y^2\right],$$

$$A = \mu\,(1-z)\,\mathrm{d}t\,, \qquad X_1 = \alpha\, x\,, \quad X_2 = \alpha\, y$$

What is the bulk metric giving the conductivity at boundary





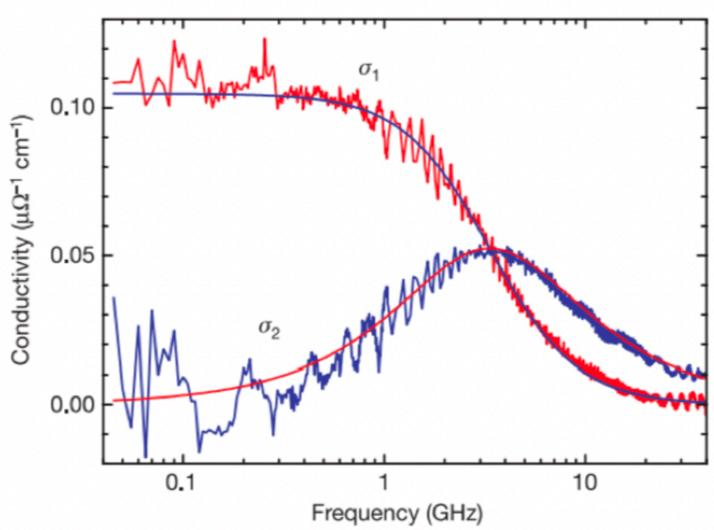
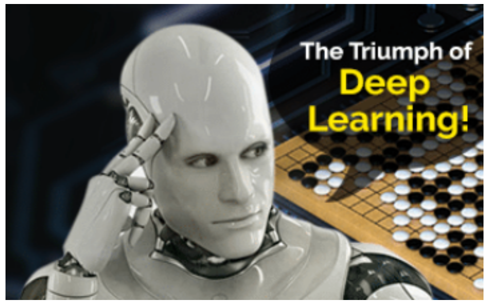
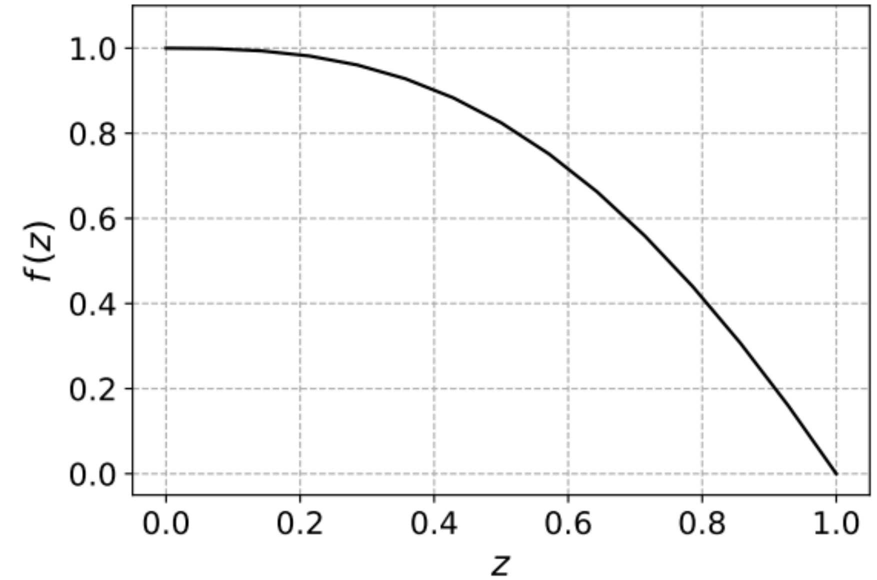




**Figure 2 | Conductivity spectrum of $UPd_2Al_3$ at temperature 2.75 K; both real and imaginary parts ($\sigma_1$ and $\sigma_2$, respectively) are shown.** The fit $(\sigma_1 + i\sigma_2 = \sigma_0(1 - i\omega\tau)^{-1}$, $\sigma_0 = 0.105\,\mu\Omega^{-1}\,\mathrm{cm}^{-1}$, $\tau = 4.8 \times 10^{-11}$ s) documents the excellent agreement of experimental data and the Drude prediction. The characteristic relaxation rate $1/\tau$ is marked by the decrease in $\sigma_1$ and the maximum in $\sigma_2$ around 3 GHz.



Holography

# Deep Learning $\boxed{\text{Bulk Spacetime from Boundary data}}$

- What is “Machine Learning” & “Deep Learning”?

  - Deep Learning 101: standard story

- Physics equation related Deep Learning I

  - Deep Learning for ODE: classical mechanics

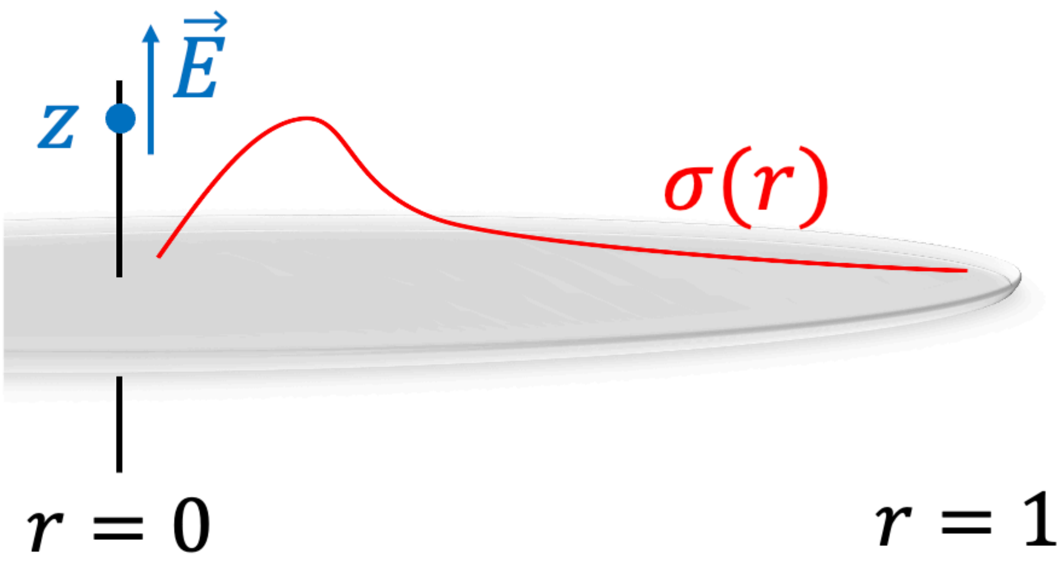
  - AdS/Deep Learning: optical conductivity

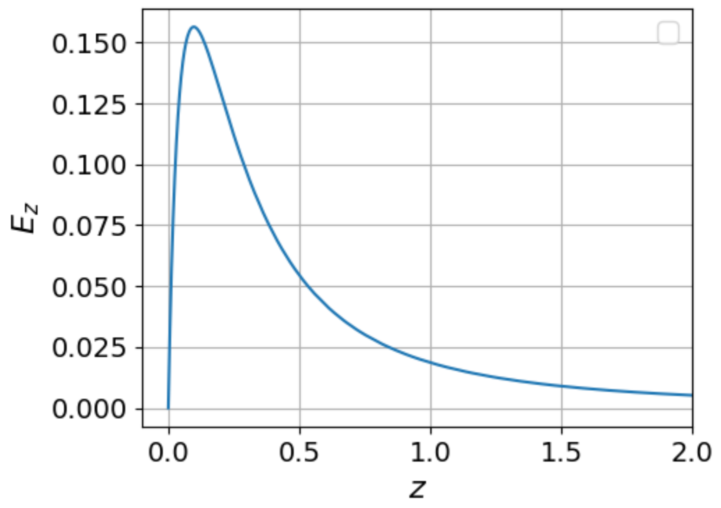
- Physics equation related Deep Learning II

  - Deep Learning for Integral: classical electrostatics

  - AdS/Deep Learning: Entanglement entropy

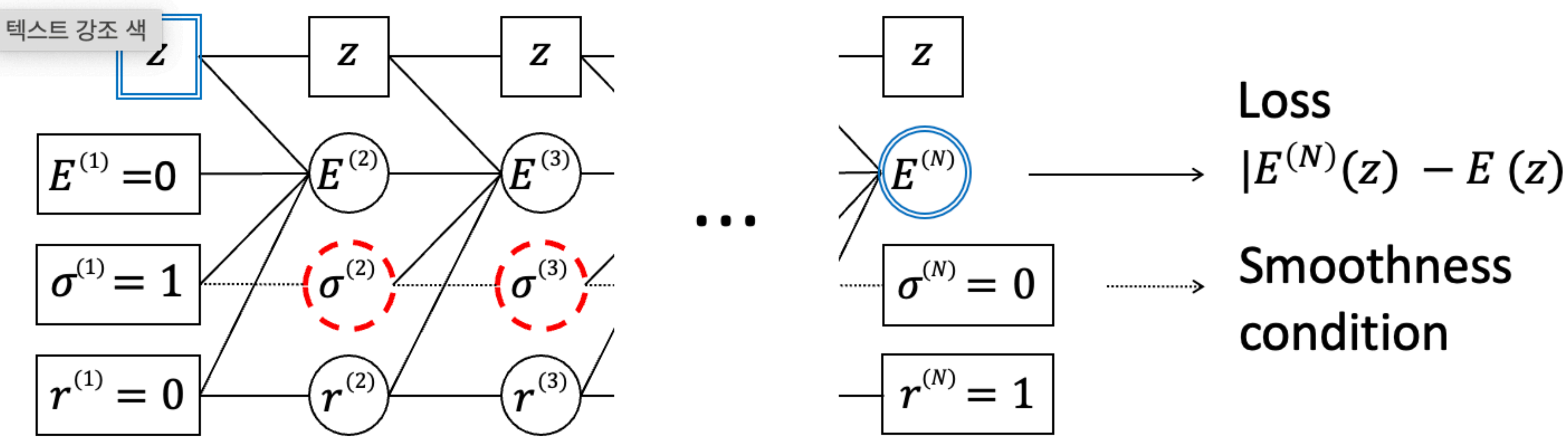
$$E(z) = \int_0^1 dr\ 2\pi r \frac{z\,\sigma(r)}{\sqrt{r^2+z^2}^3}$$



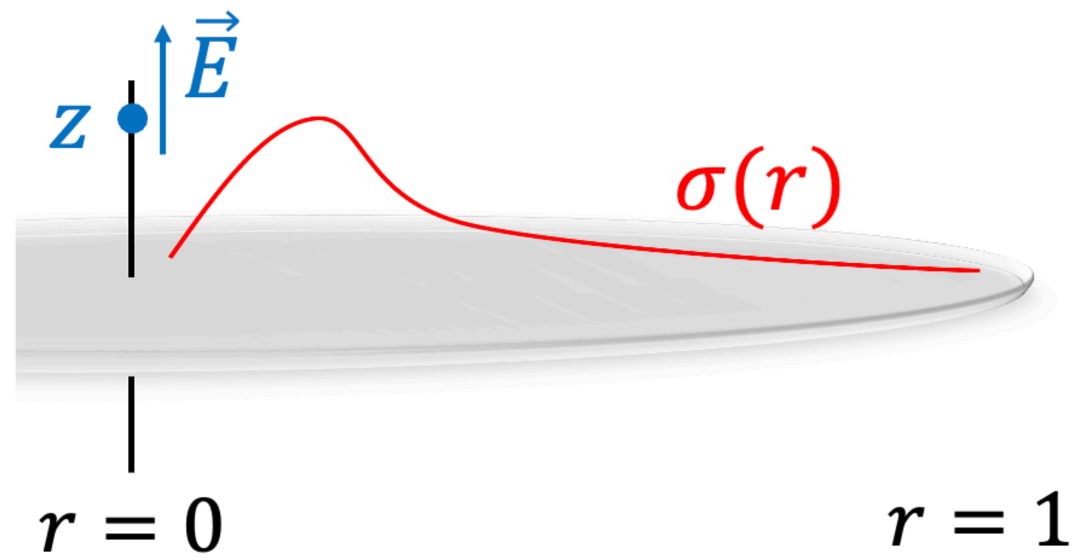



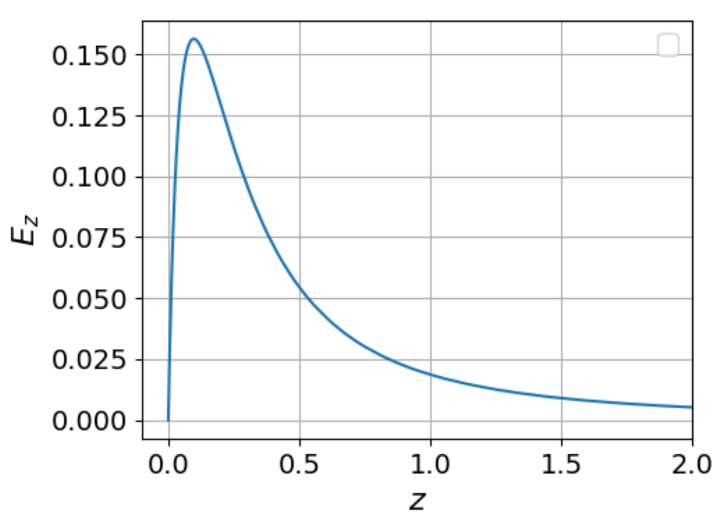
# Deep Learning for integral: electrostatics

$$E^{(n+1)} = E^{(n)} + \Delta r \cdot 2\pi r \frac{z\,\sigma(r)}{\sqrt{r^2+z^2}^3},$$



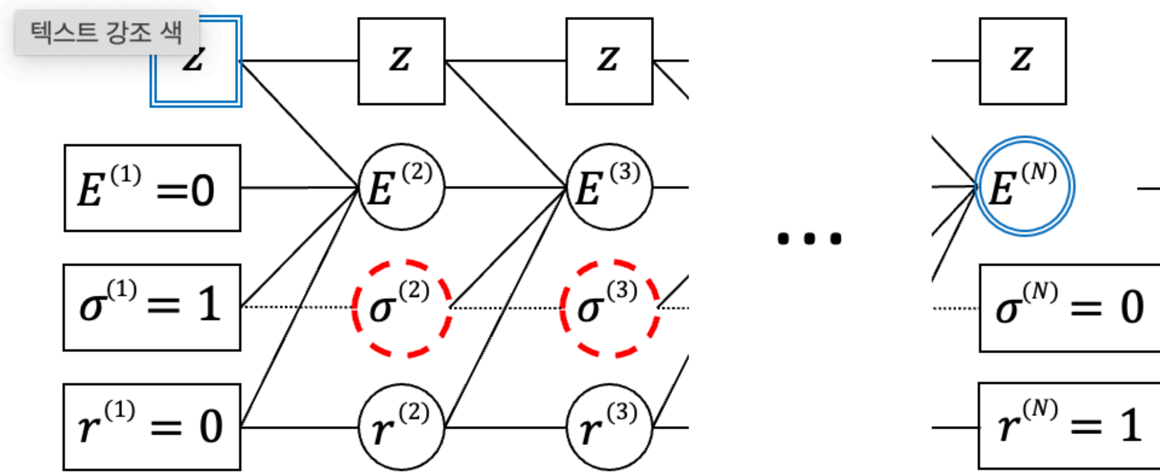

$$E(z) = \int_0^1 dr\ 2\pi r \frac{z\,\sigma(r)}{\sqrt{r^2+z^2}^3}$$



# Deep Learning for integral: electrostatics

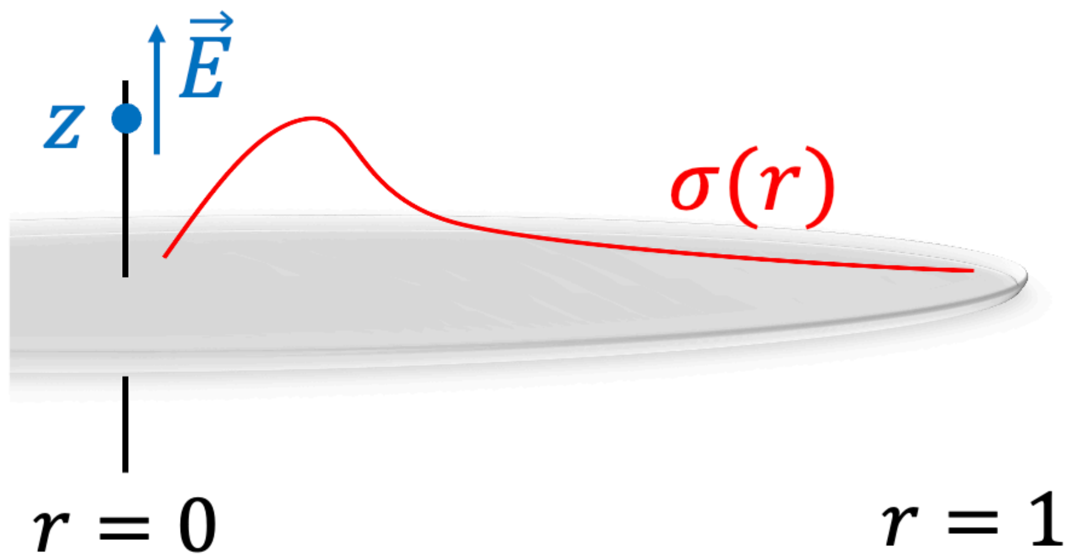
$$E^{(n+1)} = E^{(n)} + \Delta r \cdot 2\pi r \frac{z\,\sigma(r)}{\sqrt{r^2+z^2}^3}\,,$$

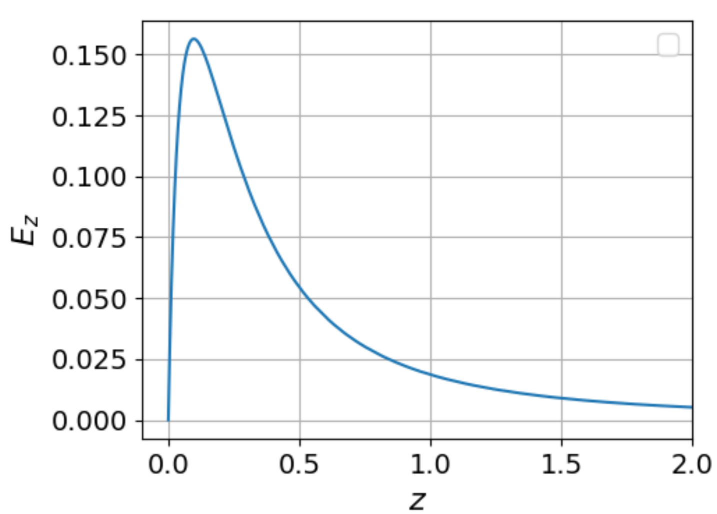


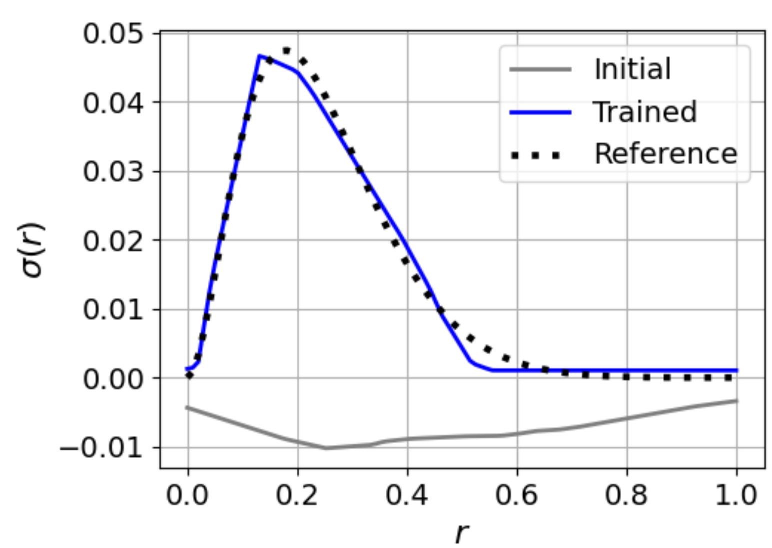

Loss
$|E^{(N)}(z) - E(z)|$

Smoothness condition

$$E(z) = \int_0^1 dr\; 2\pi r \frac{z\,\sigma(r)}{\sqrt{r^2+z^2}^3}$$



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  - AdS/Deep Learning: Entanglement entropy

# AdS/Deep learning: entanglement entropy

$$E(z) = \int_0^1 dr\; 2\pi r \frac{z\,\sigma(r)}{\sqrt{r^2+z^2}^3}$$

$$A(\alpha) = \int F[\, f_1(r;\alpha), f_2(r;\alpha), \cdots ]\, dr$$

$$B(\alpha) = \int G[\, f_1(r;\alpha), f_2(r;\alpha), \cdots ]\, dr$$

$$\ell(z_*) = \int_0^{z_*} dz \frac{2z^2}{\sqrt{z_*^4 - z^4}} \frac{1}{\sqrt{f(z)}}$$

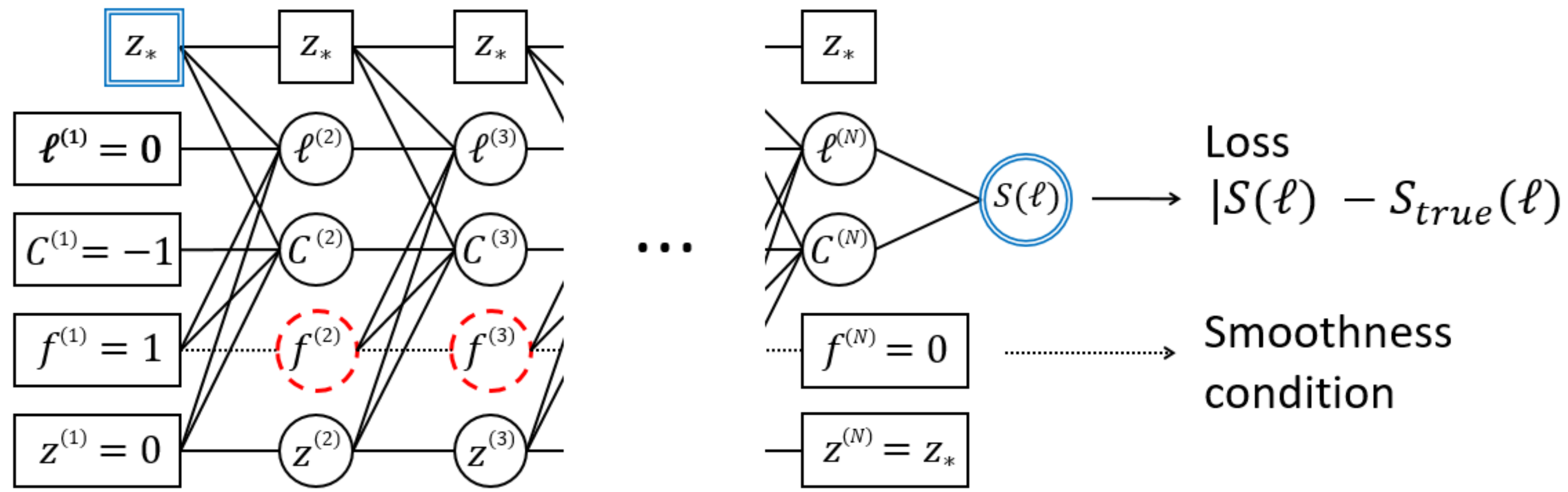
$$C(z_*) := -1 + \int_0^{z_*} dz \cdot \frac{z_*}{z^2} \left( \sqrt{1 - \frac{z^2}{z_*^2}} \sqrt{\frac{1}{f(z)}} - 1 \right)$$

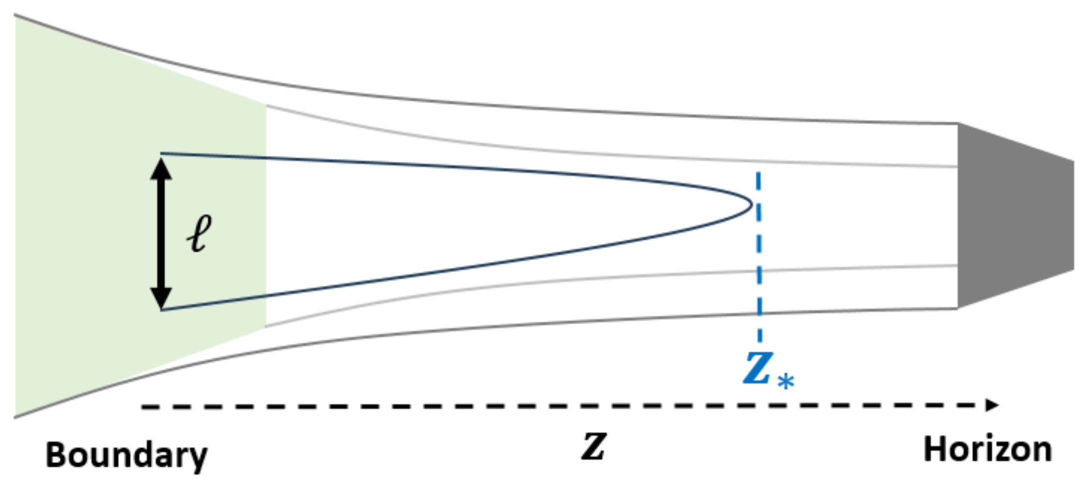
$$\bar{\sigma} := \frac{\sigma(\ell(z_*))}{s} = \frac{1}{z_*^2} + \frac{C(z_*)}{z_*} \frac{2}{\ell(z_*)} + \frac{4\pi}{\ell(z_*)^2} \left( \frac{\Gamma(\frac{3}{4})}{\Gamma(\frac{1}{4})} \right)^2$$

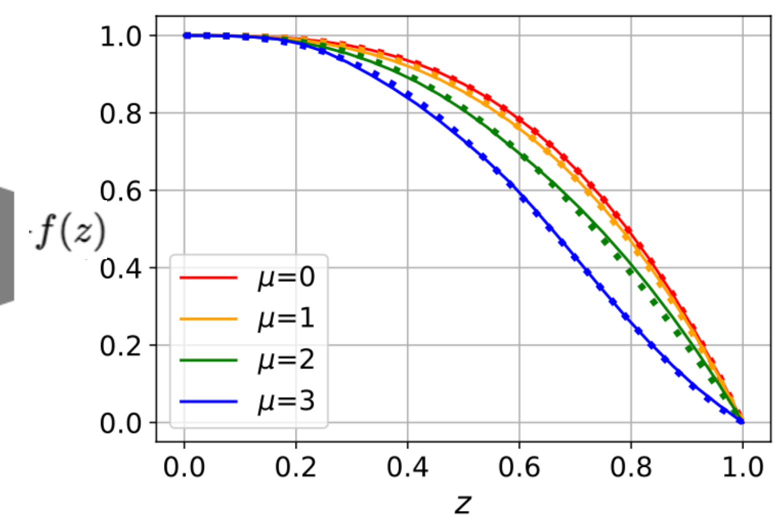
# AdS/Deep learning: entanglement entropy

$$\ell^{(n+1)} = \ell^{(n)} + \Delta z \cdot \frac{2z^2}{\sqrt{(z_*^4 - z^4)\, f(z)}}\,,$$

$$C^{(n+1)} = C^{(n)} + \Delta z \cdot \frac{z_*}{z^2}\left(\sqrt{\left(1-\frac{z^2}{z_*^2}\right)\frac{1}{f(z)}} - 1\right)$$




$$\ell(z_*) = \int_0^{z_*} dz \frac{2z^2}{\sqrt{z_*^4 - z^4}} \frac{1}{\sqrt{f(z)}}$$

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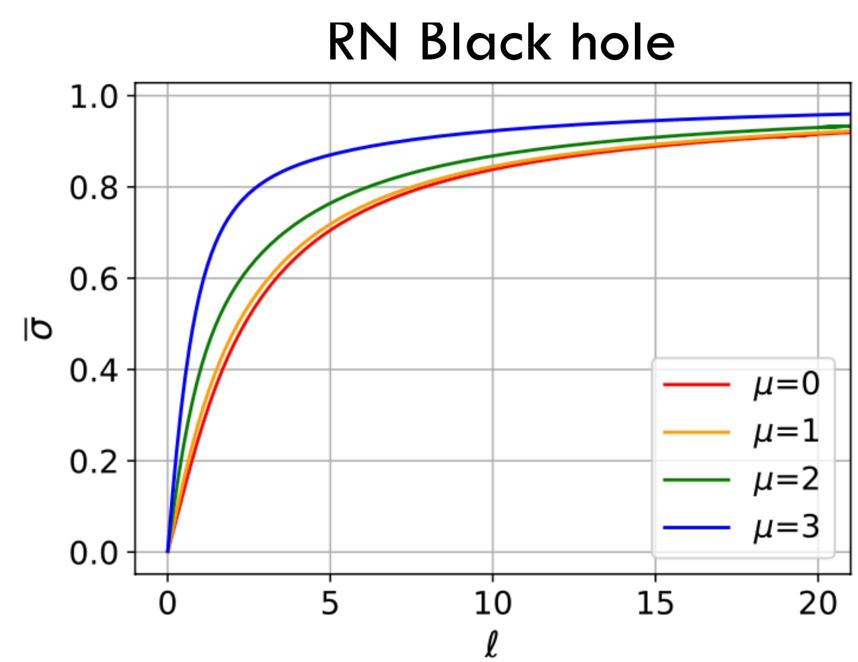


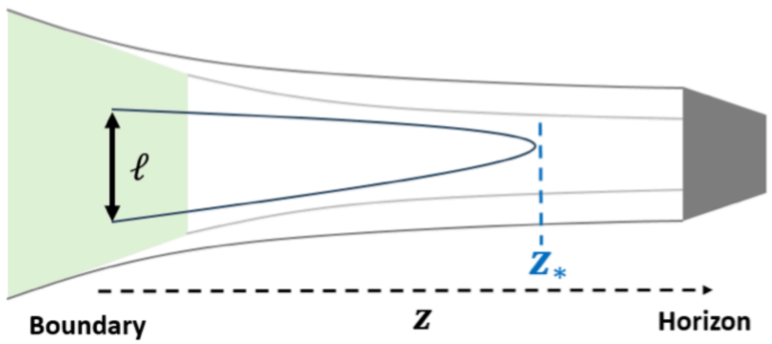

$$\bar{\sigma} := \frac{\sigma(\ell(z_*))}{s} = \frac{1}{z_*^2} + \frac{C(z_*)}{z_*}\frac{2}{\ell(z_*)} + \frac{4\pi}{\ell(z_*)^2}\left(\frac{\Gamma(\frac{3}{4})}{\Gamma(\frac{1}{4})}\right)^2$$

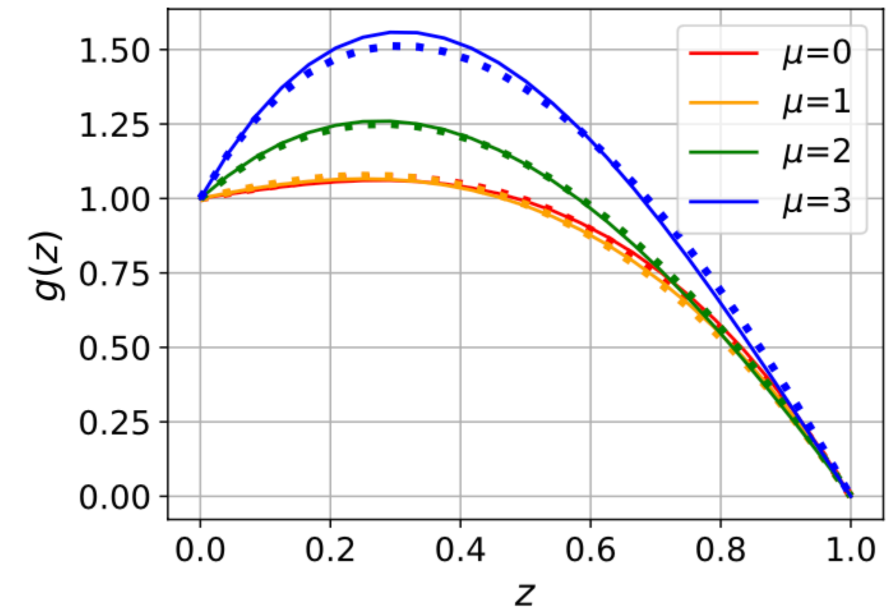
# AdS/Deep learning: entanglement entropy

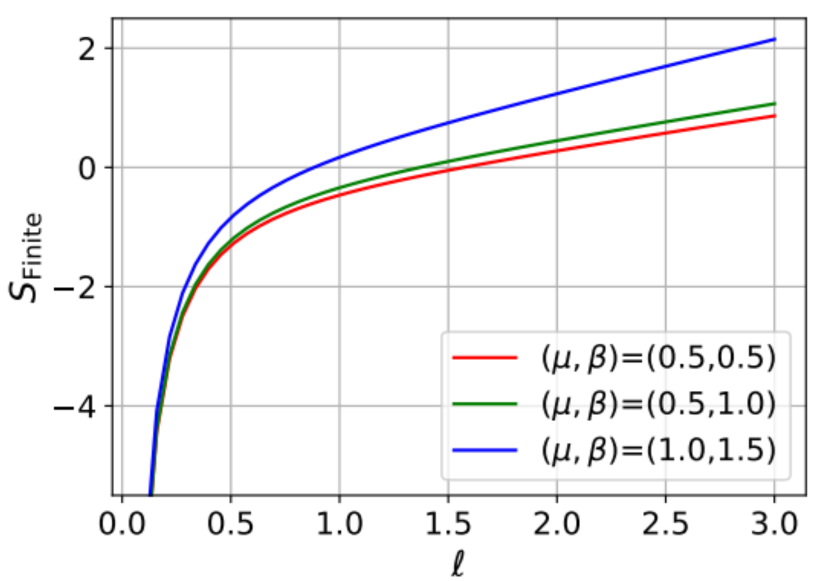
**Setup** Gubser-Rocca case

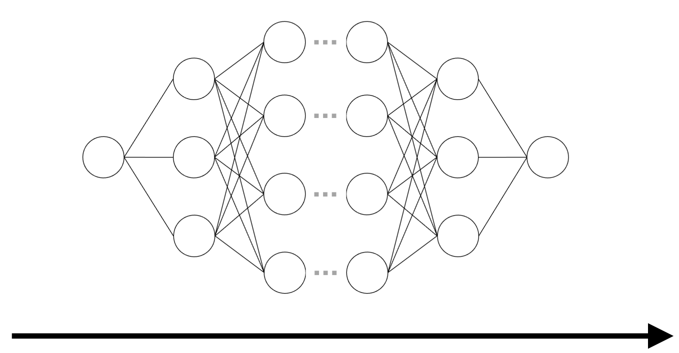
$$S = \int d^4x\sqrt{-g}\left(R + 6\cosh\phi - \frac{1}{4}e^{\phi}F^2 - \frac{3}{2}(\partial\phi)^2 - \frac{1}{2}\sum_{I=1}^{2}(\partial\psi_I)^2\right)$$

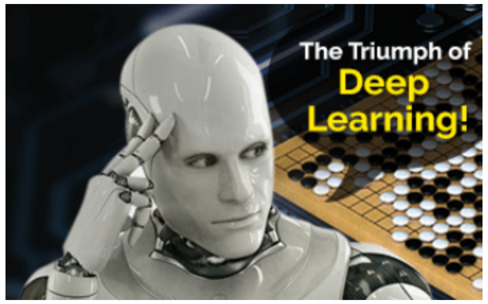
$$ds^2 = \frac{L^2}{z^2}\left[-g(z)dt^2 + \frac{dz^2}{g(z)} + h(z)(dx^2+dy^2)\right]$$

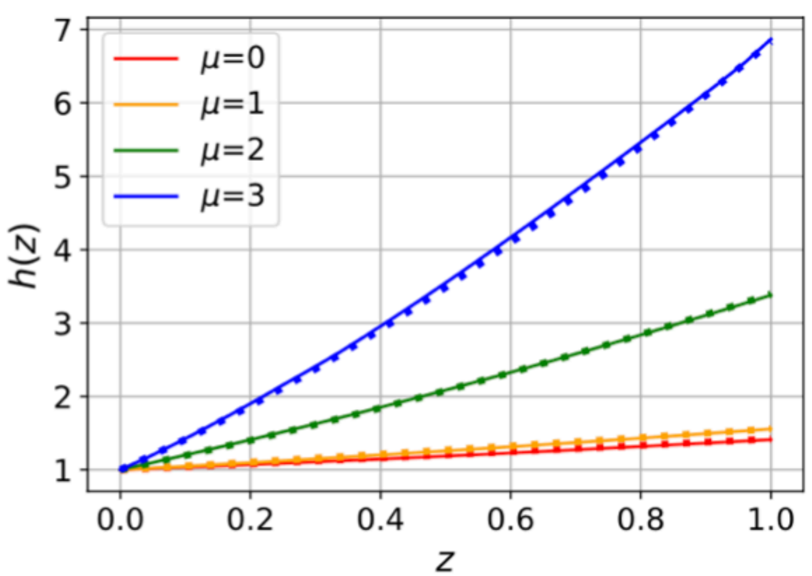




Holography

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  - AdS/Deep Learning: Entanglement entropy

# Ongoing and future work

- Methodology development

  - ResNet,

  - Neural ODE, Neural integral

  - PINN (Physics Informed Neural Network)

  - PDE

- Other physical quantities

  - ARPES: Fermionic spectral function

  - Quantum info: complexity, entanglement entropy, etc

  - Applications to other physics problems (including ODE, PDE, Integral)

- Figuring out action itself for a specific problem

  - so far, the form of the action is fixed

  - Linear T resistivity + $T^2$ Hall angle together

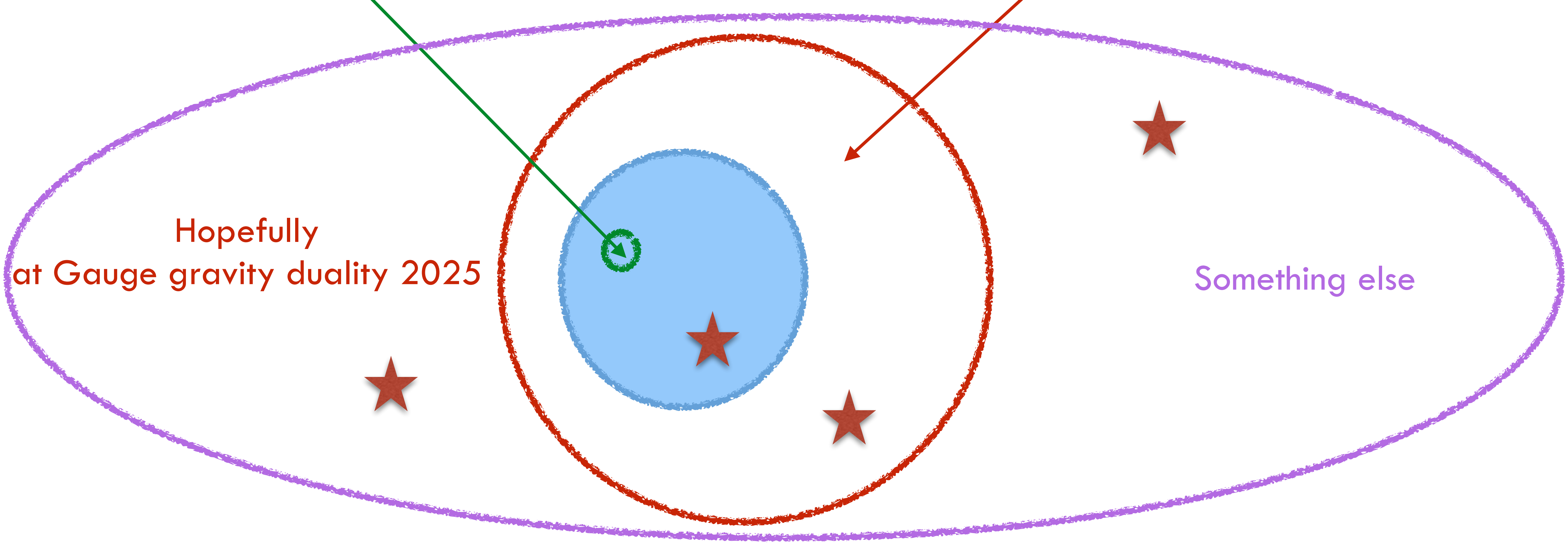
EMD(Einstein Maxwell Dilaton) model

$$S=\int \mathrm{d}^{p+1}x\,\sqrt{-g}\left[R-\frac{1}{2}\partial\phi^2-\frac{1}{4}Z(\phi)F^2+V(\phi)-\frac{1}{2}Y(\phi)\sum_{i=1}^{p-1}\partial\psi_i^2\right].$$

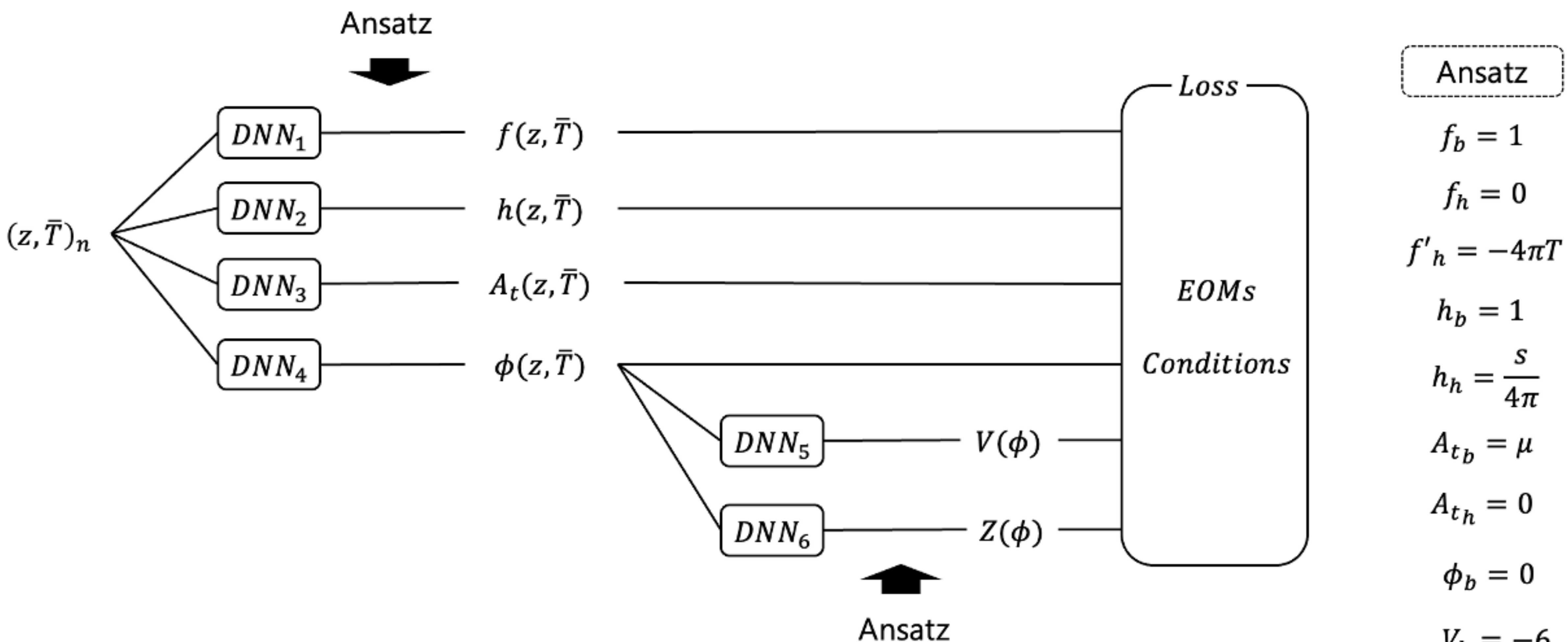
Many variations

$$Z(\phi)\sim e^{\gamma\phi},\qquad V(\phi)\sim V_0 e^{-\delta\phi},\qquad Y(\phi)\sim e^{\lambda\phi}$$

$$\mathrm{d}s^2=r^{\frac{2\theta}{p-1}}\left[-f(r)\frac{\mathrm{d}t^2}{r^{2z}}+\frac{L^2\mathrm{d}r^2}{r^2 f(r)}+\frac{\mathrm{d}\vec{x}^2}{r^2}\right],\quad A=Qr^{\zeta-z}\mathrm{d}t\,,\quad \phi=\kappa\ln r$$

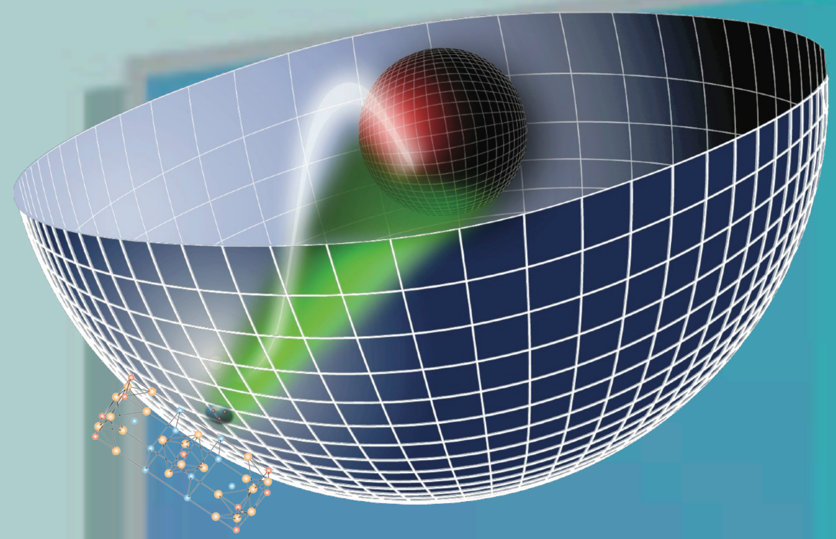


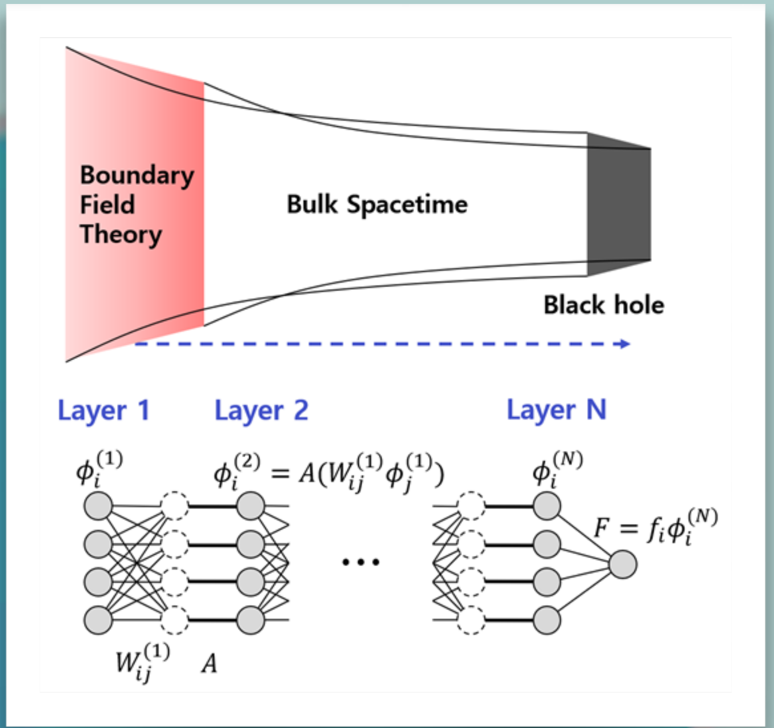
# Towards holographic strange model




$$Loss = \frac{1}{N}\sum_{n=1}^{N}\left( \sum_{i=1}^{4} \left| EOM_i((z,\bar{T})_n) \right| + \sum_j \left| Condition_j((z,\bar{T})_n) \right| \right) + L_{etc}$$

Quantum physics ~ Spacetime






Thank you