String Duality Applications on Compact Stars in Gravitational Wave Observation

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Proof of 5D A_n AGT conjecture at $\beta = 1$

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and Kilar Zhang[a,e,f,*]

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Duality between Seiberg-Witten theory and black hole superradiance

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Quasinormal modes of C-metric from SCFTs

Yang Lei,[a,1] Hongfei Shu,[b,c,d,1] Kilar Zhang[e,f,1] and Rui-Dong Zhu[a,1]

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ABCD of qq-characters

Satoshi Nawata,[a] Kilar Zhang[b,c] and Rui-Dong Zhu[d]

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Qing-Jie Yuan,[a] Shao-Ping Hu,[a] Zi-Hao Huang[a] and Kilar Zhang[a,b,c,*]

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Dark stars and gravitational waves: Topical review

Kilar Zhang[a,b], Ling-Wei Luo[c], Jie-Shiun Tsao[c], Chian-Shu Chen[d,e], Feng-Li Lin[c,*]

Dark I-Love-Q

Jing-Yi Wu,[a,b] Wei Li,[b] Xin-Han Huang[b] and Kilar Zhang[b,c,d]

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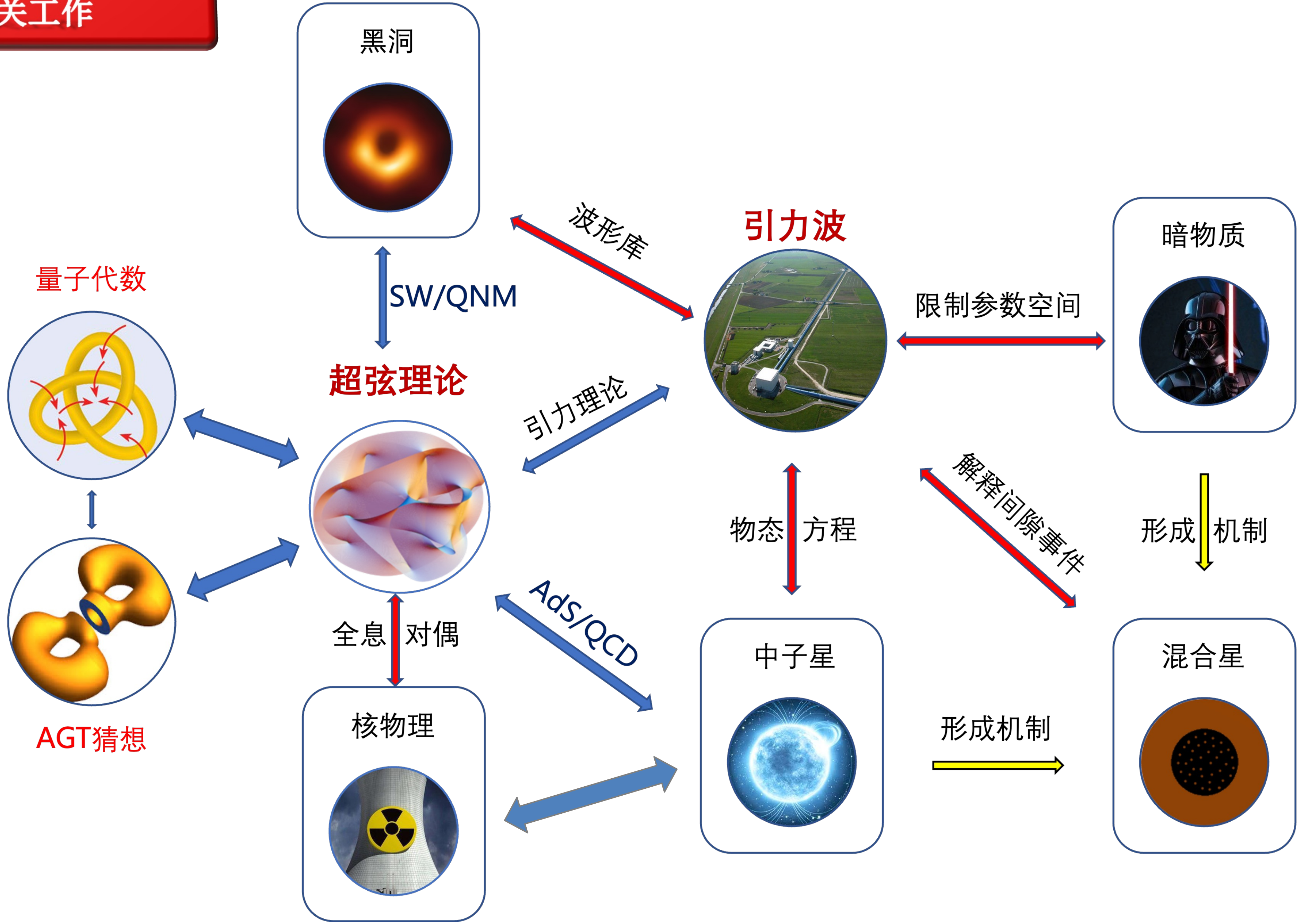
黑洞
SW/QNM
波形库
引力波
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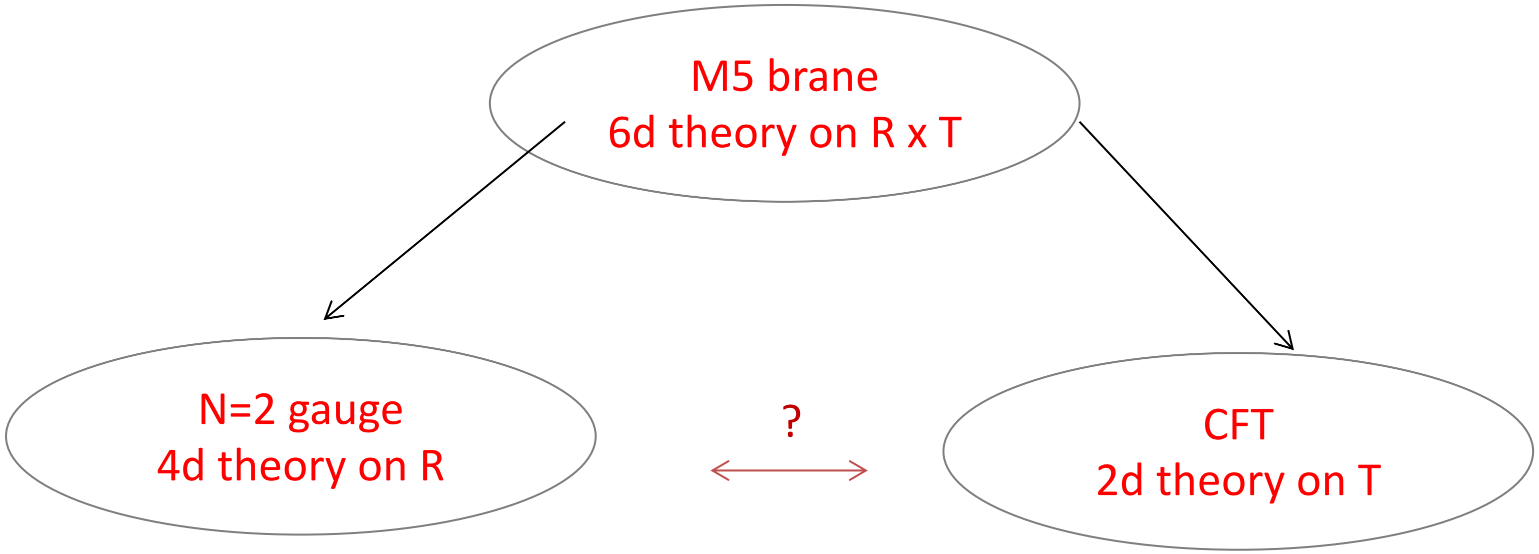
AGT猜想
全息 对偶
AdS/QCD
核物理
中子星
形成机制
混合星

演示页

Introduction

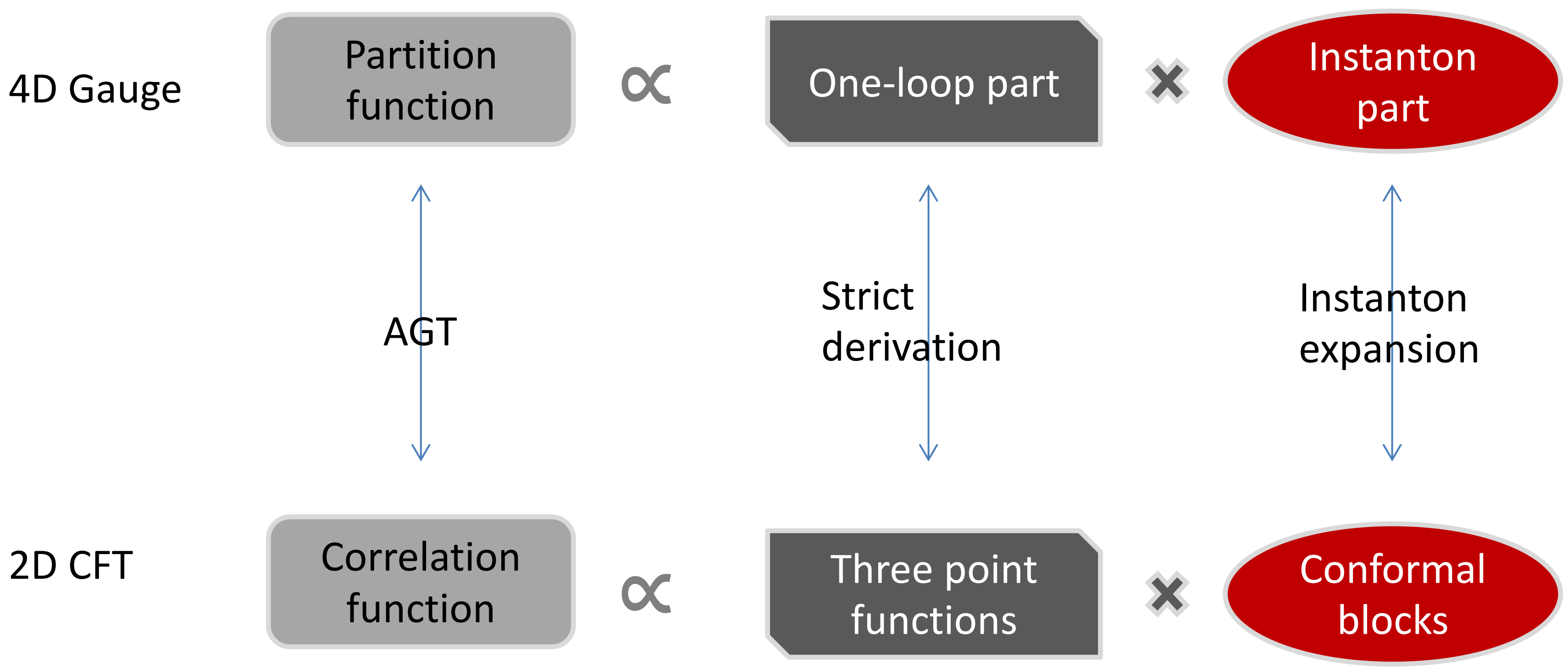
In 2002, Nekrsov performed a technique called Ω deformation in the reduction from 6D $\mathcal{N}=1$ gauge theory to 4D $\mathcal{N}=2$ gauge theory, and implied its connection with 2D conformal theory.



He found exact formulae of the partition function (Nekrasov partition function) of the $\mathcal{N}=2$ gauge theory, and showed that it reproduces the prepotential as determined by the Seiberg–Witten curve.

Introduction

AGT conjecture



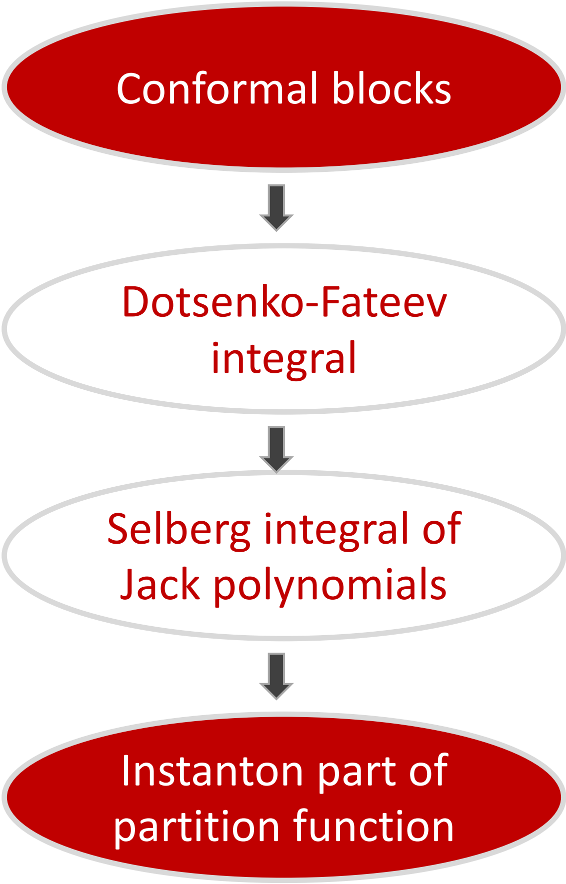
SU(2) [Alday−Gaiotto−Tachikawa '09]

SU(N) [Wyllard '09]

Introduction

We have been working on the proof of AGT conjecture through two different ways.

Direct approach



We calculated the conformal block in the form of Dotsenko−Fateev integral and reduce it in the form of Selberg integral of N Jack polynomials.

We found a formula for such Selberg average which satisfies some nontrivial consistency conditions and showed that it reproduces the SU(N) version of AGT conjecture.

$\beta = 1$ SU(2) [A. Mironov et. al. '10]

$\beta = 1$ SU(N) [Zhang Matsuo '11]

Recursive approach

2D CFT

4D Gauge

Conformal blocks

AGT conjecture

$$\langle\langle \vec{Y}|V|\vec{W}\rangle\rangle = Z$$

Partition function

Satisfy Ward identity

trivial

nontrivial

Constrained by a recursion Relation

$$\sum \hat{O}\langle\langle \vec{Y}|V|\vec{W}\rangle\rangle = 0$$

$$\sum \hat{O} Z = 0$$

$\beta = 1$ SU(N) [Kanno–Matsuo–Zhang '12]

arbitrary β SU(N) [Kanno–Matsuo–Zhang '13]

A brief review of AGT conjecture and Nekrasov formula

Nekrasov partition function

Consider lifting the $\mathcal{N}=2$ four dimensional theory to $\mathcal{N}=(1,0)$ six dimensional theory, and then compactify the six dimensional $\mathcal{N}=1$ SUSY gauge theory on the manifold with the topology **T2** $\times$ ***R4*** with the metric :

$$ds^2 = r^2 dz d\bar{z} + g_{\mu\nu}\left(dx^\mu + V^\mu dz + \bar{V}^\mu d\bar{z}\right)\left(dx^\nu + V^\nu dz + \bar{V}^\nu d\bar{z}\right)$$

where $V^\mu = \Omega^\mu_\nu x^\nu,\ \bar{V}^\mu = \bar{\Omega}^\mu_\nu x^\nu,$ and

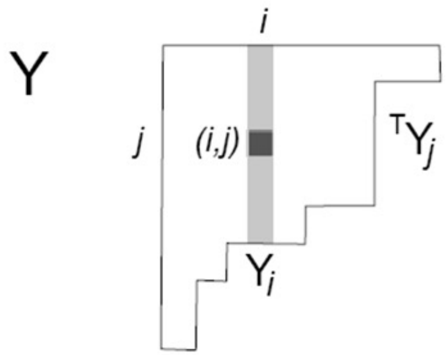
$$\Omega^{\mu\nu} = \begin{pmatrix} 0 & \epsilon_1 & 0 & 0 \\ -\epsilon_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \epsilon_2 \\ 0 & 0 & -\epsilon_2 & 0 \end{pmatrix}, \quad \bar{\Omega}^{\mu\nu} = \begin{pmatrix} 0 & \bar{\epsilon}_1 & 0 & 0 \\ -\bar{\epsilon}_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{\epsilon}_2 \\ 0 & 0 & -\bar{\epsilon}_2 & 0 \end{pmatrix}$$

The action of the four dimensional theory in the limit *$r \to 0$ is not that of the pure supersymmetric Yang–Mills* theory on *R4*. Rather, it is a deformation of the latter by the Ω–dependent terms. It is called an *$\mathcal{N}=2$ theory in the Ω-background*.

$$\beta = -\epsilon_1/\epsilon_2$$

Nekrasov partition function

With his idea of the Ω−background, Nekrasov calculated the following partition function



$$Z(\tau, a, m, \epsilon) = \int_{\phi(\infty)=a} D\Phi DAD\lambda \ldots e^{-S(\Omega)}$$

It has the important property that it gives the prepotential of the Seiberg-Witten theory in the limit $\epsilon_1 = -\epsilon_2 = \hbar \to 0$

$$F(\tau, a, m) = \lim_{\hbar \to 0} \hbar^2 \log Z_{\text{full}}(\tau, a, m; \hbar, -\hbar)$$

$$Z_{\text{full}}(q; a, m; \epsilon) = Z_{\text{tree}} Z_{1\text{loop}} Z_{\text{inst}}, \quad Z_{\text{inst}}(q; a, m; \epsilon) = \sum_{\mathbf{Y}} \mathbf{q}^{\mathbf{Y}} z(\mathbf{Y}, a, m)$$

where the instanton is labeled by a N-tuple of Young diagrams: $\mathbf{Y} := (\vec{Y}^{(1)}, \cdots, \vec{Y}^{(n)})$, (Fig. 1). The parameter a (resp. m) represents the diagonalized VEV of vector multiplets (resp. mass of hypermultiplets) whereas $q_i = e^{\pi i \tau_i}$ is the instanton expansion parameter for ith gauge group $SU(N_i)$, $\mathbf{q}^{\mathbf{Y}} := \prod_{i=1}^{n} q_i^{|\vec{Y}^{(i)}|}$. The total partition function is decomposed into a product of the contributions of the perturbative parts Z_{tree}, $Z_{1-\text{loop}}$ and non-perturbative instanton correction Z_{inst}. The latter is further decomposed into a sum of sets of Young diagrams. $\vec{Y}^{(i)} = (Y_1^{(i)}, \cdots, Y_{N_i}^{(i)})$ is a collection of N_i Young diagram which parameterizes the fixed points of instanton moduli space for i th gauge group $U(N_i)$.

Nekrasov partition function

Single gauge group case

$$Z_{\rm full}(q;a,\mu;\epsilon) = Z_{\rm tree} Z_{\rm 1loop} Z_{\rm inst}, \quad Z_{\rm inst}(q;a,m;\epsilon) = \sum_{\vec{Y}} q^{|\vec{Y}|} N^{\rm inst}_{\vec{Y}}(a,\mu),$$

$$N^{\rm inst}_{\vec{Y}}(a,\mu) = z_{\rm vect}(\vec{Y},a) \prod_{i=1}^{2N} z_{\rm fund}(\vec{Y},\mu_i) = \frac{\prod_{s=1}^{N} \prod_{k=1}^{2N} f_{Y_s}(\mu_k + a_s)}{\prod_{t,s=1}^{N} g_{Y_t,Y_s}(a_t - a_s)},$$

linear quiver gauge case

with gauge group $SU(N_1) \times \cdots \times SU(N_n)$.

$$Z^{\rm Nek} = \sum_{\vec{Y}^{(1)},\cdots,\vec{Y}^{(n)}} q_i^{|\vec{Y}^{(i)}|} \bar{V}_{\vec{Y}^{(1)}} \cdot Z_{\vec{Y}^{(1)}\vec{Y}^{(2)}} \cdots Z_{\vec{Y}^{(n-1)}\vec{Y}^{(n)}} \cdot V_{\vec{Y}^{(n)}}$$

$$\begin{aligned} Z_{\vec{Y}^{(i)}\vec{Y}^{(i+1)}} &= Z(\vec{a}^{(i)}, \vec{Y}^{(i)}; \vec{a}^{(i+1)}, \vec{Y}^{(i+1)}; \mu^{(i)}), \\ \bar{V}_{\vec{Y}^{(1)}} &= Z(\vec{\lambda}, \vec{\emptyset}; \vec{a}^{(1)}, \vec{Y}^{(1)}; \mu^{(0)}), \\ V_{\vec{Y}^{(n)}} &= Z(\vec{a}^{(n)}, \vec{Y}^{(n)}; \vec{\lambda}', \vec{\emptyset}; \mu^{(n)}), \end{aligned}$$

AGT conjecture

For a Liouville theory on a sphere, the four-point correlation function of *V at* positions *∞, 1, q, 0 is*

$$\langle V_{\beta_0}(\infty)V_{m_0}(1)V_{m_1}(q)V_{\beta_1}(0)\rangle = \int \frac{d\beta}{2\pi} C(\beta_0^*, m_0, \beta)C(\beta^*, m_1, \beta_1)\left|q^{\Delta_\beta - \Delta_{m_1} - \Delta_{\beta_1}} \mathcal{F}_{\beta_0}{}^{m_0}{}_{\beta}{}^{m_1}{}_{\beta_1}(q)\right|^2$$

$$\propto \int a^2 da \left|Z_{\beta_0}{}^{m_0}{}_{\beta}{}^{m_1}{}_{\beta_1}(q)\right|^2$$

Where C(β 1, β 2, β 3) is the three point function given by the DOZZ formula.

The function *F* carries the coordinate (*q*) dependence and reflects the contributions of the conformal descendants. It is called conformal block.

$$Z_{\text{inst}}^{U(2),N_f=4}(a, m_0, \tilde{m}_0, m_1, \tilde{m}_1) = (1-q)^{2m_0(Q-m_1)} \mathcal{F}_{\beta_0}{}^{m_0}{}_{\beta}{}^{m_1}{}_{\beta_1}(q)$$

it is Checked $\mathcal{F}_{\beta_0}{}^{m_0}{}_{\beta}{}^{m_1}{}_{\beta_1}(q)$ is the conformal block of a virasoro algebra with central charge c = 1 + 6Q2 at position ∞, 1, q, 0,

Up to order q11

AGT conjecture

SU(N) generalization

$$\langle V_{\alpha_4}(\infty)V_{\alpha_3}(1)V_{\alpha_2}(q)V_{\alpha_1}(0)\rangle$$

The conformal block of this
correlation function is written in the form,

$$\mathcal{F}_{\alpha_4,\alpha_3,\alpha_2,\alpha_1}(q)=\sum_{\vec{Y}} q^{|\vec{Y}|} N_{\vec{Y}}^{\mathrm{Toda}}(\alpha_1,\alpha_2,\alpha_3,\alpha_4)$$

Correlation functions of Toda theory and Selberg Formula

SU(N) Toda

Bosons $\phi(z) = (\phi_1(z), \cdots, \phi_N(z))$

$$\phi_i(z)\phi_j(0) \sim \delta_{ij} \ln(z)$$

Symmetry: W_N algebra

$$R_N =: \prod_{m=1}^{N} \left(Q\frac{d}{dz} - i(h_m, \partial_z \varphi) \right) := \sum_k W^{(k)}(z) \left(Q\frac{d}{dz} \right)^{N-k}$$

$$c = (N-1)(1 + N(N+1)Q^2)$$

$W^{(k)}$ satisfies W_N algebra (a nonlinear algebra) with central charge

$$V_{\vec{\alpha}}(z) =: e^{(\alpha, \phi(z))} :$$

$$Q_j^{(\pm)} = \int \frac{dz}{2\pi i} V_j^{(\pm)}(z) = \int \frac{dz}{2\pi i} : e^{\alpha_\pm (e_j, \phi(z))} :$$

Dotsenko-Fateev integral

$$Z_{\mathrm{DF}}(q) =$$

$$\left\langle\!\!\left\langle : e^{(\tilde{\alpha}_1,\phi(0))} :: e^{(\tilde{\alpha}_2,\phi(q))} :: e^{(\tilde{\alpha}_3,\phi(1))} :: e^{(\tilde{\alpha}_4,\phi(\infty))} : \prod_{a=1}^{N-1} \left(\int_0^q : e^{b(e_a,\phi(z))} : dz \right)^{N_a} \left(\int_1^\infty : e^{b(e_a,\phi(z))} : dz \right)^{\tilde{N}_a} \right\rangle\!\!\right\rangle$$

We apply *Wick's theorem* to evaluate the correlator

$$\left\langle\!\left\langle : e^{(\tilde{\alpha}_1,\phi(z_1))} : \ \ldots \ : e^{(\tilde{\alpha}_n,\phi(z_n))} : \right\rangle\!\right\rangle = \prod_{1\le i<j\le n} (z_j - z_i)^{(\tilde{\alpha}_i,\tilde{\alpha}_j)}$$

$$Z_{\mathrm{DF}}(q) = q^{(\alpha_1,\alpha_2)/\beta}(1-q)^{(\alpha_2,\alpha_3)/\beta} \prod_{a=1}^{N-1} \prod_{I=1}^{N_a} \int_0^q dz_I^{(a)} \prod_{J=N_a+1}^{N_a+\tilde{N}_a} \int_1^\infty dz_J^{(a)} \prod_{i<j}^{N_a+\tilde{N}_a} (z_j^{(a)} - z_i^{(a)})^{2\beta} \quad \times$$

$$\times \prod_i^{N_a+\tilde{N}_a} (z_i^{(a)})^{(\alpha_1,e_a)} (z_i^{(a)} - q)^{(\alpha_2,e_a)} (z_i^{(a)} - 1)^{(\alpha_3,e_a)} \prod_{a=1}^{N-2} \prod_i^{N_a+\tilde{N}_a} \prod_j^{N_{a+1}+\tilde{N}_{a+1}} (z_j^{(a+1)} - z_i^{(a)})^{-\beta} \quad .$$

Selberg integral

$$\int_{[0,1]^k} |\Delta(x)|^{2\gamma} \prod_{i=1}^{k} x_i^{\alpha-1}(1-x_i)^{\beta-1}\,\mathrm{d}x = \prod_{i=1}^{k} \frac{\Gamma(\alpha+(i-1)\gamma)\Gamma(\beta+(i-1)\gamma)\Gamma(i\gamma+1)}{\Gamma(\alpha+\beta+(i+k-2)\gamma)\Gamma(\gamma+1)}$$

When $k = 1$ the Selberg integral simplifies to the Euler beta integral

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\,\mathrm{d}x = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, \qquad \Re(\alpha) > 0,\ \Re(\beta) > 0,$$

Here we consider its $AN-1$ extension ($AN-1$ Selberg integral):

$$S_{\vec{u},\vec{v},\beta} = \int dx \prod_{a=1}^{N-1} \left[\left|\Delta(x^{(a)})\right|^{2\beta} \prod_{i=1}^{N_a} (x_i^{(a)})^{u_a} (1-x_i^{(a)})^{v_a} \right] \prod_{a=1}^{N-2} \left|\Delta(x^{(a)}, x^{(a+1)})\right|^{-\beta}$$

Selberg integral

$$I^{A_n}_{N_1,\dots,N_n}(\mathcal{O};u_1,\dots,u_n,v;\beta)$$
$$\equiv \int_{C_\beta^{N_1,\dots,N_n}[0,1]} \mathcal{O}(x^{(1)},\dots,x^{(n)}) \prod_{r=1}^{n}\prod_{i=1}^{N_r}(x_i^{(r)})^{u_r}(1-x_i^{(r)})^{v_r}$$
$$\times \prod_{r=1}^{n}|\Delta(x^{(r)})|^{2\beta}\prod_{r=1}^{n-1}|\Delta(x^{(r)},x^{(r+1)})|^{-\beta}\,\mathrm{d}x^{(1)}\cdots\mathrm{d}x^{(n)}$$

Seamus P. Albion, Eric M. Rains, and S. Ole Warnaar, 2021

For $\beta = 1$

$$C^{N_1,\dots,N_n} = C_1^{N_1}\times\cdots\times C_n^{N_n}, \quad \text{where} \quad C_r^{N_r} = \underbrace{C_r\times\cdots\times C_r}_{N_r \text{ times}}.$$

$$I^{A_n}_{N_1,\dots,N_n}(\mathscr{O};u_1,\dots,u_n,v)$$
$$\equiv \frac{1}{(2\pi\mathrm{i})^{N_1+\cdots+N_n}} \int_{C^{N_1,\dots,N_n}} \mathscr{O}(x^{(1)},\dots,x^{(n)}) \prod_{r=1}^{n}\prod_{i=1}^{N_r}(x_i^{(r)})^{u_r}(x_i^{(r)}-1)^{v_r}$$
$$\times \prod_{r=1}^{n}\Delta^2(x^{(r)})\prod_{r=1}^{n-1}\Delta^{-1}(x^{(r)},x^{(r+1)})\,\mathrm{d}x^{(1)}\cdots\mathrm{d}x^{(n)}$$

Reduction to Selberg integral

$$Z_{DF}(q) = \sum_{\vec{Y}} q^{|\vec{Y}|} \left\langle \prod_{a=1}^{N} j_{Y_a}^{(\beta)}(-r_k^{(a)} - \frac{v'_{a+}}{\beta}) \right\rangle_+ \left\langle \prod_{a=1}^{N} j_{Y_a}^{(\beta)}(\tilde{r}_k^{(a)} + \frac{v'_{a-}}{\beta}) \right\rangle_-$$

$j_{Y_a}^{(\beta)}$: Jack polynomials

$\langle \cdots \rangle_\pm$: Selberg average

$$p_k^{(a)} := \sum_i (x_i^{(a)})^k$$

Cauchy–Stanley identity

$$\exp(\beta \sum_{k=1}^{\infty} \frac{1}{k} p_k p'_k) = \sum_R j_R^{(\beta)}(p) j_R^{(\beta)}(p')$$

Jack polynomials

Jack polynomials are characterized by the fact that they are the eigenfunctions of Calogero-Sutherland Hamiltonian written in the form,

$$\mathcal{H} = \sum_{i=1}^{M} D_i^2 + \beta \sum_{i<h} \frac{z_i + z_j}{z_i - z_j}(D_i - D_j), \quad D_i := z_i \frac{\partial}{\partial z_i} .$$

The explicit form of low level ones are listed below;

$$J^{(\beta)}_{[1]}(p_k) = p_1 \ ,$$

$$J^{(\beta)}_{[2]}(p_k) = \frac{p_2 + \beta p_1^2}{\beta + 1}, \quad J^{(\beta)}_{[11]}(p_k) = \frac{1}{2}(p_1^2 - p_2) \ , \qquad)$$

$$J^{(\beta)}_{[3]}(p_k) = \frac{2p_3 + 3\beta p_1 p_2 + \beta^2 p_1^3}{(\beta+1)(\beta+2)}, \quad J^{(\beta)}_{[21]}(p_k) = \frac{(1-\beta)p_1 p_2 - p_3 + \beta p_1^3}{(\beta+1)(\beta+2)}, \quad J^{(\beta)}_{[111]}(p_k) = \frac{1}{6}p_1^3 - \frac{1}{2}p_1 p_2 + \frac{1}{3}p_3 \ .$$

Selberg integral

Schur functions,

$$\chi_\lambda(x) = \frac{\det_{1\leqslant i,j\leqslant n}\left(x_i^{\lambda_j+n-j}\right)}{\Delta(x)}$$

Seamus P. Albion, Eric M. Rains, and S. Ole Warnaar, 2021

$$\begin{aligned}
&\left\langle \prod_{r=1}^{n+1} \chi_{Y^{(r)}}\left[x^{(r)} - x^{(r-1)}\right] \right\rangle_{u_1,\dots,u_n,v}^{N_1,\dots,N_n} \\
&\quad = \prod_{r=1}^{n+1} \prod_{1\leqslant i<j\leqslant \ell_r} \frac{Y_i^{(r)} - Y_j^{(r)} + j - i}{j-i} \prod_{r,s=1}^{n+1} \prod_{i=1}^{\ell_r} \frac{(A_{r,s} - N_{s-1} + N_s - i + 1)_{Y_i^{(r)}}}{(A_{r,s} + \ell_s - i + 1)_{Y_i^{(r)}}} \\
&\quad \times \prod_{1\leqslant r<s\leqslant n+1} \prod_{i=1}^{\ell_r} \prod_{j=1}^{\ell_s} \frac{Y_i^{(r)} - Y_j^{(s)} + A_{r,s} + j - i}{A_{r,s} + j - i},
\end{aligned}$$

AGT conjecture from Selberg integral

Reduction to Selberg integral

Before we go to the details conclusion first

$$Z_{\text{inst}}(q) = Z_{\text{DF}}(q)$$

$$N^{\text{inst}}_{\vec{Y}} = N^{\text{Toda}}_{\vec{Y}}$$

$$N^{\text{inst}}_{\vec{Y}} \equiv N^{\text{inst}}_{\vec{Y}+} N^{\text{inst}}_{\vec{Y}-}, \quad N^{\text{Toda}}_{\vec{Y}} \equiv N^{\text{Toda}}_{\vec{Y}+} N^{\text{Toda}}_{\vec{Y}-},$$

$$N^{\text{inst}}_{\vec{Y}+} \equiv \frac{\prod_{s=1}^{n+1} \prod_{k=1}^{n+1} f_{Y_s}(\mu_k + a_s)}{\prod_{t,s=1}^{n+1} G_{Y_t,Y_s}(a_t - a_s)} \prod_{s=1}^{n+1} \left\{ (-1)^{|Y_s|} \sqrt{\frac{G_{Y_s,Y_s}(0)}{G_{Y_s,Y_s}(1-\beta)}} \right\},$$

$$N^{\text{Toda}}_{\vec{Y}\pm} \equiv \left\langle \prod_{a=1}^{n+1} j^{(\beta)}_{Y_a}\left(-r^{(a)}_k - \frac{v'_{a\pm}}{\beta}\right) \right\rangle_{\pm} = \prod_{a=1}^{n+1} \sqrt{\frac{G_{Y_a,Y_a}(0)}{G_{Y_a,Y_a}(1-\beta)}} \left\langle \prod_{a=1}^{n+1} J^{(\beta)}_{Y_a}\left(-r^{(a)}_k - \frac{v'_{a\pm}}{\beta}\right) \right\rangle_{\pm}$$

Known results on Selberg average

SU(2) **case:** The relevant Selberg averages for one and two Jack polynomials were obtained by Kadell.

$$\left\langle J_Y^{(\beta)}(p) \right\rangle_{u,v,\beta}^{SU(2)} \qquad \left\langle J_A^{(\beta)}(p+w) J_B^{(\beta)}(p) \right\rangle^{SU(2)}$$

SU(*n*+1) **case:**

The one-Jack Selberg integral for *SU*(*n*+1) could be calculated by the formula offered by Warnaar.

$$\left\langle J_B^{(\beta)}(p_k^{(n)}) \right\rangle_{\vec{u},\vec{v},\beta}^{SU(n+1)}$$

He also gives A_2 two Jack integral $\left\langle J_R^{(\beta)}(p_k^{(1)}) J_B^{(\beta)}(p_k^{(2)}) \right\rangle_{u,v,\beta}^{SU(3)}$

A conjecture on Selberg average

To evaluate   we need Selberg average of (n + 1) Jack polynomials. While we do not perform the integration so far, we find a formula for $\beta = 1$ which reproduces known results and satisfies consistency conditions.

The Jack polynomial for $\beta = 1$ is called Schur polynomial. $J_Y^{(\beta)}|_{\beta=1} = \chi_Y.$

Conjecture : *We propose the following formula of Selberg average for $n + 1$ Schur polynomials,*

$$
\begin{aligned}
&\left\langle \chi_{Y_1}(-p_k^{(1)} - v'_1) \ldots \chi_{Y_r}(p_k^{(r-1)} - p_k^{(r)} - v'_r) \ldots \chi_{Y_{n+1}}(p_k^{(n)}) \right\rangle^{SU(n+1)}_{\vec{u},\vec{v},\beta=1} \\
&= \prod_{s=1}^{n} \left\{ (-1)^{|Y_s|} \times \frac{[v_s + N_s - N_{s-1}]_{Y'_s}}{[N_s + N_{s-1}]_{Y'_s}} \times \prod_{1\le i<j\le N_{s-1}+N_s} \frac{(j-i+1)_{Y'_{si}-Y'_{sj}}}{(j-i)_{Y'_{si}-Y'_{sj}}} \right\} \times \prod_{1\le i<j\le N_n} \frac{(j-i+1)_{Y_{(n+1)i}-Y_{(n+1)j}}}{(j-i)_{Y_{(n+1)i}-Y_{(n+1)j}}} \\
&\times \prod_{1\le t<s\le n+1} \left\{ \frac{[v_t + u_t + \cdots + u_{s-1} + N_t - N_{t-1}]_{Y'_t}}{[v_t - v_s + u_t + \cdots + u_{s-1} + N_t - N_{t-1} - N_s]_{Y'_t}} \times \frac{[-v_s + u_t + \cdots + u_{s-1} - N_s + N_{s-1}]_{Y_s}}{[v_t - v_s + u_t + \cdots + u_{s-1} - N_{t-1} - N_s + N_{s-1}]_{Y_s}} \right. \\
&\left. \times \prod_{i=1}^{N_t} \prod_{j=1}^{N_{s-1}} \frac{v_t - v_s + u_t + \cdots + u_{s-1} + N_t - N_{t-1} - N_s + N_{s-1} + 1 - (i+j)}{v_t - v_s + u_t + \cdots + u_{s-1} + N_t - N_{t-1} - N_s + N_{s-1} + 1 + Y'_{ti} + Y_{sj} - (i+j)} \right\},
\end{aligned}
$$

Proof

$$\left\langle \prod_{r=1}^{n+1} \chi_{Y^{(r)}} \left[x^{(r-1)} - x^{(r)}\right] \right\rangle_+ = \left. \frac{\prod_{s=1}^{n+1} (-1)^{|Y^{(s)}|} \prod_{r=1}^{n+1} \prod_{s=1}^{n+1} f_{Y^{(s)}}(\mu_r + a_s)}{\prod_{r,s=1}^{n+1} G_{Y^{(r)},Y^{(s)}}(a_r - a_s)} \right|_{\beta=1},$$

and the second part,

$$\left\langle \prod_{r=1}^{n+1} \chi_{Y^{(r)}} \left[y^{(r)} - y^{(r-1)}\right] \right\rangle_- = \left. \frac{\prod_{s=1}^{n+1} (-1)^{|Y^{(s)}|} \prod_{r=n+2}^{2n+2} \prod_{s=1}^{n+1} f_{Y^{(s)}}(\mu_r + a_s)}{\prod_{r,s=1}^{n+1} G_{Y^{(r)},Y^{(s)}}(a_r - a_s)} \right|_{\beta=1}.$$

Proof

Lemma 1

$$\prod_{1\leqslant i<j\leqslant l_Y} \frac{(Y_i - Y_j + \beta(j-i))_\beta}{(\beta(j-i))_\beta} \prod_{i=1}^{l_Y} \frac{1}{(\beta(l_Y - i + 1))_{Y_i}} = \frac{1}{G_{Y,Y}(0)}$$

Lemma 2

$$\prod_{i=1}^{l_Y} (-z - \beta i + \beta)_{Y_i} = (-1)^{|Y|} f_Y(z)$$

Lemma 3

$$\prod_{i=1}^{l_Y} \prod_{j=1}^{l_W} \frac{Y_i - W_j - x + j - i}{-x + j - i} \prod_{i=1}^{l_Y} \frac{1}{(-x + l_W - i + 1)_{Y_i}} \prod_{i=1}^{l_W} \frac{1}{(x + l_Y - i + 1)_{W_i}}$$
$$= \left. \frac{(-1)^{|Y|+|W|}}{G_{Y,W}(x) G_{W,Y}(-x)} \right|_{\beta=1} .$$

Proof

Lemma 1

$$\prod_{1\leqslant i<j\leqslant l_Y} \frac{(Y_i - Y_j + \beta(j-i))_\beta}{(\beta(j-i))_\beta} \prod_{i=1}^{l_Y} \frac{1}{(\beta(l_Y - i + 1))_{Y_i}} = \frac{1}{G_{Y,Y}(0)}$$

Lemma 2

$$\prod_{i=1}^{l_Y} (-z - \beta i + \beta)_{Y_i} = (-1)^{|Y|} f_Y(z)$$

Lemma 3

$$\prod_{i=1}^{l_Y}\prod_{j=1}^{l_W} \frac{Y_i - W_j - x + j - i}{-x + j - i} \prod_{i=1}^{l_Y} \frac{1}{(-x + l_W - i + 1)_{Y_i}} \prod_{i=1}^{l_W} \frac{1}{(x + l_Y - i + 1)_{W_i}}$$
$$= \left. \frac{(-1)^{|Y|+|W|}}{G_{Y,W}(x) G_{W,Y}(-x)} \right|_{\beta=1}.$$

演示页

5D

$$Z_{\text{inst}}(\mathfrak{q}) = \sum_{\vec{Y}} \mathfrak{q}^{|\vec{Y}|} N_{5d}^{\text{inst}}(\vec{a}, \mu),$$

$$N_{5d}^{\text{inst}} = \prod_{r,s=1}^{n+1} \frac{\mathbb{F}_{Y^{(s)}}(\mu_r + a_s, q^{-1}) \mathbb{F}_{Y^{(s)}}(\mu_{n+1+r} + a_s, q)}{\mathbb{G}_{Y^{(r)}, Y^{(s)}}(a_r - a_s, q) \mathbb{G}_{Y^{(s)}, Y^{(r)}}(a_s - a_r + 1 - \beta, q^{-1})},$$

$$\mathbb{F}_Y(z, q) = \prod_{(i,j) \in Y} (1 - q^{z + \beta(i-1) - (j-1)}),$$

$$\mathbb{G}_{Y,W}(x, q) = \prod_{(i,j) \in Y} (1 - q^{x + \beta(Y'_j - i) + (W_i - j) + \beta}).$$

5D

$$[n]_q \equiv \frac{1-q^n}{1-q}.$$

$$[n_1]_q[n_2]_q \neq [m_1]_q[m_2]_q$$

q-deformed Selberg integral

$$
\begin{aligned}
&I^{A_n}_{N_1,\dots,N_n}(\mathcal{O};u_1,\dots,u_n,v;\beta)\\
&=\frac{1}{N_1!\cdots N_n!(2\pi\mathrm{i})^{N_1+\cdots+N_n}}\int_{\mathbb{T}^{N_1+\cdots+N_n}}\mathcal{O}(x^{(1)},\dots,x^{(n)})\prod_{i=1}^{N_n}\frac{(q^{a_n}/x_i^{(n)},q^{1-a_n}x_i^{(n)};q)_\infty}{(q^{v+1}/x_i^{(n)},x_i^{(n)};q)_\infty}\prod_{r=1}^{n-1}\prod_{i=1}^{N_r}(x_i^{(r)})^{a_r}\\
&\times\prod_{r=1}^{n}\prod_{1\leqslant i<j\leqslant N_r}\frac{(x_i^{(r)}/x_j^{(r)},x_j^{(r)}/x_i^{(r)};q)_\infty}{(tx_i^{(r)}/x_j^{(r)},tx_j^{(r)}/x_i^{(r)};q)_\infty}\prod_{r=1}^{n-1}\prod_{i=1}^{N_r}\prod_{j=1}^{N_{r+1}}\frac{((qt)^{1/2}x_j^{(r+1)}/x_i^{(r)};q)_\infty}{((q/t)^{1/2}x_j^{(r+1)}/x_i^{(r)};q)_\infty}\frac{\mathrm{d}x^{(1)}}{x^{(1)}}\cdots\frac{\mathrm{d}x^{(n)}}{x^{(n)}},
\end{aligned}
$$

In [11] the Selberg integral formula for $n+1$ Schur polynomials is proposed. We now find a new q-deformed Selberg integral average formula, containing a product of $n+1$ Schur functions:

$$
\begin{aligned}
&\left\langle\prod_{r=1}^{n+1}\chi_{Y^{(r)}}\left[x^{(r)}-x^{(r-1)}\right]\right\rangle^{N_1,\dots,N_n}_{u_1,\dots,u_n,v}\\
&=\prod_{r=1}^{n+1}\prod_{(i,j)\in Y^{(r)}}\frac{1-q^{-N_r+N_{r-1}+i-j}}{1-q^{-Y_i^{(r)}-Y_j^{\prime(r)}+i+j-1}}\prod_{1\le r<s\le n+1}\prod_{i=1}^{L_{Y^{(r)}}}\prod_{j=1}^{L_{Y^{(s)}}}\frac{1-q^{-Y_i^{(r)}+Y_j^{(s)}-A_{r,s}-j+i}}{1-q^{-A_{r,s}-j+i}}\\
&\times\prod_{1\le r<s\le n+1}\prod_{(i,j)\in Y^{(r)}}\frac{1-q^{-A_{r,s}+N_{s-1}-N_s+i-j}}{1-q^{-A_{r,s}-L_{Y^{(s)}}+i-j}}\prod_{(i,j)\in Y^{(s)}}\frac{1-q^{-A_{r,s}+N_r-N_{r-1}-i+j}}{1-q^{-A_{r,s}+L_{Y^{(r)}}-i+j}}.
\end{aligned}
$$

q-deformed conformal blocks

$$\left\langle\!\!\left\langle \prod_{r=1}^{n} \left\{ \prod_{i=1}^{N_r} (\mathfrak{q} x_i^{(r)}; q)_{v_{r-}} \prod_{j=1}^{\tilde{N}_r} (\mathfrak{q} y_i^{(r)}; q)_{v_{r+}} \right\} \prod_{r,s=1}^{n} \prod_{i=1}^{N_r} \prod_{j=1}^{\tilde{N}_s} \left(\mathfrak{q} x_i^{(r)} y_j^{(s)}; q \right)_{\beta}^{C_{rs}} \right\rangle_{+} \right\rangle_{-}$$

$$\sum_{\vec{Y}} \mathfrak{q}^{|\vec{Y}|} \left\langle \prod_{r=1}^{n+1} \chi_{Y^{(r)}} \left[x^{(r-1)} - x^{(r)} \right] \right\rangle_{+} \left\langle \prod_{r=1}^{n+1} \chi_{Y^{(r)}} \left[y^{(r)} - y^{(r-1)} \right] \right\rangle_{-}$$

$$\left\langle \prod_{r=1}^{n+1} \chi_{Y^{(r)}} \left[x^{(r-1)} - x^{(r)}\right] \right\rangle_+ \left\langle \prod_{r=1}^{n+1} \chi_{Y^{(r)}} \left[y^{(r)} - y^{(r-1)}\right] \right\rangle_-$$

$$= \prod_{r,s=1}^{n+1} \frac{\mathbb{F}_{Y^{(s)}}(\mu_r + a_s, q)\mathbb{F}_{Y^{(s)}}(\mu_{n+1+r} + a_s, q^{-1})}{\mathbb{G}_{Y^{(r)},Y^{(s)}}(a_r - a_s, q)\mathbb{G}_{Y^{(s)},Y^{(r)}}(a_s - a_r + 1 - \beta, q^{-1})}.$$

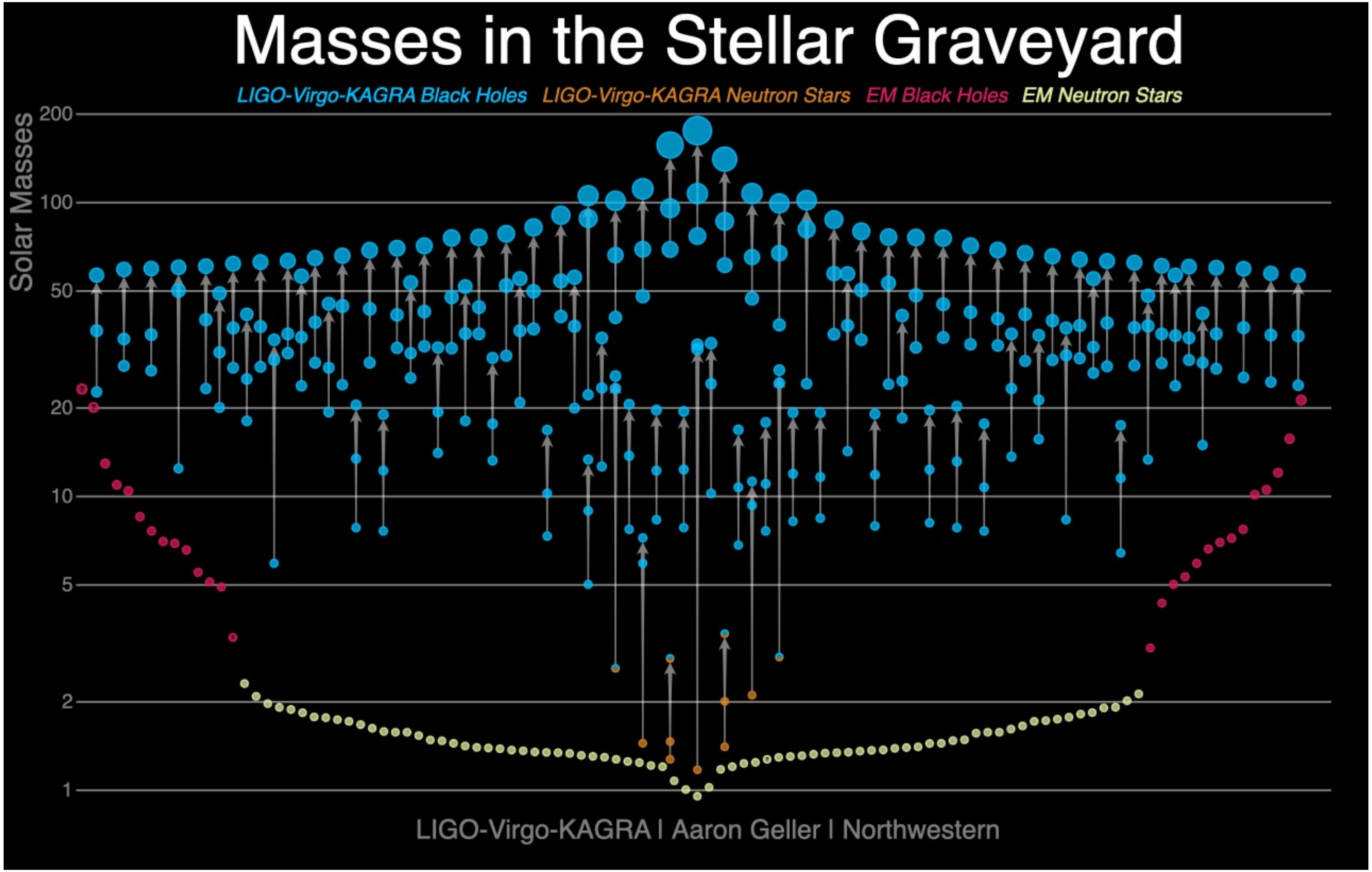
Lemma

$$\prod_{i=1}^{L_Y} \prod_{j=1}^{L_W} \frac{1 - q^{x+Y_i-W_j+j-i}}{1 - q^{x+j-i}} \prod_{(i,j)\in Y} \frac{1}{1 - q^{x+L_W-i+j}} \prod_{(i,j)\in W} \frac{1}{1 - q^{x-L_Y+i-j}}$$

$$= \left. \frac{1}{\mathbb{G}_{Y,W}(-x, q^{-1})\mathbb{G}_{W,Y}(x, q)} \right|_{\beta=1}.$$

演示页

GW



Masses in the Stellar Graveyard
LIGO-Virgo-KAGRA Black Holes
LIGO-Virgo-KAGRA Neutron Stars
EM Black Holes
EM Neutron Stars
Solar Masses
200
100
50
20
10
5
2
1
LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

Black Holes

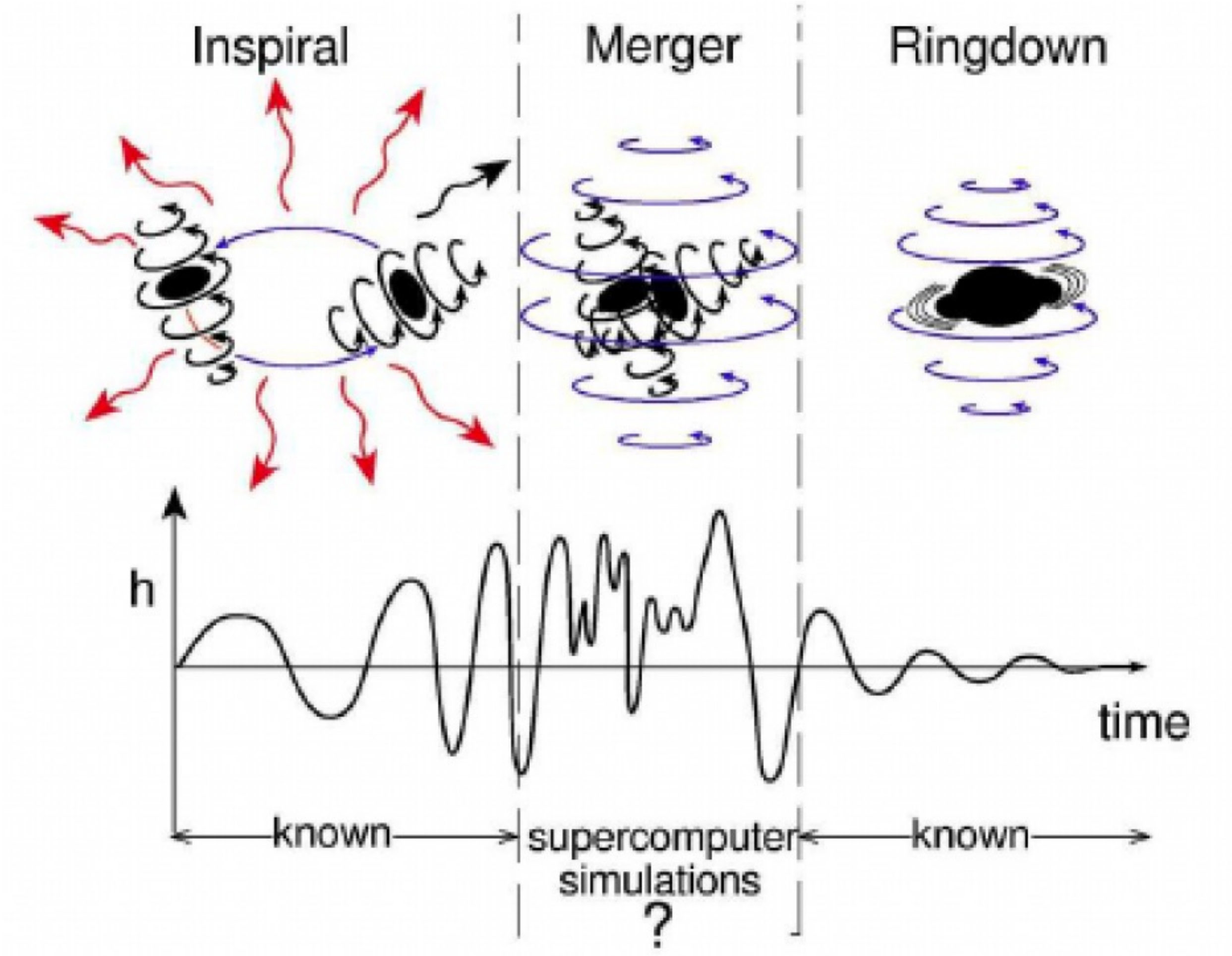


Figure: Credit: Kip Thorne

Schwarzschild BH / $SU(2)$ $N_f = 3$ $\mathcal{N} = 2$ SYM - I

- Schwarzschild metric for **static and spherically symmetric solutions** to the Einstein equation in the vacuum

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{12}$$

- Scalar ($s = 0$), electromagnetic ($s = 1$) and odd-parity gravitational ($s = 2$) **linear perturbations of the metric** are governed by the **Regge-Wheeler type equation**

$$\left[f(r)\frac{d}{dr}f(r)\frac{d}{dr} + \omega^2 - V(r)\right]\phi(r) = 0\,, \tag{13}$$

$$f(r) = \left(1 - \frac{2M}{r}\right), \qquad V(r) = f(r)\left[\frac{l(l+1)}{r^2} + (1 - s^2)\frac{2M}{r^3}\right], \tag{14}$$

boundary conditions $$\phi(r) \sim \begin{cases} e^{-i\omega(r+2M\ln(r-2M))} & r \to 2M \\ e^{+i\omega(r+2M\ln(r-2M))} & r \to \infty \end{cases}. \tag{15}$$

Schwarzschild BH / $SU(2)$ $N_f = 3$ $\mathcal{N} = 2$ SYM - II

- By the change of variable [1]

$$r = 2Mz\,, \qquad \phi(r) = \sqrt{\frac{z}{z-1}}\Phi(z)\,, \tag{16}$$

(13) becomes the equation

$$\Phi''(z) + \tilde{Q}(z)\Phi(z) = 0 \qquad \tilde{Q}(z) = \frac{1}{z^2(z-1)^2}\sum_{i=0}^{4}\tilde{A}_i z^i \tag{17}$$

$$\tilde{A}_0 = -s^2 + \frac{1}{4}\,, \quad \tilde{A}_1 = l(l+1) + s^2\,, \quad \tilde{A}_2 = -l(l+1)\,, \quad \tilde{A}_3 = 0\,, \quad \tilde{A}_4 = (2M\omega)^2\,. \tag{18}$$

- Equation (17) has two regular singularities at $z = 0, 1$ and one irregular singularity (of Poincaré rank 1) at $z = \infty$. It corresponds to the **Confluent Heun** equation.

- Accelerating black holes [Griffiths et.al. '06]

$$ds^2 = \frac{1}{(1-\alpha r\cos\theta)^2}\left(-f(r)dt^2 + \frac{dr^2}{f(r)} + \frac{r^2 d\theta^2}{P(\theta)} + P(\theta) r^2 \sin^2\theta d\phi^2\right)$$

$$f(r) = (1 - \tfrac{2M}{r} + \tfrac{Q^2}{r^2})(1-\alpha^2 r^2).$$

- $\alpha \to 0$: Reissner-Nordstrom BH, $Q, \alpha \to 0$: Schwarzchild BH.
- BH ODE at radial direction

$$\left(f(r)\frac{d}{dr}f(r)\frac{d}{dr} + \omega^2 - V(r)\right)\psi(r) = 0,$$

黑洞

黑洞

$$V_r = f(r)\left(\frac{\lambda}{r^2} - \frac{f(r)}{3r^2} + \frac{f'(r)}{3r} - \frac{f''}{6}\right)$$

- $\phi(r) = \frac{1}{\sqrt{f(r)}}\tilde{\psi}(r)$:

$$\left(\partial_r^2 + Q_r(r)\right)\tilde{\psi}(r) = 0$$

- Four regular singular points at

$$r_i = \{-\frac{1}{\alpha}, r_-, r_+, \frac{1}{\alpha}\}, \quad r_\pm = M \pm \sqrt{M^2 - Q^2}$$

- Heun equation!

- The low energy effective theory of 4D $\mathcal{N}=2$ SYM can be solved exactly via the Seiberg-Witten theory [Seiberg-Witten '94]

$$H(x,p)=E, \quad \lambda_{\mathrm{SW}}=pdx$$

- One can use the localization to compute the exact partition functions of the gauge theory, where two deformation parameters (ϵ_1,ϵ_2) are turned on [Nekrasov '02].

- In the classical limit $(\epsilon_1=\epsilon_2=0)$, one obtains the partition function of SW theory.

- In the Nekrasov-Shatashvili limit $(\epsilon_1\neq 0,\epsilon_2=0)$, the SW curve is quantized, where ϵ_1 plays the role of Planck constant $(\hat{p}=\epsilon\partial_x)$:

$$\text{Quantum SW curve}: \quad \hat{H}(x,\hat{p})\psi=E\psi \quad \text{[Nekrasov et.al '09]}$$

黑洞

- $SU(2)$ gauge theory with $N_f = 4$ flavor: $\left(\hbar^2\partial_z^2 + Q_{\rm SW}(z)\right)\psi(z) = 0$

$$Q_{\rm SW}(z) = \frac{\frac{1}{4}-a_0^2}{z^2} + \frac{\frac{1}{4}-a_1^2}{(z-1)^2} + \frac{\frac{1}{4}-a_t^2}{(z-t)^2} - \frac{\frac{1}{2}-a_0^2-a_1^2-a_t^2+a_\infty^2+u}{z(z-1)} + \frac{u}{z(z-t)}.$$

- Four regular points at $z = (0, 1, t, \infty)$.

- BH data $(r, \omega, M, \cdots) \leftrightarrow$ SW data $(z, t, a_i, \cdots)$

$$r = \frac{1}{\alpha} \leftrightarrow z = 1, \quad r = r_+ \leftrightarrow z = t, \quad t = \frac{2\alpha(r_- - r_+)}{(\alpha r_- - 1)(\alpha r_+ + 1)}, \cdots$$

$$\text{Bdy condition:} \quad \psi(z) \sim \begin{cases} (t-z)^{\frac{1}{2}-a_t} & z \to t \\ (1-z)^{\frac{1}{2}-a_1} & z \to 1 \end{cases}.$$

Need connection formula!

$$\psi^{(t),\rm in} = A\psi_+^{(1)} + B\psi_-^{(1)}, \quad \psi_-^{(1)} \sim (1-z)^{\frac{1}{2}-a_1},$$

$$A(\text{SW data}) = 0 \to A(\omega) = 0.$$

	Connection formula	Numeric data (Destounis et.al '20)
ω_α	$-0.0505984i$	$-0.00506i$
$\omega_{\rm PS}$	$0.11131 - 0.102548i$	$0.1112 - 0.1042i$

Table: $\alpha M = 0.05$, $Q = 0.3M$, $\lambda = 0.3317$, # instanton =3

	Connection formula	Numeric data (Destounis et.al '20)
$\omega_{\rm NE}$	$-0.0412141i$	$-0.0412i$
$\omega_{\rm PS}$	$0.11194 - 0.080626i$	$0.117 - 0.0814i$

Table: $\alpha M = 0.3$, $Q = 0.999M$, $\lambda = 0.3033$, # instanton =3

Seiberg-Witten theory is introduced to solve four dimensional $\mathcal{N}=2$ asymmetric gauge theory by evaluating the singularities and asymptotic behaviors. We leave the details to the references [1, 2], and here focus on SU(2) theory with flavor number $N_f=3$, which is our main concern in this paper. The quantum SW curve in this case can be rewritten in the following form after some reparameterizations:

$$\hbar^2\psi''(z)+\left(\frac{1}{z^2(z-1)^2}\sum_{i=0}^{4}\widehat{A}_iz^i\right)\psi(z)=0, \tag{2.1}$$

which is in the normal form of the so-called confluent Heun equation (A.7) [32]. The coefficients are expressed in gauge parameters by

$$\begin{aligned}
\widehat{A}_0&=-\frac{(m_1-m_2)^2}{4}+\frac{\hbar^2}{4},\\
\widehat{A}_1&=-E-m_1m_2-\frac{m_3\Lambda_3}{8}-\frac{\hbar^2}{4},\\
\widehat{A}_2&=E+\frac{3m_3\Lambda_3}{8}-\frac{\Lambda_3^2}{64}+\frac{\hbar^2}{4},\\
\widehat{A}_3&=-\frac{m_3\Lambda_3}{4}+\frac{\Lambda_3^2}{32},\\
\widehat{A}_4&=-\frac{\Lambda_3^2}{64},
\end{aligned} \tag{2.2}$$

For a massive scalar boson field ψ around a BH, it obeys the wave equation

$$\nabla^a \nabla_a \psi = \mu^2 \psi \,, \tag{3.1}$$

with $\mu = \mathcal{M}G/\hbar c$ for a particle of mass $\mathcal{M}$. Equation (3.1) is also separable in the Kerr geometry. When we assume

$$\psi = e^{-i\omega t + im\phi} S(\theta) R(r), \tag{3.2}$$

the separate equations are similar as before. For the angular part, we obtain:

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left[\sin\theta \frac{dS}{d\theta}\right] + \left[\alpha^2(\omega^2 - \mu^2)\cos^2\theta - \frac{m^2}{\sin^2\theta} + {}_sA_{\ell m}\right] S = 0 \,, \tag{3.3}$$

and for the radial part:

$$\Delta \frac{d}{dr}\left[\Delta \frac{dR}{dr}\right] + \left[\omega^2(r^2 + \alpha^2)^2 - 4aMrm\omega + \alpha^2 m^2 - \Delta(\mu^2 r^2 + \alpha^2\omega^2 + {}_sA_{\ell m})\right] R = 0 \,. \tag{3.4}$$

Or equivalently

$$\Delta(r)R''(r) + \Delta'(r)R'(r) + \left(V_T(r) - \mu^2 r^2\right) R(r)|_{s=0} = 0, \tag{3.5}$$

where we set the spin $s = 0$ because we focus only on the scalar field here. The

Comparing the potential of the wave equation in the gauge theory side, we obtain the dictionary for the massive scalar field. For the radial part, we find

$$\begin{aligned}
\Lambda_3 &= -16\mathrm{i}\sqrt{\omega^2-\mu^2}\sqrt{M^2-\alpha^2}, \\
E &= -{}_0A_{\ell m} - (2M^2-\alpha^2)\mu^2 + (8M^2-\alpha^2)\omega^2 - \frac{1}{4}, \\
m_1 &= -2\mathrm{i}M\omega, \qquad m_3 = -\frac{\mathrm{i}M(2\omega^2-\mu^2)}{\sqrt{\omega^2-\mu^2}}, \\
m_2 &= \frac{\mathrm{i}(-2M^2\omega+\alpha m)}{\sqrt{M^2-\alpha^2}}.
\end{aligned} \tag{3.7}$$

For the angular part, we have the same dictionary as the case of the massless scalar field (2.20) with $c = \alpha\sqrt{\omega^2-\mu^2}$. As in the previous section, the angular eigenvalue ${}_0A_{\ell m}$ can be expanded in c as

$${}_0A_{\ell m} = \ell(\ell+1) + \sum_{k=1}^{\infty} f_k c^{2k}, \tag{3.8}$$

and we employ up to the c^6 order in our numerical calculations. Note that if we take $\mu = 0$, the dictionary straightforwardly reduces to the identifications in the massless scalar case.

超辐射

To compute the complex eigenfrequencies for the radial part of Teukolsky equation, we need to impose the boundary conditions at the horizon and spatial infinity. In general, we obtain the following asymptotic behaviors of the field at the horizon and spatial infinity:

$$R(r \to r_+) \sim (r - r_+)^{\pm i\sigma}, \tag{4.1}$$

$$R(r \to \infty) \sim r^{-1} r^{M(\mu^2 - 2\omega^2)/q} e^{qr}, \tag{4.2}$$

where

$$\sigma = \frac{2Mr_+\omega - \alpha m}{r_+ - r_-}, \quad q = \pm\sqrt{\mu^2 - \omega^2}. \tag{4.3}$$

The sign of the exponent in (4.1) corresponds to an outgoing and ingoing wave near the horizon. Also, the sign of the real part of q in (4.2) determines the asymptotic

In terms of the field $y(z)$ solving the radial equation in Schrödinger form, the above boundary conditions are rewritten as

$$y(z \to 1) \sim (z-1)^{\frac{1}{2} \pm i\sigma}, \tag{4.4}$$

$$y(z \to \infty) \sim (\Lambda_3 z)^{\mp m_3} \, e^{\mp \frac{\Lambda_3 z}{8}}. \tag{4.5}$$

Note that the asymptotic behavior at spatial infinity is written in terms of the quantities in the gauge theory. According to the connection formula [17], asymptotic expansions at different boundaries are related each other due to crossing symmetry. Now if we impose the ingoing wave boundary condition at the horizon, $y(z \to 1) \sim (z-1)^{\frac{1}{2} - i\sigma}$, and use the connection formula studied in [13], we can derive the asymptotic behavior at spatial infinity as

$$y(z \to \infty) \sim C_1(\Lambda_3, a, \mathbf{m}) \, (\Lambda_3 z)^{+m_3} \, e^{+\frac{\Lambda_3 z}{8}} + C_2(\Lambda_3, a, \mathbf{m}) \, (\Lambda_3 z)^{-m_3} \, e^{-\frac{\Lambda_3 z}{8}}, \tag{4.6}$$

which is written as the linear combination of the asymptotic behaviors in (4.5).

The full SU(2) Nekrasov-Shatashvili free energy $\mathcal{F}^{(N_f)}(a;\mathbf{m};\Lambda_{N_f},\hbar)$ [11] has contributions from its classical, one-loop and instanton components, with this relation given explicitly as [8]

$$\begin{aligned}\partial_a\mathcal{F}^{(N_f)}(a;\mathbf{m};\Lambda_{N_f},\hbar) &= -2\,a(4-N_f)\log\left[\frac{\Lambda_{N_f}2^{-\frac{1}{(2-N_f/2)}}}{\hbar}\right]-\pi\hbar \\ &-2\,\mathrm{i}\,\hbar\log\left[\frac{\Gamma(1+\frac{2\mathrm{i}a}{\hbar})}{\Gamma(1-\frac{2\mathrm{i}a}{\hbar})}\right]-\mathrm{i}\,\hbar\sum_{j=1}^{N_f}\log\left[\frac{\Gamma(\frac{1}{2}+\frac{m_j-\mathrm{i}a}{\hbar})}{\Gamma(\frac{1}{2}+\frac{m_j+\mathrm{i}a}{\hbar})}\right]+\frac{\partial\mathcal{F}^{(N_f)}_{\mathrm{inst}}(a;\mathbf{m};\Lambda_{N_f},\hbar)}{\partial a}.\end{aligned} \tag{4.9}$$

The SU(2) instanton part $\mathcal{F}^{(N_f)}_{\mathrm{inst}}(a;\mathbf{m};\Lambda_{N_f},\hbar)$ can be obtained by removing the U(1) contribution in the U(2) instanton part $F^{(N_f)}_{\mathrm{inst}}(a;\mathbf{m};\Lambda_{N_f},\hbar)$, which is defined by

$$F^{(N_f)}_{\mathrm{inst}}(a;\mathbf{m};\Lambda_{N_f},\hbar) = -\hbar\lim_{\epsilon_2\to 0}\epsilon_2\log Z^{(N_f)}(\mathrm{i}a,\mathbf{m},\hbar,\epsilon_2). \tag{4.10}$$

The Nekrasov partition function $Z^{(N_f)}(\mathrm{i}a,\mathbf{m},\epsilon_1,\epsilon_2)$ is exact in ϵ_i, and can be written explicitly in terms of Λ_{N_f} instanton expansion [16], as a convergent series.

超辐射

Table 1. The complex frequencies $2\omega_n$ of the QNM in the Schwarzschild geometry.

	Numerical	SW-QNM
$n=0,\ s=l=0$	$0.22091 - 0.209791i$	$0.235216 - 0.205777i$
$n=0,\ s=l=1$	$0.496527 - 0.184975i$	$0.495127 - 0.18661i$
$n=1,\ s=l=1$	$0.429031 - 0.587335i$	$0.428845 - 0.588262i$
$n=0,\ s=l=2$	$0.747343 - 0.177925i$	$0.743992 - 0.0175352i$
$n=1,\ s=l=2$	$0.693422 - 0.54783i$	$0.690244 - 0.542829i$
$n=2,\ s=l=2$	$0.602107 - 0.956554i$	$0.602514 - 0.959309i$

Table 2. The complex frequencies ω_n of the QNM in the Kerr geometry.

	Numerical	SW-QNM
$n=0,\ s=l=m=0,\ \alpha/M=0.1$	$0.110533 - 0.104802i$	$0.117687 - 0.102787i$
$n=0,\ s=l=m=0,\ \alpha/M\approx 1$	$0.110245 - 0.089433i$	$0.117755 - 0.0883568i$

超辐射

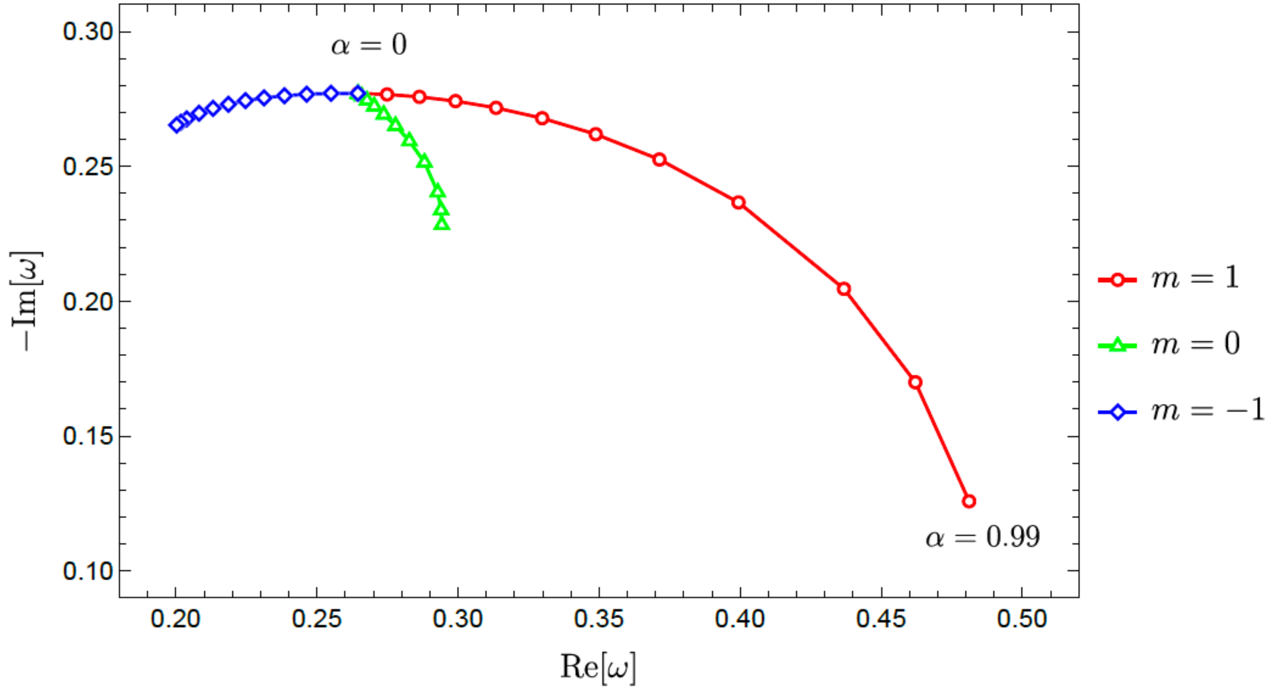


Figure 1. The QNM with $l = 1$, $m = 0, \pm 1$, and $\mu = 0.3$ as a function of the rotating parameter α in the complex frequency plane. The points denote each plot for from $\alpha = 0$ to $\alpha = 0.99$.

超辐射

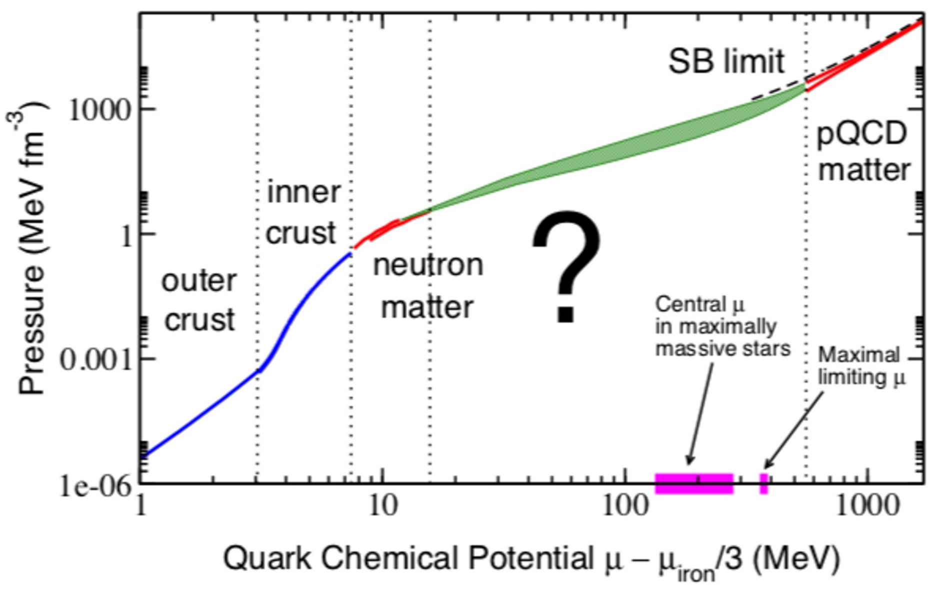
Table 3. The complex frequencies ω/μ of the quasi-bound state for $\alpha = 0.99$ and several values of $M\mu$ near the superradiant regime. The values of Continued fraction method are obtained following [37].

$M\mu$	Continued fraction method	SW-QNM
0.400	$0.976311 + 3.31172 \times 10^{-7} i$	$1.15754 + 2.12772 \times 10^{-5} i$
0.421	$0.973191 + 3.57332 \times 10^{-7} i$	$1.10408 + 2.01818 \times 10^{-5} i$
0.450	$0.968326 - 9.69046 \times 10^{-8} i$	$1.03881 + 1.83035 \times 10^{-5} i$
0.500	$0.957896 - 3.93289 \times 10^{-5} i$	$0.942834 - 8.93944 \times 10^{-4} i$
0.520	$0.952658 - 1.73567 \times 10^{-4} i$	$0.910458 - 6.81511 \times 10^{-4} i$

演示页

Neutron Stars

Matters in Neutron stars



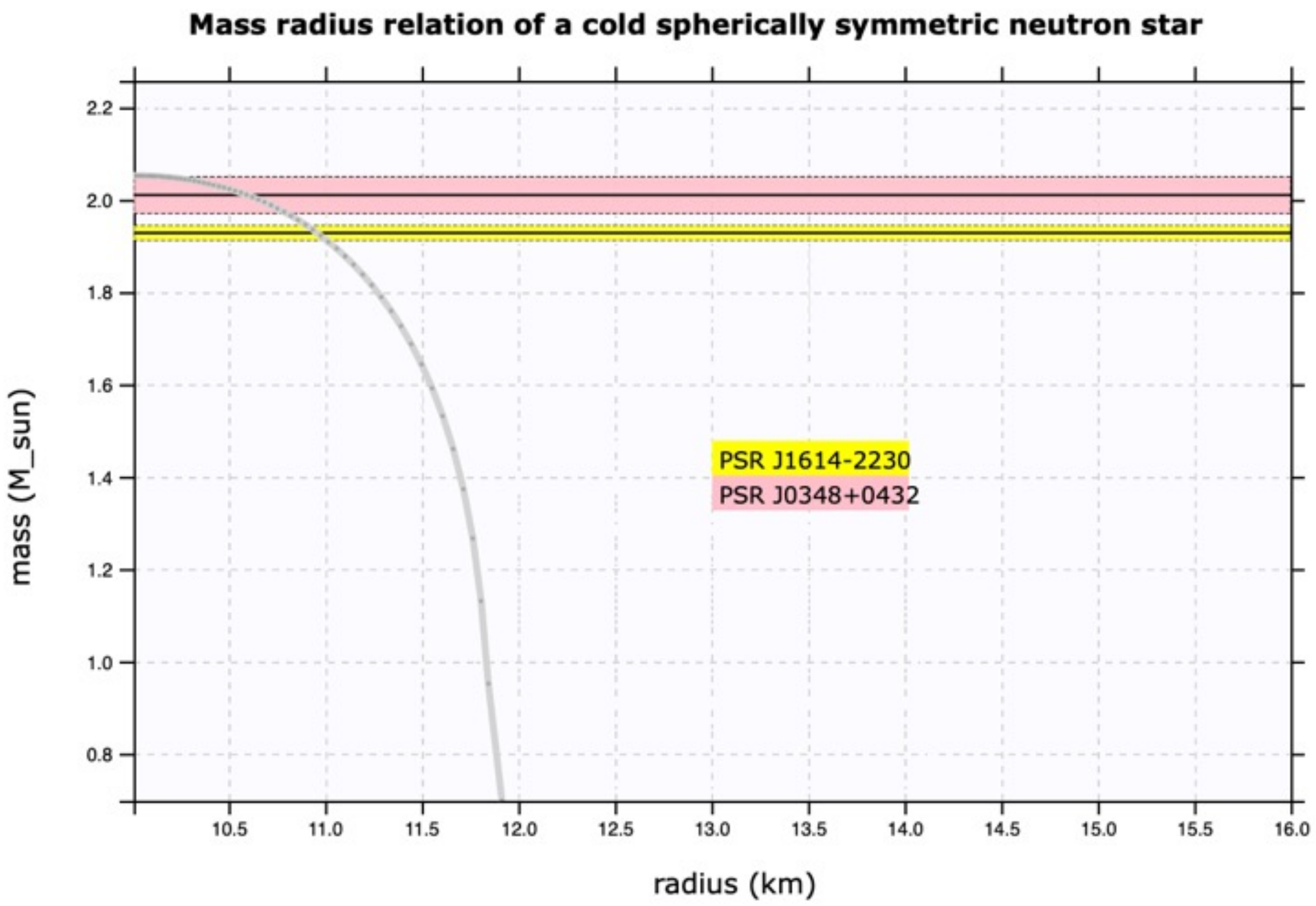
c.f. 1508.05019

The nuclear matter phase depends on non-perturbative QCD vacuum which is uncertain.

There are some phenomenological models, e.g. MIT bag model, NJL model, etc.

中子星

对星体来讲，所谓物态方程是指其内部能量密度与压强的函数关系。像白矮星、中子星等相对论性的致密天体，它们的结构计算需要在广义相对论框架中进行。在静态球对称与理想流体假设下，由爱因斯坦方程可推导出Tolman-Oppenheimer-Volkoff(TOV)方程。对致密星体给定物态方程后，其质量半径关系可由TOV方程解出。下图给出了由主流现象论中子星物态方程SLy4 带入TOV方程得到的质量半径关系。



What are TOV equations?

Neutron stars and, to some extent, also white dwarfs are relativistic objects and computations of their structure should be carried out in a general-relativistic (GR) framework.
Assuming zero space velocity, spherical symmetry, and an ideal fluid model.

The Tolman-Oppenheimer-Volkoff Equations

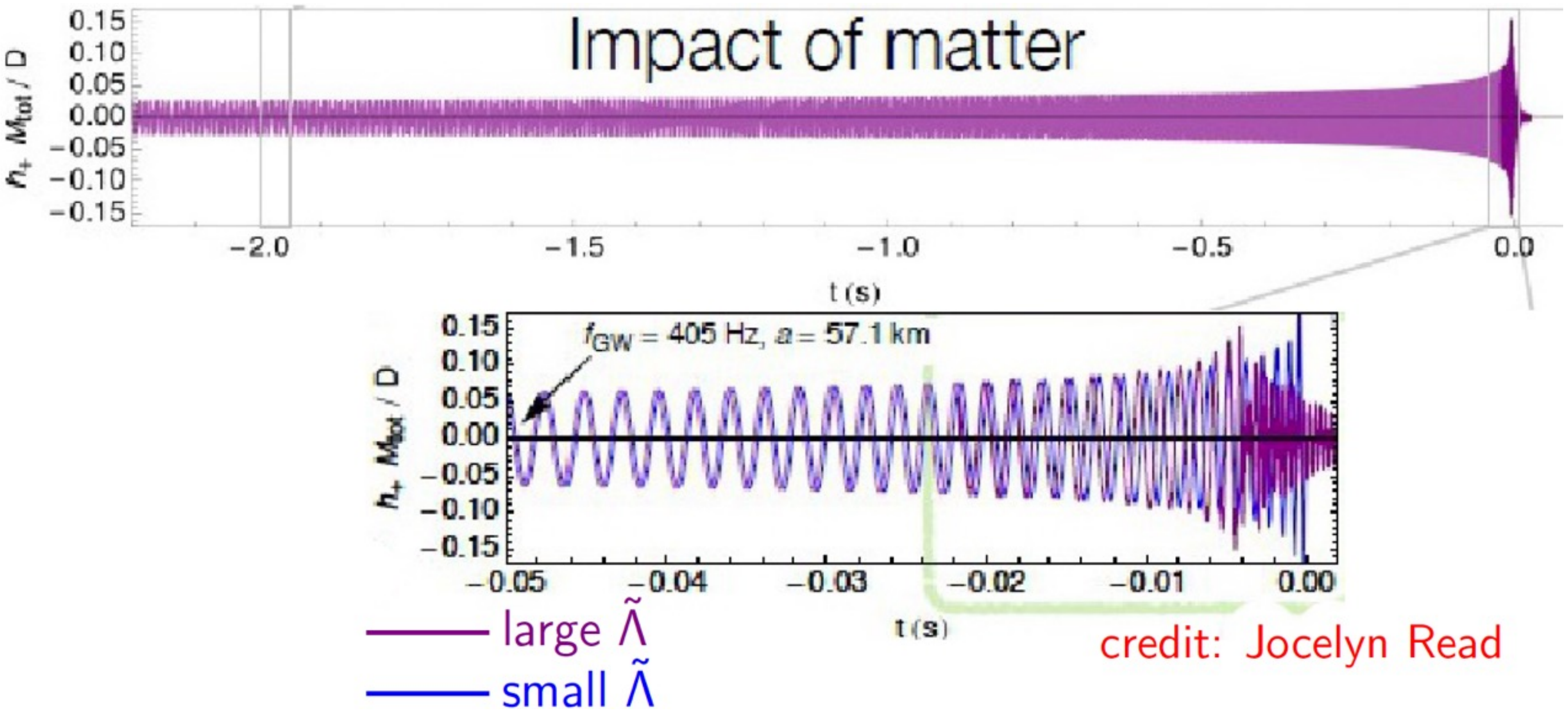
$$M' = 4\pi r^2 \rho,$$

$$p' = -G_N(\rho + p)\phi',$$

$$\phi' = \frac{M + 4\pi r^3 p}{r(r - 2G_N M)},$$

中子星

在双星合并期间，首先通过引力波频率等信息可以给出星体的总质量与质量比范围。另一方面，因为潮汐变形会使互绕加速，会使引力波波形相比点质量互绕产生相位移动，如图所示。虽然是高阶修正(正比于光速的负5次方，相当于2.5后牛顿展开)，但仍在探测器的灵敏度范围内。



中子星

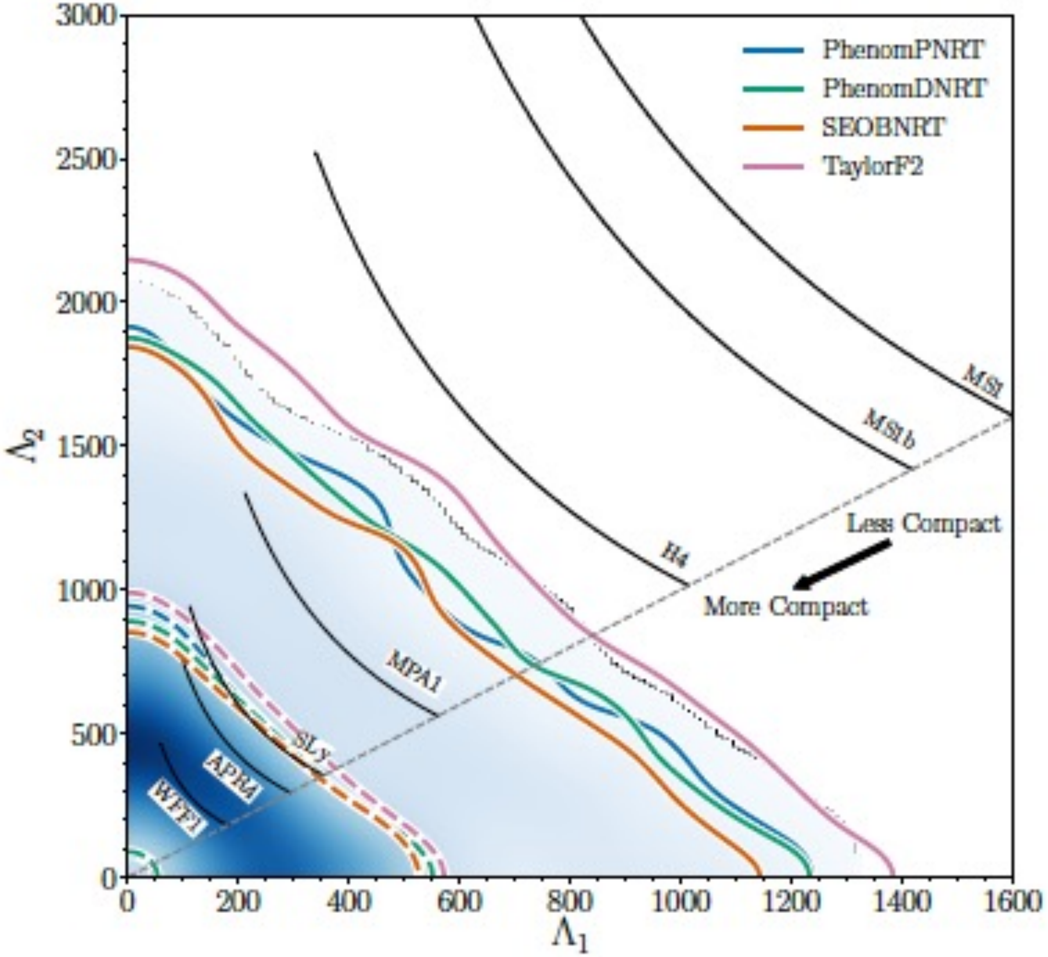


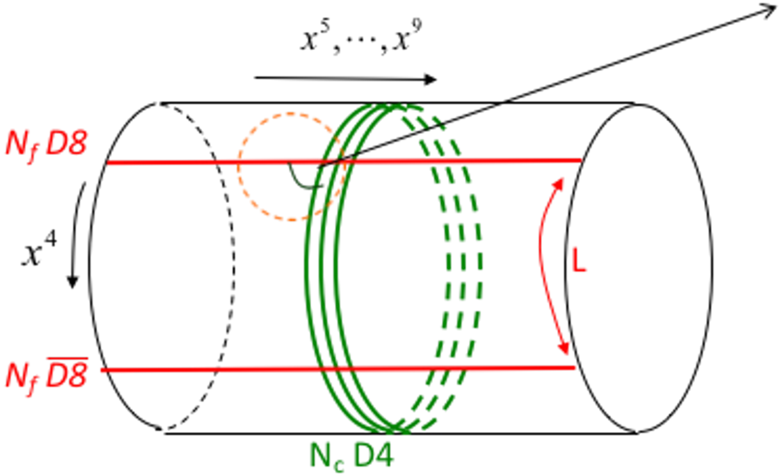
FIG. 10. PDFs for the tidal deformability parameters Λ_1 and Λ_2 using the high-spin (top) and low-spin (bottom) priors. The blue shading is the PDF for the precessing waveform PhenomPNRT. The 50% (dashed) and 90% (solid) credible regions are shown for the four waveform models. The seven black curves are the tidal parameters for the seven representative EOS models using the masses estimated with the PhenomPNRT model, ending at the $\Lambda_1 = \Lambda_2$ boundary.

Adding quarks --- Sakai-Sugimoto- model

- Weak coupling D-brane picture of SS model

N_c D4	0	1	2	3	(4)					
N_f D8/D8	0	1	2	3		5	6	7	8	9

4-8 strings: chiral quark
4-8 strings: anti-chiral quark



Massless U(Nc) QCD with N_f flavors:

$$U(N_c)\times U(N_f)_L \times U(N_f)_R$$

中子星

Thermodynamics

The on-shell D8-brane action is the grand-canonical free energy density.

$$\Omega[T,\mu;n_I,m_i,k] = \frac{T}{V} S|_{\text{on-shell}}$$

$$S[a_0,x_4;A_\mu(m_i)] := \mathcal{N}\frac{V}{T}\int_{u_c}^{\infty} du\,\mathcal{L} = S_{DBI} + S_{CS}$$

$$\text{BC:} \quad \frac{\ell}{2} = \int_{u_c}^{\infty} du\, x_4'(u)\ , \qquad \mu := \hat{a}_0(U=\infty)$$

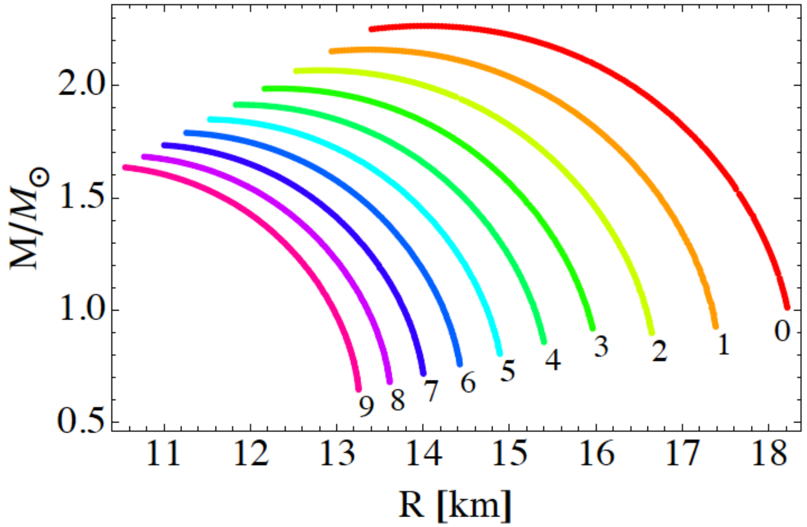
The derived thermodynamics is

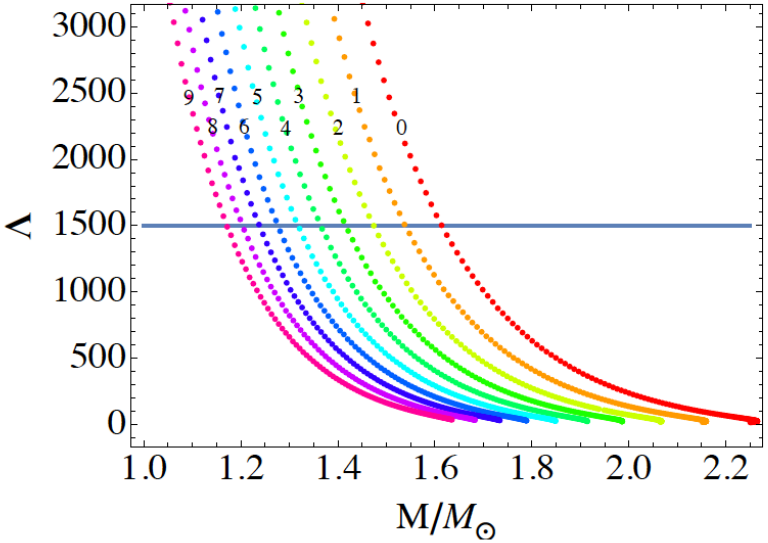
$$\text{pressure } p := -\Omega[\mu,T]\ , \qquad \text{baryon density } n := -\frac{\partial\Omega}{\partial\mu}|_T$$

$$\text{energy density } \epsilon := \Omega + n\mu \quad +Ts \quad \text{with } s := -\frac{\partial\Omega}{\partial T}|_\mu$$

中子星

利用量子色动力学的全息模型提取出了冷核物质的物态方程，只含有一个可调参数。计算了单星及双星的致密度与潮汐变形，发现结果与引力波观测数据分析相互相支持。研究首次得出了由理论推出的**top-down**模型，并与实验观测相一致。





最大问题在于，符合潮汐变形限制的模型得到的中子星最大质量不到两倍太阳质量。该研究使用的是点状瞬子模型，我们进一步应用更加复杂的瞬子气体模型。

中子星

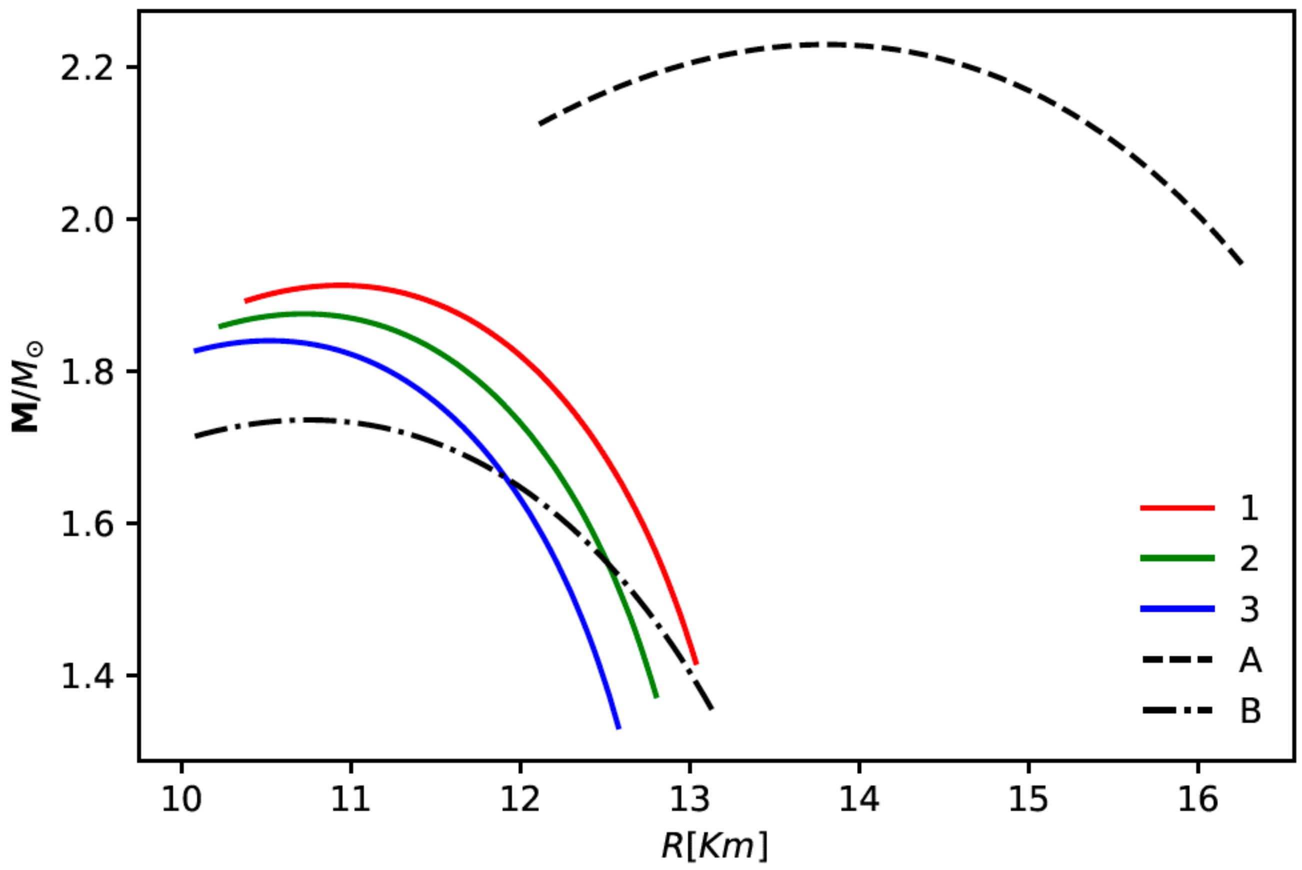


FIG. 1: Mass-Radius relations for different EoS

中子星

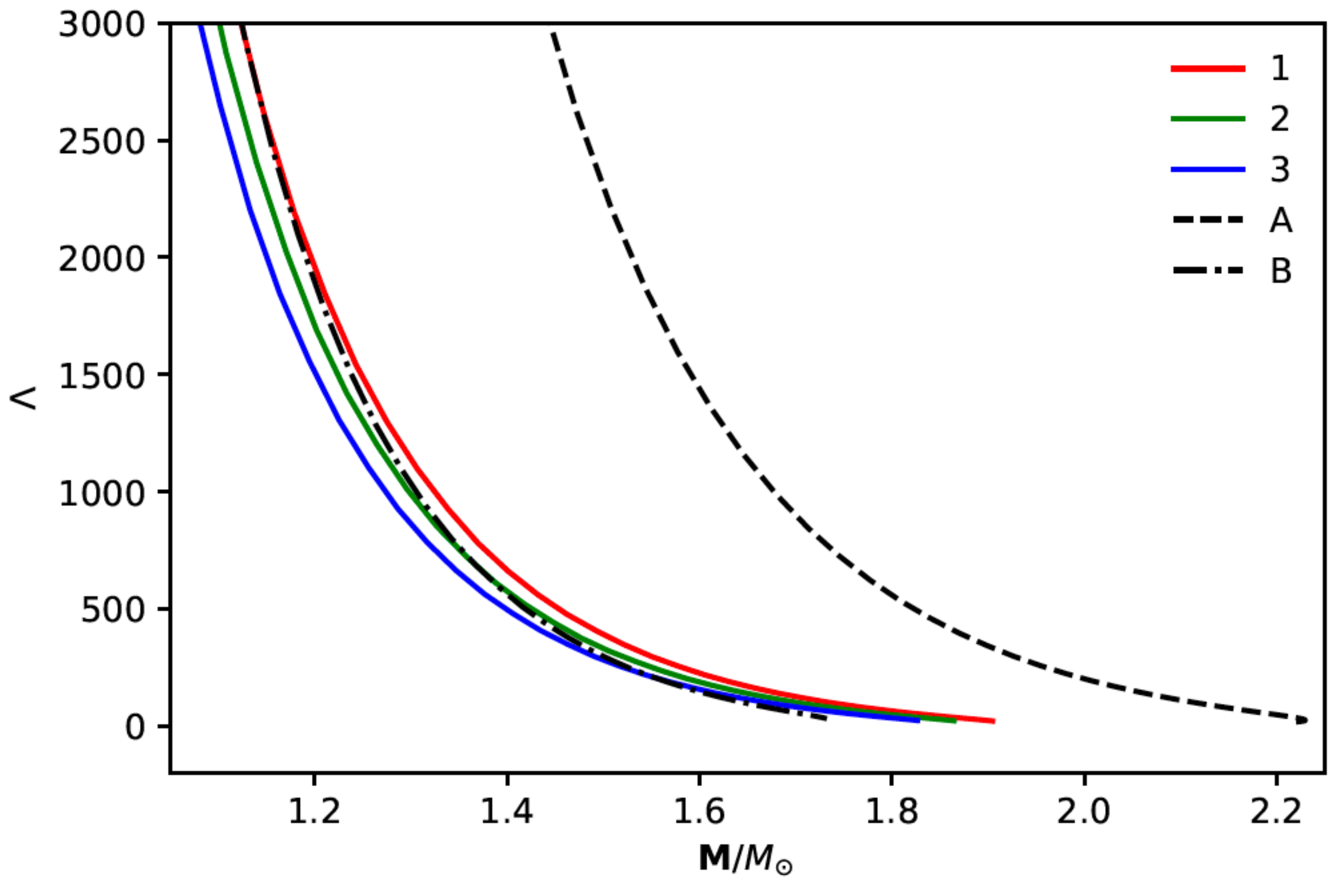


FIG. 2: TLN (Λ) vs Mass (M) for the holographic stars of EoS with the same sets of values and labels for ℓ as in Fig. 1.

中子星

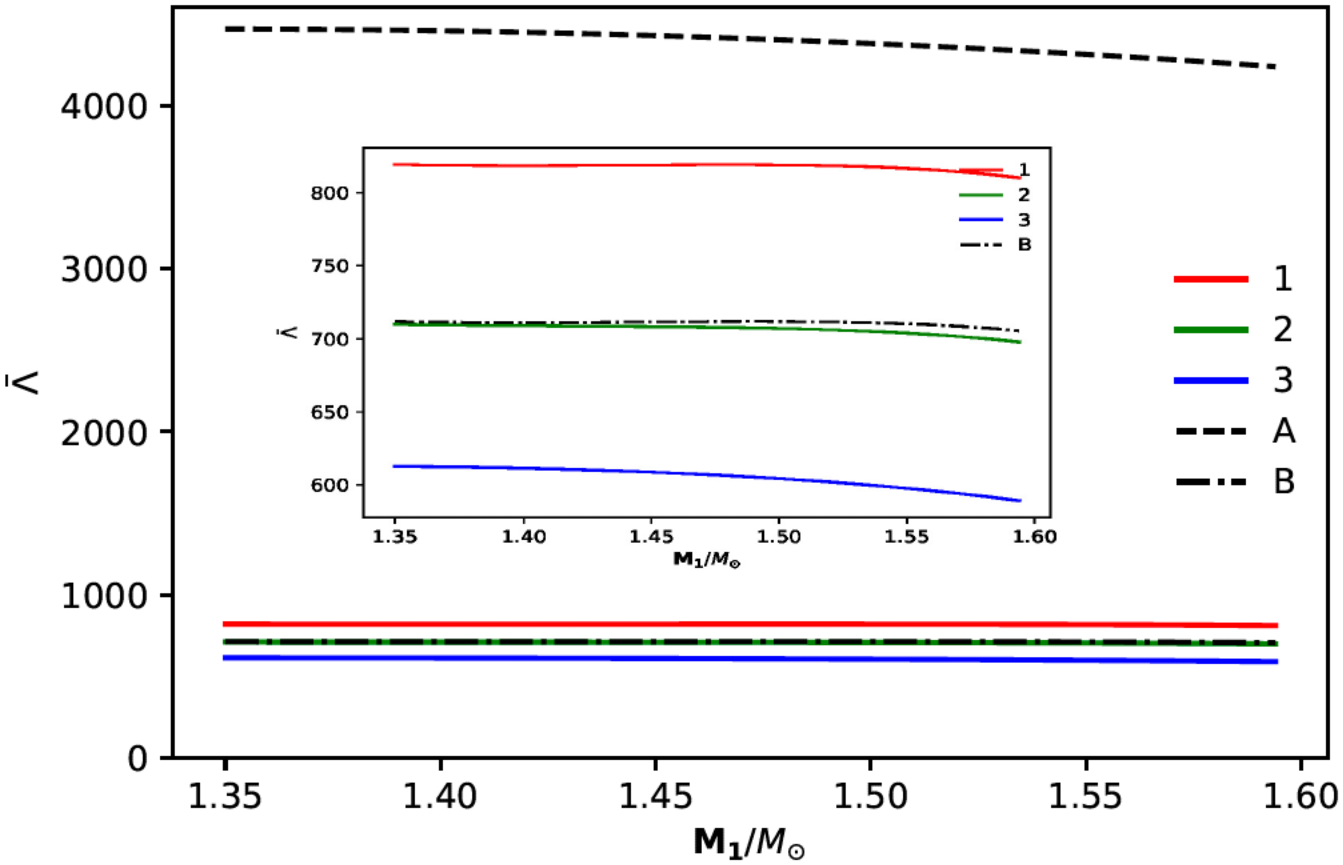


FIG. 3: Tidal deformability of the binary holographic stars of EoS vs one of the masses ($\bar{\Lambda}$ vs M_1) for partial sets of the values for ℓ used in Fig. 1.

演示页

夸克星

Extract the EoS from the holographic model built by Prof. Rong-Gen Cai, Prof. Song He, Prof. Li Li and Prof. Yuan-Xu Wang

2201.02004

Probing QCD critical point and induced gravitational wave by black hole physics

Rong-Gen Cai[b,c], Song He[a,d], Li Li[b,c,e] and Yuan-Xu Wang[a]

[a] *Center for Theoretical Physics and College of Physics, Jilin University, Changchun 130012, People's Republic of China*

[b] *CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China*

[c] *School of Fundamental Physics and Mathematical Sciences, Hangzhou Institute for Advanced Study, UCAS, Hangzhou 310024, China*

[d] *Max Planck Institute for Gravitational Physics (Albert Einstein Institute), Am Mühlenberg 1, 14476 Golm, Germany and*

[e] *Peng Huanwu Collaborative Center for Research and Education, Beihang University, Beijing 100191, China.*[*]

(Dated: February 21, 2023)

Locating the critical endpoint of QCD and the region of a first-order phase transition at finite baryon chemical potential is an active research area for QCD matter. We provide a gravitational dual description of QCD matter at finite baryon chemical potential μ_B and finite temperature using the non-perturbative approach from gauge/gravity duality. After fixing all model parameters using state-of-the-art lattice QCD data at zero chemical potential, the predicted equations of state and QCD trace anomaly relation are in quantitative agreement with the latest lattice results. We then give the exact location of the critical endpoint as well as the first-order transition line, which is within the coverage of many upcoming experimental measurements. Moreover, using the data from our model at finite μ_B, we calculate the spectrum of the stochastic gravitational wave background associated with the first-order QCD transition in the early universe, which could be observable via pulsar timing in the future.

18 Feb 2023

Dark Stars

暗星

$$\frac{\rho}{\rho_\odot} = \frac{3p}{\rho_\odot} + \mathcal{B}_4 \sqrt{\frac{p}{\rho_\odot}}$$

$$\frac{\rho}{\rho_\odot} = \frac{n+2}{n-2} \frac{p}{\rho_\odot} + \mathcal{B}_n \left(\frac{p}{\rho_\odot}\right)^{\frac{2}{n}}$$

Liouville field $\frac{m^2}{2\beta^2}\left[e^{\beta^2|\phi|^2} - 1\right]$

$$\frac{\rho}{\rho_\odot} = \mathcal{B}\left(\sigma_*^2 e^{\sigma_*^2} + e^{\sigma_*^2} - 1\right),$$

$$\frac{p}{\rho_\odot} = \mathcal{B}\left(\sigma_*^2 e^{\sigma_*^2} - e^{\sigma_*^2} - 1\right),$$

暗星

cosh-Gordon field

$$\frac{m^2}{\beta^2}[\cosh(\beta\sqrt{|\phi|^2}) - 1]$$

$$\frac{\rho}{\rho_\odot} = \mathcal{B}\left(\frac{1}{2}\sigma_* \sinh\sigma_* + \cosh\sigma_* - 1\right),$$

$$\frac{p}{\rho_\odot} = \mathcal{B}\left(\frac{1}{2}\sigma_* \sinh\sigma_* - \cosh\sigma_* + 1\right).$$

$$\frac{m^2}{\beta^2}[1 - \cos(\beta\sqrt{|\phi|^2})].$$

$$\frac{\rho}{\rho_\odot} = \mathcal{B}\left(\frac{1}{2}\sigma_* \sin\sigma_* - \cos\sigma_* + 1\right)$$

$$\frac{p}{\rho_\odot} = \mathcal{B}\left(\frac{1}{2}\sigma_* \sin\sigma_* + \cos\sigma_* - 1\right)$$

$$\frac{1}{2}m^2|\phi|^2(1 - \beta^2|\phi|^2)^2$$

$$\frac{\rho}{\rho_\odot} = \mathcal{B}\sigma_*^2(1-\sigma_*^2)(1-2\sigma_*^2),$$

$$\frac{p}{\rho_\odot} = \mathcal{B}\sigma_*^4(\sigma_*^2 - 1).$$

暗星

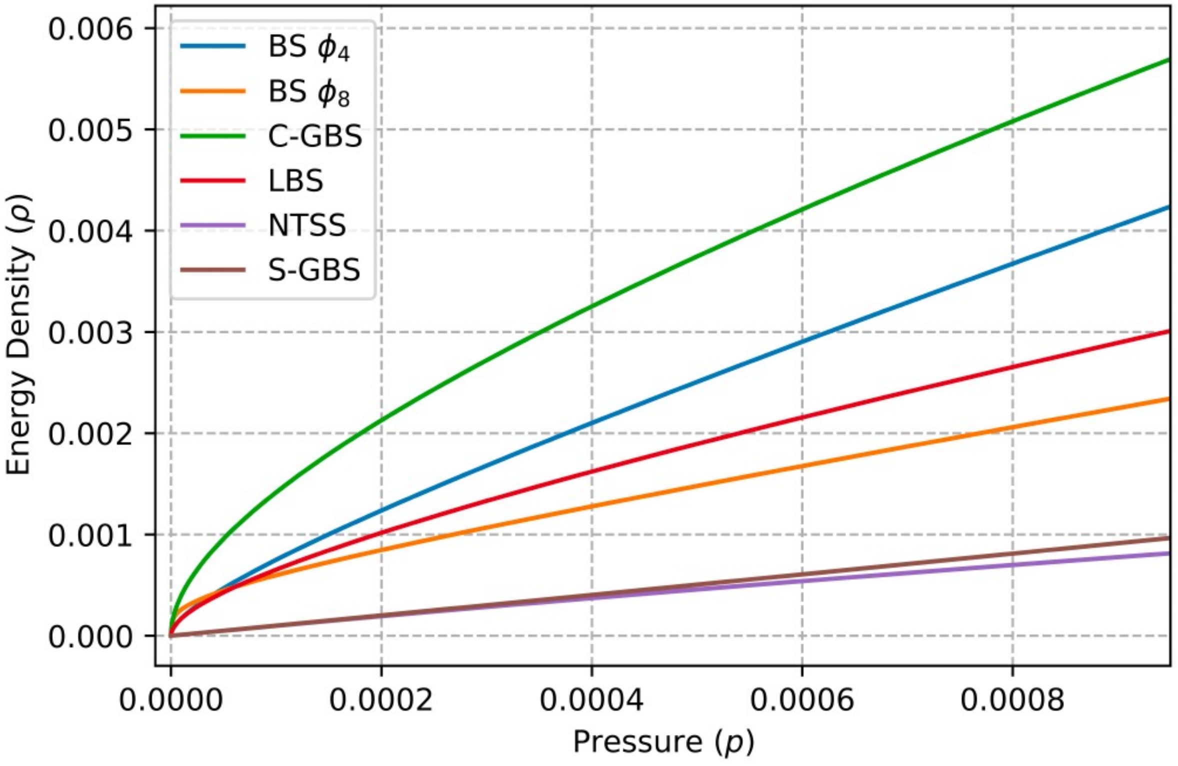
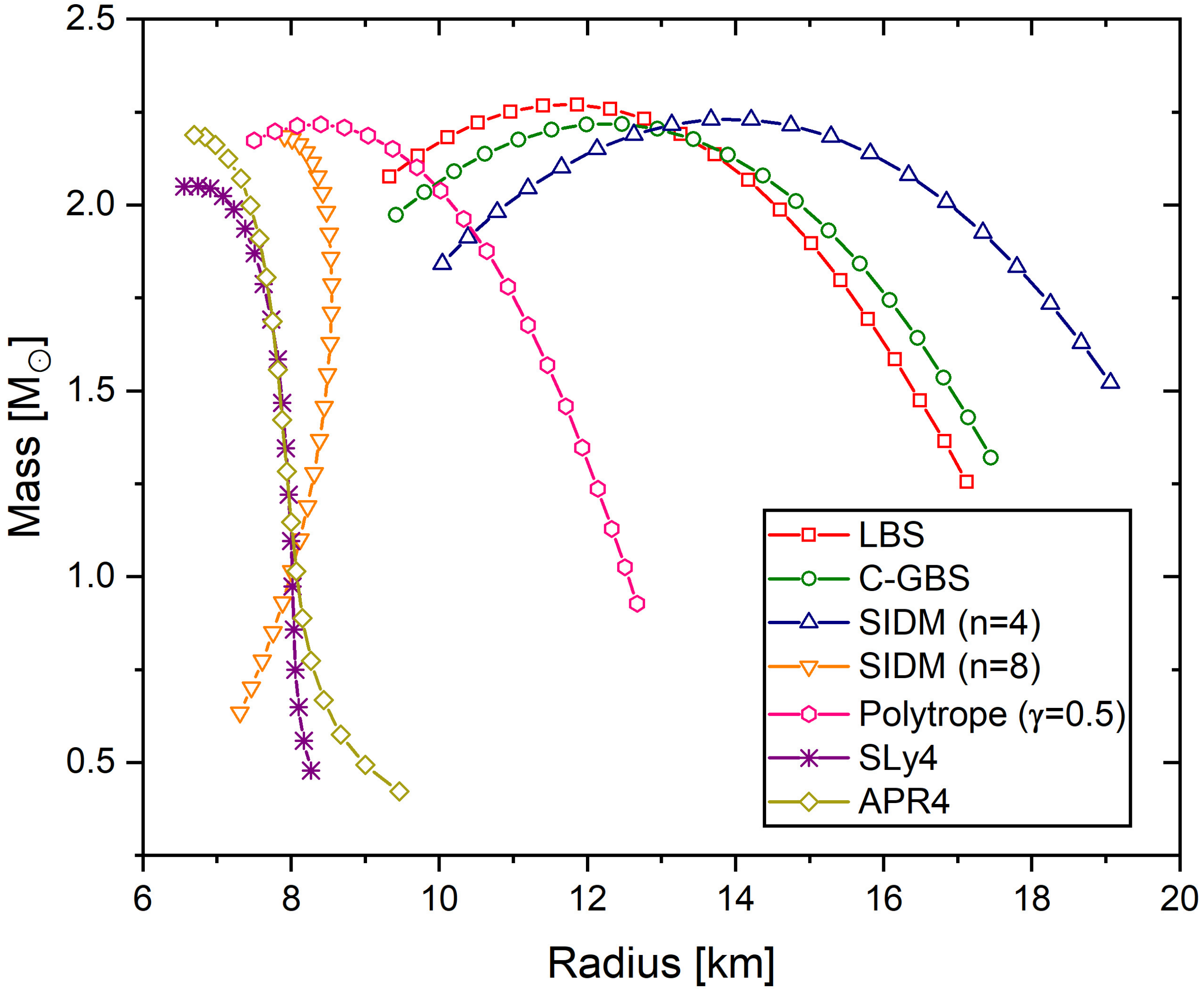


FIG. 1: An illustration of the EoSs discussed above, showing their relative behaviors. The adjusting parameter $\mathcal{B}_n$ and $\mathcal{B}$ are chosen to make the maximal masses of the boson stars have several solar masses. We consider p up to 10^{-3} since the typical range of the corresponding central pressure is between 10^{-6} to 10^{-2} as a reference. Here ρ and p are measured in the unit of $\rho_\odot$.

暗星


Mass [$M_\odot$]
Radius [km]
LBS
C-GBS
SIDM (n=4)
SIDM (n=8)
Polytrope (γ=0.5)
SLy4
APR4

暗星

Scaling Symmetry

The EoS can usually be described by a pair of parameter functions in the form of

$$\frac{\rho}{\rho_\odot} = \mathcal{B}\, f(\sigma_*)\,, \qquad (52)$$

$$\frac{p}{\rho_\odot} = \mathcal{B}\, g(\sigma_*)\,, \qquad (53)$$

where f and g are some arbitrary functions, and $\mathcal{B}$ is a control parameter. Then we can confirm that, if $\mathcal{B} \to k\mathcal{B}$, then $p \to kp$ is consitent with $\rho \to k\rho$.

And it is easy to check that the TOV condition is invariant under the symmetric transformation:

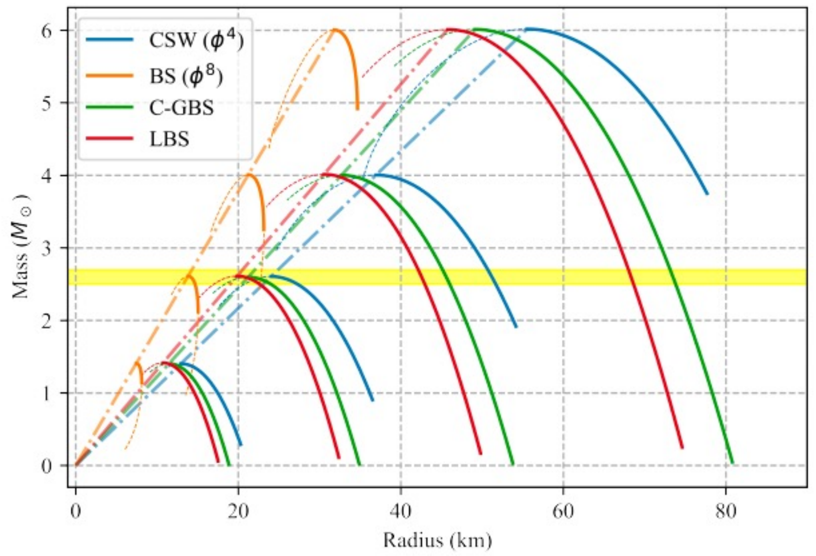
$$\rho \to k\rho\,, \qquad (54)$$

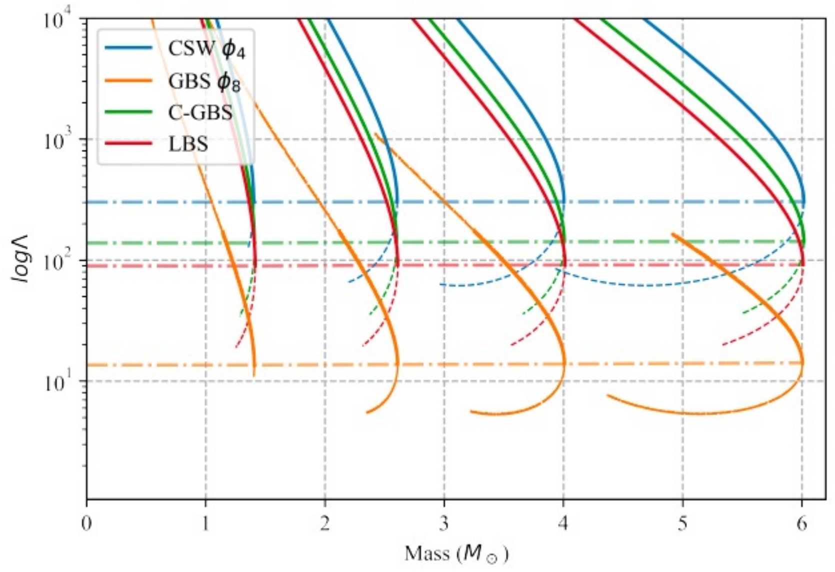
$$p \to kp\,, \qquad (55)$$

$$m \to \frac{1}{\sqrt{k}} m\,, \qquad (56)$$

$$r \to \frac{1}{\sqrt{k}} r\,. \qquad (57)$$

That is to say, if we set $\mathcal{B} \to k\mathcal{B}$, then the variables change according to the above, while the "compactness" $C = M/R$ remains the same.





暗星

Dark I-Love-Q

$$\ln y_i = a_i + b_i \ln x_i + c_i (\ln x_i)^2 + d_i (\ln x_i)^3 + e_i (\ln x_i)^4.$$

y_i	x_i	a_i	b_i	c_i	d_i	e_i
$\bar{I}$	$\bar{\lambda}^{(\mathrm{tid})}$	1.47	0.0817	0.0149	2.87×10^{-4}	-3.64×10^{-5}
$\bar{I}$	$\bar{Q}$	1.35	0.697	-0.143	9.94×10^{-2}	-1.24×10^{-2}
$\bar{Q}$	$\bar{\lambda}^{(\mathrm{tid})}$	0.194	0.0936	0.0474	-4.21×10^{-3}	1.23×10^{-4}

TABLE I. Estimated numerical coefficients for the fitting formulas of the NS and QS I-Love, I-Q and Love-Q relations.

暗星

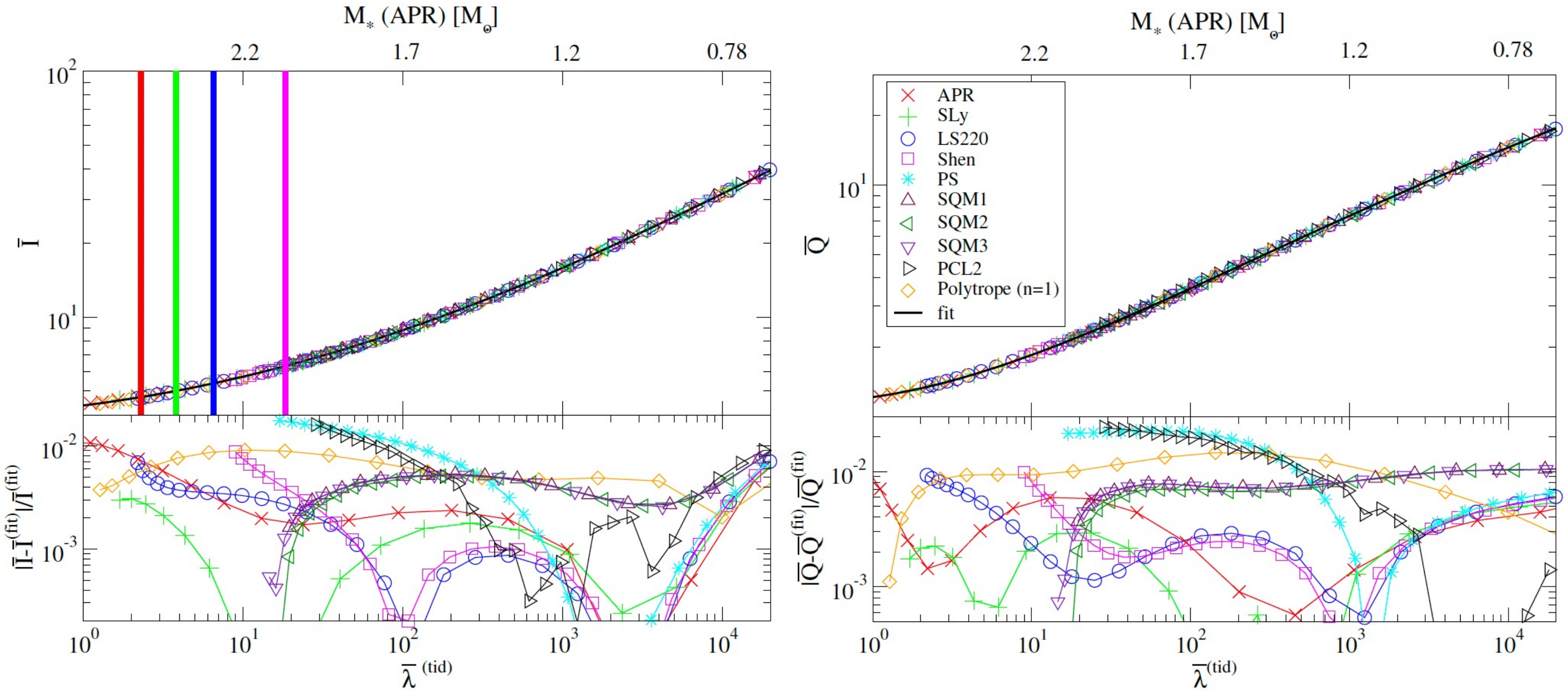
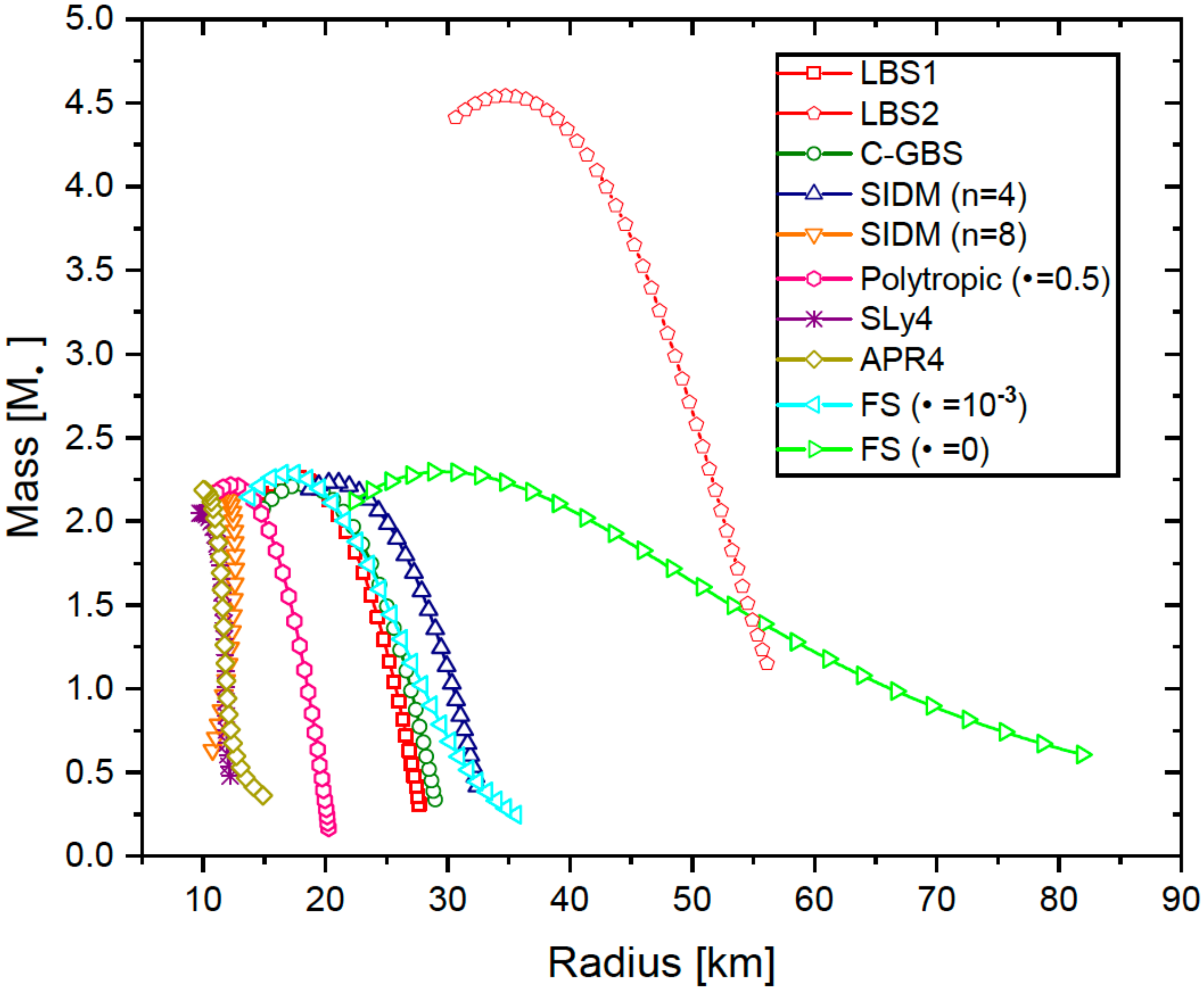


FIG. 1. (Top Left and Right) The neutron star (NS) and quark star (QS) universal I-Love and Love-Q relations for various EoSs, together with fitting curves (solid). On the top axis, we show the corresponding NS mass with an APR EoS. The thick vertical lines show the stability boundary for the APS, SLy, LS220 and Shen EoSs from left to right. The parameter varied along each curve is the NS or QS central density, or equivalently the star's compactness, with the latter increasing to the left of the plots. (Bottom Left and Right) Fractional errors between the fitting curve and numerical results.

暗星



Mass [M.]
Radius [km]
LBS1
LBS2
C-GBS
SIDM (n=4)
SIDM (n=8)
Polytropic (•=0.5)
SLy4
APR4
FS (• =10^-3)
FS (• =0)

暗星

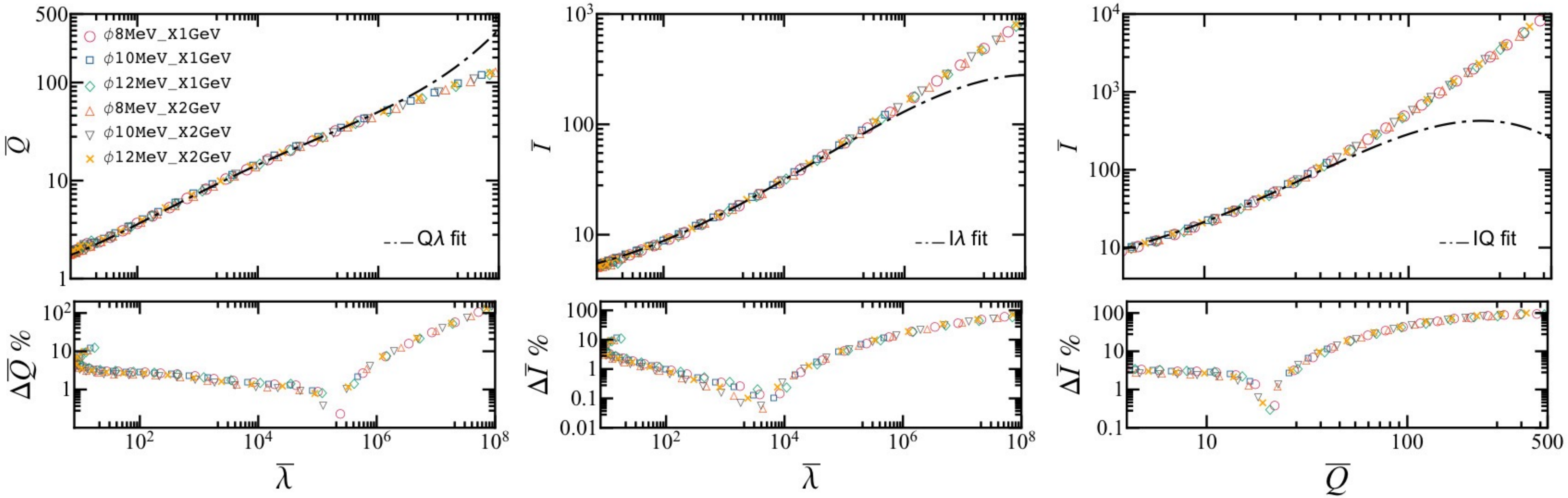
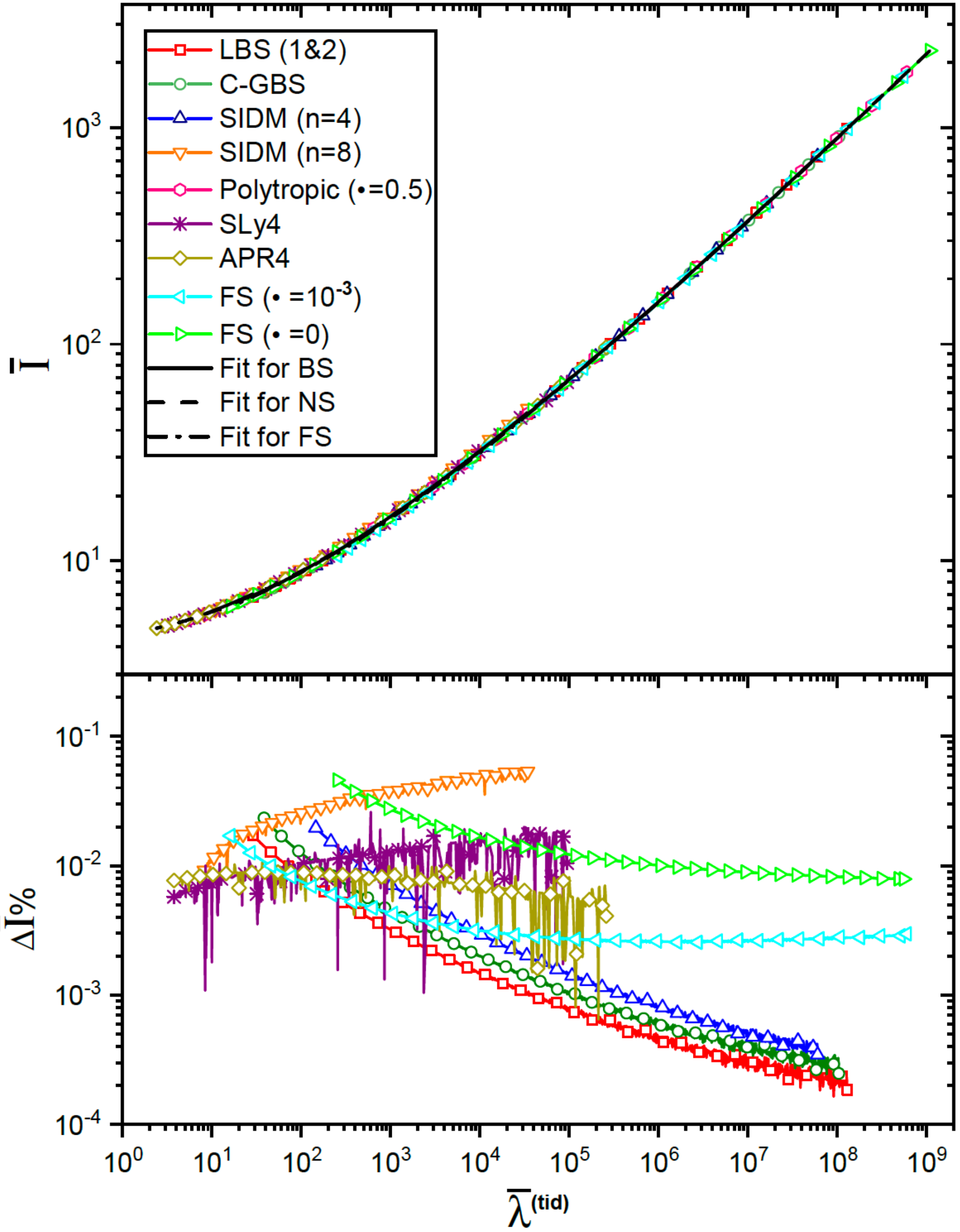


FIG. 6. I-Love-Q relations for fermion stars with a fixed coupling constant $\alpha = 10^{-3}$ and different values for m_ϕ and m_X. The bottom panels show the relative percentage errors between the numerical data and the universal relation (11) (dashed black curve).

演示页


LBS (1&2)
C-GBS
SIDM (n=4)
SIDM (n=8)
Polytropic (•=0.5)
SLy4
APR4
FS (• =10^-3)
FS (• =0)
Fit for BS
Fit for NS
Fit for FS
Ī
ΔĪ%
λ̄^(tid)

Mixed Stars

混合星

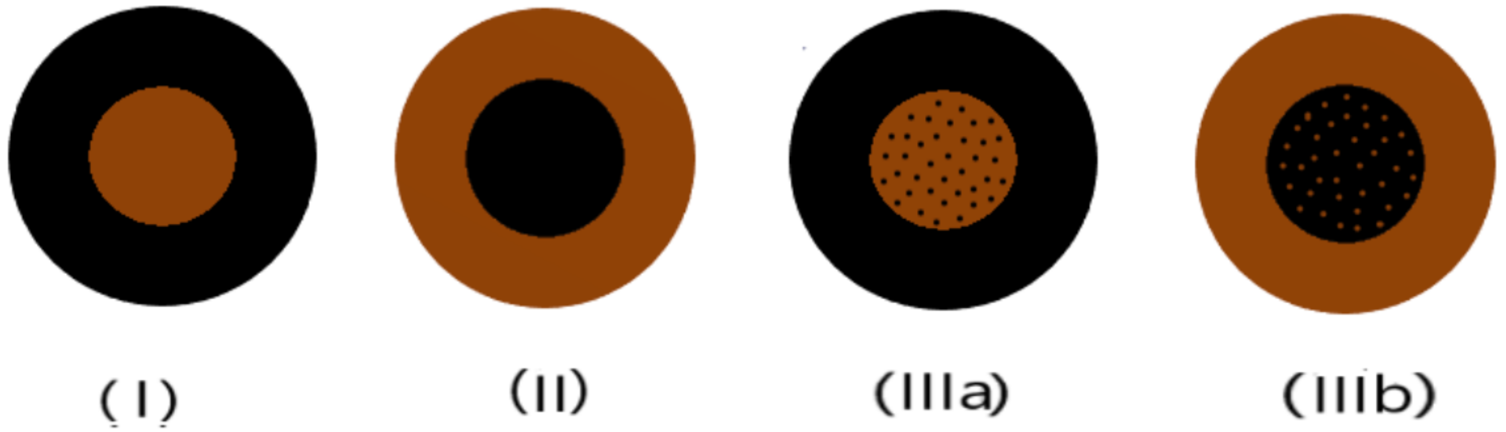
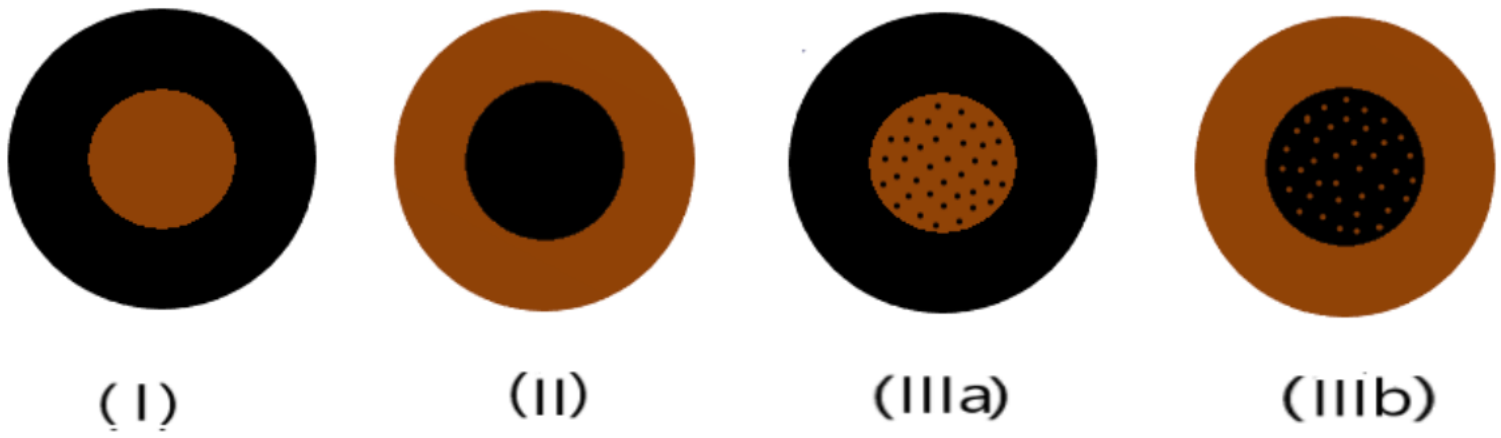


FIG. 1. Three scenarios of hybrid stars. Black color denotes dark matter and brown color denotes nuclear matter. In the first scenario we have a pure nuclear matter core and a pure dark matter crust, and swap the dark and nuclear matter in the second scenario. In the third scenarios, we have a mixed core and either a pure dark matter crust (IIIa) or a nuclear matter one (IIIb). They can form the systems of binary hybrid stars (BHS).

混合星



进一步提出了中子与暗物质构成混合星体的三种模型，能够解释中子星合并事件 GW170817以及GW190425。这三种模型考虑了暗物质与中子有相互作用的情况和无相互作用只靠引力耦合的情况。其中无相互作用时的混合星体潮汐变形的推导计算是世界首次。

混合星

Suppose the pressure reads p_W on the domain wall located at $r = r_W$. The sound speed near a density discontinuity is

$$\frac{d\rho}{dp} = \frac{1}{c_s^2} = \left.\frac{d\rho}{dp}\right|_{p \neq p_W} + \Delta\rho_p\,\delta(p - p_W), \tag{9}$$

where $\Delta\rho_p = \rho(p_W + 0) - \rho(p_W - 0)$ is the energy density jump across p_W. Yet since p decreases as r increases, equivalently $\Delta\rho_p = -\left(\rho(r_W + 0) - \rho(r_W - 0)\right) \equiv -\Delta\rho$.

to the δ-function can contribute. Therefore, this then results in

$$\left. r y'(r)\right|_{r=r_W} + r^2 4\pi e^{\lambda(r)}\left(\rho(r) + p(r)\right)\left.\frac{d\rho}{dp}\right|_{r=r_W} = 0. \tag{10}$$

Since $\frac{d\rho}{dp} = \frac{d\rho}{dr}\frac{1}{dp/dr}$, where $\frac{dp}{dr}$ can be read off from the first TOV equation (4), and $\frac{d\rho}{dr}|_{r=r_W} = \Delta\rho\,\delta(r - r_W)$, we obtain that[6]

$$\Delta y = \frac{\Delta\rho}{p + m(r_W)/(4\pi r_W^3)}, \tag{11}$$

[6] There are typos in the counterpart of (11) in [57].

混合星

Suppose the pressure reads p_W on the domain wall located at $r = r_W$. The sound speed near a density discontinuity is

$$\frac{d\rho}{dp} = \frac{1}{c_s^2} = \frac{d\rho}{dp}\bigg|_{p\neq p_W} + \Delta\rho_p\,\delta(p - p_W), \qquad (9)$$

where $\Delta\rho_p = \rho(p_W + 0) - \rho(p_W - 0)$ is the energy density jump across p_W. Yet since p decreases as r increases, equivalently $\Delta\rho_p = -(\rho(r_W + 0) - \rho(r_W - 0)) \equiv -\Delta\rho$.

to the δ-function can contribute. Therefore, this then results in

$$r y'(r)\bigg|_{r=r_W} + r^2 4\pi e^{\lambda(r)}\,(\rho(r) + p(r))\,\frac{d\rho}{dp}\bigg|_{r=r_W} = 0. \qquad (10)$$

Since $\frac{d\rho}{dp} = \frac{d\rho}{dr}\frac{1}{dp/dr}$, where $\frac{dp}{dr}$ can be read off from the first TOV equation (4), and $\frac{d\rho}{dr}|_{r=r_W} = \Delta\rho\,\delta(r - r_W)$, we obtain that[6]

$$\Delta y = \frac{\Delta\rho}{p + m(r_W)/(4\pi r_W^3)}, \qquad (11)$$

GW170817 and GW190425 as hybrid stars of dark and nuclear matter

Kilar Zhang (Taiwan, Natl. Normal U.), Guo-Zhang Huang (Taiwan, Natl. Normal U.), Jie-Shiun Tsao (Taiwan, Na
Normal U.), Feng-Li Lin (Taiwan, Natl. Normal U.) (Feb 25, 2020)

Published in: *Eur.Phys.J.C* 82 (2022) 4, 366 • e-Print: 2002.10961 [astro-ph.HE]

混合星

Comment on "Tidal Love numbers of neutron and self-bound quark stars" #1

János Takátsy (Wigner RCP, Budapest and Eotvos Lorand U., Budapest, Inst. Theor. Phys.), Péter Kovács (Wigner RCP, Budapest and Eotvos Lorand U., Budapest, Inst. Theor. Phys.) (Jun 30, 2020)

Published in: *Phys.Rev.D* 102 (2020) 2, 028501 • e-Print: 2007.01139 [astro-ph.HE]

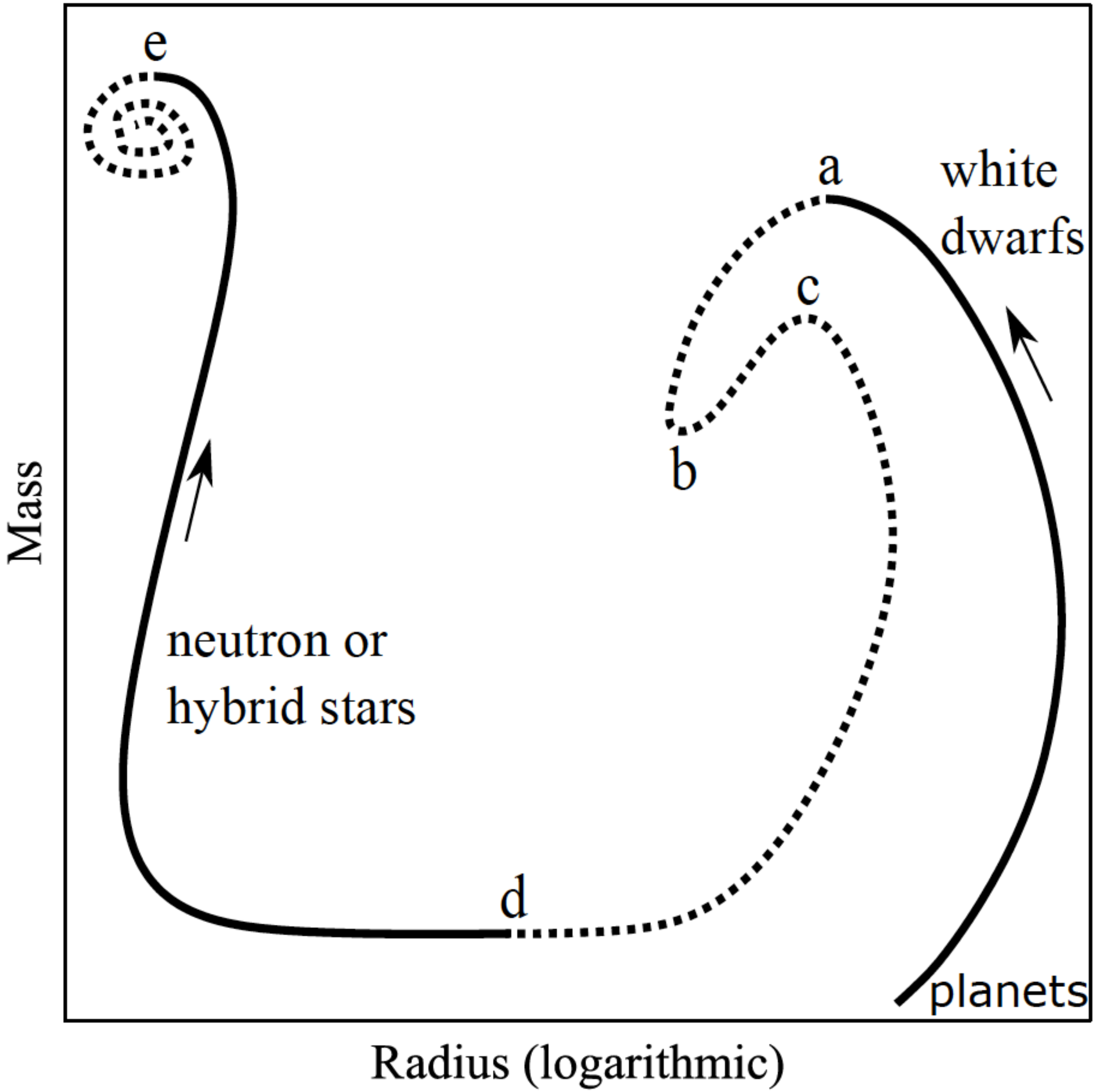
dividual tidal deformabilities. The corrected fits – as it was claimed by the authors of Ref. [1] – are negligibly different from the reported fits in Ref. [2]. We also add that the correct formula appears in Ref. [6] as well.

[5] K. Chatziioannou, S. Han, Studying strong phase transitions in neutron stars with gravitational waves, Phys. Rev. D **101**, 044019 (2020).

[6] K. Zhang, G. Z. Huang, F. L. Lin, arXiv:2002.10961.

[7] T. Damour and A. Nagar, Relativistic tidal properties of neutron stars, Phys. Rev. D **80**, 084035 (2009).

混合星



稳定性 BTM准则
压强增大 逆时针

混合星

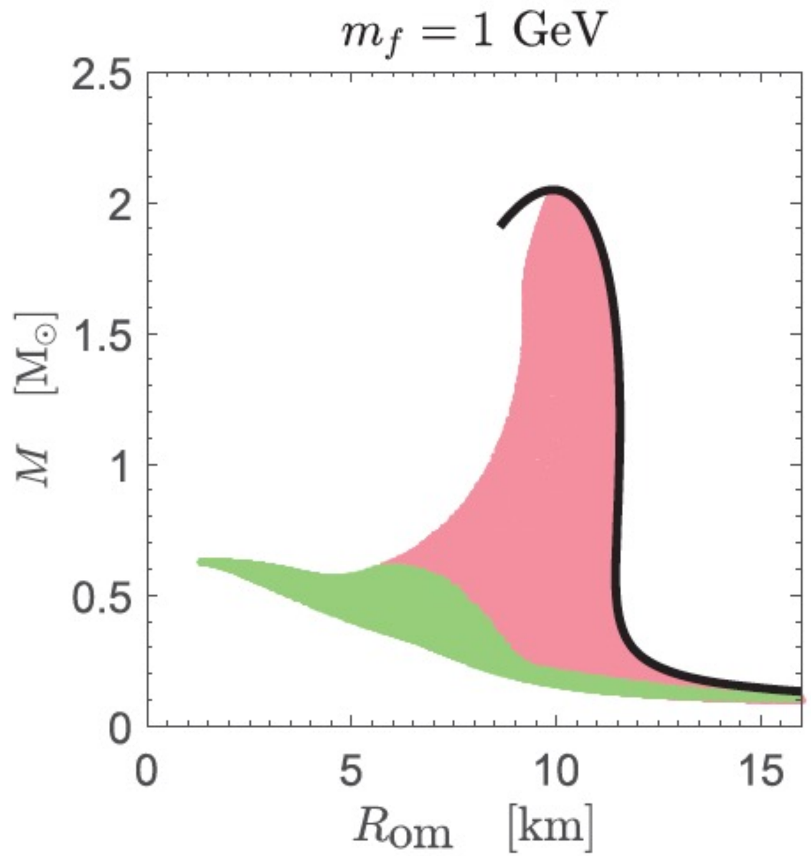


FIG. 4. The mass as a function of the visible radius, which is the radius of ordinary matter, for the stable static solutions shown in Fig. 1 (dark matter is a free Fermi gas with fermion mass $m_f = 1$ GeV). The thick black curve is for the single-fluid star with only ordinary matter, i.e., for a neutron star without dark matter. The color scheme is the same as in Fig. 1, with green indicating a dark matter halo and red a dark matter core.

稳定性 BTM准则

混合星?

演示页

Conclusions

总结

Done:

中子星物态方程 点瞬子模型

解释间隙事件

分层结构 跳变

Scaling symmetry TLN

系统求解**DM EoS** 各向同性极限 **shooting method**

稳定性 **BTM**准则 压强增大 逆时针 混合星

暗物质星**I-L-Q**关系 偏移

Undergoing:

中子星瞬子气体模型

白矮星

SW/超辐射

SW AG connection formula

演示页