TSIMF
清华三亚国际数学论坛
Gauge Gravity Duality 2024

Pole collisions and effective actions in holographic theories

Yan Liu (刘焱)

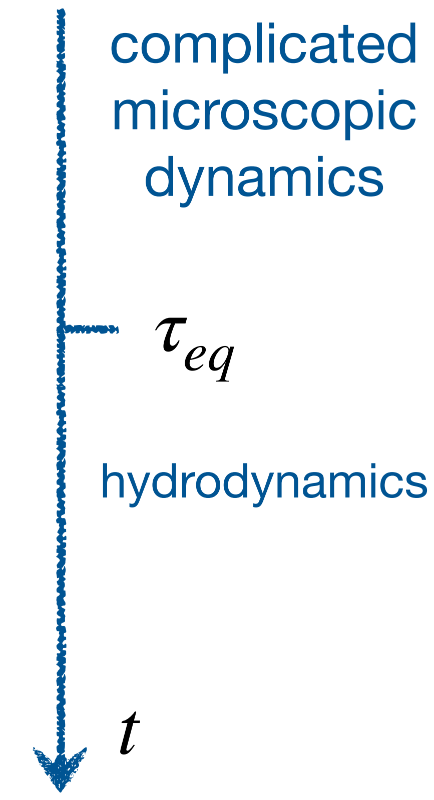
Beihang University

with Xin-Meng Wu（吴昕蒙）， 2111.07770

with Ya-Wen Sun and Xin-Meng Wu， 2411.16306

When is hydro valid?

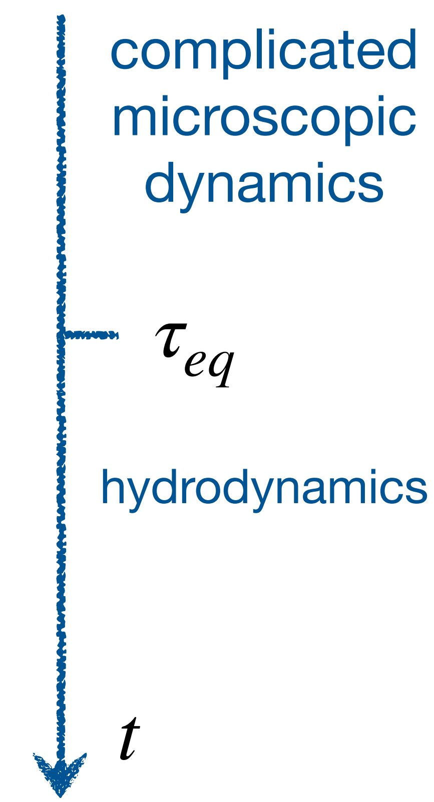
- QFT at finite temperature: important, complicated (no quasi particle, no perturbative method)
- Universal effective theory describing dynamics of conserved charges over large scales, towarding thermal equilibrium



[Kovtun, ’12]

When is hydro valid?

- QFT at finite temperature: important, complicated (no quasi particle, no perturbative method)
- Universal effective theory describing dynamics of conserved charges over large scales, towarding thermal equilibrium
- When is hydro valid?



[Kovtun, ’12]

Hydro equations

- Hydrodynamic equations: conservation equations of global symmetry

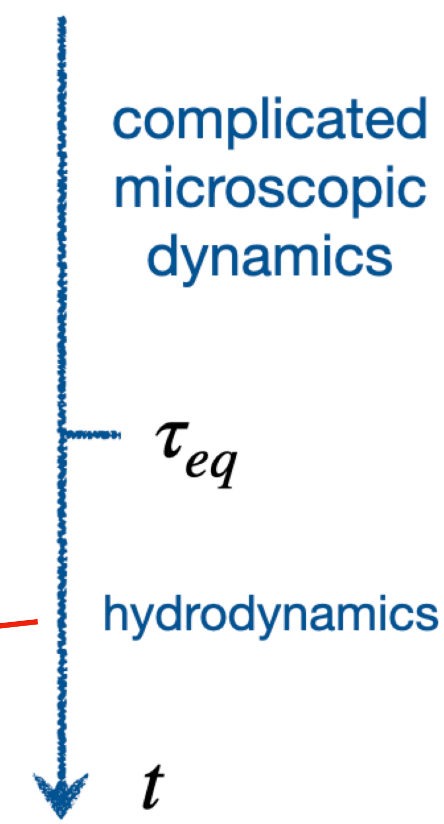
$$\frac{\partial}{\partial t}\rho_a + \frac{\partial}{\partial x^i} J_a^i = 0$$

- Constitutive equations & gradient expansion in $\frac{\ell_{eq}}{\delta \ell} \sim \ell_{eq}\partial_\mu$

$$\rho_a = \rho^{(0)}(\gamma) + \rho^{(1)}(\partial\gamma) + \rho^{(2)}(\partial^2\gamma) + \ldots$$

$$\mathbf{J}_a = \mathbf{J}^{(0)}(\gamma) + \mathbf{J}^{(1)}(\partial\gamma) + \mathbf{J}^{(2)}(\partial^2\gamma) + \ldots$$

$$T(t,\vec{x}),\ v^i(t,\vec{x}),\ \rho(t,\vec{x})$$



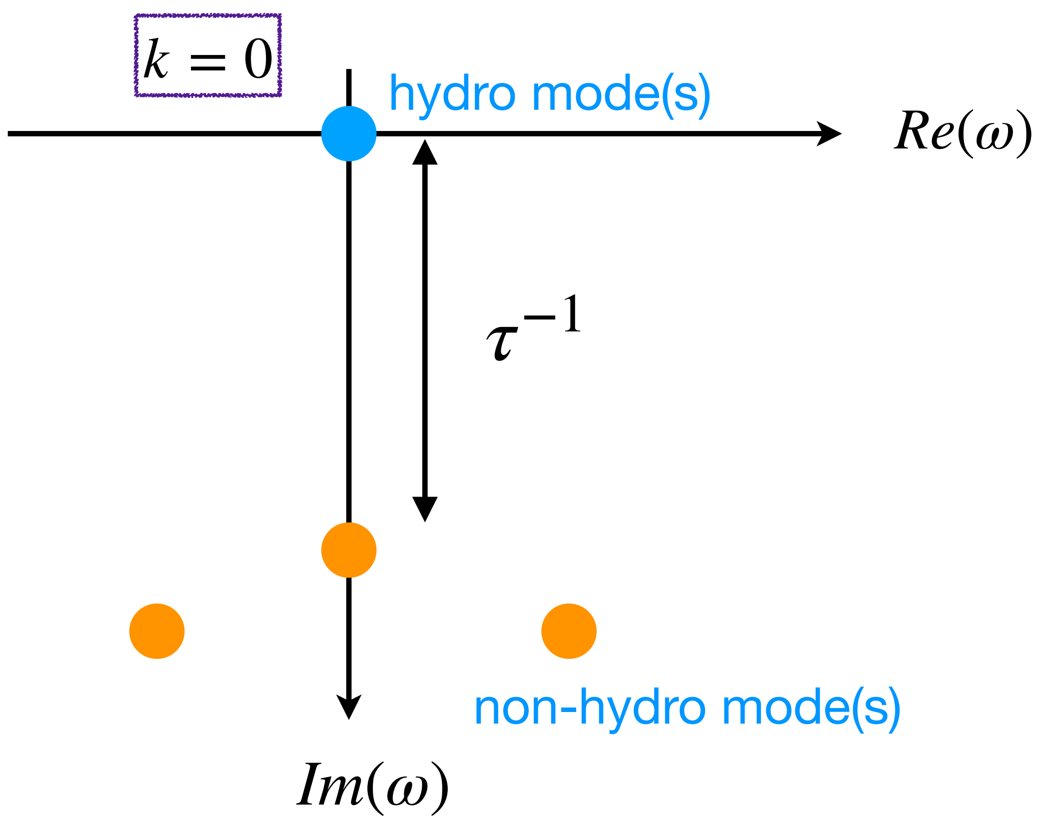
[Kovtun, '12]

Convergence of dispersion relations

- Hydro modes $\omega_i(k) \to 0 \quad \text{when} \quad k \to 0$

  Example: sound mode $\omega_{\text{sound}}(k) = \pm v_s k - i\Gamma k^2 + \ldots$

- Does the series $\omega(k) = b_1 k + b_2 k^2 + b_3 k^3 + \ldots$ converge in hydro?



-

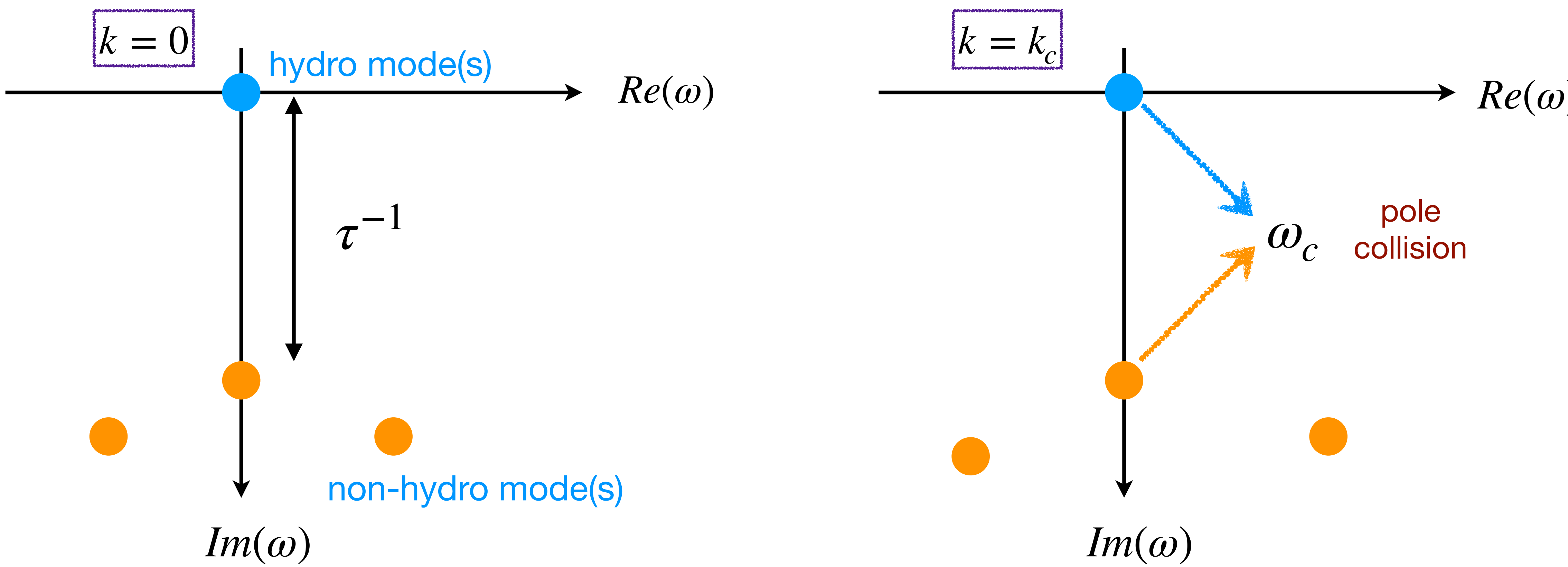
[Whiters, 2018; Grozdanov, Kovtun, Starinets, Padi, 2019, …]

Convergence of dispersion relations

- Hydro modes $\omega_i(k) \to 0 \quad \text{when} \quad k \to 0$

  Example: sound mode $\omega_{\text{sound}}(k) = \pm v_s k - i\Gamma k^2 + \ldots$

- Does the series $\omega(k) = b_1 k + b_2 k^2 + b_3 k^3 + \ldots$ converge in hydro?

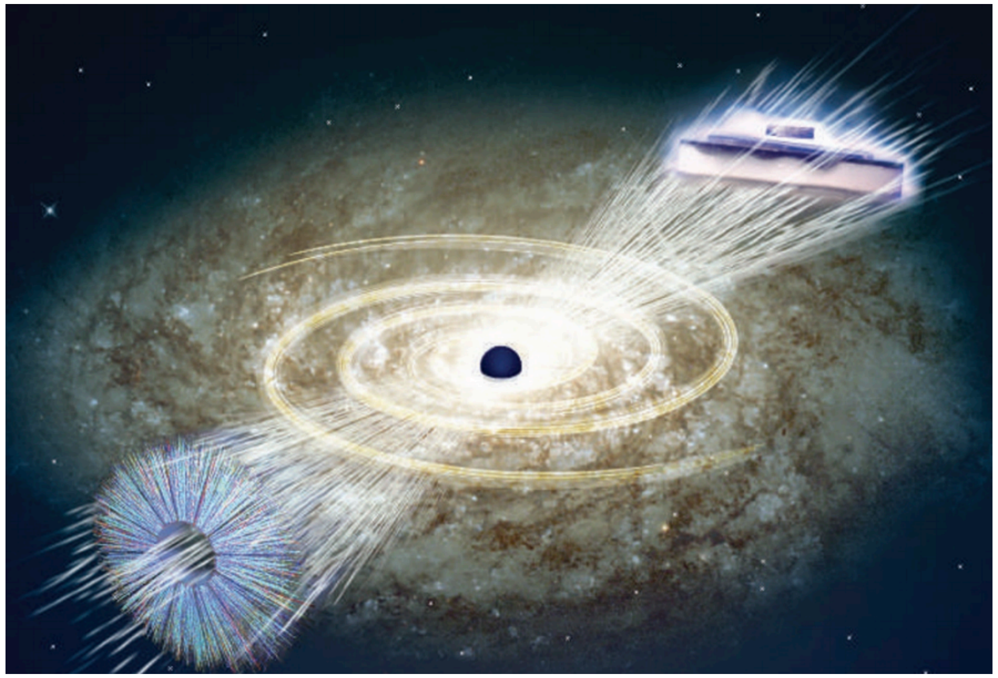


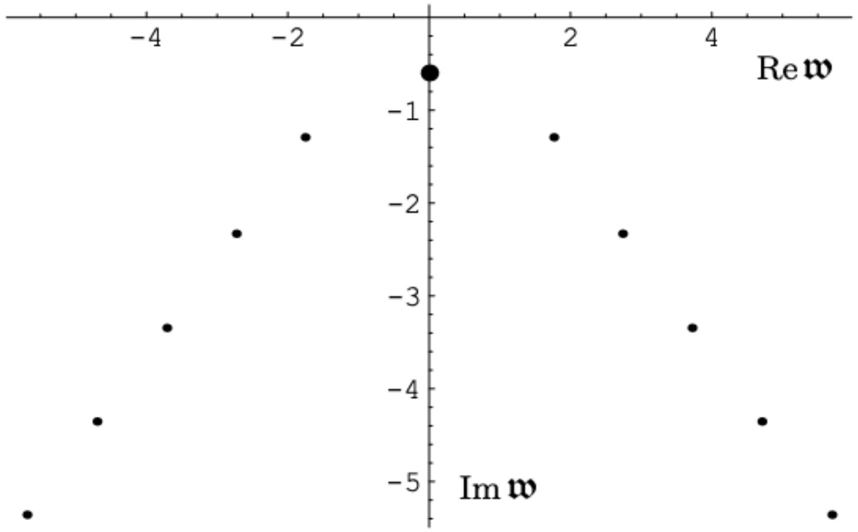
- $|k_c|$ is the radius of convergence! (equilibrium scales)

[Whiters, 2018; Grozdanov, Kovtun, Starinets, Padi, 2019, …]

QNM in holography

$$G^{-1}_{\rho\rho}(\omega, k) = 0$$



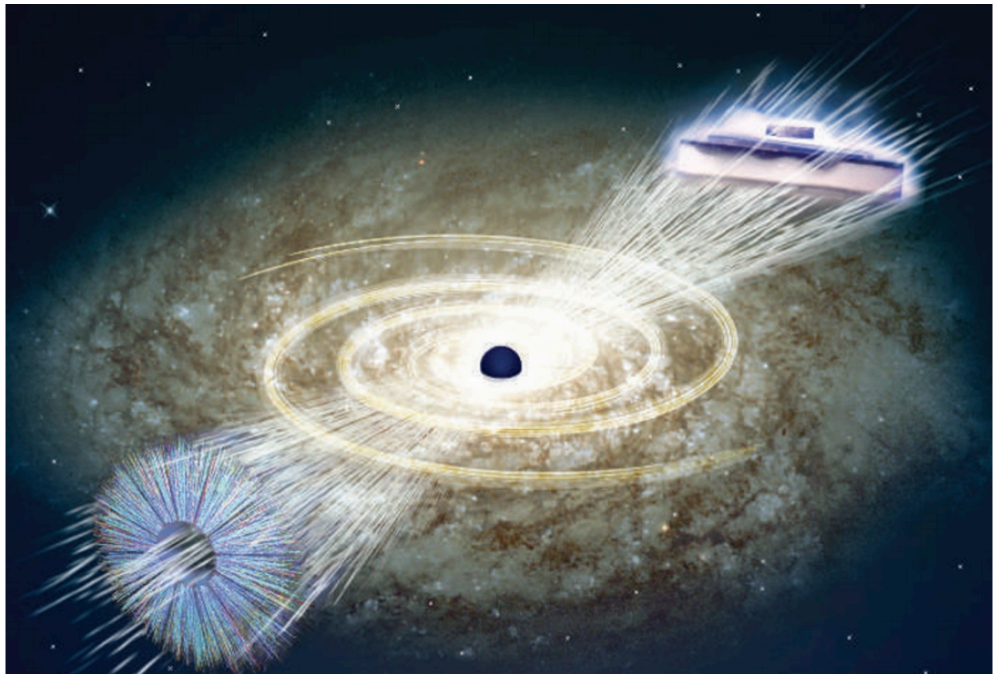


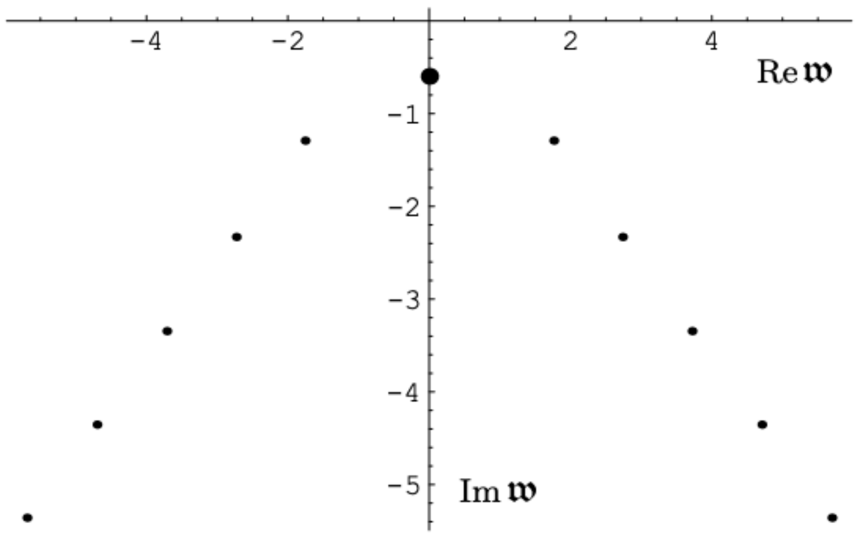
real k

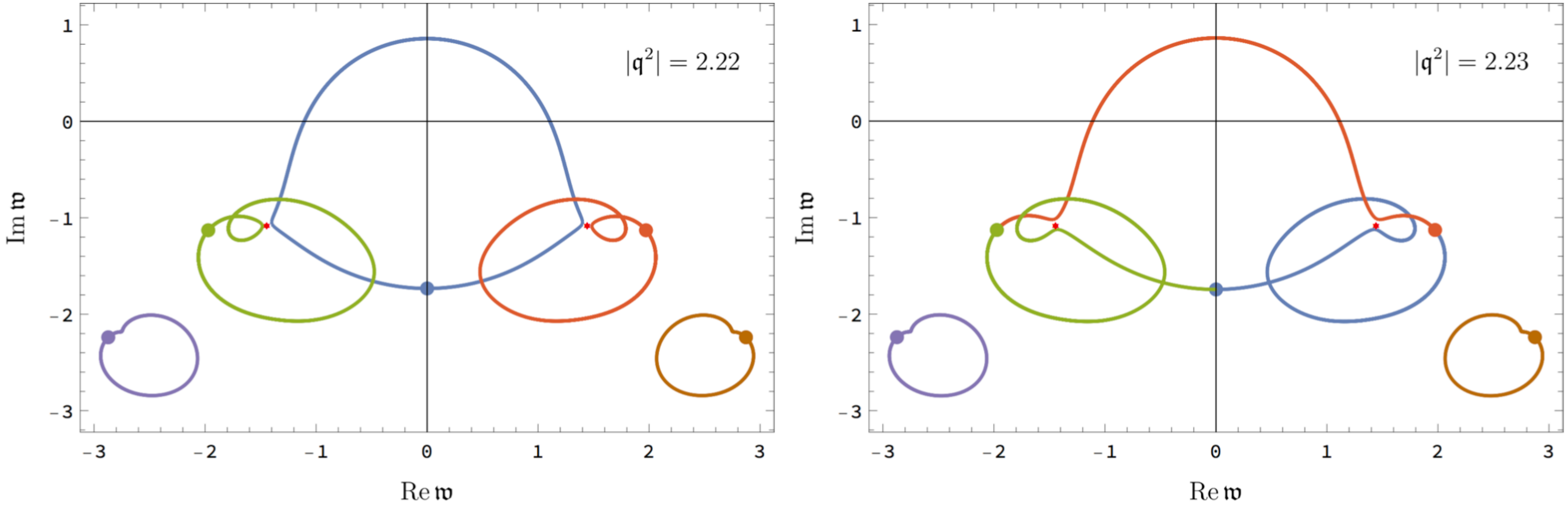
[Kovtun, Starinets ’05, …]   [talks by Landsteiner]

QNM in holography

$$G^{-1}_{\rho\rho}(\omega, k) = 0$$

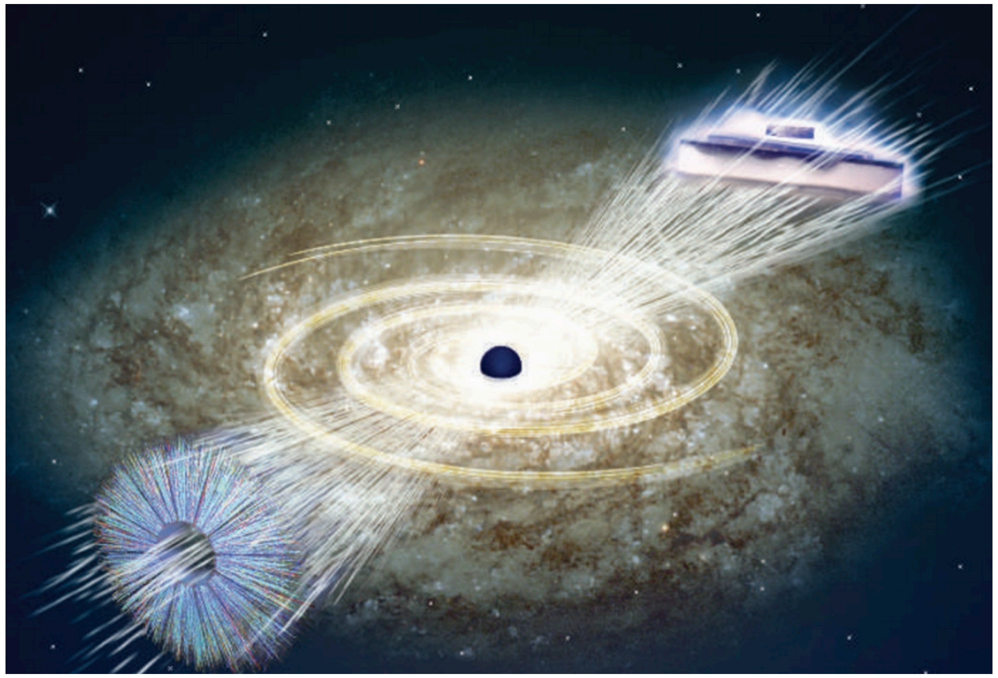


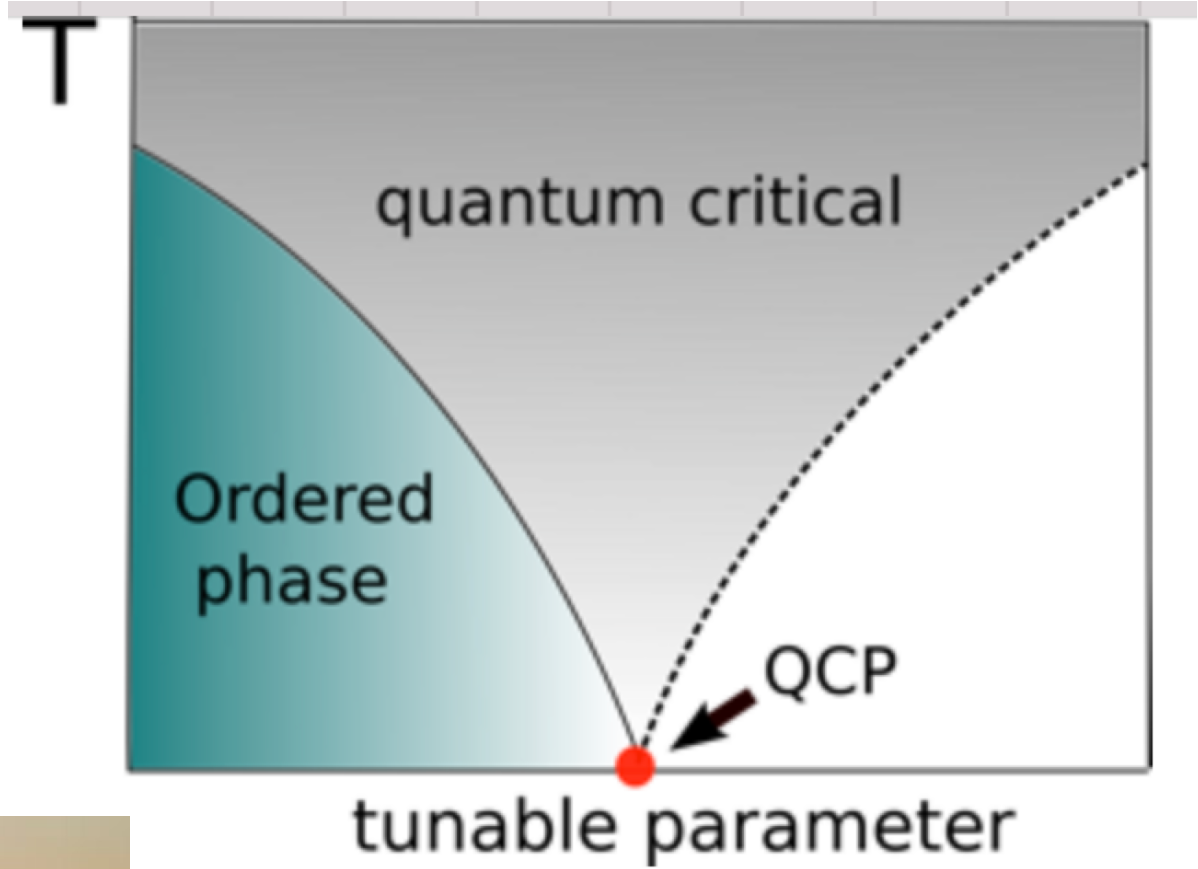




[Whiters, ’18; Grozdanov, Kovtun, Starinets, Padi, ’19, …]

Close to QCP?





[Arean, Davison, Gouteraux, Suzuki, ’21, …]

Holographic system

- Gubser-Rocha model

$$S=\int d^4x\sqrt{-g}\left(R-\frac{1}{4}e^{\alpha\phi}\,F^2-\frac{3}{2}(\partial\phi)^2+\frac{6}{L^2}\cosh\phi-\frac{1}{2}\sum_{I=1}^{2}(\partial\psi_I)^2\right),$$

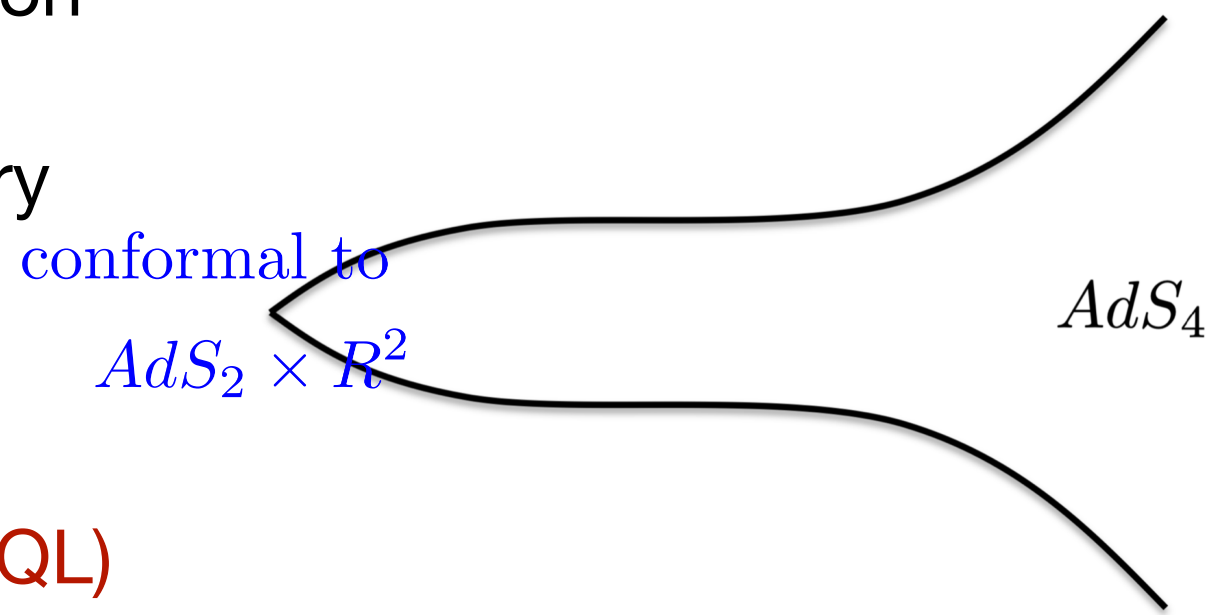
- At zero density, neutral diatonic solution with linear axion fields $\psi_I = m\,x_I$
- Charge and energy diffuse separately

[Gubser, Rocha, ’09; Andrade, Withers, ’13; …YL, Wu, ’21;…]

Holographic system

neutral diatonic black hole solution

- T=0, the near horizon geometry



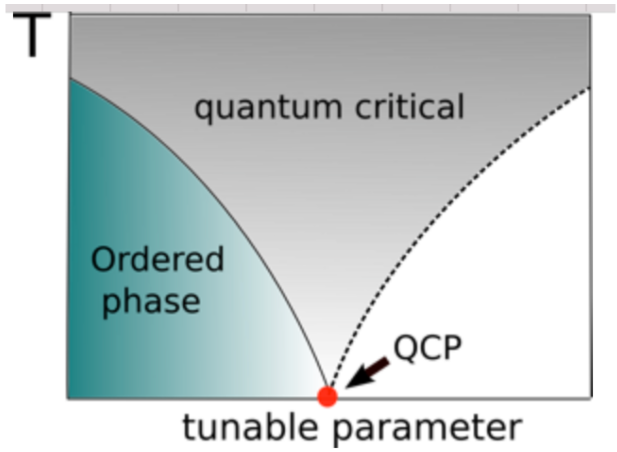
- semi-local quantum liquid (SLQL)

  infinite correlation time, finite correlation length

  [Iqbal, H.Liu, Mezei,'11; Hartnoll, Shaghoulian,'12]

- At low temperature ($T \ll m$), only near horizon regime get modified

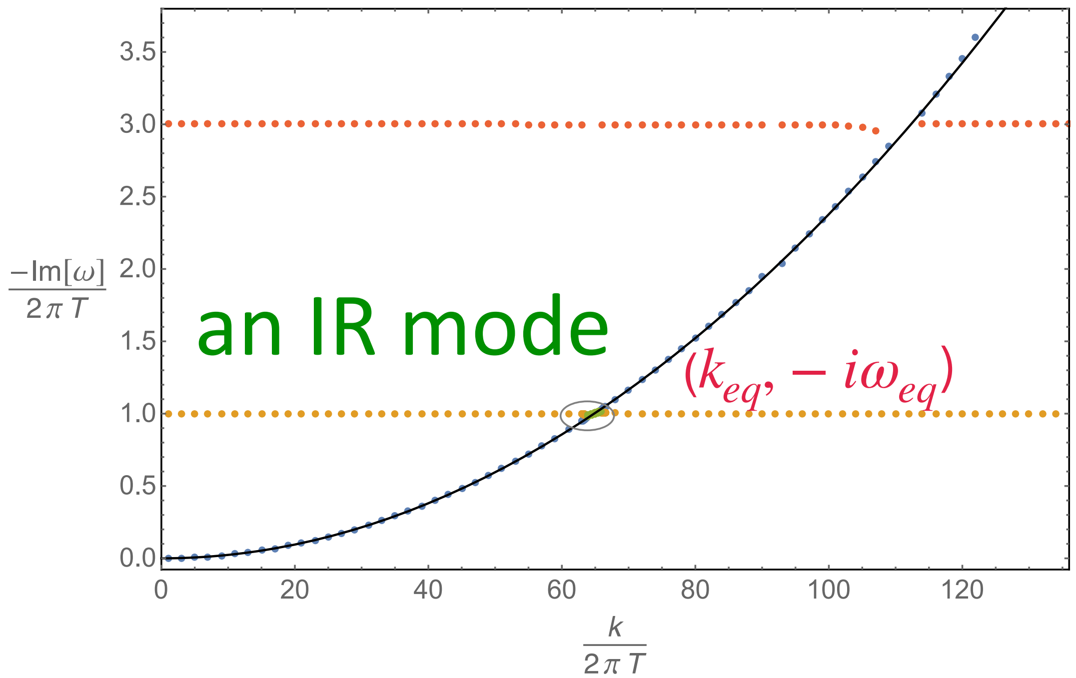
- $$s \sim T \sim \sqrt{m r_0}$$

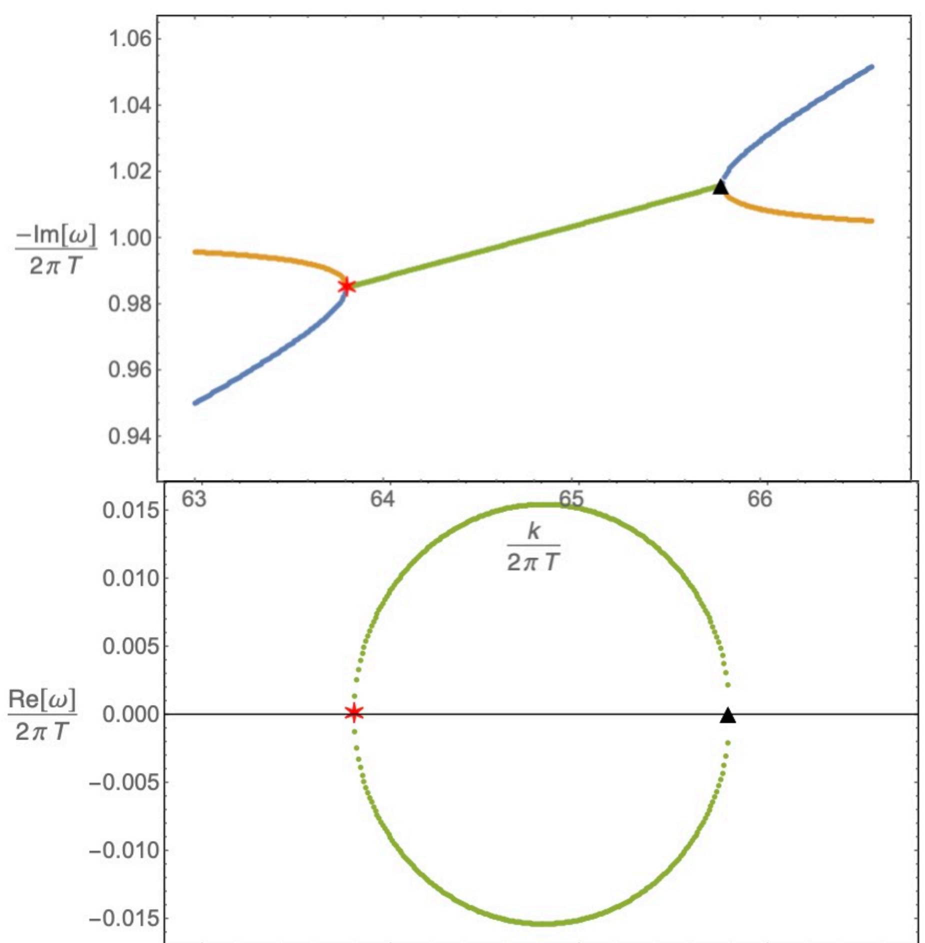


Pole collision

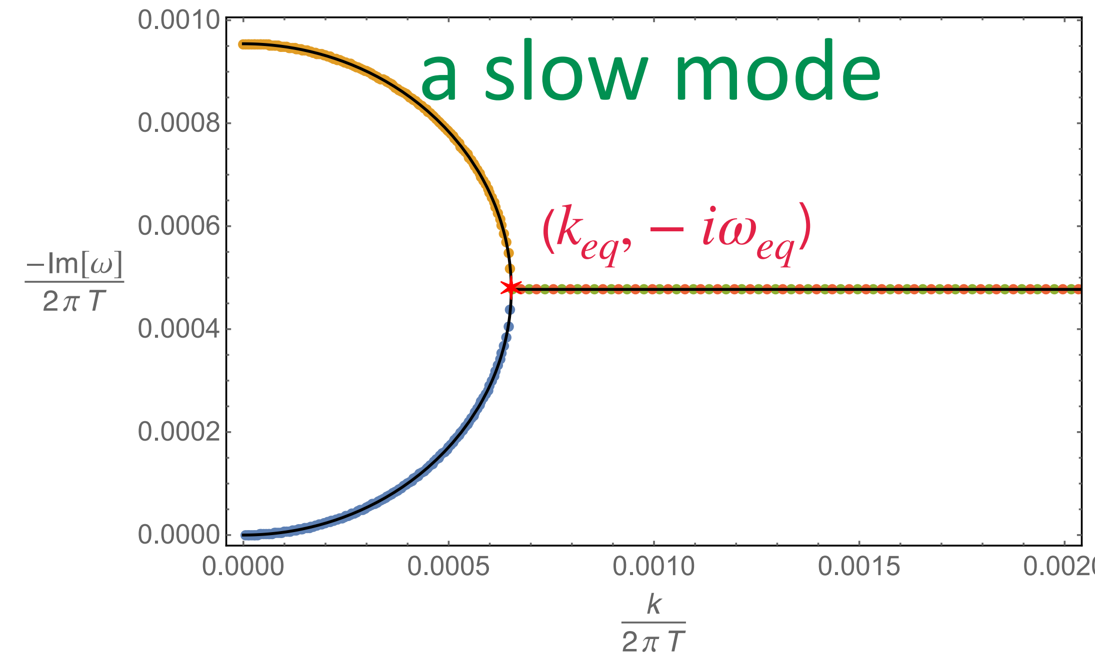
$g_{\text{eff}}^2 \sim (T/m)^\alpha$

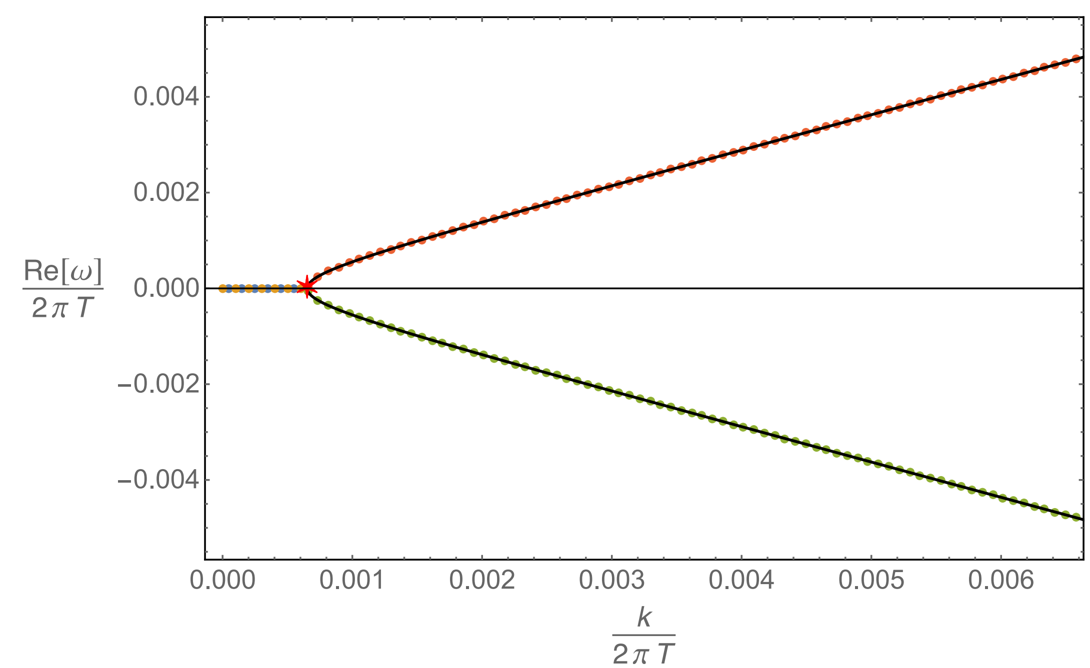
$\alpha = 0 \qquad T\tau \sim 1$





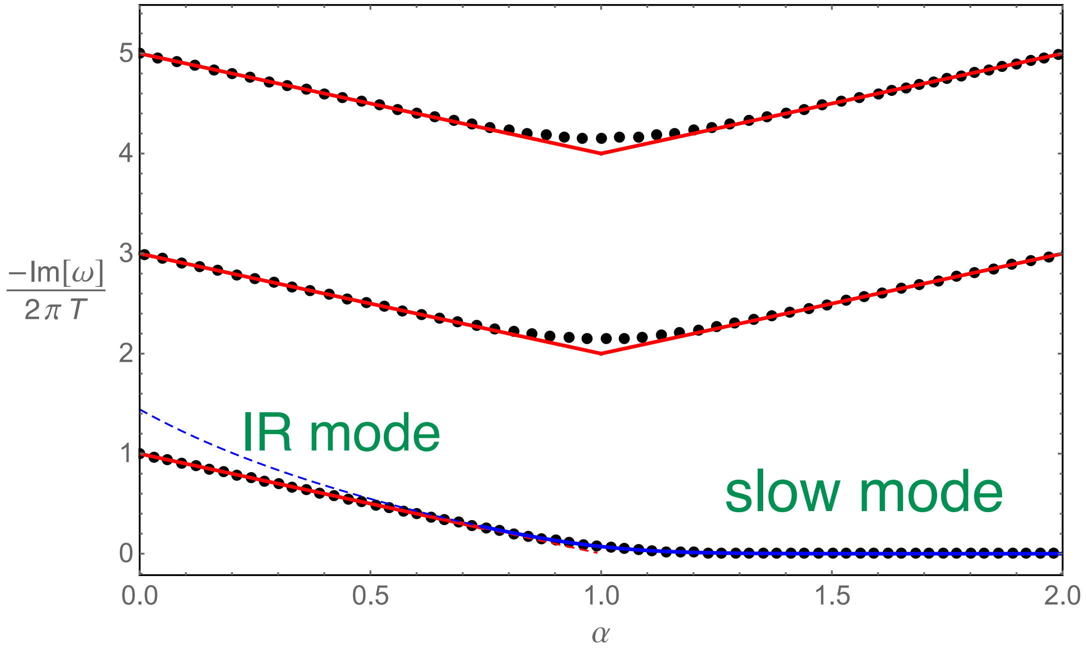
$\alpha = 2 \qquad T\tau \gg 1$



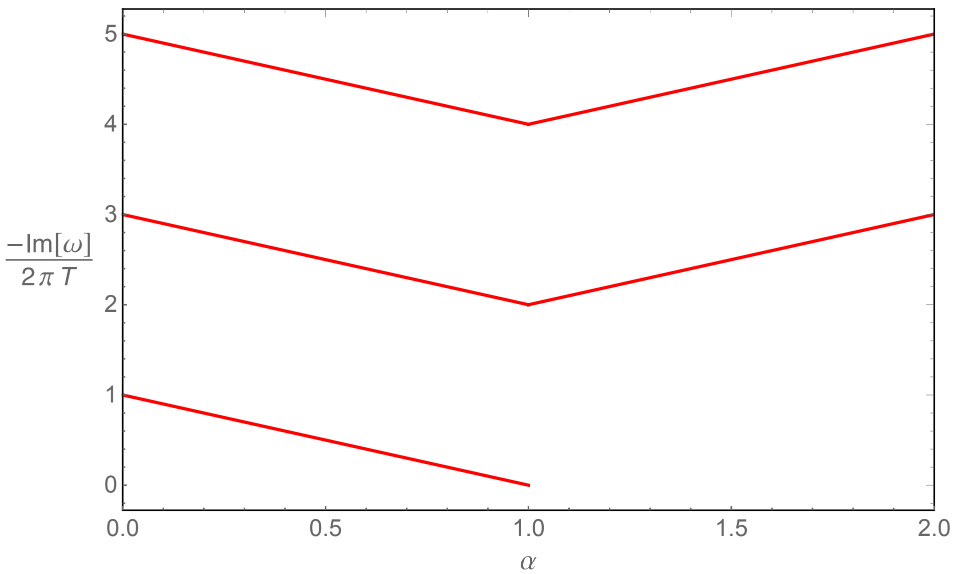


non-hydro modes

- General $\alpha \geq 0$, non-hydro modes at low T



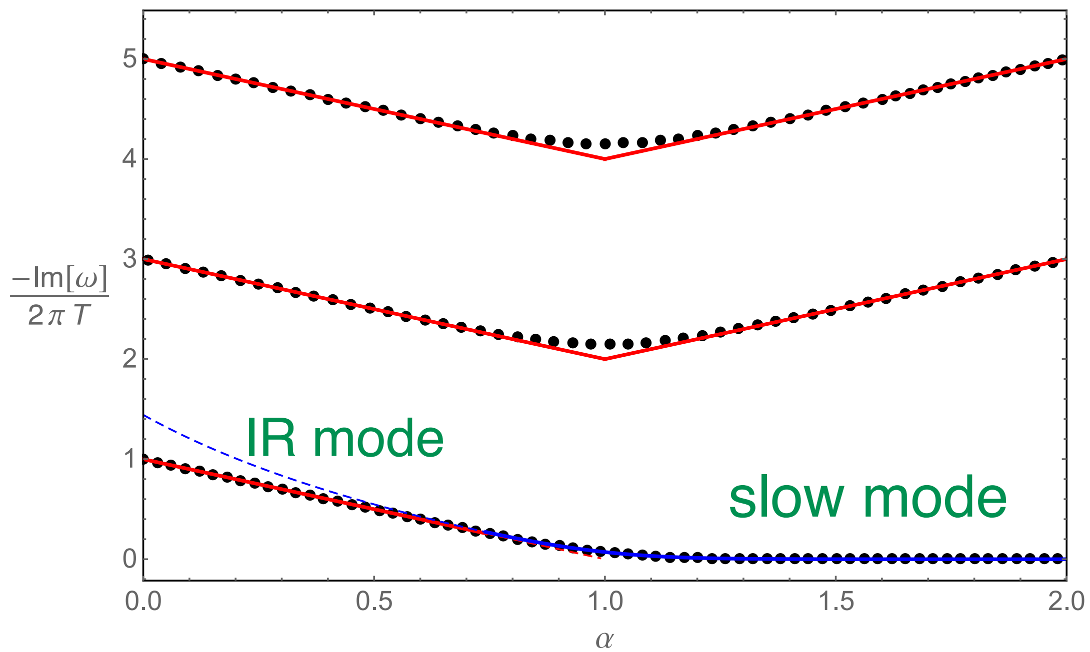
QNM



IR mode from SLQL

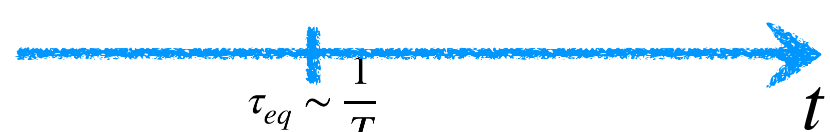
non-hydro modes

- General $\alpha \geq 0$, non-hydro modes at low T

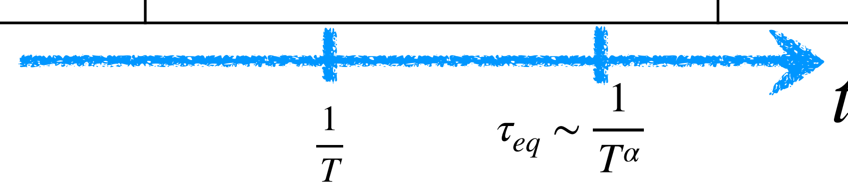


$$D_c \sim \frac{1}{m}\left(\frac{m}{2\sqrt{2}\pi T}\right)^{\alpha}$$

$$v_B^2 \sim \frac{T^2}{m^2}, \quad \tau_L \sim \frac{1}{T}$$



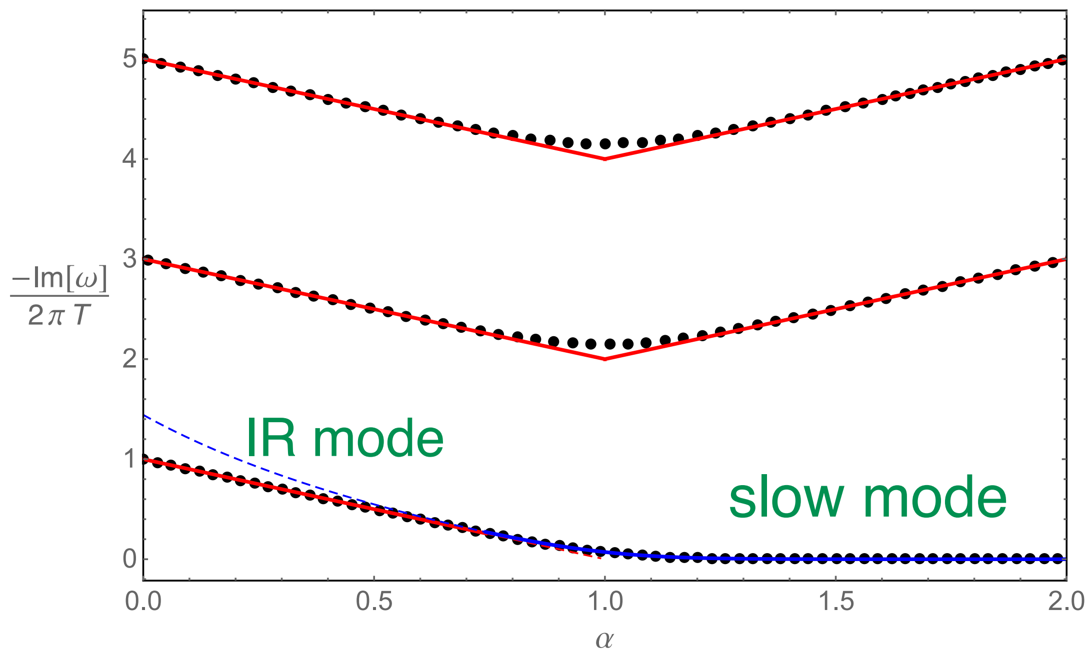
$\alpha < 1$ IR mode	$\omega_{eq} \sim T\,, \quad k_{eq} \sim m\left(\frac{T}{m}\right)^{\frac{1+\alpha}{2}}$	$v_{eq} \sim \left(\frac{T}{m}\right)^{\frac{1-\alpha}{2}}, \quad \tau_{eq} \sim \frac{1}{T}$
$\alpha > 1$ slow mode	$\omega_{eq} \sim k_{eq} \sim m\left(\frac{T}{m}\right)^{\alpha}$	$v_{eq} \sim T^0, \quad \tau_{eq} \sim \frac{m^{\alpha-1}}{T^{\alpha}}$

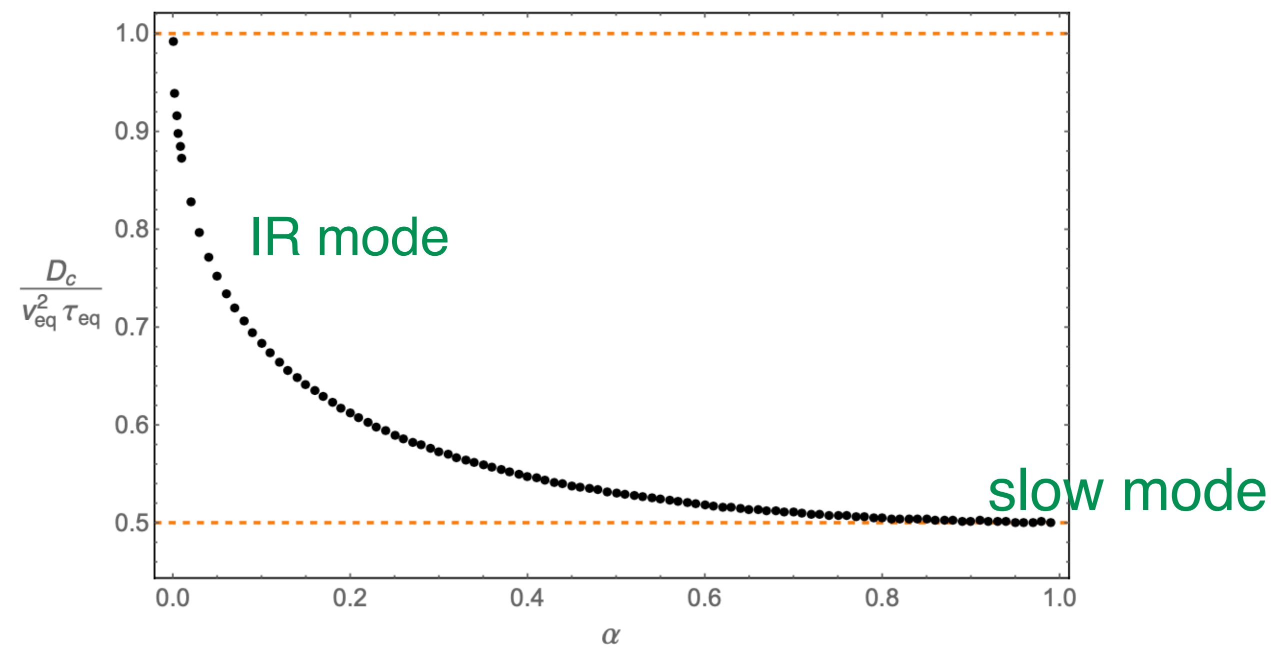


non-hydro modes

- General $\alpha \geq 0$, non-hydro modes at low T

upper bound on diffusion



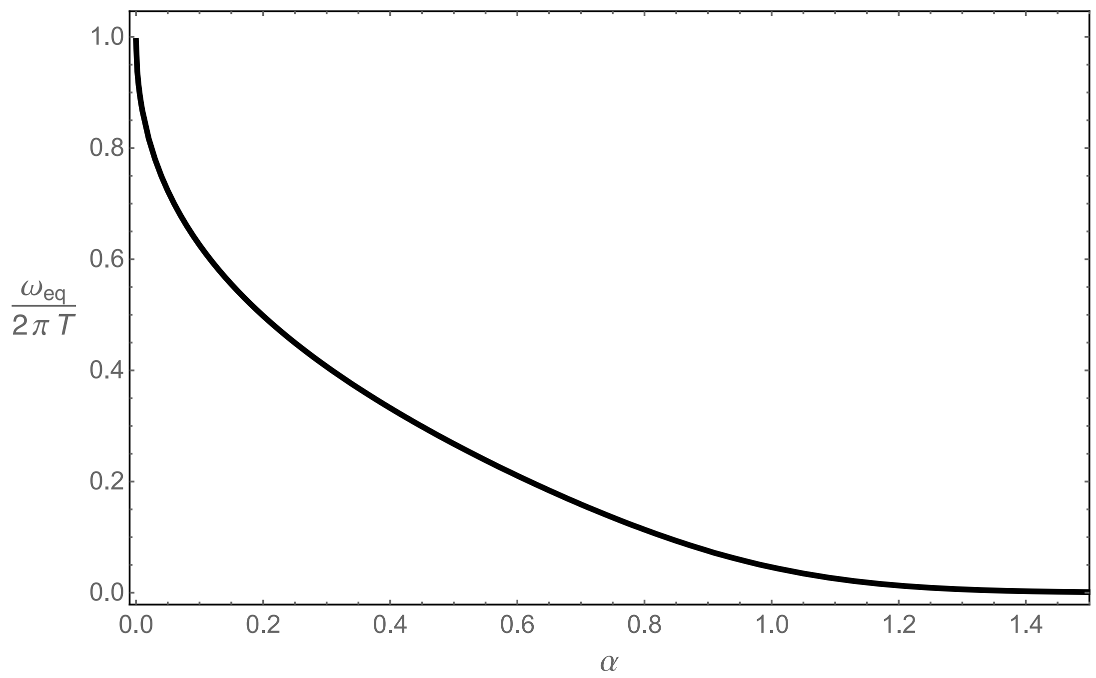


$\alpha < 1$ IR mode	$\omega_{eq} \sim T\,,\quad k_{eq} \sim m\left(\frac{T}{m}\right)^{\frac{1+\alpha}{2}}$	$v_{eq} \sim \left(\frac{T}{m}\right)^{\frac{1-\alpha}{2}}\,,\quad \tau_{eq} \sim \frac{1}{T}$
$\alpha > 1$ slow mode	$\omega_{eq} \sim k_{eq} \sim m\left(\frac{T}{m}\right)^{\alpha}$	$v_{eq} \sim T^0\,,\quad \tau_{eq} \sim \frac{m^{\alpha-1}}{T^{\alpha}}$

[Hartman, Hartnoll, Mahajan, ’17; Arean, Davison, Gouteraux, Suzuki, ’21]

The radius of convergence

- *At low temperature, hydro works better at strong coupling*



effective gauge coupling

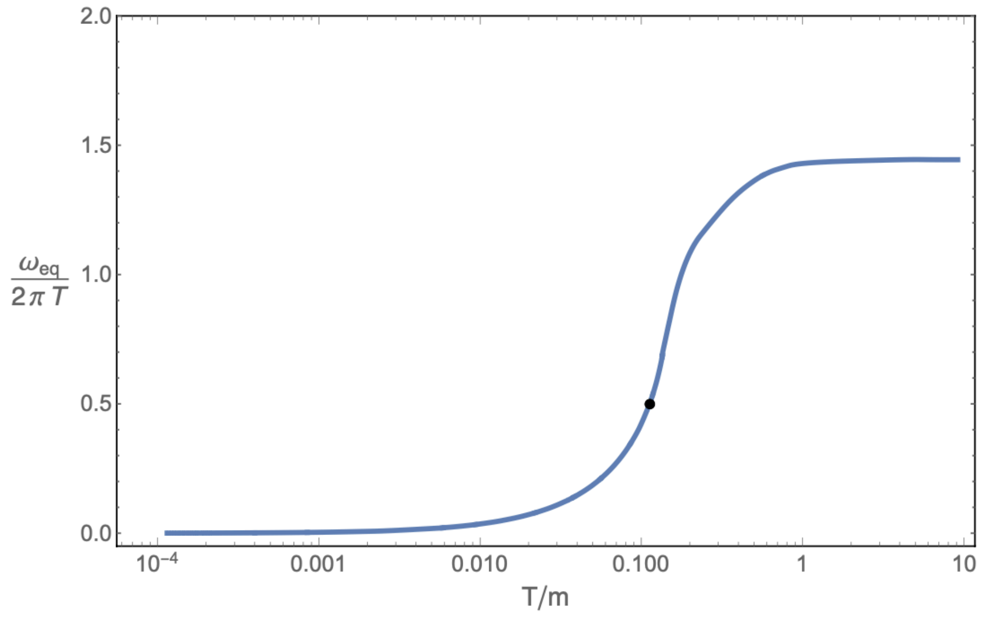
$$g_{\rm eff}^2 \sim (T/m)^\alpha$$

From quantum liquid to thermal liquid

- At general temperature,

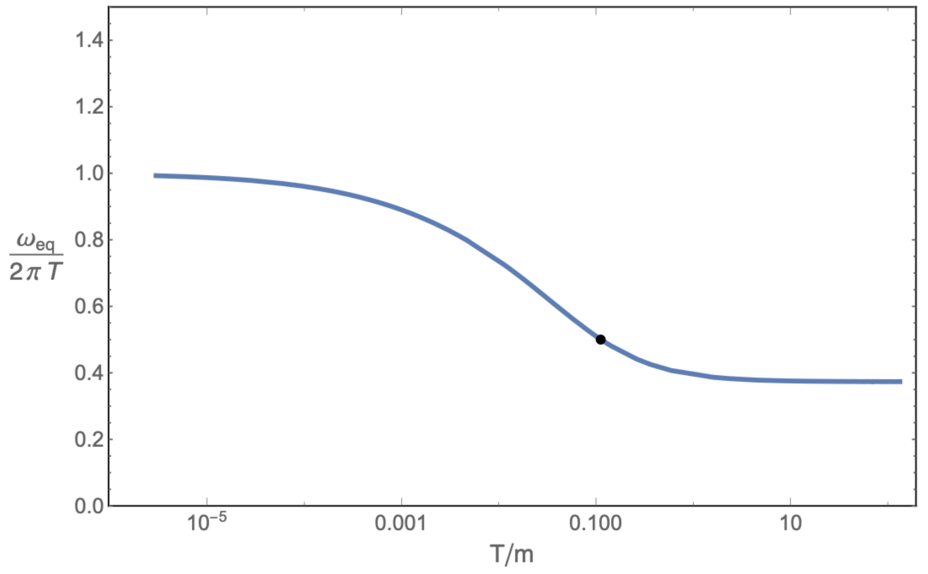
case 1: at low T, hydro mode collides with a slow mode

- Hydro works better at high T



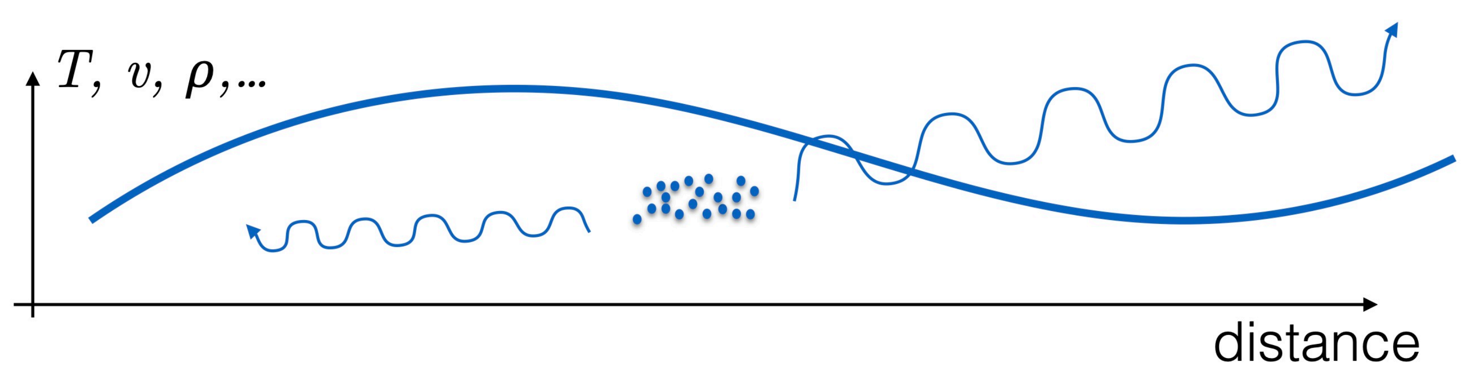
case 2: at low T, hydro mode collides with an IR mode

- Hydro works better at low T



Classical hydro is incomplete

- The incompleteness of classical hydro?
  - fluctuation effect may be important *(it is suppressed by 1/N in holography)*
  - phenomenological constraints in hydro
- Stochastic hydrodynamics (phenomenological)
- EFT from first principle (symmetry + unitarity)

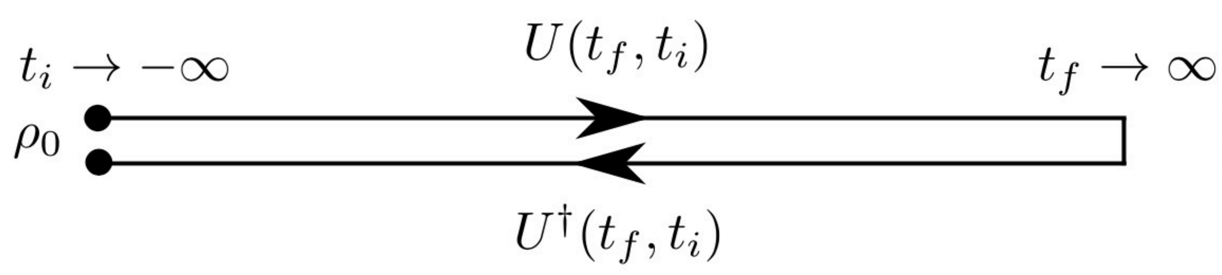


[Crossley, Glorioso, H. Liu;
Heal,Loganayagam, Rangamani, 2015]

[talks by Abbasi, Yin]

Schwinger-Keldysh effective field theory

- Should double all the degrees of freedom



$$e^{W[A_{1\mu}, A_{2\mu}]} = \mathrm{Tr}\left(\rho_0 \mathcal{P} e^{i\int d^dx\, A_{1\mu} J_1^\mu - i\int d^dx\, A_{2\mu} J_2^\mu}\right)$$

- For a conserved current

$$e^{W[A_{1\mu}, A_{2\mu}]} = \int \mathcal{D}\varphi_1 \mathcal{D}\varphi_2\, e^{iI_{\text{eff}}[B_{1\mu}, B_{2\mu}]}$$

a local action
fixed by symmetry principle

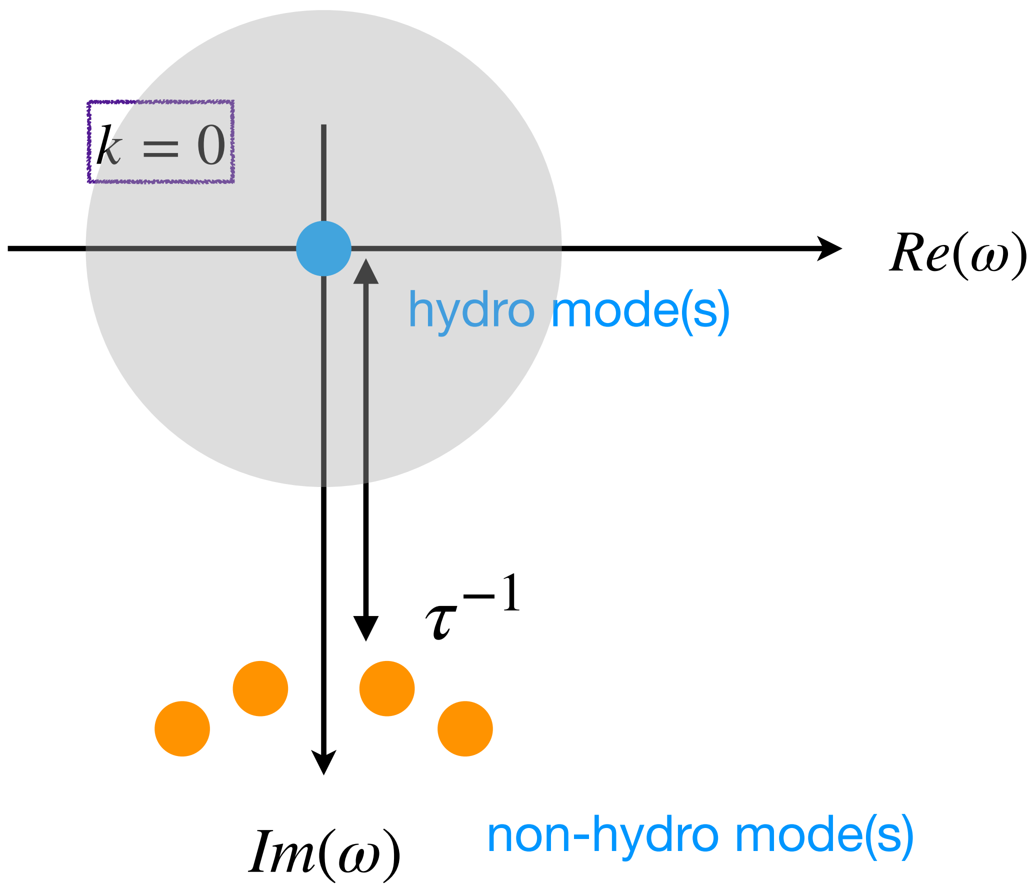
$$B_{1\mu} = A_{1\mu} + \partial_\mu \varphi_1\,, \qquad B_{2\mu} = A_{2\mu} + \partial_\mu \varphi_2$$

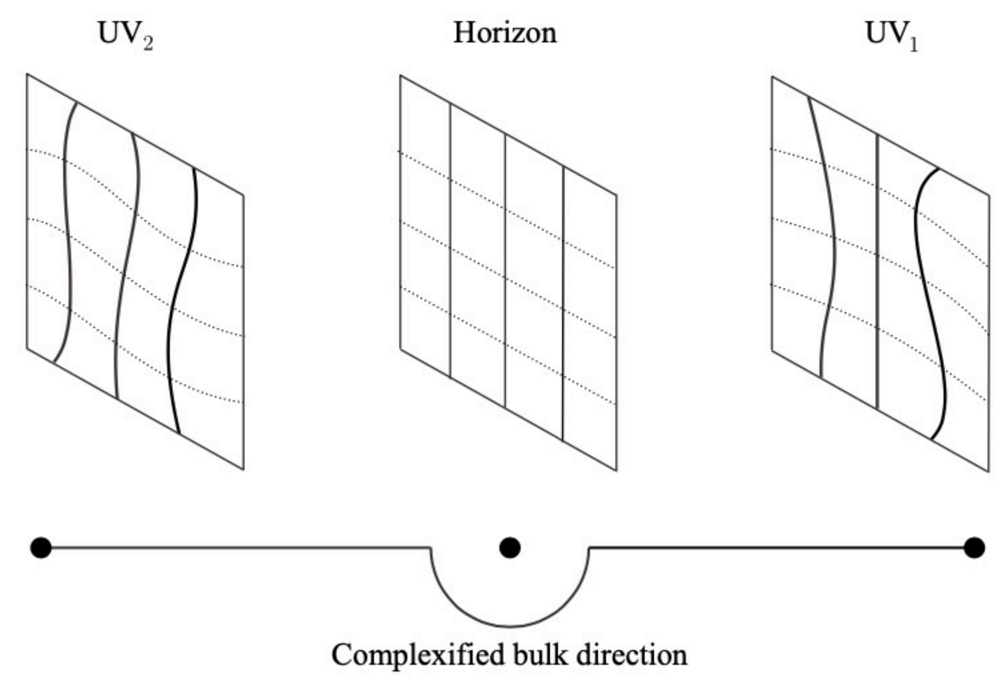
dynamical variable

[Crossley, Glorioso, H. Liu;
Heal, Loganayagam, Rangamani; 2015]

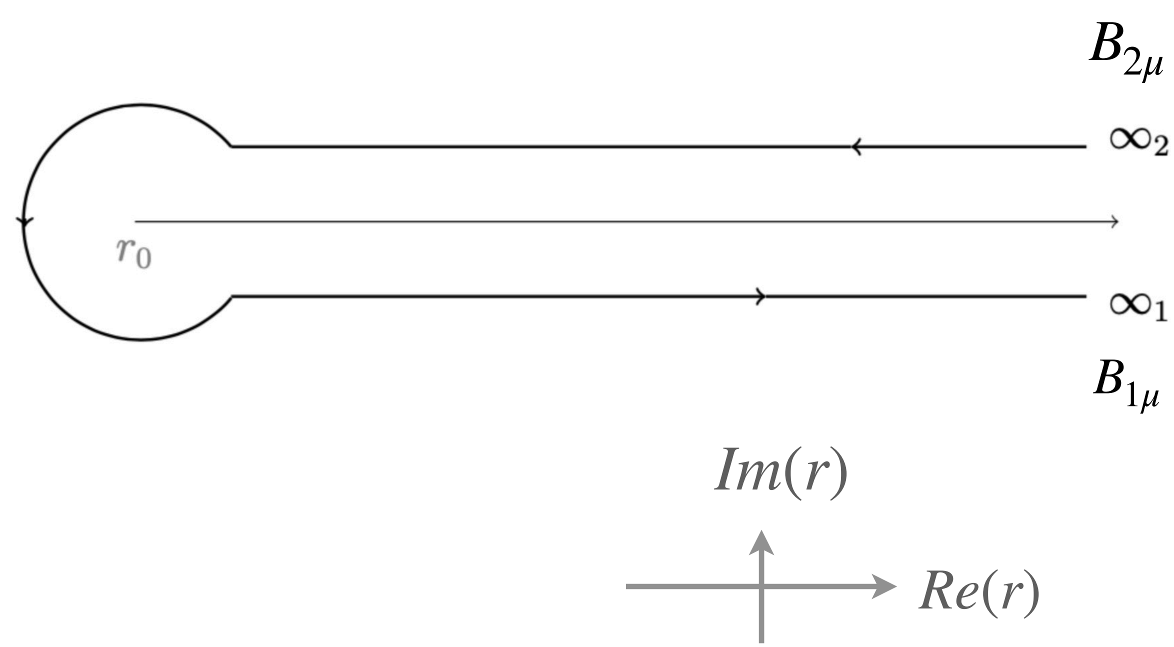
[talks by Abbasi, Yin]

Hydro EFT for diffusion from holography



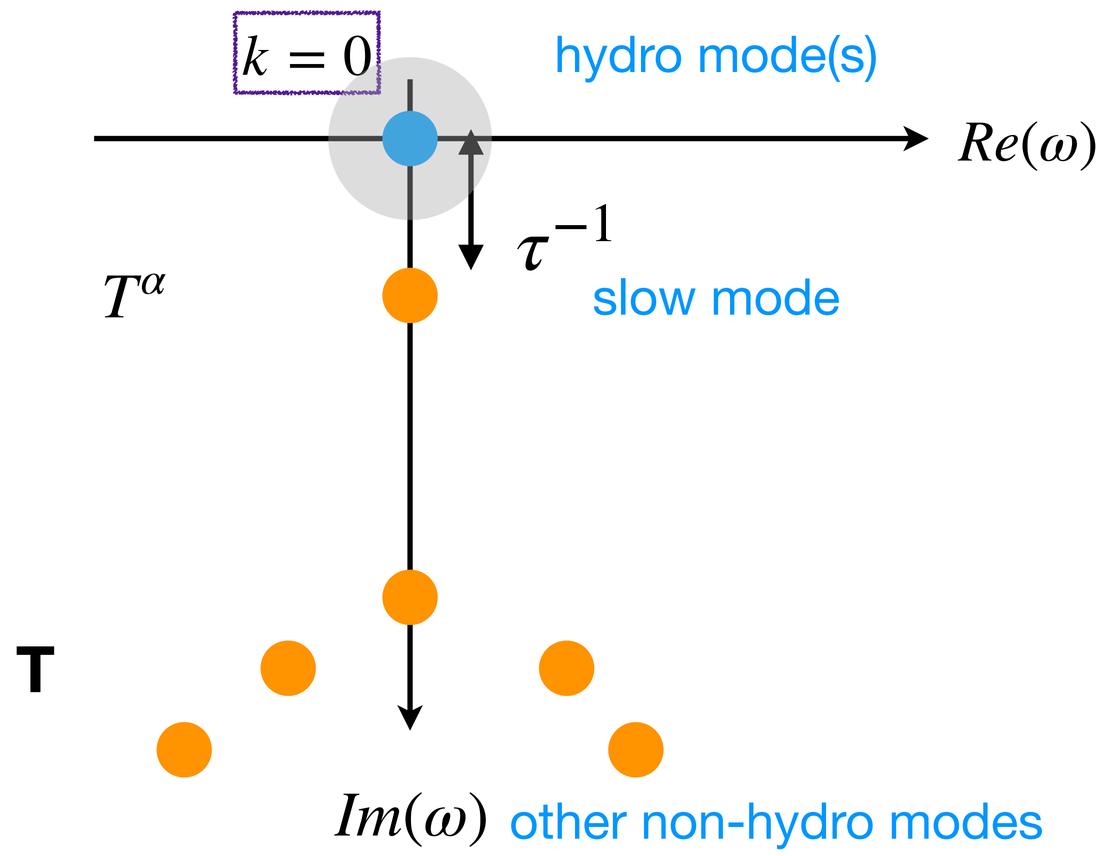


$$I_{\rm eff}[B_{a\mu}, B_{r\mu}] = \int d^{d+1}x \Big[\chi B_{a0}B_{r0} + iT\sigma B_{ai}^2 - \sigma B_{ai}\partial_0 B_{ri})\Big)\Big]$$



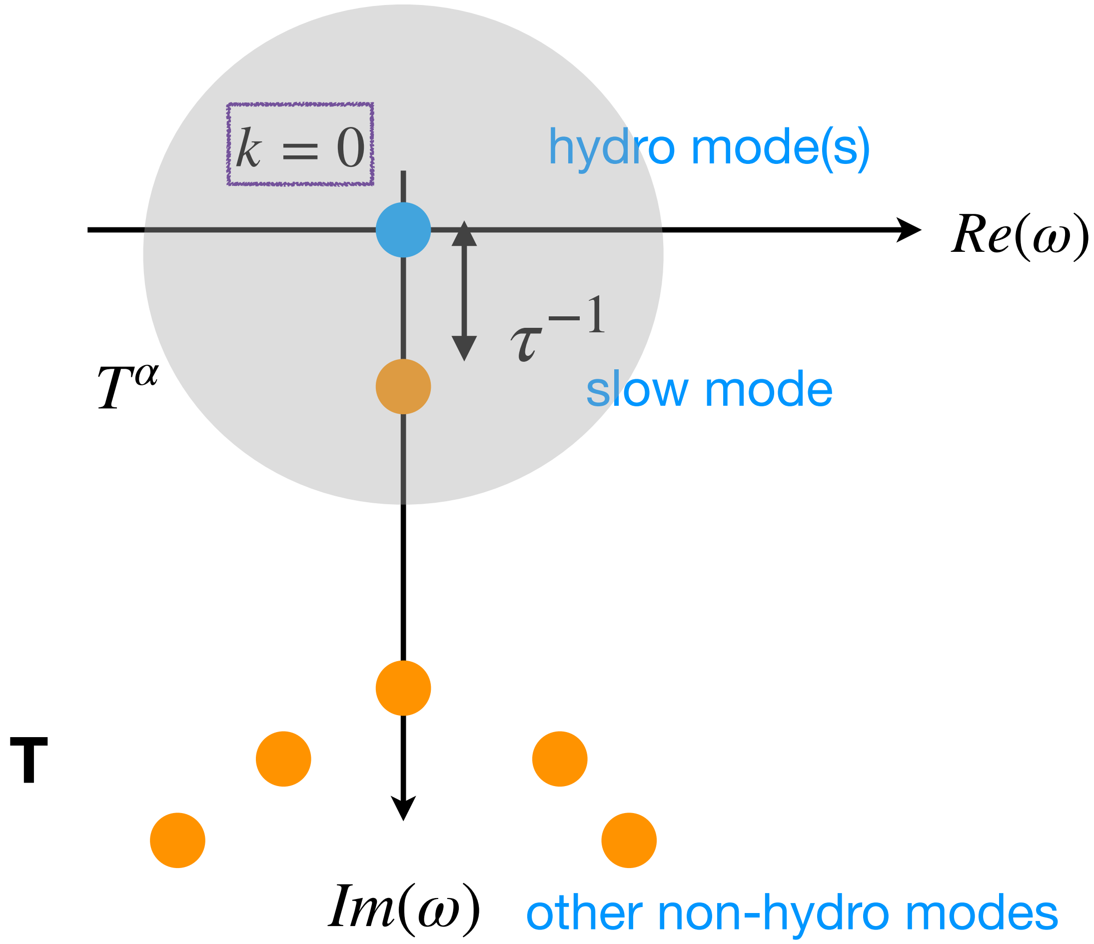
[Crossley, Glorioso, H. Liu;
de Boer, Heller, Pinzani-Fokeeva; 2018]

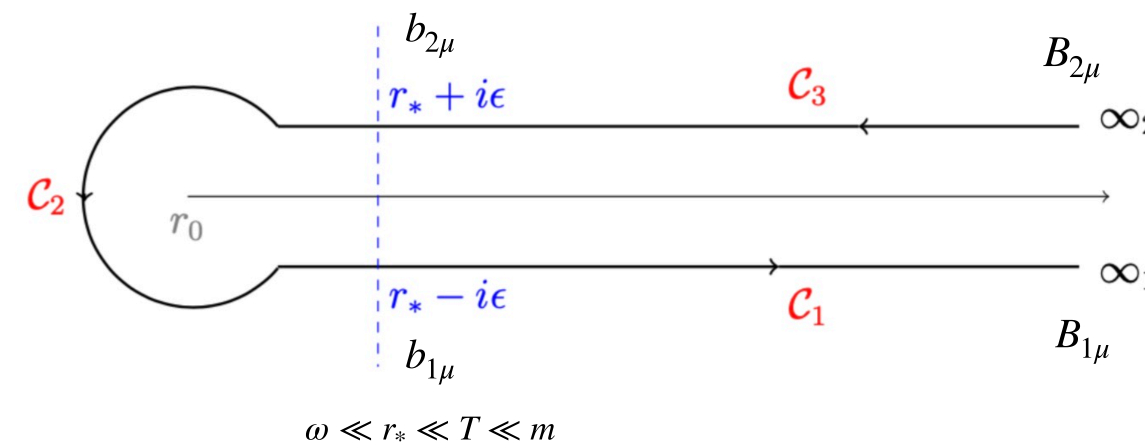
SK EFT for diffusion + slow mode



[YL, Sun, Wu, '24]

SK EFT for diffusion + slow mode

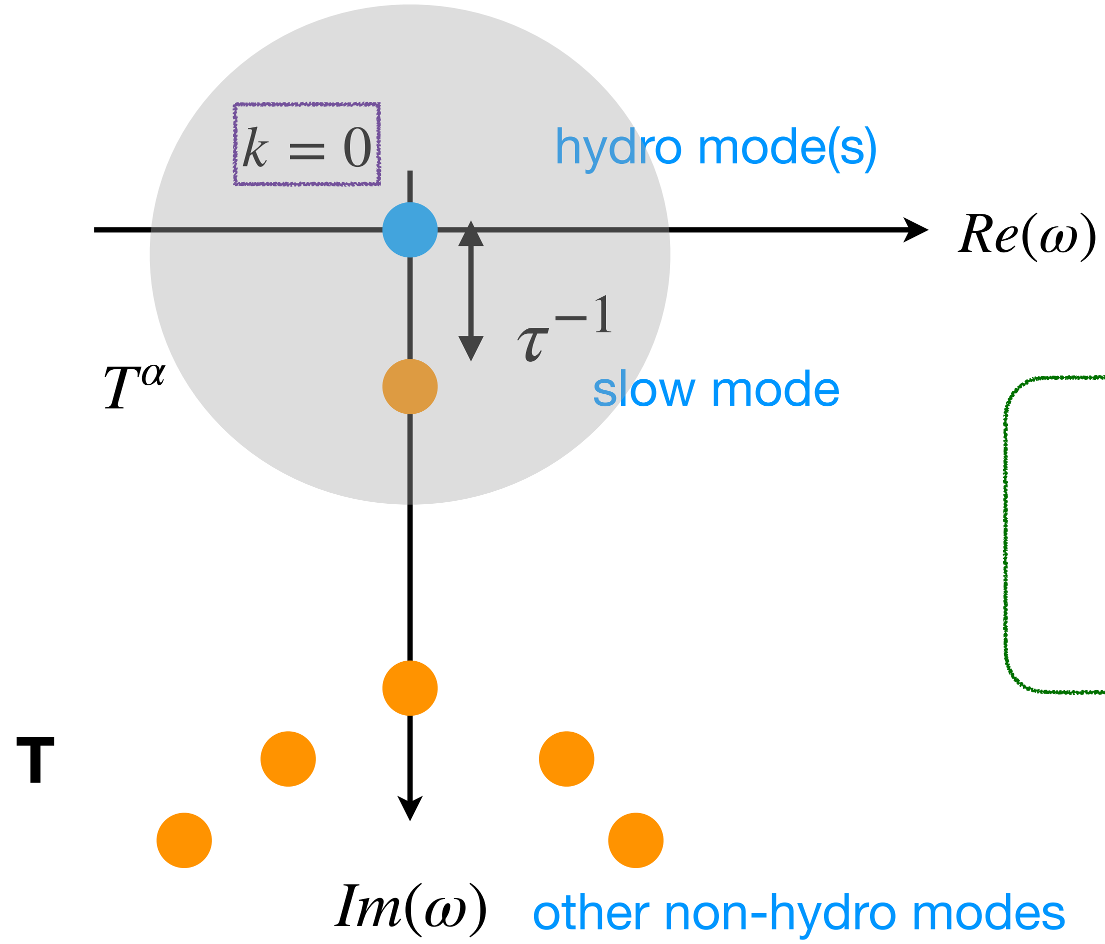




$$S = \int dv dx dy \Big[\chi B_{a0} B_{r0} + i\sigma T\, b_{ai} b_{ai} - \sigma\, b_{ai}\partial_0 b_{ri} - \frac{\sigma}{\tau}(B_{ai} - b_{ai})(B_{ri} - b_{ri})\Big],$$

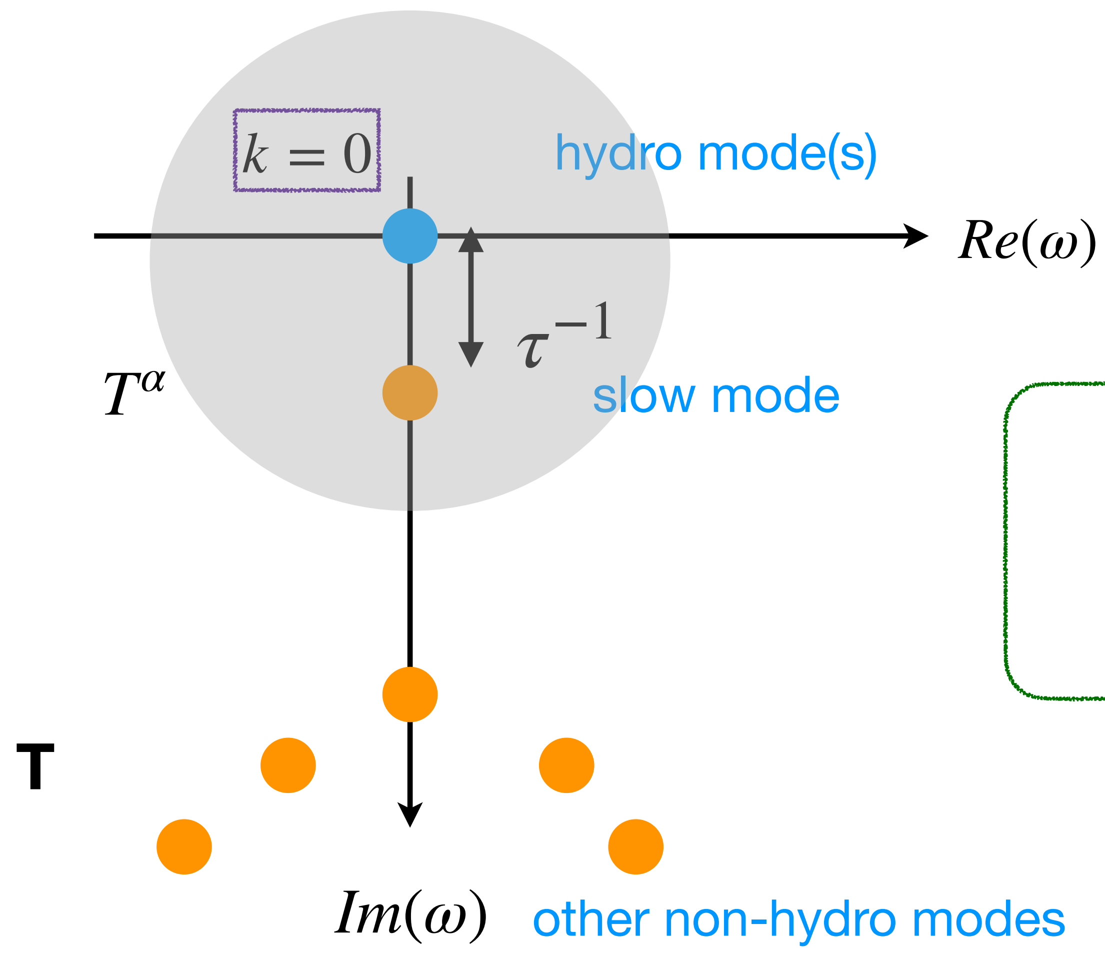
$$e^{W[A_{1\mu}, A_{2\mu}]} = \int \mathcal{D}\varphi_1 \mathcal{D}\varphi_2\, e^{iI_{\text{eff}}[B_{1\mu}, B_{2\mu}]} = \int \mathcal{D}\varphi_1 \mathcal{D}\varphi_2 \mathcal{D}v_{1i} \mathcal{D}v_{2i}\, e^{iS_{\text{eff}}[B_{1\mu}, B_{2\mu}, v_{1i}, v_{2i}]}$$

SK EFT for diffusion + slow mode



$$S = \int dv dx dy \Big[\chi B_{a0} B_{r0} + i\sigma T\, b_{ai} b_{ai} - \sigma\, b_{ai} \partial_0 b_{ri} - \frac{\sigma}{\tau} (B_{ai} - b_{ai})(B_{ri} - b_{ri}) \Big],$$

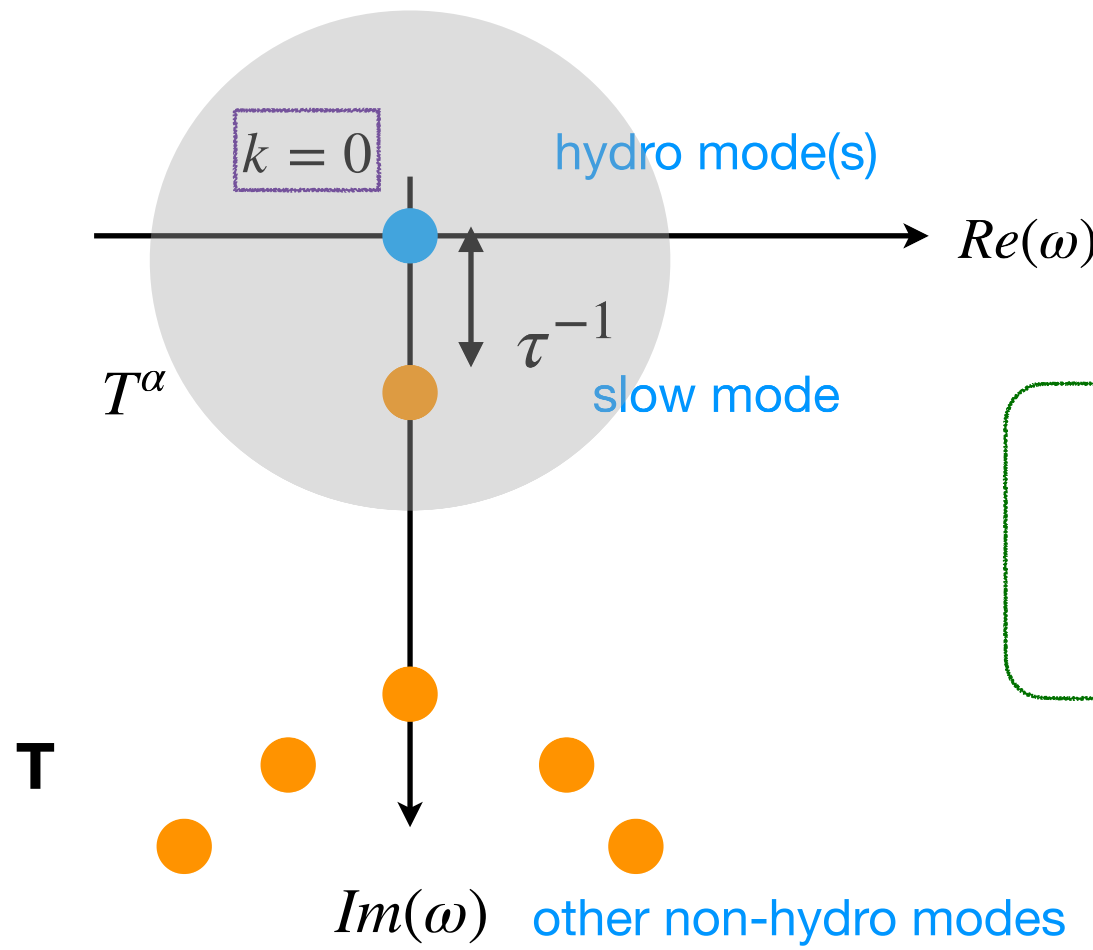
SK EFT for diffusion + slow mode



$$S=\int dvdxdy\Big[\chi B_{a0}B_{r0}+i\sigma T\,b_{ai}b_{ai} - \sigma\,b_{ai}\partial_0 b_{ri}-\frac{\sigma}{\tau}(B_{ai}-b_{ai})(B_{ri}-b_{ri})\Big],$$

(1) Integrate out b-fields, perform the limit $\omega\to 0$, recover the standard diff.

SK EFT for diffusion + slow mode



$$S=\int dvdxdy\Big[\chi B_{a0}B_{r0}+i\sigma T\,b_{ai}b_{ai} - \sigma\,b_{ai}\partial_0 b_{ri}-\frac{\sigma}{\tau}(B_{ai}-b_{ai})(B_{ri}-b_{ri})\Big],$$

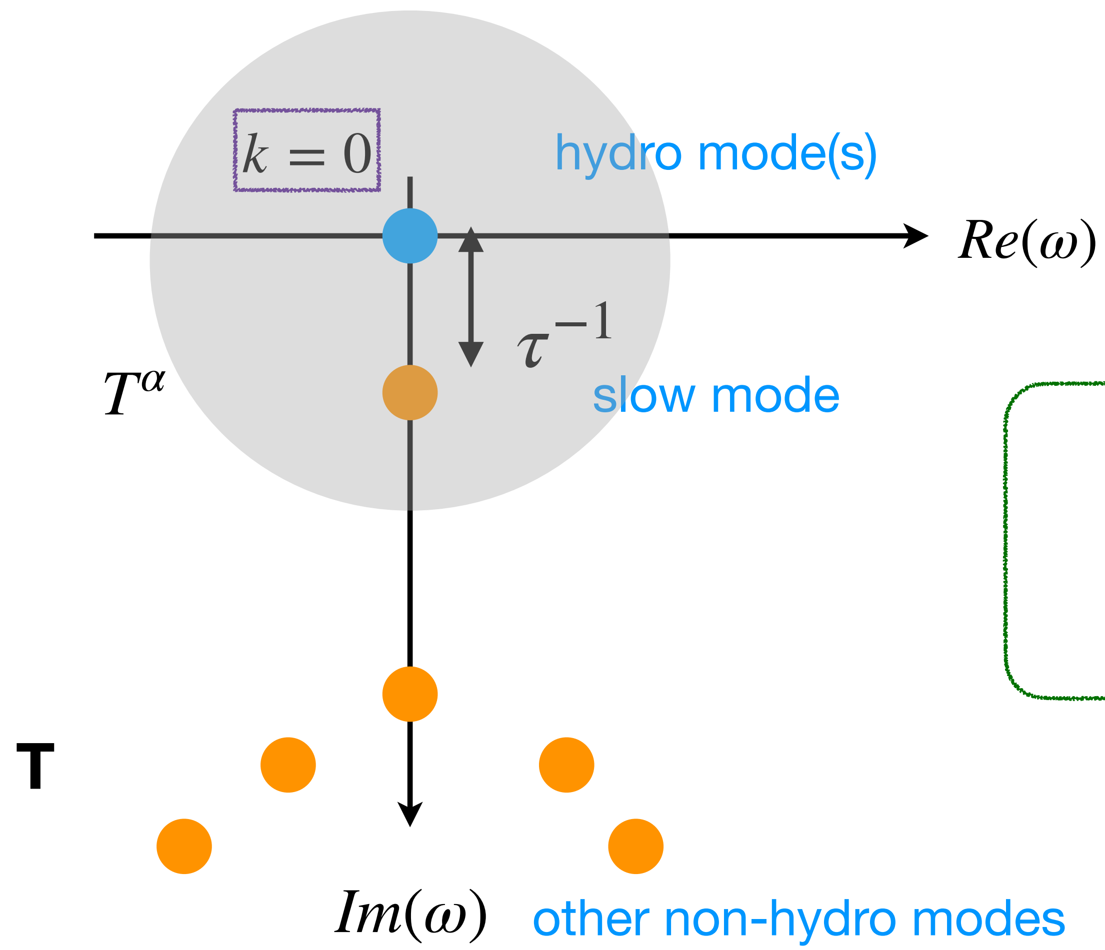
(2) Redefine $b_{ai}=B_{ai}+v_{ai}\,,\quad b_{ri}=B_{ri}+v_{ri}\,,$

we recover the SK EFT for Maxwell-Cattaneo diffusion theory by Jain and Kovtun

$$\mathcal{L}=\chi B_{a0}B_{r0}+i\sigma T(B_{ai}+v_{ai})(B_{ai}+v_{ai})-c_1 v_{ai}v_{ri}-\sigma(B_{ai}+v_{ai})\partial_0(B_{ri}+v_{ri}),$$

[Jain, Kovtun, ’23]

SK EFT for diffusion + slow mode



$$S = \int dv dx dy \Big[\chi B_{a0} B_{r0} + i\sigma T\, b_{ai} b_{ai} - \sigma\, b_{ai}\partial_0 b_{ri} - \frac{\sigma}{\tau}(B_{ai} - b_{ai})(B_{ri} - b_{ri})\Big],$$

(2) Redefine $b_{ai} = B_{ai} + v_{ai}\,, \quad b_{ri} = B_{ri} + v_{ri}\,,$

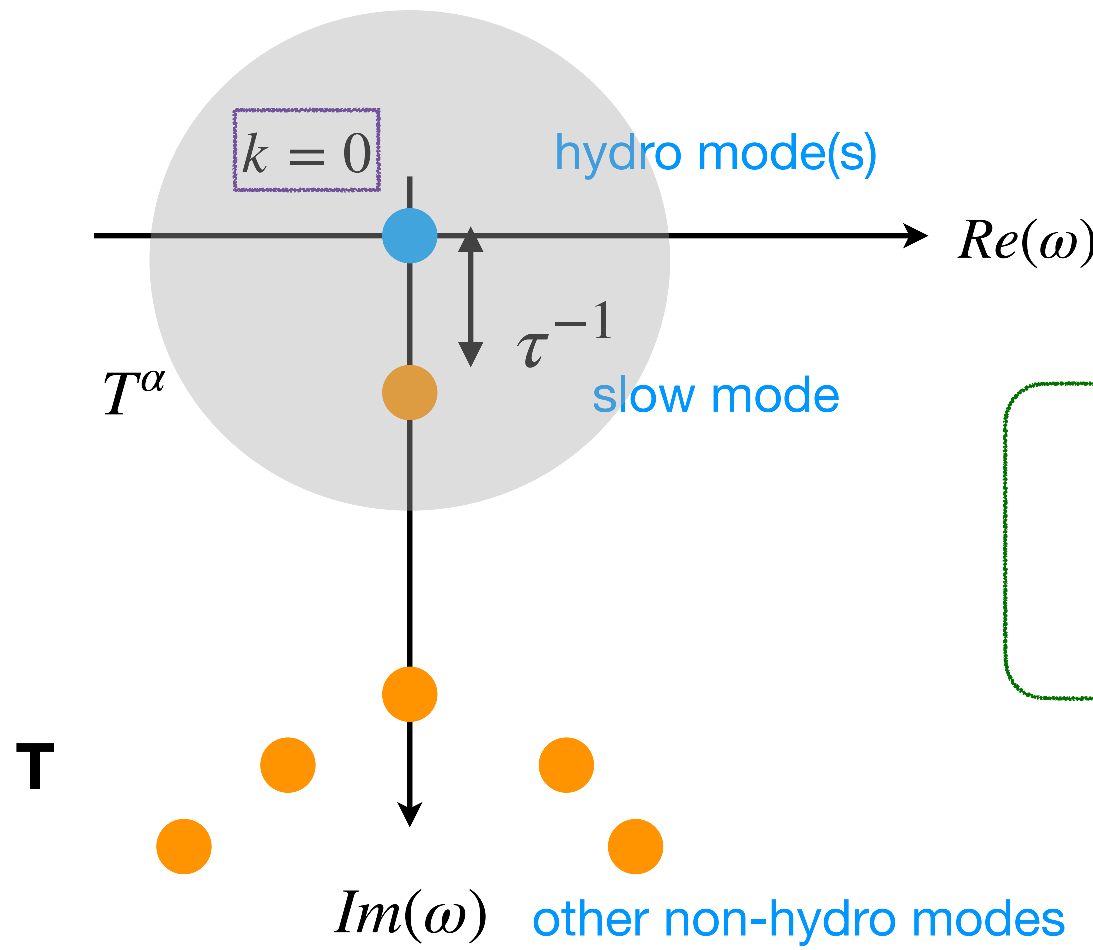
we recover the SK EFT for Maxwell-Cattaneo diffusion theory by Jain and Kovtun

$$\mathcal{L} = \chi B_{a0} B_{r0} + i\sigma T (B_{ai} + v_{ai})(B_{ai} + v_{ai}) - c_1 v_{ai} v_{ri} - \sigma (B_{ai} + v_{ai})\partial_0 (B_{ri} + v_{ri})$$

dynamical, normal mode of gauge field between “the Dirichlet wall”

[Jain, Kovtun, ’23]

SK EFT for diffusion + slow mode



$$S=\int dvdxdy\Big[\chi B_{a0}B_{r0}+i\sigma T\,b_{ai}b_{ai} - \sigma\, b_{ai}\partial_0 b_{ri}-\frac{\sigma}{\tau}(B_{ai}-b_{ai})(B_{ri}-b_{ri})\Big],$$

(3) Fluctuation-Dissipation theorem

$$G^S_{\mu\nu}(\omega,k)=i\coth\big(\frac{\beta\omega}{2}\big)\,\mathrm{Im}G^R_{\mu\nu}(\omega,k)$$

$$G^S_{tt}(\omega,k)=\frac{-i\,\delta^2W}{\delta A_{at}(-\omega,-k)\,\delta A_{at}(\omega,k)}=\frac{2i\sigma Tk^2}{|i\omega(1-i\omega\tau)-Dk^2|^2},$$

$$G^S_{xx}(\omega,k)=\frac{-i\,\delta^2W}{\delta A_{ax}(-\omega,-k)\,\delta A_{ax}(\omega,k)}=\frac{2i\sigma T\omega^2}{|i\omega(1-i\omega\tau)-Dk^2|^2},$$

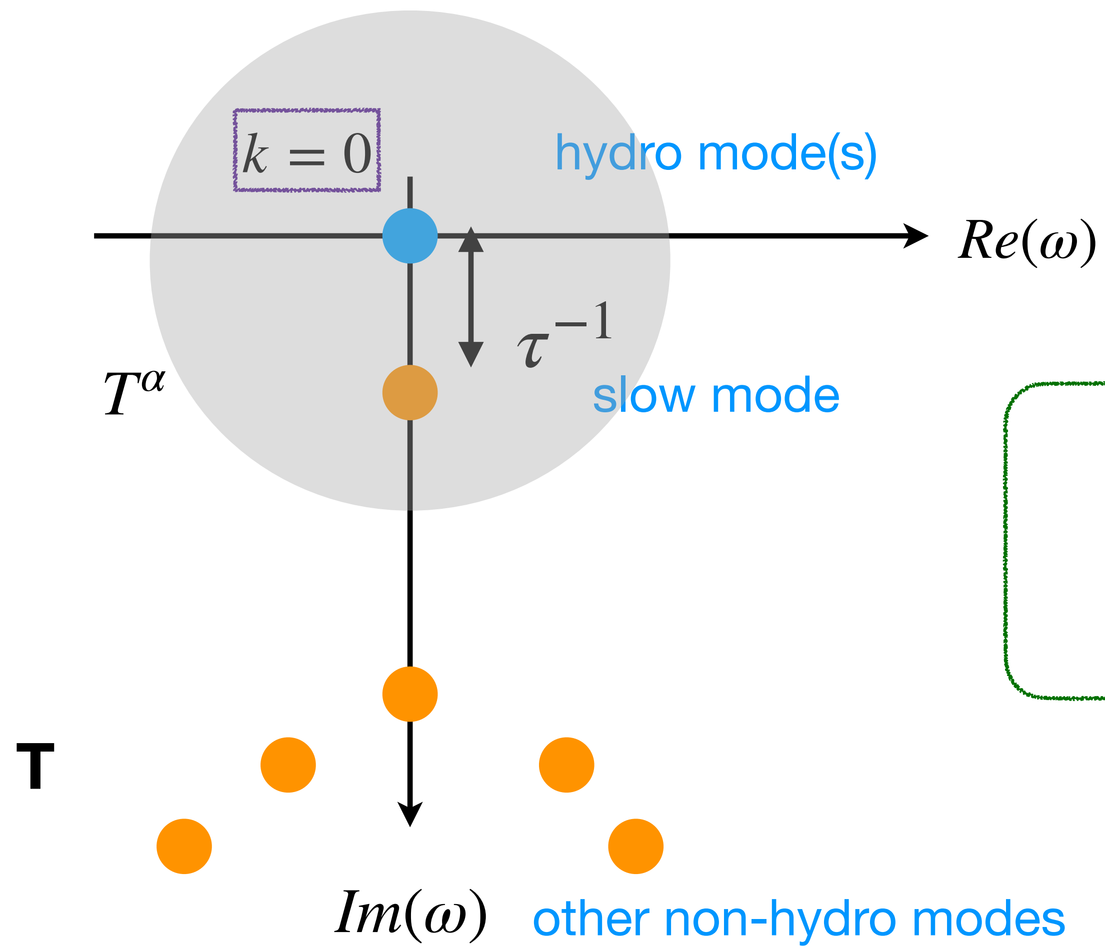
$$G^S_{yy}(\omega,k)=\frac{-i\,\delta^2W}{\delta A_{ay}(-\omega,-k)\,\delta A_{ay}(\omega,k)}=\frac{2i\sigma T}{|1-i\omega\tau|^2},$$

$$G^R_{tt}(\omega,k)=\frac{-i\,\delta^2W}{\delta A_{at}(-\omega,-k)\,\delta A_{rt}(\omega,k)}=\frac{-k^2\sigma}{i\omega(1-i\omega\tau)-Dk^2},$$

$$G^R_{xx}(\omega,k)=\frac{-i\,\delta^2W}{\delta A_{ax}(-\omega,-k)\,\delta A_{rx}(\omega,k)}=\frac{-\omega^2\sigma}{i\omega(1-i\omega\tau)-Dk^2},$$

$$G^R_{yy}(\omega,k)=\frac{-i\,\delta^2W}{\delta A_{ay}(-\omega,-k)\,\delta A_{ry}(\omega,k)}=\frac{i\omega\sigma}{1-i\omega\tau},$$

SK EFT for diffusion + slow mode



$$S=\int dvdxdy\,\Big[\chi B_{a0}B_{r0}+i\sigma T\, b_{ai}b_{ai} - \sigma\, b_{ai}\partial_0 b_{ri} - \frac{\sigma}{\tau}(B_{ai}-b_{ai})(B_{ri}-b_{ri})\Big],$$

Poles: “semi-circle” law

(3) Fluctuation-Dissipation theorem

$$G^S_{tt}(\omega,k)=\frac{-i\,\delta^2 W}{\delta A_{at}(-\omega,-k)\,\delta A_{at}(\omega,k)}=\frac{2i\sigma T k^2}{|i\omega(1-i\omega\tau)-Dk^2|^2},$$

$$G^S_{xx}(\omega,k)=\frac{-i\,\delta^2 W}{\delta A_{ax}(-\omega,-k)\,\delta A_{ax}(\omega,k)}=\frac{2i\sigma T \omega^2}{|i\omega(1-i\omega\tau)-Dk^2|^2},$$

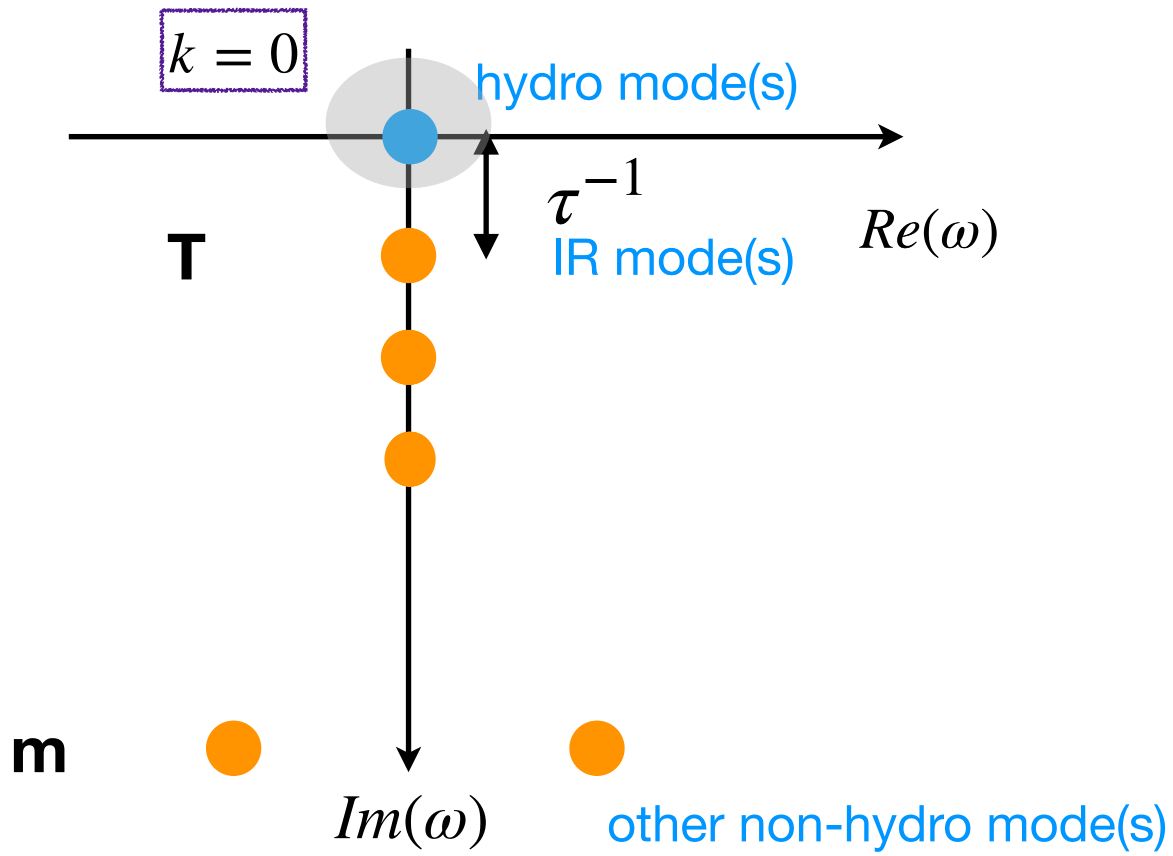
$$G^S_{yy}(\omega,k)=\frac{-i\,\delta^2 W}{\delta A_{ay}(-\omega,-k)\,\delta A_{ay}(\omega,k)}=\frac{2i\sigma T}{|1-i\omega\tau|^2},$$

$$G^R_{tt}(\omega,k)=\frac{-i\,\delta^2 W}{\delta A_{at}(-\omega,-k)\,\delta A_{rt}(\omega,k)}=\frac{-k^2\sigma}{i\omega(1-i\omega\tau)-Dk^2},$$

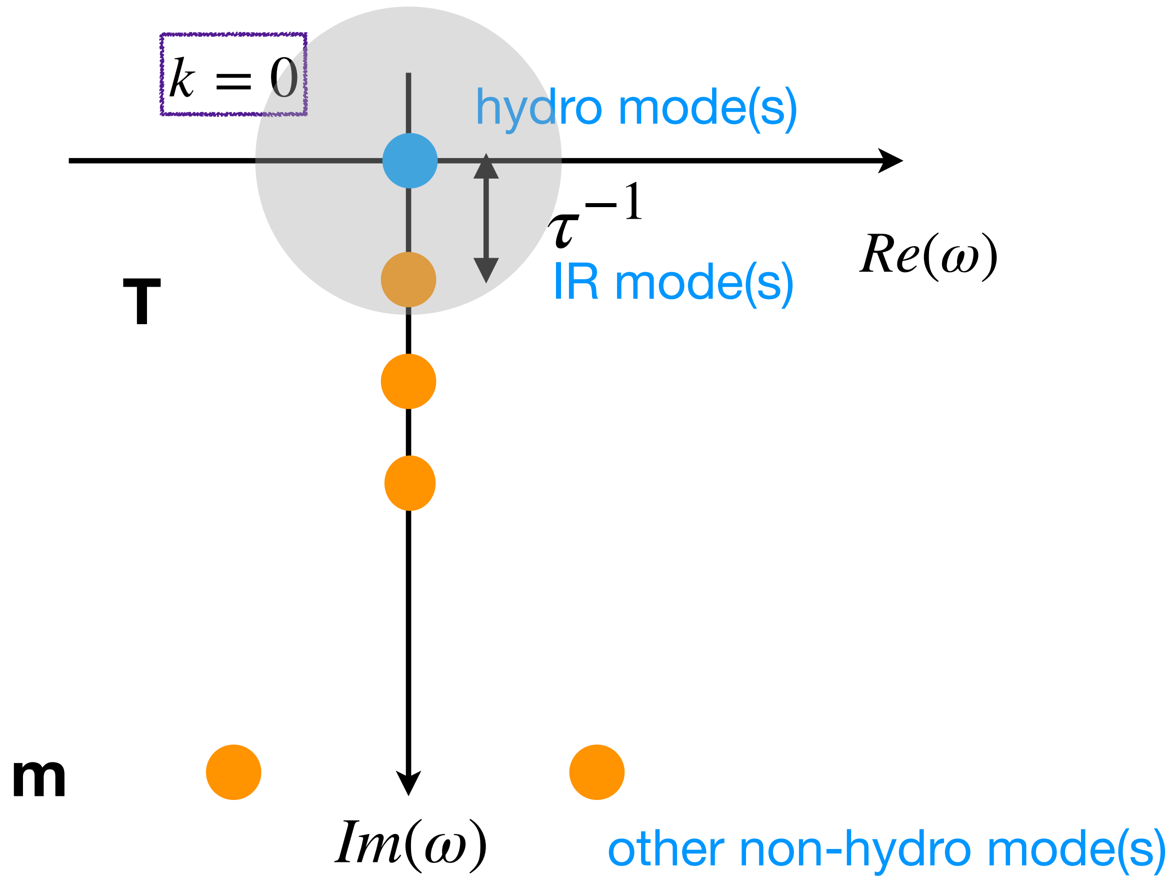
$$G^R_{xx}(\omega,k)=\frac{-i\,\delta^2 W}{\delta A_{ax}(-\omega,-k)\,\delta A_{rx}(\omega,k)}=\frac{-\omega^2\sigma}{i\omega(1-i\omega\tau)-Dk^2},$$

$$G^R_{yy}(\omega,k)=\frac{-i\,\delta^2 W}{\delta A_{ay}(-\omega,-k)\,\delta A_{ry}(\omega,k)}=\frac{i\omega\sigma}{1-i\omega\tau},$$

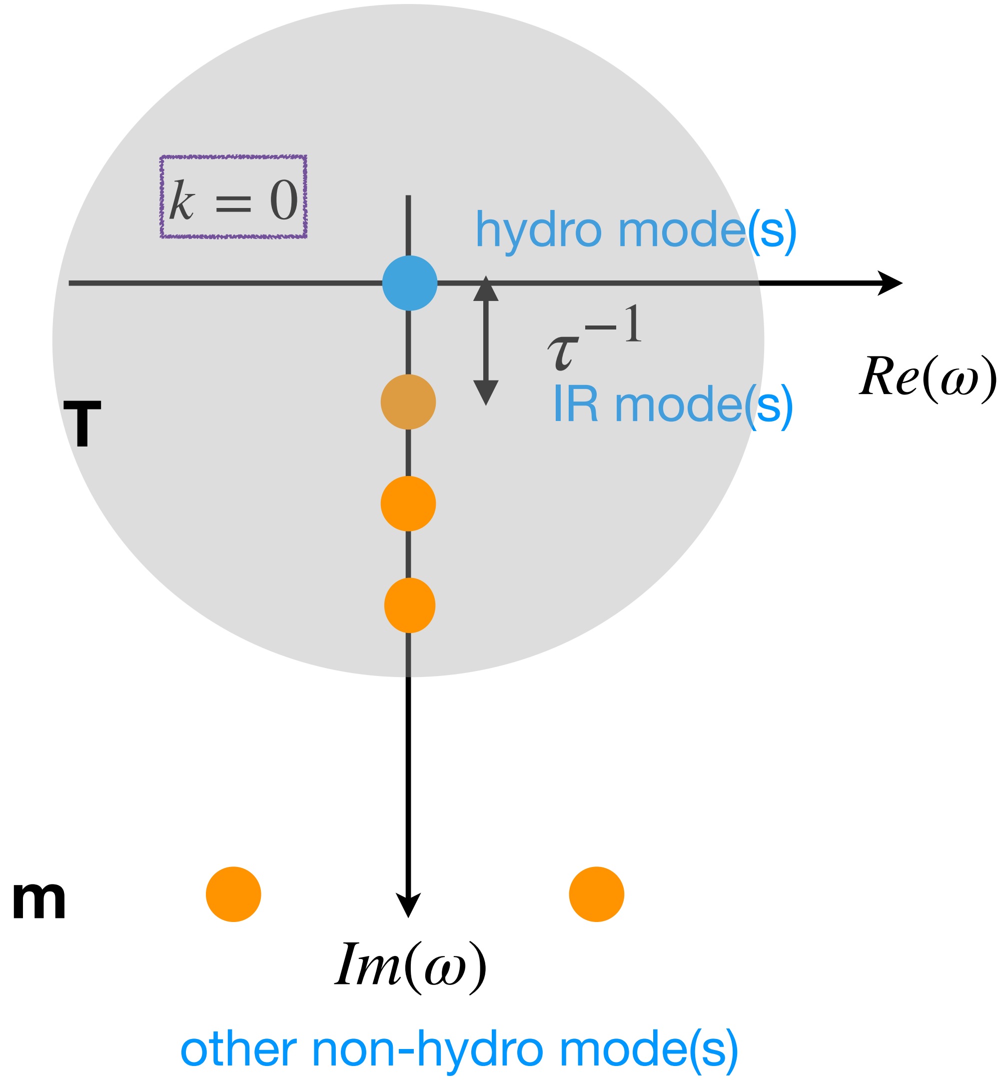
SK EFT for diffusion + IR mode

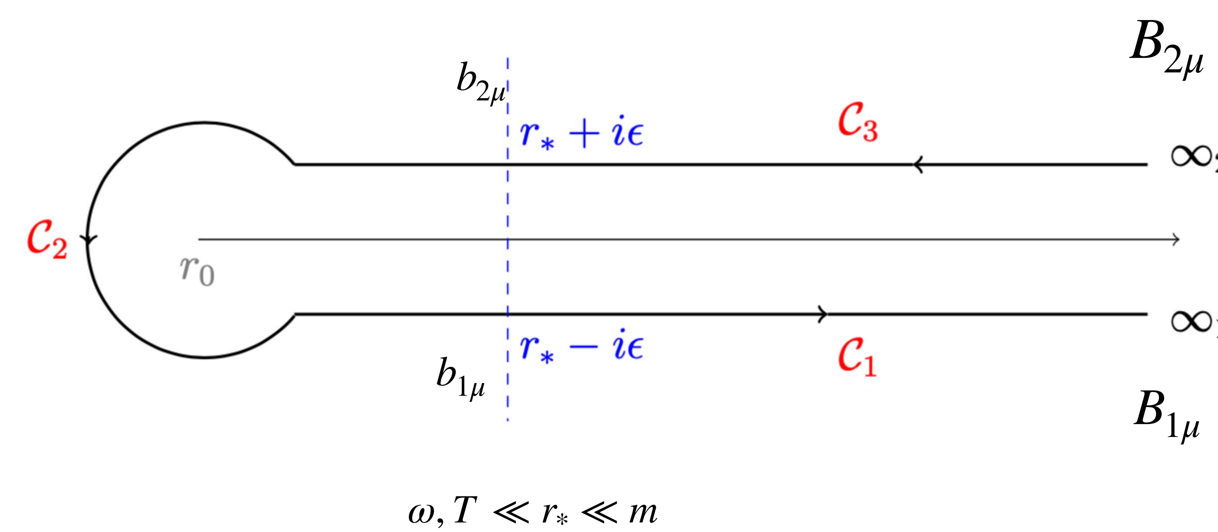


SK EFT for diffusion + IR mode



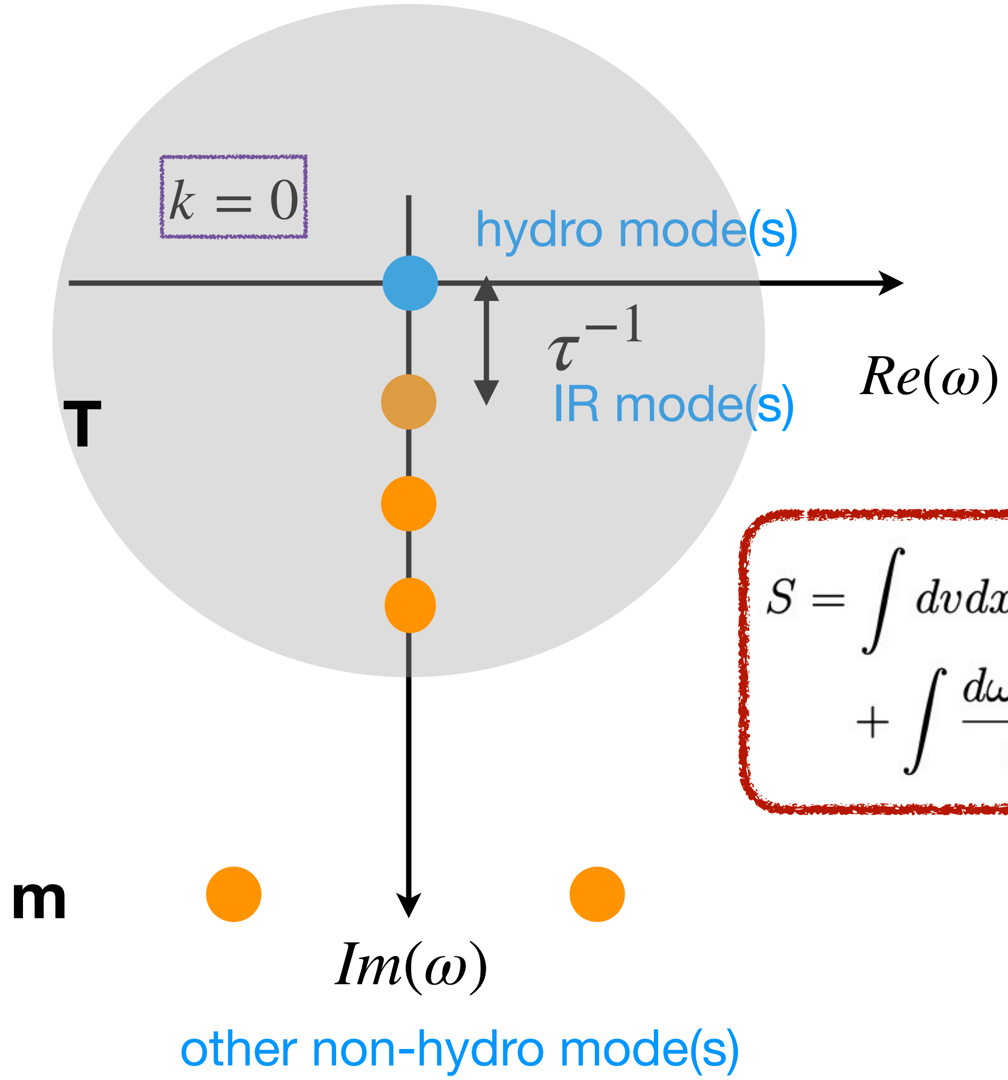
SK EFT for diffusion + IR mode





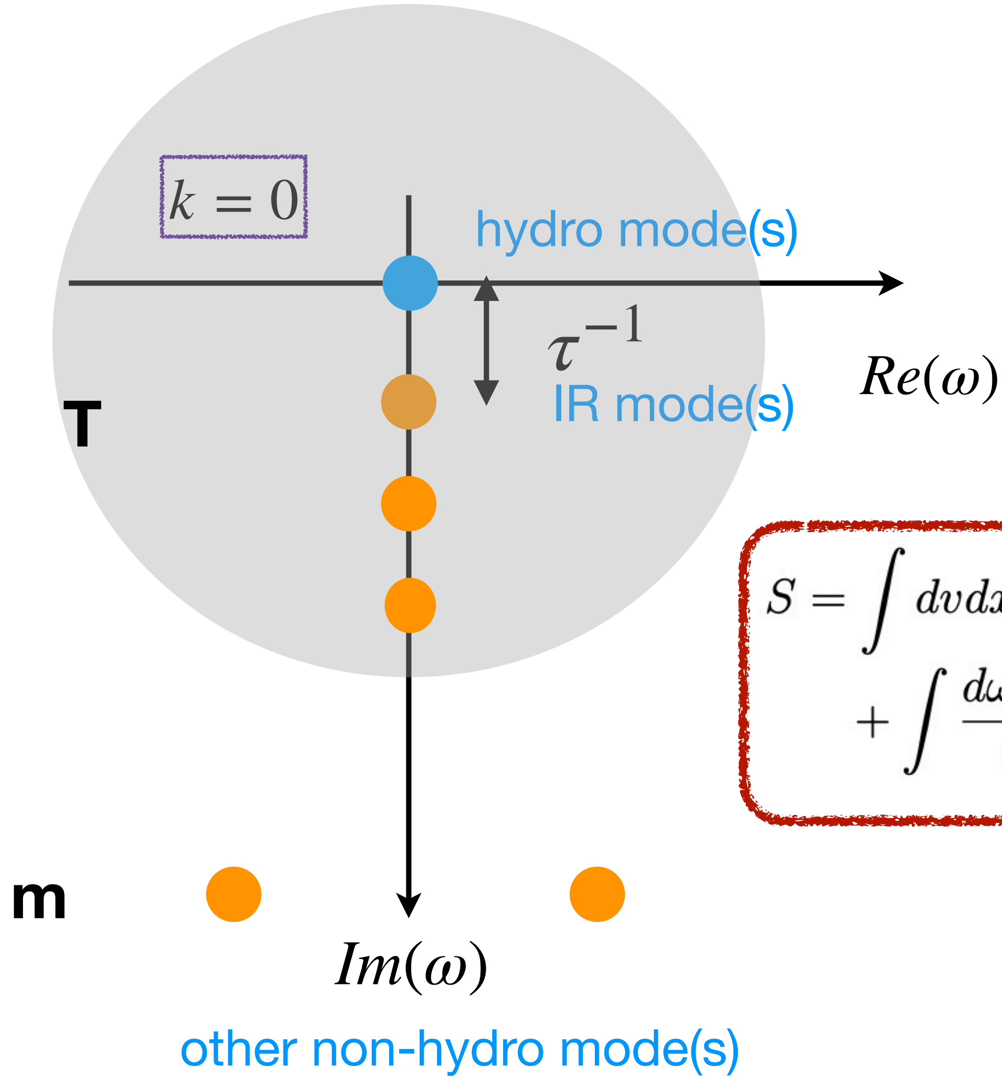
$$S = \int dv dx dy \Big[\chi B_{a0} B_{r0} - c_1 (B_{ai} - b_{ai})(B_{ri} - b_{ri})\Big] + \\ + \int \frac{d\omega dk_x dk_y}{(2\pi)^3} \Big[- 2\pi T \sigma \coth\big(\frac{\beta\omega}{2}\big)\, \mathcal{G}_{\rm IR}(\omega, T)\, b_{ai} b_{ai} - 4\pi T \sigma\, \mathcal{G}_{\rm IR}(\omega, T)\, b_{ai} b_{ri}\Big]$$

SK EFT for diffusion + IR mode



$$S = \int dv dx dy \Big[\chi B_{a0} B_{r0} - c_1 (B_{ai} - b_{ai})(B_{ri} - b_{ri})\Big] +$$
$$+ \int \frac{d\omega dk_x dk_y}{(2\pi)^3} \Big[-2\pi T\sigma \coth\left(\frac{\beta\omega}{2}\right) \mathcal{G}_{\rm IR}(\omega, T)\, b_{ai} b_{ai} - 4\pi T\sigma\, \mathcal{G}_{\rm IR}(\omega, T)\, b_{ai} b_{ri}\Big]$$

SK EFT for diffusion + IR mode

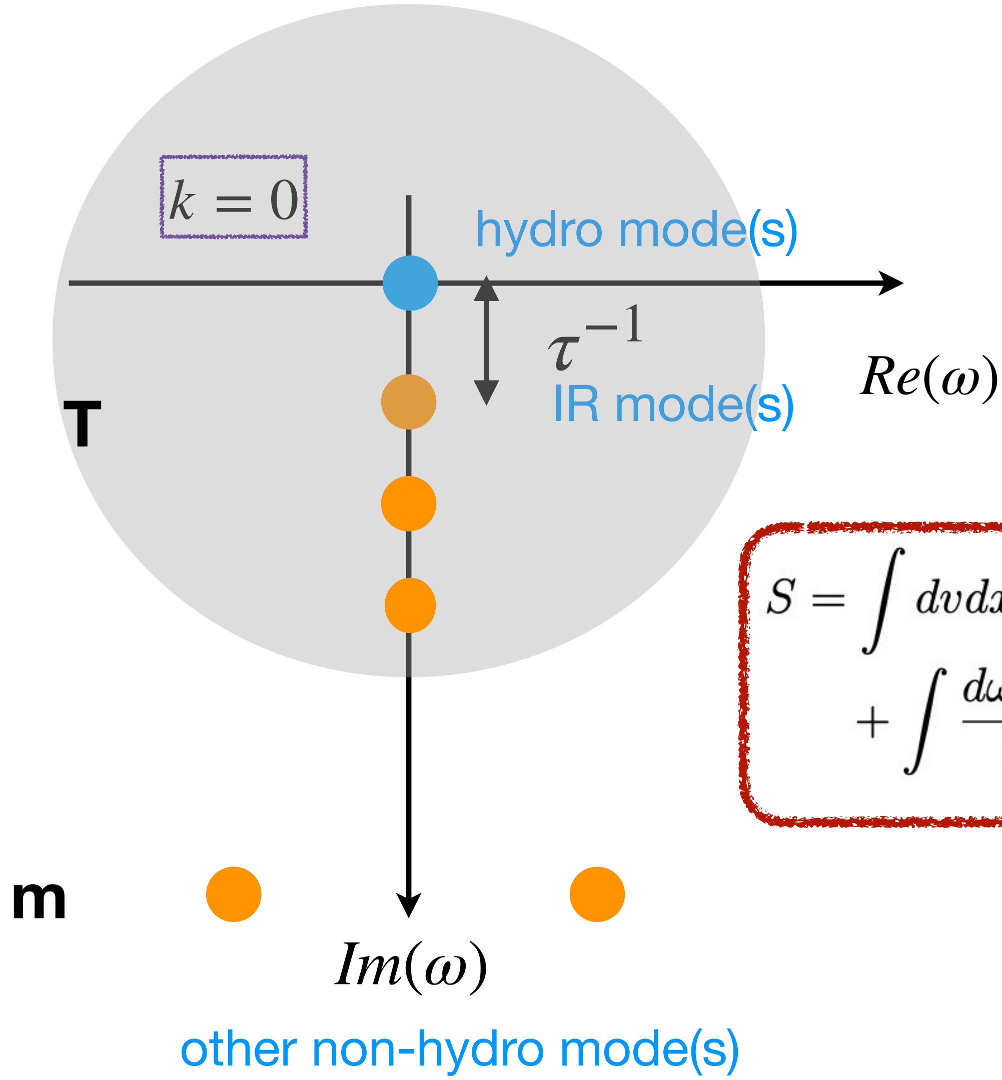


$$S=\int dvdxdy\left[\chi B_{a0}B_{r0}-c_1(B_{ai}-b_{ai})(B_{ri}-b_{ri})\right]+$$
$$+\int\frac{d\omega dk_x dk_y}{(2\pi)^3}\left[-2\pi T\sigma\coth\left(\frac{\beta\omega}{2}\right)\mathcal{G}_{\rm IR}(\omega,T)\,b_{ai}b_{ai}-4\pi T\sigma\,\mathcal{G}_{\rm IR}(\omega,T)\,b_{ai}b_{ri}\right]$$

(1) When $\omega\to 0$, $\mathcal{G}_{\rm IR}(\omega)=-\frac{i\omega}{4\pi T}$, recover the standard diff.

However, there are some differences to the previous slow mode case.

SK EFT for diffusion + IR mode



$$S=\int dvdxdy\left[\chi B_{a0}B_{r0}-c_1(B_{ai}-b_{ai})(B_{ri}-b_{ri})\right]+$$
$$+\int\frac{d\omega dk_x dk_y}{(2\pi)^3}\left[-2\pi T\sigma\coth\left(\frac{\beta\omega}{2}\right)\mathcal{G}_{\rm IR}(\omega,T)\,b_{ai}b_{ai}-4\pi T\sigma\,\mathcal{G}_{\rm IR}(\omega,T)\,b_{ai}b_{ri}\right]$$

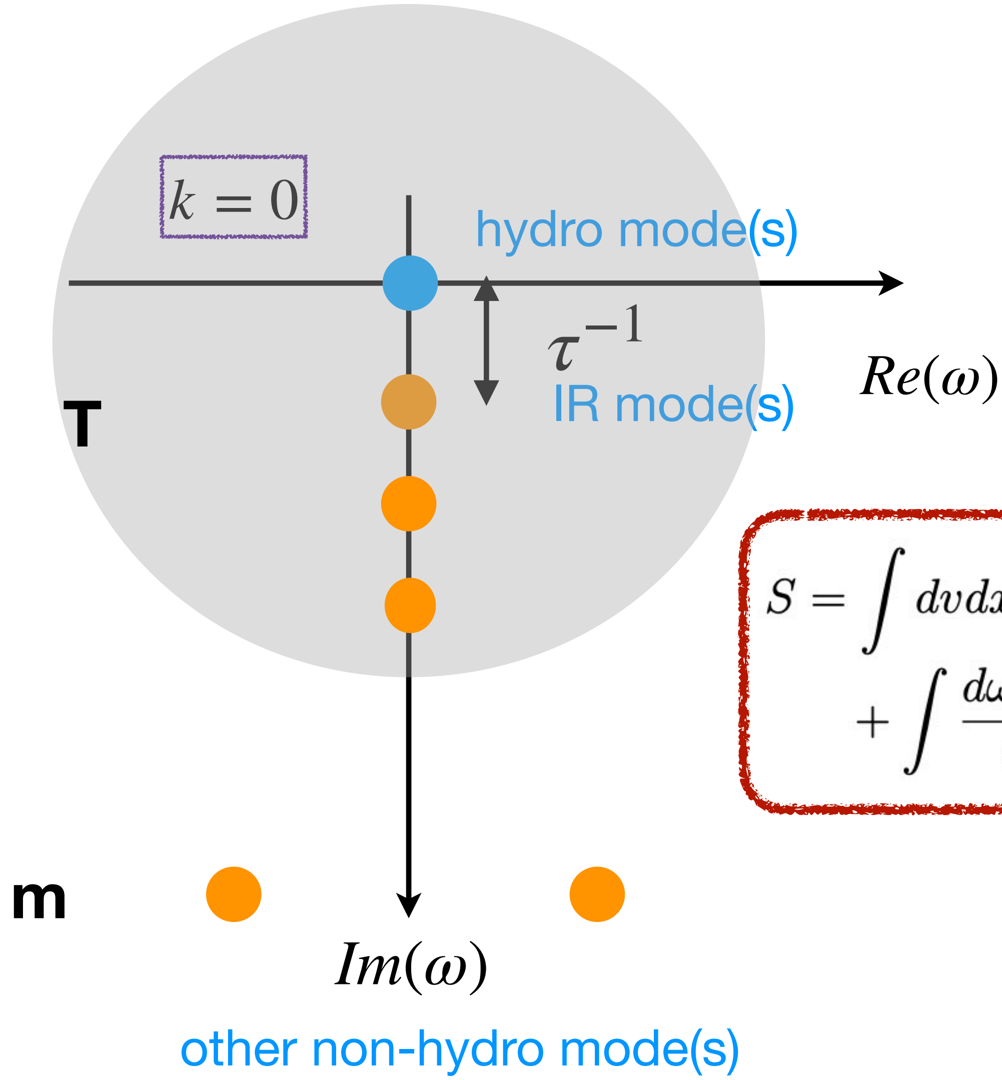
(2) One could replace the IR geometry by AdS_2 or Lifshitz geometries.

Similar to the slow mode case, one can perform a redefinition of fields

$$b_{ai}=B_{ai}+v_{ai}\,,\quad b_{ri}=B_{ri}+v_{ri}\,,$$

to obtain the effective action S[B, v].

SK EFT for diffusion + IR mode



$$S=\int dvdxdy\left[\chi B_{a0}B_{r0}-c_1(B_{ai}-b_{ai})(B_{ri}-b_{ri})\right]+$$
$$+\int\frac{d\omega dk_x dk_y}{(2\pi)^3}\left[-2\pi T\sigma\coth\left(\frac{\beta\omega}{2}\right)\mathcal{G}_{\rm IR}(\omega,T)\,b_{ai}b_{ai}-4\pi T\sigma\,\mathcal{G}_{\rm IR}(\omega,T)\,b_{ai}b_{ri}\right]$$

(3) Fluctuation-dissipation theorem

$$G^S_{\mu\nu}(\omega,k)=i\coth\left(\frac{\beta\omega}{2}\right){\rm Im}G^R_{\mu\nu}(\omega,k)$$

$$G^S_{tt}=\frac{-i\,\delta^2W}{\delta A_{at}(-\omega,-k)\,\delta A_{at}(\omega,k)}=\frac{-4i\pi T\sigma k^2\omega^2\coth(\frac{\beta\omega}{2})\,{\rm Im}(\mathcal{G}_{\rm IR})}{|(-\omega^2+4\pi TDk^2\mathcal{G}_{\rm IR})-\omega^2 4\pi T\tilde{\sigma}\mathcal{G}_{\rm IR}|^2},$$

$$G^S_{xx}=\frac{-i\,\delta^2W}{\delta A_{ax}(-\omega,-k)\,\delta A_{ax}(\omega,k)}=\frac{-4i\pi T\sigma\omega^4\coth(\frac{\beta\omega}{2})\,{\rm Im}(\mathcal{G}_{\rm IR})}{|(-\omega^2+4\pi TDk^2\mathcal{G}_{\rm IR})-\omega^2 4\pi T\tilde{\sigma}\mathcal{G}_{\rm IR}|^2}$$

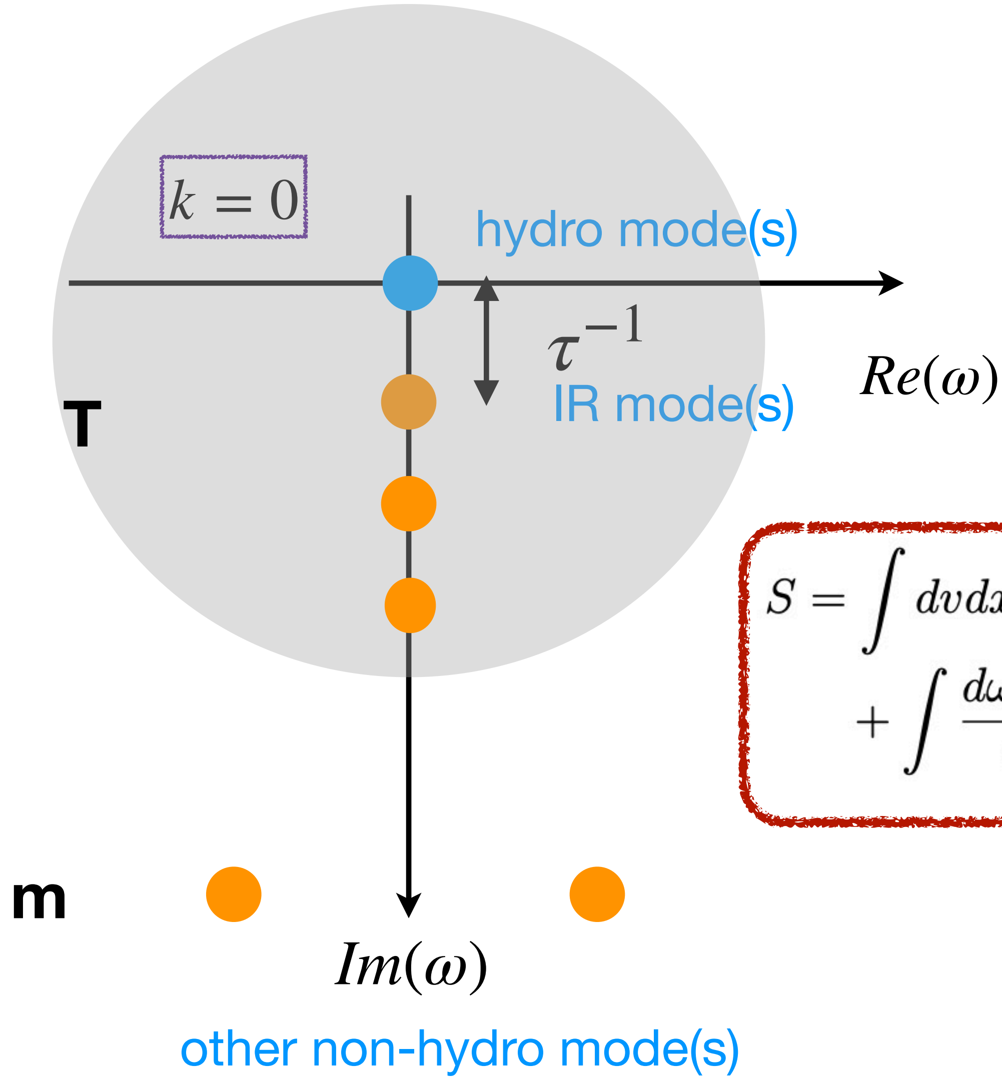
$$G^S_{yy}=\frac{-i\,\delta^2W}{\delta A_{ay}(-\omega,-k)\,\delta A_{ay}(\omega,k)}=\frac{-4i\pi T\sigma\omega^4\coth(\frac{\beta\omega}{2})\,{\rm Im}(\mathcal{G}_{\rm IR})}{|1+4\pi T\tilde{\sigma}\mathcal{G}_{\rm IR}|^2},$$

$$G^R_{tt}=\frac{-i\,\delta^2W}{\delta A_{at}(-\omega,-k)\,\delta A_{rt}(\omega,k)}=\frac{4\pi T\sigma\mathcal{G}_{\rm IR}k^2}{(-\omega^2+4\pi TDk^2\mathcal{G}_{\rm IR})-\omega^2 4\pi T\tilde{\sigma}\mathcal{G}_{\rm IR}},$$

$$G^R_{xx}=\frac{-i\,\delta^2W}{\delta A_{ax}(-\omega,k)\,\delta A_{rx}(\omega,k)}=\frac{\omega^2 4\pi T\sigma\mathcal{G}_{\rm IR}}{(-\omega^2+4\pi TDk^2\mathcal{G}_{\rm IR})-\omega^2 4\pi T\tilde{\sigma}\mathcal{G}_{\rm IR}},$$

$$G^R_{yy}=\frac{-i\,\delta^2W}{\delta A_{ay}(-\omega,-k)\,\delta A_{ry}(\omega,k)}=\frac{-4\pi T\sigma\tilde{\mathcal{G}}_{\rm IR}}{1+4\pi T\tilde{\sigma}\tilde{\mathcal{G}}_{\rm IR}},$$

SK EFT for diffusion + IR mode



$$S = \int dv dx dy \left[\chi B_{a0} B_{r0} - c_1 (B_{ai} - b_{ai})(B_{ri} - b_{ri})\right] + $$
$$+ \int \frac{d\omega dk_x dk_y}{(2\pi)^3} \left[-2\pi T \sigma \coth\left(\frac{\beta\omega}{2}\right) \mathcal{G}_{\rm IR}(\omega, T)\, b_{ai} b_{ai} - 4\pi T \sigma\, \mathcal{G}_{\rm IR}(\omega, T)\, b_{ai} b_{ri}\right]$$

Poles: (1) IR pole;
(2) standard diffusive mode

(3) Fluctuation-dissipation theorem

$$G^S_{tt} = \frac{-i\,\delta^2 W}{\delta A_{at}(-\omega,-k)\,\delta A_{at}(\omega,k)} = \frac{-4i\pi T\sigma k^2\omega^2 \coth(\frac{\beta\omega}{2})\,{\rm Im}(\mathcal{G}_{\rm IR})}{|(-\omega^2 + 4\pi T D k^2 \mathcal{G}_{\rm IR}) - \omega^2 4\pi T\tilde{\sigma}\mathcal{G}_{\rm IR}|^2},$$

$$G^S_{xx} = \frac{-i\,\delta^2 W}{\delta A_{ax}(-\omega,-k)\,\delta A_{ax}(\omega,k)} = \frac{-4i\pi T\sigma \omega^4 \coth(\frac{\beta\omega}{2})\,{\rm Im}(\mathcal{G}_{\rm IR})}{|(-\omega^2 + 4\pi T D k^2 \mathcal{G}_{\rm IR}) - \omega^2 4\pi T\tilde{\sigma}\mathcal{G}_{\rm IR}|^2}$$

$$G^S_{yy} = \frac{-i\,\delta^2 W}{\delta A_{ay}(-\omega,-k)\,\delta A_{ay}(\omega,k)} = \frac{-4i\pi T\sigma \omega^4 \coth(\frac{\beta\omega}{2})\,{\rm Im}(\mathcal{G}_{\rm IR})}{|1 + 4\pi T\tilde{\sigma}\mathcal{G}_{\rm IR}|^2},$$

$$G^R_{tt} = \frac{-i\,\delta^2 W}{\delta A_{at}(-\omega,-k)\,\delta A_{rt}(\omega,k)} = \frac{4\pi T\sigma \mathcal{G}_{\rm IR} k^2}{(-\omega^2 + 4\pi T D k^2 \mathcal{G}_{\rm IR}) - \omega^2 4\pi T\tilde{\sigma}\mathcal{G}_{\rm IR}},$$

$$G^R_{xx} = \frac{-i\,\delta^2 W}{\delta A_{ax}(-\omega,k)\,\delta A_{rx}(\omega,k)} = \frac{\omega^2 4\pi T\sigma \mathcal{G}_{\rm IR}}{(-\omega^2 + 4\pi T D k^2 \mathcal{G}_{\rm IR}) - \omega^2 4\pi T\tilde{\sigma}\mathcal{G}_{\rm IR}},$$

$$G^R_{yy} = \frac{-i\,\delta^2 W}{\delta A_{ay}(-\omega,-k)\,\delta A_{ry}(\omega,k)} = \frac{-4\pi T\sigma \tilde{\mathcal{G}}_{\rm IR}}{1 + 4\pi T\tilde{\sigma}\tilde{\mathcal{G}}_{\rm IR}},$$

Summary

- We have studied the charge diffusive hydro near a semi-local quantum liquid (conformal to $AdS_2 \times R^2$)
- Depending on the IR gauge coupling, pole collisions are different： (1) slow mode; (2) IR mode
- The upper bound for the diffusion constant is always satisfied. Different behaviors of the radii of convergence for thermal liquid
- EFT for hydro mode + slow mode
- EFT for hydro mode + IR mode

Future work

- Study other different types of hydro modes
- Study hydrodynamic system in different critical states
- Construct EFT for other sectors
- Study the effects of non-hydro modes from EFT
- …

Thank you!

Thank you!