Disordered Quantum Critical Fixed Points from Holography

Xiaoyang Huang

University of Colorado, Boulder

Gauge-Gravity duality 2024

Outline & Acknowledgement

GORDON AND BETTY MOORE FOUNDATION

1. Disorder in CMT
2. Conformal perturbation theory
3. Disordered Holographic duality

Reference [**XYH**, Sachdev and Lucas, Phys. Rev. Lett. (2023)]

Subir Sachdev

Andy Lucas

Disorder in CMT: breaking ergodicity

- Anderson localization

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. Anderson
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

$$H = t \sum_{\langle ij \rangle} \left(c_i^\dagger c_j + c_j^\dagger c_i \right) + \sum_i U_i c_i^\dagger c_i$$

$$U_i \in [-W, +W]$$

Disorder in CMT: breaking ergodicity

- Anderson localization

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

$$H = t \sum_{\langle ij \rangle} \left(c_i^\dagger c_j + c_j^\dagger c_i \right) + \sum_i U_i c_i^\dagger c_i$$

$$U_i \in [-W, +W]$$

Single particle energy is *off resonance*, so the perturbation theory is convergent

Disorder in CMT: breaking ergodicity

- Anderson localization

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

$$H = t\sum_{\langle ij\rangle}\left(c_i^\dagger c_j + c_j^\dagger c_i\right) + \sum_i U_i c_i^\dagger c_i$$

$$U_i \in [-W, +W]$$

Single particle energy is *off resonance*, so the perturbation theory is convergent

- Protect information! in Many-body system?

Disorder in CMT: breaking ergodicity

- Anderson localization

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

$$H = t \sum_{\langle ij \rangle} \left(c_i^\dagger c_j + c_j^\dagger c_i \right) + \sum_i U_i c_i^\dagger c_i$$

$$U_i \in [-W, +W]$$

Single particle energy is *off resonance*, so the perturbation theory is convergent

- Protect information! in Many-body system?

Naively, “off resonance” does not apply since there are O(N) states a state can tunnel to

Disorder in CMT: breaking ergodicity

- Anderson localization

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

$$H = t \sum_{\langle ij \rangle} \left(c_i^\dagger c_j + c_j^\dagger c_i \right) + \sum_i U_i c_i^\dagger c_i$$

$$U_i \in [-W, +W]$$

Single particle energy is *off resonance*, so the perturbation theory is convergent

- Protect information! in Many-body system?

Naively, "off resonance" does not apply since there are O(N) states a state can tunnel to

Nevertheless, people find evidences that ETH is violated, known as the many-body localization [Nandkishore and Huse (2015)]

Disorder in CMT: breaking ergodicity

- Anderson localization

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

$$H = t\sum_{\langle ij\rangle}\left(c_i^\dagger c_j + c_j^\dagger c_i\right) + \sum_i U_i c_i^\dagger c_i$$

$$U_i \in [-W, +W]$$

Single particle energy is *off resonance*, so the perturbation theory is convergent

- Protect information! in Many-body system?

Naively, “off resonance” does not apply since there are O(N) states a state can tunnel to

Nevertheless, people find evidences that ETH is violated, known as the many-body localization [Nandkishore and Huse (2015)]

- A universality class—infinite-randomness fixed point? [Pal and Huse (2010)]

Disorder in CMT: transport

- Disorder: translational symmetries are broken explicitly

Disorder in CMT: transport

- Disorder: translational symmetries are broken explicitly

  Momentum conservation equation:

$$\partial_t \pi_i = enE_i - \frac{\pi_i}{\tau_{\text{dis}}} \qquad \frac{1}{\tau_{\text{dis}}} \propto D \quad \text{Disorder strength}$$

Disorder in CMT: transport

- Disorder: translational symmetries are broken explicitly

  Momentum conservation equation:

$$\partial_t \pi_i = enE_i - \frac{\pi_i}{\tau_{\text{dis}}}$$

$$\frac{1}{\tau_{\text{dis}}} \propto D$$ Disorder strength

  Using $\pi_i = e^{-1} m J_i$

  We find the Drude formula

$$\sigma(\omega) = \frac{e^2 n}{m(\tau_{\text{dis}}^{-1} - i\omega)}$$

$D = 0 \longrightarrow \text{Re}\,\sigma \sim \delta(\omega)$

Disorder in CMT: transport

- Disorder: translational symmetries are broken explicitly

  Momentum conservation equation:

$$\partial_t \pi_i = enE_i - \frac{\pi_i}{\tau_{\text{dis}}}$$

$$\frac{1}{\tau_{\text{dis}}} \propto D$$ Disorder strength

  Using $\pi_i = e^{-1} m J_i$

  We find the Drude formula

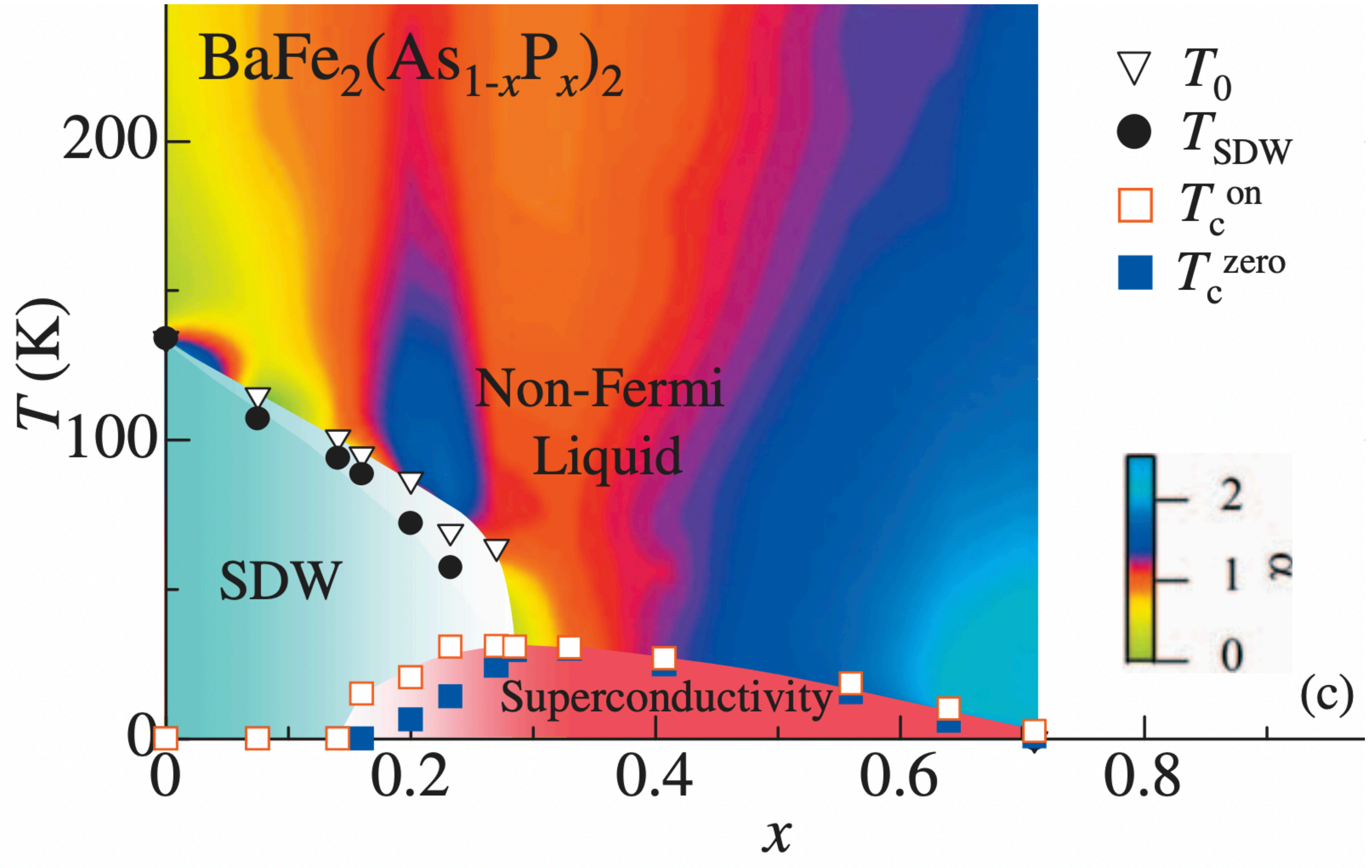
$$\sigma(\omega) = \frac{e^2 n}{m(\tau_{\text{dis}}^{-1} - i\omega)}$$

$$D = 0 \longrightarrow \text{Re}\,\sigma \sim \delta(\omega)$$

  Electrical conductivity is finite at small frequency

Disorder in CMT: transport

- Disorder is important for strange metal transport



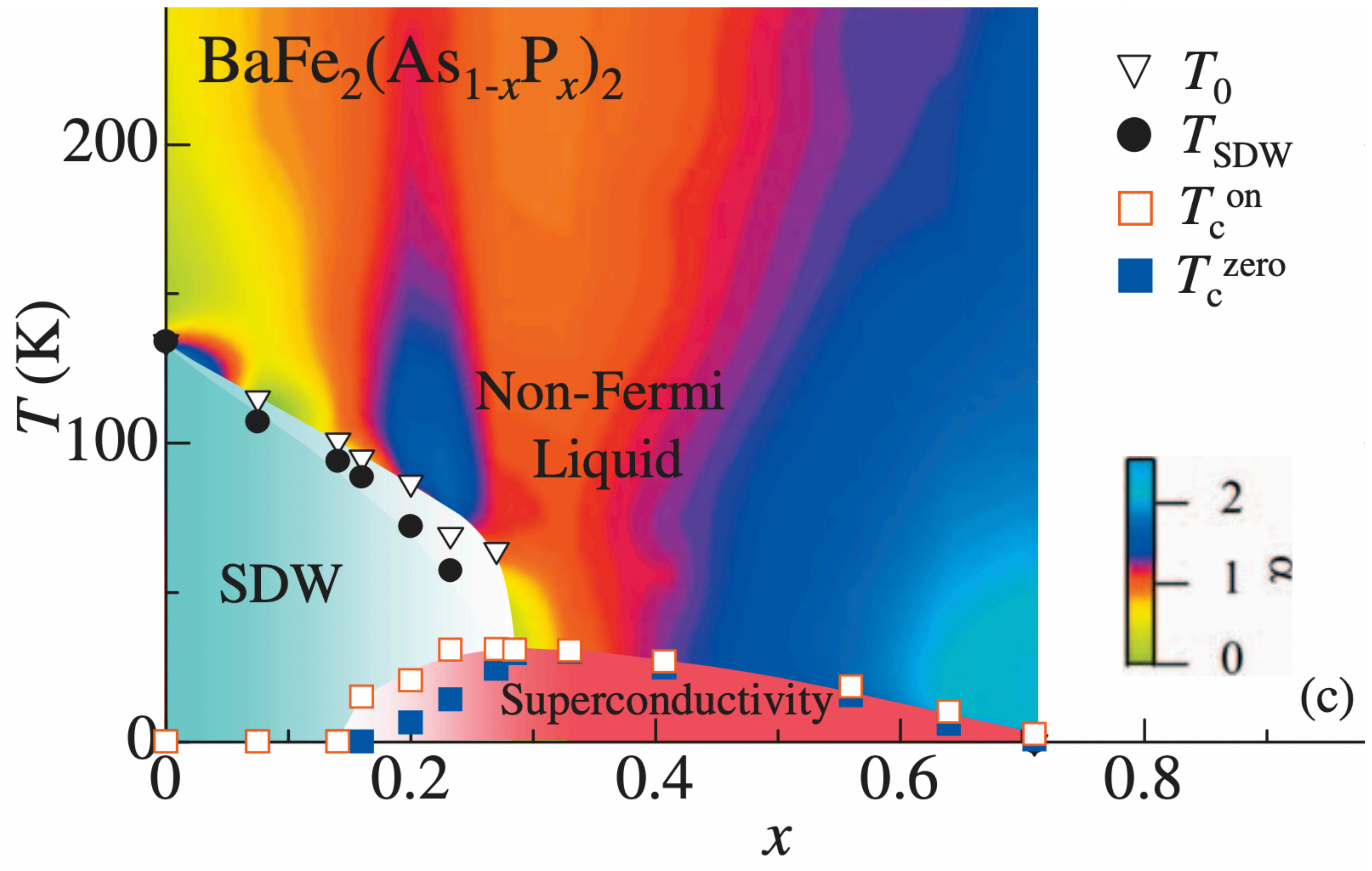
[S. Kasahara et al., PRB (2010)]

Disorder in CMT: transport

- Disorder is important for strange metal transport

$$\rho \sim T^{\alpha}$$

$$\sigma \sim \omega^{-\alpha}$$



[S. Kasahara et al., PRB (2010)]

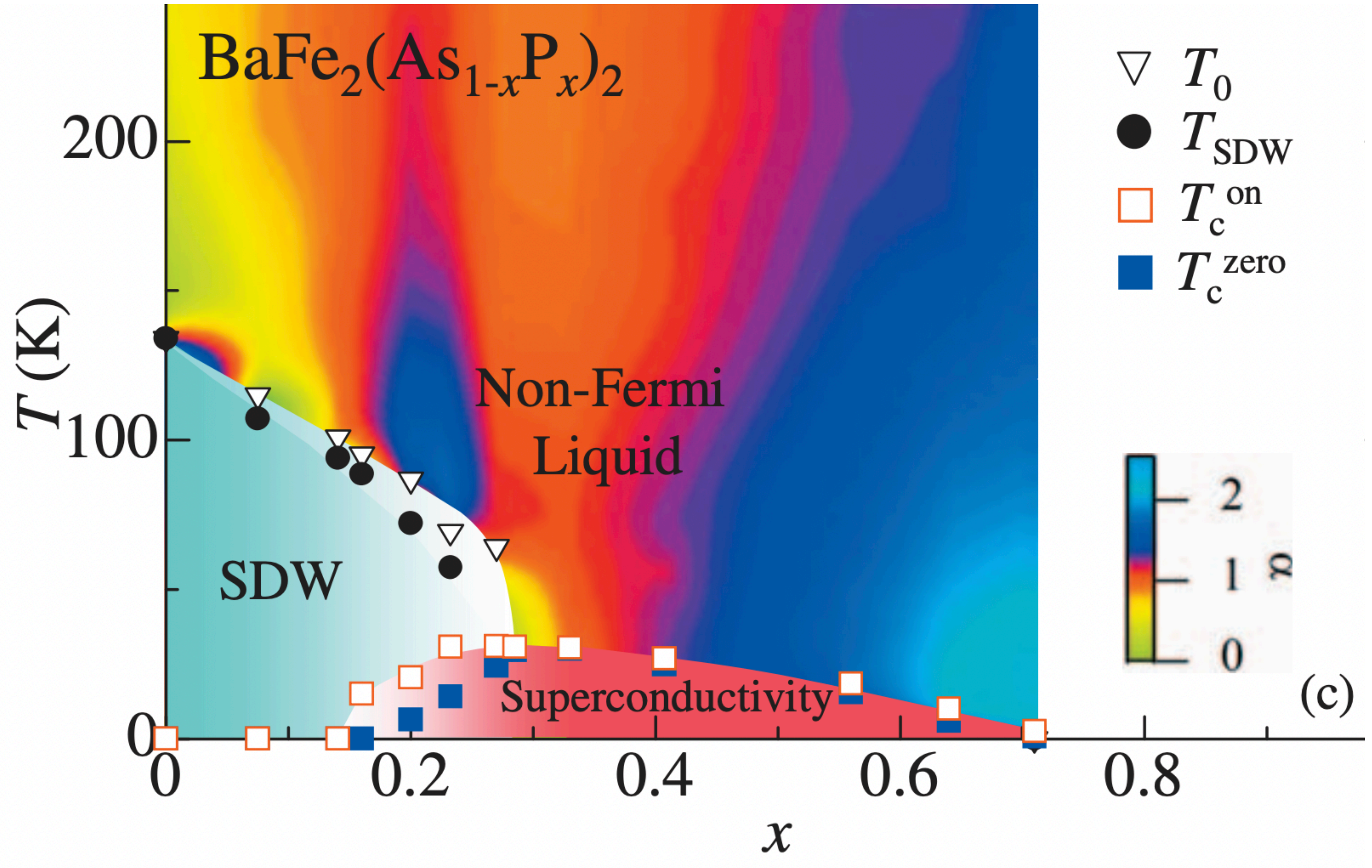
Disorder in CMT: transport

- Disorder is important for strange metal transport

$$\rho \sim T^{\alpha}$$

$$\sigma \sim \omega^{-\alpha}$$

If momentum is conserved: $\mathrm{Re}\,\sigma \sim \delta(\omega)$



[S. Kasahara et al., PRB (2010)]

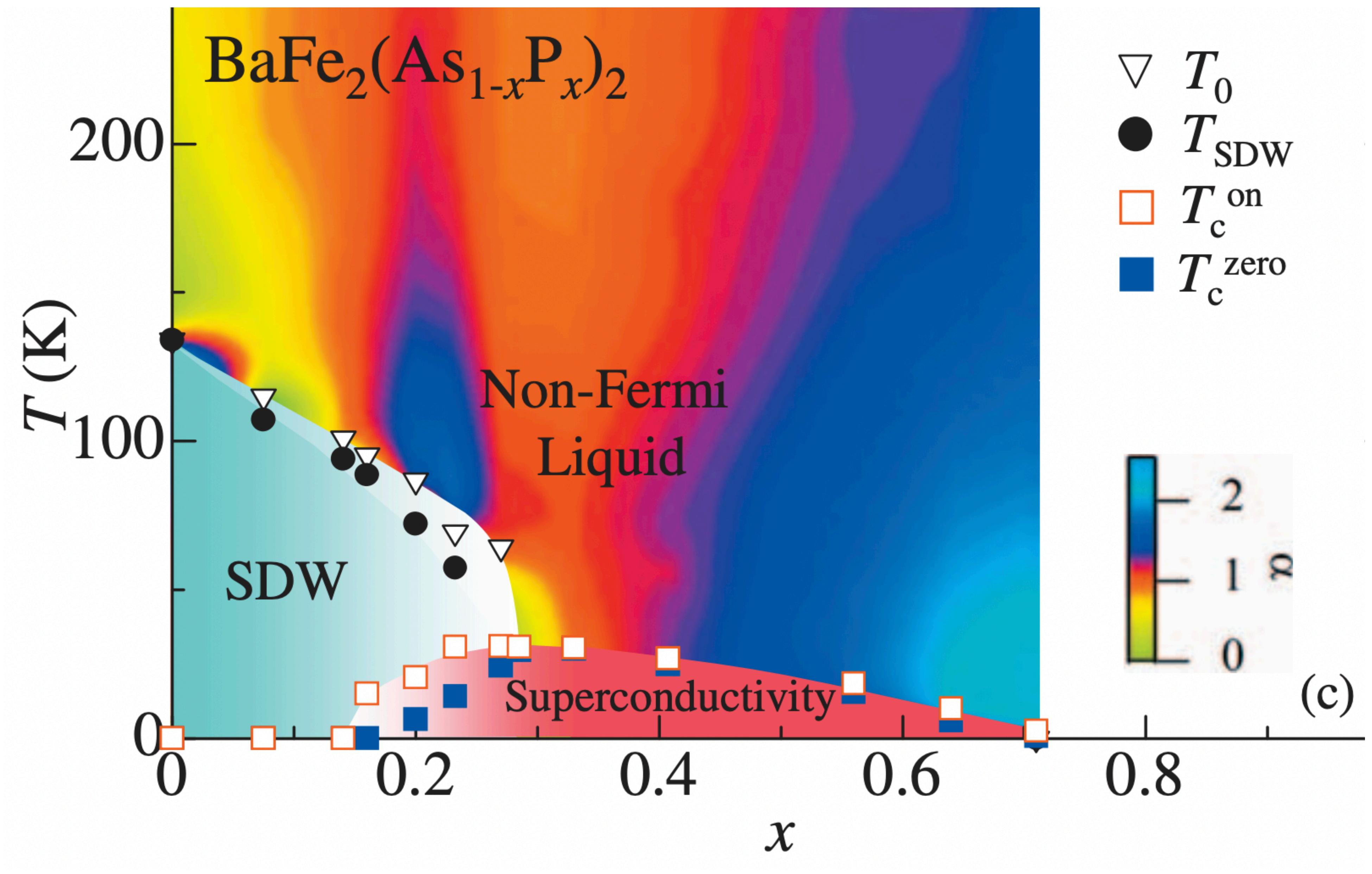
Disorder in CMT: transport

- Disorder is important for strange metal transport

$$\rho \sim T^{\alpha}$$

$$\sigma \sim \omega^{-\alpha}$$

If momentum is conserved: $\quad \mathrm{Re}\,\sigma \sim \delta(\omega)$



[S. Kasahara et al., PRB (2010)]

- Spatially random interaction

PHYSICS

Universal theory of strange metals from spatially random interactions

Aavishkar A. Patel[1,2], Haoyu Guo[3,4,5], Ilya Esterlis[4,6], Subir Sachdev[4,7]*

Strange metals—ubiquitous in correlated quantum materials—transport electrical charge at low temperatures but not by the individual electronic quasiparticle excitations, which carry charge in ordinary metals. In this work, we consider two-dimensional metals of fermions coupled to quantum critical scalars, the latter representing order parameters or fractionalized particles. We show that at low temperatures (T), such metals generically exhibit strange metal behavior with a T-linear resistivity arising from spatially random fluctuations in the fermion-scalar Yukawa couplings about a nonzero spatial average. We also find a $T \ln(1/T)$ specific heat and a rationale for the Planckian bound on the transport scattering time. These results are in agreement with observations and are obtained in the large N expansion of an ensemble of critical metals with N fermion flavors.

$$\mathrm{Re}\,\sigma(\omega) \sim \tau(\omega) \sim 1/|\omega|$$

Disorder in CMT: universality

A longstanding problem:

Controlled IR fixed points for strongly coupled systems with finite disorder

Disorder in CMT: universality

A longstanding problem:

Controlled IR fixed points for <u>strongly coupled</u> systems with <u>finite disorder</u>

which can be made more complicated by

- Non-relativistic fixed point
- Fermi surface

Disorder in CMT: universality

A longstanding problem:

Controlled IR fixed points for <u>strongly coupled</u> systems with <u>finite disorder</u>

which can be made more complicated by

- Non-relativistic fixed point
- Fermi surface

This talk: A random fixed point with a perturbative quenched quantum disorder at finite density

Model

annealed disorder: disorder is dynamical

quenched disorder:
disorder is non-dynamical

Model

annealed disorder: disorder is dynamical

quenched disorder:
disorder is non-dynamical

[Aharony and Narovlansky (2018)]

classical disorder: time-dependent

quantum disorder: time-independent

Model

annealed disorder: disorder is dynamical

quenched disorder:
disorder is non-dynamical

[Aharony and Narovlansky (2018)]

classical disorder: time-dependent

quantum disorder: time-independent ✓

Model

- annealed disorder: disorder is dynamical
- quenched disorder: disorder is non-dynamical
  [Aharony and Narovlansky (2018)]
  - classical disorder: time-dependent
  - quantum disorder: time-independent

Perturb a clean S_0 by a spatially random field $h(\vec{x})$ that couples to a scalar operator $\mathcal{O}$

$$S = S_0 + \int \mathrm{d}t \, \mathrm{d}^d x \, h(\boldsymbol{x}) \mathcal{O}(\boldsymbol{x}, t)$$

where $\overline{h(\boldsymbol{x}) h(\boldsymbol{y})} \approx D \delta(\boldsymbol{x} - \boldsymbol{y})$.

Harris criterion

Replica action: $$S_n = \sum_A^n S_{0,A} - \frac{D}{2}\sum_{AB}\int \mathrm{d}^d x\, \mathrm{d}t\, \mathrm{d}t' \mathcal{O}_A(x,t)\mathcal{O}_B\left(x,t'\right)$$

Let $[x] = -1 \quad [\mathcal{O}] = \Delta$

- Dynamical scaling exponent z $\qquad [t] = z[x] \qquad$ e.g. Galilean symmetry $z = 2$
- Hyperscaling-violation θ $\qquad d \to d - \theta \qquad$ e.g. Fermi surface $\theta = d - 1$

Harris criterion

Replica action: $$S_n = \sum_{A}^{n} S_{0,A} - \frac{D}{2} \sum_{AB} \int \mathrm{d}^d x \, \mathrm{d}t \, \mathrm{d}t' \mathcal{O}_A(x,t) \mathcal{O}_B\left(x,t'\right)$$

Let $[x] = -1 \quad [\mathcal{O}] = \Delta$

- Dynamical scaling exponent z $\quad [t] = z[x] \quad$ e.g. Galilean symmetry $z = 2$
- Hyperscaling-violation θ $\quad d \to d - \theta \quad$ e.g. Fermi surface $\theta = d - 1$

$$[D] = -2\Delta + d - \theta + 2z$$

Harris criterion

Replica action: $$S_n = \sum_A^n S_{0,A} - \frac{D}{2}\sum_{AB}\int \mathrm{d}^d x\ \mathrm{d}t\ \mathrm{d}t' \mathcal{O}_A(x,t)\mathcal{O}_B\left(x,t'\right)$$

Let $[x] = -1 \quad [\mathcal{O}] = \Delta$

- Dynamical scaling exponent z $\quad [t] = z[x] \quad$ e.g. Galilean symmetry $z = 2$
- Hyperscaling-violation θ $\quad d \to d - \theta \quad$ e.g. Fermi surface $\theta = d - 1$

$$[D] = -2\Delta + d - \theta + 2z$$

or $\quad [D] = 2\nu$

- Harris criterion: $\quad \nu < 0$ (irrelevant), $\quad \nu = 0$ (marginal), $\quad \nu > 0$ (relevant)

[Harris (1974)]

Disordered CFT

CFT: $\quad z = 1, \quad \theta = 0$

$$S_n = \sum_A^n S_{0,A} - \frac{D}{2} \sum_{AB} \int \mathrm{d}^d x \, \mathrm{d}t \, \mathrm{d}t' \mathcal{O}_A(x,t) \mathcal{O}_B\left(x, t'\right)$$

Disordered CFT

CFT: $z=1, \quad \theta=0$

$$S_n=\sum_A^n S_{0,A}-\frac{D}{2}\sum_{AB}\int \mathrm{d}^d x\ \mathrm{d}t\ \mathrm{d}t' \mathcal{O}_A(x,t)\mathcal{O}_B\left(x,t'\right)$$

- Marginal disorder $\Delta=d/2+1$ (i. e. $\nu=0$)

$$\mathcal{O}_A(x,t)\mathcal{O}_B\left(x,t'\right)\supset\frac{-\left|C_{\mathcal{O}\mathcal{O}T}\right|}{C_{TT}}\frac{1}{\left|t-t'\right|}T_{00,A}(x,t)\delta_{AB}+\cdots$$

[Aharony and Narovlansky (2018)]

Disordered CFT

CFT: $z=1, \quad \theta=0$

$$S_n=\sum_A^n S_{0,A}-\frac{D}{2}\sum_{AB}\int \mathrm{d}^d x\, \mathrm{d}t\, \mathrm{d}t' \mathcal{O}_A(x,t)\mathcal{O}_B\left(x,t'\right)$$

- Marginal disorder $\quad \Delta=d/2+1$ (i. e. $\nu=0$)

$$\mathcal{O}_A(x,t)\mathcal{O}_B\left(x,t'\right)\supset\frac{-\left|C_{\mathcal{O}\mathcal{O}T}\right|}{C_{TT}}\frac{1}{\left|t-t'\right|}T_{00,A}(x,t)\delta_{AB}+\cdots$$

[Aharony and Narovlansky (2018)]

- Lifshitz scaling

$$z=1+D\frac{\left|C_{\mathcal{O}\mathcal{O}T}\right|}{C_{TT}}\log b$$

Disordered CFT

$$z^* = 1 + D\frac{|C_{\mathcal{O}\mathcal{O}T}|}{C_{TT}}\log b$$

This Lifshitz scaling was previously found using holographic duality [Hartnoll and Santos (2014)] [Hartnoll, Ramirez and Santos (2016)]

Disordered CFT

$$z^* = 1 + D\frac{|C_{\mathcal{O}\mathcal{O}T}|}{C_{TT}}\log b$$

This Lifshitz scaling was previously found using holographic duality [Hartnoll and Santos (2014)] [Hartnoll, Ramirez and Santos (2016)]

However, a line of fixed points for a marginal disorder?

Disordered CFT

$$z^* = 1 + D\frac{|C_{\mathcal{OO}T}|}{C_{TT}}\log b$$

This Lifshitz scaling was previously found using holographic duality

[Hartnoll and Santos (2014)]

[Hartnoll, Ramirez and Santos (2016)]

However, a line of fixed points for a marginal disorder?

At this new fixed point

$$\Delta' \approx \left(\frac{d}{2}+1\right) + \left(\frac{d}{2}+1\right)(z^*-1) > \frac{d}{2}+1$$

[Ganesan and Lucas (2020)]

[Ganesan, Lucas and Radzihovsky (2022)]

The scaling dimension of the disordered operator is renormalized, resulting in an *irrelevant* disorder

$$\nu < 0$$

Disordered CFT

$$z^* = 1 + D\frac{|C_{\mathcal{O}\mathcal{O}T}|}{C_{TT}}\log b$$

This Lifshitz scaling was previously found using holographic duality

[Hartnoll and Santos (2014)]

[Hartnoll, Ramirez and Santos (2016)]

However, a line of fixed points for a marginal disorder?

At this new fixed point

$$\Delta' \approx \left(\frac{d}{2}+1\right) + \left(\frac{d}{2}+1\right)(z^*-1) > \frac{d}{2}+1$$

[Ganesan and Lucas (2020)]

[Ganesan, Lucas and Radzihovsky (2022)]

The scaling dimension of the disordered operator is renormalized, resulting in an *irrelevant* disorder

$$\nu < 0$$

Contradiction: an irrelevant disorder cannot support the Lifshitz scaling!

Disordered CFT

$$S_n = \sum_A^n S_{0,A} - \frac{D}{2} \sum_{AB} \int \mathrm{d}^d x \; \mathrm{d}t \; \mathrm{d}t' \mathcal{O}_A(x,t) \mathcal{O}_B\left(x,t'\right)$$

We are missing the renormalization of the disorder strength D

Disordered CFT

$$S_n = \sum_{A}^{n} S_{0,A} - \frac{D}{2} \sum_{AB} \int \mathrm{d}^d x \, \mathrm{d}t \, \mathrm{d}t' \mathcal{O}_A(x,t) \mathcal{O}_B\left(x,t'\right)$$

We are missing the renormalization of the disorder strength D

This is obtained by computing the beta-function

Disordered CFT

$$S_n = \sum_{A}^{n} S_{0,A} - \frac{D}{2} \sum_{AB} \int \mathrm{d}^d x \, \mathrm{d}t \, \mathrm{d}t' \mathcal{O}_A(x,t)\mathcal{O}_B\left(x,t'\right)$$

We are missing the renormalization of the disorder strength D

This is obtained by computing the beta-function

In the "matrix large N" limit of holographic duality, we have leading order $C_{TT},\ C_{\mathcal{O}\mathcal{O}},\ C_{TTT},\ C_{\mathcal{O}\mathcal{O}T}$
but $C_{\mathcal{O}\mathcal{O}\mathcal{O}} = 0$

$$e^{-S_n} = e^{-\sum_A S_{0,A}} \left[1 + \ldots - \frac{D^2}{2} \frac{|C_{\mathcal{O}\mathcal{O}T}|}{C_{TT}} \log b \sum_{ABC} \int d^d x_1 dt_1 d^d x_2 dt_2 dt_2' \, T_{00,A}(x_1,t_1)\mathcal{O}_B(x_2,t_2)\mathcal{O}_C(x_2,t_2') + \ldots \right]$$

[Aharony and Narovlansky (2018)]

Disordered CFT

$$S_n = \sum_{A}^{n} S_{0,A} - \frac{D}{2}\sum_{AB}\int \mathrm{d}^d x\,\mathrm{d}t\,\mathrm{d}t' \mathcal{O}_A(x,t)\mathcal{O}_B\left(x,t'\right)$$

We are missing the renormalization of the disorder strength D

This is obtained by computing the beta-function

In the "matrix large N" limit of holographic duality, we have leading order $C_{TT},\ C_{\mathcal{O}\mathcal{O}},\ C_{TTT},\ C_{\mathcal{O}\mathcal{O}T}$ but $C_{\mathcal{O}\mathcal{O}\mathcal{O}} = 0$

$$e^{-S_n} = e^{-\sum_A S_{0,A}}\left[1 + \ldots - \frac{D^2}{2}\frac{|C_{\mathcal{O}\mathcal{O}T}|}{C_{TT}}\log b \sum_{ABC}\int d^d x_1 dt_1 d^d x_2 dt_2 dt_2'\, T_{00,A}(x_1,t_1)\mathcal{O}_B(x_2,t_2)\mathcal{O}_C(x_2,t_2') + \ldots\right]$$

[Aharony and Narovlansky (2018)]

$$\boxed{\beta_D = -\frac{\partial \delta D}{\partial \log b} = \frac{d|C_{\mathcal{O}\mathcal{O}T}|}{C_{TT}} D^2}$$

[XYH, Sachdev and Lucas (2023)]

Disordered CFT

The above is for marginal disorder

[XYH, Sachdev and Lucas (2023)]

Disordered CFT

The above is for marginal disorder

By turning on a relevant disorder ($\nu > 0$)

$$\beta_D = \frac{d|C_{\mathcal{O}\mathcal{O}T}|}{C_{TT}} D^2 - 2\nu D$$

[XYH, Sachdev and Lucas (2023)]

Disordered CFT

The above is for marginal disorder

By turning on a relevant disorder ($\nu > 0$)

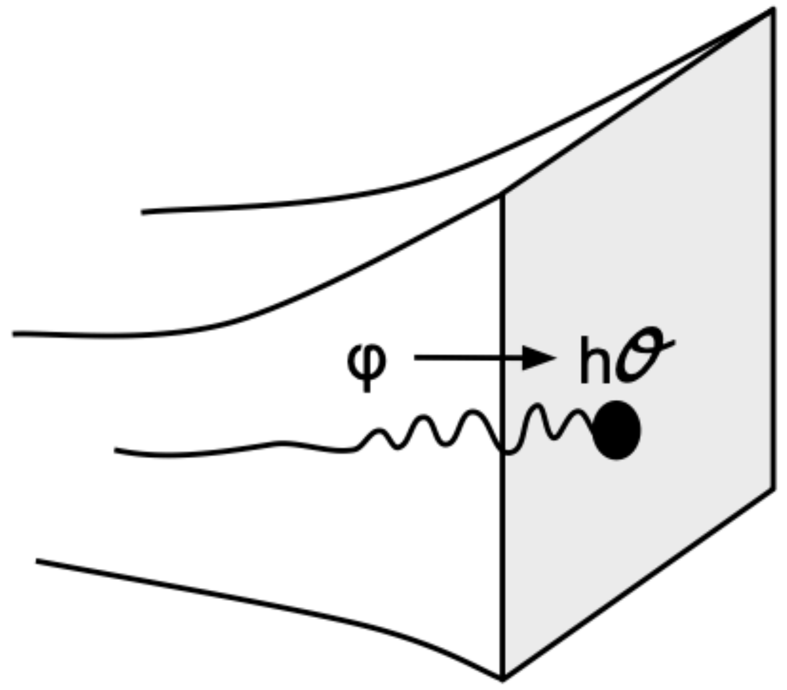
$$\beta_D = \frac{d|C_{\mathcal{O}\mathcal{O}T}|}{C_{TT}} D^2 - 2\nu D$$

We find a *Lifshitz fixed point*

$$D^* = \frac{2\nu C_{TT}}{d|C_{\mathcal{O}\mathcal{O}T}|} \qquad\qquad z^* = 1 + \frac{|C_{\mathcal{O}\mathcal{O}T}|}{C_{TT}} D^* = 1 + \frac{2\nu}{d}$$

[XYH, Sachdev and Lucas (2023)]

Disordered holography



Global symmetry on the boundary (QFT) is dual to gauge symmetry in the bulk

- stress tensor $\longleftrightarrow$ metric
- U(1) current $\longleftrightarrow$ U(1) gauge field

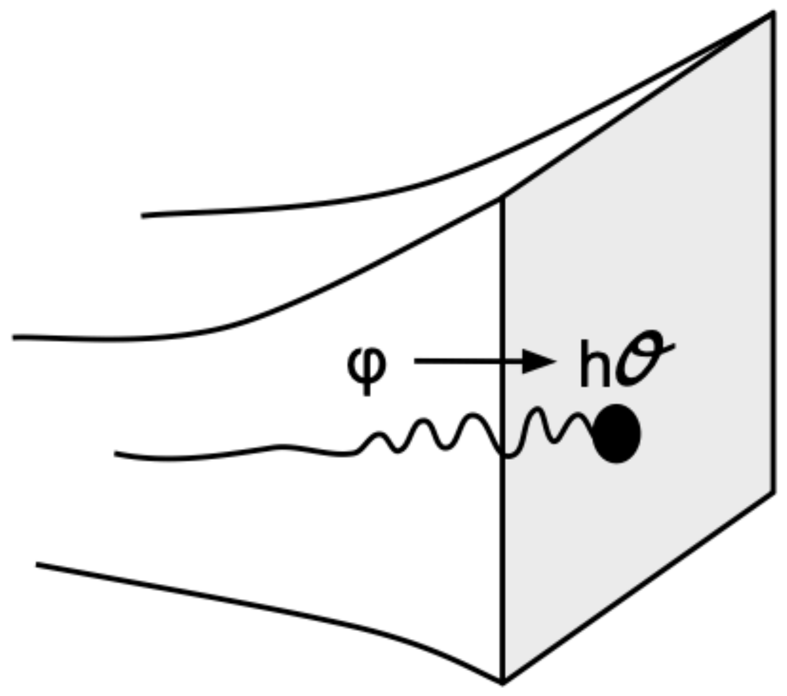
Einstein-Maxwell-Dilaton model

$$S_0 = \int d^{d+2}x\sqrt{-g}\left[\left(R - 2(\partial\Phi)^2 - V(\Phi)\right) - \frac{Z(\Phi)}{4}F^2\right]$$

This model is known to support nonzero charge density and generic z, θ

[Huijse, Sachdev and Swingle (2012)]
[Lucas, Sachdev and Schalm (2014)]
[Lucas and Sachdev (2015)]

Disordered holography



Disordered theory *without using replica trick*! [Hartnoll and Santos (2014)]

- disordered operator $\mathcal{O} \longleftrightarrow$ scalar field ψ $\qquad \psi(r \to 0, t, x) \sim r^{\#} h(x)$

The total action

$$S = S_{\mathrm{EMD}} - \int \mathrm{d}^{d+2}x \sqrt{-g}\left[\frac{1}{2}(\partial\psi)^2 + \frac{B(\Phi)}{2}\psi^2\right]$$

$B(\Phi)$ encodes the scaling dimension $[\mathcal{O}] = \Delta$

[XYH, Sachdev and Lucas (2023)]

Holographic fixed point

Solve the bulk equations of motion

[XYH, Sachdev and Lucas (2023)]

Holographic fixed point

Solve the bulk equations of motion

1) We are interested in the spatially homogeneous solution

inhomogeneity enters at $O(D^2)$

$$R_{ab} - \frac{R}{2} g_{ab} = \frac{1}{2} \left(T^{A}_{ab} + T^{\Phi}_{ab} + \overbrace{T^{\psi}_{ab}}^{\text{Stress tensors}} \right)$$

[XYH, Sachdev and Lucas (2023)]

Holographic fixed point

Solve the bulk equations of motion

1) We are interested in the spatially homogeneous solution

inhomogeneity enters at $O(D^2)$

Stress tensors

$$R_{ab} - \frac{R}{2} g_{ab} = \frac{1}{2}\left(T^A_{ab} + T^\Phi_{ab} + \overline{T^\psi_{ab}}\right)$$

2) We work in the IR scaling regime ($r \to \infty$)

$$g_{ab}(r) \sim r^{\#}$$

[XYH, Sachdev and Lucas (2023)]

Holographic fixed point

Solve the bulk equations of motion

1) We are interested in the spatially homogeneous solution

inhomogeneity enters at $O(D^2)$

$$R_{ab} - \frac{R}{2} g_{ab} = \frac{1}{2}\left(T^{A}_{ab} + T^{\Phi}_{ab} + \overbrace{T^{\psi}_{ab}}^{\text{Stress tensors}}\right)$$

2) We work in the IR scaling regime ($r \to \infty$)

$$g_{ab}(r) \sim r^{\#}$$

Solve ODEs and ignore differences that are vanishingly small at IR ($r \to \infty$)

confirmed by numerics

[XYH, Sachdev and Lucas (2023)]

Holographic fixed point

We find

$$z^* \approx z + \frac{2\nu}{d}(z-\theta), \quad \theta^* = \theta \qquad\qquad D^* \propto \nu$$

UV-IR crossover energy is non-perturbatively large $\qquad E_c \sim \left(\frac{D}{\nu}\right)^{\frac{1}{\nu}}$

[XYH, Sachdev and Lucas (2023)]

Holographic fixed point

We find

$$z^* \approx z + \frac{2\nu}{d}(z-\theta), \quad \theta^* = \theta \qquad D^* \propto \nu$$

UV-IR crossover energy is non-perturbatively large $\quad E_c \sim \left(\frac{D}{\nu}\right)^{\frac{1}{\nu}}$

1) For $\quad z = 1,\ \theta = 0$

$$z^* = 1 + \frac{2\nu}{d}$$

consistent with CFT perturbation

[XYH, Sachdev and Lucas (2023)]

Holographic fixed point

We find

$$z^* \approx z + \frac{2\nu}{d}(z-\theta), \quad \theta^* = \theta \qquad D^* \propto \nu$$

UV-IR crossover energy is non-perturbatively large $\quad E_c \sim \left(\frac{D}{\nu}\right)^{\frac{1}{\nu}}$

1) For $\quad z = 1,\ \theta = 0$

$$z^* = 1 + \frac{2\nu}{d}$$

consistent with CFT perturbation

2) For $\quad z = 1,\ \theta = 0,\ d = 2$

$$z^* = 1 + \nu, \quad \nu = \frac{16}{3\pi^2 N}$$

[Goldman, et al., (2020)]

[XYH, Sachdev and Lucas (2023)]

Summary & outlook

We find at a disordered fixed point at finite charge density

$$z^* \approx z + \frac{2\nu}{d}(z-\theta), \quad \theta^* = \theta$$

Outlook:

- Emergent scale invariance under inhomogeneous boundary condition using numerics
- Non-equilibrium fixed point?