# Three-point Functions in ABJM Theory and Integrable Boundary States

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Based on **JW** and Peihe Yang, [arXiv: 2408.03643]

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# Introduction

- In conformal field theories, higher point functions of local operators are determined by the two-point and three-point functions.
- The conformal dimensions of the operators (determining the two-point functions) and structure constants (OPE coefficients) are called conformal data.
- At weak coupling, they can be computed using perturbation theory.
- For theories with holographic duals, the strong coupling limit of conformal data can be calculated using weakly coupling gravity or string theory.

- We hope to compute the conformal dimensions and OPE coefficients non-perturbatively in the field theory side.
- It is intensively needed to non-trivially verify the prediction of AdS/CFT correspondence. [Maldacena, 97][Gubser, Klebanov, Polyakov, 98][Witten, 98]
- It is also needed when the coupling constants are around order one.

# Introduction

- Usually it is very hard to perform such non-perturbative calculations.
- In the last $20+$ years, many non-perturbative tools in the field theory have been developed. These methods include integrability, supersymmetric localization, conformal bootstrap, ...
- For $\mathcal{N}=4$ super Yang-Mills and ABJM theories, integrability is a very powerful non-perturbative tools (mainly in the planar limit).
- This also applies to some cousins (orbifolds, $\beta/\gamma$-deformations, fishnet theories...) of the $\mathcal{N}=4$ SYM and ABJM theories.

# Introduction

- $AdS_5/CFT_4$ correspondence states that four-dimensional $\mathcal{N} = 4$ super Yang-Mills theory is dual to type IIB superstring theory on $AdS_5 \times S^5$.
- In the field theory side, integrability means that, in the large N limit, the anomalous dimension matrix (a. k. a. the dilatation operator) of composition operators obtained from perturbative computations gives an integrable Hamiltonian. [Minahan, Zarembo, 02]. $\cdots$
- In the string theory side, integrability means that the worldsheet theory of type IIB superstring on $AdS_5 \times S^5$ in the free limit is a two-dimensional integrable field theory. [Bena, Polchinski, Roiban, 03]

# Introduction

- It is reasonable to expect that the integrable structure exists for an arbitrary 't Hooft coupling in the large N limit.
- The case for ABJM theory is in the same spirit but much more complicated and hard. [Minahan, Zarembo, 08][Bak, Rey, 08][Gromov, Vieira, 08], $\cdots$

# Introduction

- The spectral problem in planar $\mathcal{N}=4$ SYM and ABJM is essentially solved by using the quantum spectral curve (QSC) method. [Gromov, Kazakov, Leurent, Volin, 14], $\cdots$
- Using integrability, people also made great progress on three point functions ([Escobedo, Gromov, Sever, Vieira, 10], $\cdots$, [Basso, Komatsu, Vieira, 15], $\cdots$) in $\mathcal{N}=4$ SYM.
- However the three-point functions of single-trace operators in ABJM theory from integrability were reraly studied. Before our work, only the correlators in the $SU(2)\times SU(2)$ sector were studied [Bissi, Kristjansen, Martirosyan, Orselli, 12].
- In this talk, we will study a class of three-point functions in ABJM theory from the viewpoint of integrable boundary states.

# Integrable boundary state (IBS)

- Integrable boundary states [Piroli, Pozsgay, Vernier, 17] of spin chain play an important role in both quantum quench dynamics and AdS/CFT correspondence. (Integrable boundary states in field theory were first studied in [Ghoshal, Zamolodchikov 93].)
- In the study of quantum quench, one need to compute the overlap of $\langle \mathbf{u}_1, \cdots, \mathbf{u}_r | \Psi \rangle$, with $\langle \mathbf{u}_1, \cdots, \mathbf{u}_r |$ is normalized eigen-state of an integrable $H$ and $|\Psi\rangle$ certain state in the Hilbert space.
- Here $\mathbf{u_1}, \cdots, \mathbf{u_r}$ are Bethe roots satisfying certain Bethe ansatz equations.

# IBS

- If $|\Psi\rangle$ satisfies certain integrable conditions, then the computations of $\langle \mathbf{u}_1, \cdots, \mathbf{u}_r|\Psi\rangle$ can be greatly simplified.
- In $\mathcal{N}=4$ SYM and ABJM theory many correlation functions can be also expressed as the overlap between a boundary state $\langle\Psi|$ and a Bethe state $|\mathbf{u_1}, \cdots, \mathbf{u_r}\rangle$.
- If this boundary state $\langle\Psi|$ is integrable, we may have compact formulas for such correlation functions.

# ABJM theory

- Aharony-Bergman-Jafferis-Maldacena (ABJM) theory is a $3d$ $\mathcal{N}=6$ Chern-Simons-matter theory.
- The gauge group is $U(N)\times U(N)$ with CS levels $(k,-k)$.
- The gauge fields are denoted by $A_\mu$ and $\hat{A}_\mu$, respectively.
- The matter fields include complex scalars $Y^I$ and spinors $\psi_I$ ($I=1,\cdots,4$) in the bi-fundamental representation of the gauge group.
- ABJM gave strong evidence to support this theory to be the low energy effective theory of $N$ M2-branes putting at the tip of $\mathbf{C}^4/\mathbf{Z}_k$.

# Quiver diagram of ABJM theory

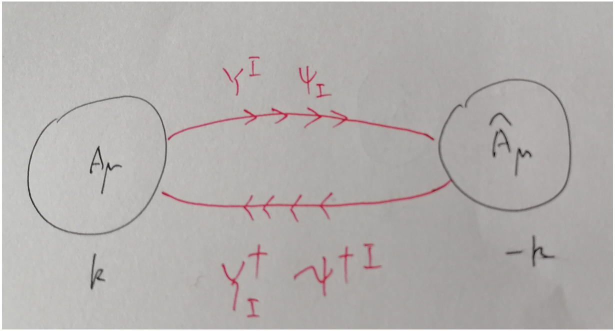


Figure: The quiver diagram of ABJM theory.

# IBS in ABJM theory

- $\langle \mathcal{D}^\circ \mathcal{D}^\circ \mathcal{O} \rangle$, here $\mathcal{D}^\circ$'s are BPS determinant operators (dual to giant gravitons), and $\mathcal{O}$ is a generic non-BPS single-trace operator. [Yang, Jiang, Komatsu, **JW**, 21]
- domain wall one-point functions. [Kristjansen, Vu, Zarembo, 21]
- $\langle W[C] O \rangle$, with $W[C]$ a certain BPS Wilson loop. [Jiang, **JW**, Yang, 23]
- $\langle \mathcal{O}^\circ \mathcal{O}^\circ \mathcal{O} \rangle$, with $\mathcal{O}^\circ$'s being BPS single-trace operators. [**JW**, Yang, 24], **this talk**
- A class of integrable matrix product states were recently constructed [Bai, Shao, 24]. Applications to the ABJM theory are to be studied.

# Single-trace operators

- The single-trace operator in the scalar sector of the ABJM theory is
$$\mathcal{O}_C = C^{J_1 \cdots J_L}_{I_1 \cdots I_L} \mathrm{Tr}(Y^{I_1} Y^{\dagger}_{J_1} \cdots Y^{I_L} Y^{\dagger}_{J_L}) \,. \quad (1)$$
- The cyclicity property of the trace can be used to choose $C$ to be invariant under the following simultaneous cyclic shift of the upper and lower indices, $C^{J_1 \cdots J_L}_{I_1 \cdots I_L} = C^{J_2 \cdots J_L J_1}_{I_2 \cdots I_L I_1}$.

# Chiral Primary Operators

- When the tensor $C$ is invariant under the respective permutations among the upper and the lower indices, and traceless,

$$C^{J_1\cdots J_L}_{I_1\cdots I_L} = C^{J_1\cdots J_L}_{(I_1\cdots I_L)} = C^{(J_1\cdots J_L)}_{I_1\cdots I_L}\,,\; C^{J_1\cdots J_L}_{I_1\cdots I_L}\delta^{I_1}_{J_1} = 0\,, \tag{2}$$

  the operator $\mathcal{O}_C$ is a $1/3$-BPS operator.

- A natural choice of such symmetric traceless tensor $C$ is in terms of polarization vectors $n_I$ and $\bar{n}^I$,

$$C^{J_1\cdots J_L}_{I_1\cdots I_L} = n_{I_1}\cdots n_{I_L}\bar{n}^{J_1}\cdots\bar{n}^{J_L}\,, \tag{3}$$

  with BPS condition $n\cdot\bar{n} = 0$.

- Notice that $\bar{n}$ does not need to be the complex conjugation of $n$.

# Two point functions

- With this choice, the BPS operator becomes

$$\mathcal{O}^{\circ}_{L}(x,n,\bar{n}) = \mathrm{tr}\left(\left(n\cdot Y \bar{n}\cdot Y^{\dagger}\right)^{L}\right). \tag{4}$$

- The two-point function of $\mathcal{O}^{\circ}_{L}$'s is constrained by symmetries to take the form,

$$\langle \mathcal{O}^{\circ}_{L_1}(x_1)\mathcal{O}^{\circ}_{L_2}(x_2)\rangle = \delta_{L_1,L_2}\mathcal{N}_{\mathcal{O}^{\circ}_{L_1}}(d_{12}d_{21})^{L_1}, \tag{5}$$

with the definitions

$$d_{ij} = \frac{n_i\cdot \bar{n}_j}{|x_{ij}|}\,,\; x_{ij} = x_i - x_j\,. \tag{6}$$

- At tree-level in the planar limit, we have

$$\mathcal{N}_{\mathcal{O}^{\circ}_{L}} = L\lambda^{2L}\,. \tag{7}$$

# Non-BPS Operators

- We consider a non-BPS operator

$$\mathcal{O}_C = C^{J_1 \cdots J_L}_{I_1 \cdots I_L} \mathrm{Tr}(Y^{I_1} Y^{\dagger}_{J_1} \cdots Y^{I_L} Y^{\dagger}_{J_L}) \,. \quad (8)$$

which can be mapped to the following state

$$|\mathcal{O}_C\rangle = C^{J_1 J_2 \cdots J_L}_{I_1 I_2 \cdots I_L} |I_1, \bar{J}_1, \cdots, I_L, \bar{J}_L\rangle \,, \quad (9)$$

of the $SU(4)$ alternating spin chain. The Hamiltonian on this spin chain is from the planar two-loop dilatation operator in the scalar sector. This Hamiltonian has been proven to be integrable [Minahan, Zarembo, 08][Bak, Rey, 08]. And the above state is taken as one of the eigen-states of this Hamiltonian.

- Generically this state can be parametrized by the solution $\mathbf{u}, \mathbf{w}, \mathbf{v}$ to the Bethe ansatz equations and zero-momentum condition,

$$|\mathcal{O}\rangle = |\mathbf{u}, \mathbf{w}, \mathbf{v}\rangle \,. \quad (10)$$

# Twisted-translated frame

- We pick up a special class of three-point functions in the scalar sector by considering the twisted-translated frame.
- We put all operators along the line $x^\mu = (0, 0, a)$.
- When we translate an operator from the origin to the point $(0, 0, a)$, we perform the following transformation,

$$Y^1 \to Y^1 + \kappa a Y^4, \; Y_4^\dagger \to Y_4^\dagger - \kappa a Y_1^\dagger. \tag{11}$$

- From now on, we choose $\kappa = 1$. The $\kappa$-depedenece can be recovered by dimensional analysis.

# Three-point functions

- Based on the conformal symmetry and R-symmetry, the normalized correlation function of three generic single-trace operators in the twisted-translated frame should take the following form,
$$\frac{\langle \hat{\mathcal{O}}_1(a_1)\hat{\mathcal{O}}_2(a_2)\hat{\mathcal{O}}_3(a_3)\rangle}{\sqrt{\mathcal{N}_{\mathcal{O}_1}\mathcal{N}_{\mathcal{O}_2}\mathcal{N}_{\mathcal{O}_3}}} = \frac{\mathcal{C}_{123}}{a_{12}^{\gamma_{12|3}}a_{23}^{\gamma_{23|1}}a_{31}^{\gamma_{31|2}}}\,, \tag{12}$$
where
$$\gamma_{ij|k} := (\Delta_i + \Delta_j - \Delta_k) - (J_i + J_j - J_k)\,, \tag{13}$$
and $J$ is a $U(1)$ R-charge which assigns charges $(1/2, 0, 0, -1/2)$ to $Y^1, \cdots, Y^4$. [Kazama, Komatsu, Nishimura, 14][Yang, Jiang, Komatsu, **JW**, 21]

# Three-point functions

- The main focus of this talk is on three-point functions of two $1/3$-BPS single-trace operators $\hat{\mathcal{O}}^{\circ}_{L_i}, i = 1, 2$ and one non-BPS operator $\hat{\mathcal{O}}_3$.
- For this special case we have

$$\frac{\langle \hat{\mathcal{O}}^{\circ}_1(a_1)\hat{\mathcal{O}}^{\circ}_2(a_2)\hat{\mathcal{O}}_3(a_3)\rangle}{\sqrt{\mathcal{N}_{\mathcal{O}^{\circ}_1}\mathcal{N}_{\mathcal{O}^{\circ}_2}\mathcal{N}_{\mathcal{O}_3}}} = \mathcal{C}_{123}\left(\frac{a_{12}}{a_{23}a_{31}}\right)^{\Delta_3 - J_3} . \qquad (14)$$

# Three point functions

- In the large $N$ limit, the tree-level three-point function

$$\langle \hat{\mathcal{O}}_1^{\circ}(a_1)\hat{\mathcal{O}}_2^{\circ}(a_2)\hat{\mathcal{O}}_3(a_3)\rangle \,, \tag{15}$$

  is computed from planar Wick contractions, where $\sum_{i=1}^{3} L_i$ pairs of fields are contracted.
- Without loss of generality, we set $a_3 = 0$ from now on.
- Let us denote the number of the contractions between operators $\mathcal{O}_i$ and $\mathcal{O}_j$ by $l_{ij|k}, k \neq i, j$. It is straightforward to see that $l_{ij|k} = L_i + L_j - L_k$.
- Notice that $l_{12|3}, l_{23|1}, l_{31|2}$ always have the same odevity. This behavior contrasts with that in $\mathcal{N} = 4$ SYM theory.

# Wick contractions

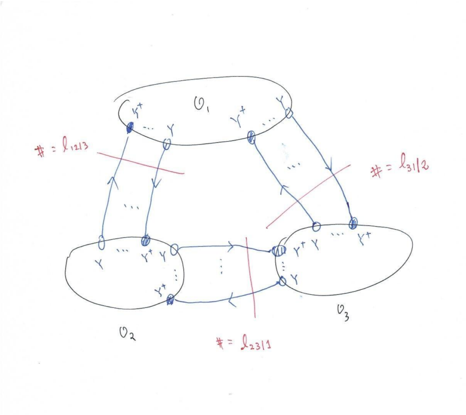


Figure: Tree-level planar Wick contractions.

# Boundary states

- The three point function $\langle \hat{\mathcal{O}}_1^\circ(a_1)\hat{\mathcal{O}}_2^\circ(a_2)\hat{\mathcal{O}}_3(a_3)\rangle$ can be expressed using the overlap between a boundary state and a Bethe state. When $l_{ij|k}$'s are even and $2 \leq l_{31|2} \leq 2L_3 - 2$, we have

$$\langle \hat{\mathcal{O}}_1^\circ(a_1)\hat{\mathcal{O}}_2^\circ(a_2)\hat{\mathcal{O}}_3(a_3)\rangle = \frac{(-1)^{\frac{l_{12|3}}{2}} L_1 L_2 \lambda^{\sum_{i=1}^3 L_i}}{N|a_1|^{l_{31|2}}|a_2|^{l_{23|1}}} \langle \mathcal{B}^{\text{even}}_{l_{31|2}}|\mathbf{u},\mathbf{w},\mathbf{v}\rangle \, . \tag{16}$$

# Boundary states

- To make the structure of $|\mathcal{B}^{\text{even}}_{l_{31|2}}\rangle$ clear, We first define

$$\begin{aligned}&\langle \bar{n}_1@\{1,2,\cdots,m\}, n_1@\{1,2,\cdots,m\}| = \\ &(\bar{n}_1)^{I_1}(n_1)_{J_1}\cdots(\bar{n}_1)^{I_m}(n_1)_{J_m}(\bar{n}_2)^{I_{m+1}}(n_2)_{J_{m+1}} \\ &\cdots(\bar{n}_2)^{I_1}(n_2)_{J_L}\langle I_1,J_1,\cdots,I_L,J_L|\,.\end{aligned} \tag{17}$$

- And

$$\begin{aligned}&U_{\text{even}}|I_1,J_1,I_2,J_2\cdots,I_{L-1},J_{L-1},I_L,J_L\rangle \\ &=|I_1,J_2,I_2,J_3,\cdots,I_{L-1},J_L,I_L,J_1\rangle\,,\end{aligned} \tag{18}$$

$$\begin{aligned}&U_{\text{odd}}|I_1,J_1,I_2,J_2\cdots,I_{L-1},J_{L-1},I_L,J_L\rangle \\ &=|I_2,J_1,I_3,J_2,\cdots,I_L,J_{L-1},I_1,J_L\rangle\,,\end{aligned} \tag{19}$$

where $U_{\text{even}}$ has already been introduced in [Jiang, **JW**, Yang, 23].

# Boundary states

- Then the boundary state $|\mathcal{B}^{\text{even}}_{l_{31|2}}\rangle$ in the considered case is,

$$\langle\mathcal{B}^{\text{even}}_{l_{23|1}}| = \langle\mathcal{B}^{\text{even},\,a}_{l_{23|1}}| + \langle\mathcal{B}^{\text{even},\,b}_{l_{23|1}}| \tag{20}$$

with

$$\langle\mathcal{B}^{\text{even},\,a}_{l}| = \sum_{j=0}^{L-1}\langle\bar{n}_1@\{1,2,\cdots,l/2\}, n_1@\{1,2,\cdots,l/2\}|(U_{\text{even}}U_{\text{odd}})^j\,, \tag{21}$$

$$\langle\mathcal{B}^{\text{even},\,b}_{l}| = \langle\mathcal{B}^{\text{even},\,a}_{l}|U_{\text{even}}\,. \tag{22}$$

# Boundary states

- When $l_{31|2} = 0$, the boundary state is

$$\langle \mathcal{B}_0^{\text{even}}| = L(\bar{n}_2)^{I_1}(n_2)_{J_1} \cdots (\bar{n}_2)^{I_L}(n_2)_{J_L}\langle I_1, J_1, \cdots, I_L, J_L| \,. \quad (23)$$

- Then boundary state for the case $l_{31|2} = 2L_3$ is

$$\langle \mathcal{B}_{2L_3}^{\text{even}}| = L\langle n_1@\{1,2,\cdots,L_3\}, \bar{n}_1@\{1,2,\cdots,L_3\}| \,. \quad (24)$$

- In these scenarios, we have either $L_2 = L_3 + L_1$ or $L_1 = L_2 + L_3$, which makes the three-point function an extremal one.
- For these cases, the computation of correlators must account for the mixing of $\mathcal{O}_2$ or $\mathcal{O}_1$ with double trace operators, even in the large $N$ limit. [D'Hoker, Freedman, Mathur, 99]
- We temporarily ignore this mixing.

# Boundary states

- When $l_{31|2}$ is odd, corresponding boundary states have similar structure.
- For example, when $l_{31|2} = 1$, we have

$$\langle \mathcal{B}_1^{\mathrm{odd}}| = \langle \mathcal{B}_1^{\mathrm{odd},\,a}| - \langle \mathcal{B}_1^{\mathrm{odd},\,b}|\,, \tag{25}$$

$$\langle \mathcal{B}_1^{\mathrm{odd},\,a}| = \sum_{l=1}^{L} \langle \bar{n}_1 @ l|\,, \tag{26}$$

$$\langle \mathcal{B}_1^{\mathrm{odd},\,b}| = \sum_{l=1}^{L} \langle n_1 @ l|, \tag{27}$$

# Integrable boundary states

- We want to know among the boundary states $|\mathcal{B}\rangle$, which are integrable.
- The result [Yang, 22][Yang, **JW**, 24] is that when $l_{31|2} = 0, 2L_3$ (special extremal) or when $l_{31|2} = 1, 2L_3 - 1$ (special next-to-extremal), the boundary state satisfies the following twisted integrable condition,

$$\tau(\lambda)|\mathcal{B}\rangle = \tau(-2-\lambda)|\mathcal{B}\rangle\,. \tag{28}$$

- Here $\tau(\lambda)$ is one of the transfer matrices (generating functions of conserved charges) of the ABJM spin chain.
- This twisted integrable condition leads to the selection rule that the necessary condition for $\langle \mathbf{u}, \mathbf{w}, \mathbf{v}|\mathcal{B}\rangle$ being non-zero is that $\mathbf{u} = -\mathbf{v}, \mathbf{w} = -\mathbf{w}$ as equations for sets $\mathbf{u}, \mathbf{w}, \mathbf{v}$.

# Special extremal correlators

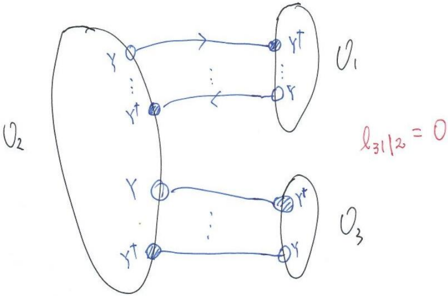


Figure: Special extremal correlators.

# Special next-to-extremal correlators

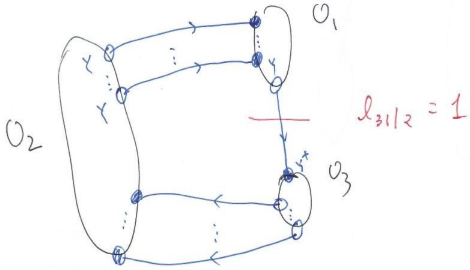


Figure: Special next-to-extremal correlators.

# Integrable boundary states

- $|\mathcal{B}_0^{\text{even}}\rangle$ is integrable known from [Yang, 22]
- We ([Yang, **JW**, 24]) managed to prove that both $|\mathcal{B}_1^{\text{odd},\,b}\rangle$ and $|\mathcal{B}_1^{\text{odd},\,a}\rangle$ satisfy the conditions (28). So they are integrable, as well as $|\mathcal{B}_1^{\text{odd}}\rangle$. This is one of our main results.

# Integrable boundary states

- In the proof, we do not use the BPS conditions, $n_1 \cdot \bar{n}_1 = n_2 \cdot \bar{n}_2 = 0$, letting along the twisted-translated frame.
- So this proof show that $|\mathcal{B}_l^{\mathrm{odd},\,a}\rangle$ and $|\mathcal{B}_l^{\mathrm{odd},\,b}\rangle$ are integrable for $l = 1$ or $l = 2L_3 - 1$ without any constraints on $n_i$'s and $\bar{n}_i$'s.

# The overlaps

- The twisted-translated frame leads to the selection rule,

$$N_{\mathbf{u}} = N_{\mathbf{v}} = N_{\mathbf{w}} \,. \tag{29}$$

- The following result from symmetry

$$\frac{\langle \hat{\mathcal{O}}_1^{\circ}(a_1)\hat{\mathcal{O}}_2^{\circ}(a_2)\hat{\mathcal{O}}_3(a_3)\rangle}{\sqrt{\mathcal{N}_{\mathcal{O}_1^{\circ}}\mathcal{N}_{\mathcal{O}_2^{\circ}}\mathcal{N}_{\mathcal{O}_3}}} = \mathcal{C}_{123}\left(\frac{a_{12}}{a_{23}a_{31}}\right)^{\Delta_3 - J_3} \,, \tag{30}$$

further leads to the constraints,

$$N_{\mathbf{u}} \leq \min(l_{31|2}, l_{23|1}) \,. \tag{31}$$

- Using this constraint and coordinate Bethe ansatz (CBA), all overlaps from the three point functions in the integrable cases are computed.

# An example

- Consider the case with $l_{31|2} = N_{\mathbf{u}} = N_{\mathbf{w}} = N_{\mathbf{v}} = 1$, the Bethe ansatz equations are

$$1 = \left(\frac{u+\frac{\mathrm{i}}{2}}{u-\frac{\mathrm{i}}{2}}\right)^{L_3} \frac{u-w+\frac{\mathrm{i}}{2}}{u-w-\frac{\mathrm{i}}{2}}, \quad (32)$$

$$1 = \frac{w-u+\frac{\mathrm{i}}{2}}{w-u-\frac{\mathrm{i}}{2}} \quad (33)$$

$$1 = \left(\frac{v+\frac{\mathrm{i}}{2}}{v-\frac{\mathrm{i}}{2}}\right)^{L_3} \frac{v-w+\frac{\mathrm{i}}{2}}{v-w-\frac{\mathrm{i}}{2}}, \quad (34)$$

and the zero-momentum condition is

$$1 = \frac{u+\frac{\mathrm{i}}{2}}{u-\frac{\mathrm{i}}{2}} \frac{v+\frac{\mathrm{i}}{2}}{v-\frac{\mathrm{i}}{2}}. \quad (35)$$

# An example

- The solutions are [Bak, Rey, 08]

$$u = -v = \frac{1}{2}\cot\frac{k\pi}{L_3+1}\,,\; w = 0\,, \tag{36}$$

with $k = 1, \cdots, L_3$.

- By constructing eigenstates via CBA and Gaudin formula for the norms [Yang, Jiang, Komatsu, **JW**, 21], we can get

$$\mathcal{C}_{123} = \frac{(-1)^{L_2+1}\mathrm{sgn}(a_1 a_2 a_{12})\sqrt{2L_1L_2L_3}\exp\frac{2\pi k \mathrm{i}}{L_3+1}}{N\sqrt{L_3+1}}\,. \tag{37}$$

# Holographic computations?

- There are some holographic computations of three point functions in ABJM, but not for the case at hand.
- The three point functions $\langle \mathcal{O}^\circ_{L_1}(x_1)\mathcal{O}^\circ_{L_2}(x_2)\mathcal{O}^\circ_{L_3}(x_3)\rangle$ of three BPS operators are computed in [Hirano, Krstjansen, Young, 12] based on [Bastianelli, Zucchini, 99].
- [Hirano, Krstjansen, Young, 12] also computed $\langle \mathcal{D}^\circ(x_1)\mathcal{D}^\circ(x_2)\mathcal{O}^\circ_{L_3}(x_3)\rangle$ using dual M-theory. However the correct computations were only done in [Yang, Jiang, Komatsu, **JW, 22**].
- As far as we know, there is no rigorous prescription for computing $\langle \mathcal{O}^\circ_{L_1}(x_1)\mathcal{O}^\circ_{L_2}(x_2)\mathcal{O}_{L_3}(x_3)\rangle$. Many a starting point is the vertex operator approach of [Buchbinder, Tseytlin, 10].

# Conclusion

- We found that the boundary state from the two BPS operators is intergrable when the correlator is special extremal ($l_{31|2} = 0, 2L_3$) or special next-to-extremal ($l_{31|2} = 1, 2L_3 - 1$).
- For these proved integrable case, we computed the three-point functions using the constraints from symmetries and CBA.
- The integrable states with $l_{31|2} = 1, 2L_3 - 1$ is of a novel type.

# Outlook

- It should be interesting to revisit the three-point function $\langle \mathcal{O}^\circ \mathcal{O}^\circ \mathcal{O} \rangle$ in $\mathcal{N} = 4$ SYM [Escobedo, Gromov, Sever, Vieira, 10] to determine when the boundary state from the two BPS operators is integrable.
- It is also desirable to compute more general three-point functions in ABJM theory to aid the development of hexagon program [Basso, komatsu, Vieira, 15] for this theory.

# Comments on bottom up holography

- In [Maldacena, Nunez, 00] with the title *"**Towards** the large N limit of pure $N = 1$ super Yang-Mills theory"* , it was mentioned in the abstract,
- *"When the gravity approximation is valid the masses of the glueballs are comparable to the masses of Kaluza-Klein states on the $5$-brane,"* $\cdots$
- I think this is still a big challenge for AdS/QCD and AdS/CMT.

**Thanks for Your Attention !**